MTH410

Quantitative Business Analysis

Module 6: Hypothesis Testing

The creation and testing of hypotheses serve as the foundation for all research studies. Before you can set out to prove something from your data, you need to focus your analysis. This focus can be achieved through the development of hypotheses. In this module we will examine data collection scenarios and learn how to create appropriate hypotheses. We will also understand the parameters surrounding accepting or rejecting our hypotheses and the types of errors we should try to avoid.

Learning Outcomes

1. Develop null and alternative hypotheses.
2. Make a conclusion and interpret the results of a one-mean hypothesis test.
3. Make a conclusion and interpret the results of a hypothesis test for a proportion.

For Your Success & Readings

This module will start to aggregate your learning in statistics and apply it to scenarios for real-life research. Through understanding and practicing the drafting of hypotheses, you will begin to understand the breadth of knowledge you can obtain from any data set, and you will see for yourself the challenge of focusing research studies.

In this module, we will cover more complex statistical calculations, so it will be important to move slowly through the readings and the example problems so that you can understand the distinction between one-tailed and two-tailed tests, and the use of the *z*-score and t-statistic. We are nearing the end of the required statistical knowledge needed for this course level. Hang on—you're almost there!

To navigate through this module successfully, keep the following in mind:

- This week, you will complete the fourth Critical Thinking Assignment. Review the assignment early in the week and contact your instructor if you have any questions or concerns.

Required

- Chapter 9 in *Introductory Business Statistics*

Recommended

- Akbar, M., Masyita, D., Febrian, E., & Buchory, H. (2018). **The impact of macroeconomics factor, capital structure and liquidity on the foreign bank's performance in Indonesia.** (https://search-proquest-com.csuglobal.idm.oclc.org/docview/2046134055/fulltextPDF/902C2C6F33064206PQ/1?accountid=38569) *Academy of Strategic Management Journal, 17*(2), 1–17.
- Taylor, C. (2018, Dec. 5). An introduction to hypothesis testing. *ThoughtCo*. Retrieved from **https://www.thoughtco.com/introduction-to-hypothesis-testing-3126336** (https://www.thoughtco.com/introduction-to-hypothesis-testing-3126336)

1. Developing Null and Alternative Hypotheses

As we have learned, samples can be used to develop point and interval estimates of population parameters. Statistical inference can also be used through hypothesis testing to determine whether a statement about the value of a population parameter should or should not be rejected.

Statisticians follow a formal process to determine whether to reject or fail to reject a **null hypothesis** based on sample data. This process, called **hypothesis testing**, consists of four steps:

The Four Steps of Hypothesis Testing

Develop the null and alternative hypotheses. The hypotheses should be stated in such a way that they are mutually exclusive. In other words, if one is true, the other must be false.

Formulate an analysis plan. The analysis plan describes how to use sample data to accept or reject the null hypothesis, which is usually done through a single test statistic.

Analyze sample data to find the value of the test statistic (mean score, proportion, t-score, z-score, etc.) described in the analysis plan.

Interpret results by applying the decision rule described in the analysis plan. If the test statistic supports the null hypothesis, fail to reject the null hypothesis; otherwise, reject the null hypothesis.

For example, if we wanted to determine whether a coin toss was fair and balanced, we would create the null hypothesis (H_0) that half of the flips would result in heads and half of the flips in tails. The alternative hypothesis (H_a) could be that the number of heads and tails would be very different. These hypotheses would be expressed as:

Suppose we flipped the coin 100 times, resulting in 60 heads and 40 tails. Given this result, we would reject the null hypothesis, as the result was not 50:50, and accept the alternative hypothesis.

There are three types of hypothesis tests: **lower tail**, **upper tail**, and **two-tailed**.

Lower Tail TestUpper Tail TestTwo-Tailed Test

For example, a newspaper article stated that the average weight of dogs is 25 lbs. Suppose you think that the average weight of dogs is less than 25 lbs. This is a lower tail test:

For example, a newspaper article stated that the average weight of dogs is 25 lbs. Suppose you think that the average weight of dogs is more than 25 lbs. This is an upper tail test:

For example, a newspaper article stated that the average weight of dogs is 25 lbs.

Suppose you think that the average weight of dogs is not 25 lbs. This is a two-tailed test:

The following table summarizes the three formats of hypothesis tests:

		Type of Test
equal: =	not equal: ≠	two-tailed
greater than or equal to: ≥	less than: <	left-tailed
Less than or equal to: ≤	greater than: >	right-tailed

Observe that the null and alternative hypotheses are opposites of each other. For example, if the null hypothesis is ,

then the alternative hypothesis is .

Ideally, the hypothesis testing should result in the acceptance of H_0 or H_a, but not both. However, when testing for the hypothesis, two types of errors can result:

Type I Error

A Type I error occurs when the researcher rejects a null hypothesis when it is true. The probability of committing a Type I error is called the **significance level**. This probability is also called **alpha**, and it is often denoted by .

Type II Error

A Type II error occurs when the researcher accepts a null hypothesis that is false. The probability of committing a Type II error is called **beta**, and it is often denoted by .

Here is a table that summarizes what a Type I and Type II error are based on the truthfulness of the null hypothesis:

		Truth	
		H_o True	H_o False
Your Findings	H_o True	Correct	Type II Error (β)
	H_o False	**Type I Error (α)**	Correct

By selecting the level of significance, you can control the probability of making a Type I error, but you cannot always control the probability of a making a Type II error. Therefore, if we accept H_0, we cannot determine how confident we can be with that decision through significance testing, so we can say that we “do not reject H_0” versus we “accept H_0.”

1.1. Identify Null and Alternative Hypotheses

Identifying the null and alternative hypotheses is an important step. The hypotheses will guide the rest of the problem-solving techniques.

EXAMPLE

A car manufacturer claims that the average gas mileage of its sedans is 30 miles per gallon or better. You question the results and believe the manufacturer overstated the results of the claim. Identify the null and alternative hypotheses.

Click "Solution" to check your thinking.

Solution

Let μ be the population mean gas mileage of the sedans produced by the manufacturer. It is a good idea to start with the claim, if there is any. The manufacturer claim is

The alternative hypothesis is that the mean gas mileage is less than 30 miles per gallon:

.

This is a left-tailed test.

EXAMPLE

A marketer believes that a different color of its packaging will increase the current average sales of $0.5 million per month. Identify the null and alternative hypotheses.

Click "Solution" to check your thinking.

Solution

Let μ be the population mean monthly sales. First identify the claim. The marketer claim is

This is a right-tailed test. The null hypothesis is thus

.

EXAMPLE

A business journal recently stated that 8% of all merchandise bought during the month of December gets returned. A business owner believes that the proportion of December returns is significantly different from 8%.

Click "Solution" to check your thinking.

Solution

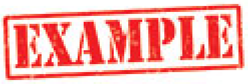
Let p be the population proportion of returned December merchandise. The business owner claim is

This is a two-tailed test. The null hypothesis is thus that the proportion of returned December merchandise is

.

1.2. Identify Type I and II Errors

When conducting hypothesis tests, there is always a possibility of two types of errors, Type I and Type II. The following are some examples on the two types of errors.



Let p be the population proportion of customers who use a new mailed coupon on a certain item. The claim is that more than 3% of the recipients use the coupon:

H_a: $p > 0.03$

This is a right-tailed test. Here are the null and alternative hypotheses:

H_0: $p \leq 0.03$
H_a: $p > 0.03$

A Type I error occurs by erroneously rejecting H_0. In this case, the Type I error occurs when the population proportion of customers who use a new mailed coupon is less than or equal to 3% ($p \leq 0.03$), yet we conclude that the proportion is more than 3%.

A Type II error occurs by erroneously failing to reject H_0. In this case, the Type II error occurs when the population proportion of customers who use a new mailed coupon is greater than 3% ($p > 0.03$), yet we fail to support that conclusion.

According to a news article, the mean price of a gallon of milk is $2.50. As a grocery store owner, you believe the average price of a gallon of milk is not $2.50. We have the following two-tailed test:

H_0: $\mu = 2.50$
H_a: $\mu \neq 2.50$

A Type I error occurs by erroneously rejecting H_0. In this case, the Type I error occurs when the population mean gallon of milk is $2.50 ($\mu = 2.50$), yet we conclude that the mean significantly differs from $2.50 ($\mu \neq 2.50$).

A Type II error occurs by erroneously failing to reject H_0. In this case, the Type II error occurs when the population mean gallon of milk significantly differs from $2.50 ($\mu \neq 2.50$), yet we fail to support that conclusion. We erroneously do not conclude that the mean price of milk significantly differs from $2.50.

2. Hypothesis Tests for the Mean

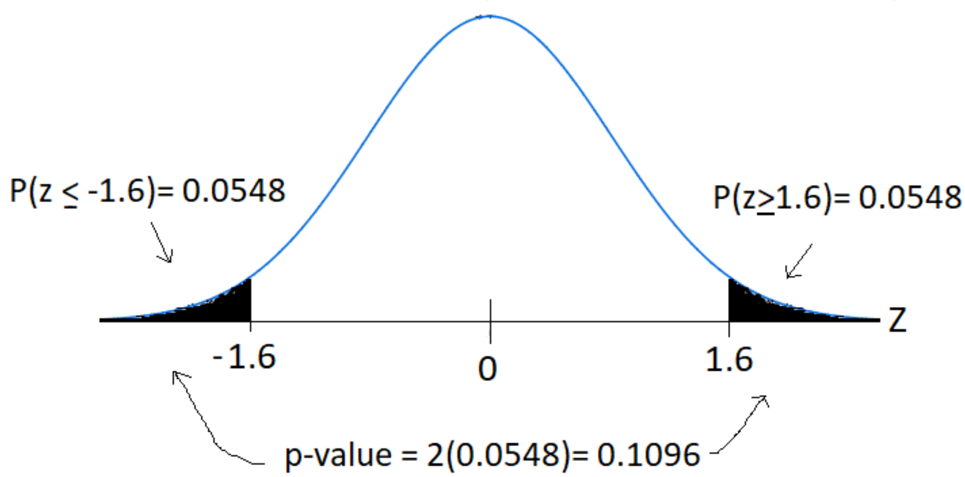
Your analysis plan should include decision rules for accepting or rejecting the null hypothesis. Statisticians describe these decision rules in two ways, with reference to a ***p*-value** or to a **rejection region**:

The strength of evidence in support of a null hypothesis is measured by the *p*-value. If the *p*-value is less than the significance level, we reject the null hypothesis.

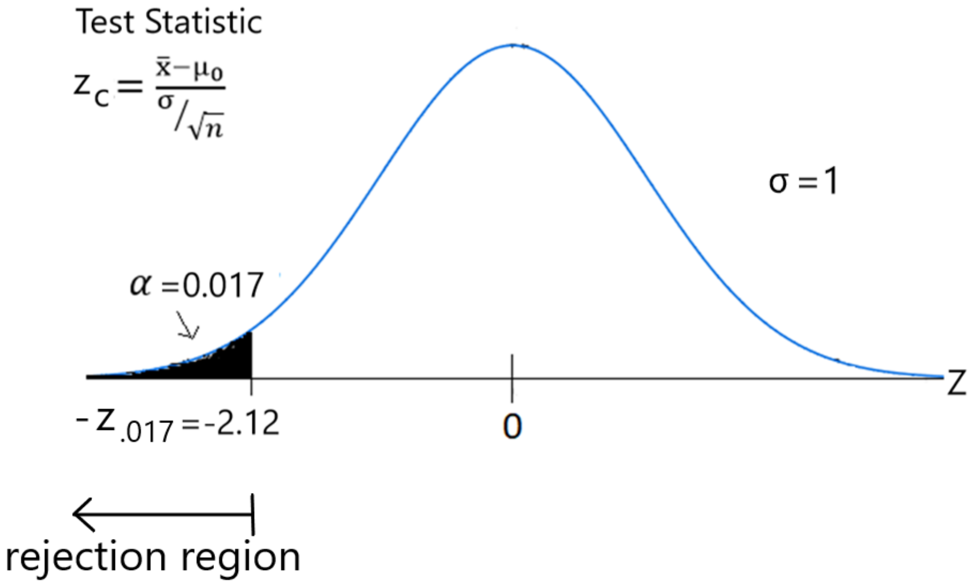
The rejection is a range of values. If the test statistic falls within the rejection region, the null hypothesis is rejected. If that outcome occurs, we say that the hypothesis has been rejected at the level of significance. The rejection region is defined so that the probability of making a Type I error is equal to the significance level.

In hypothesis testing, we encounter **one-** or **two-tailed tests** that establish acceptance or rejection of the null and alternative hypothesis. A one-tailed test means that only one end of the curve is used to test the hypothesis, while a two-tailed test considers both ends of the curve.

As shown in the graph below, the *p*-value for the two-tailed test is the sum of the two areas at both extreme ends of the distribution.



When using a one-tailed test, you are testing for the possibility of the relationship in one direction and completely disregarding the possibility of a relationship in the other direction. In this type of test, rejection of the null hypothesis occurs for values of the test statistic in one tail of its sampling distribution. If the critical value, Z_c , lies in the rejection region, then reject the null hypothesis. In the following display, the critical value formula uses the normal distribution and the sample standard deviation, σ, is known.



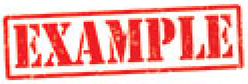
For example, if we say that the null hypothesis is that the mean is equal to 20 with a significance of 95%, a one-tailed test will test either if the mean is significantly greater than 20 or significantly less than 20, but not both. Depending on the chosen tail, the mean is either significantly greater than 20 or less than 20 if the test statistic is in the top 5% or bottom 5% of its probability distribution, resulting in a p-value less than 0.05. The one-tailed test provides more power to detect an effect in one direction by not testing the effect in the other direction.

If, however, you are using a two-tailed test, regardless of the direction of the relationship you hypothesize, you are testing for the possibility of the relationship in both directions. In this type of test, rejection of the null hypothesis occurs for values of the test statistic in either tail of its sampling distribution. For example, if you are using a significance level of 0.05, a two-tailed test takes half of your alpha to testing the statistical significance in one direction and half of your alpha to testing statistical significance in the other direction. This means that .025 is in each tail of the distribution of your test statistic. Suppose you say that the null hypothesis is that the mean is equal to 20 with a significance of 95%; a two-tailed test will test both if the mean is significantly greater than 20 *and* if the mean significantly less than 20. The mean is considered significantly different from 20 if the test statistic is in the top 2.5% or the bottom 2.5% of its probability distribution, resulting in a p-value less than 0.05.

Using hypothesis testing when analyzing data allows you to make decisions from those data. By establishing the level of significance you are willing to accept from the data, you are then able to accept or reject either H_0 or H_a as statistically significant and to draw conclusions from your data. When paired with point estimates of the population proportion or interval estimation, hypothesis testing allows us to use statistical evidence that supports or rejects our assumptions about a given set of data within our identified acceptable range of significance.

As with confidence intervals, **use the t-distribution if the sample size is less than 30 and the population standard deviation, σ, is unknown**, unless otherwise specified. Otherwise, use the normal distribution. See Table 9.4 on page 391 for a summary explanation of when to use the normal distribution versus when to use the t-distribution.

Note: In other textbooks, the t-distribution is **always** used if the value σ **is unknown,** even if the sample size is large.



A certain newspaper article claims that a college student watches less TV, on average, than the public. The national average is 29.4 hour per week with σ = 2 hours. A randomly selected sample of n = 30 students has a mean of 28.3 hours per week. Is there enough evidence to support the claim at the = 0.01 level of significance?

Click "Solution" to check your thinking.

Solution

This is a left-tailed test:

$- \quad - \quad - \quad - -3.01$

Since the sample standard deviation is known, we will use the normal distribution. The test statistic is

Method 1 – Critical Value/Rejection Region Approach

The critical value is:

Because the test statistic is less than or equal to the critical value (-.3.01 ≤ -2.33), then **reject** H_0. At the 0.01 level of significance, there is strong enough evidence to reject the claim that a college student watches an average of 29.4 hours or more.

Method 2 – *p*-value Approach

The *p*-value is this probability

The *p*-value is found in Appendix A on page 607. Appendix A only has positive z values:

Because the *p*-value is less than or equal to the level of significance (.0013 ≤ .01), **reject** H_0**.** At the 0.01 level of significance, there is strong enough evidence to reject the claim that a college student watches an average of 29.4 hours or more.

As mentioned previously, if the population standard deviation, σ is unknown and the sample size is less than 30, one uses the t-distribution.

The average time it takes for workers to complete a certain task at a large factory is five minutes. A manager claims that the average time is more than that. A random sample of 25 workers produces a sample mean of six minutes with a sample standard deviation of two minutes. Test the manager's claim at the 0.05 level of significance.

Click "Solution" to check your thinking.

Solution

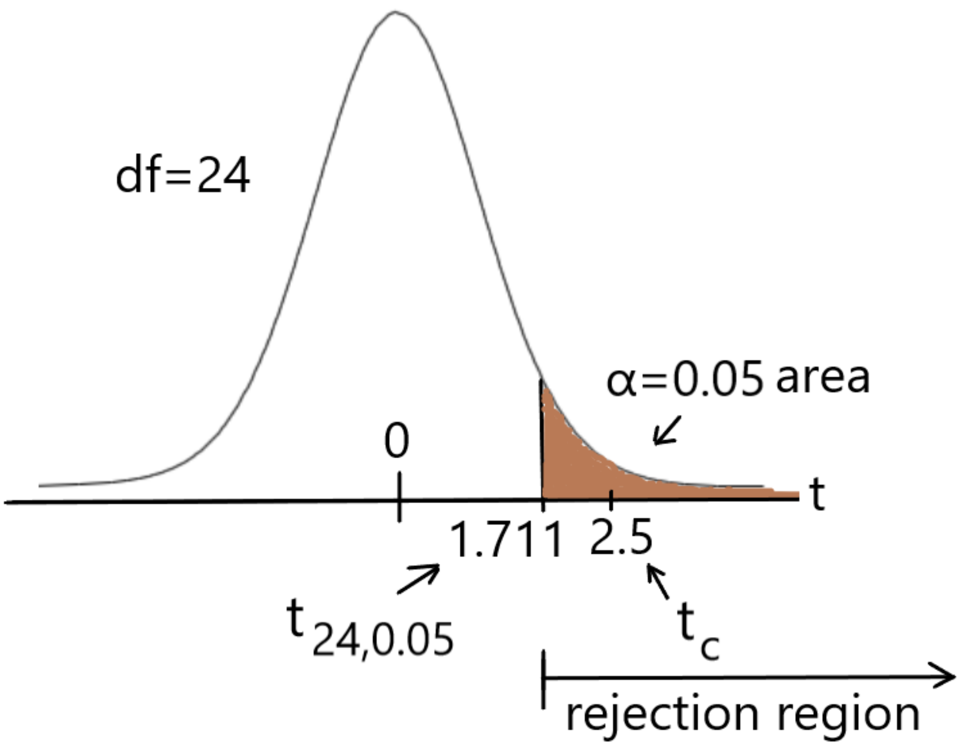
This is a right-tailed test:

– – – – 2.5

The test statistic is:

The degrees of freedom are =df=25-1=24 and the level of significance is =0.05.

The critical value is found by using Table A12 on pages 608-609 of Appendix A of the textbook (Student's t-Distribution).



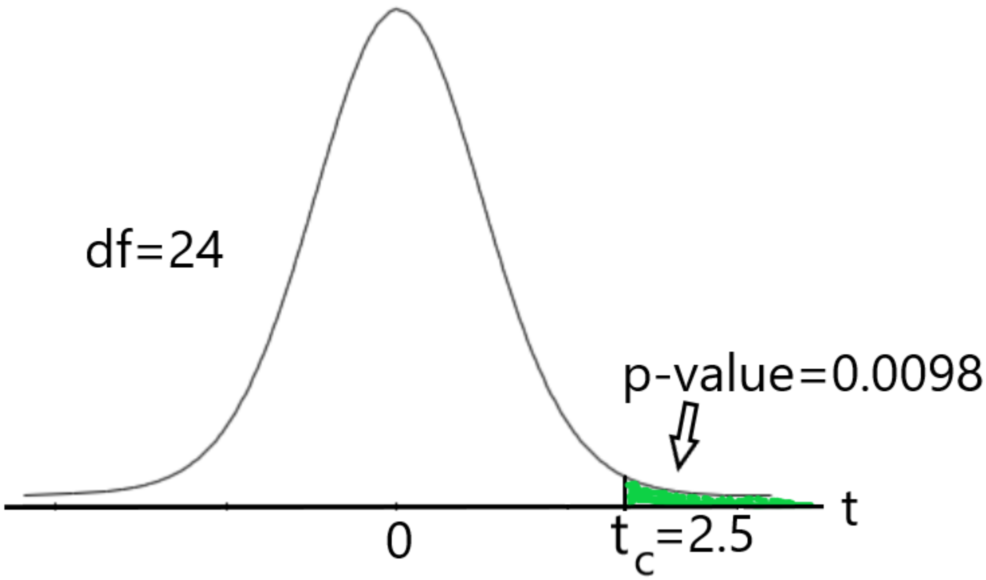
With **Excel** you may use this command, where probability is cumulative:

The Excel command will obtain :
=T.INV(0.05, 24)

Because the test statistic is greater than the critical value (2.5 > 1.711), reject At the 0.05 level of significance, there is strong enough evidence to accept the manager's claim that the average time to complete the task is more than 5 minutes.

Method 2: *p*-value Approach

Since this is a right-tail test, the *p*-value with *n*-1=25-1=24 degrees of freedom is the right-tail area:



We can use Excel to find cumulative probability:

=T.DIST(*x*, degrees of freedom, TRUE)

For example, this command will produce this left-tail probability $P(t < -2.5) = 0.00983$.

=T.DIST(-2.5, 24, TRUE)
Then by symmetry of the t-distribution: $P(t > 2.5) = P(t < -2.5) = 0.00983$

Because the *p*-value is less than the level of significance (0.0098 < 0.05), reject H_0 At the 0.05 level of significance, there is strong enough evidence to accept the manager's claim that the average time to complete the task is more than 5 minutes.

2.1. Hypothesis Tests for the Mean Using the Critical Value/Rejection Region Approach

The critical value/rejection region approach to hypothesis tests is based on the location of the test statistic relative to the critical value.

A marketing consultant claims that that a customer spends on average 20 minutes or more at a local store. A manager believes that a customer spends less than 20 minutes on average. The manager randomly chose 40 customers and observed that the sample average and sample standard deviation are 16.5 and 8 minutes, respectively. Test the claim at the 0.01 level of significance.

Click "Solution" to check your thinking.

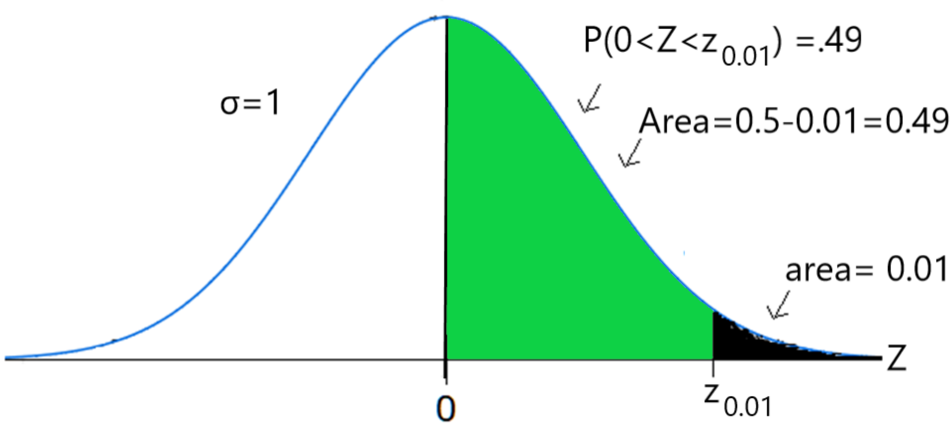
Solution

This is a left-tailed test:

Since the sample size is greater than or equal to 30, we will use the normal distribution. The test statistic is

The critical value is:

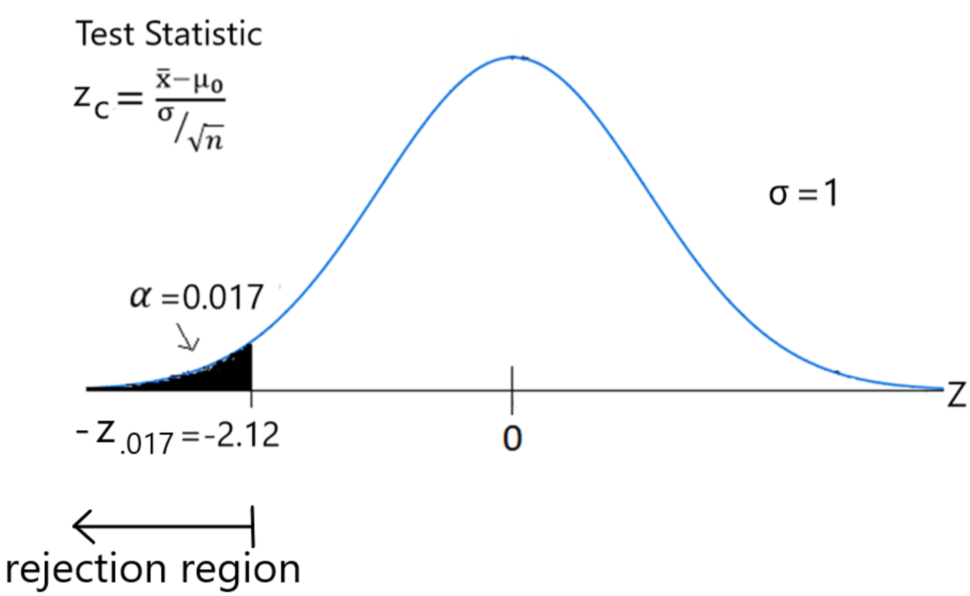
By symmetry of the normal distribution, the critical value can be found by finding the *z*-value whose right tail area is 0.01:



By using Appendix A on page 609, we look up the probability of 0.49. The probability value of 0.49 is between 0.4898 and 0.4901. We compute the average of the two z scores:

Thus, the critical value is

The critical value, , is greater than the critical value, -2.325 as shown:



The rejection region is all z values to the left of the critical value, -2.325. Thus, the critical value critical value, , is not in the rejection region. Therefore, fail to reject H_0. There is not strong enough evidence at the 0.01 level of significance to conclude that a customer spends less than 20 minutes on average.

2.2. Hypothesis Tests for the Mean Using the *p*-value Approach

The critical *p*-value approach to hypothesis tests is based on the probability of the tail area(s) of the test statistic.

EXAMPLE

A restaurant industry magazine contained an article stating that adults eat an average of 200 meals away from home per year. Because of fluctuations in the economy last year, a restaurant industry analyst believes that the average number of meals eaten away from home last year significantly differed from 200. The analyst collected a sample of 40 people from last year and obtained a sample mean and standard deviation of 190 and 30 meals, respectively. Test the claim at the 0.1 level of significance.

Click "Solution" to check your thinking.

Solution

This is a two-tailed test:

H_0: $\mu = 200$
H_a: $\mu \neq 200$

Since the sample size is greater than or equal to 30, we will use the normal distribution. The test statistic is

Since this is a two-tailed test, the p-value is the sum of the two tail areas under the standard normal distribution:

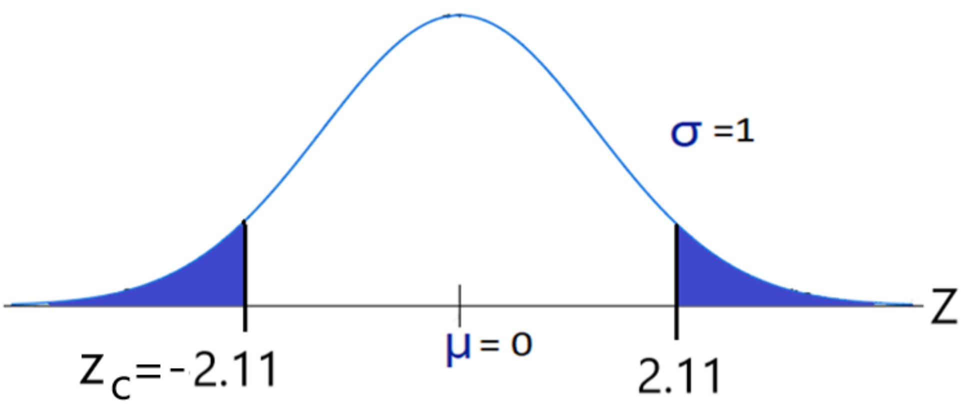


Table A11 in Appendix page 607 can help us find $P(Z > 2.11)$:

$P(Z > 2.11) = 0.5 - P(0 < Z < 2.11) = 0.5 - 0.4826 = 0.0174$

Thus, the p-value is:

Because the p-value is less than 0.1 (0.0348< 0.1), reject the null hypothesis. There is strong enough evidence at the 0.1 level of significance that the average number of meals eaten away from home last year significantly differed from 200.

3. Hypothesis Tests for Proportions

One may also construct a hypothesis test for the population proportion, p. The test statistic is based on the normal distribution:

p' is the sample proportion and p_0 is the hypothesized value. Also, q_0=1-p_0 and n is the sample size. As with confidence intervals, a requirement is that np'>5 and nq'>5. Recall that q'=1-p'.

Seven years ago, 60% of people got time off from work for personal reasons. A researcher wants to see if the percentage has changed. One hundred people were interviewed, and 53 of them got time off from work. At the =0.01 level of significance, has the percentage changed?

Click "Solution" to check your thinking.

Solution

The hypothesized value is p_0=0.60. This is a two-tailed test, since the researcher wants to know if the percentage has changed. The sample proportion is $p' = x/n = 53/100 = 0.53$

The hypotheses are

H_0: $p = 0.60$

H_a: $p \neq 0.60$

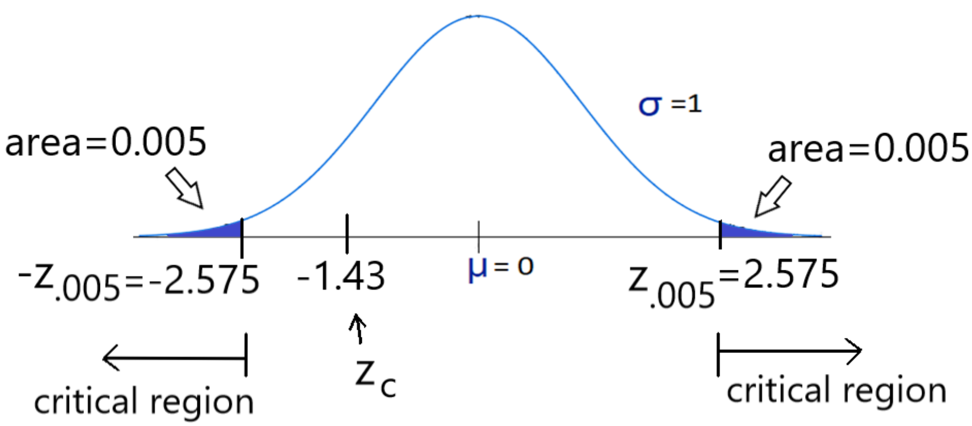
Before proceeding, we need to check that np'>5 and nq'>5:

np'=100(0.53) = 53 > 5 and nq'=100(1 - 0.53) = 100(0.47) = 47 > 5

The test statistic is:

Method 1 – Critical Value Approach

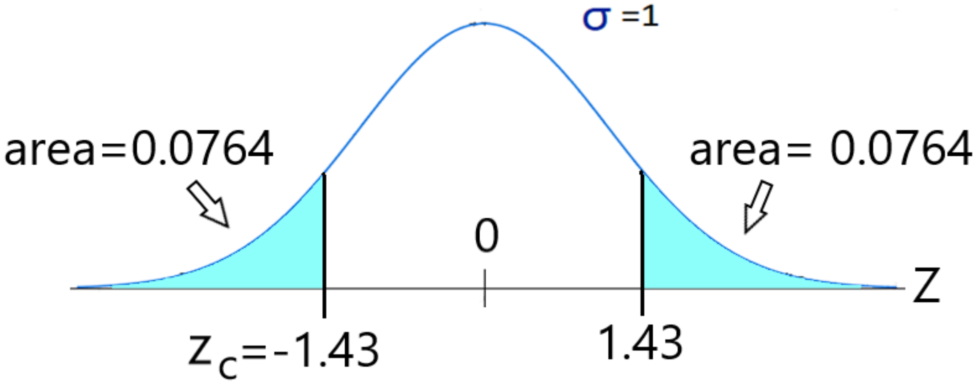
The critical values are



Because the test statistic, -1.43, is not less than -2.575 or not greater than 2.575, fail to reject the null hypothesis. In other words, because the test statistic, -1.43, is not in the critical region, fail to reject the null hypothesis. There is not strong enough evidence at the 0.01 level of significance that the percentage of workers who get time off for personal reasons differs from 60%.

Method 2 – *p*-value Approach

Since this is a two-tailed test, the *p*-value is the sum of the two tail areas under the standard normal distribution:



We used Table A11 in Appendix page 607 to find $P(Z>1.43)$:

$P(Z > 1.43) = 0.5 - P(0<Z<1.43) = 0.5 - 0.4236 = 0.0764$

Because the *p*-value greater than the level of significance (0.1528>0.01), fail to reject the null hypothesis. There is not strong enough evidence at the 0.01 level of significance that the percentage of workers who get time off for personal reasons differs from 60%.

3.1. Hypothesis Tests for Proportions Using the Critical Value/Rejection Region Approach

The critical value/rejection region approach to hypothesis tests for proportions is like the one for means. Let's look at an example.

A local newspaper reported that 60% of local supermarket shoppers believe that generic brand items are just as good as name brands. A grocery store manager believes the proportion is higher at his/her store. The manager collected a sample of 100 shoppers at his/her store and determined that 65% of those shoppers believe that generic brand items are just as good as name brands. Test the claim at the 0.025 level of significance.

Click "Solution" to check your thinking.

Solution

Let p represent the proportion of shoppers at the manager's grocery store who believe generic brands are just as good as name brands. This is a right-tailed test.

H_0: $p \leq 0.60$

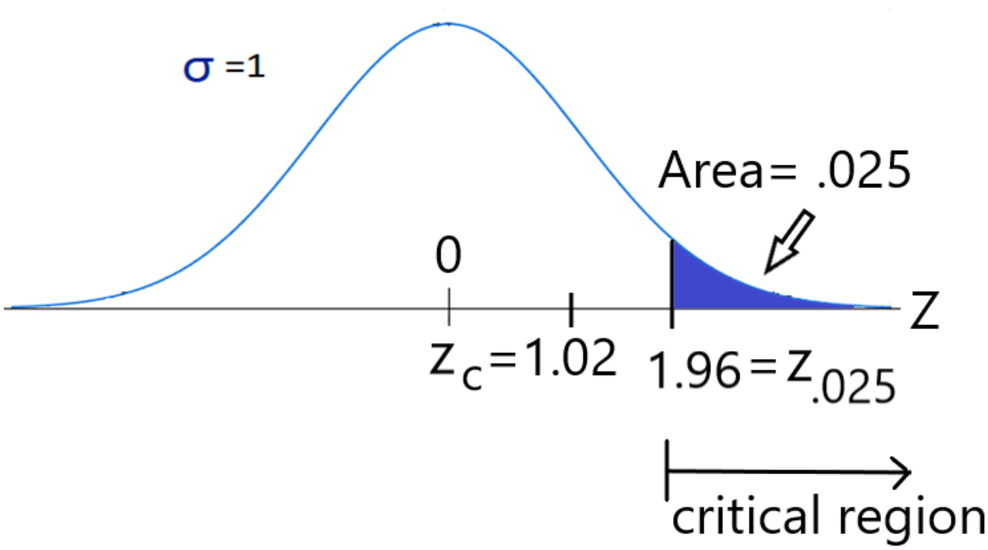
H_a: $p > 0.60$

Before proceeding, we need to check that $np' > 5$ and $nq' > 5$:

$np' = 100\,(0.65) = 65 > 5$ and $nq' = 100(1 - 0.65) = 100(0.35) = 35 > 5$

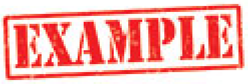
The test statistic is:

The critical value is



Since this is a right-tailed test and the test statistic is less than the critical value (1.02 < 1.96), fail to reject the null hypothesis. There is not enough evidence at the 0.025 level of significance to conclude that the proportion of customers who believe that generic brand items are just as good as name brands is higher at the manager's store.

3.2. Hypothesis Tests for the Proportions Using the *p*-value Approach



A restaurant manager claims that 75% or more of customers tip at least 15% of their meal cost. A survey of 60 customers produced the result that 68% of customers tipped at least 15%. Construct a hypothesis test at the 0.1 level of significance.

Click "Solution" to check your thinking.

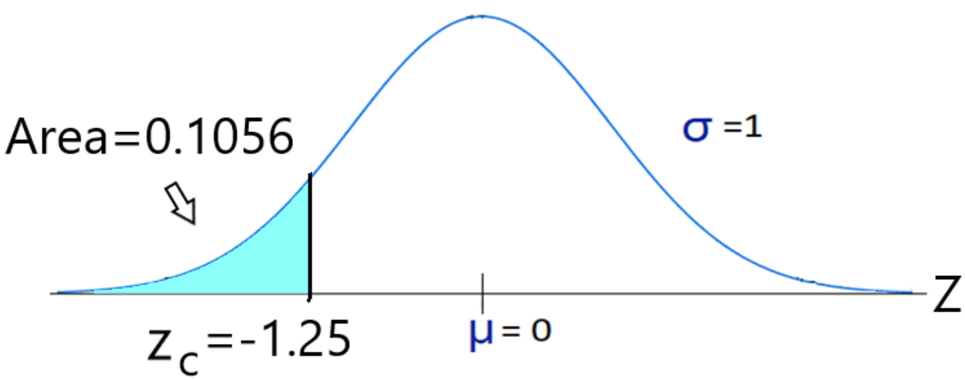
Solution

Let p represent the population proportion of customers who tip. This is a right-tailed test. The hypotheses are

H_0: $p \geq 0.75$
H_a: $p < 0.75$

The test statistic is

The *p*-value is 0.1056, the area to the left:



The *p*-value was calculated by using symmetry of the normal distribution and Table A11 on page 609:

$P(Z < -1.25) = P(Z > -1.25) = 0.5 – P(0 < Z < 1.25)= 0.5 – 0.3944 = 0.1056$

Since the *p*-value is greater than 0.10 (0.1056 > 0.10), fail to reject the null hypothesis. there is not enough evidence at the 0.10 level of significance to conclude that the proportion of customers who tip at least 15% is less than 75%.
Had the *p*-value been less than 0.10, then we would have rejected the null hypothesis.

4. Summary

In this module, we covered an introduction to hypothesis tests. we covered how to set up hypotheses and Type I and Type II errors. We then covered how to solve hypothesis test problems for the population means and proportions. There are two methods to solving hypothesis test problems, the *p*-value and the critical value/rejection region method. Both methods are useful, as you will see in the next two modules.

is the level of significance.

For the Population Mean, μ:	
Test Statistic	**When the Formula Can be Used (unless specified otherwise)**
	Normal Distribution:if the sample size, n, is 30 or more or the population standard deviation, σ, is known (s is the sample standard deviation)
	t-Distribution: if the sample size is less than 30 and the population standard deviation, σ, is unknown (degrees of freedom is n-1)

left-tailed	**right-tailed**	**two-tailed**
H_0: $\mu \geq \mu_0$ H_a: $\mu < \mu_0$	H_0: $\mu \leq \mu_0$ H_a: $\mu > \mu_0$	H_0: $\mu = \mu_0$ H_a: $\mu \neq \mu_0$

***p*-value:**
if z is used:
 	
if t is used:
	***p*-value:**
	if z is used:
	
	if t is used:
		***p*-value:**
		if z is used:
		
		if t is used:

critical value approach:	**critical value approach:**	**critical value approach:**
If z is used: Reject H_0 if	If z is used: Reject H_0 if	If z is used: Reject H_0 if or
If t is used: Reject H_0 if	If t is used: Reject H_0 if	If t is used: Reject H_0 if or
***p*-value approach:** reject H_0 if the p-value is less than		

For the Population Proportion, *p*:	
	Use if $np'>5$ and $n(1-p')>5$ ***P*' is the sample proportion**

left-tailed	**right-tailed**	**two-tailed**
$H_0: p \geq p_0$ $H_a: p < p_0$	$H_0: p \leq p_0$ $H_a: p > p_0$	$H_0: p = p_0$ $H_a: p \neq p_0$
***p*-value:**	***p*-value:**	***p*-value:**
critical value approach:	**critical value approach:**	**critical value approach:**
Reject H_0 if	Reject H_0 if	Reject H_0 if or
***p*-value approach:** reject H_0 if the *p*-value is less than		

Here is the list of the objectives that we have covered and are part of the Mastery Exercises in Knewton Alta:

- Identify the null and alternative hypotheses for an experiment with one population mean
- Distinguish between one- and two-tailed hypotheses tests and understand possible conclusions
- Differentiate between Type I and Type II errors when performing a hypothesis test
- Compute the value of the test statistic (z-value) for a hypothesis test for one population mean with a known standard deviation
- Determine the critical value(s) of a one-mean z-test at a given significance level to define a rejection region
- Make a conclusion and interpret the results of a one-mean hypothesis test (population standard deviation known) using the p-value Approach
- Compute the value of the test statistic (t-value) and degrees of freedom for a hypothesis test for one population mean with an unknown population standard deviation
- Conduct and interpret a one-mean hypothesis test using the Critical Approach with an unknown standard deviation
- Determine the p-value for a hypothesis test for the mean (population standard deviation unknown)
- Make a conclusion and interpret the results of a one-mean hypothesis test (population standard deviation unknown) using the p-value Approach
- Identify the null and alternative hypotheses for an experiment with one population proportion
- Compute the value of the test statistic (z-value) for a hypothesis test for a proportion
- Make a conclusion and interpret the results of a hypothesis test for a proportion using the Critical Value/Rejection Region Approach
- Determine the p-value for a hypothesis test for a proportion
- Make a conclusion and interpret the results for a hypothesis test for a proportion using the p-value Approach

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References

None