Dynamics of mapping and stable regimes of singulary perturbed neuron system

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Let us consider a chain of diffused connected and singularly perturbed non-linear differential equations with a delay:

$$\dot{u}_j = d\left(u_{j+1} - 2u_j + u_{j-1}\right) + \lambda\left[-1 + \alpha f\left(u_j(t-1)\right) - \beta g\left(u_j\right)\right] u_j, j = \overline{1, m} \quad (1)$$

where $m \geq 2, \lambda \gg 1, \beta > 0, \alpha > 1 + \beta,\ u_j > 0, u_0 = u_1, u_m = u_{m+1}$. The smooth functions $f(u), g(u) \in C^2(\mathbb{R}_+)$ have entry conditions:

$$f(0) = g(0) = 1,\ 0 < \beta g(u) + 1 < \alpha,\ \forall\, u \in \mathbb{R}_+;$$
$$f(u), g(u), uf'(u), ug'(u) = O(1/u),\ u \to +\infty.$$

In articles [1,2] there was made a transformation of system (1) to the system of differential equations with impulsive influences. Let us consider a mapping:

$$\Phi(z): \begin{pmatrix} z_1 \\ \vdots \\ z_{m-1} \end{pmatrix} \to \begin{pmatrix} y_1(T_0) \\ \vdots \\ y_{m-1}(T_0) \end{pmatrix}, \quad (2)$$

where the functions y_j are connected with initial variables by means of asymptotic equations $y_j \approx \ln u_{j+1} - \ln u_j$. $(y_1(t), \dots, y_{m-1}(t))^T$ is a solution, where $y_1(-0) = z_1, \dots, y_{m-1}(-0) = z_{m-1}$. $T_0 = \alpha + 1 + (\beta + 1)/(\alpha - \beta - 1)$ is the first approximation of stable cycle of single oscillator of system (1).

In articles [1,2] there was proved a statement that exponential stable points of mapping (2) are satisfied the orbital asymptotic stable cycles of system (1). An asymptotic analysis of mapping (2) shows that it has at least $n + 1$ stable points and zero point is stable for any values of d.

As the result of numerical research there were found different stable regimes of dynamic system. These results coincide with the results of theoretical asymptotic form. Also there were found the cases of coexistence of greater number of stable points. In the article [3] there were considered reconstructions of phase portraits for cases of 5 and 7 stable points for two-dimensional mapping (2).

References

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