Problem Set 1

1. **Car and Bicycle Rider**

   A car is driving along a straight line with a speed v_0. At time $t = 0$ the car is at the origin. At a later instant of time $t = t_1$ the car starts to slow down until it stops at a time $t = t_2$.

   The acceleration of the car as a function of time is given by

$$a_c = \begin{cases} 0 & 0 < t < t_1 \\ -c(t - t_1) & t_1 < t < t_2 \end{cases}$$

   where c is a positive constant which has dimensions of acceleration per unit time.

   (a) Find $v_c(t)$ and $x_c(t)$, the x-component of the velocity and the position of the car as a function of time. Express your answer in terms of some or all of the following variables: v_0, c, t, t_1 and t_2.

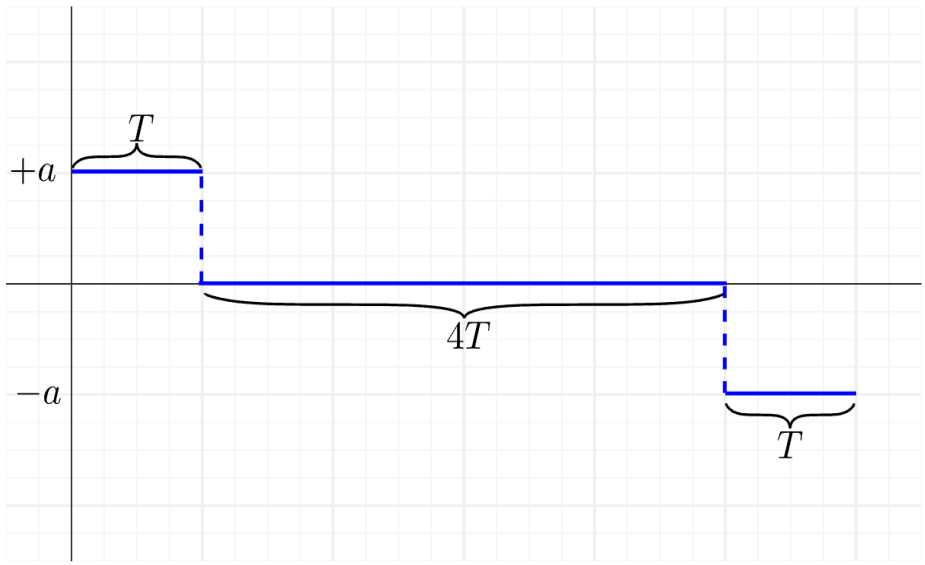
   (b) A bicycle rider is riding at a constant speed of v_b and at $t = 0$ is 17 m behind the car. The cyclist reaches the car when the car just comes to rest. The car is moving with an acceleration

$$a_c = \begin{cases} 0 & 0 < t < t_1 \\ -c(t - t_1) & t_1 < t < t_2 \end{cases}$$

   where its initial component of the velocity is $v_0 = 12$ m/s, $t_1 = 1$ s and $c = 6$ m/s^3. The car comes to rest at t_2.

   Find the speed of the bicycle to 2 significant figures.

2. **Elevator Trip**

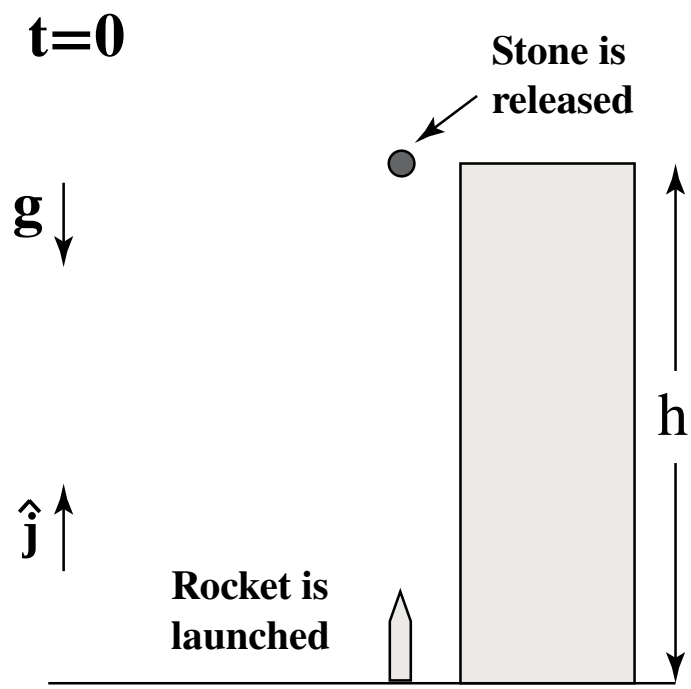


A person is standing on an elevator initially at rest at the first floor of a high building. The elevator then begins to ascend to the sixth floor, which is a known distance h above the starting point. The elevator undergoes an unknown constant acceleration of magnitude a for a given time interval T. Then the elevator moves at a constant velocity for a time interval $4T$. Finally the elevator brakes with an acceleration of magnitude a, (the same magnitude as the initial acceleration), for a time interval T until stopping at the sixth floor.

(a) Make a sketch of the velocity $v(t)$ of the elevator as it travels to the sixth floor. Your sketch should be qualitatively correct: it should have the right shape, but the vertical scale need not be accurate. *Hint: thinking about the graphical representation of v(t) leads to a much easier solution for part B.*

(b) Find the value of a, the magnitude of the acceleration, in terms of h and T.

3. **Rocket Launch**

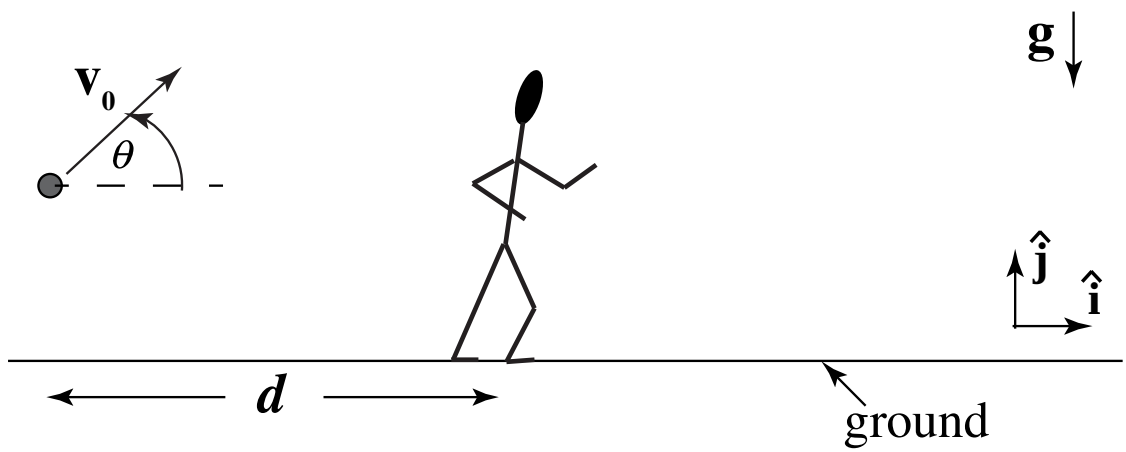


At the base of a vertical cliff, a model rocket, starting from rest, is launched upwards at $t = 0$ with a time-varying acceleration given by

$$a_y(t) = A - Bt$$

where A and B are positive constants. Also at $t = 0$, a small stone is released from rest from the top of the cliff at a height h directly above the rocket. (This height h is higher than the maximum height reached by the rocket.) The stone hits the rocket at the instant when the rocket reaches its maximum height. The gravitational acceleration of magnitude g is downward. You may neglect air resistance. Determine an expression for the initial height h from which the stone was dropped in terms of the constants A, B, and g.

4. **Throw and Catch**



A ball is thrown over the head of a person who is standing a horizontal distance d from the point where the ball was thrown. The initial ball's velocity is at an angle θ with respect to the ground and has a magnitude v_0. As soon as the ball is thrown, the person runs with a time-varying acceleration whose component along the x-axis is given by Bt, where B is a positive constant. The person catches the ball at exactly the same height it was thrown from. Assume that the air resistance is negligible and that the gravitational acceleration is directed downward and has magnitude g. Find the constant B.

Express your answer in terms of some or all of the given variables. Use θ, g, d, and v_0 as needed.

5. **Vertical Collision**

   Mary wants to throw a can straight up into the air and then hit it with a second can. She wants the collision to occur at height $h = 5.0$ m above the throw point. In addition, she knows that she needs $t_1 = 4.0$ s between successive throws. Assume that she throws both cans with the same speed. Take g to be 9.81 m/s^2.

   (a) How long it takes (in seconds) after the first can has been thrown into the air for the two cans to collide? Answer to 4 significant figures.

   (b) Find the initial speed of the cans (in meters/second) to 4 significant figures.

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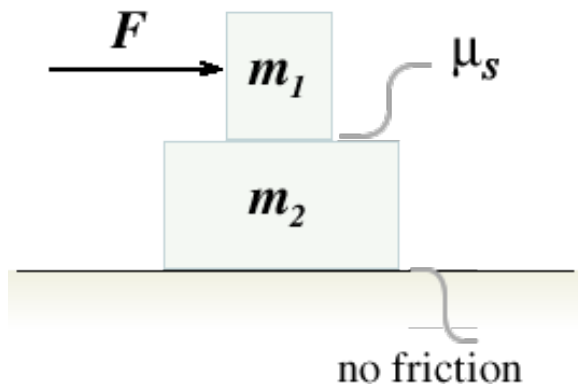
Problem Set 2

1. **Stacked Blocks**

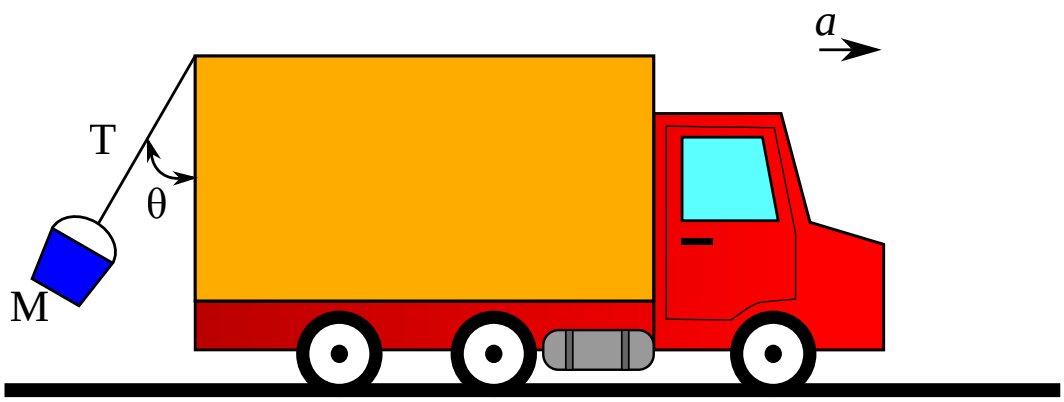


Consider two blocks that are resting one on top of the other. The lower block has mass $m_2 = 4.8$ kg and is resting on a frictionless table. The upper block has mass $m_1 = 2.2$ kg. Suppose the coefficient of static friction between the two blocks is given by $\mu_s = 0.5$.

A force of magnitude F is applied as shown. What is F_{max}, the maximum value of F for which the upper block can be pushed horizontally so that the two blocks move together without slipping? Please give your answer in Newtons.

2. **Accelerating Truck and Suspended Bucket**

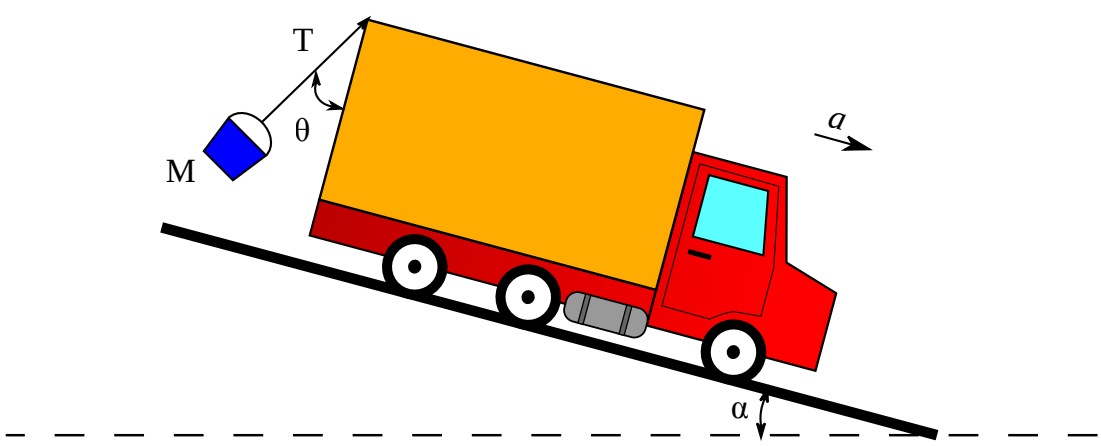
A truck is traveling in a straight line on level ground, and is accelerating uniformly with an acceleration of magnitude a. A rope (that can be considered massless and inextensible) is tied to the back of the truck. The other end of the rope is tied to a bucket of mass M. The bucket tosses wildly when the truck starts to accelerate, but later on it stays at a fixed position at a fixed distance behind the truck, with the rope hanging straight at a fixed angle, as shown in the diagram.



(a) Find the angle θ at which the rope will settle. Express your answers in terms of the given variables M, g, and a as needed.

(b) What will the tension T of the rope be once it settles into this angle? Express your answers in terms of the given variables M, g, and a as needed.

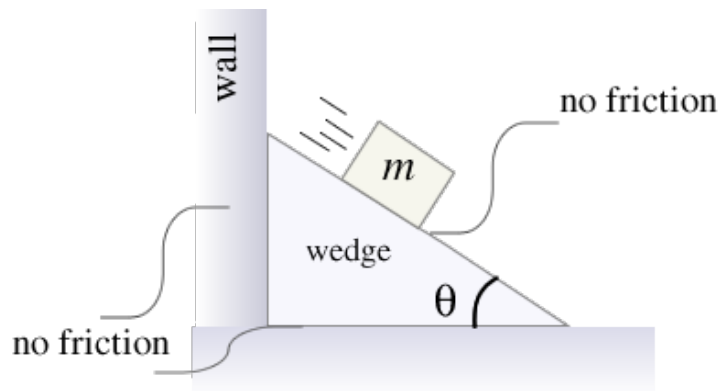
(c) The truck comes to a downhill section of road, at an angle α relative to the horizontal, as shown in the figure. Suppose that the truck continues to accelerate with an acceleration of magnitude a.



Once the rope again settles to a (possibly different) fixed angle θ relative to the truck's back door, what will that new angle be? Express your answer using the given variables M, g, α, and a as needed.

(d) What will the new tension T in the rope be? Express your answers in terms of the given variables M, g, α, and a as needed.

3. **A wedge against a wall**

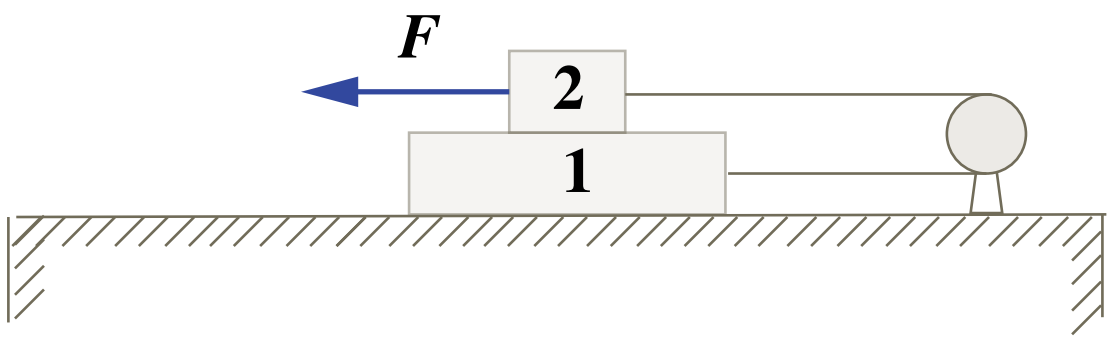


A wedge with an inclination of angle θ rests next to a wall. A box of mass m is sliding down the incline surface, as shown. There is no friction between any of the surfaces, (wedge and the box, wedge and the horizontal surface, wedge and the wall). For the answers below, you can use θ, m and g as needed.

(a) Find F_{total}, the magnitude of the sum of all forces acting on the box.

(b) Find F_{ww}, the magnitude of the force exerted by the wall on the wedge.

4. **Dragging two blocks**



Two blocks of masses m_1 and m_2 are connected by a light cord passing around a fixed, frictionless pulley. The coefficient of kinetic friction between all surfaces is μ. Find F, the magnitude of the horizontal force necessary to drag block 2 to the left at constant speed.

5. **Blocks and springs**

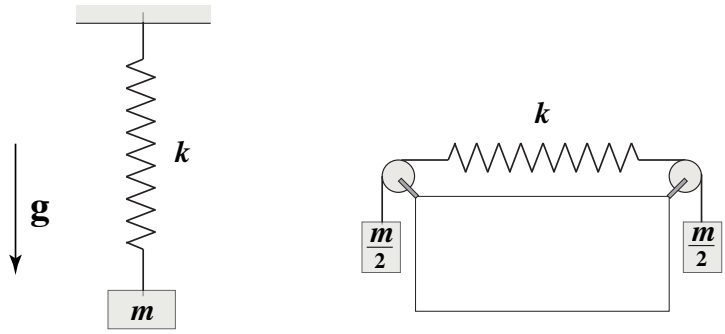
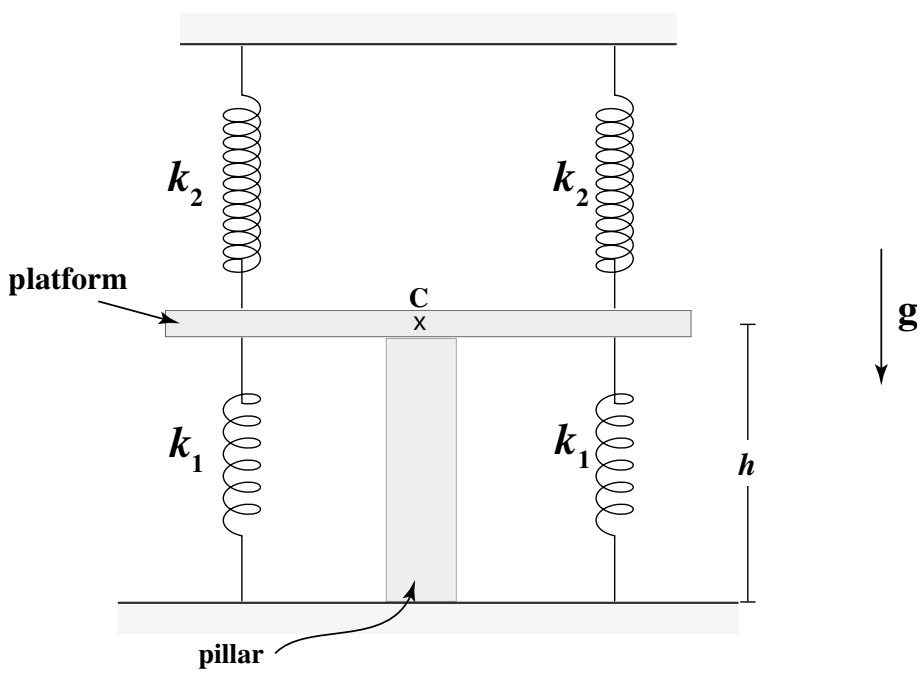


Figure A **Figure B**

(a) In Figure A above, a block of mass m is hanging from a spring attached to the ceiling. In Figure B above, two identical blocks of mass $m/2$ are hanging from two ropes that are attached to a spring that has the same spring constant k. If the spring in Figure A has stretched a distance d from its natural length, what is x, the distance the spring in Figure B has stretched with respect to its natural length?



(b) A platform of mass M and at a height h above the ground is supported by a pillar as shown. The pillar is placed right under the center of the platform, point C in the figure. Two identical springs of spring constant constant k_1 connect the

platform to the floor, and a second pair of identical springs of spring constant k_2 connect the platform to the ceiling. The springs are attached to the platform at the same distance from its center, point C. With the support pillar in place, the springs are unstretched.

When the pillar is removed, at what height h_f will the platform come to equilibrium? Express your answer in terms of h, M, g, k_1 and k_2 as needed.

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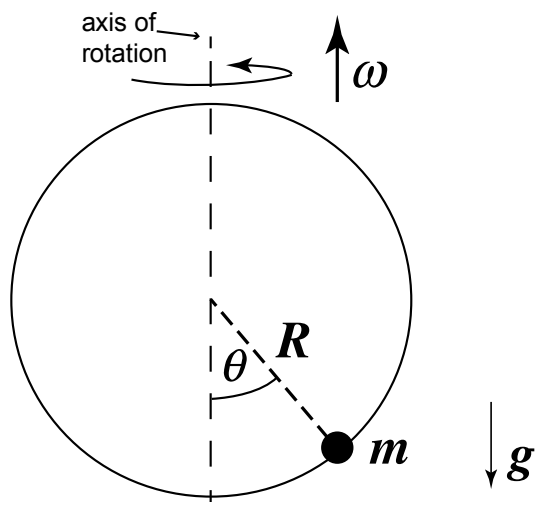
Problem Set 3

1. **Bead on a Rotating Hoop**

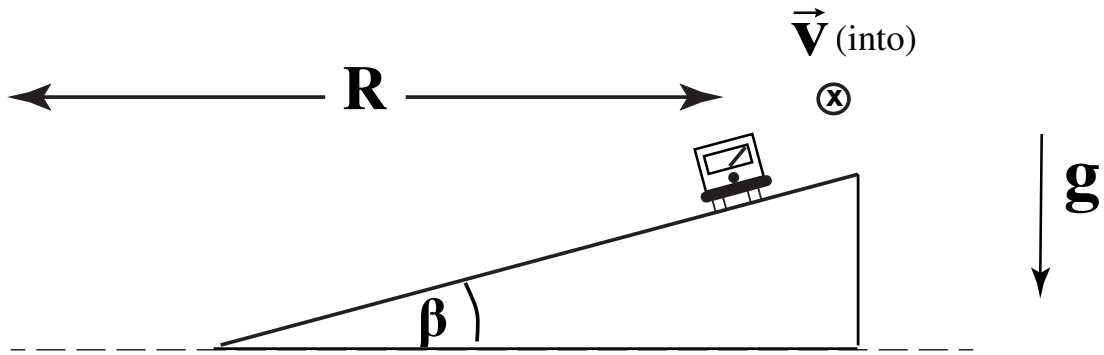
   A bead lies on a frictionless hoop of radius R that rotates around a vertical diameter with constant angular speed ω, as shown in the figure below.



   (a) What should ω be so that the bead maintains the same position on the hoop, at an angle θ with respect to the vertical? Express you answer in terms of some or all of the following: θ, R and g.

   (b) Analyzing the answer for Part A, you will find that there is a range of angular speeds, $0 < \omega < \omega_o$ for which the fixed angle $\theta = 0$ (meaning that the only balanced position is at the bottom of the hoop). Find the value of ω_o. Express you answer in terms of some or all of the following: R and g.

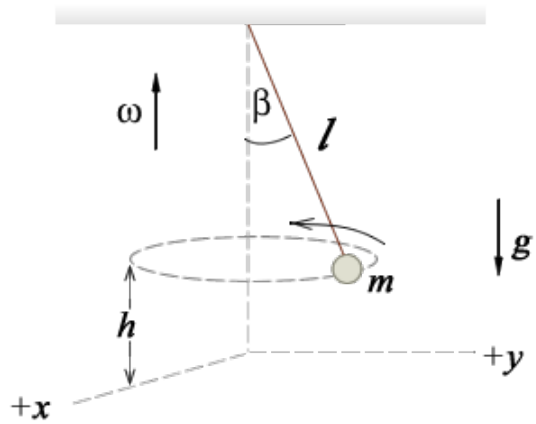
2. **Banked Turn**



A car of mass m is going around a circular turn of radius R, which is banked at an angle β with respect to the ground. Assume there is friction between the wheels and the road. Let μ_s be the coefficient of static friction and g the magnitude of the gravitational acceleration. You may neglect kinetic friction (that is, the car's tires do not slip). Derive an expression for the range of possible speeds $v_{\text{min}} \leq v \leq v_{\text{max}}$ necessary to keep the car moving in a circle without slipping up or down the embanked turn. Express your answer in terms of some or all of the following: μ_s, β, m, R and g.

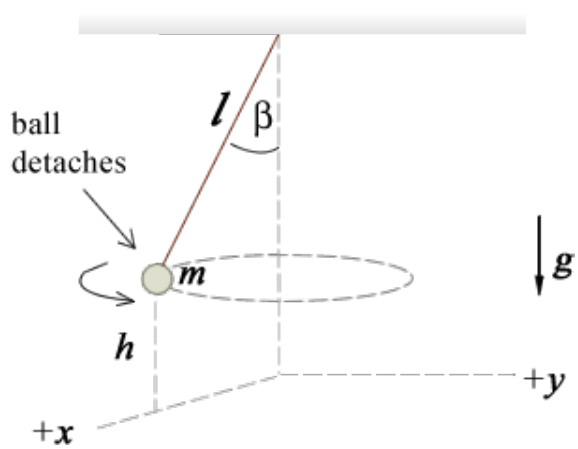
3. **Tetherball Breaking Off**

A small ball of mass m is suspended by a string of length l. The string makes an angle β with the vertical. The ball revolves in a circle with an unknown constant angular speed ω. The orbital plane of the ball is at a height h above the ground. Let g be the gravitational constant. You may ignore air resistance and the size of the ball.



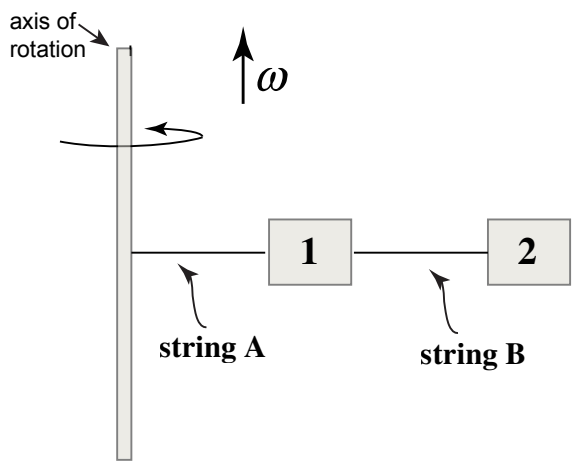
(a) Find an expression for the angular speed ω. Express you answer in terms of some or all of the following: l, β, and g.

(b) Later, the ball detaches from the string just as it passes the x-axis. It flies through the air and hits the ground at an unknown horizontal distance d from the point at which it detached from the string.



What horizontal distance d does the ball traverse before it hits the ground? Express you answer in terms of some or all of the following: l, β and h.

4. **Two Boxes Around a Shaft**

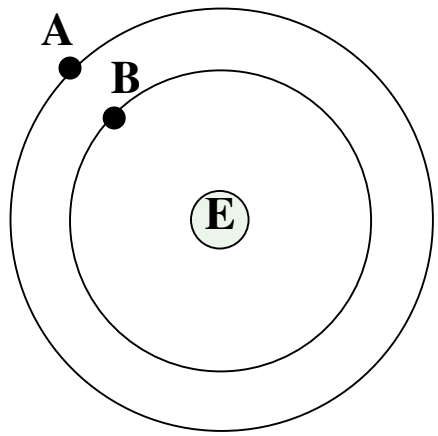


Box 1 and box 2 are whirling around a shaft with a constant angular velocity of magnitude ω. Box 1 is at a distance d from the central axis, and box 2 is at a distance $2d$ from the axis. You may ignore the mass of the strings and neglect the effect of gravity. Express your answer in terms of d, ω, m_1 and m_2, the masses of box 1 and 2.

(a) Calculate T_B, the tension in string B (the string connecting box 1 and box 2):

(b) Calculate T_A, the tension in string A (the string connecting box 1 and the shaft):

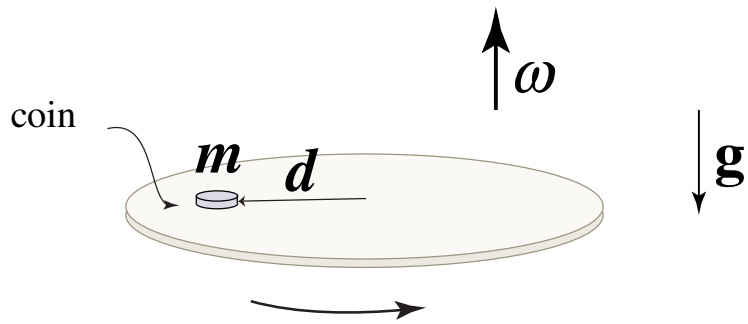
5. **Satellite**



(a) Two satellites are orbiting earth at different altitudes. Which satellite orbits at a higher speed v around earth? Assume that the orbits are circular and both satellites have the same mass.

(b) Which satellite orbits with a longer period, T, around earth? Assume that the orbits are circular and both satellites have the same mass.

6. **A coin on a rotating disk**



A coin of mass m is on a rigid disk at a distance d from the center of the disk. There is friction between the coin and the disk. The coefficient of static friction is μ_s. At time $t = 0$, the disk begins to rotate with a constant angular acceleration of magnitude α. The magnitude of the acceleration due to gravity is g.

Express you answers in terms of some or all of the given variables m, d, μ_s, α, t and g as needed.

(a) While the coin remains at rest relative to the disk, what is f_s, the magnitude of the force of static friction exerted by the disk on the coin as a function of time t?

(b) At what angular speed ω will the coin start to slip with respect to the disk?

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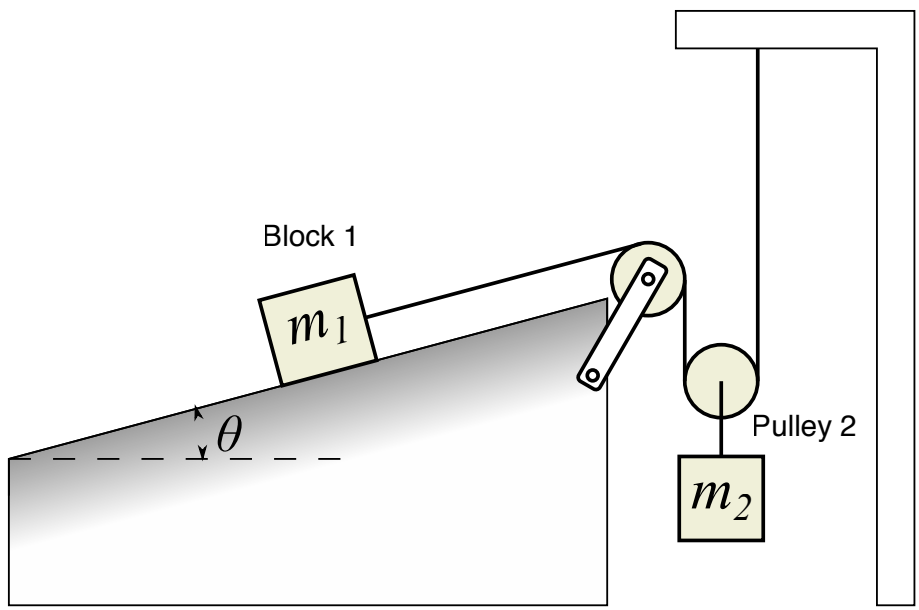
Problem Set 4

1. **Block and Pulley System**



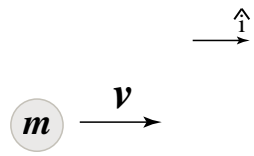
A block of mass m_1 is at rest on an inclined plane that makes an angle θ with the horizontal. The coefficient of static friction between the block and the incline surface is μ_s. A massless, inextensible string is attached to one end of the block, passes over a fixed pulley, pulley 1, around a second freely suspended pulley, pulley 2, and is finally attached to a fixed support. The left hand part of the string is parallel to the surface of the inclined plane. The sections of the string coming off the suspended pulley are vertical. The pulleys are massless, but a second block of mass m_2 is hung from the suspended pulley. Gravity acts downward.

(a) What is $m_{2,min}$, the minimum value of m_2 for which block 1 just barely slides **up** the incline? Express your answers in terms of some or all of the variables m_1, θ, μ_s and g.

(b) What is $m_{2,max}$, the maximum value of m_2 for which block 1 just barely slides **down** the incline? Express your answers in terms of some or all of the variables m_1, θ, μ_s and g.

(c) Now assume that the block on the incline plane is sliding upward. The coefficient of kinetic friction between the block and the incline surface is μ_k. Find the magnitude of the acceleration of the block on the inclined plane, a_{x1}. Express your answers in terms of some or all of the variables m_1, m_2, θ, μ_k and the acceleration of gravity g.

2. **Velocity Dependent Force**

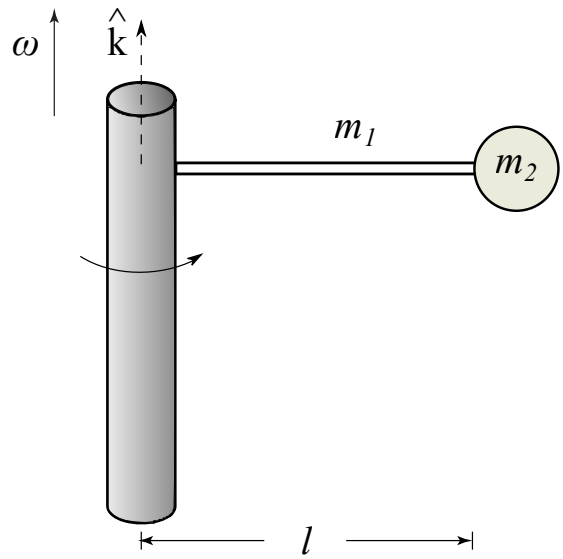


A particle of mass m moving parallel to the x-axis is acted on by a velocity dependent force directed against its motion. The force is given by:

$$\vec{F} = -be^{cv}\,\hat{i}$$

where b is a positive constant (units N), c ia also a positive constant (units s m^{-1}), and v is the speed, the magnitude of the particle's velocity (units m s^{-1}). If at $t = 0$, the particle is moving with speed v_0, find the speed $v(t)$ as a function of time t. Express your answer in terms of some or all of the following: b, m, t, c, and v_0.

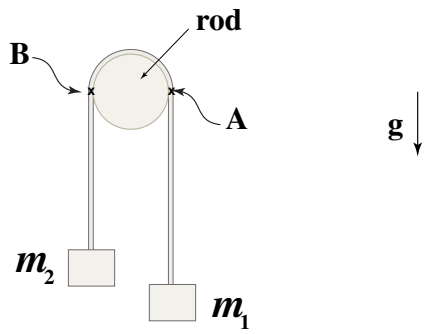
3. **Tension in Massive Rotating Rope with Object**



One end of a uniform rope of mass m_1 and length l is attached to a shaft that is rotating at constant angular velocity of magnitude ω. The other end is attached to a point-like object of mass m_2. Find $T(r)$, the tension in the rope as a function of r, the distance from the shaft.

Assume that the radius of the shaft is much smaller than l. You may also ignore the effect of gravitation - assume the shaft is rotating fast enough so that the mass and the rope are almost horizontal. Express your answer in terms of some or all of the following variables: m_1, m_2, ω, r and l.

4. **Tension in Rope Wrapped Around a Rod**



Two unequal blocks of masses m_1 and m_2, $m_1 > m_2$, are suspended by a rope over a fixed rod. The axis of the rod is perpendicular to the figure (only its cross section is shown). The coefficient of static friction between the rope and the rod is μ_s, and the coefficient of sliding friction is μ_k, $\mu_k < \mu_s$. The mass of the rope can be ignored.

For this problem , you can use the result from Chapter 8, Example 8.11 in the course textbook, titled "The Capstan", where it is shown that when the rope is about to slide the tension at point B in the rope, T_B, is related to the tension at point A in the rope, T_A, by:

$T_B = T_A e^{-\mu_s \theta}$

where θ is the angle subtended by the portion of the rope in contact with the rod. In this problem, the angle θ is $\theta = \pi$. (Note: points A and B are the points where the rope loses contact with the surface of the rod and we assume the cross section of the rod to be a perfect circle).

(a) Let T_1 be the magnitude of the force of tension exerted by the rope on block 1. Is T_A greater, less than, or qual to T_1?

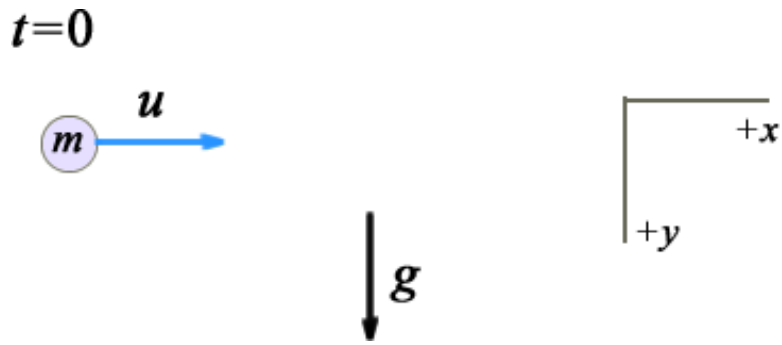
(b) What is the value of m_1 for which the rope starts sliding? Express your answer in terms of μ_s and m_2.

(c) Now assume that m_1 is large enough so that the rope starts to slip and the masses start to move. What is a, the magnitude of the acceleration of the masses after sliding has begun?

(Hint: Just when the masses start moving, the relationship between T_A and T_B becomes $T_B = T_A e^{-\mu_k \theta}$, where μ_s is replaced by μ_k. You can show this by following similar logic used in solving example 8.11 in the textbook.)

Express your answer in terms of some or all of the following: μ_k, m_1, m_2, and g.

5. **Drag Force at Low Speeds**



At low speeds (especially in liquids rather than gases), the drag force is proportional to the velocity, i.e., $\vec{F} = -C\,\vec{v}$, where C is a constant. At time $t = 0$, a small ball of mass m is projected into a liquid so that it initially has a horizontal velocity of magnitude u in the $+x$ direction as shown. (The vertical component of the velocity is zero). The gravitational acceleration is g. Consider the cartesian coordinate system shown in the figure (+x to the right and +y downwards).

(a) What is the component of the acceleration in the x direction for $t > 0$? Express your answer in terms of v_x (the component of the velocity in the x direction), C, g, m and u as needed.

(b) What is the component of the acceleration in the y direction for $t > 0$? Express your answer in terms of v_y (the component of the velocity in the y direction), C, g, m and u as needed.

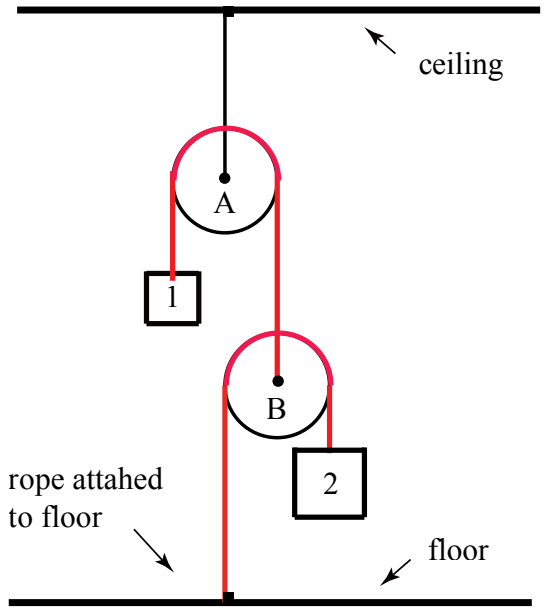
(c) Using your result from part (a), find $v_x(t)$, the horizontal component of the ball's velocity as a function of time t. Express your answer in terms of C, g, m, u and t as needed.

(d) Using your result from part (b), find $v_y(t)$, the vertical component of the ball's velocity as a function of time t. Express your answer in terms of C, g, m, u and t as needed.

(e) What is v_{xT}, the value of the horizontal component of the ball's terminal velocity? Express your answer in terms of C, g, m and u as needed.

(f) What is v_{yT}, the value of the vertical component of the ball's terminal velocity? Express your answer in terms of C, g, m and u as needed.

6. **Two Pulleys, Two Strings and Two Blocks**



Block 1 and block 2, with masses m_1 and m_2, are connected by a system of massless, inextensible ropes and massless pulleys as shown above.

Solve for the acceleration of block 2 in terms of m_1, m_2 and g. Assume that "down" is positive. Express your answer in terms of some or all of the following: g, m_1, and m_2.

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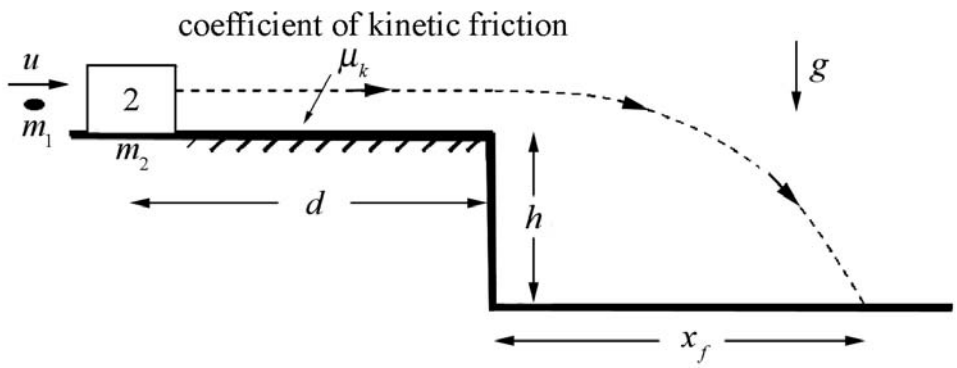
Problem Set 5

1. **Stopping a Bullet**



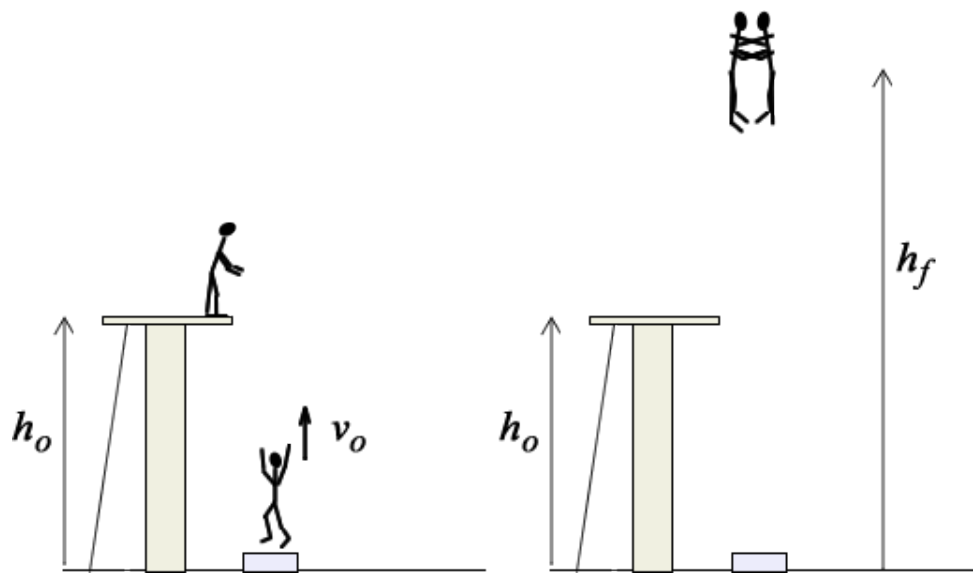
A bullet of mass m_1 traveling horizontally with speed u hits a block of mass m_2 that is originally at rest and becomes embedded in the block. After the collision, the block slides horizontally a distance d on a surface with friction, and then falls off the surface at a height h as shown.

The coefficient of kinetic friction between the block and the surface is μ_k. Assume the collision is nearly instantaneous and all distances are large compared to the size of the block. Neglect air resistance.

(a) What is u_{min}, the minimum speed of the bullet so that the block falls off the surface? Express your answer in terms of some or all of the following: m_1, m_2, μ_k, d, h and g for the gravitational constant.

(b) Assume that the initial speed of the bullet u is large enough for the block to fall off the surface. Calculate x_f, the position where the block hits the ground measured from the bottom edge of the surface. Express your answer in terms of some or all of the following: m_1, m_2, μ_k, u, d, h and g.

2. **Acrobat and Clown** An acrobat of mass m_A jumps upwards off a trampoline with an initial speed v_0. At a height h_0, the acrobat grabs a clown of mass m_B. Assume that the time the acrobat takes to grab the clown is negligibly small.



What is the maximum height h_f reached by the acrobat and clown? Write your answer in terms of some or all of the following: m_A, m_B, g, h_0, and v_0.

3. **Compressive Strength of Bones**

   The compressive force per area necessary to break the tibia in the lower leg is about $F/A = 1.6 \times 10^8$ N/m^2. The smallest cross sectional area of the tibia, about 3.2 cm^2, is slightly above the ankle. Suppose a person of mass $m = 60$ kg jumps to the ground from a height $h_0 = 2.0$ m and absorbs the shock of hitting the ground by bending the knees. Assume that there is constant deceleration during the collision with the ground, and that the person lowers their center of mass by an amount $d = 1.0$ cm from the time they hit the ground until they stop moving.

   (a) What is the collision time Δt_{col}, to 2 significant figures?

   (b) Find N_{ave}, the magnitude of the average force exerted by the ground on the person during the collision in Newtons.

   (c) What is the ratio of the average force of the ground on the person to the gravitational force on the person? Can we effectively ignore the gravitational force during the collision?

   (d) Will the person break his ankle?

4. **Center of Mass of a Rod**

A thin rod has length L and total mass M lies along the x axis.

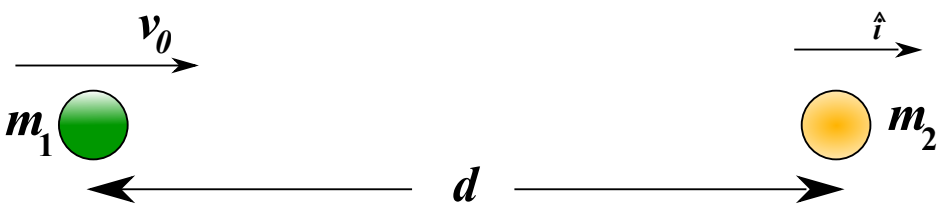
(a) Suppose the rod is uniform. Find the position of the center of mass with respect to the left end of the rod in terms of L, M, and $\hat{i}$.

(b) Now suppose the rod is not uniform but has a linear mass density that varies with the distance x from the left end according to

$$\lambda = \frac{\lambda_0}{L^3} x^3$$

where λ_0 is a constant and has SI units of kg/m. The total mass of the rod is still M. Find λ_0 and the position of the center of mass with respect to the left end of the rod. Express your answer in terms of some or all of the following: L, M, and $\hat{i}$.

5. **Two Particles Colliding**



Two small particles of mass m_1 and mass m_2 attract each other with a force that varies inversely with the cube of their separation. At time t_0, m_1 has a velocity of magnitude v_0, directed towards m_2, which is at rest a distance d away. At time t_1, the particles collide.

Calculate L, the distance travelled by particle 1 during the time interval $t_1 - t_0$. Express your answer using some or all of the following variables: m_1, m_2, t_0, t_1, v_0, and d.

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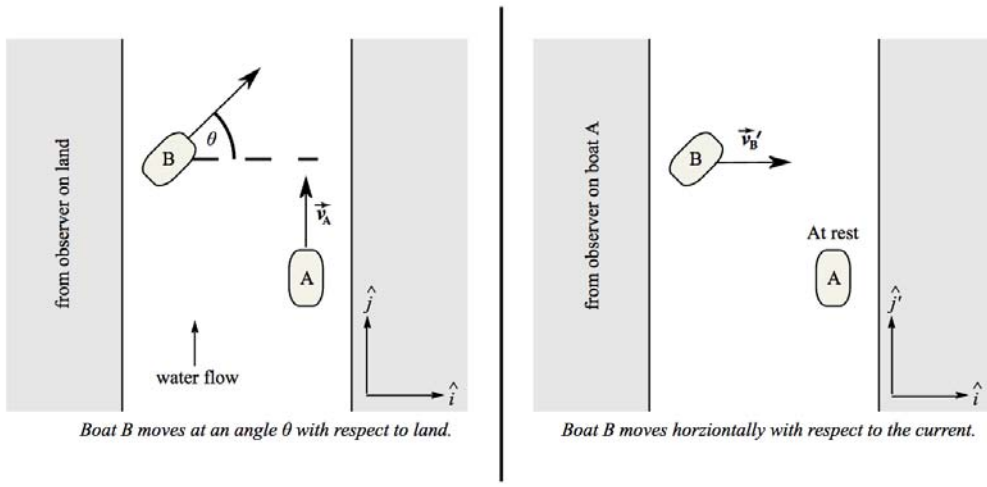
Problem Set 6

1. **Rowing Across the River**



An MIT student wants to row across the Charles River. Suppose the water is moving downstream at a constant rate of 1.0 m/s. A second boat is floating downstream with the current. From the second boat's viewpoint, the student is rowing perpendicular to the current at 0.5 m/s. Suppose the river is 800 m wide.

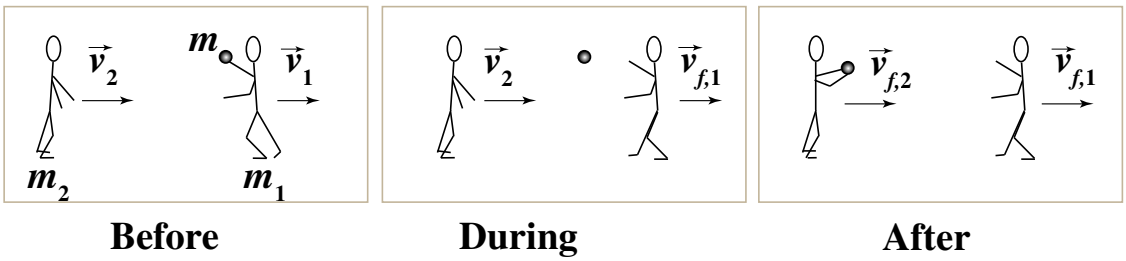
(a) What is the magnitude and direction of the velocity of the student as seen from an observer at rest along the bank of the river? The angle is measured as shown in the figure below.

(b) How far down river does the student land on the opposite bank?

(c) How long does the student take to reach the other side?

2. **Astronauts Playing Catch**

Two astronauts are playing catch in a zero gravitational field. Astronaut 1 of mass m_1 is initially moving to the right with speed v_1. Astronaut 2 of mass m_2 is initially moving to the right with speed $v_2 > v_1$. Astronaut 1 throws a ball of mass m with speed u **relative to herself** in a direction opposite to her motion. Astronaut 2 catches the ball. The final speed of astronaut 1 is $v_{f,1}$ and the final speed of astronaut 2 is $v_{f,2}$.



(a) What is the speed $v_{f,1}$ of astronaut 1 after throwing the ball? Express you answer in terms of some or all of the following: m, m_1, m_2, u, and v_1.

(b) What is the required speed u of the ball (relative to astronaut 1) such that the final speed of both astronauts are equal $v_{f,1} = v_{f,2}$? Express you answer in terms of some or all of the following: m, m_1, m_2, v_1 and v_2.

3. **Multi-stage Rocket in Empty Space**

A rocket in zero gravitational field has a mass of $m_{r,i} = 2.81 \times 10^7$ kg, which is the sum of the mass of the fuel $m_{f,i} = 2.46 \times 10^7$ kg and the dry mass of the rocket (empty of fuel) $m_{r,d} \equiv m_{r,i} - m_{f,i} = 0.35 \times 10^7$ kg. The fuel is ejected at a speed $u = 3000$ m/s relative to the rocket. The total burn time is 510 s and the fuel is burned at a constant rate.

(a) What is the final speed v_f of the rocket in meters/second after all the fuel is burned assuming it starts from rest?

(b) Now suppose that the same rocket burns the fuel in two stages, expelling the fuel in each stage at the same relative speed $u = 3000$ m/s. In stage one, the available fuel to burn is $m_{f,1,i} = 2.03 \times 10^7$ kg with burn time 150 s. The total mass of the rocket after all the fuel in stage 1 is burned is $m_{r,1,d} = m_{r,i} - m_{f,1,i} = 0.78 \times 10^7$ kg. What is the change in speed after stage one is complete?

(c) Next, the empty fuel tank and accessories from stage one are disconnected from the rest of the rocket. These disconnected parts had a mass $m = 1.4 \times 10^6$kg, hence the remaining dry mass of the rocket is $m_{r,2,d} = 2.1 \times 10^6$ kg. All the remaining fuel with mass $m_{f,2,i} = 4.3 \times 10^6$ kg is burned during stage 2 with burn time of 360 s. What is the change in speed in meters/second after stage two is complete?

(d) What is the final speed in meters/second of the rocket after both stages are complete?

(e) How does this compare to your answer to Part A for the final speed if all the fuel were burned in one stage? Does your answer make sense?

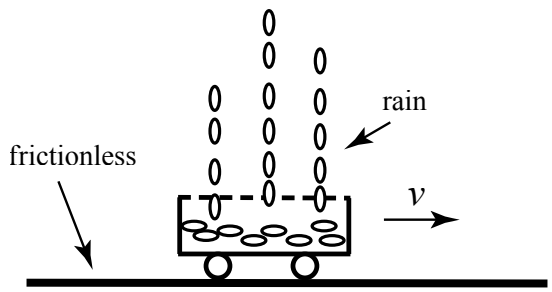
4. **Falling Drop**

A raindrop of mass m_0, starting from rest, falls under the influence of gravity. Assume that as the raindrop travels through the clouds, it gains mass at a rate proportional to the momentum of the raindrop, $\frac{dm_r}{dt} = km_r v_r$, where m_r is the instantaneous mass of the raindrop, v_r is the instantaneous velocity of the raindrop, and k is a constant with unit $[m^{-1}]$. You may neglect air resistance.

(a) Derive a differential equation for the raindrop's accelerations $\frac{dv_r}{dt}$ in terms of k, g, and the raindrop's instantaneous velocity v_r. Express your answer using some or all of the following variables: k, g for the gravitational acceleration and v_r, the raindrop's instantaneous velocity.

(b) What is the terminal speed, v_T, of the raindrop? Express your answer using some or all of the following variables: k and g for the gravitational acceleration.

5. **Moving Vehicle and Falling rain**



A vehicle of 'dry' mass m_0 , with no propulsion, is moving without friction on horizontal ground as shown in the figure above. Rain is falling vertically. While the rain is falling, for each time interval Δt, an amount of rain $\Delta m_r = b\Delta t$ hits the vehicle, sticks to it, and then moves along with zero relative velocity with respect to the vehicle. The vehicle is observed to be slowing down. In a reference system fixed to the ground, at time $t = 0$, the cart is initially moving with speed v_0.

(a) At time t, the vehicle is traveling with speed $v(t)$. For the time interval $[t, t+\Delta t]$, while the rain is falling, determine a differential relation for the change in speed of the vehicle Δv. Write your answer using some or all of the following: t, Δt, m_0 and b.

(b) Integrate your result from part a) to find the speed $v(t)$ of the cart at time t. Write your answer using some or all of the following: v_0, b, m_0 and t.

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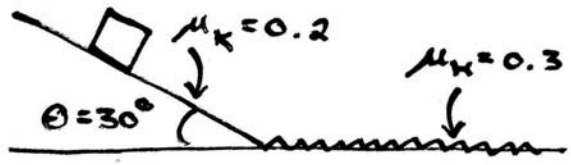
Problem Set 7

1. **Object Sliding Down an Inclined Plane**

   An object of mass $m = 4.0$ kg, starting from rest, slides down an inclined plane of length $l = 3.0$ m. The plane is inclined by an angle of $\theta = 30°$ to the ground. The coefficient of kinetic friction $\mu_k = 0.2$. At the bottom of the plane, the mass slides along a rough surface with a coefficient of kinetic friction $\mu_k = 0.3$ until it comes to rest. The goal of this problem is to find out how far the object slides along the rough surface.



   (a) What is the work done by the friction force while the mass is sliding down the inclined plane? (Is it positive or negative?)

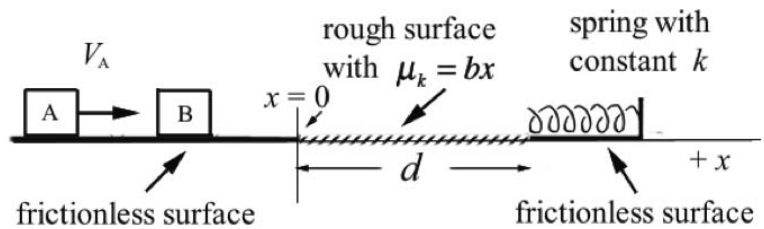
   (b) What is the work done by the gravitational force while the mass is sliding down the inclined plane? (Is it positive or negative?)

   (c) What is the kinetic energy of the mass when it just reaches the bottom of the inclined plane?

   (d) Symbolically, what is the work done by the friction force while the mass is sliding along the ground? Is this positive or negative? Express you answer in terms of some or all of the following: m, μ_k, g, and d where d is the distance it takes the object to stop measured from the bottom of the incline.

   (e) How far from the bottom of the inclined plane does the object slide along the rough surface?

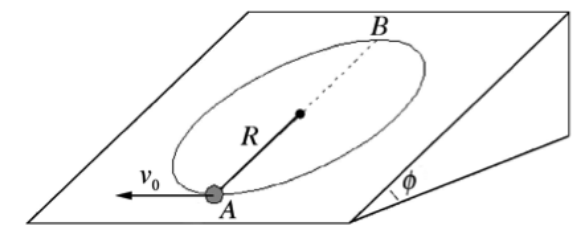
2. **Collision and Sliding on a Rough Surface**

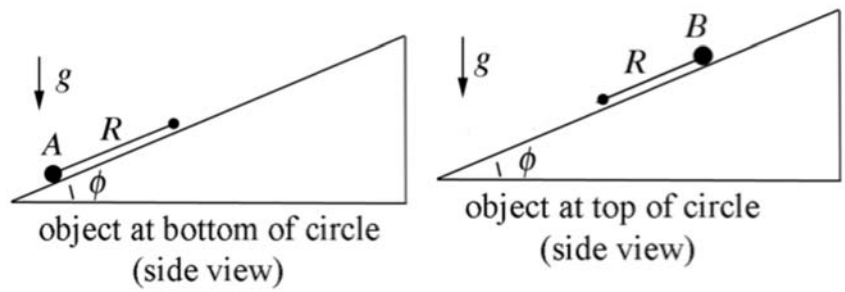


Block A of mass m_A is moving horizontally with speed v_A along a frictionless surface. It collides with block B of mass m_B that is initially at rest. The two blocks stick together after the collision. At $x = 0$, block B enters a rough surface with a coefficient of kinetic friction that increases linearly with distance $\mu_k(x) = bx$ for $0 \leq x \leq d$, where b is a positive constant.

At $x = d$, block B collides with an unstretched spring with spring constant k on a frictionless surface. The downward gravitational acceleration has magnitude g. What is the distance the spring is compressed when the blocks first comes to rest? Express you answer in terms of some or all of the following: v_A, b, d, g, k, m_A and m_B.

3. **Inclined plane** A body of mass m is attached to one end of a string of length R. The other end of the string is fixed on an inclined plane making an angle ϕ with the horizontal as shown in the figure. The body has speed v_0 at the bottom of the circle (point A). The body undergoes circular motion. There is a coefficient of sliding friction μ between the body and the plane. The downward acceleration of gravity is g. Express all answers in terms of m, ϕ, v_0, g, μ and R as needed.





(a) How much work does the friction force do on the body as it moves from the bottom of the circle (point A) to the top of the circle (point B)?

(b) What is the tension in the string when it reaches point B? Express your answer in terms of m, ϕ, v_0, g, μ and R as needed.

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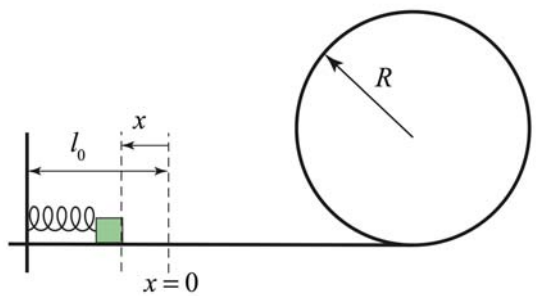
Problem Set 8

1. **Spring-Loop-the-Loop**

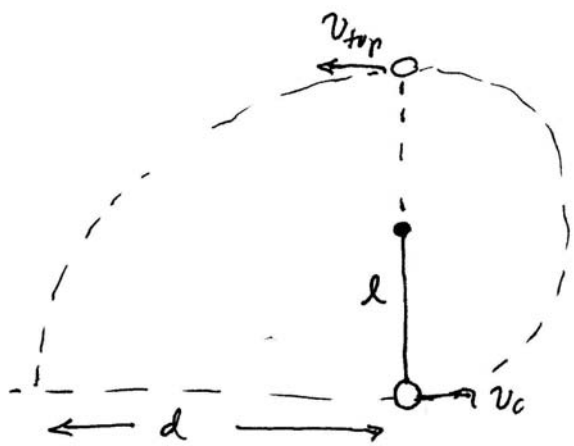
   A small block of mass m is pushed against a spring with spring constant k and held in place with a catch. The spring compresses an unknown distance x. When the catch is removed, the block leaves the spring and slides along a frictionless circular loop of radius R.



   When the block reaches the top of the loop, the force of the loop on the block (the normal force) is equal to twice the gravitational force on the mass. How far was the spring initially compressed? Write your answer using some or all of the following: g, k, R, and m.

2. **Slingshot**

A ball of negligible size and mass m hangs from a string of length l. It is hit in such a way that it then travels in a vertical circle. The initial speed of the ball after being struck is v_0. The goal of the first part of this problem is to find the tension in the string when the ball is at the top of the circle. You may assume that there are no external forces other than gravity doing work on the ball and string. Let g denote the magnitude of the gravitational constant.

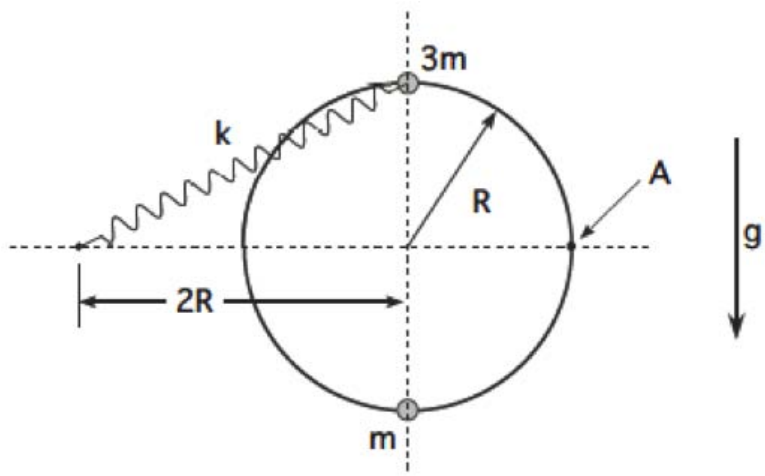


(a) Find the tension in the string when the ball is at the top of the circle. Express your answer in terms of some or all of the following: m, g, l and v_0.

(b) When the ball is exactly at the top of the circle, it detaches from the string and follows the trajectory shown in the figure above. When the ball returns to the level of the bottom of the circle, it is a distance d from the bottom of the circle. Find the distance d. Express your answer in terms of m, g, l, and v_0 as needed.

3. **Objects on a Ring**

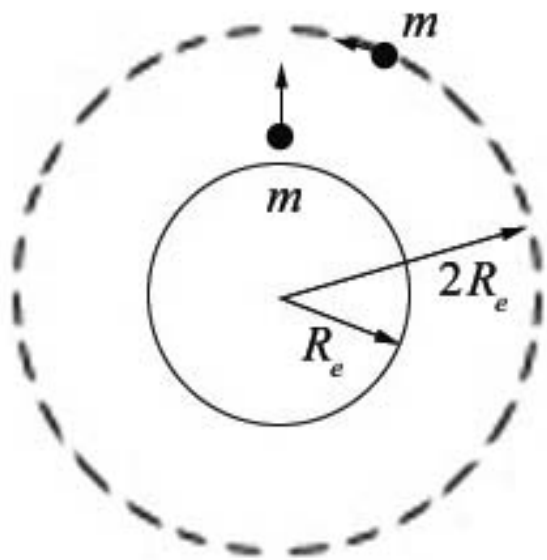
Two objects slide without friction on a circular ring of radius R oriented in a vertical plane. The heavier object (of mass $3m$) is attached to a spring with an unstretched length of zero (admittedly an unphysical assumption) and spring constant k. The fixed end of the spring is attached to a point a horizontal distance $2R$ from the center of the circle. The lighter object (of mass m) is initially at rest at the bottom of the ring. The heavier object is released from rest at the top of the ring, then collides with and sticks to the lighter object. Find the value for m that will allow the combined object (of mass $4m$) to just reach the point A on the ring, but go no higher. Express your answer in terms of some or all of the quantities k, R and the acceleration of gravity g.



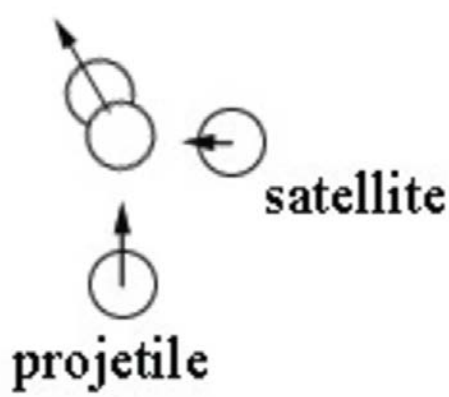
Express you answer in terms of some or all of the following: k, R and g.

4. **Orbital Collisions**

A projectile of mass m is fired vertically from the earth's surface with an initial speed that is equal to the escape velocity. The radius of the earth is R_e, the mass of the earth is m_e, and the universal gravitational constant is G.



(a) What is the initial speed of the projectile when it is launched from the surface of the earth? Express you answer in terms of some or all of the following: m, G, m_e and R_e.



(b) When the projectile is a distance $2R_e$ from the center of the earth, it collides with a satellite also of mass m that has been orbiting the earth in a circular orbit before

the collision. After the collision the two objects stick together. Assume that the collision is instantaneous, and that the velocities of the projectile and satellite are orthogonal right before the collision. What is the speed of the projectile, just before the collision, when it is a distance $2R_e$ from the center of the earth? Express you answer in terms of some or all of the following: m, G, m_e, and R_e.

(c) What is the speed of the satellite, just before the collision, when it is in a circular orbit of radius $2R_e$? Express you answer in terms of some or all of the following: m, G, m_e and R_e.

(d) What is the speed of projectile and satellite immediately after the collision? Express you answer in terms of some or all of the following: m, G, m_e and R_e.

5. **Exponential Potential Energy Diagram** A particle of mass m moves in one dimension. Its potential energy is given by

$$U(x) = -U_0 e^{-x^2/a^2}$$

where U_0 and a are constants.

(a) Draw an energy diagram showing the potential energy $U(x)$. Choose some value for the total mechanical energy E such that $-U_0 < E < 0$. Mark the kinetic energy, the potential energy and the total energy for the particle at some point of your choosing.

(b) Find the force on the particle as a function of position x. Express you answer in terms of some or all of the following: x, a, and U_0.

(c) Find the speed at the origin $x = 0$ such that when the particle reaches $x = \pm a$, it stops momentarily and reverses the direction of its motion. Express you answer in terms of some or all of the following: x, a, m and U_0.

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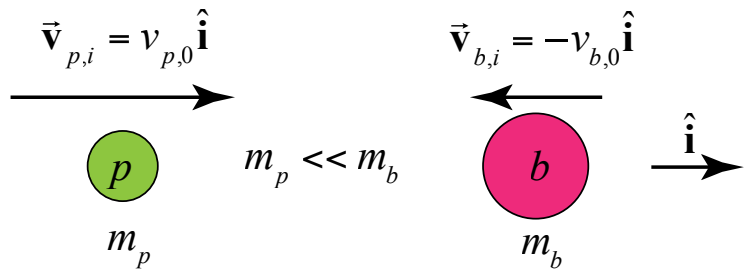
Problem Set 9

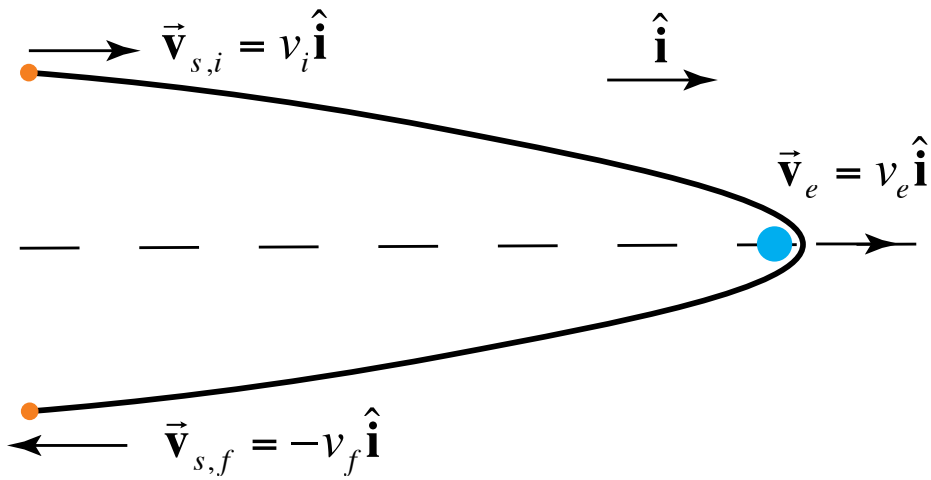
1. **Ping-Pong Ball and Bowling Ball Collision**

   A ping-pong ball and a bowling ball collide elastically on a frictionless surface. The magnitude of the initial velocity of the ping-pong ball is $v_{p,0}$ and the direction of the velocity is in the positive x-direction. The magnitude of the initial velocity of the bowling ball is $v_{b,0}$ and the direction of the velocity is in the negative x-direction. You may assume that the mass of the bowling ball is much greater than the mass of the ping-pong ball.



   After the collision, what is the component of the velocity of the ping-pong ball in the x direction? (pay attention to signs). Express your answer in some or all of the following variables: v_{p0} and v_{b0}.

2. **Satellite Fly-By** A satellite (with mass negligible compared to that of the Earth) is making a fly-by of the Earth on a hyperbolic orbit. Let the velocity of the Earth be $\vec{v}_e = v_e\hat{i}$. The satellite originally approaches from the negative x-direction, goes around the earth, and then flies off in the same direction it came from. Far from the Earth, the magnitude of the satellite's x-component of velocity is v_i when approaching and v_f when receding.

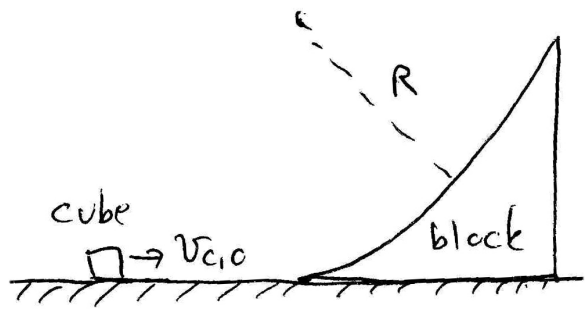


For the case where $\vec{v}_i = 4v_e\hat{i}$, what is the final speed v_f of the satellite in terms of the speed of the earth v_e?

3. **Block Sliding up Curved Surface**

A block of mass m_b sits at rest on a frictionless table; the block has a circular surface of radius R as shown in the figure. A small cube of mass $m_c < m_b$ and speed $v_{c,0}$ is incident upon the block; the cube moves without friction on the table and the block.

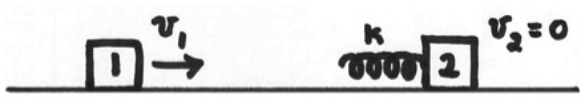
Notice that in this problem, the block does move since it is not fixed to the table



(a) What is the maximum height h above the table that is reached by the cube? Assume $v_{c,0}$ is small enough so that $h < R$. Express your answer using some or all of the following variables: $v_{c,0}$, g, m_b and m_c.

(b) After the cube reaches the maximum height, it slides back down the wedge. After the two separate again, what is the speed (magnitude of the velocity) of each? Express your answer using some or all of the following variables: $v_{c,0}$, g, m_b and m_c.

4. **Spring Blocks Motion**

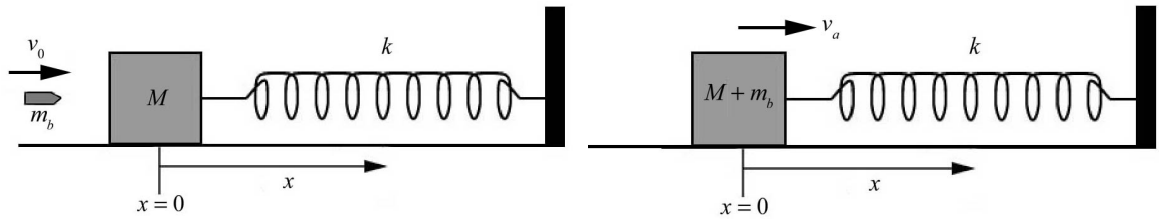


Block 1 of mass m_1 is moving with speed v_1 to the right on a horizontal frictionless table as shown in the figure above. The block collides with the free end of a spring-block system of m_2 that is initially at rest. The collision is one-dimensional. The spring has spring constant k.

What is the maximum compression of the spring? Express your answer using some or all of the following: k, m_1, m_2 and v_1.

5. **Bullet-Spring Collision**

A massless spring with spring constant k is attached at one end of a block of mass M that is resting on a frictionless horizontal table. The other end of the spring is fixed to a wall. A bullet of mass m_b is fired into the block from the left with a speed v_0 and comes to rest in the block (assume that this happens instantaneously).

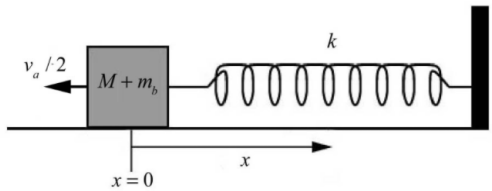


(a) How fast is the block moving immediately after the bullet comes to rest with respect to the block? Express you answer in terms of some or all of the following: M, m_b, and v_0.

(b) The block is at $x = 0$ when the bullet hits the block and the block starts moving. How far has the block travelled when it first comes to rest after this? Express you answer in terms of some or all of the following: M, m_b, k and v_0.

(c) Immediately after the bullet comes to rest with respect to the block, what is the ratio of the kinetic energy of the bullet-block to the original kinetic energy of the bullet? Express you answer in terms of some or all of the following: M, m_b and v_0.

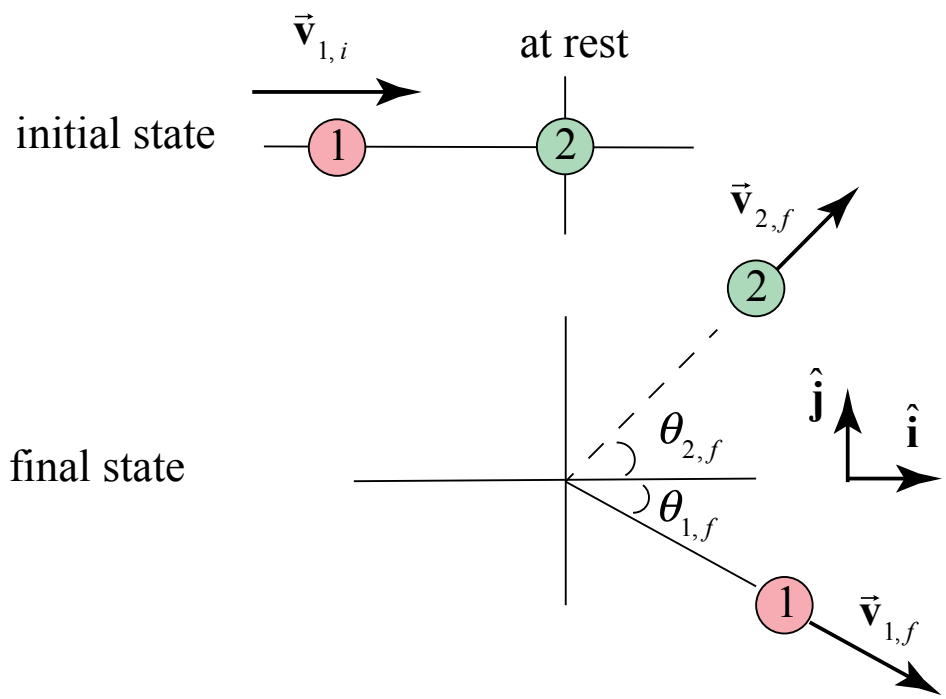
(d) Now suppose that instead of sliding on a frictionless table, the coefficient of kinetic friction between the block and the table is μ_k. Suppose that when the block first returned to the position $x = 0$, the speed of the block was found to be one half the speed immediately after the collision that you found in part a.



What is the total distance that the block traveled before returning to $x = 0$? Express you answer in terms of some or all of the following: M, m_b and v_0, μ_k, and g.

6. **Elastic 2D Collision**

Particle 1 of mass $m_1 = m$ is initially moving in the positive x-direction with a speed $v_{1,i}$ and collides elastically with Particle 2 of mass $m_2 = 2m$, which is initially at rest. After the collision Particle 1 moves with an unknown speed $v_{1,f}$, at an unknown angle $\theta_{1,f}$ with respect to the positive x-direction and particle 2 moves with an unknown speed $v_{2,f}$, at an angle $\theta_{2,f} = 45°$ with respect to the positive x-direction (notice the convention of measuring the angles shown below).



Find $\tan\theta_{1,f}$ (your answer should reduce to a simple numerical value)

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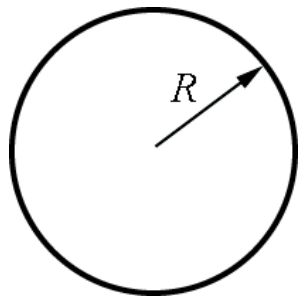
Problem Set 10

1. **Moment of Inertia: Disc and Washer**

   (a) A thin uniform disc of mass M and radius R is mounted on an axis passing through the center of the disk, perpendicular to the plane of the disc. Determine the moment of inertia about the following two parallel axes that are perpendicular to the plane of the disk.

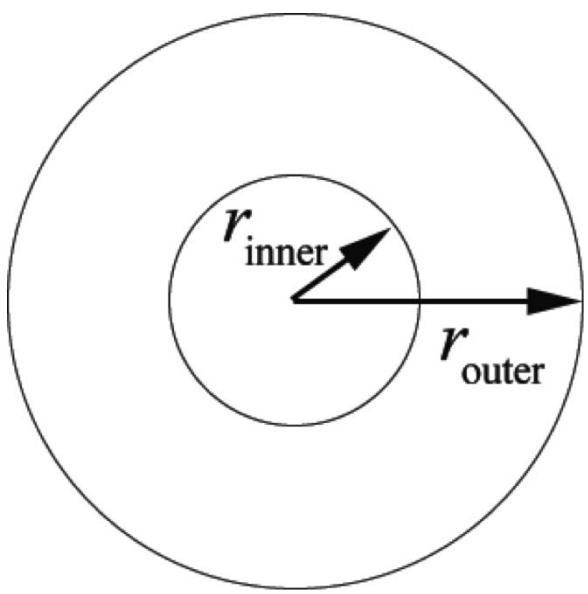


   (i) An axis passing through the center of mass of the disc.

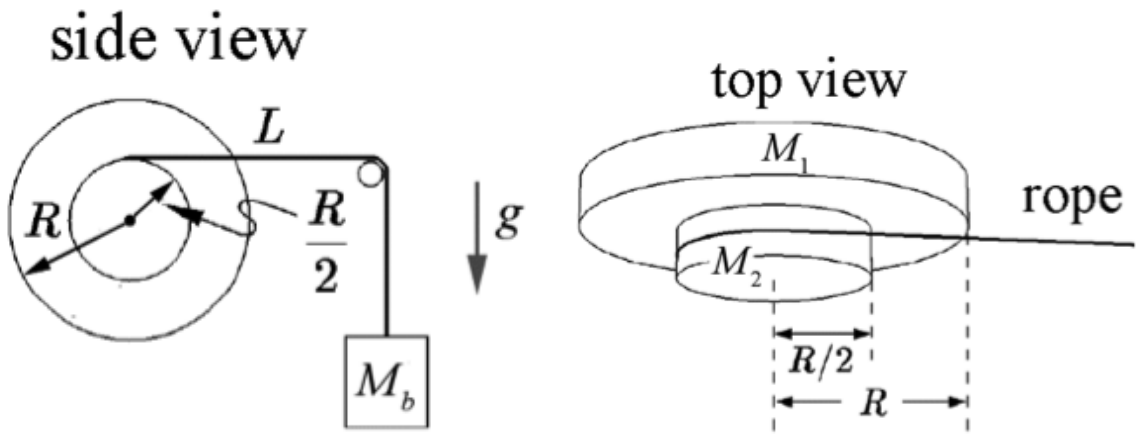
   (ii) An axis passing through a point on the rim of the disc a distance R from the center.

   Express your answers in some or all of the following variables: M and R.

   (b) A 1 inch US Standard Washer has inner radius $r_i = 13.5$ mm and an outer radius $r_o = 31.0$ mm. The washer is approximately $d = 4.0$ mm thick. The density of the washer is $\rho = 7.8 \times 10^3$ kg m^{-3}. Calculate the moment of inertia of the washer about an axis that is perpendicular to the plane of the washer and passes through its center of mass.



2. **Compound Pulley**

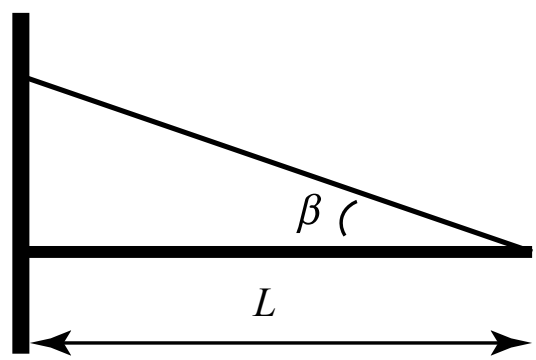


Consider a fixed wheel that consists of a uniform cylindrical disk with a radius R and a mass M_1, and a second uniform cylindrical disk of radius $R/2$ and mass M_2 that is attached concentrically to the first disk, as shown in the top view (above right). The wheel is free to rotate about a frictionless pivot at its center. A rope is wound around the smaller disk, then around a small frictionless post. The other end of the rope is attached to a block of mass M_b so that both disks rotate together as the block falls. The downward acceleration of gravity is g. The block is released from rest.

How long does it take the block to fall a distance d? Express your answer in terms of R, M_1, M_2, M_b, g, and d as needed.

3. **Suspended Rod**

A uniform rod of Length $L = 2.0$ m and mass $m = 4.0$ kg is hinged to a wall at one end and suspended from the wall by a cable that is attached to the other end of the rod at an angle of $\beta = 30°$ to the rod (see figure below). Assume the cable has zero mass. There is a contact force at the pivot on the rod. Use α to represent the angle of the pivot force (the force acting at the wall). The magnitude and direction of this force is unknown. In this problem, you will solve for the magnitude of the tension in the cable and the direction and magnitude of the pivot force.



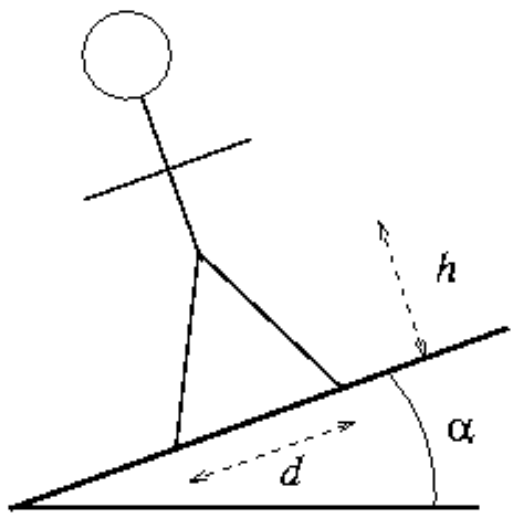
(a) What is the magnitude of the tension in the cable?

(b) What angle does the pivot force make with the beam in degrees?

(c) What is the magnitude of the pivot force?

4. **Person Standing on a Hill**

A person is standing on a hill that is sloped at an angle of α with respect to the horizontal. The person's legs are separated by a distance d, with one foot uphill and one downhill. The center of mass of the person is at a distance h above the ground, perpendicular to the hillside, midway between the person's feet. Assume that the coefficient of static friction between the person's feet and the hill is sufficiently large that the person will not slip.

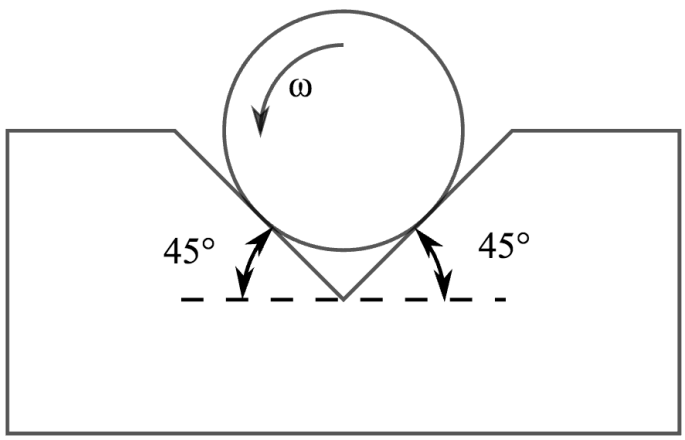


(a) What is the magnitude of the normal force on each foot ($N_1 > N_2$)? Express you answer in terms of some or all of the following: m, α, h, d and g.

(b) How far must the feet be apart so that the normal force on the upper foot is just zero? This is the moment when the person starts to rotate and fall over. Express you answer in terms of some or all of the following: m, α, h and g.

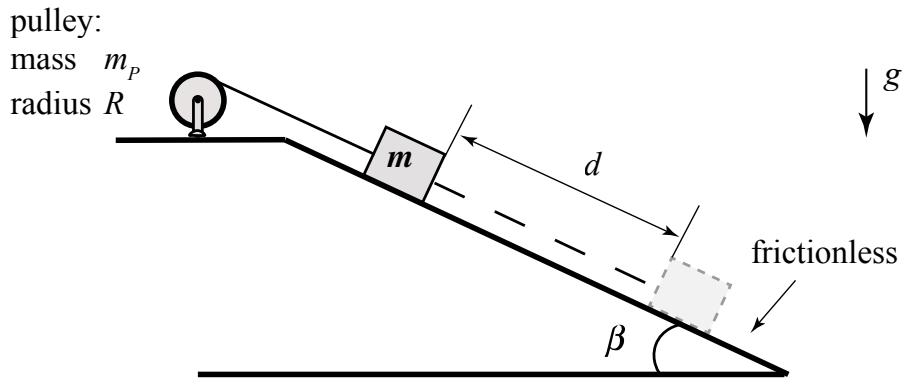
5. **A Cylinder Rolling in a V-Groove**

A cylinder of mass m and radius R is rotating in a V-shaped groove with a constant angular velocity ω_0. The coefficient of friction between the cylinder and the surface is μ.



What external torque must be applied to the cylinder to keep it rotating at a constant angular velocity? Express your answer using some or all of the following variables: m for the mass of the cylinder, g for the gravitational acceleration, R for the radius of the cylinder and μ for the coefficient of friction.

6. **A Massive Pulley and a Block on an Incline**



Consider a pulley of mass m_p and radius R that has a moment of inertia $\frac{1}{2}m_pR^2$. The pulley is free to rotate about a frictionless pivot at its center. A massless string is wound around the pulley and the other end of the rope is attached to a block of mass m that is initially held at rest on frictionless inclined plane that is inclined at an angle β with respect to the horizontal. The downward acceleration of gravity is g. The block is released from rest.

How long does it take the block to move a distance d down the inclined plane? Write your answer using some or all of the following: R, m, g, d, m_p, β.

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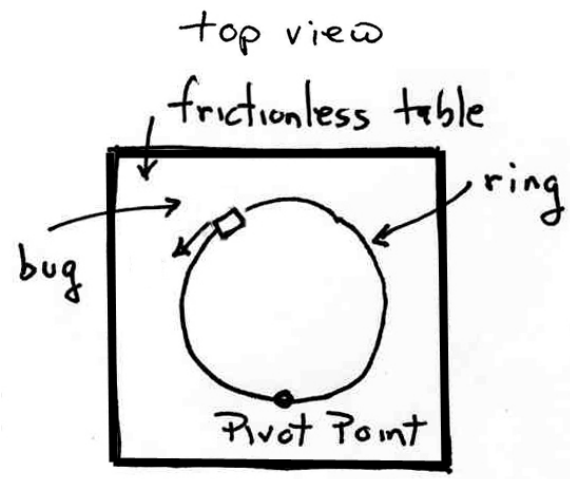
Problem Set 11

1. **Bug Walking on Pivoted Ring**

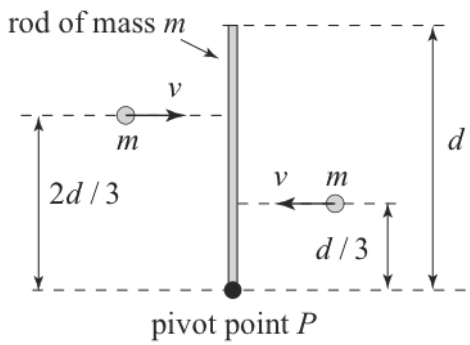
   A ring of radius R and mass m_1 lies on its side on a frictionless table. It is pivoted to the table at its rim. A bug of mass m_2 walks on the ring with constant speed v relative to the ring, starting at the pivot, when the ring is initially at rest. Take $\hat{k}$ to point out of the page.



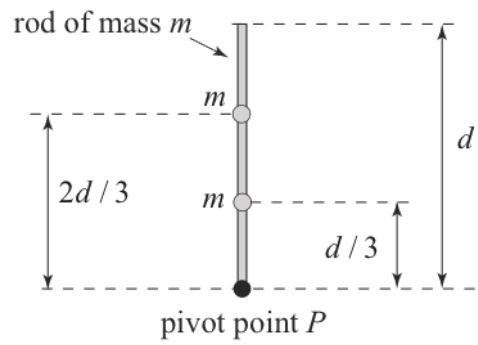
   (a) What is the angular velocity of the ring when the bug is halfway around? Express you answer in terms of some or all of the following: m_1, m_2, v, R and $\hat{k}$.

   (b) What is the angular velocity of the ring when the bug is back at the pivot? Express you answer in terms of some or all of the following: m_1, m_2, v, R and $\hat{k}$.

2. **A Rigid Rod**



view from above frictionless surface before collision



view from above frictionless surface after collision

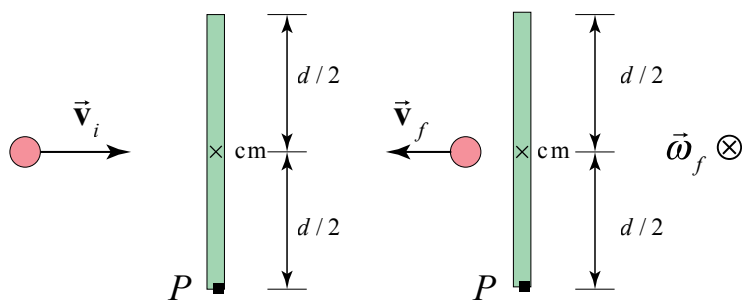
A rigid uniform rod of length d and mass m is lying at rest on a horizontal frictionless table and pivoted at the point P. A point-like object of mass m is moving to the right with speed v. It collides and sticks to the rod at a distance $2d/3$ from the pivot. A second point-like object of mass m is moving to the left (see figure) with speed v and collides with the rod at exactly the same instant as the first particle at a distance $d/3$ from the pivot. The moment of inertia of a rod for axis through the center of mass and perpendicular to the plane of the rod is $I_{cm} = \frac{1}{12}md^2$. After the collision, the rod and the two particles all rotate about the pivot point with angular speed ω_f.

(a) What is the component of the angular speed ω_f of the two particles and the rod immediately after the collision? Express your answer in terms of d, m, and v, as needed. Assume clockwise (into the page) to be positive.

(b) What is the ratio of the change in kinetic energy to the initial kinetic energy of the system, $\frac{K_f - K_i}{K_i}$? Express your answer in terms of d, m, and v, as needed.

3. **Elastic Collision Between Ball and Pivoted Rod**

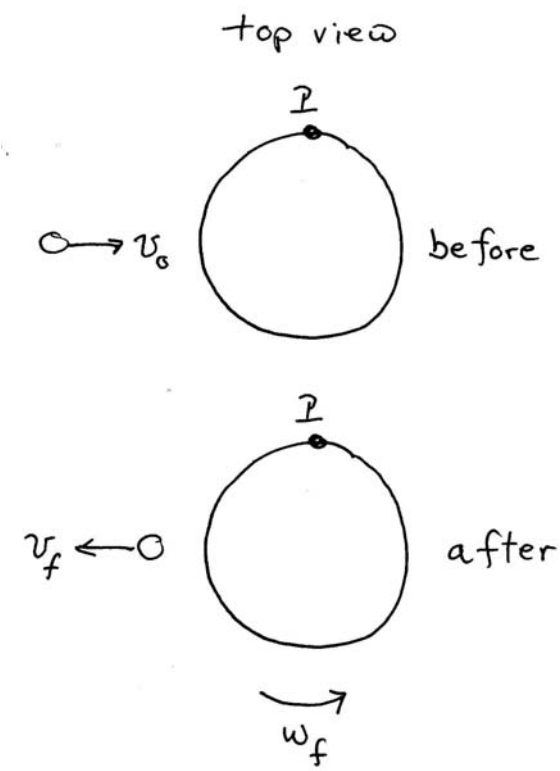
A rigid rod of length d and mass m is lying on a horizontal frictionless table and pivoted at the point P on one end (shown in the figure). A point-like object of the same mass m is moving to the right (see figure) with speed v_i. It collides elastically with the rod at the midpoint of the rod and rebounds backwards with speed v_f. After the collision, the rod rotates clockwise about its pivot point P with angular speed ω_f. The moment of inertia of a rod about the center of mass is $I_{cm} = \frac{1}{12}md^2$.



Find the angular speed ω_f. Express your answer in terms of d, m and v_i as needed.

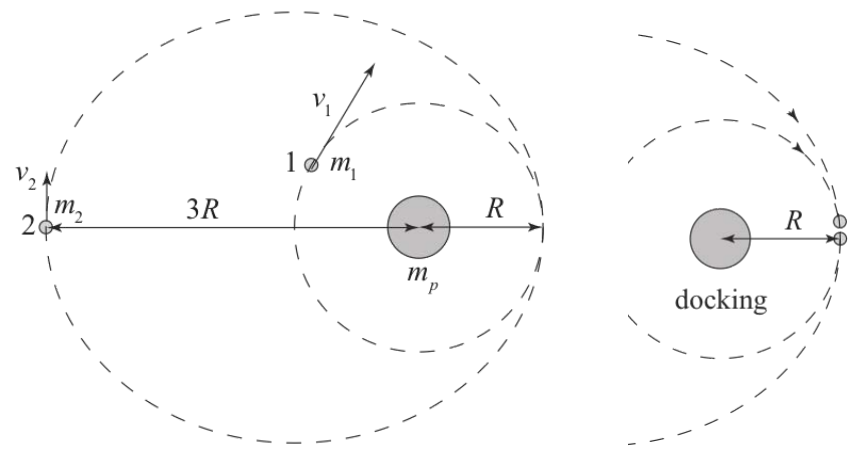
4. **Elastic Collision of Object and Pivoted Ring**

A rigid hoop of radius R and mass m_R is lying on a horizontal frictionless table and pivoted at the point P (shown in the figure below). A point-like object of mass m is moving to the right with speed v_0. It collides elastically with the hoop at its midpoint. After the collision, the object is moving with an unknown speed v_f to the left and the hoop rotates counterclockwise about its pivot point with angular speed ω_f. The moment of inertia of a hoop for axis through the center of mass and perpendicular to the plane of the hoop is $I_{\text{cm}} = m_R R^2$.



What is the speed v_f of the object immediately after the collision? Express your answer in terms of R, m, m_R, and v_0 as needed (do not use ω_f in your answer).

5. **A Spaceship and a Planet**



Spaceship 1 has mass m_1 and is moving with speed v_1 in a circular orbit of radius R around a planet of mass m_p. Spaceship 2 has mass m_2 and is moving in an elliptical orbit around the same planet. The mass of the planet is much, much greater than the mass of either spaceship. When spaceship 2 is at its furthest distance $3R$ from the planet, it is moving with speed v_2. When spaceship 2 is at its closest distance R from the planet, it is moving with speed v_p. The two spaceships are orbiting in the same plane as shown in the figures above. At a later time, both spaceships arrive nearly simultaneously at a point corresponding to the closest approach of spaceship 2. Spaceship 2 fires its rockets in order to reach the same speed v_1 as spaceship 1 in order to dock together. You may assume that the elapsed time interval for docking is very small compared to the orbital periods of the spaceships. Let G be Newton's universal constant of gravity.

What is the change in the speed, $\Delta v = v_1 - v_p$, of spaceship 2 in order for the two spaceships to dock together? (Does spaceship 2 speed up or slow down in order to dock?) Express your answer only in terms of G, R and m_p.

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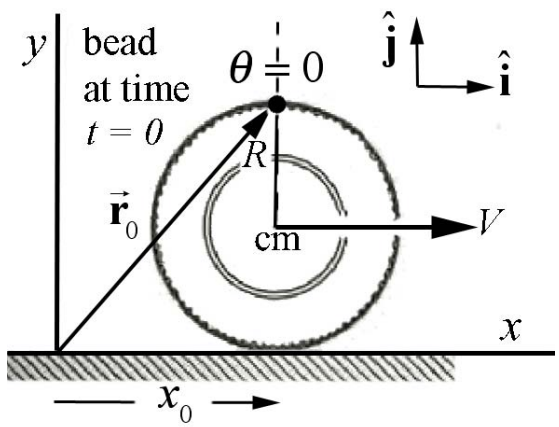
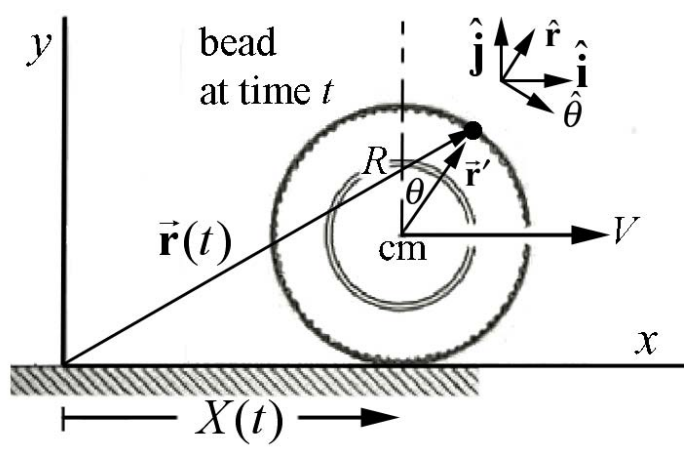
Problem Set 12

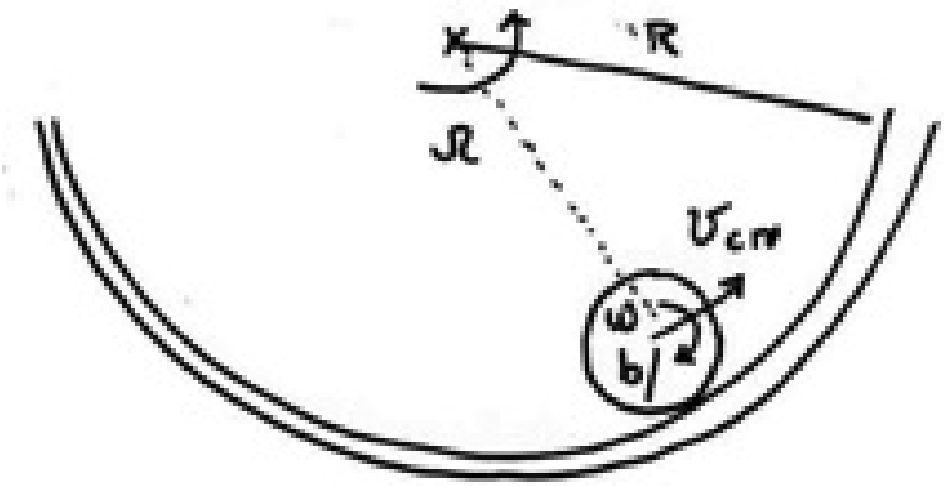
1. **Bicycle Wheel** A bicycle wheel of radius R is rolling without slipping along a horizontal surface. The center of mass of the bicycle is moving with a constant speed V in the positive x-direction. A bead is lodged on the rim of the wheel. Assume that at $t = 0$, the bead is located at the top of the wheel at $x(t = 0) = x_0$, $y(t = 0) = 2R$.





What are the x- and y-components of the position of the bead as a function of time according to an observer fixed to the ground? Express you answer in terms of some or all of the following: x_0, V, t and R.

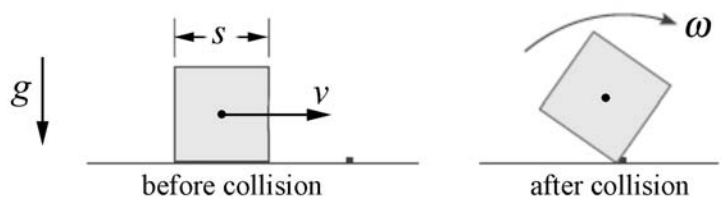
2. **Rolling Without Slipping**



A ball of radius b and mass M is rolling without slipping on the surface of a ring of radius $R > b$. The angular speed of the ball is ω. Suppose the ring is rotating with angular speed Ω as shown in the figure and the ball is rolling without slipping. What is the speed of the center of mass of the ball? Write your answer using some or all of the following: b, R, ω and Ω.

3. **Cubical Block Collision with Low Ridge**

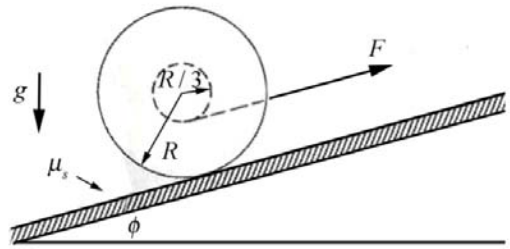
A uniform cubical block of mass m and edge length s slides without friction with speed v on a horizontal surface. It collides inelastically with a low ridge on the surface. Assume the collision is instantaneous. It subsequently pivots without friction about that ridge. The moment of inertia of a cube about an axis normal to and centered on one of its faces (and thus passing through the center of mass) is $ms^2/6$. The downward acceleration of gravity is g.



(a) Find the initial rotation rate of the cube ω_a just after the collision. Express your answer in terms of some or all of the parameters m, s, v_0 and g.

(b) Find the maximum value v_0 such that the cube does not topple over and end up resting on other side of the ridge. Express your answer in terms of some or all of the parameters m, s, v_0 and g.

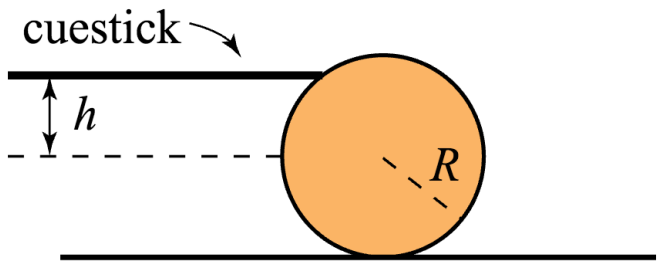
4. **Yo-yo Rolling On Inclined Plane**



A yo-yo of mass m has a spool of radius R and an axle of radius $R/3$. Its moment of inertia about an axis passing through the center of the yo-yo can be approximated by $I_0 = (1/2)mR^2$. The yo-yo is placed upright on an inclined plane that makes an angle ϕ with respect to the horizontal, and the string is pulled with a force of magnitude F in the direction shown in the figure. The axis of rotation of the yo-yo is perpendicular to the page. The downward acceleration of gravity is g. The coefficient of static friction between the yo-yo and the inclined plane is μ_s.

What is the maximum magnitude of the pulling force, F, for which the yo-yo will roll without slipping? Express your answer using some or all of the following variables: μ_s, ϕ, m, R and g.

5. **Billiards**



A spherical billiard ball of uniform density has mass m and radius R and moment of inertia about the center of mass $I_{cm} = \frac{2}{5}mR^2$. The ball, initially at rest on a table, is given a sharp horizontal impulse by a cue stick that is held an unknown distance h above the centerline (see diagram above). The coefficient of sliding fiction between the ball and the table is μ_k. The average force due to the impulse is much larger than the force of friction, so you may ignore the friction during the time the impulse is applied. The ball leaves the cue with a given speed v_0 and an unknown angular velocity ω_0. Because of its initial rotation, the ball starts to skid, but eventually acquires a maximum speed of $\frac{9}{7}v_0$ when it starts rolling without slipping. Find the ratio h/R. Your answer should simplify to a simple numerical fraction.

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