
MODULE *JustInTimePaxos*

EXTENDS *Naturals, Sequences, FiniteSets, TLC*

The set of *Paxos* replicas

CONSTANT *Replicas*

The set of *Paxos* clients

CONSTANT *Clients*

The set of possible values

CONSTANT *Values*

An empty value

CONSTANT *Nil*

Request/response types

CONSTANTS

MClientRequest,
MClientResponse,
MRepairRequest,
MRepairResponse,
MAbortRequest,
MAbortResponse,
MViewChangeRequest,
MViewChangeResponse,
MStartViewRequest

Replica statuses

CONSTANTS

SNormal,
SAborting,
SViewChange

Entry types

CONSTANTS

TValue,
TNoOp

A sequence of replicas used for deterministic primary election

VARIABLE *replicas*

$globalVars \triangleq \langle replicas \rangle$

The set of all messages on the network

VARIABLE *messages*

$messageVars \triangleq \langle messages \rangle$

Local client state

Strictly increasing representation of synchronized time

VARIABLE $cTime$

The highest known view ID for a client

VARIABLE $cViewID$

The current sequence number for a client

VARIABLE $cSeqNum$

A client response buffer

VARIABLE $cResps$

A set of all *commits* – used for model checking

VARIABLE $cCommits$

$clientVars \triangleq \langle cTime, cViewID, cSeqNum, cResps, cCommits \rangle$

Local replica state

The current status of a replica

VARIABLE $rStatus$

A replica's commit *log*

VARIABLE $rLog$

The current view ID for a replica

VARIABLE $rViewID$

The current sequence number for each session

VARIABLE $rSeqNum$

The highest known timestamp for all sessions

VARIABLE $rTimestamp$

The last known normal view

VARIABLE $rLastViewID$

The set of received view change responses

VARIABLE $rViewChanges$

The point (*client* + sequence number) in the *log* currently being aborted

VARIABLE $rAbortPoint$

The set of abort responses received

VARIABLE $rAbortResps$

$replicaVars \triangleq \langle rStatus, rLog, rViewID, rSeqNum, rTimestamp, rAbortPoint, rAbortResps \rangle$

$rLastViewID, rViewChanges, rAbortPoint, rAbortResps\rangle$

A counter used to limit the state space for model checking

VARIABLE *transitions*

$vars \triangleq \langle globalVars, messageVars, clientVars, replicaVars, transitions \rangle$

This section provides helpers for the spec.

RECURSIVE *SeqFromSet*(-)

SeqFromSet(*S*) \triangleq

IF *S* = {} THEN

$\langle \rangle$

ELSE LET *x* \triangleq CHOOSE *x* \in *S* : TRUE

IN $\langle x \rangle \circ SeqFromSet(S \setminus \{x\})$

Pick(*S*) \triangleq CHOOSE *s* \in *S* : TRUE

RECURSIVE *SetReduce*(-, -, -)

SetReduce(*Op*(-, -), *S*, *value*) \triangleq

IF *S* = {} THEN

value

ELSE

LET *s* \triangleq *Pick*(*S*)

IN *SetReduce*(*Op*, *S* \setminus {*s*}, *Op*(*s*, *value*))

Max(*s*) \triangleq CHOOSE *x* \in *s* : $\forall y \in s : x \geq y$

Sum(*S*) \triangleq LET *_op*(*a*, *b*) \triangleq *a* + *b*
IN *SetReduce*(*_op*, *S*, 0)

IsQuorum(*s*) \triangleq *Cardinality*(*s*) * 2 \geq *Cardinality*(*Replicas*)

Quorums \triangleq {*r* \in SUBSET *Replicas* : *IsQuorum*(*r*)}

Primary(*v*) \triangleq *replicas*[(*v* % *Len*(*replicas*)) + (IF *v* \geq *Len*(*replicas*) THEN 1 ELSE 0)]

IsPrimary(*r*) \triangleq *Primary*(*rViewID*[*r*]) = *r*

This section models the network.

Send a set of messages

Sends(*ms*) \triangleq *messages'* = *messages* \cup *ms*

Send a message

Send(*m*) \triangleq *Sends*({*m*})

Reply to a message with a set of responses

$$Replies(req, resps) \triangleq messages' = (messages \cup resps) \setminus \{req\}$$

Reply to a message

$$Reply(req, resp) \triangleq Replies(req, \{resp\})$$

Discard a message

$$Discard(m) \triangleq messages' = messages \setminus \{m\}$$

This section models client requests.

Client 'c' sends value 'v' to all replicas

$$\begin{aligned} ClientRequest(c, v) &\triangleq \\ &\wedge cTime' = cTime + 1 \\ &\wedge cSeqNum' = [cSeqNum \text{ EXCEPT } ![c] = cSeqNum[c] + 1] \\ &\wedge Sends(\{ \begin{array}{ll} src & \mapsto c, \\ dest & \mapsto r, \\ type & \mapsto MClientRequest, \\ viewID & \mapsto cViewID[c], \\ seqNum & \mapsto cSeqNum'[c], \\ value & \mapsto v, \\ timestamp & \mapsto cTime' \end{array} : r \in Replicas \}) \\ &\wedge \text{UNCHANGED } \langle globalVars, replicaVars, cViewID, cResps, cCommits \rangle \end{aligned}$$

Client 'c' handles a response 'm' from replica 'r'

$$\begin{aligned} HandleClientResponse(c, r, m) &\triangleq \\ &\wedge \vee \wedge m.viewID = cViewID[c] \\ &\wedge cResps' = [cResps \text{ EXCEPT } ![c] = cResps[c] \cup \{m\}] \\ &\wedge \text{LET} \\ &\quad seqNumResps \triangleq \{n \in cResps[c] : n.seqNum = m.seqNum\} \\ &\quad goodResps \triangleq \{n \in seqNumResps : n.viewID = cViewID[c] \wedge n.succeeded\} \\ &\quad isCommitted \triangleq \wedge \exists n \in goodResps : n.src = Primary(n.viewID) \\ &\quad \wedge \{n.src : n \in goodResps\} \in Quorums \\ &\text{IN} \\ &\quad \wedge \vee \wedge isCommitted \\ &\quad \quad \wedge cCommits' = [cCommits \text{ EXCEPT } ![c] = cCommits[c] \cup \\ &\quad \quad \quad \{ \text{CHOOSE } n \in goodResps : n.src = Primary(n.viewID) \}] \\ &\quad \vee \wedge \neg isCommitted \\ &\quad \quad \wedge \text{UNCHANGED } \langle cCommits \rangle \\ &\quad \quad \wedge \text{UNCHANGED } \langle cViewID, cSeqNum \rangle \\ &\vee \wedge m.viewID > cViewID[c] \\ &\quad \wedge cViewID' = [cViewID \text{ EXCEPT } ![c] = m.viewID] \\ &\quad \wedge cSeqNum' = [cSeqNum \text{ EXCEPT } ![c] = 0] \\ &\quad \wedge cResps' = [cResps \text{ EXCEPT } ![c] = \{\}] \\ &\quad \wedge \text{UNCHANGED } \langle cCommits \rangle \\ &\vee \wedge m.viewID < cViewID[c] \end{aligned}$$

$$\begin{aligned}
& \text{!}[c] = \text{Append}(r\text{Log}[r][c], e)] \\
\text{IN} \\
& \vee \wedge \text{isSequential} \\
& \quad \wedge \text{isLinear} \\
& \quad \wedge r\text{Log}' = \text{append}(\text{entry}) \\
& \quad \wedge r\text{SeqNum}' = [r\text{SeqNum} \text{ EXCEPT } ![r] = [r\text{SeqNum}[r] \text{ EXCEPT } ![c] = m.\text{seqNum}]] \\
& \quad \wedge r\text{Timestamp}' = [r\text{Timestamp} \text{ EXCEPT } ![r] = m.\text{timestamp}] \\
& \quad \wedge \text{Reply}(m, [\text{src} \mapsto r, \\
& \quad \quad \quad \text{dest} \mapsto c, \\
& \quad \quad \quad \text{type} \mapsto \text{MClientResponse}, \\
& \quad \quad \quad \text{viewID} \mapsto r\text{ViewID}[r], \\
& \quad \quad \quad \text{seqNum} \mapsto m.\text{seqNum}, \\
& \quad \quad \quad \text{index} \mapsto \text{index}, \\
& \quad \quad \quad \text{value} \mapsto m.\text{value}, \\
& \quad \quad \quad \text{succeeded} \mapsto \text{TRUE}]) \\
& \quad \wedge \text{UNCHANGED } \langle r\text{Status}, r\text{AbortPoint}, r\text{AbortResps} \rangle \\
& \vee \wedge \vee \neg \text{isSequential} \\
& \quad \vee \neg \text{isLinear} \\
& \quad \wedge \vee \wedge \text{IsPrimary}(r) \\
& \quad \quad \wedge \text{Abort}(r, c, m) \\
& \quad \vee \wedge \neg \text{IsPrimary}(r) \\
& \quad \quad \wedge \text{Reply}(m, [\text{src} \mapsto r, \\
& \quad \quad \quad \text{dest} \mapsto c, \\
& \quad \quad \quad \text{type} \mapsto \text{MClientResponse}, \\
& \quad \quad \quad \text{viewID} \mapsto r\text{ViewID}[r], \\
& \quad \quad \quad \text{seqNum} \mapsto m.\text{seqNum}, \\
& \quad \quad \quad \text{succeeded} \mapsto \text{FALSE}]) \\
& \quad \quad \wedge \text{UNCHANGED } \langle r\text{Status}, r\text{AbortPoint}, r\text{AbortResps} \rangle \\
& \quad \wedge \text{UNCHANGED } \langle r\text{Log}, r\text{SeqNum}, r\text{Timestamp} \rangle \\
& \vee \wedge m.\text{viewID} < r\text{ViewID}[r] \\
& \quad \wedge \text{Reply}(m, [\text{src} \mapsto r, \\
& \quad \quad \quad \text{dest} \mapsto c, \\
& \quad \quad \quad \text{type} \mapsto \text{MClientResponse}, \\
& \quad \quad \quad \text{viewID} \mapsto r\text{ViewID}[r], \\
& \quad \quad \quad \text{seqNum} \mapsto m.\text{seqNum}, \\
& \quad \quad \quad \text{succeeded} \mapsto \text{FALSE}]) \\
& \quad \wedge \text{UNCHANGED } \langle r\text{Status}, r\text{Log}, r\text{SeqNum}, r\text{Timestamp}, r\text{AbortPoint}, r\text{AbortResps} \rangle \\
& \wedge \text{UNCHANGED } \langle \text{globalVars}, \text{clientVars}, r\text{ViewID}, r\text{LastViewID}, r\text{ViewChanges} \rangle \\
& \text{Replica 'r' handles replica 's' repair request 'm'} \\
& \text{HandleRepairRequest}(r, s, m) \triangleq \\
& \quad \wedge m.\text{viewID} = r\text{ViewID}[r] \\
& \quad \wedge \text{IsPrimary}(r) \\
& \quad \wedge r\text{Status}[r] = \text{SNormal} \\
& \quad \wedge \text{LET } \text{index} \triangleq \text{Len}(r\text{Log}[r][m.\text{client}]) + 1 - (r\text{SeqNum}[r] - m.\text{seqNum})
\end{aligned}$$

IN
 $\wedge \vee \wedge index \leq Len(rLog[r][m.client])$
 $\wedge Reply(m, [src \mapsto r,$
 $dest \mapsto s,$
 $type \mapsto MRepairResponse,$
 $viewID \mapsto rViewID[r],$
 $client \mapsto m.client,$
 $seqNum \mapsto m.seqNum])$
 $\wedge UNCHANGED \langle rStatus, rAbortPoint, rAbortResps \rangle$
 $\vee \wedge index = Len(rLog[r][m.client]) + 1$
 $\wedge Abort(r, m.client, m)$
 $\wedge UNCHANGED \langle globalVars, clientVars \rangle$

Replica 'r' handles replica 's' repair response 'm'
 $HandleRepairResponse(r, s, m) \triangleq$
 $\wedge HandleClientRequest(r, m.client, [m \text{ EXCEPT } !.src = m.client])$

Replica 'r' handles replica 's' abort request 'm'
 $HandleAbortRequest(r, s, m) \triangleq$
 $\wedge m.viewID = rViewID[r]$
 $\wedge rStatus[r] \in \{SNormal, SAborting\}$
 $\wedge LET$
 $offset \triangleq Len(rLog[r][m.client]) + 1 - (rSeqNum[r][m.client] - m.seqNum)$
 $entry \triangleq [type \mapsto TNoOp, timestamp \mapsto m.timestamp]$
 $replace(i, e) \triangleq [j \in 1 \dots Max(\{Len(rLog[r][m.client]), i\}) \mapsto$
 $IF j = i THEN e ELSE rLog[r][m.client][j]]$

IN
 $\wedge offset \leq Len(rLog[r][m.client]) + 1$
 $\wedge rLog' = replace(offset, entry)$
 $\wedge rTimestamp' = [rTimestamp \text{ EXCEPT } ![r] = Max(\{rTimestamp[r], m.timestamp\})]$
 $\wedge rSeqNum' = [rSeqNum \text{ EXCEPT } ![r] = [rSeqNum[r] \text{ EXCEPT }$
 $![m.client] = Max(\{rSeqNum[r][m.client], m.seqNum\})]$
 $\wedge Replies(m, \{[src \mapsto r,$
 $dest \mapsto Primary(rViewID[r]),$
 $type \mapsto MAbortResponse,$
 $viewID \mapsto rViewID[r],$
 $client \mapsto m.client,$
 $seqNum \mapsto m.seqNum],$
 $[src \mapsto r,$
 $dest \mapsto m.client,$
 $type \mapsto MClientResponse,$
 $viewID \mapsto rViewID[r],$
 $seqNum \mapsto m.seqNum,$
 $succeeded \mapsto FALSE]\})$
 $\wedge UNCHANGED \langle globalVars, clientVars, rStatus, rAbortPoint,$

$rAbortResps, rViewID, rLastViewID, rViewChanges$

Replica 'r' handles replica 's' repair response 'm'

$$\begin{aligned}
& HandleAbortResponse(r, s, m) \triangleq \\
& \quad \wedge rStatus[r] = SAborting \\
& \quad \wedge m.viewID = rViewID[r] \\
& \quad \wedge IsPrimary(r) \\
& \quad \wedge m.seqNum = rAbortPoint[r].seqNum \\
& \quad \wedge rAbortResps' = [rAbortResps \text{ EXCEPT } ![r] = rAbortResps[r] \cup \{m\}] \\
& \quad \wedge \text{LET } resps \triangleq \{res.src : res \in \{resp \in rAbortResps'[r] : \\
& \quad \quad \quad \wedge resp.viewID = rViewID[r] \\
& \quad \quad \quad \wedge resp.client = rAbortPoint[r].client \\
& \quad \quad \quad \wedge resp.seqNum = rAbortPoint[r].seqNum\}\} \\
& \quad isQuorum \triangleq r \in resps \wedge resps \in Quorums \\
& \quad \text{IN} \\
& \quad \quad \vee \wedge isQuorum \\
& \quad \quad \quad \wedge rStatus' = [rStatus \text{ EXCEPT } ![r] = SNormal] \\
& \quad \quad \vee \wedge \neg isQuorum \\
& \quad \quad \quad \wedge \text{UNCHANGED } \langle rStatus \rangle \\
& \quad \wedge \text{UNCHANGED } \langle globalVars, messageVars, clientVars, rLog, rSeqNum, rTimestamp, \\
& \quad \quad rAbortPoint, rViewID, rViewChanges, rLastViewID \rangle
\end{aligned}$$

Replica 'r' requests a view change

$$\begin{aligned}
& ChangeView(r) \triangleq \\
& \quad \wedge Sends(\{[src \mapsto r, \\
& \quad \quad \quad dest \mapsto d, \\
& \quad \quad \quad type \mapsto MViewChangeRequest, \\
& \quad \quad \quad viewID \mapsto rViewID[r] + 1] : d \in Replicas\}) \\
& \quad \wedge \text{UNCHANGED } \langle globalVars, clientVars, replicaVars \rangle
\end{aligned}$$

Replica 'r' handles replica 's' view change request 'm'

$$\begin{aligned}
& HandleViewChangeRequest(r, s, m) \triangleq \\
& \quad \wedge rViewID[r] < m.viewID \\
& \quad \wedge rViewID' = [rViewID \text{ EXCEPT } ![r] = m.viewID] \\
& \quad \wedge rStatus' = [rStatus \text{ EXCEPT } ![r] = SViewChange] \\
& \quad \wedge rViewChanges' = [rViewChanges \text{ EXCEPT } ![r] = \{\}] \\
& \quad \wedge Reply(m, [src \mapsto r, \\
& \quad \quad \quad dest \mapsto Primary(m.viewID), \\
& \quad \quad \quad type \mapsto MViewChangeResponse, \\
& \quad \quad \quad viewID \mapsto m.viewID, \\
& \quad \quad \quad lastViewID \mapsto rLastViewID[r], \\
& \quad \quad \quad logs \mapsto rLog[r]]) \\
& \quad \wedge \text{UNCHANGED } \langle globalVars, clientVars, rLog, rSeqNum, rTimestamp, \\
& \quad \quad rAbortPoint, rAbortResps, rLastViewID \rangle
\end{aligned}$$

Replica 'r' handles replica 's' view change response 'm'

$$\begin{aligned}
\wedge rSeqNum' &= [rSeqNum \quad \text{EXCEPT } ![r] = [c \in Clients \mapsto 0]] \\
\wedge rTimestamp' &= [rTimestamp \quad \text{EXCEPT } ![r] = m.timestamp] \\
\wedge rStatus' &= [rStatus \quad \text{EXCEPT } ![r] = SNormal] \\
\wedge rViewID' &= [rViewID \quad \text{EXCEPT } ![r] = m.viewID] \\
\wedge rLastViewID' &= [rLastViewID \quad \text{EXCEPT } ![r] = m.viewID] \\
\wedge Discard(m) \\
\wedge \text{UNCHANGED } \langle globalVars, clientVars, rAbortPoint, rAbortResps, rViewChanges \rangle
\end{aligned}$$

$$\begin{aligned}
InitMessageVars &\triangleq \\
&\wedge messages = \{\}
\end{aligned}$$

$$\begin{aligned}
InitClientVars &\triangleq \\
&\wedge cTime = 0 \\
&\wedge cViewID = [c \in Clients \mapsto 1] \\
&\wedge cSeqNum = [c \in Clients \mapsto 0] \\
&\wedge cResps = [c \in Clients \mapsto \{\}] \\
&\wedge cCommits = [c \in Clients \mapsto \{\}]
\end{aligned}$$

$$\begin{aligned}
InitReplicaVars &\triangleq \\
&\wedge replicas = SeqFromSet(Replicas) \\
&\wedge rStatus = [r \in Replicas \mapsto SNormal] \\
&\wedge rLog = [r \in Replicas \mapsto [c \in Clients \mapsto \langle \rangle]] \\
&\wedge rSeqNum = [r \in Replicas \mapsto [c \in Clients \mapsto 0]] \\
&\wedge rTimestamp = [r \in Replicas \mapsto 0] \\
&\wedge rAbortPoint = [r \in Replicas \mapsto [client \mapsto Nil, seqNum \mapsto 0]] \\
&\wedge rAbortResps = [r \in Replicas \mapsto \{\}] \\
&\wedge rViewID = [r \in Replicas \mapsto 1] \\
&\wedge rLastViewID = [r \in Replicas \mapsto 1] \\
&\wedge rViewChanges = [r \in Replicas \mapsto \{\}]
\end{aligned}$$

$$\begin{aligned}
Init &\triangleq \\
&\wedge InitMessageVars \\
&\wedge InitClientVars \\
&\wedge InitReplicaVars \\
&\wedge transitions = 0
\end{aligned}$$

The type invariant verifies that clients do not receive two commits at the same index with different values.

$$\begin{aligned}
TypeOK &\triangleq \\
&\forall c1, c2 \in Clients : \\
&\quad \forall e1 \in cCommits[c1] : \\
&\quad \neg \exists e2 \in cCommits[c2] :
\end{aligned}$$

$$\begin{aligned} & \wedge e1.index = e2.index \\ & \wedge e1.value \neq e2.value \end{aligned}$$

$$Transition \triangleq transitions' = transitions + 1$$

$$Next \triangleq$$

$$\begin{aligned} & \vee \exists c \in Clients : \\ & \quad \exists v \in Values : \\ & \quad \quad \wedge ClientRequest(c, v) \\ & \quad \quad \wedge Transition \\ & \vee \exists r \in Replicas : \\ & \quad \wedge ChangeView(r) \\ & \quad \wedge Transition \\ & \vee \exists m \in messages : \\ & \quad \wedge m.type = MClientRequest \\ & \quad \wedge HandleClientRequest(m.dest, m.src, m) \\ & \quad \wedge Transition \\ & \vee \exists m \in messages : \\ & \quad \wedge m.type = MClientResponse \\ & \quad \wedge HandleClientResponse(m.dest, m.src, m) \\ & \quad \wedge Transition \\ & \vee \exists m \in messages : \\ & \quad \wedge m.type = MRepairRequest \\ & \quad \wedge HandleRepairRequest(m.dest, m.src, m) \\ & \quad \wedge Transition \\ & \vee \exists m \in messages : \\ & \quad \wedge m.type = MRepairResponse \\ & \quad \wedge HandleRepairResponse(m.dest, m.src, m) \\ & \quad \wedge Transition \\ & \vee \exists m \in messages : \\ & \quad \wedge m.type = MAbortRequest \\ & \quad \wedge HandleAbortRequest(m.dest, m.src, m) \\ & \quad \wedge Transition \\ & \vee \exists m \in messages : \\ & \quad \wedge m.type = MAbortResponse \\ & \quad \wedge HandleAbortResponse(m.dest, m.src, m) \\ & \quad \wedge Transition \\ & \vee \exists m \in messages : \\ & \quad \wedge m.type = MViewChangeRequest \\ & \quad \wedge HandleViewChangeRequest(m.dest, m.src, m) \\ & \quad \wedge Transition \\ & \vee \exists m \in messages : \\ & \quad \wedge m.type = MViewChangeResponse \\ & \quad \wedge HandleViewChangeResponse(m.dest, m.src, m) \\ & \quad \wedge Transition \end{aligned}$$

$$\begin{aligned}
& \forall \exists m \in \text{messages} : \\
& \quad \wedge m.type = MStartViewRequest \\
& \quad \wedge HandleStartViewRequest(m.dest, m.src, m) \\
& \quad \wedge Transition \\
& \forall \exists m \in \text{messages} : Discard(m) \\
Spec & \triangleq Init \wedge \Box[Next]_{vars}
\end{aligned}$$

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