
MODULE *JustInTimePaxos*

EXTENDS *Naturals, Sequences, FiniteSets, TLC*

The set of Paxos replicas
 CONSTANT *Replicas*

The set of Paxos clients
 CONSTANT *Clients*

An empty value
 CONSTANT *Nil*

Client request/response types
 CONSTANTS
 MWriteRequest,
 MWriteResponse,
 MReadRequest,
 MReadResponse

Server request/response types
 CONSTANTS
 MRepairRequest,
 MRepairResponse,
 MAbortRequest,
 MAbortResponse,
 MViewChangeRequest,
 MViewChangeResponse,
 MStartViewRequest

Replica roles
 CONSTANTS
 SNormal,
 SAborting,
 SViewChange

Entry types
 CONSTANTS
 TValue,
 TNoOp

VARIABLE *replicas*

globalVars \triangleq $\langle \textit{replicas} \rangle$

VARIABLE *messages*

$messageVars \triangleq \langle messages \rangle$
 VARIABLE $cTime$
 VARIABLE $cViewID$
 VARIABLE $cSeqNum$
 VARIABLE $cResps$
 VARIABLE $cWrites$
 VARIABLE $cReads$
 $clientVars \triangleq \langle cTime, cViewID, cSeqNum, cResps, cWrites, cReads \rangle$
 VARIABLE $rStatus$
 VARIABLE $rLog$
 VARIABLE $rViewID$
 VARIABLE $rSeqNum$
 VARIABLE $rLastView$
 VARIABLE $rViewChanges$
 VARIABLE $rAbortSeqNum$
 VARIABLE $rAbortResps$
 $replicaVars \triangleq \langle rStatus, rLog, rViewID, rSeqNum, rLastView, rViewChanges, rAbortSeqNum, rAbortResps \rangle$
 VARIABLE $transitions$
 $vars \triangleq \langle globalVars, messageVars, clientVars, replicaVars, transitions \rangle$

Helpers

RECURSIVE $SeqFromSet(-)$
 $SeqFromSet(S) \triangleq$
 IF $S = \{\}$ THEN $\langle \rangle$
 ELSE LET $x \triangleq$ CHOOSE $x \in S : \text{TRUE}$
 IN $\langle x \rangle \circ SeqFromSet(S \setminus \{x\})$
 $Max(s) \triangleq$ CHOOSE $x \in s : \forall y \in s : x \geq y$
 $IsQuorum(s) \triangleq Cardinality(s) * 2 \geq Cardinality(Replicas)$
 $Quorums \triangleq \{r \in \text{SUBSET } Replicas : IsQuorum(r)\}$

$$Primary(v) \triangleq replicas[(v \% Len(replicas)) + (\text{IF } v \geq Len(replicas) \text{ THEN } 1 \text{ ELSE } 0)]$$

$$IsPrimary(r) \triangleq Primary(rViewID[r]) = r$$

$$Replace(l, i, x) \triangleq [j \in 1 \dots Max(\{Len(l), i\}) \mapsto \text{IF } j = i \text{ THEN } x \text{ ELSE } l[j]]$$

Messaging helpers

$$Sends(ms) \triangleq messages' = messages \cup ms$$

$$Send(m) \triangleq Sends(\{m\})$$

$$Replies(req, resps) \triangleq messages' = (messages \cup resps) \setminus \{req\}$$

$$Reply(req, resp) \triangleq Replies(req, \{resp\})$$

$$Discard(m) \triangleq messages' = messages \setminus \{m\}$$

$$Write(c) \triangleq$$

$$\wedge cTime' = cTime + 1$$

$$\wedge cSeqNum' = [cSeqNum \text{ EXCEPT } ![c] = cSeqNum[c] + 1]$$

$$\wedge Sends(\{[src \mapsto c, \\ dest \mapsto r, \\ type \mapsto MWriteRequest, \\ viewID \mapsto cViewID[c], \\ seqNum \mapsto cSeqNum'[c], \\ timestamp \mapsto cTime'] : r \in Replicas\})$$

$$\wedge \text{UNCHANGED } \langle globalVars, replicaVars, cViewID, cResps, cWrites, cReads \rangle$$

$$Read(c) \triangleq$$

$$\wedge Sends(\{[src \mapsto c, \\ dest \mapsto r, \\ type \mapsto MReadRequest, \\ viewID \mapsto cViewID[c]] : r \in Replicas\})$$

$$\wedge \text{UNCHANGED } \langle globalVars, replicaVars, cTime, cSeqNum, cResps, cWrites, cReads \rangle$$

$$HandleWriteResponse(c, r, m) \triangleq$$

$$\wedge \vee \wedge m.viewID = cViewID[c]$$

$$\wedge \text{IF } m.seqNum \notin \text{DOMAIN } cResps[c][r] \text{ THEN}$$

$$cResps' = [cResps \text{ EXCEPT } ![c] = [cResps[c] \text{ EXCEPT } ![r] = cResps[c][r] @@ (m.seqNum :> m)]]$$

$$\text{ELSE}$$

$$cResps' = [cResps \text{ EXCEPT } ![c] = [cResps[c] \text{ EXCEPT } ![r] = [cResps[c][r] \text{ EXCEPT } ![m.seqNum]$$

$$\wedge \text{LET}$$

$$allResps \triangleq \{cResps[c][r][r1] : r1 \in \{r2 \in Replicas : r2 \in \text{DOMAIN } cResps[c][r]\}\}$$

$$isCommitted \triangleq \{r1.src : r1 \in \{r2 \in allResps : r2.succeeded\}\} \in Quorums$$

$$\begin{aligned}
& \text{IN} \\
& \quad \wedge \vee \wedge isCommitted \\
& \quad \quad \wedge cWrites' = [cWrites \text{ EXCEPT } ![c] = cWrites[c] \cup \{m\}] \\
& \quad \vee \wedge \neg isCommitted \\
& \quad \quad \wedge \text{UNCHANGED } \langle cWrites \rangle \\
& \quad \quad \wedge \text{UNCHANGED } \langle cViewID, cSeqNum \rangle \\
& \vee \wedge m.viewID > cViewID[c] \\
& \quad \wedge cViewID' = [cViewID \text{ EXCEPT } ![c] = m.viewID] \\
& \quad \wedge cSeqNum' = [cSeqNum \text{ EXCEPT } ![c] = 0] \\
& \quad \wedge cResps' = [cResps \text{ EXCEPT } ![c] = \{i \in Replicas \mapsto \{\}\}] \\
& \quad \wedge \text{UNCHANGED } \langle cWrites \rangle \\
& \vee \wedge m.viewID < cViewID[c] \\
& \quad \wedge \text{UNCHANGED } \langle cWrites \rangle \\
& \wedge Discard(m) \\
& \wedge \text{UNCHANGED } \langle globalVars, replicaVars, cTime, cSeqNum, cReads \rangle \\
& HandleReadResponse(c, r, m) \triangleq \\
& \quad \wedge \vee \wedge m.viewID = cViewID[c] \\
& \quad \quad \wedge cReads' = [cReads \text{ EXCEPT } ![c] = cReads[c] \cup \{m\}] \\
& \quad \quad \wedge \text{UNCHANGED } \langle cViewID, cSeqNum \rangle \\
& \quad \vee \wedge m.viewID > cViewID[c] \\
& \quad \quad \wedge cViewID' = [cViewID \text{ EXCEPT } ![c] = m.viewID] \\
& \quad \quad \wedge cSeqNum' = [cSeqNum \text{ EXCEPT } ![c] = 0] \\
& \quad \quad \wedge \text{UNCHANGED } \langle cReads \rangle \\
& \quad \vee \wedge m.viewID < cViewID[c] \\
& \quad \quad \wedge \text{UNCHANGED } \langle cViewID, cSeqNum, cReads \rangle \\
& \quad \wedge Discard(m) \\
& \quad \wedge \text{UNCHANGED } \langle globalVars, replicaVars, cTime, cSeqNum, cResps, cWrites \rangle
\end{aligned}$$

Server request/response handling

$$\begin{aligned}
& Repair(r, c, m) \triangleq \\
& \quad \wedge Replies(m, \{[src \mapsto r, \\
& \quad \quad \quad dest \mapsto d, \\
& \quad \quad \quad type \mapsto MRepairRequest, \\
& \quad \quad \quad viewID \mapsto rViewID[r], \\
& \quad \quad \quad client \mapsto c, \\
& \quad \quad \quad seqNum \mapsto rSeqNum[r][c] + 1] : d \in Replicas\}) \\
& Abort(r, c, m) \triangleq \\
& \quad \wedge IsPrimary(r) \\
& \quad \wedge rStatus[r] = SNormal \\
& \quad \wedge rStatus' = [rStatus \text{ EXCEPT } ![r] = SAborting] \\
& \quad \wedge rAbortResps' = [rAbortResps \text{ EXCEPT } ![r] = [rAbortResps[r] \text{ EXCEPT } ![c] = \{\}]]
\end{aligned}$$

$$\begin{aligned}
& \wedge rAbortSeqNum' = [rAbortSeqNum \text{ EXCEPT } ![r] = [rAbortSeqNum[r] \text{ EXCEPT } ![c] = m.seqNum]] \\
& \wedge Replies(m, \{[src \mapsto r, \\
& \quad \quad \quad dest \mapsto d, \\
& \quad \quad \quad type \mapsto MAbortRequest, \\
& \quad \quad \quad viewID \mapsto rViewID[r], \\
& \quad \quad \quad client \mapsto c, \\
& \quad \quad \quad seqNum \mapsto m.seqNum] : d \in Replicas\})
\end{aligned}$$

$$\begin{aligned}
& HandleWriteRequest(r, c, m) \triangleq \\
& \wedge rStatus[r] = SNormal \\
& \wedge \vee \wedge m.viewID = rViewID[r] \\
& \quad \wedge LET \\
& \quad \quad isSequential \triangleq m.seqNum = rSeqNum[r][c] + 1 \\
& \quad \quad isLinear \triangleq \forall i \in \text{DOMAIN } rLog[r] : \forall e \in rLog[r][i] : m.timestamp > e.timestamp \\
& \quad IN \\
& \quad \vee \wedge isSequential \\
& \quad \quad \wedge isLinear \\
& \quad \quad \wedge rLog' = [rLog \text{ EXCEPT } ![r] = [\\
& \quad \quad \quad \quad \quad rLog[r] \text{ EXCEPT } ![c] = \\
& \quad \quad \quad \quad \quad \quad Append(rLog[r][c], [type \mapsto TValue, \\
& \quad \quad \quad \quad \quad \quad \quad \quad value \mapsto m.value, \\
& \quad \quad \quad \quad \quad \quad \quad \quad timestamp \mapsto m.timestamp])]] \\
& \quad \quad \wedge rSeqNum' = [rSeqNum \text{ EXCEPT } ![r] = [rSeqNum[r] \text{ EXCEPT } ![c] = m.seqNum]] \\
& \quad \quad \wedge Reply(m, [src \mapsto r, \\
& \quad \quad \quad \quad \quad dest \mapsto c, \\
& \quad \quad \quad \quad \quad type \mapsto MWriteResponse, \\
& \quad \quad \quad \quad \quad seqNum \mapsto m.seqNum, \\
& \quad \quad \quad \quad \quad viewID \mapsto rViewID[r], \\
& \quad \quad \quad \quad \quad succeeded \mapsto TRUE]) \\
& \quad \vee \wedge \vee \neg isSequential \\
& \quad \quad \vee \neg isLinear \\
& \quad \quad \wedge \vee \wedge IsPrimary(r) \\
& \quad \quad \quad \wedge Abort(r, c, m) \\
& \quad \quad \vee \wedge \neg IsPrimary(r) \\
& \quad \quad \quad \wedge Repair(r, c, m) \\
& \quad \quad \wedge UNCHANGED \langle rLog \rangle \\
& \quad \vee \wedge m.viewID < rViewID[r] \\
& \quad \quad \wedge Reply(m, [src \mapsto r, \\
& \quad \quad \quad \quad \quad dest \mapsto c, \\
& \quad \quad \quad \quad \quad type \mapsto MWriteResponse, \\
& \quad \quad \quad \quad \quad seqNum \mapsto m.seqNum, \\
& \quad \quad \quad \quad \quad viewID \mapsto rViewID[r], \\
& \quad \quad \quad \quad \quad succeeded \mapsto FALSE]) \\
& \quad \quad \wedge UNCHANGED \langle rLog \rangle \\
& \wedge UNCHANGED \langle globalVars, clientVars, rStatus, rViewID, rLastView, rViewChanges \rangle
\end{aligned}$$

$$\begin{aligned}
& \text{HandleReadRequest}(r, c, m) \triangleq \\
& \quad \wedge rStatus[r] = SNormal \\
& \quad \wedge Len(rLog[r]) > 0 \\
& \quad \wedge \text{Reply}(m, [src \mapsto r, \\
& \quad \quad \quad dest \mapsto c, \\
& \quad \quad \quad type \mapsto MReadResponse, \\
& \quad \quad \quad viewID \mapsto rViewID[r], \\
& \quad \quad \quad primary \mapsto IsPrimary(r), \\
& \quad \quad \quad index \mapsto Len(rLog[r]), \\
& \quad \quad \quad checksum \mapsto rLog[r][Len(rLog[r])].checksum, \\
& \quad \quad \quad succeeded \mapsto TRUE]) \\
& \quad \wedge \text{UNCHANGED } \langle globalVars, clientVars, rStatus, rLog, rViewID, rLastView, rViewChanges \rangle
\end{aligned}$$

$$\begin{aligned}
& \text{HandleRepairRequest}(r, s, m) \triangleq \\
& \quad \wedge m.viewID = rViewID[r] \\
& \quad \wedge IsPrimary(r) \\
& \quad \wedge rStatus[r] = SNormal \\
& \quad \wedge \vee \wedge m.seqNum \leq Len(rLog[r][m.client]) \\
& \quad \quad \wedge \text{Reply}(m, [src \mapsto r, \\
& \quad \quad \quad dest \mapsto s, \\
& \quad \quad \quad type \mapsto MRepairResponse, \\
& \quad \quad \quad viewID \mapsto rViewID[r], \\
& \quad \quad \quad client \mapsto m.client, \\
& \quad \quad \quad seqNum \mapsto m.seqNum]) \\
& \quad \quad \wedge \text{UNCHANGED } \langle rStatus, rAbortResps, rAbortSeqNum \rangle \\
& \quad \vee \wedge m.seqNum = Len(rLog[r][m.client]) + 1 \\
& \quad \quad \wedge \text{Abort}(r, m.client, m) \\
& \quad \wedge \text{UNCHANGED } \langle globalVars, clientVars \rangle
\end{aligned}$$

$$\begin{aligned}
& \text{HandleRepairResponse}(r, s, m) \triangleq \\
& \quad \wedge \text{HandleWriteRequest}(r, m.client, [m \text{ EXCEPT } !.src = m.client])
\end{aligned}$$

$$\begin{aligned}
& \text{HandleAbortRequest}(r, s, m) \triangleq \\
& \quad \wedge m.viewID = rViewID[r] \\
& \quad \wedge m.seqNum \leq Len(rLog[r][m.client]) + 1 \\
& \quad \wedge rStatus[r] \in \{SNormal, SAborting\} \\
& \quad \wedge rLog' = [rLog \text{ EXCEPT } ![r] = [rLog[r] \text{ EXCEPT } ![m.client] = \text{Replace}(rLog[r][m.client], m.seqNum, [type \\
& \quad \wedge \vee \wedge m.seqNum > rSeqNum[r][m.client] \\
& \quad \quad \wedge rSeqNum' = [rSeqNum \text{ EXCEPT } ![r] = [rSeqNum[r] \text{ EXCEPT } ![m.client] = m.seqNum]] \\
& \quad \vee \wedge m.seqNum \leq rSeqNum[r][m.client] \\
& \quad \quad \wedge \text{UNCHANGED } \langle rSeqNum \rangle \\
& \quad \wedge \text{Replies}(m, \{[src \mapsto r, \\
& \quad \quad \quad dest \mapsto Primary(rViewID[r]), \\
& \quad \quad \quad type \mapsto MAbortResponse, \\
& \quad \quad \quad viewID \mapsto rViewID[r], \\
& \quad \quad \quad seqNum \mapsto m.seqNum],
\end{aligned}$$

$$\begin{aligned}
& \begin{aligned}
& [src \quad \mapsto r, \\
& dest \quad \mapsto Primary(rViewID[r]), \\
& type \quad \mapsto MWriteResponse, \\
& viewID \quad \mapsto rViewID[r], \\
& seqNum \quad \mapsto m.seqNum, \\
& succeeded \mapsto FALSE]] \\
& \wedge \text{UNCHANGED } \langle globalVars, clientVars, rStatus, rViewID, rLastView, rViewChanges \rangle
\end{aligned} \\
\text{HandleAbortResponse}(r, s, m) & \triangleq \\
& \begin{aligned}
& \wedge rStatus[r] = SAborting \\
& \wedge m.viewID = rViewID[r] \\
& \wedge IsPrimary(r) \\
& \wedge m.seqNum = rAbortSeqNum[r][m.client] \\
& \wedge rAbortResps' = [rAbortResps \text{ EXCEPT } ![r] = [rAbortResps[r] \text{ EXCEPT } ![m.client] = rAbortResps[r][m.client] \\
& \wedge \text{LET } resps \triangleq \{res.src : res \in \{resp \in rAbortResps'[r][m.client] : \\
& \quad \wedge resp.viewID = rViewID[r] \\
& \quad \wedge resp.seqNum = rAbortSeqNum[r][m.client]\}] \\
& isQuorum \triangleq r \in resps \wedge resps \in Quorums \\
& \text{IN} \\
& \quad \vee \wedge isQuorum \\
& \quad \wedge rStatus' = [rStatus \text{ EXCEPT } ![r] = [rStatus[r] \text{ EXCEPT } ![m.client] = SNormal]] \\
& \quad \vee \wedge \neg isQuorum \\
& \quad \wedge \text{UNCHANGED } \langle rStatus \rangle \\
& \wedge \text{UNCHANGED } \langle globalVars, clientVars \rangle
\end{aligned} \\
\text{ChangeView}(r) & \triangleq \\
& \begin{aligned}
& \wedge Sends(\{[src \quad \mapsto r, \\
& \quad dest \quad \mapsto d, \\
& \quad type \quad \mapsto MViewChangeRequest, \\
& \quad viewID \mapsto rViewID[r] + 1] : d \in Replicas\}) \\
& \wedge \text{UNCHANGED } \langle globalVars, clientVars, replicaVars \rangle
\end{aligned} \\
\text{HandleViewChangeRequest}(r, s, m) & \triangleq \\
& \begin{aligned}
& \wedge rViewID[r] < m.viewID \\
& \wedge rViewID' = [rViewID \text{ EXCEPT } ![r] = m.viewID] \\
& \wedge rStatus' = [rStatus \text{ EXCEPT } ![r] = SViewChange] \\
& \wedge rViewChanges' = [rViewChanges \text{ EXCEPT } ![r] = \{\}] \\
& \wedge \text{Reply}(m, [src \quad \mapsto r, \\
& \quad dest \quad \mapsto Primary(m.viewID), \\
& \quad type \quad \mapsto MViewChangeResponse, \\
& \quad viewID \mapsto m.viewID, \\
& \quad lastViewID \mapsto rLastView[r], \\
& \quad logs \quad \mapsto rLog[r]]) \\
& \wedge \text{UNCHANGED } \langle globalVars, clientVars, rLog, rSeqNum, rAbortSeqNum, rAbortResps, rLastView \rangle
\end{aligned} \\
\text{HandleViewChangeResponse}(r, s, m) & \triangleq
\end{aligned}$$

$$\begin{aligned}
InitClientVars &\triangleq \\
&\wedge cTime = 0 \\
&\wedge cViewID = [c \in Clients \mapsto 1] \\
&\wedge cSeqNum = [c \in Clients \mapsto 0] \\
&\wedge cResps = [c \in Clients \mapsto [r \in Replicas \mapsto [s \in \{\} \mapsto [index \mapsto 0, checksum \mapsto Nil]]]] \\
&\wedge cWrites = [c \in Clients \mapsto \{\}] \\
&\wedge cReads = [c \in Clients \mapsto \{\}] \\
InitReplicaVars &\triangleq \\
&\wedge replicas = SeqFromSet(Replicas) \\
&\wedge rStatus = [r \in Replicas \mapsto SNormal] \\
&\wedge rLog = [r \in Replicas \mapsto [c \in Clients \mapsto \langle \rangle]] \\
&\wedge rSeqNum = [r \in Replicas \mapsto [c \in Clients \mapsto 0]] \\
&\wedge rAbortSeqNum = [r \in Replicas \mapsto [c \in Clients \mapsto 0]] \\
&\wedge rAbortResps = [r \in Replicas \mapsto [c \in Clients \mapsto \{\}]] \\
&\wedge rViewID = [r \in Replicas \mapsto 1] \\
&\wedge rLastView = [r \in Replicas \mapsto 1] \\
&\wedge rViewChanges = [r \in Replicas \mapsto \{\}] \\
Init &\triangleq \\
&\wedge InitMessageVars \\
&\wedge InitClientVars \\
&\wedge InitReplicaVars \\
&\wedge transitions = 0
\end{aligned}$$

The type invariant checks that no read ever reads a different value than a previous write

$$Inv \triangleq \text{TRUE } \text{TODO}$$

$$Transition \triangleq transitions' = transitions + 1$$

$$\begin{aligned}
Next &\triangleq \\
&\vee \exists c \in Clients : \\
&\quad \wedge Write(c) \\
&\quad \wedge Transition \\
&\vee \exists c \in Clients : \\
&\quad \wedge Read(c) \\
&\quad \wedge Transition \\
&\vee \exists r \in Replicas : \\
&\quad \wedge ChangeView(r) \\
&\quad \wedge Transition \\
&\vee \exists m \in messages : \\
&\quad \wedge m.type = MWriteRequest \\
&\quad \wedge HandleWriteRequest(m.dest, m.src, m) \\
&\quad \wedge Transition \\
&\vee \exists m \in messages :
\end{aligned}$$

$$\begin{aligned}
& \wedge m.type = MWriteResponse \\
& \wedge HandleWriteResponse(m.dest, m.src, m) \\
& \wedge Transition \\
\vee \exists m \in messages : & \\
& \wedge m.type = MReadRequest \\
& \wedge HandleReadRequest(m.dest, m.src, m) \\
& \wedge Transition \\
\vee \exists m \in messages : & \\
& \wedge m.type = MReadResponse \\
& \wedge HandleReadResponse(m.dest, m.src, m) \\
& \wedge Transition \\
\vee \exists m \in messages : & \\
& \wedge m.type = MRepairRequest \\
& \wedge HandleRepairRequest(m.dest, m.src, m) \\
& \wedge Transition \\
\vee \exists m \in messages : & \\
& \wedge m.type = MRepairResponse \\
& \wedge HandleRepairResponse(m.dest, m.src, m) \\
& \wedge Transition \\
\vee \exists m \in messages : & \\
& \wedge m.type = MAbortRequest \\
& \wedge HandleAbortRequest(m.dest, m.src, m) \\
& \wedge Transition \\
\vee \exists m \in messages : & \\
& \wedge m.type = MAbortResponse \\
& \wedge HandleAbortResponse(m.dest, m.src, m) \\
& \wedge Transition \\
\vee \exists m \in messages : & \\
& \wedge m.type = MViewChangeRequest \\
& \wedge HandleViewChangeRequest(m.dest, m.src, m) \\
& \wedge Transition \\
\vee \exists m \in messages : & \\
& \wedge m.type = MViewChangeResponse \\
& \wedge HandleViewChangeResponse(m.dest, m.src, m) \\
& \wedge Transition \\
\vee \exists m \in messages : & \\
& \wedge m.type = MStartViewRequest \\
& \wedge HandleStartViewRequest(m.dest, m.src, m) \\
& \wedge Transition
\end{aligned}$$

$$Spec \triangleq Init \wedge \Box[Next]_{vars}$$

\ * Modification History
\ * Last modified Tue Sep 22 04:25:22 PDT 2020 by jordanhalterman
\ * Created Fri Sep 18 22:45:21 PDT 2020 by jordanhalterman