JOINT ESTIMATION OF INTERACTION AND PIPED EFFECTS IN AN ITEM RESPONSE MODEL

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1. Description

The Directed Acyclic Graph (DAG) shown below represent the data generating process of the IRT model of interest. It can be conceptualized as a wine rating context, where the rating of wine quality (R) is influenced by three factors:

1. J: Judge leniency
2. W: Wine quality
3. O_w: Wine origin

Furthermore, it is assumed that O_w differentially influences R based on the levels of another variable O_j, the origin of the judge In stats jargon, there's an interaction between O_w and O_j. In simpler conceptual terms, consider the scenario that, for instance, French wines are rated higher by French judges, in addition to the scores they should have received based on their quality alone. The simulation code of this data generating process is found in the `sim_dat()` function in `wine.R`.

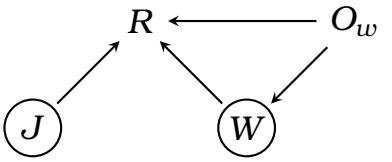

Figure 1. Underlying data-generating process of the IRT Model

2. The Original Model and its Problem

The specification of the original model is shown in the equations below. A problem found in this model is that it cannot stably recover the wine quality (`W`) and the interaction (`Int`) parameters.

$$
\begin{aligned}
R &\sim Bernoulli(p) \\
logit(p) &= W_{[Wid]} + J_{[Jid]} + Int_{[O_j, O_w]} \\
J &\sim Normal(0, \sigma_J) \\
\sigma_J &\sim Exponential(1)
\end{aligned}
$$

$$
\begin{aligned}
W_{[Wid]} &\sim Normal(a_{[O_w]}, \sigma_W) \\
a &\sim Normal(0, 1.5) \\
\sigma_W &\sim Exponential(1)
\end{aligned}
$$

3. Potential Causes

With some exploration on a simpler model (the response was modeled as normal distributions generated from latent scores), it was found that the problem seemed to arise from an identifiablity issue: the model cannot correctly attribute the right amount of effect to wine quality (which is influenced by wine origin) and the direct (interactive) influence of wine origin on rating scores. The parameter estimates float around case by case when different configurations of the interaction are set in the simulation.

4. Fixes

As illustrated in `wine2_normal.stan` (Case 4 & 5), the problem can be fixed by imposing additional constraints on the model. Two of them are imposed here to correctly identify the parameters:

1. A sum-to-zero constraint on the effects of wine origin on wine quality. That is, the effect through the path $O_w \rightarrow W$.
2. One of a term in the interaction matrix (`(2, 2)` in the case here) is constraint to zero as the reference.

5. ToDo

Test whether the conclusion also holds with logit models (binary/ordinal response models).