

SUPERLU – FULL SET OF RESULTS

How is the error computed:

Relative forward error:

$$\text{RFE} = \frac{\|x - x_{\text{exact}}\|_{\infty}}{\|x\|_{\infty}}$$

Normwise relative backward error:

$$\text{NBRE} = \frac{\|Ax - b\|_{\infty}}{\| |A| |x| + |b| \|_{\infty}}$$

Name	Rows	Columns	NNZ un	NNZ L+U	NNZ inc	av. RFE	av. NBRE	Failure rate
TAMU 500	56010	56010	410882	1288292.56	3.14x	$5.63 \cdot 10^{-4}$	$9.76 \cdot 10^{-7}$	0 %
Case 118	214492	214492	2343288	8612421.75	3.68x	$2.59 \cdot 10^{-10}$	$1.17 \cdot 10^{-15}$	0 %
RTS – 1 TP	2237	2237	11297	55810.35	4.94x	$4.34 \cdot 10^{-6}$	$1.81 \cdot 10^{-16}$	5 %
RTS – 2 TP	4766	4766	23762	125639.97	5.29x	$9.00 \cdot 10^{-6}$	$2.02 \cdot 10^{-16}$	5 %
ACTIVSg200	4644	4644	21630	133847.2	6.19x	$3.69 \cdot 10^{-7}$	$1.78 \cdot 10^{-5}$	0 %
ACTIVSg2000	55667	55667	268299	1911570.3	7.12x	$5.38 \cdot 10^{-7}$	$2.73 \cdot 10^{-15}$	0 %
ACTIVSg10k	238072	238072	1111990.8	7252607.2	6.52x	$2.74 \cdot 10^{-8}$	$5.78 \cdot 10^{-6}$	0 %
ACTIVSg70k	1640411	1640411	7671693	52579080	6.85x	0.0072	1.045	0 %

Table 1: The error levels achieved by SuperLU_dist; *NNZ L+U* – combined nonzeros in the *L* and *U* factors (measure of the fill-in level); *NNZ inc.* – increase in non zeros inside *L* and *U*; *Failure rate* – ratio of the matrices for which SuperLU failed to the total number of matrices in a test case.

The computations performed by SuperLU can be divided into 8 phases:

1. Equilibration (scaling),
2. Row permutation,
3. Column permutation,
4. Symbolic factorization,
5. Data distribution (through MPI),
6. (Full) factorization,
7. Solve,
8. Iterative refinement.

TAMU 500 (CPU, correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.05	0.26	0.01	0.10	0.28	0.01	0.03	1.45
6	0.00	0.05	0.27	0.01	0.05	0.26	0.02	0.04	1.42
21	0.01	0.05	0.27	0.01	0.06	0.31	0.02	0.05	1.48
42	0.01	0.05	0.27	0.01	0.09	0.26	0.02	0.05	1.49

Case 118 (CPU, correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.02	0.17	1.20	0.09	0.55	1.67	0.06	0.24	7.82
6	0.01	0.17	1.24	0.10	0.30	3.67	0.12	0.35	9.76
21	0.01	0.17	1.26	0.10	0.29	2.65	0.10	0.28	8.61
42	0.01	0.18	1.26	0.10	0.42	1.91	0.08	0.23	7.93

RTS - One Time Period (CPU, correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.00	0.01	0.00	0.01	0.05	0.00	0.01	0.13
6	0.00	0.01	0.01	0.00	0.00	0.04	0.01	0.02	0.11
21	0.00	0.01	0.01	0.00	0.00	0.03	0.00	0.01	0.10
42	0.00	0.01	0.01	0.00	0.01	0.03	0.00	0.02	0.12

RTS - Two Time Period (CPU, correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.01	0.02	0.00	0.01	0.08	0.00	0.01	0.23
6	0.00	0.01	0.03	0.00	0.01	0.07	0.01	0.03	0.21
21	0.00	0.01	0.03	0.00	0.01	0.05	0.01	0.02	0.18
42	0.00	0.01	0.03	0.00	0.01	0.05	0.01	0.03	0.20

ACTIVSg200 (correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.00	0.02	0.00	0.01	0.03	0.00	0.00	0.13
6	0.00	0.00	0.02	0.00	0.00	0.06	0.01	0.01	0.18
21	0.00	0.00	0.02	0.00	0.01	0.04	0.01	0.01	0.15
42	0.00	0.00	0.02	0.00	0.01	0.05	0.01	0.01	0.17

ACTIVSg2000 (CPU, correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.04	0.41	0.02	0.12	0.42	0.01	0.04	1.85
6	0.00	0.04	0.42	0.02	0.06	0.52	0.03	0.06	1.82
21	0.00	0.04	0.42	0.02	0.06	0.54	0.04	0.08	1.86
42	0.02	0.04	0.42	0.02	0.10	0.40	0.03	0.06	1.77

ACTIVSg10k (CPU, correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.01	0.17	1.92	0.08	0.47	1.41	0.05	0.30	7.22
6	0.01	0.17	1.96	0.08	0.25	2.29	0.09	0.45	8.05
21	0.01	0.17	1.98	0.08	0.25	1.41	0.07	0.34	6.99
42	0.01	0.17	1.98	0.08	0.37	1.02	0.06	0.24	6.71

ACTIVSg70k (CPU, correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.09	1.04	15.69	0.55	3.51	10.89	0.37	1.12	51.66
6	0.05	1.04	16.34	0.56	1.68	17.22	0.60	1.05	57.21
21	0.05	1.06	16.75	0.56	1.67	12.09	0.40	0.52	51.60
42	0.08	1.07	16.76	0.57	2.67	7.71	0.31	0.67	49.13

Table 2: SuperLU results computed on the CPU (without GPU acceleration) with correct right-hand sides. Times are reported in seconds.

TAMU 500 (CPU, default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.05	0.27	0.01	0.12	0.29	0.01	0.03	1.43
6	0.00	0.05	0.27	0.01	0.06	0.24	0.02	0.04	1.37
21	0.00	0.05	0.27	0.01	0.06	0.32	0.02	0.05	1.45
42	0.00	0.05	0.27	0.01	0.09	0.26	0.02	0.05	1.44

Case 118 (CPU, default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.02	0.17	1.20	0.09	0.59	1.68	0.06	0.25	7.57
6	0.01	0.17	1.25	0.10	0.34	3.76	0.12	0.34	9.70
21	0.01	0.17	1.26	0.10	0.31	3.40	0.10	0.32	9.26
42	0.01	0.18	1.26	0.10	0.43	1.91	0.08	0.23	7.75

RTS - One Time Period (CPU, default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.00	0.01	0.00	0.01	0.05	0.00	0.01	0.13
6	0.00	0.01	0.01	0.00	0.00	0.04	0.01	0.02	0.11
21	0.00	0.01	0.01	0.00	0.00	0.03	0.01	0.02	0.11
42	0.00	0.01	0.01	0.00	0.01	0.03	0.01	0.02	0.11

RTS - Two Time Period (CPU, default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.01	0.02	0.00	0.01	0.08	0.00	0.01	0.22
6	0.00	0.01	0.03	0.00	0.01	0.07	0.01	0.03	0.21
21	0.00	0.01	0.03	0.00	0.01	0.05	0.01	0.02	0.18
42	0.00	0.01	0.03	0.00	0.01	0.05	0.01	0.03	0.19

ACTIVSg200 (default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.00	0.02	0.00	0.01	0.03	0.00	0.01	0.18
6	0.00	0.00	0.02	0.00	0.00	0.05	0.01	0.07	0.22
21	0.00	0.00	0.02	0.00	0.01	0.04	0.01	0.05	0.18
42	0.00	0.00	0.02	0.00	0.01	0.04	0.01	0.04	0.19

ACTIVSg2000 (CPU, default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.04	0.41	0.02	0.12	0.42	0.01	0.08	1.82
6	0.00	0.04	0.42	0.02	0.06	0.65	0.04	0.17	2.01
21	0.00	0.04	0.42	0.02	0.06	0.57	0.03	0.18	1.93
42	0.00	0.04	0.42	0.02	0.09	0.40	0.03	0.14	1.77

ACTIVSg10k (CPU, default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.01	0.17	1.91	0.08	0.47	1.40	0.05	0.43	7.06
6	0.01	0.17	1.96	0.08	0.25	2.16	0.09	0.60	7.86
21	0.01	0.17	1.98	0.08	0.25	1.45	0.07	0.41	6.92
42	0.01	0.17	1.98	0.08	0.37	1.02	0.05	0.33	6.48

ACTIVSg70k (CPU, default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.09	1.04	15.68	0.55	3.54	10.93	0.36	1.29	50.35
6	0.05	1.04	16.36	0.56	1.73	17.26	0.59	1.92	56.81
21	0.04	1.05	16.78	0.56	1.69	11.99	0.40	0.94	50.62
42	0.05	1.09	16.79	0.57	2.62	7.86	0.31	0.78	47.07

Table 3: SuperLU results computed on the CPU (without GPU acceleration) with default right-hand sides. Times are reported in seconds.

TAMU 500 (GPU, correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.05	0.26	0.01	0.11	0.78	0.01	0.03	2.14
2	0.00	0.05	0.27	0.01	0.10	1.47	0.03	0.06	2.73
6	0.00	0.05	0.27	0.01	0.06	2.13	0.02	0.04	3.30

Case 118 (GPU, correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.02	0.17	1.20	0.09	0.54	2.23	0.06	0.23	8.33
2	0.02	0.17	1.24	0.10	0.48	2.97	0.12	0.38	9.31
6	0.01	0.17	1.24	0.10	0.31	5.74	0.13	0.40	11.88

RTS - One Time Period (GPU, correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.00	0.01	0.00	0.01	0.53	0.00	0.00	0.59
2	0.00	0.00	0.01	0.00	0.00	0.77	0.01	0.02	0.84
6	0.00	0.01	0.01	0.00	0.00	1.93	0.01	0.02	2.00

RTS - Two Time Period (GPU, correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.01	0.02	0.00	0.01	0.58	0.00	0.01	0.71
2	0.00	0.01	0.03	0.00	0.01	0.78	0.01	0.02	0.91
6	0.00	0.01	0.03	0.00	0.01	1.95	0.01	0.03	2.09

ACTTIVS200 (GPU, correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.00	0.02	0.00	0.01	0.52	0.00	0.00	0.62
2	0.00	0.00	0.02	0.00	0.01	0.77	0.00	0.00	0.87
6	0.00	0.00	0.02	0.00	0.00	1.91	0.01	0.01	2.03

ACTTIVS2000 (GPU, correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.04	0.41	0.02	0.13	0.91	0.01	0.04	2.38
2	0.00	0.04	0.42	0.02	0.12	1.22	0.03	0.07	2.59
6	0.00	0.04	0.42	0.02	0.07	2.41	0.04	0.07	3.73

ACTTIVS10k (GPU, correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.01	0.16	1.91	0.08	0.48	1.84	0.05	0.33	7.51
2	0.01	0.16	1.96	0.08	0.43	2.55	0.11	0.61	8.66
6	0.01	0.16	1.96	0.08	0.27	3.96	0.10	0.53	9.78

ACTTIVS70k (GPU, correct RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.09	1.05	15.70	0.55	3.40	10.19	0.37	0.77	50.49
2	0.07	1.04	16.17	0.56	2.90	11.77	0.76	1.05	53.28
6	0.05	1.06	16.26	0.56	1.86	20.90	0.64	1.11	61.15

Table 4: SuperLU results computed on the GPU with correct right-hand sides. Each rank is using a separate GPU. Times are reported in seconds.

TAMU 500 (GPU, default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.05	0.26	0.01	0.11	0.79	0.01	0.03	1.97
2	0.00	0.05	0.27	0.01	0.10	1.41	0.03	0.07	2.62
6	0.00	0.05	0.27	0.01	0.06	2.15	0.02	0.05	3.28

Case 118 (GPU, default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.02	0.17	1.20	0.09	0.59	2.26	0.06	0.25	8.14
2	0.02	0.17	1.24	0.10	0.54	2.95	0.12	0.41	9.22
6	0.01	0.17	1.24	0.10	0.33	5.34	0.13	0.37	11.31

RTS - One Time Period (GPU, default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.01	0.01	0.00	0.01	0.54	0.00	0.00	0.62
2	0.00	0.01	0.01	0.00	0.00	0.78	0.01	0.02	0.85
6	0.00	0.00	0.01	0.00	0.00	1.93	0.01	0.02	2.00

RTS - Two Time Period (GPU, default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.01	0.02	0.00	0.01	0.58	0.00	0.01	0.72
2	0.00	0.01	0.03	0.00	0.01	0.79	0.01	0.02	0.92
6	0.00	0.01	0.03	0.00	0.01	1.96	0.01	0.03	2.09

ACTTIVS200 (GPU, default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.00	0.02	0.00	0.01	0.54	0.00	0.01	0.70
2	0.00	0.00	0.02	0.00	0.01	0.77	0.00	0.04	0.90
6	0.00	0.00	0.02	0.00	0.00	1.93	0.01	0.06	2.09

ACTTIVS2000 (GPU, default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.00	0.04	0.41	0.02	0.12	0.93	0.01	0.08	2.34
2	0.00	0.04	0.42	0.02	0.11	1.13	0.03	0.16	2.53
6	0.00	0.04	0.42	0.02	0.06	2.40	0.04	0.16	3.76

ACTTIVS10k (GPU, default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.01	0.17	1.91	0.08	0.49	1.85	0.05	0.43	7.62
2	0.01	0.17	1.96	0.08	0.45	2.43	0.11	0.77	8.58
6	0.01	0.17	1.96	0.08	0.27	4.14	0.10	0.67	9.93

ACTTIVS70k (GPU, default RHS)									
Ranks	Equil	Row perm	Col perm	Sym factor	Dist	Factor	Solve	Ref	TOTAL
1	0.09	1.04	15.71	0.55	3.71	10.29	0.37	1.13	49.67
2	0.07	1.04	16.19	0.56	3.28	11.86	0.77	1.71	53.05
6	0.05	1.06	16.28	0.56	1.92	18.91	0.64	1.39	58.10

Table 5: SuperLU results computed on the GPU with default right-hand sides. Each rank is using a separate GPU. Times are reported in seconds.