Replica roles

CONSTANTS

SNormal,

SAborting,

SViewChange

MStartViewRequest

Entry types

CONSTANTS

TValue,

TNoOp

VARIABLE replicas

 $globalVars \stackrel{\triangle}{=} \langle replicas \rangle$ 

VARIABLE messages

```
messageVars \triangleq \langle messages \rangle
Variable cTime
VARIABLE cViewID
Variable cSeqNum
Variable cResps
VARIABLE cWrites
VARIABLE cReads
clientVars \triangleq \langle cTime, cViewID, cSegNum, cResps, cWrites, cReads \rangle
Variable rStatus
VARIABLE rLoq
VARIABLE rViewID
VARIABLE rSegNum
Variable rLastView
Variable rViewChanges
{\tt VARIABLE}\ rAbortSeqNum
VARIABLE rAbortResps
replicaVars \stackrel{\triangle}{=} \langle rStatus, rLog, rViewID, rSeqNum, rLastView, rViewChanges, rAbortSeqNum, rAbortResps \rangle
Variable transitions
vars \triangleq \langle global Vars, message Vars, client Vars, replica Vars, transitions \rangle
 Helpers
RECURSIVE SeqFromSet(_)
SeqFromSet(S) \triangleq
  If S = \{\} Then \langle \rangle
   ELSE LET x \stackrel{\triangle}{=} \text{CHOOSE } x \in S : \text{TRUE}
          IN \langle x \rangle \circ SeqFromSet(S \setminus \{x\})
Max(s) \stackrel{\triangle}{=} \text{ CHOOSE } x \in s : \forall y \in s : x \geq y
```

 $IsQuorum(s) \triangleq Cardinality(s) * 2 \geq Cardinality(Replicas)$ 

 $Quorums \triangleq \{r \in SUBSET \ Replicas : IsQuorum(r)\}$ 

```
Primary(v) \stackrel{\triangle}{=} replicas[(v\%Len(replicas)) + (\text{IF } v \geq Len(replicas) \text{ THEN } 1 \text{ ELSE } 0)]
IsPrimary(r) \stackrel{\triangle}{=} Primary(rViewID[r]) = r
Replace(l, i, x) \triangleq [j \in 1 ... Max(\{Len(l), i\}) \mapsto \text{if } j = i \text{ Then } x \text{ else } l[j]]
 Messaging helpers
Sends(ms) \stackrel{\triangle}{=} messages' = messages \cup ms
Send(m) \triangleq Sends(\{m\})
Replies(req, resps) \triangleq messages' = (messages \cup resps) \setminus \{req\}
Reply(req, resp) \stackrel{\Delta}{=} Replies(req, \{resp\})
Discard(m) \stackrel{\triangle}{=} messages' = messages \setminus \{m\}
Write(c) \triangleq
     \wedge cTime' = cTime + 1
     \land cSeqNum' = [cSeqNum \ EXCEPT \ ![c] = cSeqNum[c] + 1]
     \land Sends(\{[src
                                \mapsto c,
                   dest
                                \mapsto r,
                                \mapsto MWriteRequest,
                   type
                   viewID
                                \mapsto c ViewID[c],
                   seqNum \mapsto cSeqNum'[c],
                   timestamp \mapsto cTime' | : r \in Replicas \}
     ∧ UNCHANGED ⟨globalVars, replicaVars, cViewID, cResps, cWrites, cReads⟩
Read(c) \triangleq
       \land Sends(\{[src
                                  \mapsto c,
                                  \mapsto r,
                                  \mapsto MReadRequest,
                    viewID \mapsto cViewID[c]]: r \in Replicas\}
       \land UNCHANGED \langle globalVars, replicaVars, cTime, cSeqNum, cResps, cWrites, cReads <math>\rangle
HandleWriteResponse(c, r, m) \triangleq
     \land \lor \land m.viewID = cViewID[c]
           \land IF m.seqNum \notin DOMAIN <math>cResps[c][r] THEN
                  cResps' = [cResps \ EXCEPT \ ![c] = [cResps[c] \ EXCEPT \ ![r] = cResps[c][r] @@ (m.seqNum:> m)]]
                  cResps' = [cResps \ EXCEPT \ ![c] = [cResps[c] \ EXCEPT \ ![r] = [cResps[c][r] \ EXCEPT \ ![m.seqNum]]
           \wedge LET
                                     \stackrel{\triangle}{=} \{cResps[c][r][r1] : r1 \in \{r2 \in Replicas : r2 \in DOMAIN \ cResps[c][r]\}\}
                   isCommitted \triangleq \{r1.src : r1 \in \{r2 \in allResps : r2.succeeded\}\} \in Quorums
```

```
\land \ \lor \ \land \ isCommitted
                         \land cWrites' = [cWrites \ EXCEPT \ ![c] = cWrites[c] \cup \{m\}]
                      \lor \land \neg isCommitted
                         \land UNCHANGED \langle cWrites \rangle
                   \land UNCHANGED \langle cViewID, cSeqNum \rangle
        \lor \land m.viewID > cViewID[c]
           \land cViewID' = [cViewID \text{ EXCEPT } ! [c] = m.viewID]
           \wedge cSeqNum' = [cSeqNum \ EXCEPT \ ![c] = 0]
           \land cResps' = [cResps \ EXCEPT \ ![c] = [i \in Replicas \mapsto \{\}]]
           \land UNCHANGED \langle cWrites \rangle
        \lor \land m.viewID < cViewID[c]
           \land UNCHANGED \langle cWrites \rangle
     \wedge Discard(m)
     \land UNCHANGED \langle globalVars, replicaVars, cTime, cSeqNum, cReads <math>\rangle
HandleReadResponse(c, r, m) \stackrel{\Delta}{=}
     \land \lor \land m.viewID = cViewID[c]
           \land cReads' = [cReads \ \texttt{EXCEPT} \ ![c] = cReads[c] \cup \{m\}]
           \land UNCHANGED \langle cViewID, cSeqNum \rangle
        \lor \land m.viewID > cViewID[c]
           \land cViewID' = [cViewID \text{ EXCEPT } ! [c] = m.viewID]
           \wedge cSeqNum' = [cSeqNum \ EXCEPT \ ![c] = 0]
           \land UNCHANGED \langle cReads \rangle
        \vee \wedge m.viewID < cViewID[c]
           \land UNCHANGED \langle cViewID, cSeqNum, cReads \rangle
     \wedge Discard(m)
     \land UNCHANGED \langle globalVars, replicaVars, cTime, cSeqNum, cResps, cWrites <math>\rangle
 Server request/response handling
Repair(r, c, m) \triangleq
     \land Replies(m, \{[src
                                  \mapsto d,
                        dest
                        type
                                  \mapsto MRepairRequest,
                        viewID \mapsto rViewID[r],
                        seqNum \mapsto rSeqNum[r][c] + 1] : d \in Replicas\}
Abort(r, c, m) \triangleq
     \wedge IsPrimary(r)
     \wedge rStatus[r]
                           = SNormal
     \land rStatus'
                           = [rStatus]
                                               EXCEPT ![r] = SAborting]
     \land rAbortResps'
                           = [rAbortResps \ EXCEPT \ ![r] \ = [rAbortResps[r] \ EXCEPT \ ![c] = \{\}]]
```

```
 \land rAbortSeqNum' = [rAbortSeqNum \ \ \texttt{EXCEPT} \ ![r] = [rAbortSeqNum[r] \ \ \texttt{EXCEPT} \ ![c] = m.seqNum]] 
     \land Replies(m, \{[src
                                   \mapsto r,
                                   \mapsto d.
                        dest
                                  \mapsto MAbortRequest,
                        type
                        viewID \mapsto rViewID[r],
                        client \mapsto c,
                        seqNum \mapsto m.seqNum]: d \in Replicas})
HandleWriteRequest(r, c, m) \stackrel{\Delta}{=}
     \land rStatus[r] = SNormal
    \land \lor \land m.viewID = rViewID[r]
           \wedge LET
                  isSequential \stackrel{\triangle}{=} m.seqNum = rSeqNum[r][c] + 1
                                \stackrel{\triangle}{=} \forall i \in \text{DOMAIN } rLog[r] : \forall e \in rLog[r][i] : m.timestamp > e.timestamp
             IN
                  \lor \land isSequential
                     \land \ is Linear
                     \wedge rLog' = [rLog]
                                             EXCEPT ![r] = [
                                   rLog[r] EXCEPT ![c] =
                                         Append(rLog[r][c], [type]
                                                                                \mapsto TValue,
                                                                  value
                                                                                \mapsto m.value,
                                                                  timestamp \mapsto m.timestamp])]]
                     \land rSeqNum' = [rSeqNum \ EXCEPT \ ![r] = [rSeqNum[r] \ EXCEPT \ ![c] = m.seqNum]]
                     \land Reply(m, [src
                                                  \mapsto r,
                                     dest
                                                  \mapsto c,
                                                  \mapsto MWriteResponse,
                                     type
                                     seqNum \mapsto m.seqNum,
                                     viewID \mapsto rViewID[r],
                                     succeeded \mapsto TRUE
                  \lor \land \lor \neg isSequential
                        \vee \neg isLinear
                     \wedge \vee \wedge IsPrimary(r)
                           \wedge Abort(r, c, m)
                        \lor \land \neg IsPrimary(r)
                           \land Reply(m, [src]
                                                        \mapsto r,
                                            dest
                                                        \mapsto c,
                                                        \mapsto MWriteResponse,
                                            type
                                            segNum \mapsto m.segNum,
                                            viewID
                                                        \mapsto rViewID[r],
                                            succeeded \mapsto FALSE)
                     \land UNCHANGED \langle rLog \rangle
        \lor \land m.viewID < rViewID[r]
           \land Reply(m, [src
                                       \mapsto r,
                           dest
                                       \mapsto c,
                                       \mapsto MWriteResponse,
                           type
```

```
segNum \mapsto m.segNum,
                           viewID
                                      \mapsto rViewID[r],
                           succeeded \mapsto FALSE
           \land UNCHANGED \langle rLog \rangle
     \land UNCHANGED \langle globalVars, clientVars, rStatus, rViewID, rLastView, rViewChanges <math>\rangle
HandleReadRequest(r, c, m) \stackrel{\Delta}{=}
     \wedge rStatus[r] = SNormal
     \wedge Len(rLog[r]) > 0
     \land Reply(m, [src
                                 \mapsto r,
                     dest
                                 \mapsto c,
                                 \mapsto MReadResponse,
                     type
                                 \mapsto rViewID[r],
                     viewID
                     primary \mapsto IsPrimary(r),
                     index
                                 \mapsto Len(rLog[r]),
                     checksum \mapsto rLog[r][Len(rLog[r])].checksum,
                     succeeded \mapsto TRUE
     \land UNCHANGED \langle globalVars, clientVars, rStatus, rLog, rViewID, rLastView, rViewChanges <math>\rangle
HandleRepairRequest(r, s, m) \stackrel{\Delta}{=}
     \land m.viewID = rViewID[r]
     \wedge IsPrimary(r)
     \land rStatus[r] = SNormal
     \wedge LET index \stackrel{\triangle}{=} Len(rLog[r][m.client]) + 1 - (rSeqNum[r] - m.seqNum)
            \land \lor \land index \leq Len(rLog[r][m.client])
                  \land Reply(m, [src])
                                  dest
                                            \mapsto s,
                                            \mapsto MRepairResponse,
                                  type
                                  viewID \mapsto rViewID[r],
                                  client \mapsto m.client,
                                  seqNum \mapsto m.seqNum)
                  \land UNCHANGED \langle rStatus, rAbortResps, rAbortSeqNum \rangle
               \lor \land index = Len(rLog[r][m.client]) + 1
                  \wedge Abort(r, m.client, m)
     \land UNCHANGED \langle globalVars, clientVars \rangle
HandleRepairResponse(r, s, m) \triangleq
     \land Handle Write Request (r, m.client, [m \text{ EXCEPT } !.src = m.client])
HandleAbortRequest(r, s, m) \stackrel{\Delta}{=}
     \land m.viewID = rViewID[r]
     \land \mathit{rStatus}[r] \in \{\mathit{SNormal}, \mathit{SAborting}\}
     \wedge LET index \stackrel{\triangle}{=} Len(rLog[r][m.client]) + 1 - (rSeqNum[r] - m.seqNum)
            \land index \leq Len(rLog[r][m.client]) + 1
```

```
\land rLog' = [rLog \ EXCEPT \ ![r] = [rLog[r] \ EXCEPT \ ![m.client] = Replace(rLog[r][m.client], index, [typ])
                          \land \lor \land m.seqNum > rSeqNum[r][m.client]
                                        \land rSeqNum' = [rSeqNum \ EXCEPT \ ![r] = [rSeqNum[r] \ EXCEPT \ ![m.client] = m.seqNum]]
                                 \lor \land m.seqNum \le rSeqNum[r][m.client]
                                        \land UNCHANGED \langle rSeqNum \rangle
                          \land Replies(m, \{[src
                                                                                               \mapsto r,
                                                                                               \mapsto Primary(rViewID[r]),
                                                                                               \mapsto MAbortResponse,
                                                                    type
                                                                    viewID
                                                                                               \mapsto rViewID[r],
                                                                    segNum
                                                                                             \mapsto m.seqNum],
                                                                   [src]
                                                                                               \mapsto r,
                                                                    dest
                                                                                               \mapsto Primary(rViewID[r]),
                                                                                               \mapsto MWriteResponse,
                                                                    type
                                                                                             \mapsto rViewID[r],
                                                                    viewID
                                                                    seqNum \mapsto m.seqNum,
                                                                    succeeded \mapsto FALSE[\})
           \land \  \, \text{UNCHANGED} \ \langle \textit{globalVars}, \ \textit{clientVars}, \ \textit{rStatus}, \ \textit{rViewID}, \ \textit{rLastView}, \ \textit{rViewChanges} \rangle
HandleAbortResponse(r, s, m) \stackrel{\Delta}{=}
           \wedge rStatus[r] = SAborting
           \land m.viewID = rViewID[r]
           \wedge IsPrimary(r)
           \land m.seqNum = rAbortSeqNum[r][m.client]
           \land rAbortResps' = [rAbortResps \ EXCEPT \ ![r] = [rAbortResps[r] \ EXCEPT \ ![m.client] = rAbortResps[r][m.client]
           \land LET resps \stackrel{\triangle}{=} \{res.src : res \in \{resp \in rAbortResps'[r][m.client] : resps \stackrel{\triangle}{=} \{res.src : resps \in rAbortResps'[r][m.client] : resps \mapsto rabortResps'[r][m.client] : resps'[r][m.client] : resps'[r][m.client] : 
                                                                                          \land resp.viewID = rViewID[r]
                                                                                          \land resp.seqNum = rAbortSeqNum[r][m.client]\}\}
                              isQuorum \stackrel{\triangle}{=} r \in resps \land resps \in Quorums
                IN
                          \lor \land isQuorum
                                 \land rStatus' = [rStatus \ EXCEPT \ ![r] = [rStatus[r] \ EXCEPT \ ![m.client] = SNormal]]
                          \lor \land \neg isQuorum
                                 \land UNCHANGED \langle rStatus \rangle
           \land UNCHANGED \langle globalVars, clientVars \rangle
ChangeView(r) \triangleq
           \land Sends(\{[src
                                                             \mapsto r,
                                                            \mapsto MViewChangeRequest,
                                        viewID \mapsto rViewID[r] + 1] : d \in Replicas \})
           \land UNCHANGED \langle globalVars, clientVars, replicaVars \rangle
Handle View Change Request(r, s, m) \stackrel{\Delta}{=}
           \land rViewID[r] < m.viewID
                                                           = [rViewID \ EXCEPT \ ![r] = m.viewID]
           \land rViewID'
                                                           = [rStatus \ EXCEPT \ ![r] = SViewChange]
           \wedge rStatus'
```

```
\land rViewChanges' = [rViewChanges \ EXCEPT \ ![r] = \{\}]
     \land Reply(m, [src])
                                      \mapsto r,
                      dest
                                      \mapsto Primary(m.viewID),
                                      \mapsto MViewChangeResponse,
                      type
                      viewID
                                      \mapsto m.viewID,
                      lastViewID \mapsto rLastView[r],
                                      \mapsto rLog[r])
     \land UNCHANGED \langle globalVars, clientVars, rLoq, rSeqNum, rAbortSeqNum, rAbortResps, rLastView <math>\rangle
Handle View Change Response(r, s, m) \stackrel{\triangle}{=}
     \wedge IsPrimary(r)
     \land rViewID[r]
                             = m.viewID
                             = SViewChange
     \wedge rStatus[r]
     \land rViewChanges' = [rViewChanges \ EXCEPT \ ![r] = rViewChanges[r] \cup \{m\}]
                                     \stackrel{\triangle}{=} \{ v \in \mathit{rViewChanges'}[r][\mathit{m.client}] : \land v.\mathit{viewID} = \mathit{rViewID}[r] \}
     \land LET viewChanges

\stackrel{\triangle}{=} \{v.src : v \in viewChanges\} 

\stackrel{\triangle}{=} r \in viewSources \land viewSources \in Quorums

              view Sources \\
              is Quorum
                                     \triangleq \{v.lastViewID : v \in viewChanges\}
              last \, Views
              lastView
                                        (CHOOSE v1 \in lastViews : \forall v2 \in lastViews : v2 \leq v1)
                                     \triangleq \begin{bmatrix} c \in \mathit{Clients} \mapsto \{v1.logs[c] : v1 \in \{v2 \in \mathit{viewChanges} : v2.last\mathit{View} = last\mathit{View} \} \end{bmatrix}
              viewLogs
              mergeEnts(es)
                   If es = \{\} \lor \exists e \in es : r.type = TNoOp \text{ Then}
                        [type \mapsto TNoOp]
                    ELSE
                        CHOOSE e \in es : e.type \neq TNoOp
                                  \stackrel{\Delta}{=} Max(\{Len(l): l \in ls\})
              range(ls)
              entries(ls, i) \triangleq \{l[i] : l \in \{k \in ls : i \leq Len(k)\}\}
              mergeLogs(ls) \stackrel{\triangle}{=} [i \in 1 .. range(ls) \mapsto mergeEnts(entries(ls, i))]
        ΙN
              \lor \land isQuorum
                 \land Replies(m, \{[src
                                                \mapsto r,
                                                \mapsto d,
                                               \mapsto MStartViewRequest,
                                      type
                                      viewID \mapsto rViewID[r],
                                                \mapsto [c \in Clients \mapsto mergeLogs(viewLogs[c])] : d \in Replicas\})
              \lor \land \neg isQuorum
                 \wedge Discard(m)
     \land UNCHANGED \langle globalVars, clientVars, rStatus, rViewID, rLog, rSeqNum, rAbortSeqNum, rAbortResps, r
HandleStartViewRequest(r, s, m) \stackrel{\Delta}{=}
     \land \lor rViewID[r] < m.viewID
         \lor \land rViewID[r] = m.viewID
            \land rStatus[r] = SViewChange
```

EXCEPT ![r] = m.log]

 $= [rStatus \quad EXCEPT \ ![r] \quad = SNormal]$ 

 $\wedge rLog'$ 

 $\wedge rStatus'$ 

= [rLog

```
\wedge Discard(m)
     \land UNCHANGED \langle globalVars, clientVars, rViewChanges \rangle
InitMessageVars \stackrel{\triangle}{=}
     \land messages = \{\}
InitClientVars \stackrel{\triangle}{=}
     \wedge c Time
                          = 0
     \land cViewID
                          = [c \in Clients \mapsto 1]
     \wedge cSeqNum
                          = [c \in Clients \mapsto 0]
     \land cResps = [c \in Clients \mapsto [r \in Replicas \mapsto [s \in \{\} \mapsto [index \mapsto 0, checksum \mapsto Nil]]]]
     \land cWrites
                        = [c \in Clients \mapsto \{\}]
     \land\ cReads
                        = [c \in Clients \mapsto \{\}]
InitReplicaVars \triangleq
     \land replicas
                              = SeqFromSet(Replicas)
     \land \mathit{rStatus}
                              = [r \in Replicas \mapsto SNormal]
     \wedge rLog
                              = [r \in Replicas \mapsto [c \in Clients \mapsto \langle \rangle]]
     \wedge rSeqNum
                              = [r \in Replicas \mapsto [c \in Clients \mapsto 0]]
     \land rAbortSeqNum = [r \in Replicas \mapsto [c \in Clients \mapsto 0]]
                             = [r \in Replicas \mapsto [c \in Clients \mapsto \{\}]]
     \wedge rAbortResps
     \land rViewID
                              = [r \in Replicas \mapsto 1]
                              = [r \in Replicas \mapsto 1]
     \land \mathit{rLastView}
     \land rViewChanges = [r \in Replicas \mapsto \{\}]
Init \triangleq
     \land \ InitMessageVars
     \land \ InitClientVars
     \land InitReplica Vars
     \wedge transitions = 0
 The type invariant checks that no read ever reads a different value than a previous write
Inv \stackrel{\triangle}{=} TRUE TODO
Transition \stackrel{\triangle}{=} transitions' = transitions + 1
Next \triangleq
     \vee \exists c \in Clients:
           \wedge Write(c)
           \land Transition
```

 $\land rViewID' = [rViewID \quad \text{EXCEPT } ![r] = m.viewID]$  $\land rLastView' = [rLastView \quad \text{EXCEPT } ![r] = m.viewID]$ 

 $\vee \exists c \in Clients:$ 

```
\wedge Read(c)
     \land \ \mathit{Transition}
\vee \exists r \in Replicas :
     \land Change View(r)
     \land Transition
\vee \exists m \in messages :
     \land m.type = MWriteRequest
     \land Handle WriteRequest(m.dest, m.src, m)
     \land Transition
\vee \exists m \in messages :
     \land m.type = MWriteResponse
     \land Handle WriteResponse (m.dest, m.src, m)
     \land Transition
\vee \exists m \in messages :
     \land m.type = MReadRequest
     \land HandleReadRequest(m.dest, m.src, m)
     \land Transition
\vee \exists m \in messages :
     \land \ m.type = MReadResponse
     \land HandleReadResponse(m.dest, m.src, m)
     \land \ Transition
\vee \exists m \in messages :
     \land \ m.type = MRepairRequest
     \land HandleRepairRequest(m.dest, m.src, m)
     \land \ Transition
\vee \exists m \in messages :
     \land m.type = MRepairResponse
     \land HandleRepairResponse(m.dest, m.src, m)
     \land Transition
\vee \exists m \in messages :
     \land m.type = MAbortRequest
     \land HandleAbortRequest(m.dest, m.src, m)
     \land Transition
\vee \exists m \in messages :
     \land m.type = MAbortResponse
     \land HandleAbortResponse(m.dest, m.src, m)
     \land \ Transition
\vee \exists m \in messages :
     \land m.type = MViewChangeRequest
     \land Handle View Change Request (m.dest, m.src, m)
     \land Transition
\vee \exists m \in messages :
     \land m.type = MViewChangeResponse
     \land Handle View Change Response (m.dest, m.src, m)
```

 $\land Transition$ 

```
 \begin{tabular}{ll} $\vee \; \exists \; m \in messages: \\ $\wedge \; m.type = MStartViewRequest \\ $\wedge \; HandleStartViewRequest(m.dest, \; m.src, \; m)$ \\ $\wedge \; Transition \\ \\ Spec \; \triangleq \; Init \; \wedge \; \Box [Next]_{vars} \\ \end{tabular}
```

- \ \* Created Fri Sep 18 22:45:21 PDT 2020 by jordanhalterman