THE UNIVERSITY OF MELBOURNE
SCHOOL OF COMPUTING AND INFORMATION SYSTEMS
COMP90020 DISTRIBUTED ALGORITHMS

Tutorial Week 3: Logical Clocks

Solutions

10.

- **LC1:** L_i is incremented before each event is issued at process p_i, $L_i := L_i + 1$
- **LC2:**
  (a) When a process p_i sends a message m_i, it piggybacks on m the value $t = L_i$
  (b) On receiving (m, t), a process p_j computes $L_j := max(L_j, t)$ and then applies **LC1** before time stamping the event *receive(m)*
- p_0 = $(s_1 = 1, s_2 = 2, r_5 = 5)$
- p_1 = $(r_2 = 3, s_5 = 4)$
- p_2 = $(r_1 = 2, a = 3, s_4 = 4, r_3 = 5, r_6 = 8)$
- p_3 = $(s_3 = 1, r_4 = 5, b = 6, s_6 = 7)$

11. Concurrent events aren't related by $a \rightarrow b$ or $b \rightarrow a$. We use $a||b$ to denote concurrent events. In the above example, $L(s_3) < L(a)$ whilst $s_3||a$.

12.

- Consider e and e' are successive events in the same process (LC1) or related to by $e = send(m)$ and $e' = recv(m)$ (LC2), then it must be the case that $L(e) < L(e')$. **(1)**
- Now assume $e_i \rightarrow e_j \Rightarrow L(e_i) < L(e_j)$ for all connected pairs of events i, j where $i < j$ in a sequence of length N or less. Following, e and e' are connected by a chain of events $e_1, e_2, ..., e_{N+1}$ at $m \geq 1$ processes such that $e = e_1$ and $e' = e_{N+1}$. Then it must be the case that $e \rightarrow e_N$, so $L(e) < L(e_N)$ by the induction hypothesis. Then by (LC1) and (LC2), $L(e_N) < L(e')$. Therefore by the transitive property, $L(e) < L(e_N) < L(e'), \forall N > 1$. **(2)**.

Combining **(1)** and **(2)**: $e \rightarrow e' \Rightarrow L(e) < L(e')$.

13.

- **VC1:** Initially, $V_i[j] = 0$ for $i, j = 1, 2, ..., N$.
- **VC2:** Just before p_i timestamps an event, it sets $V_i[i] := V_i[i] + 1$.
- **VC3:** p_i includes the value $t = V_i$ in every message it sends.
- **VC4:** When p_i receives a timestamp t in a message, it sets $V_i[j] := max(V_i[j], t[j])$ for $j = 1, 2, ..., N$. Taking the component wise maximum of the two vector timestamps.
- p_0 = $(s_1 = [1, 0, 0, 0], s_2 = [2, 0, 0, 0], r_5 = [3, 2, 0, 0])$
- p_1 = $(r_2 = [2, 1, 0, 0], s_5 = [2, 2, 0, 0])$

- p_2 = ($r_1 = [1, 0, 1, 0], a = [1, 0, 2, 0], s_4 = [1, 0, 3, 0], r_3 = [1, 0, 4, 1], r_6 = [1, 0, 5, 4]$)
- p_3 = ($s_3 = [0, 0, 0, 1], r_4 = [1, 0, 3, 2], b = [1, 0, 3, 3], s_6 = [1, 0, 3, 4]$)

14.

(a) Consider:

- Initially by VC1, $V_i[j] = 0$ for $i, j = 1, 2, ..., N$, therefore $V_j[i] \leq V_i[i]$
- Then by VC2, p_i is the source of any increment to $V[i]$. This occurs before timestamping any event. p_j increments $V_j[i]$ only as it receives messages containing a timestamp with a larger $V[i]$. Therefore $V_j[i] \leq V_i[i]$ follows.

(b) This is very similar to *exercise 13* and we can use a similar approach.

- Consider e and e' are successive events in the same process (VC2), or there is a message such that $e = send(m)$ and $e' = recv(m)$ (VC3,VC4). Then the result follows from VC2-VC4. **(1)**
- Now assume $e_i \rightarrow e_j V(e_i) < V(e_j)$ for all pairs of events $i, j = 1, 2, ..., N$ where $i < j$ in a sequence of events of length N or less. Following, $e = e_1$ and $e' = e_{N+1}$. Then it must be the case that $e \rightarrow e'$, so $V(e) < V(e_N)$ by the induction hypothesis. Then by VC2-VC4, $V(e_N) < V(e')$. Therefore by the transitive property, $V(e) < V(e_N) < V(e'), \forall N > 1$. **(2)**

Combining **(1)** and **(2)**: $e \rightarrow e' \Rightarrow V(e) < V(e')$.

(c) Let e and e' be concurrent ($e||e'$) and e occur at p_i and e' at p_j. Because these events are concurrent, it must be the case that no message from or to (p_i, p_j) has propogated to (p_j, p_i) at or after (e, e'). Using the finding in (a), we know that $V_j[i] < V_i[i]$ and also $V_j[j] > V_i[i]$. Hence neither $V(e') \geq V(e)$, nor $V(e') \leq V(e)$ can be true by our definitions of ordering. Therefore, if $V(e) < V(e')$ the two events cannot be concurrent and must be related by happened-before or $\neg e||e'$. If they are related by happened before, either $e \rightarrow e'$ or $e' \rightarrow e$. Of these two, we will show only $e \rightarrow e'$ is possible.
Suppose $e' \rightarrow e$, then either e' happened in the same process as e or there is a corresponding chain of messages between e' and e'. We have shown in 14(b) that $a \rightarrow a' \Rightarrow V(a) < V(a')$. So $e' \rightarrow e$ should imply $V(e) > V(e')$. However this contradicts $V(e') > V(e)$, therefore it cannot be the case that $e \rightarrow e'$.Hence, $V(e') > V(e) \Rightarrow e \rightarrow e'$. We can extend this further, by 14(b), $V(e') > V(e) \Leftrightarrow e \rightarrow e'$