



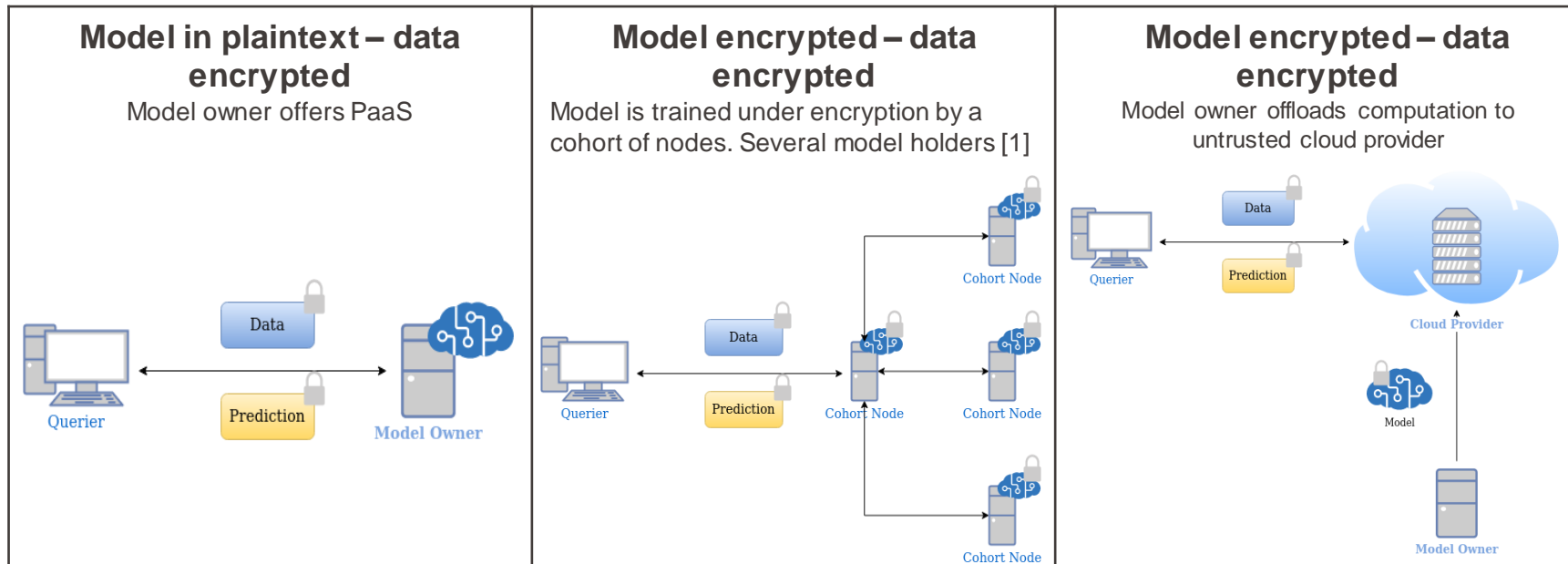
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Privacy-preserving inference on DNNs with MHE

July 7, 2022



- Enable efficient privacy-preserving inference on Deep Neural Networks
- Different scenarios



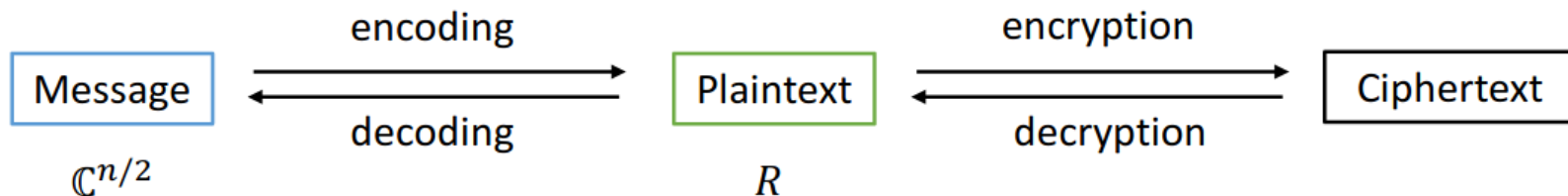


1. How to design an efficient and consistent **data packing** for network evaluation?
2. How to efficiently compute **convolutions on encrypted data** in a layer agnostic way?
3. How to **approximate non linear operations** in the network, and how does this impact the training phase and accuracy of models?
4. How to compute homomorphic **matrix multiplication** in an efficient way?
5. How to handle **data size which does not fit** into one ciphertext?

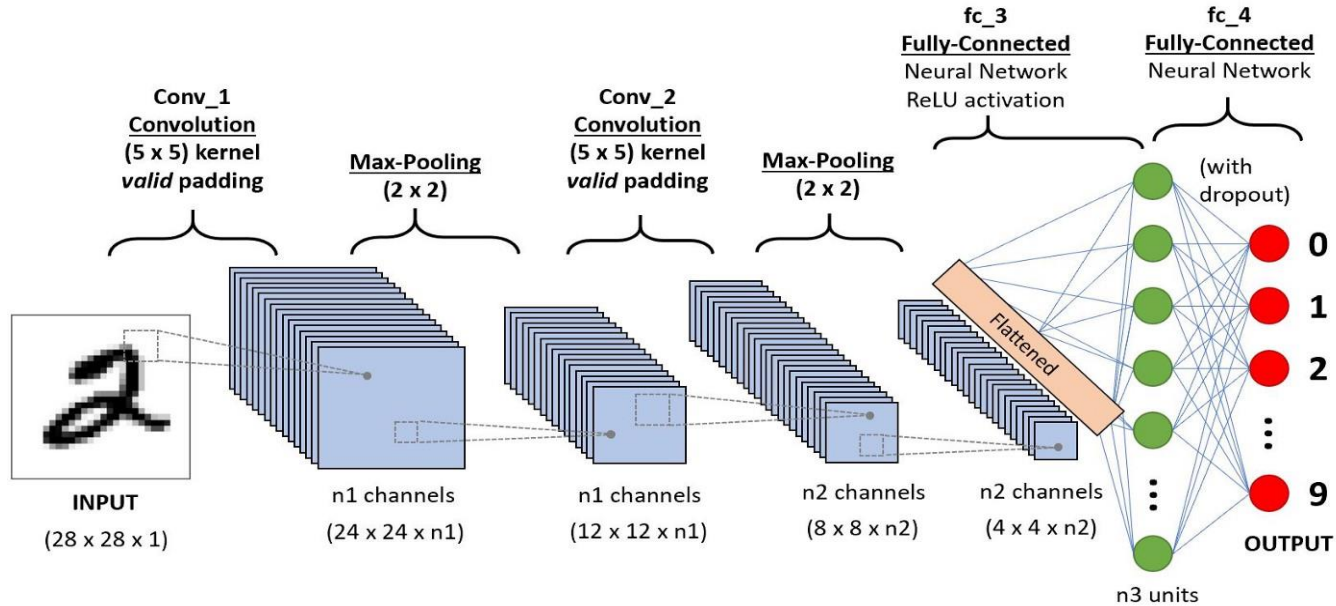


Background – CKKS scheme

- Based on RLWE hardness
- Leveled encryption schemes
- Suited for computation over floating point numbers



Background – Convolutional Neural Network



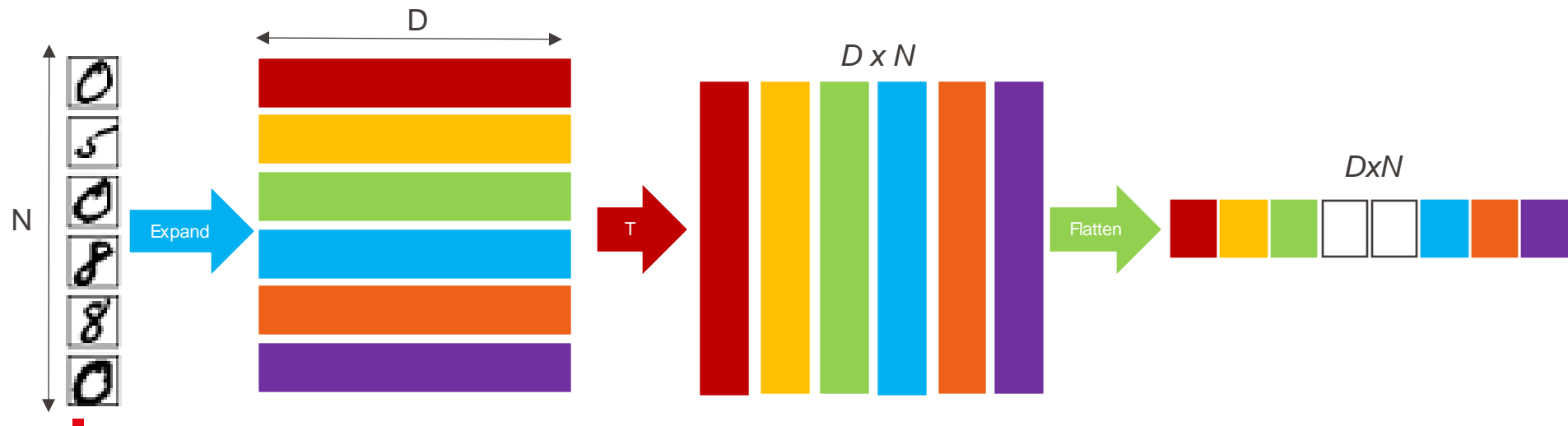


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Methods - Data packing

- Querier holds batch of N images:
 - Pre-processing tasks (normalization, padding, etc...)
 - Images are represented as vectors of size D (feature number)
 - The $N \times D$ matrix is transposed
 - The $N \times D$ matrix is row-flattened
- Benefits:
 - Very simple
 - Fully compatible with matrix multiplication approach





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Methods - Convolutional layer «linearization»

- Transform the convolutional layer into a sparse layer and use matrix multiplication (Toeplitz matrix representation)
- Example: transformation of a filter operating on a 3x3 matrix with stride 1

$$\begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix} = \begin{bmatrix} k_1 & k_2 & 0 & k_3 & k_4 & 0 & 0 & 0 & 0 \\ 0 & k_1 & k_2 & 0 & k_3 & k_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_1 & k_2 & 0 & k_3 & k_4 & 0 \\ 0 & 0 & 0 & 0 & k_1 & k_2 & 0 & k_3 & k_4 \end{bmatrix}$$



Methods - Convolutional layer «linearization» cont'd

- Generalized for f kernels of $K \times K$ size, with M input channels

$$\begin{bmatrix} \langle m(k_{1,ch1}) | \dots | m(k_{1,chM}) \rangle \\ \langle m(k_{2,ch1}) | \dots | m(k_{2,chM}) \rangle \\ \langle m(k_{f,ch1}) | \dots | m(k_{f,chM}) \rangle \end{bmatrix}$$

- Benefits:
 - Can represent any convolution
 - Self-consistent for subsequent convolutional or linear layers



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Methods - Non linear operations in network

- Approximate non-linear activation functions with polynomials using Minimax approximation
 - Polynomial activations introduced new challenges in training phase:
 - Weight explosion
 - Slow convergence
 - Partially solved with fine-tuned weight initialization and learning rate
 - Polynomial activations might introduce errors during encrypted inference with respect to the original function:
 - For each layer, record the interval of intermediate results and approximate the function only on the interval
 - Lower degree of approximation needed
 - Higher accuracy
- Average Pooling and Sum Pooling in place of Max Pooling



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Methods - Matrix Multiplication

- Input matrix is transposed and row-flattened (same as data packing)
- Weight matrix in diagonal form
- Element wise multiplications, rotations and additions
- Benefits:
 - Linear complexity
 - Fully compatible with data packing
 - Easily Parallelizable

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} w_{00} & w_{01} & w_{02} \\ w_{10} & w_{11} & w_{12} \\ w_{20} & w_{21} & w_{22} \end{bmatrix} = \begin{bmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{bmatrix}$$

Transpose and row-flatten

$$\begin{aligned} & [w_{00}w_{00}w_{00}w_{11}w_{11}w_{11}w_{22}w_{22}w_{22}] \odot [a_{00}a_{10}a_{20}a_{01}a_{11}a_{21}a_{02}a_{12}a_{22}] \\ & + [w_{10}w_{10}w_{10}w_{21}w_{21}w_{21}w_{02}w_{02}w_{02}] \odot [a_{01}a_{11}a_{21}a_{02}a_{12}a_{22}a_{00}a_{10}a_{20}] \\ & + [w_{20}w_{20}w_{20}w_{01}w_{01}w_{01}w_{12}w_{12}w_{12}] \odot [a_{02}a_{12}a_{22}a_{00}a_{10}a_{20}a_{01}a_{11}a_{21}] \\ & = [b_{00}b_{10}b_{20}b_{01}b_{11}b_{21}b_{02}b_{12}b_{22}] \end{aligned}$$

Same format

$$\sum_{j=0}^{d-1} \text{Diag}_j(W) \odot \text{Rotate}_{d_j}(\text{Flatten}(A^T)) \rightarrow O(d) \text{ rotations}$$



- Optimized version for $O\left(\frac{d}{2}\right)$
- Half the (intermediate) size of the matrices by packing real values in complex form
- Later remove the imaginary part with 1 addition and 1 rotation

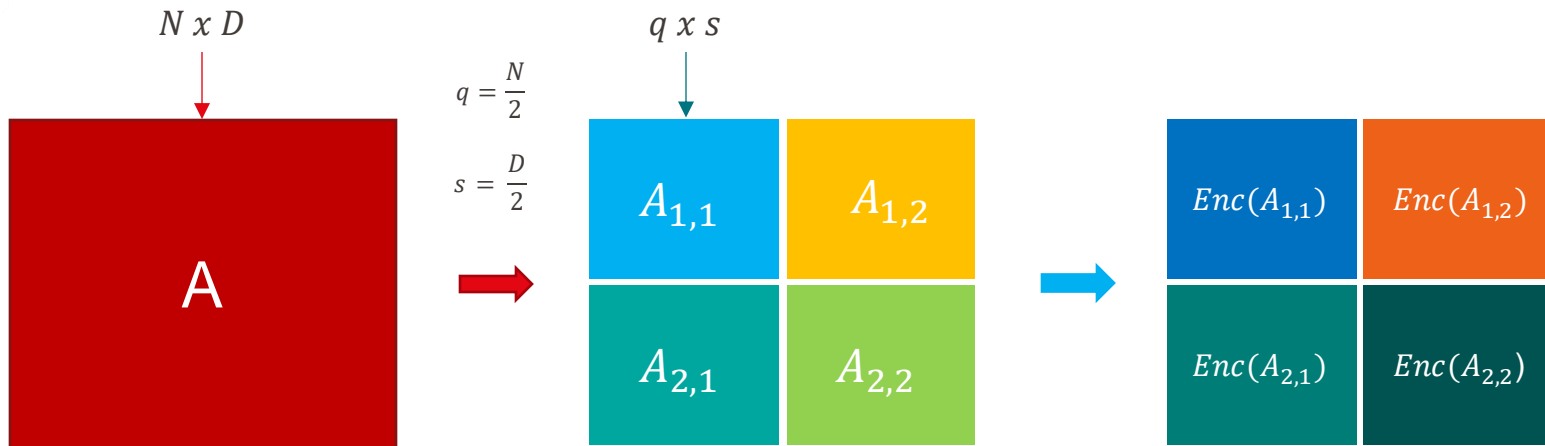
$$\begin{pmatrix} a_{1,1} & \dots & a_{1,m} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \dots & a_{n,m} \end{pmatrix} \times \begin{pmatrix} b_{1,1} & \dots & b_{1,h} \\ \vdots & \ddots & \vdots \\ b_{m,1} & \dots & b_{m,h} \end{pmatrix} \rightarrow \begin{pmatrix} a_{1,1} - ia_{1,2} & \dots & a_{1,m-1} - ia_{1,m} \\ \vdots & \ddots & \vdots \\ a_{n,1} - ia_{n,2} & \dots & a_{n,m-1} - ia_{n,m} \end{pmatrix} \times \begin{pmatrix} b_{1,1} + ib_{2,1} & \dots & b_{1,m} + ib_{2,m} \\ \vdots & \ddots & \vdots \\ b_{n-1,1} + ib_{n,1} & \dots & b_{n-1,m-1} + ib_{n,m} \end{pmatrix}$$



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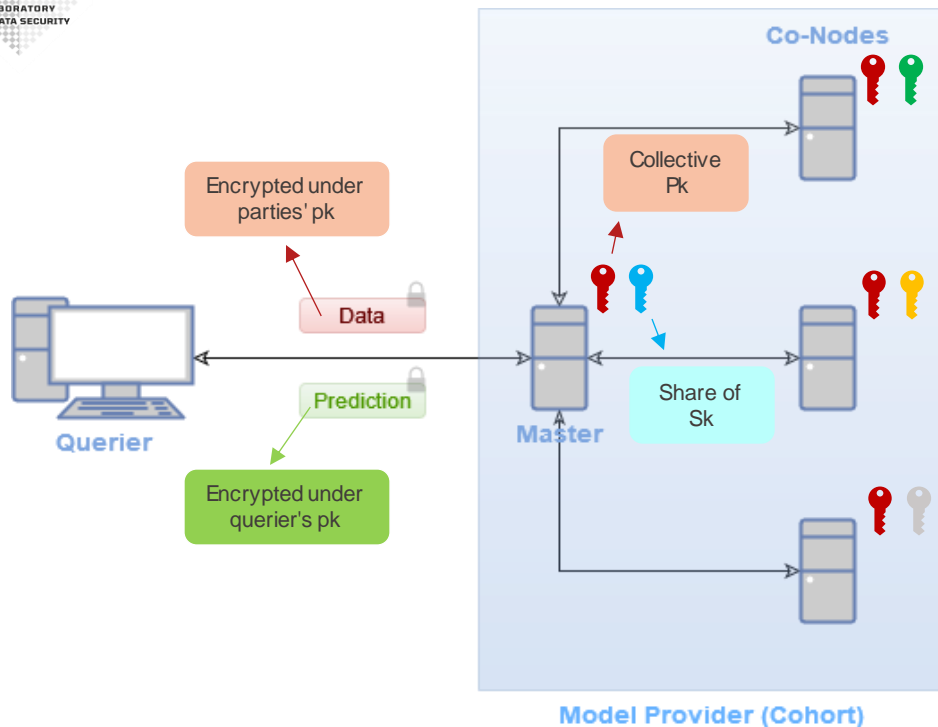
Methods – Block Matrix Arithmetics



- Each block (sub-matrix) is encrypted/encoded independently
- Operations can be carried out on each block using block matrix arithmetics
- Benefits:
 - Smaller ciphertexts/plaintexts
 - Easily parallelizable
 - Flexible: block splits can be adjusted to maximize latency or throughput



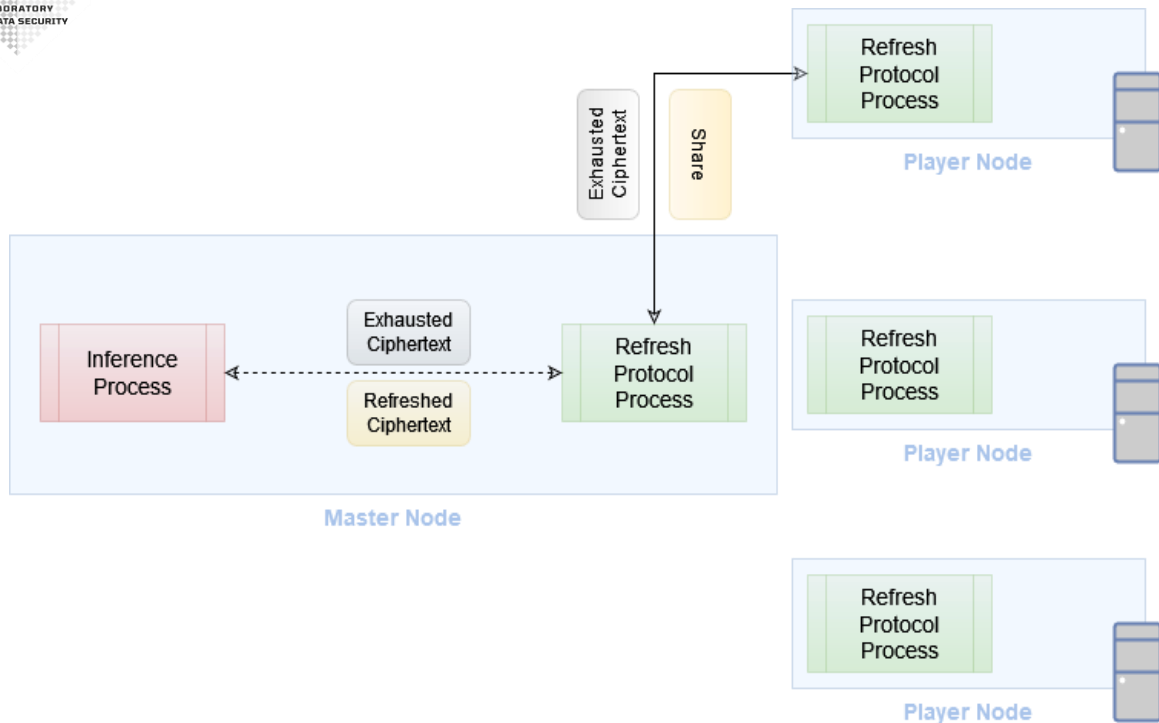
Methods – Distributed protocols for MHE



- Assumption: model is trained encrypted by a cohort of nodes.
- Very efficient: 1 Round Protocols!
- Reference: Mouchet et al.²

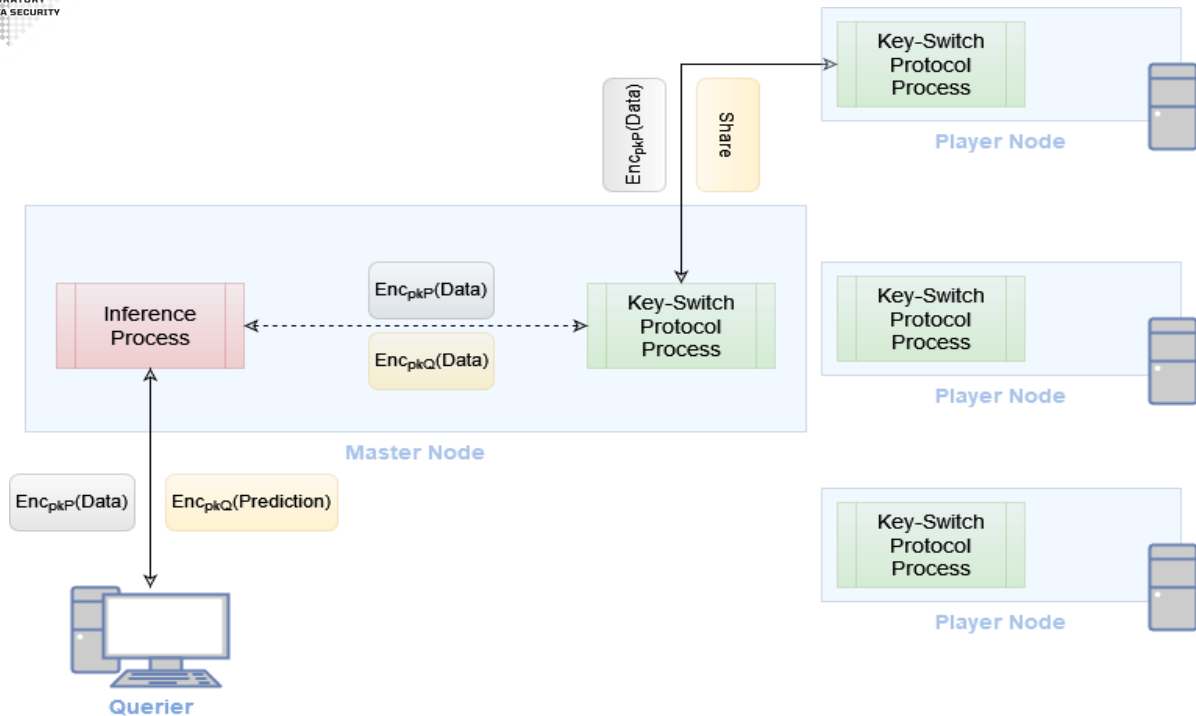


Methods – Distributed protocols for MHE-Refresh



- Distributed Bootstrapping generates a freshly encrypted ciphertext from a "low-level" ciphertext

Methods – Distributed protocols for MHE – Collective Key Switch



- Collective Key Switch Protocol is invoked to generate an encryption of the result under querier's public key



- Hardware specifications:
 - 2x Intel Xeon E5-2680 2.5 GHz (48 threads)
 - 16 GB RAM DDR4
- Parameters for CKKS scheme:

Set	LogN (bits)	LogQP (bits)	Levels	Scale (bits)
CryptoNets	14	329	7	30
NN20	15	874	18	35
NN50	15	629	11	35
NN50-light	14	436	10	31
NN-Central-Btp	15	768	13	25

Experiment 1 - CryptoNets



- We benchmarked our framework on the Microsoft CryptoNets³ model
 - Training and inference on MNIST dataset
 - Scenario is “model in clear – data encrypted”

Model	Batching	Batch	Latency(s)	Throughput(im/s)
CryptoNets	Y	4096	250	16.54
Faster Cryptonets ⁵	N	1	39.1	0.02
LoLa ⁴	N	1	2.2	0.5
MiniONN ⁶	N	1	1.28	0.78
Ours	Y	83	10.5	7.9

[3] CryptoNets: Applying Neural Networks to Encrypted Data with High Throughput and Accuracy

[4] Low Latency Privacy Preserving Inference

[5] Faster CryptoNets: Leveraging Sparsity for Real-World Encrypted Inference

[6] Oblivious Neural Network Predictions via MiniONN transformations

Experiment 2 – ZAMA NN



- We further tested our framework on deeper and more complex networks, using the models proposed by ZAMA⁸
 - Training and inference on MNIST dataset
 - Model is encrypted
 - Two models tested → 20 and 50 layers
 - Two bootstrapping approaches → centralized and distributed

■ [8] [Programmable Bootstrapping Enables Efficient Homomorphic Inference of Deep Neural Networks](#)



Experiment 2 – ZAMA

NN – 20 layers

Setting	Batch	Latency (s)	Throughput (im/s)	Communication (MB/cipher)
1PC	292	639.6	0.45	-
MPC-5	292	365	0.85	21
MPC-10	292	417	0.70	21
Baseline-PC	1	115.52	0.008	-
Baseline-AWS	1	17.96	0.055	-

Experiment 2 – ZAMA

NN – 50 layers



Parameters with
smaller ring size!

Setting	Batch	Latency (s)	Throughput (im/s)	Communication (MB/cipher)
IPC	292	1343	0.21	-
MPC-3	292	811	0.36	21
MPC-5	146	759	0.19	21
MPC-10	146	1007	0.14	21
Baseline-PC	1	233.55	0.004	-
Baseline-AWS	1	37.69	0.026	-



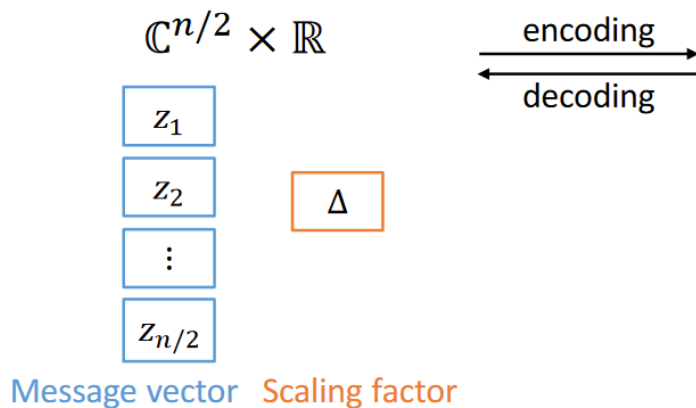
- Results are promising...
 - CKKS allows for data packing and SIMD operations for high throughput
- ...but optimization is hard!
 - Finding optimal data packing approach is crucial for performance
 - Data packing and block matrices splitting greatly influence latency and throughput
 - Dealing with non-linearities is complex
 - Introducing polynomial activations during training might cause slow or no convergence
 - Polynomial approximations must be fine-tuned (tradeoff between accuracy – efficiency)
 - CKKS parametrization is hard!
 - A bad choice for the parameters of the scheme might completely disrupt performance and accuracy!

Privacy-preserving inference on DNNs with MHE

Backup
Slides

July 7, 2022

Background – CKKS scheme – Encoding & Decoding



$$R = \mathbb{Z}[X]/(X^n + 1)$$

$$m(X) \approx$$

$$\begin{array}{c}
 \Delta \cdot z_1 \\
 \Delta \cdot z_2 \\
 \vdots \\
 \Delta \cdot z_{n/2}
 \end{array}$$

Plaintext (Encoded message)

i-th slot of plaintext

$$m(\zeta_j) \approx \Delta \cdot z_j \text{ for some roots } \zeta_j \text{ of } X^n + 1 = 0$$

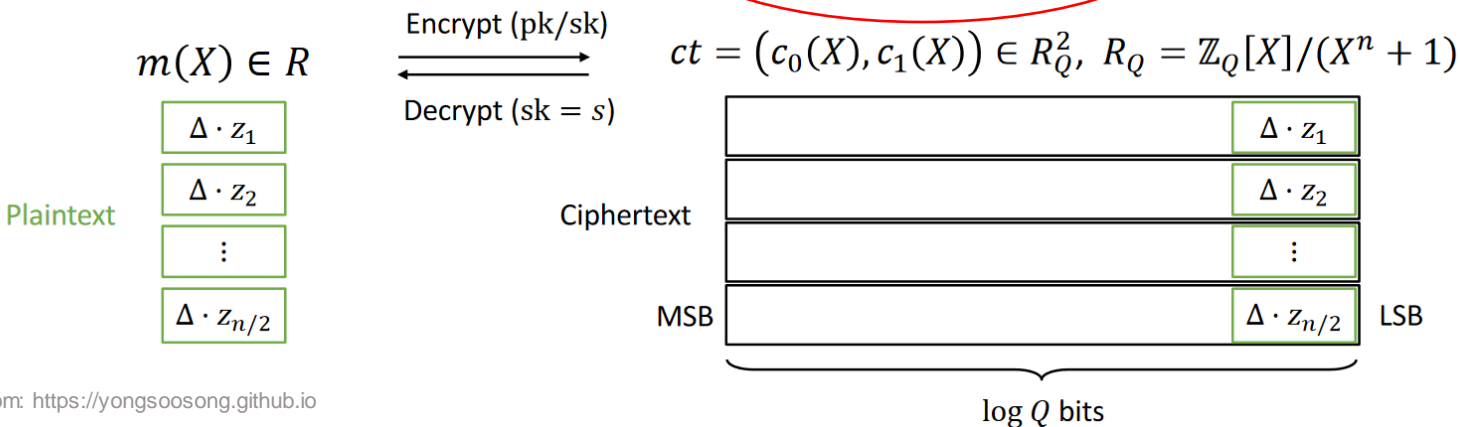


Background – CKKS scheme – Encryption & Decryption

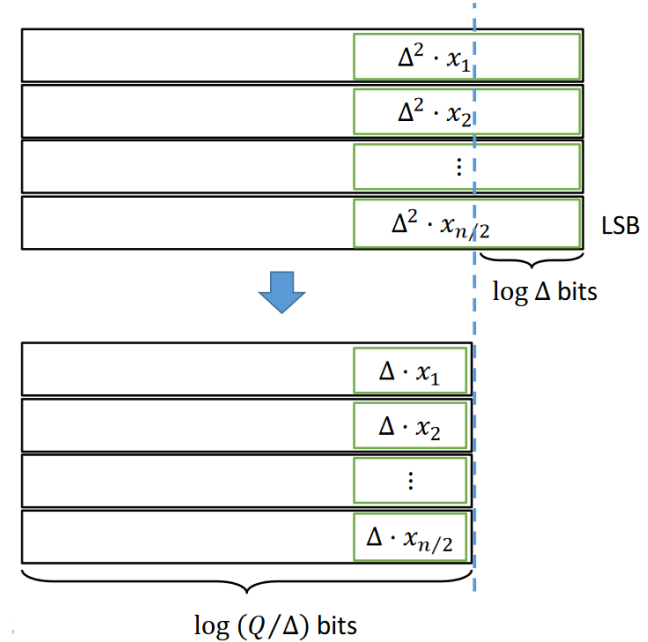
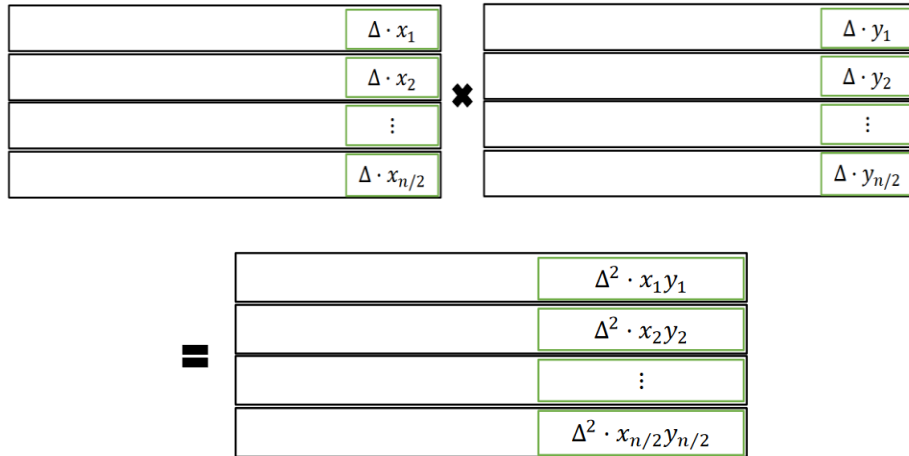
Ring structure $R_q = \mathbb{Z}_q[x]/(x^n + 1)$.

Modulus Chain

Residue Number System (RNS) : $\mathbb{Z}_q \cong \mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2} \times \dots \times \mathbb{Z}_{p_L}$.



Background – CKKS scheme – Multiplication & Rescaling



Background – CKKS scheme – Bootstrapping

