

DUE DATE SELECTION PROCEDURES FOR JOB-SHOP SIMULATION

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Abstract—This paper introduces a comprehensive classification of due date selection procedures from which three major categories are derived. Emphasis is centered on providing a summary of selection procedures in a form that can be readily used by both researchers and practitioners. The paper concludes with a review and critique of recent discrete digital computer simulation studies concerning the impact of due date assignment procedures on shop performance and priority scheduling rules.

INTRODUCTION

Many computer simulation studies of job shops in the past two decades have been largely concerned with the experimental evaluation of priority dispatching rules. Most of these studies can be characterized by random assignment of due dates to arriving jobs or by some arbitrary selection of a due date assignment policy and its parameters.

Recent studies [4, 13, 32, 42] indicate that the scheduling rules, as well as the due date selection policies, simultaneously control the performance of the shop. Thus, in order to foster valid conclusions from studies of simulated job-shops, the researchers are expected today to test their conjectures over a wide spectrum of commonly used due date assignment policies. This article is directed at providing a survey of due date assignment procedures in a form that can be readily used by both simulation researchers and practitioners.

The paper is centered on the current approaches that have been implemented and are being used in simulation experiments or in industry. Descriptions of these procedures are presented along with a classification scheme, a summary of several representative studies, and a discussion of major research issues concerning the selection of due dates in job-shop simulation and the impact of due date procedures on priority rules and shop performance.

THE DUE DATE ASSIGNMENT PROBLEM

In the industrial environment due dates are essential for coordinating the activities of both vendors and customers. The customer needs a due date so that plans can be made for utilizing the product at a certain time in the future. Due date information is a significant input for the manufacturing management system of the vendors since it serves as a basis for the production planning and control system as well as for issuing orders to subcontractors, suppliers of raw material and other exogenous agents.

That due date assignment problems are of considerable importance to industrial engineering is incontrovertible. Due dates usually can be found whenever a product or a service is sold without an immediate delivery to the customer. Since it is a fundamental means of coordinating the activities of different organizations, the exactness in which due dates have to be met is significant for all the parties involved.

In simulation studies of hypothetical job-shops the need for a due date assignment procedure is twofold. First, due dates represent delivery commitments by the shop; actual performance can be evaluated in light of the given due dates. Most simulation studies have considered due date related performance measures such as average tardiness or the distribution

of jobs lateness. The second need for due date assignment procedures in simulation studies stems from the fact that many scheduling rules are related to the jobs due dates. For example, the EDD (early due date) or the S/OPN (Slack time per remaining operation) scheduling rules exploit due date information in order to reduce the chance of late deliveries. In the next section we present a classification of due date selection procedures. Then in the following section we survey simulation studies which have been used in generation and evaluation of these procedures.

CLASSIFICATION OF DUE DATE SELECTION PROCEDURES

Three basic categories of due date selection procedures may be identified. The three major categories are divided into several subclasses, and procedures are described for each subclass. The names used for each procedure are those names commonly found in the literature. In some cases the names of authors are used.

I. Direct Procedures, that is, procedures that can be related to currently available information about a given job or about the state of the shop. Examples of Direct Due Date assignment procedures are detailed at the appendix.

I.1. Simple Assignment Procedures—procedures which are independent of the dynamic state of the shop or of the characteristics of the incoming job.

(1) CON: Common allowance—the allowance for flow time is a constant[11].

(2) RDM: Random allowance—the allowance for flow time is assigned at random[11].

I.2. Job Dependent Assignment Procedures—procedures that are based on information related to a specific job such as its total work content or the number of operations.

(1) NOP: Number of operations—the allowance for flow time is proportional to the number of operations of the job[11].

(2) TWK: Total work—the allowance for flow time is proportional to the total processing time of the job[11].

(3) VTWK: Variant of TWK—the allowance for flow time is proportional to the total processing time of a job raised to a given power[13].

(4) TWNO: Total work and number of operations—the allowance for flow time is proportional to both the number of operations and the total processing time of the job[2].

I.3. State Dependent Assignment Procedures—procedures that take into consideration the current state of the shop. Relevant factors may be the total number of jobs in the shop or the current shop utilization.

(1) CNU: Current number of jobs—the allowance for flow time is proportional to the current number of jobs in the shop and to the total processing time of the job[39].

(2) CSU: Current shop utilization—the allowance for flow time is proportional to the current shop utilization level and to the total processing time of the job[39].

(3) TOP: Total number of operations—the allowance for flow time is proportional to the total number of operations of jobs currently in the shop and to the total processing time of the job[39].

(4) TWQE: Total work and jobs in queue—the allowance for flow time is proportional to the current number of jobs in queues waiting to be processed on the same machines which lie on-route of the job[13].

(5) SPRT: Sum of the processing times—the allowance for flow time is proportional to the total processing times of all the jobs currently in the shop[39].

I.4. Combination Assignment Procedures—procedures which combine some ideas from I.1, I.2, and I.3 in order to overcome some of their deficiencies.

(1) SLK: Slack time—the allowance for flow time equals the total processing time of the job plus a constant[4].

(2) TWRD: Total work and random allowance—the allowance for flow time is proportional to the total processing time of the job plus a random value drawn from a uniform distribution[35].

II. Heuristic Procedures, that is, procedures which involve more complex methodologies for the selection of due dates. These are usually based on initial simulation runs of the shop or on detailed information provided by a production data base.

II.1. Simulation-Based Assignment Procedures—procedures that utilize conditional estimates of individual job flow times, or delay, derived from prior simulation studies.

(1) **TWMQ**: Total work and mean queue—the allowance for flow time is proportional to the total processing time of the job and to the mean waiting time per job as determined by simulation experiments[13].

(2) **TWODT**: Total work and operations delay time—the allowance for flow time equals the sum of the total processing time and the product of the estimated mean delay time per operation and the number of operations of the job[40].

(3) **Reiter (1966)**—an industrial production planning and control system including a subsystem for setting due dates. By adding the total processing time, set-up times and an average delay as a function of existing shop load, the subsystem estimates the total flow time for each job. The average delays are generated by a simulation model which estimates weekly average delays for each work center[28].

(4) **Tilak (1974)**—an industrial procedure for a plant producing bottled and canned soft drinks. Changeover costs are minimized when jobs with the same set-up of the production line are assigned a common due date[38].

(5) **Weeks & Fryer (1976)**—a methodology for estimating the proportionality coefficient of the TWK procedure. The objective is to minimize a total cost function including job earliness, job tardiness, flow time, due date and labor transfer component costs[42].

II.2. Materials Requirement Planning Related Assignment Procedures—procedures usually applied by companies operating computer based MRP systems for production and inventory management. The MRP system uses simultaneous trial and error fitting of jobs along with production schedule changes in order to specify feasible delivery dates.

(1) **Orlicky (1975)**—a general approach to setting due dates where incoming orders are matched with the master production schedule. The MRP system tries to fit the job into the schedule and later indicates which job may be accepted with the customer's delivery date request and which jobs should be renegotiated with the customers for delayed shipment[24].

(2) **Hutchings (1976)**—a computerized shop scheduling and control system which has been applied in several foundries. Upon the receipt of an order, the computer system evaluates the potential delays due to actual and potential bottle-neck work centers relevant to this order. Results indicate whether the plant had the capacity to start molding in time to meet the customer's requested delivery date and, if not, the earliest feasible due date for that order[19].

III. Analytic Procedures, that is procedures that have been developed from normative, analytic investigation.

III.1. Deterministic Shop Assignment Procedures—procedures based on results obtained in scheduling studies of static and dynamic deterministic systems.

(1) **Seidmann, Smith and Panwalkar (1981)**—a scheduling procedure for selecting optimal distinct due dates and optimal sequence for a single machine problem. The performance measure is the sum of the penalties for earliness and tardiness and the penalties associated with the assignment of due dates[32].

(2) **Panwalkar, Smith and Seidmann (1982)**—a polynomial bound algorithm for the n job, one machine scheduling problem in which all jobs have a common due date. It finds the optimal value of the common due date and an optimal sequence which minimizes the total penalty based on the due date and the earliness or the tardiness of each job[26].

III.2. Stochastic-Shop Assignment Procedures—procedures developed for dynamic systems where the interarrival times and the production times are randomly distributed.

(1) **Reinitz (1963)**—a methodology based on dynamic programming and Markovian Decision Process. Having obtained the distribution of time in the shop the due date assignment procedure maintains the percentage of tardy jobs at a desired level[27].

(2) **Oral & Malovin (1973)**—a simple procedure based on analytic results from queuing theory. The allowance for flow time under the SPT priority rule equals the sum of the total processing time of the job and its expected waiting time. Conway's formula[11, p. 166] is used to compute the expected waiting time for every operation of that job[23].

(3) **Heard (1976)**—a due date assignment procedure which minimizes an expected penalty function of job lateness and flow time. Research methodology used by Heard combined mathematical analysis, sequential control and stochastic dynamic programming[18].

(4) Bertrand (1981)—the mean quoted lead time of a new job is based on preliminary evaluation of anticipated waiting times and on capacity congestion en route of that job. Whenever congestion is forecasted—the appropriate due date is extended until a period is found with adequate free capacity[6].

(5) Seidmann & Smith (1981)—a study of the constant due date assignment policy in a stochastic dynamic environment. The objective is to minimize the expected aggregate cost per job subject to several restrictive assumptions. The structure of the optimal solution obtained was found to be independent of the specific probability distribution function of both the interarrival job times and the total shop time[31].

DISCUSSION OF THE LITERATURE ON DUE DATE ASSIGNMENT

Research of job-shop systems has been largely concerned with the basic decision problem of determining the order in which jobs are selected from a queue for processing at a work center. No attempt will be made here to provide a comprehensive review of the existing literature on job-shop scheduling. Reviews of this research can be seen in Conway *et al.*[11], Elmaghraby[14], Spinner[36], Day & Hottenstein[12], Baker[3], Panwalkar & Iskander[25], Coffman[8], Rinnooy Kan[29], Lenstra[21], Bertrand & Wotermann[7] and French[17].

In this section, we shall focus on some of the major thrusts that appear in simulation models for due date assignment. As noticed by several researchers, the main contribution of many simulation studies is not the numerical results but the insight, or the identification and verification of specific effects of given decision policies. The general job shop simulation model shared by most studies cited here is illustrated by Fig. 1. Usually, jobs arrived in the shop at random time intervals. The characteristic of each job were generated on arrival. They included the number of operations, the machining routing, the estimated processing times and the quoted due date. In some studies job file parameters were generated beforehand by different programs and were recorded on a magnetic tape. This tape was used as an input to the main program where the simulation process took place. Figure 2 depicts this procedure.

Simulation studies related to due date assignment procedures[5, 10, 11, 16, 39, 41, 42] indicate that shop performance, and the relative effectiveness of the priority rules, are affected by the method used for due date assignment as well as by the tightness of the due dates. Conway[10] performed a simulated study which was principally concerned with comparing and evaluating a large number of priority disciplines and the effects on job lateness of several due date assignment procedures. The study pointed out that due dates related to job attributes of total processing time or number of operations performed better in terms of mean tardiness than constant or random due dates.

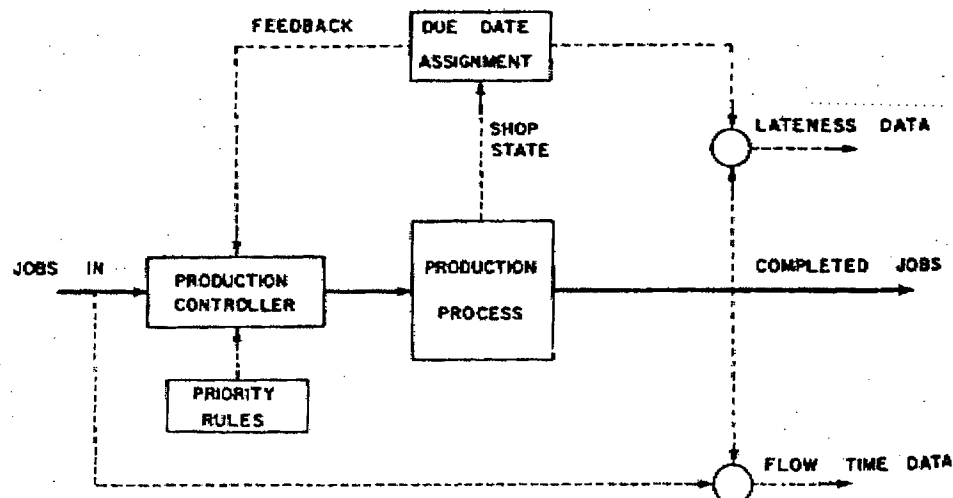


Fig. 1.

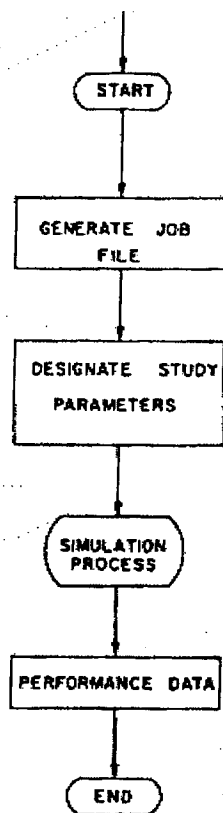


Fig. 2.

Another simulation of due date selection rules was presented by Ulgen[39]. He simulated a dynamic shop with nine machine groups where the routing of each job was fixed and known when the jobs arrived at the shop. Due dates were generated by the TWK procedure and by four variations of that procedure: CSU, CNU, TOP and SPRT. It was observed that the CNU procedure was significantly better than the other procedures in minimizing the average lateness with SPT (shortest imminent processing time), FCFS (first come first served) and EDD (earliest due date) priority rules at low utilization levels. As the utilization level increases, the average lateness under CNU approaches to that under the other due date procedures. Moreover, at high utilization levels the percentage of tardy jobs with EDD sequencing is significantly better with CNU, TOP or SPRT due dates with respect to the performance of EDD rule under TWK due date procedure. This behaviour suggests that due date assignment procedures based upon current shop load could be more effective in improving the shop performance.

A different approach to incorporating the current shop status in a procedure for lead time estimation is to utilize information about the instantaneous number of jobs currently in specified queues. One way of doing so was presented by Eilon & Chowdbury[13] who examined four procedures for dynamic assignment of due dates.

The effectiveness of three priority rules in meeting due dates was determined by a simulation model of a shop having five non-identical machines. When notional cost functions were used, a job priority rule was found to be the most effective. Parameters in the due date assignment procedures strongly affected the mean lateness and the percentage of tardy jobs. The TWQE due date procedure, which utilizes current shop conditions, was found to give the best results under varying shop loading.

In several cases where due dates can be selected, an implicit approach is to try to select lead times to be as tight as possible. Short lead times are desirable since they provide better

customer service than loose due dates and tend to incur lower in process inventory levels. A simulation model with eight machines was used by Elvers[16] in order to investigate the interaction between the tightness of the due dates and the relative performance of ten scheduling rules. Each job had one to four operations with randomly generated estimated times. Six distinct sets of due dates were assigned by the TWK procedure. These sets were computed by taking the total estimated job time and multiplying it by 1, 2, 3, 4, 5, 6 and 7 respectively. For the purpose of the experiments in this study, Elvers assumed that the goal was to minimize the number, or percentage of tardy jobs. The local scheduling rules which emphasize minimizing processing times, such as LWKR (least work remaining) or SPT appeared to be the most effective when tight due dates were established. Contrarily with loose due dates the rules which aim at minimizing slack, such as S/OPN (current slack per number of remaining operation) or MST (minimum slack time), gave the best performance. It seems, therefore, that some priority rules operate better when loose due dates are assigned while other priority rules are better when tight due dates are set.

Obviously in a competitive market, quoted short lead times are likely to attract more customers than long lead times. However, tight due dates are not desired from production planning considerations and in practice most due dates probably lie somewhere in between the marketing and the scheduling requirements. A description of this problem was first given by Jones[20]. This work was later extended in a paper by Weeks & Fryer[42]. Their paper presented a methodology for assigning minimum cost due dates. Component costs included mean job flow time, mean job lateness, mean job earliness, mean job due date and mean labor transfer costs. Due dates for arriving jobs were assigned by TWK where the estimated job times were multiplied by 1.5, 3.0 and 7.0 respectively. Regression analyses of simulation experiments with hypothetical labor and machine constrained job shop employing various sequencing and labor assignment rules were used to estimate the required tightness of the due dates.

Another approach to incorporating mean lead time considerations into the selection model is the hybrid use of analytic results from scheduling theory along with computer simulation, as discussed in Baker & Bertrand[4, 5]. Their first paper[4] compared the CON, SLK and TWK due date assignment procedures for a static and a dynamic single machine model. Jobs were sequenced according to EDD (early due date) priority rule with the objective of making the due dates as tight as possible subject to the constraint that no job will be tardy. The CON and TWK was found to be dominated most of the time and TWK usually produced tighter due dates than SLK. In a complementary study Baker & Bertrand[5] treated the aggregate tightness of jobs due dates as a policy decision. Their objective was to sequence the jobs in order to minimize the average tardiness. Analytical and empirical results indicated that it might be advantageous to use an adaptive production control scheme; SPT and EDD sequencing procedures should be invoked with tight and loose due dates respectively.

While the primary concern of the previous research cited above have been directed to the effects of due date tightness, the following studies investigated the feasibility of due date assignments based on conditional estimates of job flow times. Weeks[41] used analysis of variance of simulation results to suggest that managers should establish attainable or predictable due dates based on expected job flow times and current shop congestion information. Performance measures used were mean earliness, mean lateness and mean tardiness. Additionally, Weeks[41], noted that better due date performance could be obtained with due date oriented priority rules such as S/OPN (current slack per number of remaining operations) and when the shop model is not elaborate and complex.

Finally, simulation studies by Bertrand[6] have indicated that by using time phased shop utilization information further improvements with attainable due dates could be realized. The time phased work load information considered the instantaneous shop congestion and the aggregate future periodic work load per machine due to the remaining operations of the jobs in the shop. Simulation experiments with five non identical machines found that a minimum waiting allowance per operation was vital for reducing the variance of the due date deviations. Due date and sequencing procedures based on time phased information reduced the variance of due date deviations and kept the mean due date deviation at a constant level.

The preceding review of scheduling research has been limited to those computer simulation

studies which treated the due date as a decision variable, or at least recognized its importance. Several simulation results were in conflict indicating, for example, that different priority rules work best for some given performance measures. It may be conjectured that these contradictory conclusions resulted from using divergent experimental conditions in the research. Also, a recent study[1] pointed out that dynamic ratio type priority rules, such as S/OPN which has been extensively used in those studies, tend to exhibit abnormal reactions when a job becomes tardy. These studies, nonetheless identified several major control policies, interaction effects and variables which manifested the need for joint approaches to future research and implementations.

SUMMARY

The assignment of due dates to jobs in a simulation study of dynamic job shops is an important factor that may have been given insufficient attention. There is evidence that some job scheduling simulations reported in the past have had inconsistent results due to different methods used to assign the due dates. One can conclude that the results from these studies, i.e. the ranking of scheduling rules according to effectiveness, must be viewed in light of the respective due date assignment procedure used, as well as other basic assumptions that are made.

In simulation studies the assignment of due dates is done in an environment that differs greatly from the environment in a production control department. One difference is in the setting of due dates. In the operating situation, each job has many characteristics that may be combined to produce a due date; many (if not all) of the non-quantitative factors associated with real jobs are not present in simulation studies. To compensate for the lack of these factors, arbitrary assumptions are made out of necessity. One of these assumptions concerns the procedure for assigning due dates. This assumption may have little relationship to reality, but it can have significant effect upon the conclusions of the simulation study.

One who is performing a general study of scheduling that is not directly related to a specific scheduling environment desires results that are applicable. It might seem that there exists at least one due date assignment procedure that is completely unbiased by not favoring one scheduling rule over another. The random assignment of due dates seems to have this characteristic; however, the absence of bias itself constitutes a bias in the results unless, in the environment of application, the due dates appear to have been randomly assigned. It seems that the only way to obtain some measure of generality is to repeat the simulation with different methods of assigning due dates.

A number of procedures for assigning due dates have been described and referenced in this paper. Most recent studies indicate the preferability of using attainable due dates based upon current shop load information and the job characteristics. The evident interaction existing between the performance of the priority rules and the due date assignment procedures necessitates the simultaneous selection of both. No absolute assessment of the value of each procedure can be made; it is the authors' view that each procedure may have applicability under some set of realistic circumstances. Obviously the circumstances vary considerably from one application to another.

The increasing deployment of modern Computer Aided Manufacturing facilities and Flexible Manufacturing Systems that are operated with real time computer supervision will undoubtedly facilitate the development and implementation of advanced software control strategies including due date selection and production scheduling functions. It is suggested that the compilation and classification of due date assignment procedures presented in this paper be used as a guide so that procedures that differ greatly in their attributes can be evaluated and applied to real-world problems.

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APPENDIX

Examples of direct due date assignment procedures

Direct due date assignment procedures commonly used in computer simulation models are defined here. Only the direct procedures are given since these definitions can be given in brief form.

The following notation is used in the definitions that follow:

- $CJ(t)$ number of jobs in the shop at time t .
- $CS(t)$ shop utilization at time t .
- d_i the due date of job i .
- F_i weighting factors and constants ($i=1,2,\dots$); the specific values of the weighting factors depend on the particular application involved.
- i job index.
- j operation index.
- J_i a specific value of j ; the operation for which the job i is in queue at the present time.
- k machine index.
- M_i the total number of operations for job i ; $1 \leq j \leq M_i$.
- $N_{ij}(t)$ the number of jobs, at time t , in the queue corresponding to the j -th operation of job i .
- $NP(t)$ total number of operations of current jobs at time t .
- P_{ij} the processing time for the j -th operation of job i .

- $PS(t)$ The sum of P_{ij} taken over all the jobs in the shop at time t (sum of operations times taken over all the operations of current jobs).
- $R(U, V)$ a random variable, uniformly distributed between U and V ($V > U$)
- t present time.
- T_{ij} the time at which job i becomes ready for its j -th operation. T_{i1} is the arrival time of job i at the shop.

The assignment procedures are presented below.

Abbreviation	Definition
1. CNU	$d_i = T_{i1} + F_1 \cdot CJ(T_{i1}) \cdot \left[\sum_{j=1}^{M_i} P_{ij} \right]$
2. CON	$d_i = T_{i1} + F_1$
3. CSU	$d_i = T_{i1} + F_1 \cdot \left[\sum_{j=1}^{M_i} P_{ij} \right] \cdot CS(T_{i1})$
4. NOP	$d_i = T_{i1} + F_1 \cdot M_i$
5. RDM	$d_i = T_{i1} + F_1 \cdot R(F_2, F_3)$ (Note: $F_3 > F_2$)
6. SLK	$d_i = T_{i1} + \sum_{j=1}^{M_i} P_{ij} + F_1$
7. TOP	$d_i = T_{i1} + F_1 \cdot NP(T_{i1}) \cdot \left[\sum_{j=1}^{M_i} P_{ij} \right]$
8. SPRT	$d_i = T_{i1} + F_1 \cdot PS(T_{i1}) \cdot \left[\sum_{j=1}^{M_i} P_{ij} \right]$
9. TWK	$d_i = T_{i1} + F_1 \cdot \left[\sum_{j=1}^{M_i} P_{ij} \right]$
10. TWNO	$d_i = T_{i1} + F_1 \cdot \left[\sum_{j=1}^{M_i} P_{ij} \right] + F_2 \cdot M_i$
11. TWQE	$d_i = T_{i1} + F_1 \cdot \left[\sum_{j=1}^{M_i} P_{ij} \right] + F_2 \cdot \left[\sum_{j=1}^{M_i} N_{ij}(T_{ij}) \right]$
12. TWRD	$d_i = T_{i1} + F_1 \cdot \left[\sum_{j=1}^{M_i} P_{ij} \right] + F_2 \cdot R(0,1)$
13. VTWK	$d_i = T_{i1} + F_1 \cdot \left[\sum_{j=1}^{M_i} P_{ij} \right]^{F_2}$