
MODULE *JustInTimePaxos*

EXTENDS *Naturals, Reals, Sequences, FiniteSets, TLC*

The set of *Paxos* replicas
 CONSTANT *Replicas*

The set of *Paxos* clients
 CONSTANT *Clients*

The set of possible values
 CONSTANT *Values*

An empty value
 CONSTANT *Nil*

Request/response types
 CONSTANTS
 MClientRequest,
 MClientReply,
 MReconcileRequest,
 MReconcileReply,
 MRepairRequest,
 MRepairReply,
 MViewChange,
 MViewChangeReply,
 MStartView

Replica statuses
 CONSTANTS
 SNormal,
 SRepair,
 SViewChange

The set of all messages on the network
 VARIABLE *messages*

The total number of messages sent
 VARIABLE *messageCount*

The total number of steps executed
 VARIABLE *stepCount*

$messageVars \triangleq \langle messages, messageCount, stepCount \rangle$

Local client state

Strictly increasing representation of synchronized time
 VARIABLE $cTime$

The highest known view ID for a client
 VARIABLE $cViewID$

Client request IDs
 VARIABLE $cReqID$

A client response buffer
 VARIABLE $cReps$

A set of all commits - used for model checking
 VARIABLE $cCommits$

$clientVars \triangleq \langle cTime, cViewID, cReqID, cReps, cCommits \rangle$

Local replica state

The current status of a replica
 VARIABLE $rStatus$

The current view ID for a replica
 VARIABLE $rViewID$

A replica's commit log
 VARIABLE $rLog$

A replica's sync index
 VARIABLE $rSyncIndex$

The view ID for the log
 VARIABLE $rLogViewID$

The set of view change replies
 VARIABLE $rViewChangeReps$

$replicaVars \triangleq \langle rStatus, rViewID, rLog, rSyncIndex, rLogViewID, rViewChangeReps \rangle$

$vars \triangleq \langle messageVars, clientVars, replicaVars \rangle$

This section provides utilities for implementing the spec.

Creates a sequence from set 'S'
 RECURSIVE $SeqFromSet(-)$

$SeqFromSet(S) \triangleq$

IF $S = \{\}$ THEN

$\langle \rangle$

ELSE LET $x \triangleq$ CHOOSE $x \in S$: TRUE

IN $\langle x \rangle \circ SeqFromSet(S \setminus \{x\})$
 RECURSIVE $SetReduce(-, -, -)$
 $SetReduce(Op(-, -), S, value) \triangleq$
 IF $S = \{\}$ THEN
 $value$
 ELSE
 LET $s \triangleq$ CHOOSE $s \in S : \text{TRUE}$
 IN $SetReduce(Op, S \setminus \{s\}, Op(s, value))$

Computes the greatest vlue in set 'S'
 $Max(S) \triangleq$ CHOOSE $x \in S : \forall y \in S : x \geq y$

Computes the sum of numbers in set 'S'
 $Sum(S) \triangleq$ LET $_op(a, b) \triangleq a + b$
 IN $SetReduce(_op, S, 0)$

The values of a sequence
 $Range(s) \triangleq \{s[i] : i \in \text{DOMAIN } s\}$

This section provides helpers for the protocol.

A sorted sequence of replicas
 $replicas \triangleq SeqFromSet(Replicas)$

The primary index for view 'v'
 $PrimaryIndex(v) \triangleq (v \% Len(replicas)) + (\text{IF } v \geq Len(replicas) \text{ THEN } 1 \text{ ELSE } 0)$

The primary for view 'v'
 $Primary(v) \triangleq replicas[PrimaryIndex(v)]$

Quorum is the quorum for a given view
 $Quorum(v) \triangleq$

LET
 $quorumSize \triangleq Len(replicas) \div 2 + 1$
 $index(i) \triangleq PrimaryIndex(v) + (i - 1)$
 $member(i) \triangleq \text{IF } index(i) > Len(replicas) \text{ THEN } replicas[index(i) \% Len(replicas)] \text{ ELSE } replicas[index(i)]$
 IN
 $\{member(i) : i \in 1 \dots quorumSize\}$

A boolean indicating whether the given set is a quorum
 $IsQuorum(S) \triangleq Cardinality(S) * 2 \geq Cardinality(Replicas)$

A boolean indicating whether the given set is a quorum that includes the given replica
 $IsLocalQuorum(r, S) \triangleq IsQuorum(S) \wedge r \in S$

This section models the network.

Send a set of messages

$$\begin{aligned}
Sends(ms) &\triangleq \\
&\wedge messages' = messages \cup ms \\
&\wedge messageCount' = messageCount + Cardinality(ms) \\
&\wedge stepCount' = stepCount + 1
\end{aligned}$$

Send a message

$$Send(m) \triangleq Sends(\{m\})$$

Ack a message

$$\begin{aligned}
Ack(m) &\triangleq \\
&\wedge messages' = messages \setminus \{m\} \\
&\wedge messageCount' = messageCount + 1 \\
&\wedge stepCount' = stepCount + 1
\end{aligned}$$

Ack a message and send a set of messages

$$\begin{aligned}
AckAndSends(m, ms) &\triangleq \\
&\wedge messages' = (messages \cup ms) \setminus \{m\} \\
&\wedge messageCount' = messageCount + Cardinality(ms) \\
&\wedge stepCount' = stepCount + 1
\end{aligned}$$

Ack and send a message

$$AckAndSend(m, n) \triangleq AckAndSends(m, \{n\})$$

Reply to a message with a set of responses

$$Replies(req, reps) \triangleq AckAndSends(req, reps)$$

Reply to a message

$$Reply(req, resp) \triangleq AckAndSend(req, resp)$$

This section models client requests.

Client 'c' sends value 'v' to all replicas

$$\begin{aligned}
ClientRequest(c, v) &\triangleq \\
&\wedge cTime' = cTime + 1 \\
&\wedge cReqID' = [cReqID \text{ EXCEPT } ![c] = cReqID[c] + 1] \\
&\wedge Sends(\{ \begin{array}{ll} src & \mapsto c, \\ dest & \mapsto r, \\ type & \mapsto MClientRequest, \\ viewID & \mapsto cViewID[c], \\ reqID & \mapsto cReqID'[c], \\ value & \mapsto v, \\ timestamp & \mapsto cTime' \end{array} : r \in Quorum(cViewID[c]) \}) \\
&\wedge UNCHANGED \langle replicaVars, cViewID, cReps, cCommits \rangle
\end{aligned}$$

Client 'c' handles a response 'm' from replica 'r'
 $HandleClientReply(c, r, m) \triangleq$

If the reply view ID does not match the request view ID , update the client's view.

$\wedge \vee \wedge m.viewID \neq m.req.viewID$
 $\wedge \vee \wedge cViewID[c] < m.viewID$
 $\wedge cViewID' = [cViewID \text{ EXCEPT } ![c] = m.viewID]$
 $\vee \wedge cViewID[c] \geq m.viewID$
 $\wedge \text{UNCHANGED } \langle cViewID \rangle$
 $\wedge Ack(m)$
 $\wedge \text{UNCHANGED } \langle cReps, cCommits \rangle$

If the request and reply views match and the reply view matches the client's view,
 aggregate the replies for the associated client request.

$\vee \wedge m.viewID = m.req.viewID$
 $\wedge m.viewID = cViewID[c]$
 $\wedge \vee \wedge m.succeeded$
 $\wedge cReps' = [cReps \text{ EXCEPT } ![c] =$
 $(cReps[c] \setminus \{n \in cReps[c] : \wedge n.src = m.src$
 $\wedge n.viewID = cViewID[c]$
 $\wedge n.req.reqID = m.req.reqID$
 $\wedge \neg n.succeeded\}) \cup \{m\}]$
 $\vee \wedge \neg m.succeeded$
 $\wedge \neg \exists n \in cReps[c] : \wedge n.src = m.src$
 $\wedge n.viewID = cViewID[c]$
 $\wedge n.req.reqID = m.req.reqID$
 $\wedge n.succeeded$
 $\wedge cReps' = [cReps \text{ EXCEPT } ![c] = cReps[c] \cup \{m\}]$
 $\wedge \text{LET } reps \triangleq \{n \in cReps'[c] : \wedge n.viewID = cViewID[c]$
 $\wedge n.req.reqID = m.req.reqID\}$
 $isQuorum \triangleq \{n.src : n \in \{n \in reps : n.succeeded\}\} = Quorum(cViewID[c])$
 $isCommitted \triangleq \wedge \forall n \in reps : n.succeeded$
 $\wedge Cardinality(\{n.checksum : n \in reps\}) = 1$
 $hasPrimary \triangleq \exists n \in reps : n.src = Primary(cViewID[c]) \wedge n.succeeded$

IN

If a quorum of successful replies have been received and the checksums
 match, add the primary reply to commits.

$\vee \wedge isQuorum$
 $\wedge isCommitted$
 $\wedge \text{LET } commit \triangleq \text{CHOOSE } n \in reps : n.src = Primary(cViewID[c])$
 $\text{IN } cCommits' = [cCommits \text{ EXCEPT } ![c] = cCommits[c] \cup \{commit\}]$
 $\wedge Ack(m)$

If some reply failed or was returned with an incorrect checksum,
 send a *ReconcileRequest* to the inconsistent node to force it to
 reconcile its *log* with the primary's *log*.

$\vee \wedge \neg isCommitted$
 $\wedge \vee \wedge hasPrimary$

$$\begin{aligned}
& \wedge \text{LET } \textit{primaryRep} \triangleq \text{CHOOSE } n \in \textit{reps} : \wedge n.\textit{src} = \textit{Primary}(c\textit{ViewID}[c]) \\
& \quad \wedge n.\textit{succeeded} \\
& \quad \textit{retryReps} \triangleq \{n \in \textit{reps} : \\
& \quad \quad \wedge n.\textit{src} \neq \textit{Primary}(c\textit{ViewID}[c]) \\
& \quad \quad \wedge n.\textit{checksum} \neq \textit{primaryRep}.\textit{checksum}\} \\
& \text{IN } \textit{AckAndSends}(m, \{[\textit{src} \mapsto c, \\
& \quad \textit{dest} \mapsto r, \\
& \quad \textit{type} \mapsto \textit{MReconcileRequest}, \\
& \quad \textit{viewID} \mapsto c\textit{ViewID}[c], \\
& \quad \textit{reqID} \mapsto m.\textit{req}.\textit{reqID}, \\
& \quad \textit{index} \mapsto \textit{primaryRep}.\textit{index}] : n \in \textit{retryReps}\}) \\
& \quad \vee \wedge \neg \textit{hasPrimary} \\
& \quad \wedge \textit{Ack}(m) \\
& \quad \wedge \text{UNCHANGED } \langle c\textit{Commits} \rangle \\
& \quad \text{If a quorum has not yet been reached, wait for more replies.} \\
& \quad \vee \wedge \neg \textit{isQuorum} \\
& \quad \wedge \textit{isCommitted} \\
& \quad \wedge \textit{Ack}(m) \\
& \quad \wedge \text{UNCHANGED } \langle c\textit{Commits} \rangle \\
& \quad \wedge \text{UNCHANGED } \langle c\textit{ViewID} \rangle \\
& \quad \wedge \text{UNCHANGED } \langle \textit{replicaVars}, c\textit{Time}, c\textit{ReqID} \rangle \\
& \textit{HandleReconcileReply}(c, r, m) \triangleq \textit{HandleClientReply}(c, r, m)
\end{aligned}$$

This section models the replica protocol.

Replica 'r' handles client 'c' request 'm'

$\textit{HandleClientRequest}(r, c, m) \triangleq$

Client requests can only be handled while in the *SNormal* status.

$\wedge r\textit{Status}[r] = \textit{SNormal}$

If the client's view matches the replica's view, process the client's request.

$\wedge \vee \wedge m.\textit{viewID} = r\textit{ViewID}[r]$

$\wedge \text{LET } \textit{lastTimestamp} \triangleq \text{Max}(\{r\textit{Log}[r][i].\textit{timestamp} : i \in \text{DOMAIN } r\textit{Log}[r]\} \cup \{0\})$

IN

If the request timestamp is greater than the highest *log* timestamp,
append the entry to the *log* and return a successful response with
the appended entry index.

$\wedge \vee \wedge m.\textit{timestamp} > \textit{lastTimestamp}$

$\wedge r\textit{Log}' = [r\textit{Log} \text{ EXCEPT } ![r] =$

$\textit{Append}(r\textit{Log}[r], [\textit{value} \mapsto m.\textit{value},$

$\textit{timestamp} \mapsto m.\textit{timestamp}])]$

$\wedge \textit{Reply}(m, [\textit{src} \mapsto r,$

$\textit{dest} \mapsto c,$

$\textit{req} \mapsto m,$

$\textit{type} \mapsto \textit{MClientReply},$

$viewID \mapsto rViewID[r],$
 $index \mapsto Len(rLog'[r]),$
 $checksum \mapsto rLog'[r],$
 $value \mapsto m.value,$
 $timestamp \mapsto m.timestamp,$
 $succeeded \mapsto TRUE]$

If the request timestamp matches the highest *log* timestamp, treat the request as a duplicate. Return a successful response indicating the entry was appended.

$\vee \wedge m.timestamp = lastTimestamp$
 $\wedge Reply(m, [src \mapsto r,$
 $dest \mapsto c,$
 $req \mapsto m,$
 $type \mapsto MClientReply,$
 $viewID \mapsto rViewID[r],$
 $index \mapsto Len(rLog[r]),$
 $checksum \mapsto rLog[r],$
 $value \mapsto m.value,$
 $timestamp \mapsto m.timestamp,$
 $succeeded \mapsto TRUE])$

$\wedge UNCHANGED \langle rLog \rangle$

If the request timestamp is less than the highest *log* timestamp, reject the request.

$\vee \wedge m.timestamp < lastTimestamp$
 $\wedge Reply(m, [src \mapsto r,$
 $dest \mapsto c,$
 $req \mapsto m,$
 $type \mapsto MClientReply,$
 $viewID \mapsto rViewID[r],$
 $index \mapsto Len(rLog[r]),$
 $checksum \mapsto rLog[r],$
 $value \mapsto m.value,$
 $timestamp \mapsto m.timestamp,$
 $succeeded \mapsto FALSE])$

$\wedge UNCHANGED \langle rLog \rangle$

$\wedge UNCHANGED \langle rViewID, rStatus, rViewChangeReps \rangle$

If the client's view is greater than the replica's view, reject the client's request with the outdated view *ID* and enter the view change protocol.

$\vee \wedge m.viewID > rViewID[r]$
 $\wedge rViewID' = [rViewID \text{ EXCEPT } ![r] = m.viewID]$
 $\wedge rStatus' = [rStatus \text{ EXCEPT } ![r] = SViewChange]$
 $\wedge rViewChangeReps' = [rViewChangeReps \text{ EXCEPT } ![r] = \{\}]$
 $\wedge Replies(m, \{[src \mapsto r,$
 $dest \mapsto c,$
 $req \mapsto m,$

$$\begin{aligned}
& \text{type} \mapsto MClientReply, \\
& \text{viewID} \mapsto rViewID[r], \\
& \text{succeeded} \mapsto \text{FALSE}], \\
& [\text{src} \mapsto r, \\
& \text{dest} \mapsto \text{Primary}(m.viewID), \\
& \text{type} \mapsto MViewChangeReply, \\
& \text{viewID} \mapsto m.viewID, \\
& \text{logViewID} \mapsto rLogViewID[r], \\
& \text{log} \mapsto rLog[r]]\}) \\
& \wedge \text{UNCHANGED } \langle rLog \rangle \\
& \text{If the client's view is less than the replica's view, reject the client's request} \\
& \text{with the updated view ID to force the client to retry.} \\
& \vee \wedge m.viewID < rViewID[r] \\
& \wedge \text{Reply}(m, [\text{src} \mapsto r, \\
& \text{dest} \mapsto c, \\
& \text{req} \mapsto m, \\
& \text{type} \mapsto MClientReply, \\
& \text{viewID} \mapsto rViewID[r], \\
& \text{succeeded} \mapsto \text{FALSE}]) \\
& \wedge \text{UNCHANGED } \langle rViewID, rStatus, rLog, rViewChangeReps \rangle \\
& \wedge \text{UNCHANGED } \langle clientVars, rLogViewID, rSyncIndex \rangle \\
\text{HandleReconcileRequest}(r, c, m) & \triangleq \\
& \wedge rStatus[r] = SNormal \\
& \wedge rViewID[r] = m.viewID \\
& \wedge \vee \wedge rSyncIndex[r] \geq m.index \\
& \wedge \text{Reply}(m, [\text{src} \mapsto r, \\
& \text{dest} \mapsto c, \\
& \text{req} \mapsto m, \\
& \text{type} \mapsto MReconcileReply, \\
& \text{viewID} \mapsto rViewID[r], \\
& \text{index} \mapsto m.index, \\
& \text{checksum} \mapsto [i \in 1 \dots m.index \mapsto rLog[r][i]], \\
& \text{value} \mapsto rLog[r][m.index].value, \\
& \text{timestamp} \mapsto rLog[r][m.index].timestamp, \\
& \text{succeeded} \mapsto \text{TRUE}]) \\
& \wedge \text{UNCHANGED } \langle rStatus \rangle \\
& \vee \wedge rSyncIndex[r] < m.index \\
& \wedge \text{Primary}(rViewID[r]) \neq r \\
& \wedge rStatus' = [rStatus \text{ EXCEPT } ![r] = SRepair] \\
& \wedge \text{AckAndSend}(m, [\text{src} \mapsto r, \\
& \text{dest} \mapsto \text{Primary}(rViewID[r]), \\
& \text{req} \mapsto m, \\
& \text{type} \mapsto MRepairRequest, \\
& \text{viewID} \mapsto rViewID[r],
\end{aligned}$$

$$\begin{aligned}
& index \mapsto m.index]) \\
& \wedge \text{UNCHANGED } \langle clientVars, rViewID, rLog, rLogViewID, rSyncIndex, rViewChangeReps \rangle \\
\text{HandleRepairRequest}(r, s, m) & \triangleq \\
& \wedge rStatus[r] = SNormal \\
& \wedge rViewID[r] = m.viewID \\
& \wedge Primary(rViewID[r]) = r \\
& \wedge Reply(m, [src \mapsto r, \\
& \quad dest \mapsto s, \\
& \quad req \mapsto m.req, \\
& \quad type \mapsto MRepairReply, \\
& \quad viewID \mapsto rViewID[r], \\
& \quad index \mapsto m.index, \\
& \quad log \mapsto [i \in 1 \dots m.index \mapsto rLog[r][i]]) \\
& \wedge \text{UNCHANGED } \langle clientVars, replicaVars \rangle \\
\text{HandleRepairReply}(r, s, m) & \triangleq \\
& \wedge rStatus[r] = SRepair \\
& \wedge rViewID[r] = m.viewID \\
& \wedge rStatus' = [rStatus \text{ EXCEPT } ![r] = SNormal] \\
& \wedge rLog' = [rLog \text{ EXCEPT } ![r] = m.log \circ SubSeq(rLog[r], Len(m.log), Len(rLog[r]))] \\
& \wedge rSyncIndex' = [rSyncIndex \text{ EXCEPT } ![r] = Len(rLog'[r])] \\
& \wedge Reply(m, [src \mapsto r, \\
& \quad dest \mapsto m.req.src, \\
& \quad req \mapsto m.req, \\
& \quad type \mapsto MReconcileReply, \\
& \quad viewID \mapsto rViewID[r], \\
& \quad index \mapsto m.index, \\
& \quad checksum \mapsto m.log, \\
& \quad value \mapsto m.log[m.index].value, \\
& \quad timestamp \mapsto m.log[m.index].timestamp, \\
& \quad succeeded \mapsto TRUE]) \\
& \wedge \text{UNCHANGED } \langle clientVars, rViewID, rLogViewID, rViewChangeReps \rangle \\
\text{Replica 'r' requests a view change} \\
\text{ChangeView}(r) & \triangleq \\
& \wedge Sends(\{[src \mapsto r, \\
& \quad dest \mapsto d, \\
& \quad type \mapsto MViewChange, \\
& \quad viewID \mapsto rViewID[r] + 1] : d \in Replicas\}) \\
& \wedge \text{UNCHANGED } \langle clientVars, replicaVars \rangle \\
\text{Replica 'r' handles replica 's' view change request 'm'} \\
\text{HandleViewChange}(r, s, m) & \triangleq \\
& \wedge \vee \wedge rViewID[r] < m.viewID \\
& \wedge rViewID' = [rViewID \text{ EXCEPT } ![r] = m.viewID]
\end{aligned}$$

$$\begin{aligned}
& \wedge rStatus' = [rStatus \text{ EXCEPT } ![r] = SViewChange] \\
& \wedge rViewChangeReps' = [rViewChangeReps \text{ EXCEPT } ![r] = \{\}] \\
& \wedge Reply(m, [src \mapsto r, \\
& \quad \quad \quad dest \mapsto Primary(m.viewID), \\
& \quad \quad \quad type \mapsto MViewChangeReply, \\
& \quad \quad \quad viewID \mapsto m.viewID, \\
& \quad \quad \quad logViewID \mapsto rLogViewID[r], \\
& \quad \quad \quad log \mapsto rLog[r]]) \\
& \vee \wedge rViewID[r] \geq m.viewID \\
& \wedge Ack(m) \\
& \wedge \text{UNCHANGED } \langle rViewID, rStatus, rViewChangeReps \rangle \\
& \wedge \text{UNCHANGED } \langle clientVars, rLog, rLogViewID, rSyncIndex \rangle
\end{aligned}$$

Replica 'r' handles replica 's' view change reply 'm'

$HandleViewChangeReply(r, s, m) \triangleq$

The view change protocol is run by the primary for the view.

$$\begin{aligned}
& \wedge Primary(m.viewID) = r \\
& \wedge rViewID[r] = m.viewID \\
& \wedge rStatus[r] = SViewChange \\
& \wedge rViewChangeReps' = [rViewChangeReps \text{ EXCEPT } ![r] = rViewChangeReps[r] \cup \{m\}] \\
& \wedge \text{LET } viewChanges \triangleq \{v \in rViewChangeReps'[r] : v.viewID = rViewID[r]\} \\
& \text{IN}
\end{aligned}$$

In order to ensure the new view is initialized with the latest view, a quorum of view change replies must be received to guarantee the last activated view is present in the set of replies.

If view change replies have been received from a majority of the replicas, initialize the view using the *log* from the highest activated view.

$$\begin{aligned}
& \vee \wedge IsLocalQuorum(r, \{v.src : v \in viewChanges\}) \\
& \quad \wedge \text{LET } latestViewID \triangleq Max(\{v.logViewID : v \in viewChanges\}) \\
& \quad \quad latestChange \triangleq \text{CHOOSE } v \in viewChanges : \\
& \quad \quad \quad \wedge v.logViewID = latestViewID \\
& \quad \quad \quad \wedge v.src \in Quorum(latestViewID) \\
& \quad \text{IN } AckAndSends(m, \{[src \mapsto r, \\
& \quad \quad \quad dest \mapsto d, \\
& \quad \quad \quad type \mapsto MStartView, \\
& \quad \quad \quad viewID \mapsto rViewID[r], \\
& \quad \quad \quad log \mapsto latestChange.log] : d \in Replicas\})
\end{aligned}$$

If view change replies have not yet been received from a quorum, record the view change reply and discard the message.

$$\begin{aligned}
& \vee \wedge \neg IsLocalQuorum(r, \{v.src : v \in viewChanges\}) \\
& \quad \wedge Ack(m) \\
& \wedge \text{UNCHANGED } \langle clientVars, rStatus, rViewID, rLog, rLogViewID, rSyncIndex \rangle
\end{aligned}$$

Replica 'r' handles replica 's' start view request 'm'

$HandleStartView(r, s, m) \triangleq$

To activate a view, the replica must either not know of the view or already be participating in the view change protocol for the view.

$$\begin{aligned} \wedge \vee rViewID[r] < m.viewID \\ \vee \wedge rViewID[r] = m.viewID \\ \wedge rStatus[r] = SViewChange \end{aligned}$$

If the replica is part of the quorum for the activated view, update the *log* and record the activated view for use in the view change protocol.

$$\begin{aligned} \wedge \vee \wedge r \in Quorum(m.viewID) \\ \wedge rLog' = [rLog \text{ EXCEPT } ![r] = m.log] \\ \wedge rLogViewID' = [rLogViewID \text{ EXCEPT } ![r] = m.viewID] \\ \wedge rSyncIndex' = [rSyncIndex \text{ EXCEPT } ![r] = Len(m.log)] \\ \vee \wedge r \notin Quorum(m.viewID) \\ \wedge \text{UNCHANGED } \langle rLog, rLogViewID, rSyncIndex \rangle \end{aligned}$$

Update the replica's view *ID* and status and clean up view change state.

$$\begin{aligned} \wedge rViewID' = [rViewID \text{ EXCEPT } ![r] = m.viewID] \\ \wedge rStatus' = [rStatus \text{ EXCEPT } ![r] = SNormal] \\ \wedge \text{LET } viewChanges \triangleq \{v \in rViewChangeReps[r] : v.viewID = rViewID[r]\} \\ \text{IN } rViewChangeReps' = [rViewChangeReps \text{ EXCEPT } ![r] = rViewChangeReps[r] \setminus viewChanges] \\ \wedge Ack(m) \\ \wedge \text{UNCHANGED } \langle clientVars \rangle \end{aligned}$$

$$\begin{aligned} InitMessageVars &\triangleq \\ \wedge messages &= \{\} \\ \wedge messageCount &= 0 \\ \wedge stepCount &= 0 \end{aligned}$$

$$\begin{aligned} InitClientVars &\triangleq \\ \wedge cTime &= 0 \\ \wedge cViewID &= [c \in Clients \mapsto 1] \\ \wedge cReqID &= [c \in Clients \mapsto 0] \\ \wedge cReps &= [c \in Clients \mapsto \{\}] \\ \wedge cCommits &= [c \in Clients \mapsto \{\}] \end{aligned}$$

$$\begin{aligned} InitReplicaVars &\triangleq \\ \wedge rStatus &= [r \in Replicas \mapsto SNormal] \\ \wedge rViewID &= [r \in Replicas \mapsto 1] \\ \wedge rLog &= [r \in Replicas \mapsto \langle \rangle] \\ \wedge rSyncIndex &= [r \in Replicas \mapsto 0] \\ \wedge rLogViewID &= [r \in Replicas \mapsto 1] \\ \wedge rViewChangeReps &= [r \in Replicas \mapsto \{\}] \end{aligned}$$

$$\begin{aligned} Init &\triangleq \\ \wedge InitMessageVars \\ \wedge InitClientVars \end{aligned}$$

$\wedge \text{InitReplicaVars}$

This section specifies the invariants for the protocol.

The type invariant asserts that the leader's *log* will never contain a different value at the same index as a client commit.

$Inv \triangleq$
 $\forall c \in \text{Clients} :$
 $\forall e \in c\text{Commits}[c] :$
 $\neg \exists r \in \text{Replicas} :$
 $\wedge r\text{Status}[r] = \text{SNormal}$
 $\wedge r\text{ViewID}[r] \geq e.\text{viewID}$
 $\wedge r \in \text{Quorum}(r\text{ViewID}[r])$
 $\wedge r\text{Log}[r][e.\text{index}].\text{value} \neq e.\text{value}$

$\text{NextClientRequest} \triangleq$
 $\exists c \in \text{Clients} :$
 $\exists v \in \text{Values} :$
 $\text{ClientRequest}(c, v)$

$\text{NextChangeView} \triangleq$
 $\exists r \in \text{Replicas} :$
 $\text{ChangeView}(r)$

$\text{NextHandleClientRequest} \triangleq$
 $\exists m \in \text{messages} :$
 $\wedge m.\text{type} = \text{MClientRequest}$
 $\wedge \text{HandleClientRequest}(m.\text{dest}, m.\text{src}, m)$

$\text{NextHandleClientReply} \triangleq$
 $\exists m \in \text{messages} :$
 $\wedge m.\text{type} = \text{MClientReply}$
 $\wedge \text{HandleClientReply}(m.\text{dest}, m.\text{src}, m)$

$\text{NextHandleReconcileRequest} \triangleq$
 $\exists m \in \text{messages} :$
 $\wedge m.\text{type} = \text{MReconcileRequest}$
 $\wedge \text{HandleReconcileRequest}(m.\text{dest}, m.\text{src}, m)$

$\text{NextHandleReconcileReply} \triangleq$
 $\exists m \in \text{messages} :$
 $\wedge m.\text{type} = \text{MReconcileReply}$
 $\wedge \text{HandleReconcileReply}(m.\text{dest}, m.\text{src}, m)$

$\text{NextHandleRepairRequest} \triangleq$

$$\begin{aligned}
& \exists m \in \text{messages} : \\
& \quad \wedge m.type = MRepairRequest \\
& \quad \wedge HandleRepairRequest(m.dest, m.src, m) \\
NextHandleRepairReply & \triangleq \\
& \exists m \in \text{messages} : \\
& \quad \wedge m.type = MRepairReply \\
& \quad \wedge HandleRepairReply(m.dest, m.src, m) \\
NextHandleViewChange & \triangleq \\
& \exists m \in \text{messages} : \\
& \quad \wedge m.type = MViewChange \\
& \quad \wedge HandleViewChange(m.dest, m.src, m) \\
NextHandleViewChangeReply & \triangleq \\
& \exists m \in \text{messages} : \\
& \quad \wedge m.type = MViewChangeReply \\
& \quad \wedge HandleViewChangeReply(m.dest, m.src, m) \\
NextHandleStartView & \triangleq \\
& \exists m \in \text{messages} : \\
& \quad \wedge m.type = MStartView \\
& \quad \wedge HandleStartView(m.dest, m.src, m) \\
NextDropMessage & \triangleq \\
& \exists m \in \text{messages} : \\
& \quad \wedge Ack(m) \\
& \quad \wedge \text{UNCHANGED } \langle clientVars, replicaVars \rangle \\
Next & \triangleq \\
& \vee NextClientRequest \\
& \vee NextChangeView \\
& \vee NextHandleClientRequest \\
& \vee NextHandleClientReply \\
& \vee NextHandleReconcileRequest \\
& \vee NextHandleReconcileReply \\
& \vee NextHandleRepairRequest \\
& \vee NextHandleRepairReply \\
& \vee NextHandleViewChange \\
& \vee NextHandleViewChangeReply \\
& \vee NextHandleStartView \\
& \vee NextDropMessage \\
Spec & \triangleq Init \wedge \Box[Next]_{vars}
\end{aligned}$$

\ * Modification History
\ * Last modified Wed Sep 30 12:21:00 PDT 2020 by jordanhalterman

\ * Created *Fri Sep 18 22:45:21 PDT 2020* by *jordanhalterman*