



Privacy-preserving inference on DNNs with MHE

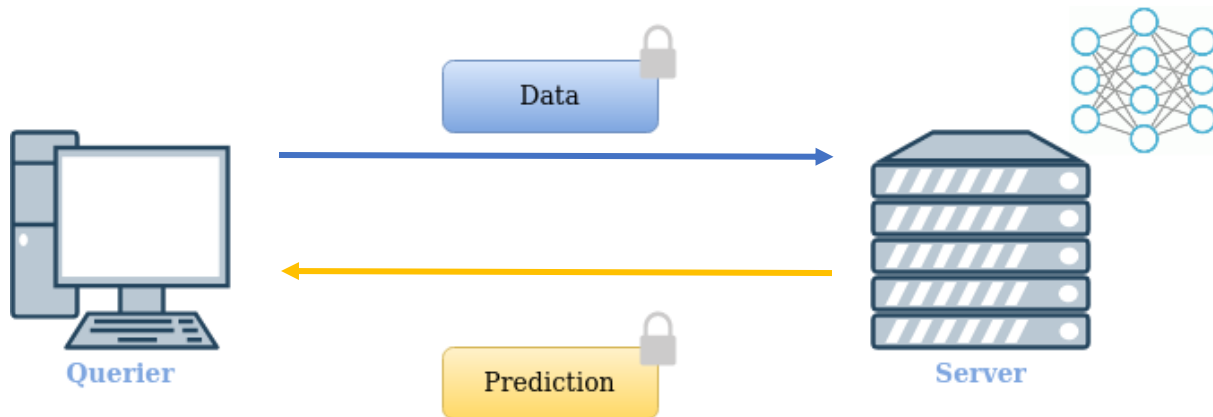
Pictures for publication

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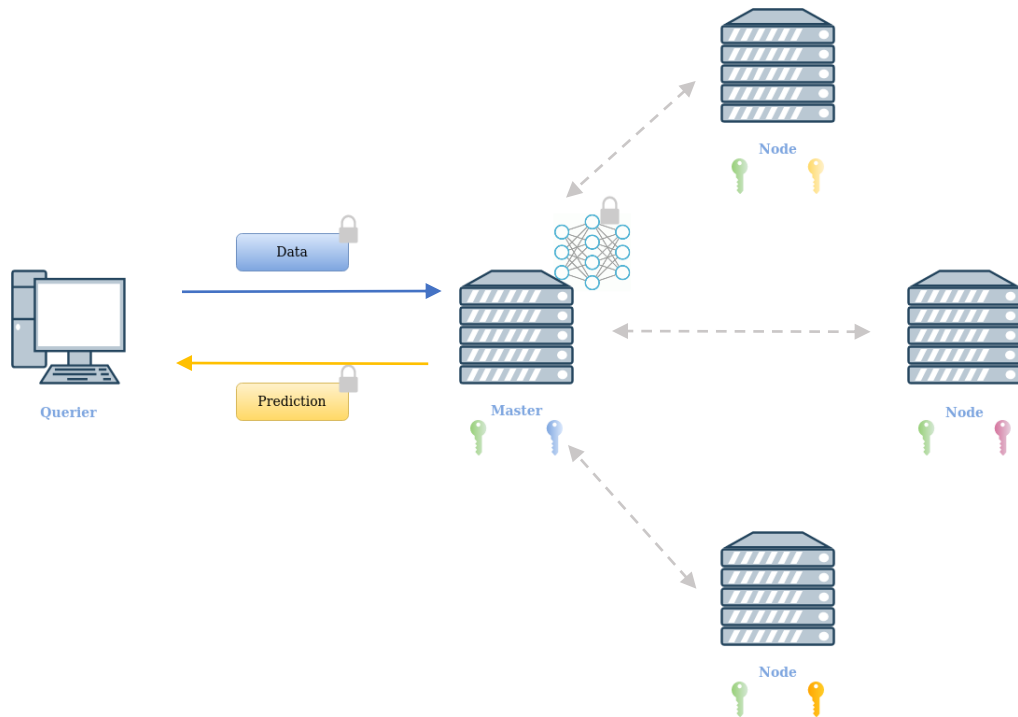


- We can present 4 experiments:
 1. Cryptonet: Model in clear – data encrypted (usual HE inference)
 2. Cryptonet: Model encrypted – data in clear (model is exported to client. Model owner offers an oblivious decryption service)
 3. NN20: Model encrypted – data encrypted (Poseidon setting with MHE training and distributed protocols)
 4. NN50: Model in clear – data encrypted (usual HE inference with Centralized Bootstrapping)

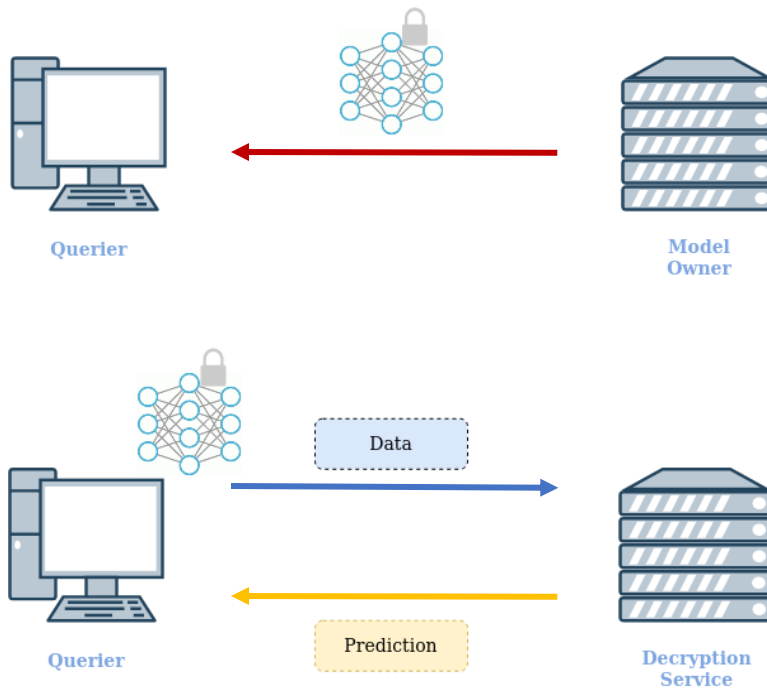
Experiments – Scenario 1



Experiments – Scenario 2



Experiments – Scenario 3





Methods - Matrix Multiplication

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} w_{00} & w_{01} & w_{02} \\ w_{10} & w_{11} & w_{12} \\ w_{20} & w_{21} & w_{22} \end{bmatrix} = \begin{bmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{bmatrix}$$

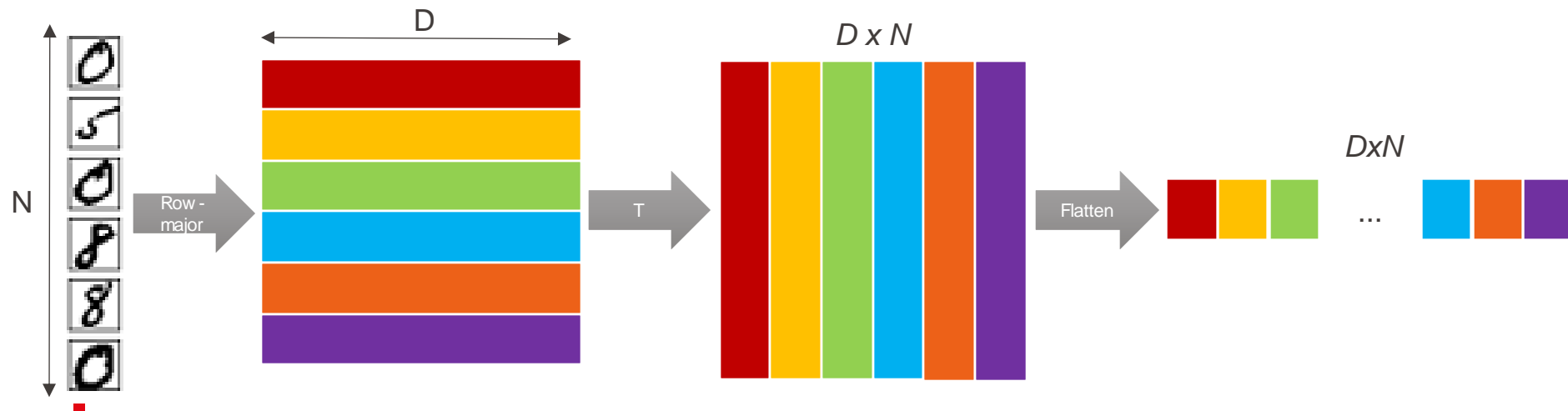
$$\begin{aligned} & [w_{00}w_{00}w_{00}w_{00}w_{11}w_{11}w_{11}w_{11}w_{22}w_{22}w_{22}w_{22}] \odot [a_{00}a_{10}a_{20}a_{01}a_{11}a_{21}a_{02}a_{12}a_{22}] \\ & + [w_{10}w_{10}w_{10}w_{10}w_{21}w_{21}w_{21}w_{21}w_{02}w_{02}w_{02}w_{02}] \odot [a_{01}a_{11}a_{21}a_{02}a_{12}a_{22}a_{00}a_{10}a_{20}] \\ & + [w_{20}w_{20}w_{20}w_{20}w_{01}w_{01}w_{01}w_{01}w_{12}w_{12}w_{12}w_{12}] \odot [a_{02}a_{12}a_{22}a_{00}a_{10}a_{20}a_{01}a_{11}a_{21}] \\ & = [b_{00}b_{10}b_{20}b_{01}b_{11}b_{21}b_{02}b_{12}b_{22}] \end{aligned}$$

Same format

$$\sum_{j=0}^{d-1} \text{Diag}_j(W) \odot \text{Rotate}_{d_j}(\text{Flatten}(A^T)) \rightarrow O(d) \text{ rotations}$$

Methods – Input matrix packing

- Querier holds batch of N images:
 - Pre-processing tasks (normalization, padding, etc...)
 - Images are represented as vectors of size D (row-major ordering)
 - The $N \times D$ matrix is transposed
 - The $N \times D$ matrix is row-flattened





Methods – Input matrix packing – complex trick

Input ciphertext gets packed with complex trick and then replicated for the multiplication (red)

$$\begin{bmatrix} a_{00} & \cdots & a_{0d} \\ \vdots & \ddots & \vdots \\ a_{n0} & \cdots & a_{nd} \end{bmatrix} \rightarrow [a_{00} \quad a_{10} \quad \cdots \quad a_{n0} \quad a_{0d} \quad \cdots \quad a_{nd}]$$

↓

$$\text{Slots} \left\{ \begin{array}{l} [a_{00} + ia_{01} \quad \cdots \quad a_{n0} + ia_{n1} \quad \cdots \quad a_{0d-1} + ia_{0d} \quad \cdots \quad a_{0d} + ia_{00} \quad \cdots \quad a_{nd} + ia_{n0}], \\ [a_{00} + ia_{01} \quad \cdots \quad a_{n0} + ia_{n1} \quad \cdots \quad a_{0d-1} + ia_{0d} \quad \cdots \quad a_{0d} + ia_{00} \quad \cdots \quad a_{nd} + ia_{n0}], \\ [a_{00} + ia_{01} \quad \cdots \quad a_{n0} + ia_{n1} \quad \cdots \quad a_{0d-1} + ia_{0d} \quad \cdots \quad a_{0d} + i0 \quad \cdots \quad a_{nd} + i0 \quad \cdots 0] \end{array} \right\}$$



Methods – Weight matrix diagonal packing

Weight matrix gets diagonalized (generalized diagonals for non-square) and the diagonals are then packed together with complex trick. Additionally every element in the diagonal is replicated a number of times equal to the number of rows of the input matrix for multiplication correctness

$$\begin{bmatrix} w_{00} & w_{01} & w_{02} & w_{03} \\ w_{10} & w_{11} & w_{12} & w_{13} \\ w_{20} & w_{21} & w_{22} & w_{23} \end{bmatrix} \rightarrow \begin{matrix} \xleftrightarrow{\text{Rows}(A) \times \text{Cols}(W)} \\ \xleftrightarrow{\text{Rows}(A)} \\ \begin{matrix} d_0 = (w_{00} - iw_{10} & \cdots & w_{03} - iw_{13} & \cdots) \\ d_1 = (w_{20} - i0 & \cdots & w_{23} - i0 & \cdots) \end{matrix} \end{matrix} \updownarrow (\text{Rows}(W) + 1)/2$$

Methods - Convolutional layer «linearization»



- Transform the convolutional layer into a sparse layer and use matrix multiplication (Toeplitz matrix representation)
- Example: transformation T of a filter operating on a 3x3 matrix with stride 1

$$\begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix} = \begin{bmatrix} k_1 & k_2 & 0 & k_3 & k_4 & 0 & 0 & 0 & 0 \\ 0 & k_1 & k_2 & 0 & k_3 & k_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_1 & k_2 & 0 & k_3 & k_4 & 0 \\ 0 & 0 & 0 & 0 & k_1 & k_2 & 0 & k_3 & k_4 \end{bmatrix}$$



Methods - Convolutional layer «linearization» cont'd

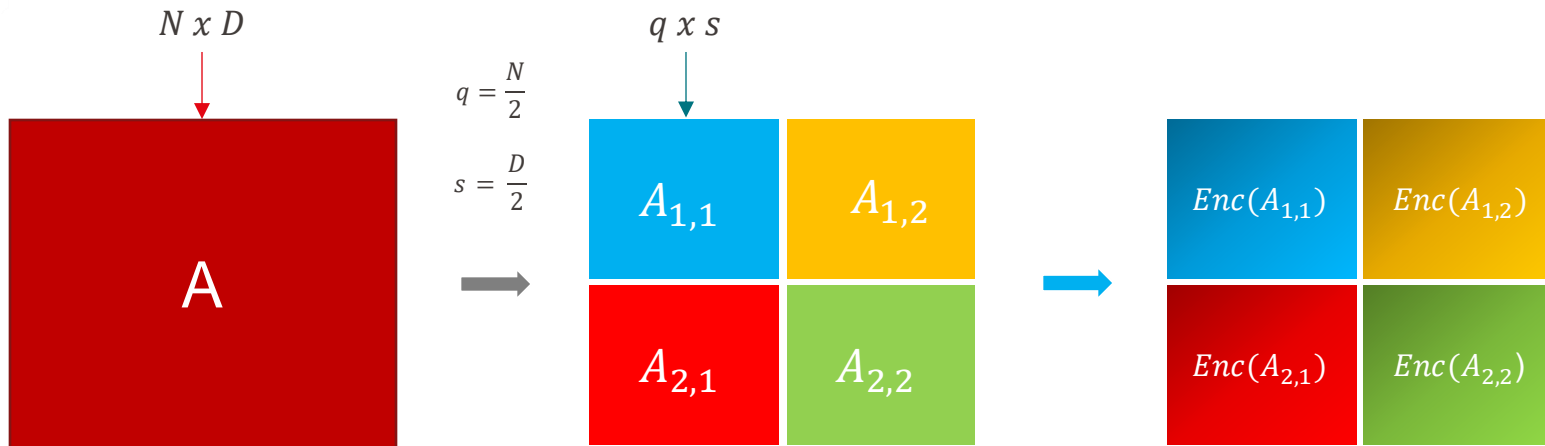
- Generalized for f kernels of $K \times K$ size, with M input channels

$$\begin{bmatrix} T(k_1^{(ch_1)}) & \dots & T(k_1^{(ch_M)}) \\ \vdots & \ddots & \vdots \\ T(k_f^{(ch_1)}) & \dots & T(k_f^{(ch_M)}) \end{bmatrix}$$

- Benefits:
 - Can represent any convolution
 - Self-consistent for subsequent convolutional or linear layers



Methods – Block Matrix Arithmetics



- Each block (sub-matrix) is encrypted/encoded independently
- Operations can be carried out on each block using block matrix arithmetics
- Benefits:
 - Smaller ciphertexts/plaintexts
 - Easily parallelizable
 - Flexible: block splits can be adjusted to maximize latency or throughput