MTH410

Quantitative Business Analysis

Module 3: Probability

Do you ever wonder what the probability is of winning the lottery? Did you know that you can calculate it? In this module we will learn how to calculate probability, the relationships and assignments of probability, and how one can apply the Addition and Multiplication Rules.

Learning Outcomes

1. Apply basic addition and multiplication formulas for probability calculations.
2. Explain and apply the basic factors of probability for decision making.
3. Develop probabilities based on data sets.

For Your Success & Readings

At first glance, calculating probabilities seems easy. While calculating probabilities is not hard, it can be confusing, as the problems become more complicated. By diagramming the easier problems and then mathematically calculating the probabilities, you will be able to gain a good foundational grasp of how to use the formulas for more difficult probability calculations.

As in the last module, don't let the formulas intimidate you. Practice the textbook examples to understand the types of variables and take notes or keep a record that you can refer back to later in the course.

This week's discussion question asks you to consider if you use probability in your profession or real life.

Also, remember that there is a Live Classroom scheduled this week. While your attendance is optional, this is an excellent opportunity to discuss any questions, concerns, or other course-related topics with your instructor.

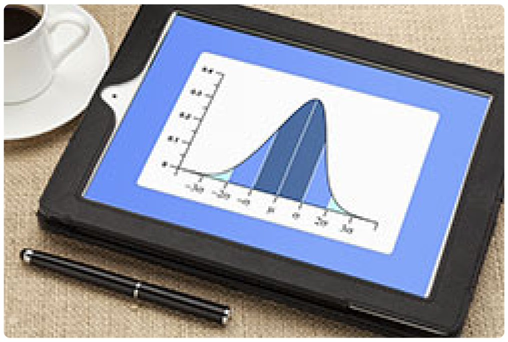
Required

- Sections 3.1-3.3, 3.4 up to page 157 in *Introductory Business Statistics*.

Recommended

- Opperman, A. (2018, Dec. 22). Bayes' theorem: The holy grail of data science. Retrieved from **https://towardsdatascience.com/bayes-theorem-the-holy-grail-of-data-science-55d93315defb** (https://towardsdatascience.com/bayes-theorem-the-holy-grail-of-data-science-55d93315defb)
- Simpson, A. (2018). **Category-theoretic structure for independence and conditional independence** (https://www-sciencedirect-com.csuglobal.idm.oclc.org/science/article/pii/S1571066118300318). *Electronic Notes in Theoretical Computer Science, 336*, 281–297.

1. Basics of Probability



Probability is a measure of “likeliness”—in other words, how likely is it that the event will happen? The more likely it is that the event will occur, the higher the probability number. Probability theory dictates that an event cannot be less than zero, nor greater than 1, as the probability of something definitely happening is 1, while the probability of nothing happening is zero. Probability can only be measured before an event occurs. Once an event has occurred, it is a certainty and no longer has probability. Percentages are probabilities that have been converted to a percentage and thus range from 0% to 100%, but the two are not interchangeable as solutions. Therefore, be sure to note what the question is asking for when writing your answer.

Glossary of Probability Terms

Use the tabs below to review six important terms related to probability:

Experiment

The **Experiment** is the activity or measurement that results in an outcome, such as rolling a die.

Sample Space

The **Sample Space** is all possible outcomes of an experiment. For a six-sided die, the sample space is {1, 2, 3, 4, 5, 6}, where the number of outcomes in the sample space is denoted by $n(S)$ and is equal to six.

Event

The **Event** is one or more of the possible outcomes of an experiment, basically a subset of the Sample Space. Rolling the die may give the event that you roll a 5. E={5} and $n(E)$ is the number of outcomes in the event, or one in this case. As another example, let E be the event of rolling an even number. Then E= {2,4,6} and $n(E)$=3.

Probability

The **Probability** of event A is a number between 0 and 1 that expresses the chance the event will occur.

Complement

The **Complement** of event A is all outcomes that are not in A. The complement of event A is labeled as A' The relationship between A and A' in terms of probabilities is: $P(A) + P(A') = 1$.

For example, suppose A is the event that it rains, and there is a 60% chance of rain today. Thus, $P(A)$= 0.6. Then, A' is the event that it does not rain and P(it does not rain) = $P(A')$ = 1-0.6 = 0.4.

As another example, suppose a box has chips numbered from 1 to 100. To find the probability that a chip is not a multiple of 10, then observe that there are 10 chips that are multiples of 10: P(not a multiple of 10)=1-P(multiple of 10) = 1- 10/100 = 1- 1/10 = 9/10.

Equally Likely

Equally likely means that each outcome of the experiment occurs with equal probability. For example, if a die is tossed, each event is equally likely with probability 1/6. An example of outcomes that are not equally likely is a shopping survey where the sample space is all shoppers in the US, yet only those shoppers in one store are randomly surveyed. Shoppers from other stores have no chance of being chosen.

The **union** of sets A and B is the event containing all sample points belonging to A or B or both. The union is denoted by $A \cup B$ The union $A \cup B$ is also written as A **or** B. For example, suppose $A = \{1,2,3,4,5,6\}$ and $B = \{4,5,6,7,8,9,10\}$. Then $A \cup B = \{1,2,3,4,5,6,7,8,9,10\}$. Note that the elements 4, 5, and 6 are not listed twice in the union.

The **intersection** of sets A and B is the event containing the sample points belonging to both A and B. The intersection is denoted as $A \cap B$. The intersection $A \cap B$ is also written as A **and** B. For example, suppose $A=\{1,2,3,4,5,6\}$ and $B=\{4,5,6,7,8,9,10\}$. Then $A \cap B = \{4,5,6\}$.

As another example, suppose a box has 10 red sports shirts, numbered 1 though 10, and 6 yellow sports shirts, numbered from 1 to 6. A shirt is randomly chosen. Suppose R and Y are the events of drawing a red and yellow chip, respectively. Suppose O and E are the events of drawing an odd and even shirt, respectively. Then a yellow shirt numbered 5 is an element of $Y \cap O$.

The notation $n(E)$ denotes the number of elements in the set E. For example, if $E=\{a, b, y\}$, then $n(E)=3$ elements in set E.

The choice of addition laws in the next page will depend on whether the events are mutually exclusive. **Mutually exclusive** events are those that have no elements in common, $A \cap B=\emptyset$. The notation $\emptyset$ is the **empty set**. The empty set, $\emptyset$, is the set with no elements in it. For example, $A=\{1,2,3\}$ and $B=\{5,6,7\}$ are mutually exclusive since they have no elements in common, $A \cap B=\emptyset$. As another example, suppose a pet store sells cats and dogs. The set of cats and the set of dogs are mutually exclusive because no pet can be both.

If events A and B are mutually exclusive, then $P(A \text{ and } B)=P(A \cap B)=0$. For example, if A and B are the set of cats and dogs, respectively, then $P(A \text{ and } B)=P(A \cap B)=0$. This is because no pet can be both a cat and a dog, $A \cap B=\emptyset$.

Classical Probability

An important type of probability is classical probability. **Classical probability** describes the theoretical probability of an event occurring given that there are equally likely outcomes. The probability that the event occurs is the number of possible outcomes in which the event occurs divided by the total number of possible outcomes. So, $P(E) = n(E)/n(S)$.

Classical probability can be used when all experimental probabilities are equally likely.

For example, consider tossing a die once. Let E denote the event that an even number appears. Notice that each outcome from 1 to 6 has an equally likely chance of occurring. Since there are three even numbers between 1 and 6, then $n(E) = 3$. Thus, $P(E) = n(E)/n(S) = 3/6 = 1/2 = 0.5$

As another example, suppose a box has red chips numbered 1, 2, green chips numbered 1, 2, 3, and purple chips numbered 1, 2, 3, 4. A chip is randomly chosen. To find the probability of a purple chip, observe that each of the 9 chips has an equal chance of being chosen. Thus, by the classical probability formula we get:

$P(\text{purple}) = 4/9$

See the following explanation of classical probability: **Statistics How To: Classical Probability: Definition and Examples** (https://www.statisticshowto.datasciencecentral.com/classical-probability-definition/)

If the outcomes are **not equally likely**, the classical probability formula does not apply:

A room has only 1 person. What is the probability that the person was born in the month of June? The sample space is all 12 months S = {January, February, ..., November, December}. The event is the set E = {June}. However, the classical probability formula $P(E) = n(E)/n(S) = 1/12$ is incorrect because each outcome (month) is not equally likely. For example, a person is more likely born in January than in June because January has 1 more day than June. The correct way to do the problem, assuming we know the person was not born in a leap year, is $P(\text{June}) = 30/365 = 30/365 = 6/73 = 0.0822$.

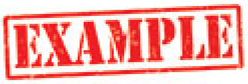
Empirical probability describes the observed probability of an event occurring in a very large number of trials. This is also known as the Law of Large Numbers. The **Law of Large Numbers** states that for a very large number of trials, the probability of the event occurring is the number of trials in which the event occurs divided by the total number of trials.

The difference between classical and empirical probability is that empirical probability is based on experimental observations. Empirical probability is used when experimental data are available from which to base estimates that can be repeated but are not as certain as they are in classical probability. For example, suppose a business owner surveyed many customers to determine the percentage of those who prefer the new service hours. The percentage is obtained experimentally, by conducting a survey. Hence, this is an example of empirical probability.

Counting Rules, Factorial Notation

The Counting Rule for Multiple-Step Experiments states that the number of ways in which a series of successive experiments can occur is found by multiplying the number of ways in which each experiment can occur. So, if the first event can occur ways and the second event occurs ways and the third event occurs ways, then the total number of ways this series can occur is the product

.



Suppose a men's formal wear store owner sells 5 types of dress pants, 6 types of shirts, 7 types of ties, and 9 types of suits. How many types of formal wear are there?

Click "Solution" to check your thinking.

Solution

By the Counting Rule for Multiple-Step Experiments, there are types of formal wear.

is the product of all positive integers from n down through 1:

by definition.

For example,

Factorial notation also occurs in sequences. For example, the first three terms of the sequence are

It turns out, by the Counting Rule for Multiple-Step Experiments, that $n!$ is the number of ways to arrange n objects on a line.

For example, if four customers are to wait in line at the cafeteria, then the number of ways that they can be arranged in the line is computed as follows by the Counting Rule for Multiple-Step Experiments:

There are four ways to choose the person that will be first in line. Once that has been done, that leaves three people who can be second in line. Similarly, there are two people who can be third in line. Finally, that leaves one person who can be last. Hence, by the Counting Rule for Multiple-Step Experiments, the number of ways that four customers can wait in line at the cafeteria is .

1.1. More on Probability for Equally Likely Outcomes

When computing probability, it is sometimes useful if we list all the elements of the sample space. Recall that the sample space is the set of all possible outcomes when a probability experiment is performed once.

A red and a blue six-sided die are tossed at the same time. Find the probability that the sum of the two dice is 5.

Click "Solution" to check your thinking.

Solution

In this case, it would be beneficial to list all of the possible outcomes, the sample space:

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

For example, (5,3) can denote that the red die is a 5 and the blue die is a 3. Also, the sum of the result (5,3) is 5+3=8. Observe that there are a total of 36 possible outcomes. In other words, the sample space consists of 36 elements.

Note that the sum of 5 is observed diagonally along the grid as shown:

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

For example, (3,2) means that the red die is 3 and the blue die is 2. Also, 3+2=5. As shown above, there are 4 outcomes whose sum is 5. Also, each of the 36 outcomes are equally likely. Thus, the probability of the dice adding up to 5 is:

To find the probability that the sum is not 5, use the rule of complements:

To find the probability that the red die is a 2, observe that the second row consists of all outcomes whose red die is a 2:
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)

Thus, the probability of that the red die is a 2 is as follows:

1.2. Factorial Notation Examples

Recall that factorial notation may be used when arranging objects on a line. In addition, factorial notation is useful in other counting problems and in probability formulas. You will see factorial notation in Module 4 when we cover the binomial distribution.

EXAMPLE

Suppose that the k^{th} term is given by

Find the first five terms, .

Click "Solution" to check your thinking.

Solution

The first term is found by replacing k by 1 in the formula for :

The second term is found by replacing k by 2 in the formula for :

The other three terms are similarly found by replacing k by 3, 4, and 5 in the formula for , respectively:

EXAMPLE

Suppose that the k^{th} term of the sequence is given by

Find the first four terms, .

Click "Solution" to check your thinking.

Solution

The first term is found by replacing *k* by 1 in the formula for :

The second term is found by replacing *k* by 2 in the formula for :

The third term is found by replacing *k* by 3 in the formula for :

Finally, find the fourth term by replacing *k* by 4 in the formula for :

2. Addition Rules

Addition Rule for Mutually Exclusive Events

Recall that **mutually exclusive** events are those that have no elements in common, $A \cap B = \emptyset$.

If A and B are mutually exclusive events, then:

P(A or B) = $P(A \cup B) = P(A) + P(B)$.

For example, suppose a card is randomly chosen from a 52-card deck. Let
H = the event that a card is Hearts
S = the event that a card is Spades

The events H and S are mutually exclusive since a card cannot be both Hearts and Spades. Thus, the probability that a card is Hearts or Spades is:

$P(H$ or $S) = P(H \;\; S) = P(H) + P(S) = 13/52 + 13/52 = 26/52 = 0.5$

Addition Rule

If A and B are events, then:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

The Addition Rule is a generalization of the one for mutually exclusive events because if A and B are mutually exclusive, then $P(A \;\; B) = 0$.

For example, suppose a card is randomly chosen from a 52-card deck. Let

H = the event that a card is Hearts
S = the event that a card is a Queen

Then the probability that a card is Hearts or Queen is:
P(Hearts Queen) = P(Hearts) + P(Queen) - P(Hearts Queen) = 13/52 + 4/52 - 1/52 = 16/52 = 4/13 = 0.308

The events H and Q are **not** mutually exclusive since a card can be both a Hearts and Queen at the same time, the Queen of Hearts. Thus, we must use the regular Addition Rule rather than the one for mutually exclusive events.

EXAMPLE A business sells blouses and dresses from two different clothing companies:

	Company *A*	Company *B*	Total
Blouses	56	49	105
Dresses	78	83	161
Total	134	132	266

(a) Find the probability that an item is from company *A* or is a dress.

(b) Find the probability that an item is from Company *A* or *B*.

Click "Solution" to check your thinking.

Solution

(a) Using the Addition Rule, we get

$P(A \cup \text{Dress}) = P(A) + P(\text{Dress}) - P(A\ \ \text{Dress}) = 134/266 + 161/266 – 78/266 = 217/266 = 0.8158$

(b) Since an item cannot be from both companies, then *A* and *B* are mutually exclusive:

$P(A \cup B)= 134/266 + 132/266 =1$

The answer of 1 was expected since all items are from either company *A* or company *B*.

In part (a) of the above example, we can find the probability that an item is **not** from company *A* and is **not** a Dress. We can use the rule of complements:

P(**not** from company *A* and is **not** a Dress)= 1 - *P*(from company *A* or is a Dress) = 1 - $P(A\ \ \text{Dress})$ = 1-0.8158 = 0.1842

2.1. Mutually Exclusive Versus Not Mutually Exclusive Events

As previously mentioned, mutually exclusive events cannot occur simultaneously. This means that *P*(*A* and *B*)=0.

Suppose *A* and *B* are events and *P*(*A* or *B*)=0.60. In addition, assume *P*(*A*)=0.40 and *P*(*B*)=0.20. Are *A* and *B* mutually exclusive?

Click "Solution" to check your thinking.

Text

By the Addition Rule:

P(*A* or *B*)=*P*(*A*)+*P*(*B*)-*P*(and *B*),

we have

0.60 = 0.40 + 0.20 - *P*(*A* and *B*)

0 = -*P*(*A* and *B*)

P(*A* and *B*)=0

Hence, *A* and *B* are mutually exclusive.

Suppose a card is randomly drawn from a deck of 52. Let *A* be the event that an Ace is drawn. Let *R* be the event that a red card is drawn. Events *A* and *R* are not mutually exclusive because there are cards that can be both red and an Ace (Ace of diamonds and Ace of hearts). In fact, we can find the probability of selecting an Ace or a red card. There are 4 ace cards and 13 red cards. Using the Addition Rule of probability, we get:

P(*A* or *R*) = *P*(*A*)+*P*(*R*)-*P*(*A* and *R*) = 4/52 + 13/52 - 2/52 = 15/52 = 0.288

In addition, if K is the event that a King is drawn, then A and K are mutually exclusive. This is because a card cannot be both an Ace and a King. Thus, $P(A \text{ and } K)=0$. Using the Addition Rule of probability and the fact that A and K are mutually exclusive, we get

$P(A \text{ or } K) = P(A)+P(K) = 4/52 + 4/52 = 8/52 = 2/13 = 0.154$

Finding $P(A \text{ or } B)$ is easier to compute if we know that A and B are mutually exclusive.

2.2. More on the Addition Formulas

In this section, we cover different ways to compute probability using the Addition Formulas.

EXAMPLE

Twenty-one percent of cars of Brand A have leather seats. Fifteen percent of cars of Brand A have navigation. Thirty-five percent of cars of Brand A have leather seats or navigation. Find the percentage of cars of Brand A having both leather seats and navigation.

Click "Solution" to check your thinking.

Solution

By the Addition Rule, we get:

P(Leather ∪ Navigation) = P(Leather) + P(Navigation) - P(Leather ∩ Navigation)

0.35 = 0.21 + 0.15 - P(Leather ∩ Navigation)

0.35 = 0.36 - P(Leather ∩ Navigation)

P(Leather ∩ Navigation) = 0.36 - 0.35 = 0.01

The following is a probability example that uses a table.

EXAMPLE

A business sells blouses, dresses, and shoes from three different clothing companies:

	Company A	Company B	Company C	Total
Blouses	56	49	56	161
Dresses	78	83	61	222
Shoes	60	76	70	206
Total	194	208	187	589

Find the probability that an item is from company A or company B.

Click "Solution" to check your thinking.

Solution

Notice that an item of clothing cannot come from both companies. Thus, using the Addition Rule for disjoint events, we get $P(A \cup B) = P(A) + P(B) = 194/589 + 208/589 = 402/589 = 0.6825$

3. Multiplication Rules

Multiplication rules of probability help to determine the probability that two or more of the events will occur. These rules depend on whether events are independent or dependent.

Independent

Events are **independent** events if the occurrence of one event has no effect on the probability that another will occur. For example, the event that the motor on a car fails is independent of the event that the carpet of the car gets one or more stains. As another example, suppose one die is rolled on trial 1 and two dice are rolled on trial 2. The outcomes of the two trials are independent since the outcome of the first trial will not affect what happens on the second trial.

Dependent

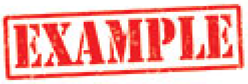
Events are **dependent** if the occurrence of one event changes the probability that another will occur. For example, the event that it snows is dependent on the event that it is cloudy. As another example, getting an A on an exam is dependent on whether the student studies. A third example is drawing a card from a deck without replacement on trial 1 and then drawing a card again. The outcomes of the two trials are dependent since the outcome of the first trial will affect what happens on the second trial. For example, if an Ace is obtained on trial 1, then there is one less Ace in the deck. This will affect the probability of getting an Ace on trial 2.

Multiplication Rule for Independent Events

If A and B are independent events, then:

.

This can be extended for multiple events that are independent by multiplying their probabilities together.



Suppose a die and coin are tossed at the same time. Find the probability that tails is obtained on the coin and a six is obtained on the die.

Click "Solution" to check your thinking.

Solution

Let T and S be the events that tails and a six are obtained, respectively. T and S are independent. Thus,

Let be the event that a couple have a boy was in the first birth. Suppose and are similarly defined. Find the probability that the couple has three boys.

Click "Solution" to check your thinking.

Solution

B_1, B_2, and B_3 are independent of each other since each gender has no effect on the gender result of the others. The probability of having three boys is thus:

The probability of an event can be influenced by whether a related event has already occurred. We may already have a probability that event A may occur, but if we gather new information and learn that a related event B has already occurred, we would take advantage of that information by calculating a new probability for event A. This new probability is called a **conditional probability, $P(B|A)$.**

$P(B|A)$ denotes the probability of event B given that event A has occurred. For example, P(snow | winter) means the probability of snow given that it is winter. We know that P(snow | winter) is different from P(snow | summer).

The **Multiplication Rule** is:

For example, if $P(A \text{ and } B)=0.43$ and $P(A|B)=0.72$, then to find $P(B)$ use:

Dividing both sides by $P(B)$, we get $P(B) = 0.43/0.72 = 0.597$.

EXAMPLE

Suppose a box has 7 red chips and 5 blue chips. A chip is randomly selected and *not* put into the box. A chip is randomly chosen again. Find the probability of a red chip on trial 1 and red chip on trial 2.

Click "Solution" to check your thinking.

Solution

The trials are not independent because the outcome of trial 1 affects the possible outcomes of trial 2. This is an example of **sampling without replacement**. Let r_1 denote the event of a red chip on trial 1. Let r_2 denote the event of a red chip on trial 2. By the multiplication rule, we get:

The Multiplication Rule is a generalization of the one for independent events. This is because if A and B are **independent**, then $P(B|A) = P(B)$. Then we would have

For example, if events A and B are independent and $P(A) = 0.45$ and $P(B) = 0.53$, then $P(A|B) = P(A) = 0.45$. This is because the likelihood of A is unaffected by the occurrence or non-occurrence of B.

EXAMPLE

Suppose a box has 7 red chips and 5 blue chips. A chip is randomly selected and put into the box. A chip is randomly chosen again. Find the probability of a red chip on trial 1 and red chip on trial 2.

Click "Solution" to check your thinking.

Solution

The trials are independent because the outcome of trial 1 does not affect the possible outcomes of trial 2. This is an example of **sampling with replacement**. Let r_1 denote the event of a red chip on trial 1. Let r_2 denote the event of a red chip on trial 2. By the multiplication rule, we get:

Compare this method with the one involving sampling without replacement.

From the definition $P(A|B) = P(A)P(B|A)$, the conditional probability $P(B|A)$ is easily obtained by dividing both sides by $P(A)$, provided that $P(A)$ is not zero:

Conditional Probability Formula:

EXAMPLE

A sales manager gave her trainees two exams. Thirty percent of the trainees passed both exams and 70% of the trainees passed the first test. What percent of those trainees who passed the first test also passed the second test?

Click "Solution" to check your thinking.

Solution

Independent or Mutually Exclusive?

Events A and B are independent if .

Events A and B are mutually exclusive if .

3.1. More on the Multiplication Formulas

In this section, we cover different ways to compute probability using the Multiplication Formulas.

EXAMPLE

A die is tossed. Find the probability that a 2 is tossed given that an even number was rolled.

Click "Solution" to check your thinking.

Solution

EXAMPLE

In game of cards, suppose a player needs to draw two cards of the same suit to win. Of the 52 cards, there are 13 cards in each suit. There are 4 suits: spades, clubs, hearts and diamonds. Suppose the player first draws a spade. Now the player wishes to draw a second spade. Since one spade has already been chosen, there are now 12 spades remaining in a deck of 51 cards. Thus, the conditional probability P (spade on second draw| spade on first draw) is as follows:

EXAMPLE

A business sells blouses, dresses, and shoes from three different clothing companies:

	Company A	Company B	Company C	Total
Blouses	56	49	56	161
Dresses	78	83	61	222
Shoes	60	76	70	206
Total	194	208	187	589

(a) Find the probability that an item is from company A and is a dress
(b) Out of all of the dresses, find the probability that an item is from company *A*.

Click "Solution" to check your thinking.

Solution

(a) Let *D* denote the event that a dress is chosen. There are 78 items that are both from company *A* and are dresses out of a total of 589 items:

(b) We are given that an item is a dress. Thus, ~~we~~ will use part (b) and the conditional probability formula:

3.2. Independent Versus Not Independent Events

Recall that events A and B are independent if the occurrence or nonoccurrence of A affects the probability of B, or vice versa. The following is an example involving conditional probabilities and events that are not independent:

A box has 6 red chips, numbered from 1 to 6. The same box has 8 blue chips, numbered from 7 to 14. Thus, the box initially has a total of 14 chips. Suppose a chip is randomly selected but not put back into the box. Suppose a chip is randomly picked again, without being replaced. Finally suppose a chip is randomly picked from the box for the third time. What is the probability of obtaining blue chips on all three trials?

Click "Solution" to check your thinking.

Solution

Let B_1, B_2, and B_3 denote the events that a blue chip is obtained on trials 1, 2, and 3, respectively. We want to find $P(B_1 \text{ and } B_2 \text{ and } B_3)$.

Notice that trials are **not independent** since the outcome of a previous trial will affect the result of the next trial. This is because the trials are **without replacement**. By the product rule, recall

$P(A \text{ and } B) = P(A \mid B)P(B)$

Thus, using the product rule above, we get

$P(B_1 \text{ and } B_2 \text{ and } B_3) = P((B_1 \text{ and } B_2) \text{ and } B_3) = P(B_3 \mid B_1 \text{ and } B_2)P(B_1 \text{ and } B_2) = (6/12)\, P(B_1 \text{ and } B_2)$

The 6/12 term is the probability of a blue chip on the third trial given that the other two prior trials were blue. Here is an explanation of how the numerator and denominator of 6/12 were obtained:

Since the other 2 trials were blue, then there are 6 (8-2=6) blue chips on trial 3. Since the 2 chips were not replaced, there is a total of 12 (14-2=12) chips on trial 3.

So far, we have:

$P(B_1 \text{ and } B_2 \text{ and } B_3) = (6/12)\, P(B_1 \text{ and } B_2)$

Using the same formula mentioned above for the product rule, we have:

$P(B1 \text{ and } B2 \text{ and } B_3) = (6/12)\, P(B_1 \text{ and } B_2) = (6/12)P(B_1)P(B_2 \mid B_1) = (6/12)(8/14)(7/13)$

The 7/13 is the probability of blue on trial 2 given that a blue was chosen on trial 1. On trial 2, there are 7 (8-1=7) blue chips if a blue chip was obtained on trial 1.

The final answer is thus:

$P(B_1 \text{ and } B_2 \text{ and } B_3) = (6/12)(8/14)(7/13) = (6/12)(7/13)(8/14)= 0.154$

Recall that if events A and B are independent, then the formula

$P(A \text{ and } B) = P(A \mid B)P(B)$

becomes
$P(A \text{ and } B) = P(A)P(B)$

This is because the probability of A is unaffected by the occurrence or nonoccurrence of B:

$P(A \mid B) = P(A)$

The computation of $P(A \text{ and } B)$ is easier to compute if A and B are independent. For example, if the chips were placed back into the box (**trials with replacement**), the answer would be easier to compute. This is because we would have **independent trials**:

$P(\mathrm{B}_1 \text{ and } B_2 \text{ and } B_3) = P(\mathrm{B}_1)P(\mathrm{B}_2)P(\mathrm{B}_3) = (8/14)(8/14)(8/14) = 0.187$

4. Summary

Probability forms the basis of inferential statistics. We covered the Counting Rule for Multiple-Step Experiments. The counting rule is useful in many probability computations. We also covered the notion of probability and classical probability. Among the two most important probability rules are the Addition and the Multiplication Rules.

Here is the list of the objectives that we have covered and are part of the Mastery Exercises in Knewton Alta:

- Understand definitions of events, outcomes, trials, independent/dependent events, and mutually exclusive events
- Use and, or, and not notation to describe events
- Use conditional probability notation to describe events
- Compute basic probability in a situation where there are equally-likely outcomes
- Compute probability involving and, or, and not
- Compute probability using the complement rule
- Understand mutually exclusive events
- Find the conditional probabilities of independent and mutually exclusive events
- Distinguish between independent or mutually exclusive events given conditional probability information
- Use the multiplication rule for conditional probabilities
- Use the multiplication rule for independent event probabilities
- Use the addition rule for probabilities
- Use the addition rule for mutually exclusive event probabilities
- Use factorial notation

Check Your Understanding

Answer each scenario and then type your answer in the blanks pace. Round your answer as each problem instructs.

Embedded Media Content! Please use a browser to view this content.