## EXTENDS Naturals, Sequences, FiniteSets, TLC

The set of Paxos replicas CONSTANT Replicas

The set of *Paxos* clients CONSTANT *Clients* 

The maximum clock interval CONSTANT MaxClockInterval

An empty value CONSTANT Nil

The set of values to write CONSTANT Values

Client request/response types+ CONSTANTS ClientRequest,

 ${\it ClientResponse}$ 

Server request/response types

CONSTANTS

SlotLookup SlotLookupRep GapCommitGapCommitRep

ViewChangeRequest, ViewChangeReq
ViewChangeResponse, ViewChange
StartViewRequest, StartView
SyncPrepareRequest, SyncPrepare
SyncCommitRequest SyncCommit

Replica roles
CONSTANTS
Normal,
ViewChanging,
Recovering

Variable replicas

Variable messages

```
messageVars \stackrel{\triangle}{=} \langle messages \rangle
{\tt VARIABLE}\ global Time
VARIABLE time
VARIABLE requestID
VARIABLE responses
VARIABLE commits
clientVars \triangleq \langle globalTime, time \rangle
Variable status
VARIABLE viewID
VARIABLE log
replica Vars \stackrel{\Delta}{=} \langle status, viewID, log \rangle
vars \triangleq \langle messageVars, clientVars, replicaVars \rangle
InitMessageVars \triangleq
     \land \mathit{messages} = \{\}
InitClientVars \triangleq
     \land qlobalTime = 0
     \land time = [c \in Clients \mapsto 0]
     \land requestID = [c \in Clients \mapsto 0]
     \land responses = [c \in Clients \mapsto [r \in Replicas \mapsto [s \in \{\} \mapsto [index \mapsto 0, value \mapsto Nil, checksum \mapsto Nil]]]]
     \land commits = [c \in Clients \mapsto [s \in \{\} \mapsto [index \mapsto 0, value \mapsto Nil, checksum \mapsto Nil]]]
InitReplicaVars \stackrel{\triangle}{=}
      \land replicas = Seq(Replicas)
     \land status = [r \in Replicas \mapsto Normal]
     \land viewID = [r \in Replicas \mapsto 0]
     \land log = [r \in Replicas \mapsto \langle \rangle]
Init \triangleq
     \land InitMessageVars
     \land InitClientVars
     \land InitReplica Vars
```

 $Quorums \triangleq \{r \in \text{SUBSET } Replicas : Cardinality(r) * 2 > Cardinality(Replicas)\}$ 

```
Primary(v) \stackrel{\triangle}{=} replicas[v\%Len(replicas)] + (\text{if } v \geq Len(replicas) \text{ THEN 1 ELSE } 0)
IsPrimary(r) \stackrel{\Delta}{=} Primary(viewID[r]) = r
   Messaging helpers
SendMessage(m) \triangleq messages' = messages \cup \{m\}
SendMessages(ms) \stackrel{\triangle}{=} messages' = messages \cup ms
AdvanceTime(c) \triangleq
          \land globalTime' = globalTime + 1
          \land IF time[c] < globalTime <math>\land globalTime - time[c] > MaxClockInterval THEN
                         time' = [time \ EXCEPT \ ![c] = globalTime' - MaxClockInterval]
                         time' = [time \ EXCEPT \ ![c] = time[c] + 1]
CurrentTime(c) \stackrel{\Delta}{=} time'[c]
SendClientRequest(c, v) \triangleq
          \wedge AdvanceTime(c)
          \land requestID' = [requestID \ EXCEPT \ ![c] = requestID[c] + 1]
          \land SendMessages(\{[source
                                                           target
                                                                                      \mapsto r,
                                                           type
                                                                                      \mapsto ClientRequest,
                                                           requestID \mapsto requestID'[c],
                                                           value
                                                                                      \mapsto v,
                                                           timestamp \mapsto CurrentTime(c) | : r \in Replicas \}
          \land UNCHANGED \langle \rangle
IsCommitted(acks) \triangleq
          \exists msgs \in \text{Subset } acks :
               \land \{m.source : m \in msgs\} \in Quorums
               \land \exists m1 \in msgs : \forall m2 \in msgs : m1.viewID = m2.viewID \land m1.checksum \cap m2.checksum = \{\}
               \wedge \exists m \in msgs : m.primary
HandleClientResponse(c, r, m) \stackrel{\Delta}{=}
          \land m.requestID \notin commits[c]
          \land \lor \land m.requestID \notin responses[c][r]
                         \land \ responses' = [responses \ \ \texttt{EXCEPT} \ ![c] = [responses[c] \ \ \texttt{EXCEPT} \ ![r] = responses[c][r] \ @@ \ (m.request Iallies) = responses[c][r] \ @@
                 \lor \land m.requestID \in responses[c][r]
                        Do not overwrite a response from a newer view
                        \land responses[c][r][m.requestID].viewID \leq m.viewID
```

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\land LET committed \stackrel{\triangle}{=} IsCommitted({responses'[c][x]: x \in Replicas})
                  \lor \land committed
                     \land commits' = [commits \ EXCEPT \ ![c] = commits[c] \cup \{m\}]
                  \vee \wedge \neg committed
                     \land UNCHANGED \langle commits \rangle
     \land UNCHANGED \langle \rangle
 Server request/response handling
HandleClientRequest(r, c, m) \stackrel{\Delta}{=}
     \land status[r] = Normal
     \wedge \vee \wedge \vee Len(log[r]) = 0
               \vee \wedge Len(log[r]) \neq 0
                  \land \ m.timestamp > log[r][Len(log[r])].timestamp
           \wedge log' = [log \ EXCEPT \ ![r] = Append(log[r], m)]
            \land SendMessage([source]
                                              \mapsto r,
                                 target
                                 type
                                              \mapsto ClientResponse,
                                 requestID \mapsto m.requestID,
                                 viewID
                                             \mapsto viewID[r],
                                 primary \mapsto IsPrimary(r),
                                 index
                                              \mapsto Len(log'[r]),
                                 checksum \mapsto \{log'[r][i].timestamp : i \in DOMAIN \ log'[r]\},
                                 succeeded \mapsto TRUE
        \vee \wedge Len(log[r])
                                \neq 0
           \land m.timestamp \leq log[r][Len(log[r])].timestamp
           \land SendMessage([source]
                                              \mapsto r,
                                 target
                                              \mapsto c,
                                 type
                                              \mapsto ClientResponse,
                                 requestID \mapsto m.requestID,
                                              \mapsto viewID[r],
                                 view
                                 primary \mapsto IsPrimary(r),
                                              \mapsto Len(log[r]),
                                 checksum \mapsto \{log[r][i].timestamp : i \in DOMAIN \ log[r]\},
                                 succeeded \mapsto FALSE
     \land UNCHANGED \langle \rangle
Handle View Change Request(r, s, m) \stackrel{\Delta}{=}
```

 $\land$  UNCHANGED  $\langle \rangle$ 

 $\land$  UNCHANGED  $\langle \rangle$ 

 $Handle View Change Response(r, s, m) \stackrel{\Delta}{=}$ 

 $HandleStartViewRequest(r, s, m) \triangleq$ 

```
\land UNCHANGED \langle \rangle
HandleSyncPrepareRequest(r, s, m) \stackrel{\Delta}{=}
     ∧ UNCHANGED ⟨⟩
HandleSyncPrepareResponse(r, s, m) \stackrel{\triangle}{=}
     \land UNCHANGED \langle \rangle
HandleSyncCommitRequest(r, s, m) \stackrel{\Delta}{=}
     ∧ UNCHANGED ⟨⟩
ReceiveMessage(m) \triangleq
    \lor \land m.type = ClientRequest
        \land HandleClientRequest(m.target, m.source, m)
     \lor \land m.type = ClientResponse
        \land Handle Client Response (m.target, m.source, m)
     \lor \land m.type = ViewChangeRequest
        \land Handle View Change Request (m.target, m.source, m)
     \lor \land m.type = ViewChangeResponse
        \land Handle View Change Response (m.target, m.source, m)
     \lor \land m.type = StartViewRequest
        \land HandleStartViewRequest(m.target, m.source, m)
     \lor \land m.type = SyncPrepareRequest
        \land HandleSyncPrepareRequest(m.target, m.source, m)
     \lor \land m.type = SyncPrepareResponse
        \land HandleSyncPrepareResponse(m.target, m.source, m)
    \lor \land m.type = SyncCommitRequest
        \land HandleSyncCommitRequest(m.target, m.source, m)
Next \triangleq
     \lor \exists c \in Clients:
         \exists v \in Values:
           SendClientRequest(c, v)
     \vee \exists m \in messages : ReceiveMessage(m)
Spec \stackrel{\triangle}{=} Init \wedge \Box [Next]_{vars}
\ * Modification History
\ * Last modified Sun Sep 20 16:51:08 PDT 2020 by jordanhalterman
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