GROWTH CURVE MODELING WITH BAYES

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1. Description

Growth curve models are often applied to situations where there are repeated observations for the same subjects at different time points, i.e., longitudinal data. The structure of the data that can be fitted with growth curve models resembles something like this:

Sid	T_x	t_0	t_1	t_2
1	CBT	0.01	0.24	0.42
2	CBT	-0.74	1.38	-1.12
3	ACT	-0.70	0.25	0.69
4	ACT	-0.26	-0.29	1.01
1	CBT	0.035	0.18	0.16
2	CBT	0.75	1.11	0.68
3	ACT	-1.33	0.03	-2.33
4	ACT	2.08	0.38	-0.60

Here, the Sid column indicates the id of the participants; T_x indicates the treatment received; and the last three columns record the *outcome* of the treatment at three different time points. The intervals between adjacent time points should be constant.

1.1. **A Standard Model.**

A commonly seen formulation of the growth curve model is shown in (1).

$$\begin{aligned} y &\sim Normal(\mu, \sigma) \\ \mu &= \alpha + \alpha_{T_x} + \alpha_{Sid} + \beta_t time + \beta_{t:\,T_x} time \end{aligned} \tag{1}$$

The α parameters are intercepts, which estimate the effects discarding the time dimension. The β parameters are *slopes*. Or more intuitively, the *rates of changes*, which are the amounts of increases added to the outcome when moving one unit of time upwards. The effects of the treatments are found in the $\beta_{t:\,T_x}$ parameters, which estimate the rate of change for each treatment **in addition to the global rate of change**, estimated through the parameter β_t.

Solomon Kurz provides a working example for fitting a growth curve model of this kind in his blog post[1].

1.2. **A Simplified Model.** The model in (1) is parameterized in a somewhat **contrast-oriented** way. This is the classic way to parameterize a model in many fields, which prefer to use contrasts to understand the effects of interest (e.g., how much **better** is *ACT* compared to *CBT*.). This method, however, makes it harder to connect the effects to real-world entities.

Below, I provide a simplified specification of the model from an **estimation perspective**. Instead of asking how much better is *ACT* than *CBT*, the estimation perspective asks how good each of them is. If we need to compare the two, simply subtract the estimated effect of *CBT* from that of *ACT*. The meaning of the parameters changes but should now become more intuitive. This simplified model is shown in (2). Two modifications are applied here.

1.2.1. *Dropping the global intercept α.* When the global intercept is dropped, the effect originally presented in it will be picked up by the remaining intercepts. This makes more sense to me, as the global intercept is rather abstract and its meaning changes greatly according to what has been modeled. For the case here, the original global intercept gets picked up by the treatment and the subject intercepts.

1.2.2. *Dropping the global rate of change β_t.* Similar to the global intercept, when the global rate of change is eliminated from the model, its effect will be collected by the remaining slopes. In the current example, it will be picked up by $\beta_{t:\,T_x}$. So now, the interpretation of $\beta_{t:\,T_x}$ simplifies to "the rate of change for each treatment".

$$\begin{aligned} y &\sim Normal(\mu, \sigma) \\ \mu &= \alpha_{T_x} + \alpha_{Sid} + \beta_{t:\,T_x} time \end{aligned} \tag{2}$$

2. Bayesian Growth Curve Models

To explore the fitting of growth curve models in *Stan*, the example provided in Solomon Kurz's post mentioned above is refitted with *Stan* and `lme4::lmer()`. The code is available in `verify.R` and `m0.stan` on GitHub[2].

The scripts `simulation.R`, `fit.R`, and `m1.stan`[3] further explore the simplified model discussed above in the Bayesian modeling framework. A simulation is set up to test if the *Stan* and *lmer* models both correctly recover the parameters. The models work as expected.

[1] https://solomonkurz.netlify.app/blog/2021-04-22-effect-sizes-for-experimental-trials-analyzed-with-multilevel-growth-models-two-of-two

[2] https://github.com/liao961120/stom/tree/main/inst/cases/growth_curve_model

[3] Also found at the location above.