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Software Implementation of the Krylov Methods Based Reconstruction for the 3D Cone Beam CT Operator

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Krylov subspace methods

- converge faster compared to classical methods, see (Liesen and Strakoš 2012),
- do not need an explicit construction of the problem matrix,
- have not yet been systematically studied in the context of CT reconstruction.

CGLS algorithm with delayed residual computation

input: Projection data $\mathbf{b}$, initial vector $\mathbf{x}_0$, relative discrepancy tolerance ERR, maximum number of iterations K, projection operator $\mathbf{A}$ and backprojection operator $\mathbf{A}^\top$.

begin

allocate $\mathbf{x}$, $\mathbf{d}_x$ and $\mathbf{r}_x$;
allocate $\mathbf{e}_b$ and $\mathbf{p}_b$;
$\mathrm{NB}_0 = \|\mathbf{b}\|_2$;
$\mathbf{x} = \mathbf{x}_0$;
$\mathbf{p}_b = \mathbf{A}\mathbf{x}$;
$\mathbf{e}_b = \mathbf{b} - \mathbf{p}_b$;
$\mathbf{r}_x = \mathbf{A}^\top \mathbf{e}_b$;
$\mathbf{d}_x = \mathbf{r}_x$;
$\mathrm{NR2}_{\mathrm{old}} = \|\mathbf{r}_x\|_2^2$;
$\mathbf{p}_b = \mathbf{A}\mathbf{d}_x$;
$\mathrm{NP2} = \|\mathbf{p}_b\|_2^2$;
$\alpha = \mathrm{NR2}_{\mathrm{old}}/\mathrm{NP2}$;
$\mathbf{x} = \mathbf{x} + \alpha \mathbf{d}_x$;
$\mathbf{e}_b = \mathbf{e}_b - \alpha \mathbf{p}_b$;
$\mathrm{NB} = \|\mathbf{e}_b\|_2$;
$i = 1$;
while $\mathrm{NB}/\mathrm{NB}_0 > \mathrm{ERR}$ & $i < \mathrm{K}$ **do**
  $\mathbf{r}_x = \mathbf{A}^\top \mathbf{e}_b$;
  $\mathrm{NR2}_{\mathrm{now}} = \|\mathbf{r}_x\|_2^2$;
  $\beta = \mathrm{NR2}_{\mathrm{now}}/\mathrm{NR2}_{\mathrm{old}}$;
  $\mathbf{d}_x = \mathbf{d}_x + \beta \mathbf{r}_x$;
  $\mathrm{NR2}_{\mathrm{old}} = \mathrm{NR2}_{\mathrm{now}}$;
  $\mathbf{p}_b = \mathbf{A}\mathbf{d}_x$;
  $\mathrm{NP2} = \|\mathbf{p}_b\|_2^2$;
  $\alpha = \mathrm{NR2}_{\mathrm{old}}/\mathrm{NP2}$;
  $\mathbf{x} = \mathbf{x} + \alpha \mathbf{d}_x$;
  $\mathbf{e}_b = \mathbf{e}_b - \alpha \mathbf{p}_b$;
  $\mathrm{NB} = \|\mathbf{e}_b\|_2$;
  $i = i + 1$;
end

end

Result: Vector $\mathbf{x}$, number of iterations i, final norm of discrepancy NB.

Algorithm 1: CGLS with delayed residual computation.

Software package for CBCT reconstruction

- Many different projectors, backprojectors and reconstruction techniques,
- written in C++ and OpenCL,
- open source, GNU GPL3 license,
- stable commit for Fully3D conference 3d97384,
- public GIT repository `https://bitbucket.org/kulvait/cbct`.

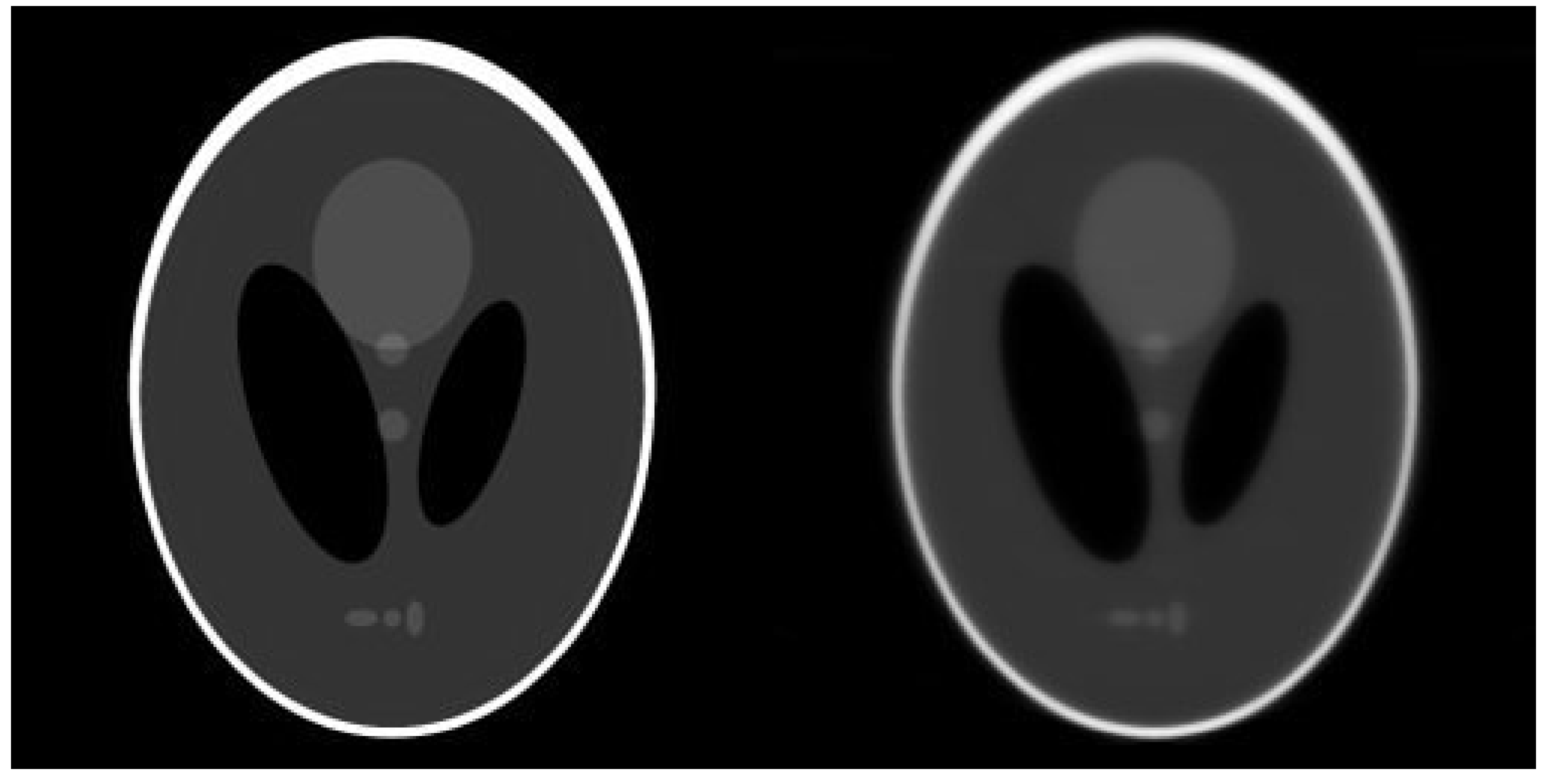
Test problem

- 3D Shepp-Logan phantom with a center slice identical to the 2D phantom,
- volume of $256 \times 256 \times 64 = 4M$ voxels $0.86\,\mathrm{mm} \times 0.86\,\mathrm{mm} \times 3.44\,\mathrm{mm}$,
- projected by CBCT operator with 496 view angles onto the detector 616×480, producing $139M$ values,
- problem is to reconstruct the original volume using given projection data,
- we use the software to compare classical scheme PSIRT (Gregor and Benson 2008) with CGLS (Krylov).

Results

Speed of CBCT operators Using the GeForce RTX 2080 Ti card, we were able to increase the speed of projector to $\approx 2.8\,\mathrm{s}$ and backprojector to $\approx 1.0\,\mathrm{s}$, so that the reconstruction of the test problem takes $\approx 2\,\mathrm{min}\,35\,\mathrm{s}$ using 40 CGLS iterations.

PSIRT converges much slower than CGLS

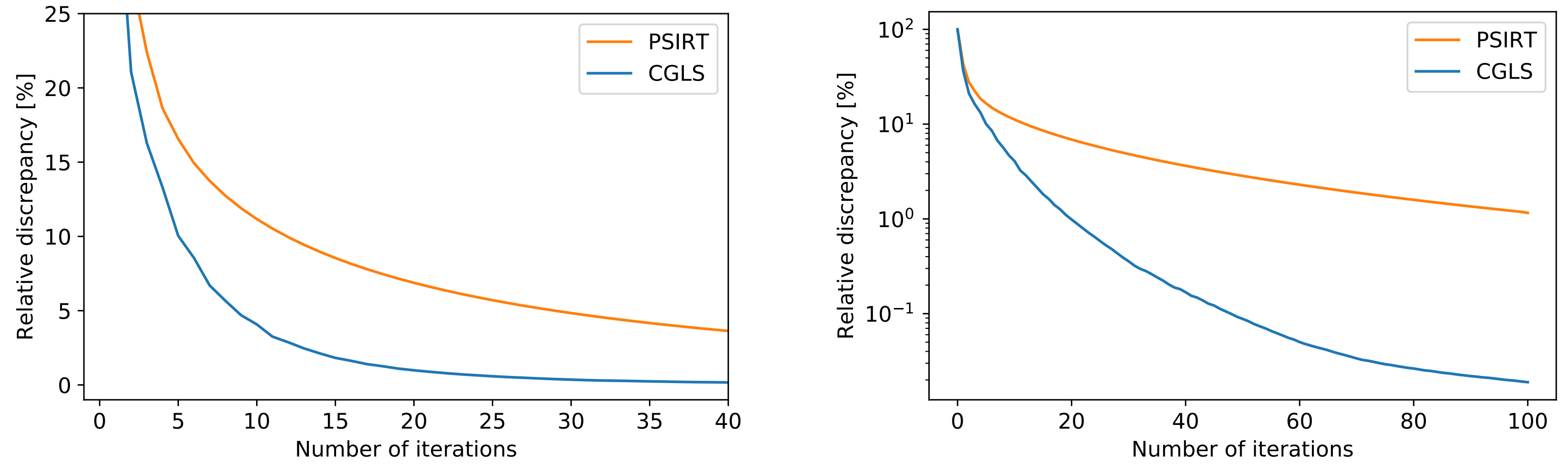


Result after 40 iterations of CGLS (left) and PSIRT (right).

Convergence properties To measure speed of convergence, we use the relative discrepancy

$$e = \frac{\|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2}{\|\mathbf{b}\|_2}.$$

In 40 iterations, this metric was $e_{\mathrm{CGLS}} = 0.18\%$ for CGLS versus $e_{\mathrm{PSIRT}} = 3.67\%$ for PSIRT.



Comparison of the speed of convergence in terms of relative norm of discrepancy for CGLS and PSIRT.

Projectors implemented

- Cutting voxel projector, an inovative projector that finds voxel cuts projected onto given pixels, will be introduced elsewhere,
- TT projector, see (Long, Fessler, and Balter 2010),
- Siddon projector, see (Siddon 1985).

CBCT perfusion processing

- Contrast agent is injected into the patient and its dynamic is examined during $\approx 60\,\mathrm{s}$ C-arm CT scan,
- the time dynamic in each volume voxel is reconstructed using temporal functions,
- this is a challenging problem due to the higher noise characteristics, slow rotation speed and lower frame rate of C-arm CT compared to conventional CT,
- algebraic reconstruction of these data provides better results than analytical reconstruction, see (Bannasch et al. 2018),
- is not used in clinical practice because current implementations of algebraic methods are slow,
- Krylov subspace methods might speed up algebraic reconstruction so that perfusion data can be processed in diagnostically acceptable times.

Discussion

- Proper preconditioning strategies could further increase the speed of Krylov methods,
- enforcing box conditions is difficult and requires restarted Krylov methods or other approaches,
- Tikhonov regularization can be applied to the reconstruction problem without speed penalties,
- GLSQR is also implemented in the package, providing the same results as CGLS,
- incorporating prior knowledge using the initial vector $\mathbf{x}_0$ is possible and there are promising results with prior knowledge based reconstruction.

Conclusion

- With CGLS, we have reduced the running time of medium sized CBCT reconstruction to $\approx 2\,\mathrm{min}\,35\,\mathrm{s}$,
- preconditioning, software and hardware improvements can further improve this number,
- in clinical applications, when using Krylov methods, the time difference between algebraic and analytical reconstruction may soon be negligible.

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References

Bannasch, Sebastian et al. (2018). "Time separation technique: Accurate solution for 4D C-Arm-CT perfusion imaging using a temporal decomposition model". In: *Medical Physics* 45.3, pp. 1080–1092. DOI: 10.1002/mp.12768.

Gregor, Jens and Thomas Benson (2008). "Computational Analysis and Improvement of SIRT". In: *IEEE Transactions on Medical Imaging* 27.7, pp. 918–924. DOI: 10.1109/tmi.2008.923696.

Liesen, Jörg and Zdeněk Strakoš (Dec. 14, 2012). *Krylov Subspace Methods: Principles and Analysis.* Oxford University Press. 391 pp. ISBN: 0199655413. DOI: 10.1093/acprof:oso/9780199655410.001.0001.

Long, Yong, Jeffrey A Fessler, and James M Balter (2010). "3D Forward and Back-Projection for X-Ray CT Using Separable Footprints". In: *IEEE Transactions on Medical Imaging* 29.11, pp. 1839–1850. DOI: 10.1109/tmi.2010.2050898.

Siddon, Robert L. (1985). "Fast calculation of the exact radiological path for a three-dimensional CT array". In: *Medical Physics* 12.2, pp. 252–255. DOI: 10.1118/1.595715.

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