> **How is the error computed:**
>
> Relative forward error:
> $$\text{RFE} = \frac{\|x - x_{\text{exact}}\|_\infty}{\|x\|_\infty}$$
>
> Normwise relative backward error:
> $$\text{NBRE} = \frac{\|Ax - b\|_\infty}{\||A||x| + |b|\|_\infty}$$

| Name | Rows | Columns | NNZ un | NNZ L+U | NNZ inc | av. RFE | av. NBRE | Failure rate |
|---|---|---|---|---|---|---|---|---|
| TAMU 500 | 56010 | 56010 | 410882 | 1288292.56 | 3.14x | $5.63 \cdot 10^{-4}$ | $9.76 \cdot 10^{-7}$ | 0 % |
| Case 118 | 214492 | 214492 | 2343288 | 8612421.75 | 3.68x | $2.59 \cdot 10^{-10}$ | $1.17 \cdot 10^{-15}$ | 0 % |
| RTS – 1 TP | 2237 | 2237 | 11297 | 55810.35 | 4.94x | $4.34 \cdot 10^{-6}$ | $1.81 \cdot 10^{-16}$ | 5 % |
| RTS – 2 TP | 4766 | 4766 | 23762 | 125639.97 | 5.29x | $9.00 \cdot 10^{-6}$ | $2.02 \cdot 10^{-16}$ | 5% |
| ACTIVSg200 | 4644 | 4644 | 21630 | 133847.2 | 6.19x | $3.69 \cdot 10^{-7}$ | $1.78 \cdot 10^{-5}$ | 0 % |
| ACTIVSg2000 | 55667 | 55667 | 268299 | 1911570.3 | 7.12x | $5.38 \cdot 10^{-7}$ | $2.73 \cdot 10^{-15}$ | 0 % |
| ACTIVSg10k | 238072 | 238072 | 1111990.8 | 7252607.2 | 6.52x | $2,74 \cdot 10^{-8}$ | $5.78 \cdot 10^{-6}$ | 0 % |
| ACTIVSg70k | 1640411 | 1640411 | 7671693 | 52579080 | 6.85x | 0.0072 | 1.045 | 0 % |

Table 1: The error levels achieved by SuperLU_dist; *NNZ L+U* – combined nonzeros in the *L* and *U* factors (measure of the fill-in level); *NNZ inc.*- increase in non zeros inside *L* and *U*; *Failure rate* – ratio of the matrices for which SuperLU failed to the total number of matrices in a test case.

The computations performed by SuperLU can be divided into 8 phases:

1. Equilibration (scaling),

2. Row permutation,

3. Column permutation,

4. Symbolic factorization,

5. Data distribution (through MPI),

6. (Full) factorization,

7. Solve,

8. Iterative refinement.

| TAMU 500 (CPU, correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.00 | 0.05 | 0.26 | 0.01 | 0.10 | 0.28 | 0.01 | 0.03 | 1.45 |
| 6 | 0.00 | 0.05 | 0.27 | 0.01 | 0.05 | 0.26 | 0.02 | 0.04 | 1.42 |
| 21 | 0.01 | 0.05 | 0.27 | 0.01 | 0.06 | 0.31 | 0.02 | 0.05 | 1.48 |
| 42 | 0.01 | 0.05 | 0.27 | 0.01 | 0.09 | 0.26 | 0.02 | 0.05 | 1.49 |

| Case 118 ( CPU, correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.02 | 0.17 | 1.20 | 0.09 | 0.55 | 1.67 | 0.06 | 0.24 | 7.82 |
| 6 | 0.01 | 0.17 | 1.24 | 0.10 | 0.30 | 3.67 | 0.12 | 0.35 | 9.76 |
| 21 | 0.01 | 0.17 | 1.26 | 0.10 | 0.29 | 2.65 | 0.10 | 0.28 | 8.61 |
| 42 | 0.01 | 0.18 | 1.26 | 0.10 | 0.42 | 1.91 | 0.08 | 0.23 | 7.93 |

| RTS - One Time Period (CPU, correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 | 0.05 | 0.00 | 0.01 | 0.13 |
| 6 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.04 | 0.01 | 0.02 | 0.11 |
| 21 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.03 | 0.00 | 0.01 | 0.10 |
| 42 | 0.00 | 0.01 | 0.01 | 0.00 | 0.01 | 0.03 | 0.00 | 0.02 | 0.12 |

| RTS - Two Time Period (CPU, correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.00 | 0.01 | 0.02 | 0.00 | 0.01 | 0.08 | 0.00 | 0.01 | 0.23 |
| 6 | 0.00 | 0.01 | 0.03 | 0.00 | 0.01 | 0.07 | 0.01 | 0.03 | 0.21 |
| 21 | 0.00 | 0.01 | 0.03 | 0.00 | 0.01 | 0.05 | 0.01 | 0.02 | 0.18 |
| 42 | 0.00 | 0.01 | 0.03 | 0.00 | 0.01 | 0.05 | 0.01 | 0.03 | 0.20 |

| ACTIVSg200 (correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.00 | 0.00 | 0.02 | 0.00 | 0.01 | 0.03 | 0.00 | 0.00 | 0.13 |
| 6 | 0.00 | 0.00 | 0.02 | 0.00 | 0.00 | 0.06 | 0.01 | 0.01 | 0.18 |
| 21 | 0.00 | 0.00 | 0.02 | 0.00 | 0.01 | 0.04 | 0.01 | 0.01 | 0.15 |
| 42 | 0.00 | 0.00 | 0.02 | 0.00 | 0.01 | 0.05 | 0.01 | 0.01 | 0.17 |

| ACTIVSg2000 (CPU, correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.00 | 0.04 | 0.41 | 0.02 | 0.12 | 0.42 | 0.01 | 0.04 | 1.85 |
| 6 | 0.00 | 0.04 | 0.42 | 0.02 | 0.06 | 0.52 | 0.03 | 0.06 | 1.82 |
| 21 | 0.00 | 0.04 | 0.42 | 0.02 | 0.06 | 0.54 | 0.04 | 0.08 | 1.86 |
| 42 | 0.02 | 0.04 | 0.42 | 0.02 | 0.10 | 0.40 | 0.03 | 0.06 | 1.77 |

| ACTIVSg10k (CPU, correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.01 | 0.17 | 1.92 | 0.08 | 0.47 | 1.41 | 0.05 | 0.30 | 7.22 |
| 6 | 0.01 | 0.17 | 1.96 | 0.08 | 0.25 | 2.29 | 0.09 | 0.45 | 8.05 |
| 21 | 0.01 | 0.17 | 1.98 | 0.08 | 0.25 | 1.41 | 0.07 | 0.34 | 6.99 |
| 42 | 0.01 | 0.17 | 1.98 | 0.08 | 0.37 | 1.02 | 0.06 | 0.24 | 6.71 |

| ACTIVSg70k (CPU, correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.09 | 1.04 | 15.69 | 0.55 | 3.51 | 10.89 | 0.37 | 1.12 | 51.66 |
| 6 | 0.05 | 1.04 | 16.34 | 0.56 | 1.68 | 17.22 | 0.60 | 1.05 | 57.21 |
| 21 | 0.05 | 1.06 | 16.75 | 0.56 | 1.67 | 12.09 | 0.40 | 0.52 | 51.60 |
| 42 | 0.08 | 1.07 | 16.76 | 0.57 | 2.67 | 7.71 | 0.31 | 0.67 | 49.13 |

Table 2: SuperLU results computed on the CPU (without GPU acceleration) with correct right-hand sides. Times are reported in seconds.

### TAMU 500 (CPU, default RHS)

| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.05 | 0.27 | 0.01 | 0.12 | 0.29 | 0.01 | 0.03 | 1.43 |
| 6 | 0.00 | 0.05 | 0.27 | 0.01 | 0.06 | 0.24 | 0.02 | 0.04 | 1.37 |
| 21 | 0.00 | 0.05 | 0.27 | 0.01 | 0.06 | 0.32 | 0.02 | 0.05 | 1.45 |
| 42 | 0.00 | 0.05 | 0.27 | 0.01 | 0.09 | 0.26 | 0.02 | 0.05 | 1.44 |

### Case 118 ( CPU, default RHS)

| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.02 | 0.17 | 1.20 | 0.09 | 0.59 | 1.68 | 0.06 | 0.25 | 7.57 |
| 6 | 0.01 | 0.17 | 1.25 | 0.10 | 0.34 | 3.76 | 0.12 | 0.34 | 9.70 |
| 21 | 0.01 | 0.17 | 1.26 | 0.10 | 0.31 | 3.40 | 0.10 | 0.32 | 9.26 |
| 42 | 0.01 | 0.18 | 1.26 | 0.10 | 0.43 | 1.91 | 0.08 | 0.23 | 7.75 |

### RTS - One Time Period (CPU, default RHS)

| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 | 0.05 | 0.00 | 0.01 | 0.13 |
| 6 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.04 | 0.01 | 0.02 | 0.11 |
| 21 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.03 | 0.01 | 0.02 | 0.11 |
| 42 | 0.00 | 0.01 | 0.01 | 0.00 | 0.01 | 0.03 | 0.01 | 0.02 | 0.11 |

### RTS - Two Time Period (CPU, default RHS)

| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.01 | 0.02 | 0.00 | 0.01 | 0.08 | 0.00 | 0.01 | 0.22 |
| 6 | 0.00 | 0.01 | 0.03 | 0.00 | 0.01 | 0.07 | 0.01 | 0.03 | 0.21 |
| 21 | 0.00 | 0.01 | 0.03 | 0.00 | 0.01 | 0.05 | 0.01 | 0.02 | 0.18 |
| 42 | 0.00 | 0.01 | 0.03 | 0.00 | 0.01 | 0.05 | 0.01 | 0.03 | 0.19 |

### ACTIVSg200 (default RHS)

| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | 0.02 | 0.00 | 0.01 | 0.03 | 0.00 | 0.01 | 0.18 |
| 6 | 0.00 | 0.00 | 0.02 | 0.00 | 0.00 | 0.05 | 0.01 | 0.07 | 0.22 |
| 21 | 0.00 | 0.00 | 0.02 | 0.00 | 0.01 | 0.04 | 0.01 | 0.05 | 0.18 |
| 42 | 0.00 | 0.00 | 0.02 | 0.00 | 0.01 | 0.04 | 0.01 | 0.04 | 0.19 |

### ACTIVSg2000 (CPU, default RHS)

| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.04 | 0.41 | 0.02 | 0.12 | 0.42 | 0.01 | 0.08 | 1.82 |
| 6 | 0.00 | 0.04 | 0.42 | 0.02 | 0.06 | 0.65 | 0.04 | 0.17 | 2.01 |
| 21 | 0.00 | 0.04 | 0.42 | 0.02 | 0.06 | 0.57 | 0.03 | 0.18 | 1.93 |
| 42 | 0.00 | 0.04 | 0.42 | 0.02 | 0.09 | 0.40 | 0.03 | 0.14 | 1.77 |

### ACTIVSg10k (CPU, default RHS)

| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.01 | 0.17 | 1.91 | 0.08 | 0.47 | 1.40 | 0.05 | 0.43 | 7.06 |
| 6 | 0.01 | 0.17 | 1.96 | 0.08 | 0.25 | 2.16 | 0.09 | 0.60 | 7.86 |
| 21 | 0.01 | 0.17 | 1.98 | 0.08 | 0.25 | 1.45 | 0.07 | 0.41 | 6.92 |
| 42 | 0.01 | 0.17 | 1.98 | 0.08 | 0.37 | 1.02 | 0.05 | 0.33 | 6.48 |

### ACTIVSg70k (CPU, default RHS)

| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.09 | 1.04 | 15.68 | 0.55 | 3.54 | 10.93 | 0.36 | 1.29 | 50.35 |
| 6 | 0.05 | 1.04 | 16.36 | 0.56 | 1.73 | 17.26 | 0.59 | 1.92 | 56.81 |
| 21 | 0.04 | 1.05 | 16.78 | 0.56 | 1.69 | 11.99 | 0.40 | 0.94 | 50.62 |
| 42 | 0.05 | 1.09 | 16.79 | 0.57 | 2.62 | 7.86 | 0.31 | 0.78 | 47.07 |

Table 3: SuperLU results computed on the CPU (without GPU acceleration) with default right-hand sides. Times are reported in seconds.

| TAMU 500 (GPU, correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.00 | 0.05 | 0.26 | 0.01 | 0.11 | 0.78 | 0.01 | 0.03 | 2.14 |
| 2 | 0.00 | 0.05 | 0.27 | 0.01 | 0.10 | 1.47 | 0.03 | 0.06 | 2.73 |
| 6 | 0.00 | 0.05 | 0.27 | 0.01 | 0.06 | 2.13 | 0.02 | 0.04 | 3.30 |

| Case 118 (GPU, correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.02 | 0.17 | 1.20 | 0.09 | 0.54 | 2.23 | 0.06 | 0.23 | 8.33 |
| 2 | 0.02 | 0.17 | 1.24 | 0.10 | 0.48 | 2.97 | 0.12 | 0.38 | 9.31 |
| 6 | 0.01 | 0.17 | 1.24 | 0.10 | 0.31 | 5.74 | 0.13 | 0.40 | 11.88 |

| RTS - One Time Period (GPU, correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 | 0.53 | 0.00 | 0.00 | 0.59 |
| 2 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.77 | 0.01 | 0.02 | 0.84 |
| 6 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 1.93 | 0.01 | 0.02 | 2.00 |

| RTS - Two Time Period ( GPU, correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.00 | 0.01 | 0.02 | 0.00 | 0.01 | 0.58 | 0.00 | 0.01 | 0.71 |
| 2 | 0.00 | 0.01 | 0.03 | 0.00 | 0.01 | 0.78 | 0.01 | 0.02 | 0.91 |
| 6 | 0.00 | 0.01 | 0.03 | 0.00 | 0.01 | 1.95 | 0.01 | 0.03 | 2.09 |

| ACTTIVS200 (GPU, correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.00 | 0.00 | 0.02 | 0.00 | 0.01 | 0.52 | 0.00 | 0.00 | 0.62 |
| 2 | 0.00 | 0.00 | 0.02 | 0.00 | 0.01 | 0.77 | 0.00 | 0.00 | 0.87 |
| 6 | 0.00 | 0.00 | 0.02 | 0.00 | 0.00 | 1.91 | 0.01 | 0.01 | 2.03 |

| ACTTIVS2000 (GPU, correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.00 | 0.04 | 0.41 | 0.02 | 0.13 | 0.91 | 0.01 | 0.04 | 2.38 |
| 2 | 0.00 | 0.04 | 0.42 | 0.02 | 0.12 | 1.22 | 0.03 | 0.07 | 2.59 |
| 6 | 0.00 | 0.04 | 0.42 | 0.02 | 0.07 | 2.41 | 0.04 | 0.07 | 3.73 |

| ACTTIVS10k (GPU, correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.01 | 0.16 | 1.91 | 0.08 | 0.48 | 1.84 | 0.05 | 0.33 | 7.51 |
| 2 | 0.01 | 0.16 | 1.96 | 0.08 | 0.43 | 2.55 | 0.11 | 0.61 | 8.66 |
| 6 | 0.01 | 0.16 | 1.96 | 0.08 | 0.27 | 3.96 | 0.10 | 0.53 | 9.78 |

| ACTTIVS70k (GPU, correct RHS) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ranks | Equil | Row perm | Col perm | Sym factor | Dist | Factor | Solve | Ref | TOTAL |
| 1 | 0.09 | 1.05 | 15.70 | 0.55 | 3.40 | 10.19 | 0.37 | 0.77 | 50.49 |
| 2 | 0.07 | 1.04 | 16.17 | 0.56 | 2.90 | 11.77 | 0.76 | 1.05 | 53.28 |
| 6 | 0.05 | 1.06 | 16.26 | 0.56 | 1.86 | 20.90 | 0.64 | 1.11 | 61.15 |

Table 4: SuperLU results computed on the GPU with correct right-hand sides. Each rank is using a separate GPU. Times are reported in seconds.