$$\dot{u_j} = d(u_{j+1} - 2u_j + u_{j-1}) + \lambda[-1 + \alpha f(u_j(t-1)) - \beta g(u_j)]u_j, \quad j = \overline{1,m}, \tag{1}$$

$$\underline{u_0 = u_1,\ u_m = u_{m+1}},$$

$$u_j = u_j(t) > 0,\ m \geq 2,\ \lambda >> 1,\ \beta > 0,\ \alpha > 1 + \beta,$$

$$f(u), g(u) \in C^2(\mathbb{R}_+): \quad \mathbb{R}_+ = \{u \in \mathbb{R} : u \geqslant 0\},$$

$$0 < \beta g(u) < \alpha,\ f(0) = g(0) = 1, \quad \forall u \in \mathbb{R}_+;$$

$$f(u), g(u), uf'(u), ug'(u), u^2 f''(u), u^2 g''(u) = O(1/u), \quad u \to +\infty.$$

$$u_1 = \exp\left(\frac{x}{\varepsilon}\right),\ u_j = \exp\left(\frac{x}{\varepsilon} + \sum_{k=1}^{j-1} y_k\right), \quad j = \overline{2,m}, \quad \varepsilon = \frac{1}{\lambda} << 1.$$

$$\begin{gathered} \dot{y_j} = d[\exp y_{j+1} + \exp(-y_j) - \exp y_j - \exp(-y_{j-1})], \\ y_j(+0) = \frac{\alpha - 1}{\alpha - \beta - 1} y_j(-0), \quad y_j(1+0) = y_j(1-0) - \frac{\alpha}{\alpha - 1} y_j(+0), \\ y_j(\alpha + 0) = (1+\beta) y_j(\alpha - 0), \quad y_j(\alpha + 1 + 0) = y_j(\alpha + 1 - 0) - \frac{\alpha}{1+\beta} y_j(\alpha + 0), \\ y_0 = y_m = 0, \quad j = \overline{1, m-1}. \end{gathered} \tag{2}$$

$$\dot{x} = -1 + \alpha f\left(\exp\left(\frac{x(t-1)}{\varepsilon}\right)\right) - \beta g\left(\exp\left(\frac{x}{\varepsilon}\right)\right)$$

$$\Phi(z): \begin{pmatrix} z_1 \\ \vdots \\ z_{m-1} \end{pmatrix} \to \begin{pmatrix} y_1(T_0) \\ \vdots \\ y_{m-1}(T_0) \end{pmatrix}, \tag{3}$$

$$y_1(-0) = z_1,\ \ldots,\ y_{m-1}(-0) = z_{m-1}$$

$$T_0 = \alpha + 1 + (\beta + 1)/(\alpha - \beta - 1)$$

Theorem 1. *For any stable point z_* of mapping (3) there exists a stable relaxational cycle with period T_0 in system (1).*

$$\begin{pmatrix} y_1(-0) \\ \vdots \\ y_{m-1}(-0) \end{pmatrix} \to \begin{pmatrix} y_1(-0+h) \\ \vdots \\ y_{m-1}(-0+h) \end{pmatrix} \to \cdots \to \begin{pmatrix} y_1(T_0) \\ \vdots \\ y_{m-1}(T_0) \end{pmatrix}.$$

$$y_j(t+h) = y_j(t) + \frac{h}{6}\,(k_1 + 2k_2 + 2k_3 + k_4), \quad j = \overline{1, m-1}\,,$$

$$\begin{aligned}
&k_1 = d(\exp(y_{j+1}) + \exp(-y_j) - \exp(y_j) - \exp(-y_{j-1})),\\
&k_2 = d(\exp(y_{j+1} + \tfrac{h}{2}k_1) + \exp(-y_j + \tfrac{h}{2}k_1) - \exp(y_j + \tfrac{h}{2}k_1) - \exp(-y_{j-1} + \tfrac{h}{2}k_1)),\\
&k_3 = d(\exp(y_{j+1} + \tfrac{h}{2}k_2) + \exp(-y_j + \tfrac{h}{2}k_2) - \exp(y_j + \tfrac{h}{2}k_2) - \exp(-y_{j-1} + \tfrac{h}{2}k_2)),\\
&k_4 = d(\exp(y_{j+1} + hk_3) + \exp(-y_j + hk_3) - \exp(y_j + hk_3) - \exp(-y_{j-1} + hk_3)).
\end{aligned}$$

$$y_j(+0) = \frac{\alpha - 1}{\alpha - \beta - 1} y_j(-0), \quad y_j(1+0) = y_j(1-0) - \frac{\alpha}{\alpha - 1} y_j(+0),$$

$$y_j(\alpha + 0) = (1+\beta) y_j(\alpha - 0), \quad y_j(\alpha + 1 + 0) = y_j(\alpha + 1 - 0) - \frac{\alpha}{1+\beta} y_j(\alpha + 0),$$

$$y_0 = y_m = 0, \quad j = \overline{1, m-1}.$$

$$U_1 = \{(\alpha, \beta) : \alpha > (\beta+1)(\beta+2),\ \beta > 0\}$$

$$U_2 = \{(\alpha, \beta) : 1 < \alpha < 2,\ 0 < \beta < (\alpha-1)(2-\alpha)\}$$

$$U_3 = \{(\alpha, \beta) : 2\beta + 2 < \alpha < (\beta+1)(\beta+2),\ \beta > 0\}$$

$$U_4 = \{(\alpha, \beta) : \max(2, 1+\beta) < \alpha < 2\beta + 2,\ \beta > 0\}$$

$$U_5 = \{(\alpha, \beta) : 1 < \alpha < 2,\ (\alpha-1)(2-\alpha) < \beta < \alpha - 1\}$$

Theorem 2. *При любых фиксированных значениях α и β, удовлетворяющих включению $(\alpha, \beta) \in U_1 \bigcup \ldots \bigcup U_5$, и при всех достаточно малых $d > 0$ отображение Φ имеет не менее m экспоненциально устойчивых неподвижных точек*

$$O_{k_0}(d) = (z_{1,k_0}(d), z_{2,k_0}(d), \ldots, z_{m-1,k_0}(d)), \quad k_0 = \overline{0, m-1},$$

компоненты которых при $d \to 0$ в случае $(\alpha, \beta) \in U_1 \bigcup U_3 \bigcup U_4$ допускают асимптотику

$$z_{j,k_0} = \ln\frac{1}{d} - \ln\frac{\beta+1}{\alpha - \beta - 1} + \ln(k_0 + 1 - j) + o(1), \quad j = \overline{1, k_0},$$

$$z_{j,k_0} = -\ln\frac{1}{d} - \ln\frac{\beta+1}{\alpha-\beta-1} + \ln(j-k_0) + o(1), \quad j = \overline{k_0+1, m-1},$$

а в случае $(\alpha,\beta) \in U_2 \bigcup U_5$ *— асимптотику*

$$z_{j,k_0} = -(\alpha-1)\ln\frac{1}{d} - (\alpha-1)\ln(k_0+1-j) + \alpha\ln(\alpha-1) + o(1), \quad j = \overline{1, k_0},$$

$$z_{j,k_0} = (\alpha-1)\ln\frac{1}{d} + (\alpha-1)\ln(j-k_0) - \alpha\ln(\alpha-1) + o(1), \quad j = \overline{k_0+1, m-1},$$

Лемма 1. *Нулевая неподвижная точка отображения (3) экпоненциально устойчива при любых значениях параметров* $(\alpha,\beta) \in U_1 \bigcup \ldots \bigcup U_5$ *и при любом* $d > 0$.

$$m = 2$$

$$\dot{y} = d(e^{-y} - e^{y})$$

$$\Phi(z) : z \to y(T_0)$$

$$m = 3$$

$$\begin{cases} \dot{y_1} = d(e^{y_2} + e^{-y_1} - e^{y_1} - 1) \\ \dot{y_2} = d(1 + e^{-y_2} - e^{y_2} - e^{-y_1}) \end{cases}$$

$$\Phi(z) : \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \to \begin{pmatrix} y_1(T_0) \\ y_2(T_0) \end{pmatrix}$$

$$m = 4$$

$$\begin{cases} \dot{y_1} = d(e^{y_2} + e^{-y_1} - e^{y_1} - 1) \\ \dot{y_2} = d(e^{y_3} + e^{-y_2} - e^{y_2} - e^{-y_1}) \\ \dot{y_3} = d(1 + e^{-y_3} - e^{y_3} - e^{-y_2}) \end{cases}$$

$$\Phi(z) : \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} \to \begin{pmatrix} y_1(T_0) \\ y_2(T_0) \\ y_3(T_0) \end{pmatrix}$$

$$m = 2$$

$$\Phi(z) : z \to y(T_0)$$

$$m = 3$$

$$\Phi(z) : \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \to \begin{pmatrix} y_1(T_0) \\ y_2(T_0) \end{pmatrix}$$

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$$\alpha = 1.1, \quad \beta = 0.05$$

$$\alpha = 5.0, \quad \beta = 0.4, \quad d = 0.005$$

$$\alpha = 1.9, \quad \beta = 0.1$$

$$\alpha = 5.0, \quad \beta = 0.4$$

$$\alpha = 4.0, \quad \beta = 2.3$$

$$\alpha = 5.0, \quad \beta = 3.0, \quad d = 0.055$$

$$\Phi(z) : \begin{pmatrix} z_1 \\ \vdots \\ z_n \end{pmatrix} \to \begin{pmatrix} y_1(T_0) \\ \vdots \\ y_n(T_0) \end{pmatrix}$$

$$y_1(-0) = z_1, \ \ldots \ , \ y_n(-0) = z_n$$

$$T_0 = \alpha + 1 + (\beta + 1)/(\alpha - \beta - 1)$$