$\begin{aligned} & \text{Variable } replicas \\ & globalVars \ \triangleq \ \langle replicas \rangle \\ & \text{Variable } messages \end{aligned}$

Entry types CONSTANTS TValue, TNoOp,

```
messageVars \triangleq \langle messages \rangle
Variable cTime
VARIABLE cViewID
Variable cSeqNum
Variable cResps
VARIABLE cWrites
VARIABLE cReads
clientVars \triangleq \langle cTime, cViewID, cSegNum, cResps, cWrites, cReads \rangle
Variable rStatus
VARIABLE rLoq
VARIABLE rViewID
VARIABLE rSegNum
Variable rLastView
Variable rViewChanges
{\tt VARIABLE}\ rAbortSeqNum
VARIABLE rAbortResps
replicaVars \stackrel{\triangle}{=} \langle rStatus, rLog, rViewID, rSeqNum, rLastView, rViewChanges, rAbortSeqNum, rAbortResps \rangle
Variable transitions
vars \triangleq \langle global Vars, message Vars, client Vars, replica Vars, transitions \rangle
 Helpers
RECURSIVE SeqFromSet(_)
SeqFromSet(S) \triangleq
  If S = \{\} Then \langle \rangle
   ELSE LET x \stackrel{\triangle}{=} \text{CHOOSE } x \in S : \text{TRUE}
          IN \langle x \rangle \circ SeqFromSet(S \setminus \{x\})
Max(s) \stackrel{\triangle}{=} \text{ CHOOSE } x \in s : \forall y \in s : x \geq y
```

 $IsQuorum(s) \triangleq Cardinality(s) * 2 \geq Cardinality(Replicas)$

 $Quorums \triangleq \{r \in SUBSET \ Replicas : IsQuorum(r)\}$

```
IsPrimary(r) \stackrel{\triangle}{=} Primary(rViewID[r]) = r
Replace(l, i, x) \triangleq [j \in 1 ... Max(\{Len(l), i\}) \mapsto \text{if } j = i \text{ Then } x \text{ else } l[j]]
    Messaging helpers
Sends(ms) \stackrel{\triangle}{=} messages' = messages \cup ms
Send(m) \triangleq Sends(\{m\})
Replies(req, resps) \triangleq messages' = (messages \cup resps) \setminus \{req\}
Reply(req, resp) \stackrel{\Delta}{=} Replies(req, \{resp\})
Discard(m) \stackrel{\triangle}{=} messages' = messages \setminus \{m\}
 Write(c) \triangleq
            \wedge cTime' = cTime + 1
            \land cSeqNum' = [cSeqNum \ EXCEPT \ ![c] = cSeqNum[c] + 1]
            \land Sends(\{[src
                                                                           \mapsto c,
                                            dest
                                                                           \mapsto r,
                                                                           \mapsto MWriteRequest,
                                            type
                                            viewID
                                                                           \mapsto c ViewID[c],
                                            seqNum \mapsto cSeqNum'[c],
                                            timestamp \mapsto cTime' | : r \in Replicas \}
            ∧ UNCHANGED ⟨globalVars, replicaVars, cViewID, cResps, cWrites, cReads⟩
 Read(c) \triangleq
                \land Sends(\{[src
                                                                               \mapsto c,
                                                dest
                                                                               \mapsto r,
                                                                               \mapsto MReadRequest,
                                                viewID \mapsto cViewID[c]]: r \in Replicas\}
                \land UNCHANGED \langle globalVars, replicaVars, cTime, cSeqNum, cResps, cWrites, cReads <math>\rangle
 ChecksumsMatch(c1, c2) \triangleq
            \wedge Len(c1) = Len(c2)
            \land \neg \exists i \in \text{DOMAIN } c1: c1[i] \neq c2[i]
IsCommitted(acks) \triangleq
           \exists msgs \in \text{SUBSET} acks:
                 \land \{m.src : m \in msgs\} \in Quorums
                 \land \exists \ m1 \in msgs: \forall \ m2 \in msgs: m1.viewID = m2.viewID \land ChecksumsMatch(m1.checksum, \ m2.checksum, \ m2.checksum, \ m2.checksum, \ m2.checksum, \ m3.viewID \land ChecksumsMatch(m3.checksum, \ m3.checksum, \ m3.checksum
                 \wedge \exists m \in msgs : m.primary
```

 $Primary(v) \stackrel{\triangle}{=} replicas[(v\%Len(replicas)) + (\text{IF } v \geq Len(replicas) \text{ THEN } 1 \text{ ELSE } 0)]$

```
Handle WriteResponse(c, r, m) \stackrel{\Delta}{=}
           \land \neg \exists \ w \in \mathit{cWrites}[\mathit{c}] : w.\mathit{seqNum} = \mathit{m.seqNum}
           \land \lor \land m.seqNum \notin DOMAIN \ cResps[c][r]
                          \land cResps' = [cResps \ \texttt{EXCEPT} \ ![c] = [cResps[c] \ \texttt{EXCEPT} \ ![r] = cResps[c][r] \ @@ (m.seqNum :> m)]] 
                         \land UNCHANGED \langle cWrites \rangle
                  \lor \land m.seqNum \in \text{DOMAIN } cResps[c][r]
                          Do not overwrite a response from a newer view
                         \land cResps[c][r][m.seqNum].viewID \le m.viewID
                          \land cResps' = [cResps \ \texttt{EXCEPT} \ ![c] = [cResps[c] \ \texttt{EXCEPT} \ ![r] = [cResps[c][r] \ \texttt{EXCEPT} \ ![m.seqNum] = [cResps[c][r] \ *[m.seqNum] = [cResps[c][r] \ *[m.seqNum
                         \land LET committed \stackrel{\triangle}{=} IsCommitted(\{cResps'[c][x][m.seqNum]: x \in \{x \in Replicas: m.seqNum \in DOM \}\})
                              IN
                                        \vee \wedge committed
                                              \land cWrites' = [cWrites \ EXCEPT \ ![c] = cWrites[c] \cup \{m\}]
                                        \lor \land \neg committed
                                              \land UNCHANGED \langle cWrites \rangle
           \wedge Discard(m)
           \land UNCHANGED \langle globalVars, replicaVars, cTime, cSeqNum, cReads <math>\rangle
HandleReadResponse(c, r, m) \stackrel{\Delta}{=}
           \land \lor \land m.primary
                         \land m \notin cReads[c]
                         \land cReads' = [cReads \ \texttt{EXCEPT} \ ![c] = cReads[c] \cup \{m\}]
                  \lor \land \neg m.primary
                         \land UNCHANGED \langle cReads \rangle
           \wedge Discard(m)
           \land UNCHANGED \langle globalVars, replicaVars, cTime, cSeqNum, cResps, cWrites <math>\rangle
  Server request/response handling
Repair(r, c, m) \triangleq
           \land Replies(m, \{[src
                                                      dest
                                                                            \mapsto d,
                                                                           \mapsto MRepairRequest,
                                                      type
                                                     viewID \mapsto rViewID[r],
                                                     seqNum \mapsto rSeqNum[r][c] + 1] : d \in Replicas\}
Abort(r, c, m) \triangleq
           \wedge IsPrimary(r)
           \wedge rStatus[r]
                                                            = SNormal
                                                                                                       EXCEPT ![r] = SAborting]
           \wedge rStatus'
                                                            = [rStatus]
           \land rAbortResps' = [rAbortResps \ EXCEPT \ ![r] = [rAbortResps[r] \ EXCEPT \ ![c] = \{\}]]
           \land rAbortSeqNum' = [rAbortSeqNum \ Except \ ![r] = [rAbortSeqNum[r] \ Except \ ![c] = m.seqNum]]
           \land Replies(m, \{[src
```

```
dest
                                  \mapsto d,
                        type
                                  \mapsto MAbortRequest,
                        viewID \mapsto rViewID[r],
                        client \mapsto c,
                       seqNum \mapsto m.seqNum]: d \in Replicas})
Handle WriteRequest(r, c, m) \stackrel{\Delta}{=}
     \wedge rStatus[r] = SNormal
     \land \lor \land m.viewID = rViewID[r]
           \land m.seqNum = rSeqNum[r][c] + 1
           \land LET entry \stackrel{\triangle}{=} [type \mapsto TValue, value \mapsto m.value, timestamp \mapsto m.timestamp]
             IN rLog' = [rLog \ EXCEPT \ ![r] = Append(rLog[r], \ entry)]
           \land rSeqNum' = [rSeqNum \ EXCEPT \ ![r] = m.seqNum]
           \land Reply(m, [src
                           dest
                           type
                                      \mapsto MWriteResponse,
                          seqNum \mapsto m.seqNum,
                                     \mapsto rViewID[r],
                          viewID
                          succeeded \mapsto TRUE)
        \lor \land m.viewID = rViewID[r]
           \land m.seqNum > rSeqNum[r][c] + 1
           \land \lor \land \mathit{IsPrimary}(r)
                 \wedge Abort(r, c, m)
              \lor \land \neg IsPrimary(r)
                 \wedge Repair(r, c, m)
           \land UNCHANGED \langle rLog \rangle
        \lor \land m.viewID < rViewID[r]
           \land Reply(m, [src
                           dest
                                      \mapsto c,
                                      \mapsto MWriteResponse,
                           type
                          seqNum \mapsto m.seqNum,
                          viewID
                                      \mapsto rViewID[r],
                          succeeded \mapsto FALSE)
           \land UNCHANGED \langle rLog \rangle
     ∧ UNCHANGED ⟨globalVars, clientVars, rStatus, rViewID, rLastView, rViewChanges⟩
HandleReadRequest(r, c, m) \stackrel{\Delta}{=}
     \land rStatus[r] = SNormal
     \wedge Len(rLog[r]) > 0
     \land Reply(m, [src])
                                \mapsto r,
                    dest
                                \mapsto c,
                    type
                                \mapsto MReadResponse,
                    viewID
                                \mapsto rViewID[r],
                    primary \mapsto IsPrimary(r),
                    index
                                \mapsto Len(rLog[r]),
```

```
checksum \mapsto rLog[r][Len(rLog[r])].checksum,
                   succeeded \mapsto TRUE)
    ∧ UNCHANGED ⟨qlobalVars, clientVars, rStatus, rLoq, rViewID, rLastView, rViewChanges⟩
HandleRepairRequest(r, s, m) \stackrel{\Delta}{=}
    \land m.viewID = rViewID[r]
    \wedge IsPrimary(r)
    \wedge rStatus[r] = SNormal
    \land \lor \land m.seqNum \le Len(rLog[r][m.client])
          \land Reply(m, [src
                                   \mapsto r,
                         dest
                                   \mapsto MRepairResponse,
                         type
                         viewID \mapsto rViewID[r],
                         client \mapsto m.client,
                         seqNum \mapsto m.seqNum])
          \land UNCHANGED \langle rStatus, rAbortResps, rAbortSeqNum \rangle
       \lor \land m.seqNum = Len(rLog[r][m.client]) + 1
          \wedge Abort(r, m.client, m)
    \land UNCHANGED \langle globalVars, clientVars \rangle
HandleRepairResponse(r, s, m) \stackrel{\Delta}{=}
    \land Handle Write Request (r, m.client, [m \text{ EXCEPT } !.src = m.client])
HandleAbortRequest(r, s, m) \stackrel{\Delta}{=}
    \land m.viewID = rViewID[r]
    \land m.seqNum \leq Len(rLog[r][m.client]) + 1
    \land rStatus[r] \in \{SNormal, SAborting\}
    \land LET entry \triangleq [type \mapsto TNoOp]
       IN rLog' = [rLog \ EXCEPT \ ![r] = [rLog[r] \ EXCEPT \ ![m.client] = Replace(rLog[r][m.client], m.seqNum,
    \land \lor \land m.seqNum > rSeqNum[r][m.client]
          \land rSeqNum' = [rSeqNum \ Except \ ![r] = [rSeqNum[r] \ Except \ ![m.client] = m.seqNum]]
       \lor \land m.seqNum \le rSeqNum[r][m.client]
          \land UNCHANGED \langle rSeqNum \rangle
    \land Replies(m, \{[src
                                  \mapsto Primary(rViewID[r]),
                       dest
                                  \mapsto MAbortResponse,
                       type
                      viewID
                                  \mapsto rViewID[r],
                      seqNum \mapsto m.seqNum,
                      [src]
                                  \mapsto r,
                      dest
                                  \mapsto Primary(rViewID[r]),
                                  \mapsto MWriteResponse,
                      type
                                 \mapsto rViewID[r],
                      viewID
                      seqNum \mapsto m.seqNum,
                      succeeded \mapsto FALSE[\})
    \land UNCHANGED \langle qlobalVars, clientVars, rStatus, rViewID, rLastView, rViewChanges <math>\rangle
```

```
HandleAbortResponse(r, s, m) \stackrel{\Delta}{=}
           \land rStatus[r] = SAborting
           \land m.viewID = rViewID[r]
           \wedge IsPrimary(r)
           \land m.seqNum = rAbortSeqNum[r][m.client]
           \land rAbortResps' = [rAbortResps \ EXCEPT \ ![r] = [rAbortResps[r] \ EXCEPT \ ![m.client] = rAbortResps[r][m.client]
           \land LET resps \stackrel{\triangle}{=} \{res.src : res \in \{resp \in rAbortResps'[r][m.client] : resps \stackrel{\triangle}{=} \{res.src : resps \in rabortResps'[r][m.client] : resps \stackrel{\triangle}{=} \{re
                                                                                            \land resp.viewID = rViewID[r]
                                                                                            \land resp.seqNum = rAbortSeqNum[r][m.client]\}
                              isQuorum \stackrel{\triangle}{=} r \in resps \land resps \in Quorums
                IN
                          \lor \land isQuorum
                                 \land rStatus' = [rStatus \ EXCEPT \ ![r] = [rStatus[r] \ EXCEPT \ ![m.client] = SNormal]]
                          \lor \land \neg isQuorum
                                 \land UNCHANGED \langle rStatus \rangle
           \land UNCHANGED \langle globalVars, clientVars \rangle
ChangeView(r) \triangleq
           \land Sends(\{[src
                                                              \mapsto r,
                                                          \mapsto MViewChangeRequest,
                                         viewID \mapsto rViewID[r] + 1] : d \in Replicas \})
           \land UNCHANGED \langle globalVars, clientVars, replicaVars \rangle
Handle View Change Request(r, s, m) \stackrel{\Delta}{=}
           \land rViewID[r] < m.viewID
                                                            = [rViewID \ EXCEPT \ ![r] = m.viewID]
           \land rViewID'
           \wedge rStatus'
                                                            = [rStatus \ EXCEPT \ ![r] = SViewChange]
           \land rViewChanges' = [rViewChanges \ EXCEPT \ ![r] = \{\}]
           \land Reply(m, [src
                                                                               \mapsto r,
                                               dest
                                                                               \mapsto Primary(m.viewID),
                                                                               \mapsto MViewChangeResponse,
                                               type
                                               viewID
                                                                               \mapsto m.viewID,
                                               lastViewID \mapsto rLastView[r],
                                                                                \mapsto rLog[r]
                                               logs
           \land UNCHANGED \langle globalVars, clientVars, rLog, rSeqNum, rAbortSeqNum, rAbortResps, rLastView <math>\rangle
Handle View Change Response(r, s, m) \stackrel{\Delta}{=}
           \wedge IsPrimary(r)
                                                            = m.viewID
           \wedge rViewID[r]
                                                            = SViewChange
           \wedge rStatus[r]
           \land rViewChanges' = [rViewChanges \ EXCEPT \ ![r] = rViewChanges[r] \cup \{m\}]
                                                                             \triangleq \{v \in rViewChanges'[r][m.client] : \land v.viewID = rViewID[r]\}
           \land Let viewChanges
                                                                            \triangleq \{v.src : v \in viewChanges\}
                              viewSources
                                                                            \stackrel{\triangle}{=} r \in \textit{viewSources} \land \textit{viewSources} \in \textit{Quorums}
                              is Quorum
                                                                            \stackrel{\triangle}{=} \{v.lastViewID : v \in viewChanges\}
                              last Views
```

```
last View
                                      (CHOOSE v1 \in lastViews : \forall v2 \in lastViews : v2 \leq v1)
                                       [c \in Clients \mapsto \{v1.logs[c] : v1 \in \{v2 \in viewChanges : v2.lastView = lastView\}]
              viewLogs
              mergeEnts(es)
                  If es = \{\} \lor \exists e \in es : r.type = TNoOp \text{ then}
                       [type \mapsto TNoOp]
                   ELSE
                       CHOOSE e \in es : e.type \neq TNoOp
                                 \stackrel{\Delta}{=} Max(\{Len(l): l \in ls\})
              range(ls)
              entries(ls, i) \triangleq \{l[i] : l \in \{k \in ls : i \leq Len(k)\}\}
              mergeLogs(ls) \stackrel{\Delta}{=} [i \in 1 .. range(ls) \mapsto mergeEnts(entries(ls, i))]
       IN
             \lor \land isQuorum
                \land Replies(m, \{[src
                                              \mapsto r,
                                    type
                                             \mapsto MStartViewRequest,
                                    viewID \mapsto rViewID[r],
                                             \mapsto [c \in Clients \mapsto mergeLogs(viewLogs[c])] : d \in Replicas\})
             \lor \land \neg isQuorum
                \wedge Discard(m)
     \land UNCHANGED \langle globalVars, clientVars, rStatus, rViewID, rLog, rSeqNum, rAbortSeqNum, rAbortResps, r
HandleStartViewRequest(r, s, m) \stackrel{\Delta}{=}
     \land \lor rViewID[r] < m.viewID
        \lor \land rViewID[r] = m.viewID
            \land rStatus[r] = SViewChange
                       = [rLog
     \wedge rLog'
                                       EXCEPT ![r] = m.log]
     \wedge rStatus'
                      = [rStatus \quad EXCEPT \ ![r] \quad = SNormal]
     \land rViewID' = [rViewID \quad \text{EXCEPT } ! [r] = m.viewID]
     \land rLastView' = [rLastView \ EXCEPT \ ![r] = m.viewID]
     \wedge Discard(m)
     \land UNCHANGED \langle globalVars, clientVars, rViewChanges \rangle
InitMessageVars \triangleq
     \land messages = \{\}
InitClientVars \triangleq
     \wedge c Time
     \land \ c \textit{ViewID}
                        = [c \in Clients \mapsto 1]
                       = [c \in Clients \mapsto 0]
     \wedge cSeqNum
     \land cResps = [c \in Clients \mapsto [r \in Replicas \mapsto [s \in \{\} \mapsto [index \mapsto 0, checksum \mapsto Nil]]]]
     \land cWrites
                      = [c \in Clients \mapsto \{\}]
     \land\ cReads
                      = [c \in Clients \mapsto \{\}]
InitReplicaVars \stackrel{\Delta}{=}
```

```
\land replicas
                              = SeqFromSet(Replicas)
     \land \mathit{rStatus}
                              = [r \in Replicas \mapsto SNormal]
     \wedge rLoq
                              = [r \in Replicas \mapsto [c \in Clients \mapsto \langle \rangle]]
                              = [r \in Replicas \mapsto [c \in Clients \mapsto 0]]
      \wedge rSegNum
      \land rAbortSeqNum = [r \in Replicas \mapsto [c \in Clients \mapsto 0]]
      \wedge rAbortResps
                              = [r \in Replicas \mapsto [c \in Clients \mapsto \{\}]]
     \land rViewID
                              = [r \in Replicas \mapsto 1]
                              = [r \in Replicas \mapsto 1]
      \land \mathit{rLastView}
      \land rViewChanges = [r \in Replicas \mapsto \{\}]
Init \stackrel{\triangle}{=}
     \land InitMessageVars
     \land InitClientVars
     \land InitReplica Vars
      \land transitions = 0
```

```
The type invariant checks that no read ever reads a different value than a previous write
Inv \triangleq
    \land \forall c1, c2 \in Clients:
         \neg \exists r \in cReads[c1]:
             \exists w \in cWrites[c2]:
                \land r.index = w.index
                \land \neg ChecksumsMatch(r.checksum, w.checksum)
    \land \forall c1, c2 \in Clients:
         \neg \exists r1 \in cReads[c1]:
             \exists r2 \in cReads[c2]:
                \wedge r1.index = r2.index
                \land \neg ChecksumsMatch(r1.checksum, r2.checksum)
Transition \triangleq transitions' = transitions + 1
Next \triangleq
     \vee \exists c \in Clients:
          \wedge Write(c)
          \land Transition
     \vee \exists c \in Clients:
          \wedge Read(c)
          \land Transition
     \vee \exists r \in Replicas :
          \wedge Change View(r)
          \land Transition
     \vee \exists m \in messages :
          \land m.type = MWriteRequest
          \land Handle WriteRequest(m.dest, m.src, m)
```

```
\land \ Transition
     \vee \exists m \in messages :
          \land \ m.type = MWriteResponse
          \land Handle WriteResponse (m.dest, m.src, m)
          \land Transition
     \vee \exists m \in messages :
          \land m.type = MReadRequest
          \land HandleReadRequest(m.dest, m.src, m)
          \land Transition
     \vee \exists m \in messages :
          \land m.type = MReadResponse
          \land HandleReadResponse(m.dest, m.src, m)
          \land Transition
     \vee \exists m \in messages :
          \land m.type = MRepairRequest
          \land HandleRepairRequest(m.dest, m.src, m)
          \land \ Transition
     \vee \exists m \in messages :
          \land \ m.type = MRepairResponse
          \land HandleRepairResponse(m.dest, m.src, m)
          \land \ Transition
     \vee \exists m \in messages :
          \land \ m.type = MAbortRequest
          \land HandleAbortRequest(m.dest, m.src, m)
          \land \ Transition
     \vee \exists m \in messages :
          \land \ m.type = \mathit{MAbortResponse}
          \land HandleAbortResponse(m.dest, m.src, m)
          \land Transition
     \vee \exists m \in messages :
          \land m.type = MViewChangeRequest
          \land Handle View Change Request (m.dest, m.src, m)
          \land Transition
     \vee \exists m \in messages :
          \land m.type = MViewChangeResponse
          \land Handle View Change Response (m.dest, m.src, m)
          \land \ Transition
     \vee \exists m \in messages :
          \land m.type = MStartViewRequest
          \land HandleStartViewRequest(m.dest, m.src, m)
          \land Transition
Spec \triangleq Init \wedge \Box [Next]_{vars}
```

- $\backslash \ * \ \operatorname{Modification} \ \operatorname{History}$
- \ * Last modified Tue Sep 22 03:38:33 PDT 2020 by jordanhalterman
- \ * Created Fri Sep 18 22:45:21 PDT 2020 by jordanhalterman