Orbital Ephemerides of the Sun, Moon, and Planets

James G. Williams
California Institute of Technology

CHAPTER 8: Orbital Ephemerides of the Sun, Moon, and Planets

E. Myles Standish and James G. Williams

8.1 Fundamental Ephemerides

The fundamental planetary and lunar ephemerides of *The Astronomical Almanac*, starting in the year 2003, are DE405/LE405 of Caltech's Jet Propulsion Laboratory (JPL). They replace JPL's DE200 which have been used in the almanac since 1984. Previous to 1984, the fundamental ephemerides were based upon analytical "theories"; these are described in Section 8.2. DE405/LE405 result from a least-squares adjustment of a previously existing ephemeris to a variety of observational data (measurements), followed by a numerical integration of the dynamical equations of motion which describe the gravitational physics of the solar system. These fundamental ephemerides are the bases for computing the planetary and lunar positions and other related phenomena that are listed in the almanac.

For the final (integration) phase of the ephemeris creation process, there are three main ingredients, each of which constitutes a major phase itself:

- the equations of motion describing the gravitational physics which govern the dynamical motions of the bodies,
- a method for integrating the equations of motion , and
- the initial conditions and dynamical constants; i.e., the starting positions and velocities of the bodies at some initial epoch along with the values for various constants which affect the motion (e.g., planetary masses).

It is mainly the accuracy of the third component, the initial conditions and dynamical constants, which determines the accuracy of modern-day ephemerides, since the other two components (the physics and the integration method) are both believed to be sufficiently complete and accurate. The values of the initial conditions and constants are determined by the least squares adjustment of them to the observational data (measurements), and the accuracy of this adjustment, and thus of the ephemerides themselves, depends primarily upon the distribution, variety, and accuracy of the observational data. This crucial part of the ephemeris creation process is given in Section 8.5.

It is assumed that the equations of motion accurately describe the basic physics which govern the motion of the major bodies of the solar system - at least to the presently observable accuracy. For the motion of the Moon, the fitting of the lunar ephemeris to the lunar laser-ranging observations is used to estimate the constants involved and to help distinguish various models of the lunar interior, Earth-raised tides, etc. The equations of motion are given in Section 8.3.

Numerical integration of the equations of motion is the only known method capable of computing fundamental ephemerides at an accuracy which is comparable to that of the modern-day observations; analytical theories have not been able to attain such high accuracy. The computer program which was used to integrate the equations of motion for DE405/LE405 has been demonstrated to be sufficiently accurate (Newhall *et al.*, 1983).

The reference frame for the ephemerides is the International Celestial Reference Frame (ICRF: Ma *et al.*, 1998). The advantages for using the frame are many; they are discussed in Section 8.6.

The independent variable of the equations of motion, and, thus, of the fundamental ephemerides themselves, may be termed, "$\mathrm{T_{eph}}$". It is a fully rigorous relativistic coordinate time, implicitly defined by the equations of motion themselves. $\mathrm{T_{eph}}$ may be considered *similar* to previously-defined "ET" or to "TDB", since their average rates are the same, even though there are basic differences in the definitions. The time recently defined by the IAU, "TCB", is mathematically equivalent to $\mathrm{T_{eph}}$; TCB and $\mathrm{T_{eph}}$ differ by only a constant rate. (For a discussion, see Standish, 1998b).

This chapter describes the previous ephemerides used in the *Astronomical Almanac* [Section 8.2]; presents the equations of motion used in computing the present ephemeris, DE405 [8.3]; gives some detail about the numerical integration program [8.4]; describes the observational data to which DE405 was adjusted [8.5]; discusses the reference frame of the ephemerides [8.6]; gives some of the formulae used in reducing the observational data [8.7]; presents the values of the initial conditions and constants of DE405 [8.8]; estimates the accuracy of the ephemerides [8.9]; gives sets of Keplerian elements useful for producing approximate planetary coordinates [8.10]; and shows the sources where DE405/LE405 may be found on CDrom and over the Internet [8.11].

8.2 Previous Ephemerides used in the Astronomical Almanacs

8.2.1 Ephemerides Prior to 1984

Before 1984, the ephemerides for the Sun, Mercury, Venus, and Mars were based on the theories and tables of Newcomb (1898). Computerized evaluations of the tables were used from 1960 through 1980. From 1981 to 1983, the ephemerides were based on the evaluations of the theories themselves. The ephemerides of the Sun were derived from the algorithm given by S. Newcomb in *Tables of the Sun* (Newcomb, 1898). Newcomb's theories of the inner planets (1895–1898) served as the basis for the heliocentric ephemerides of Mercury, Venus, and Mars. In the case of Mars, the corrections derived by F.E. Ross (1917) were applied.

Ephemerides of the outer planets, Jupiter, Saturn, Uranus, Neptune, and Pluto, were computed from the heliocentric rectangular coordinates obtained by numerical integration (Eckert *et al.*, 1951). Although perturbations by the inner planets (Clemence, 1954) were included in the printed geocentric ephemerides of the outer planets, they were omitted from the printed heliocentric ephemerides and orbital elements.

The lunar ephemeris, designated by the serial number $j = 2$, was calculated directly from E.W. Brown's algorithm instead of from his *Tables of Motion of the Moon* (1919). This was specified in the *Improved Lunar Ephemeris*. To obtain a strictly gravitational ephemeris expressed in the measure of time defined by Newcomb's *Tables of the Sun*, the fundamental arguments of Brown's tables were amended by removing the empirical term and by applying to the Moon's mean longitude the correction, $-8''.72 - 26''.74\,T - 11''.22\,T^2$, where T is measured in Julian centuries from 1900 January 0.5 (JED 2415020.0. In addition, this ephemeris was based on the IAU (1964) System of Astronomical Constants, and was further improved in its precision by transformation corrections (Eckert *et al.*, 1966, 1969). The expressions for the mean longitude of the Moon and its perigee were adjusted to remove the implicit partial correction for aberration (Clemence *et al.*, 1952).

8.2.2 Ephemerides from 1984 through 2002

During the years, 1984–2003, the JPL ephemerides, DE200/LE200, were the bases of the ephemerides in the Astronomical Almanac. The equations of motion were essentially those used in JPL's DE102 (Newhall *et al.*, 1983); they are also described in the Explanatory Supplement (1992). The observations to which these ephemerides were fit is documented in Standish (1990). The reference frame of DE200 is the mean equator and dynamical equinox of J2000, as determined and described in Standish (1982).

8.2.3 Ephemerides starting in 2003

Starting in 2003, the fundamental planetary and lunar ephemerides of the Astronomical Almanac are DE405/LE405, referenced, as noted above, to the International Celestial Reference Frame (ICRF). The remainder of this chapter describes the creation of DE405.

8.3 Equations of Motion

Equations of motion describe the forces upon the planets, Sun and Moon which affect their motions and the torques upon the Moon which affect its orientation. It is believed that the equations described here are correct and complete to the level of accuracy of the observational data. I.e., given the accuracy of the observations, there is nothing to suggest that other forces or different forces are present in the solar system. The uncertainties existing in the planets' and Moon's motions are certainly explainable, considering the uncertainties in the observations and in the fitted initial conditions and dynamical constants.

The major elements of this section were developed at JPL over the past few decades. Just the formulae are given here; also included are references to their descriptions, previously published by those responsible for their development.

The equations of motion used for the creation of DE405/LE405 included contributions from: (a) point-mass interactions among the Moon, planets, and Sun; (b) general relativity (isotropic, parametrized post-Newtonian); (c) Newtonian perturbations of selected asteroids; (d) action upon the figure of the Earth from the Moon and Sun; (e) action upon the figure of the Moon from the Earth and Sun; (f) physical libration of the Moon, modeled as a solid body with tidal and rotational distortion, including both elastic and dissipational effects, (g) the effect upon the Moon's motion caused by tides raised upon the Earth by the Moon and Sun, and (h) the perturbations of 300 asteroids upon the motions of Mars, the Earth, and the Moon.

8.3.1 Point-Mass Interactions

The principal gravitational force on the nine planets, the Sun, and the Moon is modeled by considering those bodies to be point masses in the isotropic, parametrized post-Newtonian (PPN) n-body metric (Will, 1974). The n-body equations were derived by Estabrook (1971a) from the variation of a time-independent Lagrangian action integral formulated in a non-rotating solar-system-barycentric Cartesian coordinate frame. Also included are Newtonian gravitational perturbations from 300 asteroids, chosen because they have the most pronounced effect on the Earth–Mars range over the time span covered by the accurate spacecraft ranging observations.

For each body i, the point-mass acceleration is given by

$$
\begin{aligned}
\ddot{\mathbf{r}}_{i_{\text{point mass}}} = \sum_{j\neq i} \frac{\mu_j(\mathbf{r}_j-\mathbf{r}_i)}{r_{ij}^3}\Bigg\{1 &- \frac{2(\beta+\gamma)}{c^2}\sum_{k\neq i}\frac{\mu_k}{r_{ik}} - \frac{2\beta-1}{c^2}\sum_{k\neq j}\frac{\mu_k}{r_{jk}} \\
&+ \gamma\left(\frac{v_i}{c}\right)^2 + (1+\gamma)\left(\frac{v_j}{c}\right)^2 - \frac{2(1+\gamma)}{c^2}\dot{\mathbf{r}}_i\cdot\dot{\mathbf{r}}_j \\
&- \frac{3}{2c^2}\left[\frac{(\mathbf{r}_i-\mathbf{r}_j)\cdot\dot{\mathbf{r}}_j}{r_{ij}}\right]^2 + \frac{1}{2c^2}(\mathbf{r}_j-\mathbf{r}_i)\cdot\ddot{\mathbf{r}}_j\Bigg\} \qquad (8-1) \\
&+ \frac{1}{c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}^3}\{[\mathbf{r}_i-\mathbf{r}_j]\cdot[(2+2\gamma)\dot{\mathbf{r}}_i-(1+2\gamma)\dot{\mathbf{r}}_j]\}(\dot{\mathbf{r}}_i-\dot{\mathbf{r}}_j) \\
&+ \frac{(3+4\gamma)}{2c^2}\sum_{j\neq i}\frac{\mu_j\ddot{\mathbf{r}}_j}{r_{ij}} \quad + \sum_{m=1}^{3}\frac{\mu_m(\mathbf{r}_m-\mathbf{r}_i)}{r_{im}^3} \quad + \sum_{c,s,m}\mathbf{F}
\end{aligned}
$$

where $\mathbf{r_i}$, $\dot{\mathbf{r_i}}$, and $\ddot{\mathbf{r_i}}$ are the solar-system-barycentric position, velocity, and acceleration vectors of body i; $\mu_j = Gm_j$, where G is the gravitational constant and m_j is the mass of body j; $r_{ij} = |\mathbf{r_j} - \mathbf{r_i}|$; β is the PPN parameter measuring the nonlinearity in superposition of gravity; γ is the PPN parameter measuring space curvature produced by unit rest mass (in this integration, as in general relativity, $\beta = \gamma = 1$); $v_i = |\dot{\mathbf{r_i}}|$; and c is the velocity of light.

The quantity $\ddot{\mathbf{r}}_\mathbf{j}$ appearing in two terms on the right-hand side of (8-1) denotes the barycentric acceleration of each body j due to Newtonian effects of the remaining bodies and the asteroids. Thus, the right-hand side of the equation is dependent upon the left-hand side, and so, rigorously, the computation should be iterated. However, use of Newtonian accelerations for these terms is sufficiently accurate.

In the next-to-last term on the right-hand side of (8-1), quantities employing the index m refer to the "Big3" asteroids, Ceres, Pallas, and Vesta. The last term represents forces upon the Earth, Moon, and Mars, only, from 297 other asteroids, grouped according to three taxonomic classes (C, S, M). The asteroid forces were computed in the following way:

> The orbits of the Big3 asteroids were integrated under the forces of themselves, the nine planets, the Sun, and the Moon, using initial conditions from the "dastcom" file maintained by JPL's Solar System Dynamics Group. These orbits were fit with Chebychev polynomials and used in the succeeding integration of the 297 non-Big3 asteroids. For the eventual integration of DE405, periodic mean orbits were used for the Big3 asteroids which had been fit to their integrated orbits.
>
> The 297 individual orbits were integrated under the gravitational forces of the Sun, planets, and the Big3 asteroids. At each step, the force vectors of these non-Big3 asteroids upon the Earth, Moon and Mars were computed, summed, and stored in 3 temporary files, one for the sums of the 218 C-class asteroids, one for the sums of the 58 S-type asteroids, and one for the sums of the 21 M-type asteroids. Also stored in the same temporary files were the contributions from these non-Big3 asteroids to the Solar System's center of mass. The vectors from these temporary files were fit with Chebychev polynomials and stored in a final file to be used in the main planetary and lunar integration.
>
> The radii, masses, and taxonomic classes of the 297 asteroids used in DE405/LE405 are listed in Section 8.8.

8.3.2 Solar-System Barycenter

In the n-body metric, all dynamical quantities are expressed with respect to a center of mass whose definition is modified from the usual Newtonian formulation. The solar-system barycenter is given by Estabrook (1971b) as

$$\sum_i \mu_i^* \mathbf{r}_i = 0, \tag{8-2}$$

where

$$\mu_i^* = \mu_i \left\{ 1 + \frac{1}{2c^2} v_i^2 - \frac{1}{2c^2} \sum_{j \neq i} \frac{\mu_j}{r_{ij}} \right\}$$

and where μ_i is the gravitational constant times the mass of the i^{th} body; v_i is the barycentric speed of the i^{th} body.

During the process of numerical integration the equations of motion for only the Moon and planets were actually evaluated and integrated. The barycentric position and velocity of the Sun were obtained from the equations of the barycenter. It should be noted that each of the two barycenter equations depends upon the other, requiring an iteration during the evaluation of the solar position and velocity.

8.3.3 Figure Effects

Long-term accuracy of the integrated lunar orbit requires the inclusion of the figures of the Earth, Moon, and Sun in the mathematical model. In DE405 the gravitational effects due to figures include:

1 The force of attraction between the zonal harmonics (through fourth degree) of the Earth and the point-mass Moon, Sun, Venus, and Jupiter;

2 the force of attraction between the zonal harmonics (through fourth degree) and the second- through fourth-degree tesseral harmonics of the Moon and the point-mass Earth, Sun, Venus, and Jupiter;

3 The dynamical form-factor of the Sun (J_2).

The contribution to the inertial acceleration of an extended body arising from the interaction of its own figure with an external point mass is expressed in the $\xi\eta\zeta$ coordinate system, where the ξ-axis is directed outward from the extended body to the point mass; the $\xi\zeta$-plane contains the figure (rotational) pole of the extended body, and the η-axis completes the right-handed system.

In that system (see, e.g., Moyer, 1971),

$$\begin{bmatrix} \ddot{\xi} \\ \ddot{\eta} \\ \ddot{\zeta} \end{bmatrix} = -\frac{\mu}{r^2}\left\{ \sum_{n=2}^{n_1} J_n \left(\frac{\mathcal{R}}{r}\right)^n \begin{bmatrix} (n+1)P_n(\sin\phi) \\ 0 \\ -\cos\phi P_n'(\sin\phi) \end{bmatrix} \right. \tag{8-3}$$

$$\left. + \sum_{n=2}^{n_2} \left(\frac{\mathcal{R}}{r}\right)^n \sum_{m=1}^{n} \begin{bmatrix} -(n+1)\ P_n^m(\sin\phi)\ [+C_{nm}\cos m\lambda + S_{nm}\sin m\lambda] \\ m\sec\phi\ P_n^m(\sin\phi)\ [-C_{nm}\sin m\lambda + S_{nm}\cos m\lambda] \\ \cos\phi\ P_n^{m\prime}(\sin\phi)\ [+C_{nm}\cos m\lambda + S_{nm}\sin m\lambda] \end{bmatrix} \right\}$$

where μ is the gravitational constant G times the mass of the point body; r is the center-of-mass separation between the two bodies; n_1, and n_2 are the maximum degrees of the zonal and tesseral expansions, respectively; $P_n(\sin\phi)$ is the Legendre polynomial of degree n; $P_n^m(\sin\phi)$ is the associated Legendre function of degree n and order m; J_n is the zonal harmonic for the extended body; C_{nm}, S_{nm} are the tesseral harmonics for the extended body; $\mathcal{R}$ is the equatorial radius of the extended body; ϕ is the latitude of the point mass relative to the body-fixed coordinate system in which the harmonics are expressed; and λ is the east longitude of the point mass in the same body-fixed coordinate system.

The primes denote differentiation with respect to the argument $\sin\phi$. The accelerations are transformed into the solar-system-barycentric Cartesian system by application of the appropriate rotation matrix.

The interaction between the figure of an extended body and a point mass also induces an inertial acceleration of the point mass. If $\ddot{\mathbf{r}}_{fig}$ denotes the acceleration given in (8-3) when expressed in solar-system-barycentric coordinates, then the corresponding acceleration of the point mass, $\ddot{\mathbf{r}}_{pm}$, is

$$\ddot{\mathbf{r}}_{PM} = -\frac{m_{fig}}{m_{pm}}\ddot{\mathbf{r}}_{fig} \tag{8-4}$$

where m_{fig} and m_{pm} are the masses of the extended body and point mass, respectively.

The orientation of the Earth includes precession, obliquity change (Lieske *et al.*, 1977), and 18.6-year nutation. The correction for precession, $d\Phi_Y/dt$, was set to -0."3/cty, and the obliquity rate correction, -$d\Phi_X/dt$, was taken from (Williams 1994). In addition, offsets to the Earth's orientation at J2000, Φ_X and Φ_Y, were determined through the strength of the lunar laser ranging data. The values for all four parameters are given in Table 8.8.3.

8.3.4 Lunar Gravity Coefficients

For the Moon, the 2^{nd} degree gravity field is time-varying, and the harmonic gravity coefficients are computed from the moment-of-inertia tensor, where the time dependence has been accounted for. For near principal axis alignment, the 1^{st} order relations for the coefficients are given by

$$\begin{aligned} J_2^m(t) &= \frac{I_{33}(t) - \frac{1}{2}[I_{11}(t) + I_{22}(t)]}{m\mathcal{R}^2} \\ C_{22}^m(t) &= \frac{I_{22}(t) - I_{11}(t)}{4m\mathcal{R}^2} \end{aligned}$$

$$\begin{aligned} C_{21}(t) &= -\,I_{13}(t)/m\mathcal{R}^2 \\ S_{21}(t) &= -\,I_{32}(t)/m\mathcal{R}^2 \\ S_{22}(t) &= -\,I_{21}(t)/2m\mathcal{R}^2 \end{aligned} \qquad (8-5)$$

where the I_{ij} are the components of the moment-of-inertia tensor, to be described later; m is the lunar mass and $\mathcal{R}$, the lunar radius.

8.3.5 Lunar Physical Libration: Coordinates

The orientation of the Moon is given by a set of Euler angles, ϕ, θ, and ψ, which relate the Moon-centered, rotating lunar system to the reference frame of the ephemerides (nominally, the ICRF). The elastic Moon is distorted by tides and rotation, but the mean principal axes are well-defined. This Moon-centered mean-principal-axis frame is here called the selenographic frame. The angles are ϕ, the angle from the x-axis of the ephemeris reference frame along the xy-plane to the intersection of the lunar equator; θ, the inclination of the lunar equator upon the xy-plane; and ψ, the longitude from that intersection along the lunar equator to the prime meridian.

Instead of integrating the second derivatives of the Euler angles, their first derivatives are expressed in terms of the selenocentric, body-fixed angular velocity vector, $\boldsymbol{\omega}$:

$$\begin{aligned} &\dot\phi = (\omega_x \sin\psi + \omega_y \cos\psi)/\sin\theta \\ &\dot\theta = \omega_x \cos\psi - \omega_y \sin\psi \\ &\dot\psi = \omega_z - \dot\phi\cos\theta \end{aligned} \qquad (8-6)$$

The first derivatives of the Euler angles and the first derivatives of $\boldsymbol{\omega}$ are integrated, as seen in the following subsection.

8.3.6 Physical Libration Differential Equations

The moment of inertia tensor, $\mathbf{I}$, has a rigid-body contribution plus two variable contributions due to tidal and rotational distortions. The distortions are functions of the lunar position and rotational velocities, computed at time, $t - \tau_m$, where τ_m is a time-lag of about 4 hours in DE405, as determined from the fits to the lunar laser-ranging data. The time delay allows for dissipation when flexing the Moon (Williams *et al.*, 2001).

The angular momentum vector can be expressed in inertial or body-fixed coordinates; it is the product of the moment of inertia tensor (given below) and the angular velocity vector: $\mathbf{L} = \mathbf{I}\,\boldsymbol{\omega}$. The time derivative of the angular momentum vector is equal to the torque. In an inertial coordinate system: $\mathbf{N} = d\mathbf{L}/dt$, but in a rotating system,

$$\mathbf{N} \;=\; \frac{d}{dt}(\mathbf{I}\,\boldsymbol{\omega}) \;+\; \boldsymbol{\omega}\times\mathbf{I}\,\boldsymbol{\omega}. \qquad (8-7)$$

where the second term on the right side puts the time derivative into the body-fixed system.

The complete equation of motion for the lunar librations is then given by

$$\dot{\boldsymbol{\omega}} \;=\; \mathbf{I}^{-1}\,\{\Sigma_i \mathbf{N}_{fig-pm} \;+\; \mathbf{N}_{fig-fig} \;-\; \dot{\mathbf{I}}\,\boldsymbol{\omega} \;-\; \boldsymbol{\omega}\times\mathbf{I}\,\boldsymbol{\omega}\}, \qquad (8-8)$$

where the torques have been separated into torques upon the lunar figure from external point masses and from the Earth's extended figure. These are discussed later in the section.

8.3.7 The Moment of Inertia Tensor

The three parts of the moment of inertia tensor represent the rigid body, the tidal distortion, and the spin distortion, where the latter two include both elasticity and dissipation. The body-fixed selenographic reference frame uses the rigid body principal axes. In this frame, the tensor is

$$
\begin{aligned}
\mathbf{I} \;=& \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix} \\
&- \frac{k_{2m} m_e R_m^5}{r^5} \begin{bmatrix} x^2 - \frac{1}{3}r^2 & xy & xz \\ xy & y^2 - \frac{1}{3}r^2 & yz \\ xz & yz & z^2 - \frac{1}{3}r^2 \end{bmatrix} \\
&+ \frac{k_{2m} R_m^5}{3G} \begin{bmatrix} \omega_x^2 - \frac{1}{3}(\omega^2 - n^2) & \omega_x\omega_y & \omega_x\omega_z \\ \omega_x\omega_y & \omega_y^2 - \frac{1}{3}(\omega^2 - n^2) & \omega_y\omega_z \\ \omega_x\omega_z & \omega_y\omega_z & \omega_z^2 - \frac{1}{3}(\omega^2 + 2n^2) \end{bmatrix}
\end{aligned}
\qquad (8-9)
$$

where the coordinates and rotational velocities, $\mathbf{r}$ and $\boldsymbol{\omega}$, are evaluated at time $t - \tau_m$, and where k_{2m} is the lunar potential Love number; m_e is the mass of the Earth; R_m is the equatorial radius of the Moon; r is the Earth-Moon distance; x, y, z are the components of the Earth-Moon vector aligned with the rigid body principal axes; $\omega_x, \omega_y, \omega_z$ are the components of $\boldsymbol{\omega}$ in the selenographic system; and n is the average lunar mean motion.

8.3.8 Time Derivative of the Inertia Tensor

$$
\begin{aligned}
\dot{\mathbf{I}} \;=& \frac{5 k_{2m} m_e R_m^5 \mathbf{r}\cdot\dot{\mathbf{r}}}{r^7} \begin{bmatrix} x^2 - \frac{1}{3}r^2 & xy & xz \\ xy & y^2 - \frac{1}{3}r^2 & yz \\ xz & yz & z^2 - \frac{1}{3}r^2 \end{bmatrix} \\
&- \frac{k_{2m} m_e R_m^5}{r^5} \begin{bmatrix} 2(x\dot{x} - \frac{1}{3}\mathbf{r}\cdot\dot{\mathbf{r}}) & x\dot{y} + \dot{x}y & x\dot{z} + \dot{x}z \\ x\dot{y} + \dot{x}y & 2(y\dot{y} - \frac{1}{3}\mathbf{r}\cdot\dot{\mathbf{r}}) & y\dot{z} + \dot{y}z \\ x\dot{z} + \dot{x}z & y\dot{z} + \dot{y}z & 2(z\dot{z} - \frac{1}{3}\mathbf{r}\cdot\dot{\mathbf{r}}) \end{bmatrix} \\
&+ \frac{k_{2m} R_m^5}{3\ G} \begin{bmatrix} 2(\omega_x\dot{\omega}_x - \frac{1}{3}\boldsymbol{\omega}\cdot\dot{\boldsymbol{\omega}}) & \omega_x\dot{\omega}_y + \dot{\omega}_x\omega_y & \omega_x\dot{\omega}_z + \dot{\omega}_x\omega_z \\ \omega_x\dot{\omega}_y + \dot{\omega}_x\omega_y & 2(\omega_y\dot{\omega}_y - \frac{1}{3}\boldsymbol{\omega}\cdot\dot{\boldsymbol{\omega}}) & \omega_y\dot{\omega}_z + \dot{\omega}_y\omega_z \\ \omega_x\dot{\omega}_z + \dot{\omega}_x\omega_z & \omega_y\dot{\omega}_z + \dot{\omega}_y\omega_z & 2(\omega_z\dot{\omega}_z - \frac{1}{3}\boldsymbol{\omega}\cdot\dot{\boldsymbol{\omega}}) \end{bmatrix}
\end{aligned}
\qquad (8-10)
$$

8.3.9 Principal Moments

The rigid-body principal moments are A, B, and C, with the C axis aligned toward the pole.

$$
\begin{aligned}
A &= \frac{2(1 - \beta_L\gamma_L)}{(2\beta_L - \gamma_L + \beta_L\gamma_L)} M_m \mathcal{R}_m^2 J_2^{\text{rigid}} \\
B &= \frac{2(1 + \gamma_L)}{(2\beta_L - \gamma_L + \beta_L\gamma_L)} M_m \mathcal{R}_m^2 J_2^{\text{rigid}} \\
C &= \frac{2(1 + \beta_L)}{(2\beta_L - \gamma_L + \beta_L\gamma_L)} M_m \mathcal{R}_m^2 J_2^{\text{rigid}}
\end{aligned}
\qquad (8-11)
$$

where β_L and γ_L are defined by the relations, $\beta_L = (C - A)/B$ and $\gamma_L = (B - A)/C$; a_m is the semi-major axis of the lunar orbit; and $J_2^{\text{rigid}} = J_2^{\text{input}} + k_{2m}\frac{M_e}{M_m}\left(\frac{\mathcal{R}_m}{a_m}\right)^3$. Note that J_2^{input} does not have physical significance. The quantities, J_2^{input}, β_L, γ_L, μ_m, the 3^{rd} degree harmonics, k_{2m}, and τ_m are quantities which are solved for in the least squares data fits and then input into the integration program; $\mathcal{R}_m$ and the 4^{th} degree harmonics are input, but not solved for; A, B, C, and $J_2^{\rm rigid}$ are derived from the preceding relations. For further discussion, see Newhall and Williams, (1997).

8.3.10 Figure – Point Mass Torques upon the Moon

From a single point-mass (body) acting upon the figure of the Moon, the lunar torque is given by

$$\mathbf{N}_{fig-pm} = M_m \mathbf{r}_{pm} \times \ddot{\mathbf{r}}_{fig} \qquad (8-12)$$

where $\mathbf{r}_{pm}$ is the selenocentric position of the point mass and $\ddot{\mathbf{r}}_{fig}$ is the inertial acceleration due to the point-mass. Torques are computed for the figure of the Moon interacting with the Earth, Sun, Venus, and Jupiter.

8.3.11 Figure-Figure Torque upon the Moon

Yoder (1979) and Eckhardt (1981) showed that figure-figure torques are important for the Moon. Three significant terms of the torque upon the figure of the Moon from the Earth's figure (J_{2e}) are

$$\begin{aligned} \mathbf{N}_{fig-fig} = \frac{15}{2} \frac{\mu_e \mathcal{R}_e^2 J_{2e}}{r_e^5} \{ & (1 - 7\sin^2\phi)[\hat{\mathbf{r}}_\mathbf{e} \times \mathbf{I}\hat{\mathbf{r}}_\mathbf{e}] \\ & + 2\sin\phi\, [\hat{\mathbf{r}}_\mathbf{e} \times \mathbf{I}\hat{\mathbf{P}}_\mathbf{e} + \hat{\mathbf{P}}_\mathbf{e} \times \mathbf{I}\hat{\mathbf{r}}_\mathbf{e}] \\ & - \frac{2}{5}[\hat{\mathbf{P}}_\mathbf{e} \times \mathbf{I}\hat{\mathbf{P}}_\mathbf{e}] \ \} \end{aligned} \qquad (8-13)$$

where $\hat{\mathbf{P}}_\mathbf{e}$ is the direction vector of the Earth's pole and $\hat{\mathbf{r}}_\mathbf{e}$ is the direction vector of the Earth from the Moon, both expressed in the selenographic reference frame; $\mathbf{I}$ is the lunar moment of inertia tensor; $\mathcal{R}_e$ is the equatorial radius of the Earth in AU's; and for this equation, ϕ is defined by $\sin\phi = \hat{\mathbf{r}}_\mathbf{e} \cdot \hat{\mathbf{P}}_\mathbf{e}$. Figure-figure acceleration of the lunar orbit is not considered.

8.3.12 Acceleration of the Moon from Earth Tides

The tides raised upon the Earth by the Sun and Moon affect, in turn, the motion of the Moon. For this, let

$G =$ gravitational constant,

for the distorted body (in this case, the Earth),

$M =$ mass
$R =$ radius
$\dot{\theta} =$ sidereal rotation rate
$k_{2j} =$ frequency dependent Love numbers (j = 0,1,2 ⇒ slow zonal, diurnal, semidiurnal tides)
$\tau_j =$ frequency dependent time delays (j = 0,1,2 ⇒ slow zonal, diurnal, semidiurnal tidal dissipation)

for the perturbed body (in this case, the Moon),

$m =$ mass
$\mathbf{r} =$ position (centered on mass M)

for the tide-raising body (in this case, the Moon and Sun),

$m' =$ mass
$\mathbf{r}' =$ position (centered on mass M)

For each frequency band, back-date the position of the tide-raising body by the time-delay, τ_j, and rotate that vector, about the distorted body's rotational pole, through the lag angle, $\dot{\theta}\tau_j$, giving

$$\mathbf{r}_j^* = \mathcal{R}_3(\dot{\theta}\tau_j)\mathbf{r}'(t - \tau_j). \tag{8 – 14}$$

For the acceleration of the perturbed body with respect to the distorted body (Moon with respect to Earth), break $\mathbf{r}$ and $\mathbf{r}^*$ into equatorial and polar components (with respect to the distorted body): $\mathbf{r} = \boldsymbol{\rho} + \mathbf{z}$ and $\mathbf{r}^* = \boldsymbol{\rho}^* + \mathbf{z}^*$.

$$\begin{aligned}\ddot{\mathbf{r}}(\text{tide}) = \frac{3}{2}\frac{Gm'(1+\frac{m}{M})\mathcal{R}^5}{r^5} \Bigg\{ & \frac{k_{20}}{r_0^{*5}}\Big([2z_0^{*2}\mathbf{z} + \rho_0^{*2}\boldsymbol{\rho}] - \frac{5[(zz_0^*)^2 + \frac{1}{2}(\rho\rho_0^*)^2]\mathbf{r}}{r^2} + r_0^{*2}\mathbf{r}\Big) \\ & + \frac{k_{21}}{r_1^{*5}}\Big(2[(\boldsymbol{\rho}\cdot\boldsymbol{\rho}_1^*)\mathbf{z}_1^* + zz_1^*\boldsymbol{\rho}_1^*] - \frac{10zz_1^*(\boldsymbol{\rho}\cdot\boldsymbol{\rho}_1^*)\mathbf{r}}{r^2}\Big) \\ & + \frac{k_{22}}{r_2^{*5}}\Big([2(\boldsymbol{\rho}\cdot\boldsymbol{\rho}_2^*)\boldsymbol{\rho}_2^* - \rho_2^{*2}\boldsymbol{\rho}] - \frac{5[(\boldsymbol{\rho}\cdot\boldsymbol{\rho}_2^*)^2 - \frac{1}{2}(\rho\rho_2^*)^2]\mathbf{r}}{r^2}\Big) \Bigg\}\end{aligned} \tag{8 – 15}$$

Note that $\rho\rho_i^* = |\rho||\rho_i^*|$; it is *not* $\rho\cdot\rho_i^*$. The acceleration from both Sun- and Moon-raised tides is computed. Tides raised on the Earth by the Moon do not influence the mutual barycentric motion, and the effect of Sun-raised tides on the barycentric motion is not considered. The tidal acceleration due to tidal dissipation is implicit in the above acceleration. The explicit conversion of the three Love numbers and time delays to tidal $\dot{n}$ requires a separate theory.

8.4 The Numerical Integration of DE405/LE405

The numerical integration of (8-1), (8-6), and (8-8) was carried out using a variable-step-size, variable-order Adams method (Krogh, 1972). The actual order of the 33 equations at any instant is determined by a specified error bound and by the behavior of backward differences of accelerations.

It has proved numerically more suitable to integrate the lunar ephemeris relative to the Earth rather than to the solar-system barycenter. The solar-system barycentric Earth and Moon states are replaced by the quantities $\mathbf{r_{em}}$ and $\mathbf{r_B}$, given by

$$\mathbf{r_{em}} = \mathbf{r_m} - \mathbf{r_e} \tag{8 – 16}$$

and

$$\mathbf{r_B} = \frac{\mu_\mathbf{e}\mathbf{r_e} + \mu_\mathbf{m}\mathbf{r_m}}{\mu_\mathbf{e} + \mu_\mathbf{m}}, \tag{8 – 17}$$

where the subscripts e and m denote the Earth and Moon, respectively. Note that $\mathbf{r_{em}}$ is the difference of solar-system barycentric vectors and is distinguished from a geocentric vector by the relativistic transformation from the barycenter to geocenter. (The vector $\mathbf{r_B}$ can be interpreted as representing the coordinates of the Newtonian Earth–Moon barycenter relative to the solar-system barycenter. It has no physical significance and does not appear in force calculations; it is solely a vehicle for improving the numerical behavior of the differential equations.)

8.4.1 Estimated Integration Error

The method of error control used in the integration puts a limit on the absolute value of the estimated error in velocity of each equation at the end of every integration step. Step size and integration orders were adjusted on the basis of estimated error. The limits selected for DE405 were $2\times10^{-17} au/day$ in each component of the equations of motion for the planets and Moon, and 2×10^{-15} rad d^{-1} for each component of the libration equations.

Integrations prior to DE405 were performed on a Univac mainframe computer in double precision, with a 60-bit mantissa; DE405 was integrated on a VAX Alpha in quadruple precision. In all cases, the integration error has been significantly less than the estimated error resulting from the uncertainties in the adjustment of the initial conditions and constants to the observational data. These latter errors are discussed later in Section 8.9.

8.4.2 Adopted Constants

The integration requires the input of numerical values for a number of parameters. Some of these parameters, such as the initial positions and velocities of the planets and Moon, result from the least-squares fits and are different in each fit. Other parameters, such as some of the masses and the Earth's zonal harmonics, come from outside sources and are only rarely changed for these present purposes. Some parameters, such as the mass of the Earth–Moon system, can be derived from the data, but for convenience are changed only when statistically significant improvements can be made over the standard values. The lists of initial conditions and constants are given in Section 8.8.

The list of the initial conditions and dynamical constants used in the integration of the equations of motion used for the creation of DE405 is given in Section 8.8. Also given in Section 8.8 is a list of constants used in the reduction of the observational data which were determined in the least squares solution.

8.5 Observational Data Fit by the Planetary and Lunar Ephemerides

In creating modern ephemerides, the majority of the effort is directed toward the set of observational data (measurements) to which the ephemerides are fit and to the fitting process itself. This section describes the observational data used in the creation of DE405/LE405. These observational data and their references are available over the internet at URL#1.

Table 8.5.1 presents the different types of observational data fit by DE405, along with the time-span, the bodies observed, the coordinates measured, the inherent accuracy, and the number of observations. These different sets of observations are discussed briefly here, with some of the unique features of their reductions presented in Section 8.7. References to the data can be found in the website mentioned above.

8.5.1 Optical Data

Classical ephemerides over the past centuries have been based entirely upon optical observations: almost exclusively, meridian circle transit timings. With the advent of planetary radar, spacecraft missions, VLBI, etc., the situation for the four inner planets has changed dramatically.

All of the optical observations for the Sun, Mercury, Venus, and Mars were omitted from the least squares adjustment leading to DE405. The newer and more accurate data types determine these orbits far more accurately (by orders of magnitude) than do the optical data. Since relatively large systematic errors are known to remain in the optical observations, even after the many corrections described in Standish (1990), and since at that time there was a large uncertainty connected with the frame-tie between the FK5 and ICRF reference frames, inclusion of the optical observations for the inner planets could have been detrimental for their ephemerides. Thus, the initial conditions for the inner four planets were adjusted to ranging data primarily, with the VLBI observations serving to orient the whole inner planetary system onto the ICRF.

For Jupiter, the initial conditions were fit to a number of different types of observations, not all of which seemed to be fully consistent with each other. Since that time, however, subsequent analyses of the Jupiter ephemeris, using more recent measurements from the Galileo spacecraft, indicate that the errors are no larger than about 150 km ($0\overset{''}{.}05$).

For the outermost four planets, the optical observations are effectively the only observations and are expected to remain so for a number of years.

8.5.2 Meridian Transits

Transit observations are differential in nature – the planetary observations undergo the same processing as those of the observed stars, both being related to the standard catalog of the epoch. The observations are published as geocentric apparent right ascensions and declinations, taken at the time of meridian passage. For comparison, then, one obtains a computed position from the ephemerides by iterating to find the time at which the local apparent hour angle of the planet is zero. The formulation for computing apparent places is essentially identical to that described in Chapter 7.

8.5.3 Photographic and CCD Astrometry

Photographic and CCD astrometric observations are handled in the same way. The observations based upon the 1950 reference frame (those of Pluto) were transformed onto the J2000 frame using the precession and equinox corrections of Fricke (1971, 1982). Those based upon the (J2000) FK5 were then transformed onto the ICRF using preliminary corrections supplied by Morrison (1996).

8.5.4 Occultation Timings

The timings were supplied in "normal point" form - corrections to the specific ephemeris used in the reductions. The reductions were made by those modeling the Uranian rings and the Neptunian disk and transmitted to the author by Nicholson (1992).

8.5.5 Astrolabe

Astrolabe observations are effectively timings of an object when it's apparent altitude above the horizon attains a certain pre-determined value. These observations are described in Debarbat and Guinot (1970).

8.5.6 Radiometric Emission Measurements

The radiometric emission from the Jovian and Saturnian satellites and from the disks of Uranus and Neptune were measured differentially against the then-existing radio reference catalogue, a precursor to the IERS (International Earth Rotation Service) Radio Catalogue, which is itself the precursor of the ICRF. The observations were taken at the Very Large Array (VLA) in Soccoro, New Mexico by Muhleman *et al.* (1985, 1986, 1988) and by Berge *et al.* (1988). They are presented in "normal point" form - corrections to the specific ephemeris used in the reductions.

8.5.7 Ranging Data

A "range" measurement is the actual round-trip time of the electromagnetic signal from when it leaves the transmitter until the time it arrives at the receiver. The signal is returned from a spacecraft transponder or from a planet, having bounced off of a spot on the surface where the normal to the surface points toward the Earth. The timing is done at the antennae by atomic clocks in what may be considered UTC.

The ranging observations reflected from the surface of Mercury, Venus, or Mars, are accurate to a level of about 100 meters. However, for ephemeris purposes, variations in the topography of a planet's surface introduce a noise-like uncertainty. This enters directly into the process of deducing the distance from the spot on the surface to the center of mass of the planet. Methods to reduce these uncertainties are described in Section 8.7.

Ranging observations to spacecraft can be accurate to a level of only a couple of meters, though these observations are also affected by a number of factors, including the delay of the signal through the solar corona, delays in the electronic circuitry, and uncertainties in the spacecraft's position - either from orbital uncertainties or from a lander's position on a planet's rotating surface.

In any case, the computations of all ranging measurements are very similar. The formulation is given in Section 8.7.

8.5.8 Orbiter Range Points

Orbiter range points are received in the form of "normal points", a representation of modified distance measurements. The original round-trip range and Doppler measurements are reduced using the JPL Orbit Determination Program (Moyer, 1971). This reduction is an adjustment for all relevant parameters, except for those of the planetary ephemeris, including the spacecraft orbit, the planet's mass and gravity field, etc. As such, the resultant range residuals represent derived corrections to the nominal planetary ephemeris used in the reduction. These residuals are then added to the geometric (instantaneous Earth–planet) range in order to give a pseudo "observed" range point.

References for the sources of the normal points are given in the website URL#1.

8.5.9 Lander Range Data

Lander range data are two-way ranging measurements taken of landed spacecraft on the surface of a planet. These data are reduced using the formulae given in Section 8.7 with the reflection point on the planet's surface being represented by the location of the lander on the planet. For this, one needs the surface coordinates of the landers and the orientation of the planet in order to orient the lander into the ephemeris reference frame.

8.5.10 VLBI Data

VLBI measurements of a spacecraft with respect to background sources from a radio source catalogue may be combined with the planetocentric spacecraft trajectory in order to yield a positional determination of the planet with respect to the reference frame of the radio source catalogue. For DE405, there were 2 such points from the Phobos spacecraft nearing its encounter with Mars and 18 from the Magellan spacecraft in its orbit around Venus. Venus is relatively free from the asteroid perturbations that introduce uncertainties into the ephemeris of Mars, so that its ephemeris can be more accurately projected in time. Therefore, the Venus orbit, (which connects Venus into the inner planetary system), in conjunction with the VLBI observations of Venus, is important in orienting the inner planet system onto the radio frame.

8.5.11 Lunar Laser Range Data

The Lunar Laser Range (LLR) data consist of time-of-flight measurements of the laser pulses from McDonald Observatory, Haleakala, or L'Observatoire de la Cte d'Azur (OCA) to any one of four retroreflectors on the Moon and back again. The retroreflectors are at the Apollo 11, 14, and 15 landing sites and on the Lunokhod 2 vehicle. In the solutions for DE405, there were 11218 range points, distributed from 1969 to 1996.

The LLR data are deposited in the National Space Science Data Center and the International Laser Ranging Service. During the least-squares fit the ranges have been weighted according to the instrumental errors that accompany each point. The simple post-fit rms residual in one-way range was about 30 *cm* in 1970 and improved to about 3 *cm* throughout the 1990's.

Table 8.5.1. Observational data fit by DE405. The columns contain the observatory/source, the time coverage, the bodies measured, the components measured, the *a priori* uncertainties of a measurement, the number of observations and the group totals.

Observatory	years	bodies	coords.	*a priori* sigmas	#	group totals
OPTICAL MERIDIAN TRANSITS						

Washington	1911–1994	Sun, ..., Nep	r.a., dec.	1″.0/0″.5	14242	
Herstmonceux	1957–1982	Sun, ..., Nep	r.a., dec.	1″.0/0″.5	2851	17093

PHOTOELECTRIC MERIDIAN TRANSITS

La Palma	1984–1993	Mar, ..., Plu	r.a., dec.	0″.25	6410	
Bordeaux	1985–1996	Sat, Ura, Nep	r.a., dec.	0″.25	854	
Tokyo	1986–1988	Mar, ..., Nep	r.a., dec.	0″.5	498	
Flagstaff - USNO	1995	Plu	r.a., dec.	0″.1	20	7782

PHOTOGRAPHIC ASTROMETRY OF PLUTO

(Pre-disc)	1914–1927	Pluto	r.a., dec.	0″.5	28	
Lowell	1930–1951				620	
Yerkes-McD	1930–1953				310	
(Nrml pts.)	1930–1958				66	
MacDonald	1949–1953				56	
Yerkes	1962–1963				42	
Palomar	1963–1965				8	
Dyer	1964–1981				44	
Bordeaux	1967				24	
Asiago	1969–1978				350	
Torino	1973–1982				74	
Copenhagen	1975–1978				150	
Flagstaff	1980–1994				16	
Lick	1980–1985				56	
La Silla	1988–1989				58	1902

CCD ASTROMETRY OF URANUS, NEPTUNE AND PLUTO

Flagstaff - USNO	1995–1996	Ura, Nep	r.a., dec.	0″.20	313	
Flagstaff - USNO	1995–1996	Plu	r.a., dec.	0″.20	63	
Bordeaux	1995–1996	Plu	r.a., dec.	0″.20	13	389

OCCULTATION TIMINGS

Uranus rings	1977–1983	Ura	r.a., dec.	0″.14	14	
Neptune disk	1968–1985	Nep	r.a., dec.	0″.27	18	32

ASTROLABE

Quito	1969	Sat	r.a., dec.	0″.3–1″.6	1	
Algiers	1969–1973	Mar,Sat			48	
SanFernando	1970–1978	Mar,Jup,Sat			338	
Besançon	1971–1973	Sat			44	
Paris	1971–1978	Mar,Sat			146	
OCA	1972–1981	Mar,Jup,Sat			202	
Santiago	1975–1985	Ura			284	1063

RADAR RANGING

Arecibo	1967–1982	Mer,Ven	range	10 km	284	
Haystack	1966–1971	Mer,Ven		1.5	188	
Goldstone13	1964–1970	Ven		1.5	9	
Gldstn 13–14	1970–1977	Mer,Ven		1.5	23	

Goldstone14	1970–1993	Mer,Ven		1.5	376	880
MARS RADAR–RANGING CLOSURE POINTS						
Goldstone14	1971–1993	Mars	diff. range	1.5	65	65
RADIO ASTROMETRY OF THERMAL EMISSION						
VLA	1987	Jup, ..., Nep	r.a., dec.	0.″03–0.″1	10	10
SPACECRAFT MEASUREMENTS						
Mariner 9	1971–1972	Mars	range	35–120 m	629	
Pioneer 10	1973	Jupiter	range	3 km	1	
Pioneer 11	1974			12 km	1	
Viking Lander	1976–1980	Mars	range	7 m	1018	
	1980–1981			12 m	264	
Voyager 1 OD	1979	Jupiter	r.a., dec.	0.″01, 0.″05	2	
			range	3 km	1	
Voyager 2 OD	1979		range	3 km	1	
Phobos OD	1989	Mars	range	0.5 km	1	
Phobos VLBI	1989	Mars	r.a., dec.	0.″01–0.″1	2	
Ulysses VLBI & OD	1992	Jupiter	r.a., dec.	0.″003, 0.″006	2	
			range	3 km	1	
Magellan VLBI	1990–1994	Venus	r.a., dec.	0.″003–0.″01	18	
Galileo OD	1995	Jupiter	r.a., dec.	0.″05, 0.″2	2	
			range	20 km	1	1956
FRAME-TIE DETERMINATION						
ICRF frame-tie	1988	Earth	$\hat{\mathbf{r}}_\oplus$, $\hat{\mathbf{h}}_\oplus$	0.″003	6	6
LUNAR LASER RANGING						
	1969–1996	Moon	range	2–30 cm	11218	11218
NOMINAL VALUES						
					1	1
TOTAL						42410

8.6 The Orientation of DE405/LE405

In the past, the 1950-based ephemerides of JPL have been aligned onto the (1950) FK4 reference frame (Fricke and Kopff, 1963). Starting with DE200, the ephemerides were aligned onto their own mean equator and dynamical equinox of J2000 (see Standish, 1982).

The JPL planetary and lunar ephemerides starting with DE400 have been aligned onto the International Celestial Reference Frame (ICRF) - formerly, the "IERS frame". This choice of the ICRF is advantageous for a number of reasons:

The ICRF is now the standard reference frame in astronomy.

The timing and polar motion data used for the orientation of the Earth, distributed by the IERS, are now referred to the ICRF.

The ephemerides are now fit to a number of VLBI observations which were referenced to the ICRF. Among all of the ephemeris observations which are explicitly given in a celestial reference system, these ICRF-based ones are the most accurate.

The ICRF is stable; it is accessible; it will not show any net rotation when the coordinates are improved in the future.

8.6.1 Adjustment of DE405 onto the ICRF Reference Frame

DE405 was adjusted to fit the ICRF-based VLBI observations of the Magellan Spacecraft orbiting Venus (18 points, 1990–94) and of the Phobos Spacecraft as it approached Mars (2 points, 1989) as described in the previous section. These data serve to align the ephemerides automatically onto the ICRF during the least-squares adjustment. The inherent accuracy of each of these data points is about 3-10 milliarcseconds.

Ephemerides of the outer planets rely almost entirely upon optical observations. These were initially referenced to various stellar catalogues, then transformed onto the FK4 using the formulae of Schwan (1983), then onto the FK5 system by applying the equinox offset and motion parameters of Fricke (1982), and finally onto the ICRF using a tentative set of transformation tables supplied by Morrison (1996).

8.7 Various formulae used in the reduction of the Observational Data

In the course of reducing various types of observations, coming from many different sources, it is necessary to apply a number of modeling formulae and corrections to the initially received measurements. This section presents some of the details.

8.7.1 Phase Corrections

For the planetary disk observations, the planet is usually not fully illuminated and therefore shows a crescent or gibbous disk. For the outer planets where the phase angle, i, is always small, the following formula, chosen simply from empirical considerations, was applied to the published transit observations:

$$\begin{vmatrix} \Delta\alpha \\ \Delta\delta \end{vmatrix} = \begin{vmatrix} \sin\Theta \\ \cos\Theta \end{vmatrix} B_k \sin 2i, \qquad (8-18)$$

where i is the angle between the Earth and the Sun as seen from the planet; Θ is the position angle of the mid-point of the illuminated edge, measured clockwise from north. Phase corrections were of prime importance for the inner planets when their optical observations were included in earlier ephemerides. For the outer planets, the phase corrections for only Jupiter are of much significance.

8.7.2 Corrections to precession and equinox drift

The optical residuals in right ascension and declination were reduced using the 1976 IAU (Fricke, 1971, 1982) values for precession and equinox drift. More recent estimates for precession had indicated a further correction to the IAU value of $\Delta p = -0.''3/cty$. Also, any existing secular-like trends, such as equinox drift, were modeled by the quantity, Δk, using the standard formulae,

$$\begin{aligned} \Delta\alpha &= (\Delta k + \Delta n \sin\alpha \tan\delta) T_{2000} \\ \Delta\delta &= (\Delta n \cos\alpha) T_{2000}, \end{aligned} \qquad (8-19)$$

where T_{2000} is the time in centuries past J2000, $\Delta n = \Delta p \sin\varepsilon$, and ε is the obliquity.

In DE405, the value of Δp was set to $-0.''3/cty$; that for Δk was determined to be $-0.''062/cty$ in the solution.

8.7.3 Computation of Ranges

A range measurement is the actual round-trip time of the electromagnetic signal from when it leaves the transmitter until the time it arrives at the receiver. The timing is done by the (UTC) clock at the antenna; thus, the observation is essentially the UTC times of transmission and of reception.

For an observation received at the time t_R (expressed in units of $\mathrm{T_{eph}}$, the time which is the independent variable of the equations of motion), the round-trip light-time is given by the difference $UTC(t_R) - UTC(t_R - \tau_d - \tau_u)$, where

$$\tau_u = |\mathbf{r}_\mathrm{B}(t_R - \tau_d) - \mathbf{r}_\mathrm{A}(t_R - \tau_d - \tau_u)|/c + \Delta\tau_u[rel] + \Delta\tau_u[cor] + \Delta\tau_u[tropo] \qquad (8-20)$$

and

$$\tau_d = |\mathbf{r}_\mathrm{A}(t_R) - \mathbf{r}_\mathrm{B}(t_R - \tau_d)|/c + \Delta\tau_d[rel] + \Delta\tau_d[cor] + \Delta\tau_d[tropo], \qquad (8-21)$$

and where τ_u and τ_d are the light-times (in $\mathrm{T_{eph}}$ units) of the upleg and the downleg, respectively; $\mathbf{r}_\mathrm{A}$ is the solar-system barycentric position of the antenna on the Earth's surface; $\mathbf{r}_\mathrm{B}$ is solar-system barycentric position of the "bounce point", either a responding spacecraft or the reflection point on the planet's surface; c is the velocity of light; and the three $\Delta\tau'$s are the corrections to the light-times due to relativity, the electron content of the solar corona, and the Earth's troposphere, respectively. The two formulae are each solved iteratively, first for τ_d then for τ_u.

8.7.4 Antenna Location

The location of the antenna is computed in a straightforward manner using the proper formulae (precession, nutation, UT1, and polar motion) with which one orients the Earth, and in particular the observing station, onto the ICRF, the reference frame of the ephemerides. The formulation was the same as that given by the IERS standards document of J2000 (URL#2) though the geological drifts of the stations were being omitted during the time of DE405's creation.

8.7.5 Time Delay for Relativity

The time-delay due to relativity, given by Shapiro (1964), is obtained by integrating along the signal path over the value of the potential. For each leg of the signal path, the delay is given by the formulae

$$\Delta\tau_{rel} = \frac{(1+\gamma)GM}{c^3} \ln\left|\frac{e+p+q}{e+p-q}\right|, \qquad (8-22)$$

where γ is the PPN parameter of general relativity and where e, p, and q are the heliocentric distance of the Earth, the heliocentric distance of the planet, and the geocentric distance of the planet, respectively. These distances are evaluated at $t - \tau_d$ for the planet, at $t - \tau_d - \tau_u$ for the Earth during the upleg, and at t for the Earth during the downleg. This formula has been augmented since the creation of DE405 (see Moyer, 2000) in order to allow for the increased length of the signal path due to the path's bending. Further, it is now necessary to apply this correction for not only the Sun's potential but also for that of Jupiter, since the latter alters Earth-Mars ranging by a meter or so.

8.7.6 Time Delay for the Solar Corona

The delay from the solar corona (see Muhleman and Anderson, 1981) is obtained by integrating along the signal path from point P_1 to point P_2 over the density of ionized electrons, N_e (cm^{-3}),

$$\Delta\tau_{cor} = \frac{40.3}{cf^2}\int_{P_1}^{P_2} N_\mathrm{e}\, ds, \qquad (8-23)$$

where c is the speed of light (cm/sec), f is the frequency (Hz), and s is the linear distance (cm). The density is given by

$$N_{\mathrm{e}} = \frac{A}{r^6} + \frac{ab/\sqrt{a^2 \sin^2 \beta + b^2 \cos^2 \beta}}{r^2}, \qquad (8-24)$$

where r is the heliocentric distance expressed in units of the solar radius and β is the solar latitude. The values for the constants, A, a, and b are included in Table 8.8.4. When near conjunction the major uncertainties of ranging observations are due to uncertainty in N_{e}, a number which is highly time-dependent. Since the delay is frequency dependent, the total delay may be calibrated using dual-frequency ranging; otherwise, the parameters in the preceding formula must represent some sort of mean values.

For some ranging, measurements were taken in two separate frequencies, so that one can remove the effect entirely since it is frequency-dependent and may therefore be solved-for.

8.7.8 Time Delay for the Troposphere

The delay from the Earth's troposphere for radio frequencies is discussed by Chao (1970). For each leg, it is

$$\Delta\tau_{tropo} = 7\,\mathrm{nsec}/(\cos z + 0.0014/(0.045 + \cot z)), \qquad (8-25)$$

where z is the zenith distance at the antenna.

8.7.9 Modeling the Surface of Mercury

For radar observations, the point of reflection on the surface of Mercury was approximated by a triaxial ellipsoid. Following Anderson *et al.* (1996), a set of fully normalized Legendre functions to the second degree was adjusted to fit the surface:

$$\begin{aligned} R_0 = &+ c_{10}\sqrt{3}\sin\phi + (c_{11}\cos\lambda + s_{11}\sin\lambda)\sqrt{3}\cos\phi + c_{20}\frac{\sqrt{5}}{2}(3\sin^2\phi - 1) \\ &+ (c_{21}\cos\lambda + s_{21}\sin\lambda)\sqrt{15}\sin\phi\cos\phi + (c_{22}\cos 2\lambda + s_{22}\sin 2\lambda)\frac{\sqrt{15}}{2}\cos^2\phi \end{aligned} \qquad (8-26)$$

where λ is the longitude and ϕ is the latitude of the echo point on the surface. The least squares adjustment for DE405 yielded

$R_0 \equiv 2,439,760m$	$c_{10} = +920 \pm 523m$	$c_{11} = +186 \pm 38m$	$s_{11} = -245 \pm 38m$
$c_{21} = +79 \pm 157m$	$s_{21} = +744 \pm 166m$	$c_{22} = +292 \pm 32m$	$s_{22} = +345 \pm 34m$

Since the latitude, ϕ, of the radar echo point on Mercury is always within 12 degrees of the equator, c_{20} is highly correlated ($\gg 0.99$) with R_0 and was therefore omitted from the solution. For the other parameters, they serve merely as a smoothing function for determining the Mercury ephemeris. For more definitive values in regard to the shape of Mercury, see Anderson *et al.* (1996).

8.7.10 Modeling the Surface of Venus

The surface of Venus was modeled using a topographical map fit to measurements from the Pioneer Venus Orbiter spacecraft (Pettengill *et al*., 1980). The map is referenced to a sphere whose radius was determined to be $6052.26 \pm 0.03km$ in the least squares adjustment. With such modeling, the radar residuals of Venus are reduced from over $2km$ to less than $1km$.

8.7.11 Modeling the Surface of Mars – Closure Points

The severe topography on the surface of Mars shows variations of many kilometers. These introduce scatter into the observations which is not entirely random, thus presenting the danger of systematic errors.

Therefore, the radar ranging to Mars was used only in the "closure point" mode. Closure points are pairs of days during which the observed points on the surface of Mars are nearly identical with respect to their longitudes and latitudes on Mars. Since the same topographical features are observed during each of the two days, the uncertainty introduced by the topography may be eliminated by subtracting the residuals of one day from the corresponding ones of the other day. The remaining difference is then due to only the ephemeris drift between the two days. The closure points for Mars have *a priori* uncertainties of about $100m$ or less when the points are within 0.2 degrees of each other on the martian surface.

8.7.12 Viking Lander Computations

The range data from the Viking Landers on the surface of Mars were reduced using the formulae given above with the reflection point on the planet's surface being represented by the location of the lander on Mars. For this, one needs the Martian coordinates of the landers as well as a set of angles used to express the orientation of Mars within the ephemeris reference frame. The position of the lander, expressed in the frame of the ephemeris, is given by

$$\mathbf{r} = \mathbf{r}_x(-\varepsilon)\mathbf{r}_z(-\Omega)\mathbf{r}_x(-I)\mathbf{r}_z(-\Omega_q')\mathbf{r}_x(-I_q')\mathbf{r}_z(-V')\mathbf{r}_0, \qquad (8-27)$$

where ε is the obliquity of the ecliptic; Ω and I are the node and inclination of the mean Martian orbit upon the (J2000) ecliptic; Ω_q and I_q are the mean node and inclination of the Martian equator upon the mean orbit; V is the longitude of the Martian prime meridian measured along the equator from the intersection of the orbit; and $\mathbf{r}_0$, the Mars-fixed coordinates of the lander, and where

$$\Omega_q' = \Omega_q - \Delta\psi, \qquad I_q' = I_q + \Delta\varepsilon \quad \text{and} \quad V' = V + \Delta\psi \cos I_q', \qquad (8-28)$$

where $\Delta\psi$ and $\Delta\varepsilon$ express the nutation of Mars, computed from the formulation of Lyttleton *et al.* (1979). The Mars-fixed coordinates of the lander are computed from the cylindrical coordinates,

$$\mathbf{r}_0^T = [u\cos\lambda, u\sin\lambda, v]^T. \qquad (8-29)$$

The values for the parameters used in the reductions are given in Section 8.8. The values for ε, Ω, and I were adopted: those for Ω_q, I_q, and V, as well as the coordinates of the landers, were estimated in the least-squares adjustments. For a more extensive treatment including parameter values, see Folkner *et al.* (1998).

8.8 The Initial Conditions and Constants of DE405/LE405

The starting conditions for DE405/LE405 were the result of two successive least-squares adjustments. The first adjustment was a solution for all of the relevant parameters whose values were best determined from the ephemeris data. Then, some of the parameters were rounded off (at a negligible level) for the sake of appearance and ease of use. These rounded values were then forced as known constants into the second solution which produced a minor adjustment to the other parameters.

This chapter gives the numerical values of all of the parameters that were used in the final integration of DE405/LE405. The starting epoch was Julian Ephemeris Date 2440400.5 (June 28, 1969) – chosen as being the last "0400" date before the start of the Lunar Laser Ranging observations. Table 8.8.1 gives the cartesian coordinates of the heliocentric planets, the Solar-System-Barycentric Sun, and the geocentric Moon. The orientation parameters of the Moon are given in Table 8.8.2, the dynamical constants (those in the equations of motion) are presented in Table 8.8.3, a number of parameters used in the reduction of the observational data (but not in the equations of motion) are shown in Table 8.8.4, and Table 8.8.5 gives the radii and taxonomic classes of the 297 *non*-Big3 asteroids used in DE405.

Table 8.8.1.

Initial Conditions at JED (T_{eph}) 2440400.5 (June 28, 1969). Coordinates and velocities [$au, au/day$] with respect to the International Celestial Reference Frame ("ICRF"; ∼ J2000 equator & equinox): heliocentric planets, solar-system barycentric Sun, and geocentric Moon.

Mercury	0.35726020644727541518	-0.09154904243051842990	-0.08598103998694037053
	0.00336784566219378527	0.02488934284224928990	0.01294407158679596663
Venus	0.60824943318560406033	-0.34913244319590053792	-0.19554434578540693592
	0.01095242010990883868	0.01561250673986247038	0.00632887645174666542
EM Bary	0.11601490913916648627	-0.92660555364038517604	-0.40180627760698804496
	0.01681162005220228976	0.00174313168798203152	0.00075597376713614610
Mars	-0.11468858243909270380	-1.32836653083348816476	-0.60615518941938081574
	0.01448200480794474793	0.00023728549236071136	-0.00028374983610239698
Jupiter	-5.38420940699214597622	-0.83124765616108382433	-0.22509475703354987777
	0.00109236329121849772	-0.00652329419119226767	-0.00282301226721943903
Saturn	7.88988993382281817537	4.59571072692601301962	1.55843151672508969735
	-0.00321720349109366378	0.00433063223355569175	0.00192641746379945286
Uranus	-18.26990081497826660524	-1.16271158021904696130	-0.25036954074255487461
	0.00022154016562741063	-0.00376765355824616179	-0.00165324380492239354
Neptune	-16.05954509192446441763	-23.94294829087950141524	-9.40042278035400838599
	0.00264312279157656145	-0.00150349208075879462	-0.00068127100487234772
Pluto	-30.48782211215555045830	-0.87324543019672926542	8.91129698418475509659
	0.00032256219593323324	-0.00314874792755160542	-0.00108017793159369583
Sun	0.00450250884530125842	0.00076707348146464055	0.00026605632781203556
	-0.00000035174423541454	0.00000517762777222281	0.00000222910220557907
Moon	-0.00080817732791148419	-0.00199463000162039941	-0.00108726266083810178
	0.00060108481665912983	-0.00016744546061515148	-0.00008556214497398616

Table 8.8.2.

J2000 angular coordinates of the lunar physical librations [$rad, rad/day$].

ϕ_M, θ_M, ψ_M	0.00512995970515812456	0.38239065587686011507	1.29414222411027863099
$\omega_{xM}, \omega_{yM}, \omega_{zM}$	0.00004524704499022800	-0.00000223092763198743	0.22994485870136698411

Table 8.8.3.

Dynamical Constants used in the Integration of DE405/LE405.

Defining Constants, Scale Factor		
k^2	0.01720209895^2	Gauss' (gravitational) constant
c	299792.458 [km/sec]	speed of light
au	149597870.691 [km/au]	scale factor
Planetary Masses		
GM_s/GM_1	6023600.	$mass^{-1}$ for Mercury
GM_s/GM_2	408523.71	$mass^{-1}$ for Venus

GM_s/GM_B	328900.5614	$mass^{-1}$ for EM-Bary
GM_s/GM_4	3098708.	$mass^{-1}$ for Mars
GM_s/GM_5	1047.3486	$mass^{-1}$ for Jupiter
GM_s/GM_6	3497.898	$mass^{-1}$ for Saturn
GM_s/GM_7	22902.98	$mass^{-1}$ for Uranus
GM_s/GM_8	19412.24	$mass^{-1}$ for Neptune
GM_s/GM_9	135200000.	$mass^{-1}$ for Pluto
GM_e/GM_m	81.30056	Earth-Moon mass ratio

Asteroid Masses and Densities

m_{Ceres}	4.7×10^{-10} GM_s	Mass of Ceres
m_{Pallas}	1.0×10^{-10} GM_s	Mass of Pallas
m_{Vesta}	1.3×10^{-10} GM_s	Mass of Vesta
ρ_C	1.8	Mean density of C-type asteroids
ρ_S	2.4	Mean density of S-type asteroids
ρ_M	5.0	Mean density of M-type asteroids

Relativity, $\dot{G}$, $J_2(Sun)$

β	1.0	PPN parameter
γ	1.0	PPN parameter
$\dot{G}$	0.0	rate of change of the gravitational constant
J_{2s}	2×10^{-7}	dynamical form-factor of the Sun

Earth Parameters

$\mathcal{R}_e$	6378.137 $[km]$	radius of the Earth
J_{2e}	0.001082626	zonal harmonics of the Earth
J_{3e}	-0.000002533	
J_{4e}	-0.000001616	
k_{2e0}	0.34	potential Love number for the rigid Earth tide
k_{2e1}	0.30	for the Earth's tidal deformation
k_{2e2}	0.30	for the Earth's rotational deformation
τ_{e0}	0.0	time-lag for the rigid Earth tide
τ_{e1}	0.01290895939 [days]	for the Earth's tidal deformation
τ_{e2}	0.00694178558 [days]	for the Earth's rotational deformation
Φ_X	0.006358 ["]	X-axis offset at J2000
Φ_Y	-0.015571 ["]	Y-axis offset at J2000
$d\Phi_X/dt$	0.000244 ["/yr]	negative obliquity correction : $-\Delta\varepsilon$
$d\Phi_Y/dt$	-0.001193 ["/yr]	precession correction : $+\Delta p \sin\varepsilon$

Moon Parameters

$\mathcal{R}_m$	1738.0 $[km]$	radius of the Moon
β_L	0.0006316121	lunar moment parameters
γ_L	0.0002278583	
k_{2m}	0.0299221167	potential love number of the Moon
τ_m	0.1667165558 [days]	time-lag for the lunar solid-body tide
* J_{2m}	0.000204312007	*input* zonal harmonics of the Moon
J_{3m}	0.000008785470	
J_{4m}	-0.000000145383	
$C_{3,1m}$	0.000030803810	*input* tesseral harmonics of the Moon

$C_{3,2m}$	0.000004879807
$C_{3,3m}$	0.000001770176
$S_{3,1m}$	0.000004259329
$S_{3,2m}$	0.000001695516
$S_{3,3m}$	-0.000000270970
$C_{4,1m}$	-0.000007177801
$C_{4,2m}$	-0.000001439518
$C_{4,3m}$	-0.000000085479
$C_{4,4m}$	-0.000000154904
$S_{4,1m}$	0.000002947434
$S_{4,2m}$	-0.000002884372
$S_{4,3m}$	-0.000000788967
$S_{4,4m}$	0.000000056404

* Note, as described in Section 8.3, the values used for J_{2m}, $C_{2,2m}$, $C_{2,1m}$, $S_{2,1m}$, and $S_{2,2m}$ are computed from the (time-varying) lunar moment-of-inertia tensor. The J_{2m} listed here is input; the rigid value, used in the force and torque equations, is derived from this input value and from the potential Love number, k_{2m}. The rigid value of $C_{2,2m}$ is computed from J_{2m}, β_L, and γ_L.

Table 8.8.4.

Auxiliary Constants used in the Observational Reductions for DE405/LE405.

Mars Orientation Parameters

ε	23.4392811 [^{o}]	Mean obliquity of the ecliptic
$\dot{\varepsilon}$	0.0 [$^o/day$]	
Ω	49.6167 [^{o}]	Node of Mars' mean orbit upon ecliptic
$\dot{\Omega}$	0.0 [$^o/day$]	
I	1.85137 [^{o}]	Inclination of Mars' mean orbit upon ecliptic
$\dot{I}$	0.0 [$^o/day$]	
Ω_q	35.4371 [^{o}]	Node of Mars' equator upon Mars' orbit
$\dot{\Omega}_q$	-5.76×10^{-6} [$^o/day$]	
I_q	25.1886 [^{o}]	Inclination of Mars' equator upon Mars' orbit
$\dot{I}_q$	-0.1094×10^{-6} [$^o/day$]	
V	1.3361259 [^{o}]	Longitude of Mars' prime meridian
$\dot{V}$	350.89198512444 [$^o/day$]	

Viking Spacecraft Coordinates

λ_1	311.82160 [^{o}]	longitude of Viking Lander # 1
u_1	3136.520 [km]	axial distance
v_1	1284.469 [km]	z-height
λ_1	134.05322 [^{o}]	longitude of Viking Lander # 2
u_1	2277.366 [km]	axial distance
v_1	2500.054 [km]	z-height

Solar Corona Parameters [see (8-24)]

A	1.06×10^8 [cm^{-3}]
a	4.89×10^5 [cm^{-3}]
b	3.91×10^5 [cm^{-3}]

8.8.1 Asteroids

The initial conditions for all 300 asteroids came from a file maintained by the Solar System Dynamics Group at JPL. The radii from the same file were used to compute the masses of each of the 297 *non*-Big3 individual asteroids from the formula, $GM = 6.27 \times 10^{-22} \mathrm{R}^3 \rho_k$, where R is the radius of the asteroid in kilometers and where ρ_k is the density of the kth taxonomic class of asteroids (S, C, or M), given in Table 8.8.5.

Table 8.8.5.
The 297 *non*-Big3 individual asteroids used in the integration of DE405. The taxonomic class and the radius in km are given.

0010	Hygiea	C	214.5	0109	Felicitas	C	45.8	0238	Hypatia	C	78.0
0013	Egeria	C	107.5	0111	Ate	C	69.5	0240	Vanadis	C	54.0
0019	Fortuna	C	100.0	0112	Iphigenia	C	37.8	0241	Germania	C	84.5
0024	Themis	C	99.0	0114	Kassandra	C	51.5	0247	Eukrate	C	68.5
0031	Euphrosyne	C	124.0	0117	Lomia	C	77.0	0259	Aletheia	C	92.5
0034	Circe	C	59.0	0120	Lachesis	C	89.0	0266	Aline	C	56.5
0035	Leukothea	C	54.0	0121	Hermione	C	108.5	0268	Adorea	C	71.0
0036	Atalante	C	54.5	0127	Johanna	C	61.0	0275	Sapientia	C	57.5
0038	Leda	C	60.0	0128	Nemesis	C	97.0	0276	Adelheid	C	63.5
0041	Daphne	C	91.0	0130	Elektra	C	94.5	0283	Emma	C	75.0
0045	Eugenia	C	107.0	0134	Sophrosyne	C	53.5	0303	Josephina	C	51.5
0046	Hestia	C	65.5	0137	Meliboea	C	75.0	0304	Olga	C	34.2
0047	Aglaja	C	68.5	0139	Juewa	C	81.0	0308	Polyxo	C	74.0
0048	Doris	C	112.5	0140	Siwa	C	57.0	0313	Chaldaea	C	50.5
0049	Pales	C	77.0	0141	Lumen	C	67.5	0324	Bamberga	C	114.0
0050	Virginia	C	44.0	0143	Adria	C	46.4	0326	Tamara	C	50.0
0051	Nemausa	C	76.5	0144	Vibilia	C	73.0	0329	Svea	C	40.2
0052	Europa	C	156.0	0145	Adeona	C	77.5	0334	Chicago	C	85.0
0053	Kalypso	C	59.5	0146	Lucina	C	68.5	0335	Roberta	C	46.8
0054	Alexandra	C	85.5	0147	Protogeneia	C	68.5	0336	Lacadiera	C	36.0
0056	Melete	C	58.5	0150	Nuwa	C	78.5	0344	Desiderata	C	69.0
0058	Concordia	C	48.9	0154	Bertha	C	96.0	0345	Tercidina	C	50.0
0059	Elpis	C	86.5	0156	Xanthippe	C	63.0	0350	Ornamenta	C	61.5
0062	Erato	C	49.6	0159	Aemilia	C	65.5	0356	Liguria	C	67.5
0065	Cybele	C	120.0	0160	Una	C	42.5	0357	Ninina	C	55.0
0070	Panopaea	C	63.5	0162	Laurentia	C	52.5	0358	Apollonia	C	45.9
0072	Feronia	C	44.6	0163	Erigone	C	38.2	0360	Carlova	C	60.5
0074	Galatea	C	61.5	0164	Eva	C	55.0	0362	Havnia	C	49.0
0076	Freia	C	95.0	0165	Loreley	C	80.0	0363	Padua	C	48.5
0078	Diana	C	62.5	0168	Sibylla	C	77.0	0365	Corduba	C	55.0
0081	Terpsichore	C	62.0	0171	Ophelia	C	60.5	0366	Vincentina	C	49.0
0083	Beatrix	C	42.1	0173	Ino	C	79.5	0372	Palma	C	97.5
0084	Klio	C	41.5	0175	Andromache	C	53.5	0373	Melusina	C	49.8
0085	Io	C	78.5	0176	Iduna	C	62.5	0375	Ursula	C	108.0
0086	Semele	C	63.5	0185	Eunike	C	82.5	0377	Campania	C	47.2
0087	Sylvia	C	135.5	0187	Lamberta	C	67.5	0381	Myrrha	C	62.0
0088	Thisbe	C	116.0	0191	Kolga	C	52.5	0386	Siegena	C	86.5
0090	Antiope	C	62.5	0194	Prokne	C	87.0	0388	Charybdis	C	60.0
0091	Aegina	C	57.0	0195	Eurykleia	C	44.9	0393	Lampetia	C	53.0
0093	Minerva	C	85.5	0200	Dynamene	C	66.0	0404	Arsinoe	C	50.5
0094	Aurora	C	106.0	0203	Pompeja	C	60.0	0405	Thia	C	64.5

0095	Arethusa	C	72.5
0096	Aegle	C	87.0
0098	Ianthe	C	54.5
0099	Dike	C	39.5
0102	Miriam	C	43.0
0104	Klymene	C	63.5
0105	Artemis	C	61.5
0106	Dione	C	73.5
0107	Camilla	C	118.5
0431	Nephele	C	48.9
0442	Eichsfeldia	C	33.8
0444	Gyptis	C	85.0
0449	Hamburga	C	44.3
0451	Patientia	C	115.0
0454	Mathesis	C	42.2
0455	Bruchsalia	C	43.8
0466	Tisiphone	C	60.5
0469	Argentina	C	64.5
0476	Hedwig	C	60.5
0481	Emita	C	58.0
0488	Kreusa	C	79.0
0489	Comacina	C	72.0
0490	Veritas	C	60.5
0491	Carina	C	50.5
0498	Tokio	C	42.4
0505	Cava	C	57.5
0506	Marion	C	54.5
0508	Princetonia	C	73.5
0511	Davida	C	168.5
0514	Armida	C	55.0
0521	Brixia	C	60.5
0535	Montague	C	38.5
0536	Merapi	C	79.0
0545	Messalina	C	57.5
0554	Peraga	C	49.2
0566	Stereoskopia	C	87.5
0568	Cheruskia	C	44.9
0595	Polyxena	C	57.0
0596	Scheila	C	58.5
0602	Marianna	C	65.0
0618	Elfriede	C	62.0
0626	Notburga	C	52.0
0635	Vundtia	C	50.0
0654	Zelinda	C	66.0
0663	Gerlinde	C	52.0
0683	Lanzia	C	58.0
0690	Wratislavia	C	72.5
0691	Lehigh	C	46.3
0694	Ekard	C	46.4
0702	Alauda	C	101.0
0704	Interamnia	C	166.5
0705	Erminia	C	69.5
0709	Fringilla	C	49.8
0205	Martha	C	41.8
0206	Hersilia	C	56.5
0209	Dido	C	74.5
0210	Isabella	C	45.0
0211	Isolda	C	74.0
0212	Medea	C	70.0
0213	Lilaea	C	42.3
0225	Henrietta	C	62.0
0233	Asterope	C	54.0
0751	Faina	C	57.5
0762	Pulcova	C	71.0
0769	Tatjana	C	51.0
0772	Tanete	C	61.5
0773	Irmintraud	C	49.5
0776	Berbericia	C	88.5
0780	Armenia	C	48.5
0788	Hohensteina	C	54.5
0790	Pretoria	C	88.0
0791	Ani	C	53.5
0804	Hispania	C	80.5
0814	Tauris	C	58.0
0895	Helio	C	73.5
0909	Ulla	C	60.0
0914	Palisana	C	39.5
1015	Christa	C	50.5
1021	Flammario	C	51.5
1093	Freda	C	60.0
1467	Mashona	C	56.0
0003	Juno	S	133.5
0005	Astraea	S	62.5
0006	Hebe	S	96.0
0007	Iris	S	101.5
0008	Flora	S	70.5
0009	Metis	S	85.0
0011	Parthenope	S	81.0
0012	Victoria	S	58.5
0014	Irene	S	83.5
0015	Eunomia	S	136.0
0017	Thetis	S	46.6
0018	Melpomene	S	74.0
0020	Massalia	S	75.5
0023	Thalia	S	55.5
0025	Phocaea	S	39.1
0026	Proserpina	S	49.4
0027	Euterpe	S	65.5
0028	Bellona	S	63.0
0029	Amphitrite	S	109.5
0030	Urania	S	52.0
0032	Pomona	S	41.3
0037	Fides	S	56.0
0039	Laetitia	S	79.5
0040	Harmonia	S	55.5
0042	Isis	S	53.5
0407	Arachne	C	48.8
0409	Aspasia	C	84.0
0410	Chloris	C	64.0
0412	Elisabetha	C	46.6
0419	Aurelia	C	66.5
0420	Bertholda	C	73.0
0423	Diotima	C	108.5
0424	Gratia	C	45.2
0426	Hippo	C	67.0
0071	Niobe	S	43.6
0080	Sappho	S	40.9
0089	Julia	S	79.5
0103	Hera	S	47.6
0115	Thyra	S	41.8
0124	Alkeste	S	39.8
0148	Gallia	S	52.0
0181	Eucharis	S	53.5
0192	Nausikaa	S	53.5
0196	Philomela	S	73.0
0221	Eos	S	55.0
0230	Athamantis	S	57.5
0236	Honoria	S	45.2
0287	Nephthys	S	35.0
0328	Gudrun	S	23.5
0346	Hermentaria	S	55.0
0349	Dembowska	S	71.5
0354	Eleonora	S	81.0
0385	Ilmatar	S	47.0
0387	Aquitania	S	53.0
0389	Industria	S	40.8
0416	Vaticana	S	44.8
0433	Eros	S	10.0
0471	Papagena	S	67.5
0532	Herculina	S	112.5
0674	Rachele	S	50.5
0980	Anacostia	S	44.5
1036	Ganymed	S	20.5
0016	Psyche	M	132.0
0021	Lutetia	M	49.8
0022	Kalliope	M	93.5
0069	Hesperia	M	71.5
0075	Eurydike	M	29.1
0077	Frigga	M	35.5
0092	Undina	M	66.0
0097	Klotho	M	43.5
0110	Lydia	M	44.5
0129	Antigone	M	62.5
0135	Hertha	M	41.0
0201	Penelope	M	35.2
0216	Kleopatra	M	70.0
0224	Oceana	M	35.0
0250	Bettina	M	42.8
0322	Phaeo	M	36.9

0712	Boliviana	C	66.0	0043	Ariadne	S	32.6	0337	Devosa	M	31.6
0713	Luscinia	C	54.5	0044	Nysa	S	36.6	0338	Budrosa	M	31.1
0739	Mandeville	C	55.0	0057	Mnemosyne	S	58.0	0347	Pariana	M	27.1
0740	Cantabia	C	47.2	0063	Ausonia	S	54.0	0369	Aeria	M	31.1
0747	Winchester	C	89.0	0068	Leto	S	63.5	0849	Ara	M	49.0

8.9 The Positional Errors of the Planetary and Lunar Ephemerides

Estimates of ephemeris errors are based upon a number of factors, including the accuracies of the observational data, the stability to various parameter and data combinations tested in the least squares adjustments, the observed abilities to predict positions into the future, knowledge of the uncertainties associated with various relevant factors, etc. However, one may get a more definitive assessment comparing two ephemerides, assuming that the differences are due mainly to the older ephemeris, the newer one having benefitted from refined modeling and enhanced data sets.

This section first discusses the accuracies of the ephemerides, showing how the different data types contribute to the various ephemeris parameters; it then presents comparisons of DE200 and of DE405 with a recent ephemeris, DE409.

8.9.1 Inner 4 planets : ephemerides from ranging and VLBI

The ephemerides of the four innermost planets along with the Moon and the Sun are all well-known with respect to each other (intra-planet distances and angles) because of the accurate ranging observations to which the ephemerides are adjusted. Furthermore, the mean motion of this whole inner-body system is also well-determined by fitting to the ranging observations. For explanations of this latter fact, see Williams and Standish (1989) or Standish and Williams (1990).

The orientation of the inner planet system is provided by VLBI observations of the Magellan Spacecraft orbiting Venus and the Phobos Spacecraft on its approach to Mars. These observations link the spacecraft to the ICRF (International Celestial Reference System), and thus the planet to the ICRF, given that the planetocentric spacecraft orbit is sufficiently well-determined. An additional frame-tie (Folkner *et al.*, 1993) was determined using the well-established Earth-based positions of the VLBI antennae, linked to the LLR (lunar laser ranging) telescopes by means of ground surveys. The positions of these telescopes, in turn, are connected to the Moon's orbit using the LLR data, and since the Moon's orbit is sensitive to the position of the strongly perturbing Sun, it is tied to the heliocentric Earth orbit.

Radar ranging observations have inherent accuracies of less than 100 meters, though the unmodeled uncertainties introduced by the surface topographies are a few hundred meters and more. Spacecraft orbits, on the other hand, have shown accuracies of 10 meters or less. The VLBI observations have uncertainties of a few milliarcseconds; with about 20 of them, the uncertainty of the orientation of the whole inner planet ephemeris system of DE405 is $0''\!.001$–$0''\!.002$ which is the equivalent of a kilometer or so at the typical distances between the inner planets.

Thus, during the present decade or so, relative distances between two of the inner-solar-system bodies are accurate to 100–200 meters; relative angles between the inner bodies (e.g., Earth-Sun-planet angle) are accurate to less than $0''\!.001$. The orientation is accurate to $0''\!.001$–$0''\!.002$. Away from the present epoch of accurate ranging and VLBI, the accuracies deteriorate due to the uncertainties in the mean motions caused by the perturbative effects of the asteroids.

8.9.2 Uncertainties from asteroid perturbations

The masses of the asteroids are not well-known, and therefore it has not been possible to model their perturbations with full accuracy. Williams (1984) estimated that due to the asteroids, the mean motions of the inner planets have uncertainties on the order of $0''\!.02$/century – a couple of kilometers per decade.

8.9.3 Outer planets : reliance on the classical optical observations

In contrast to those of the inner planets, the ephemerides of the outer planets rely almost entirely upon optical observations in which *systematic* errors over the past have been at the level of $0\overset{''}{.}1$ or so. As a result, the outer planet ephemerides are much less accurate than those of the planets observed with ranging. Modern CCD measurements now show single (presumably *random*) observation measurement errors of about $0\overset{''}{.}1$ so that present-day plane-of-sky positions (directions) are determined to a few hundredths of an arcsecond. For times away from the present, the accuracy deteriorates significantly, especially with the outermost planets.

8.9.4 Planetary Positional Errors in the Almancs, 1984–2002

A comparison between DE200, created before 1980, and DE409, created in 2003, shows differences which must, for the most part, be attributed to the errors in DE200; thus, these differences give good estimates of the errors in the positions of the *Astronomical Almanac* for the years, 1984–2002. Plots of the differences are given in Figs. 8.9.1a–8.9.10c.

8.9.5 Planetary Position Uncertainties in 2003

One can also compare DE405 with DE409. Importantly, this more recent ephemeris includes both ranging and VLBI measurements of the MGS (1999–) and the Odyssey (2002–) spacecraft in orbit around Mars (Konopliv, 2002, 2003 and Border, 2002, 2003), and also the continuing CCD measurements of the outer planets and their satellites (Stone, 1998, 2000). As such, DE409 is believed to be more accurate than DE405, and the differences certainly represent the sizes of the uncertainties in DE405, if not estimates of the actual errors themselves. The differences, DE405-DE409, are shown in Figs. 8.9.11a–8.9.20c.

It may be immediately noticed that the later differences, DE405–DE409, are significantly smaller than those of DE200–DE405. As the observational data accumulate, the convergence of the ephemerides becomes a good indicator of the inherent errors and uncertainties.

The comparisons of both DE200 and DE405 with DE409 are discussed in further detail in Standish (2004).

8.10 Keplerian Elements for Approximate Positions of the Major Planets

Lower accuracy formulae for planetary positions have a number of important applications when one doesn't need the full accuracy of an integrated ephemeris. They are often used in observation scheduling, telescope pointing, and prediction of certain phenomena as well as in the planning and design of spacecraft missions.

Approximate positions of the nine major planets may be found by using Keplerian formulae with their associated elements and rates. Such elements are not intended to represent any sort of mean; they are simply the result of being adjusted for a best fit. As such, it must be noted that the elements are not valid outside the given time-interval over which they were fit.

The elements are given below in Table 8.10.2 or in Tables 8.10.3 and 8.10.4, depending upon the time-interval over which they were fit and within which they are to be used.

Formulae for using them are given here.

8.10.1 Formulae for using the Keplerian elements

Keplerian elements given in the tables below are

$a_o, \dot{a}$: semi-major axis [au, au/century]

$e_o, \dot{e}$	:	eccentricity [radians, radians/century]
$I_o, \dot{I}$	:	inclination [degrees, degrees/century]
$L_o, \dot{L}$	:	mean longitude [degrees, degrees/century]
$\varpi_o, \dot{\varpi}$	:	longitude of perihelion [degrees, degrees/century] ($\varpi = \omega + \Omega$)
$\Omega_o, \dot{\Omega}$	:	longitude of the ascending node [degrees, degrees/century]

In order to obtain the coordinates of one of the planets at a given Julian Ephemeris Date, $\mathrm{T_{eph}}$,

1. Compute the value of each of that planet's six elements: $a = a_o + \dot{a}\mathrm{T}$, etc., where T, the number of centuries past J2000.0, is $\mathrm{T}=(\mathrm{T_{eph}}\text{-}2451545.0)/36525$.

2. Compute the argument of perihelion, ω, and the mean anomaly, M :

$$\omega = \varpi - \Omega \;\; ; \;\; M = L \; - \; \varpi \; + \; b\mathrm{T}^2 \; + \; c\cos(f\mathrm{T}) \; + \; s\sin(f\mathrm{T}) \qquad (8-30)$$

   where the last three terms must be added to M for Jupiter through Pluto when using the formulae for 3000 BC to 3000 AD.

3. Modulus the mean anomaly so that $-180^{\mathrm{o}} \leq M \leq +180^{\mathrm{o}}$ and then obtain the eccentric anomaly, E, from the solution of Kepler's equation (see below):

$$M \;=\; E - e^* \sin E, \qquad (8-31)$$

   where $e^* \;=\; 180/\pi \; e \;=\; 57.29578 \; e$.

4. Compute the planet's heliocentric coordinates in its orbital plane, $\mathbf{r}'$, with the x'-axis aligned from the focus to the perihelion:

$$x' = a(\cos E - e) \quad ; \quad y' = a\sqrt{1-e^2}\;\sin E \quad ; \quad z' = 0. \qquad (8-32)$$

5. Compute the coordinates, $\mathbf{r}_{ecl}$, in the J2000 ecliptic plane, with the x-axis aligned toward the equinox:

$$\mathbf{r}_{ecl} \;= \mathcal{M}\mathbf{r}' \;\equiv\; \mathcal{R}_z(-\Omega)\mathcal{R}_x(-I)\mathcal{R}_z(-\omega)\mathbf{r}' \qquad (8-33)$$

   so that

$$\begin{aligned} x_{ecl} &= (\cos\omega\cos\Omega - \sin\omega\sin\Omega\cos I)\;\; x' \;+ (-\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos I)\;\; y' \\ y_{ecl} &= (\cos\omega\sin\Omega + \sin\omega\cos\Omega\cos I)\;\; x' \;+ (-\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos I)\;\; y' \\ z_{ecl} &= (\sin\omega\sin I)\;\; x' \;+ (\cos\omega\sin I)\;\; y' \end{aligned} \qquad (8-34)$$

6. If desired, obtain the equatorial coordinates in the "ICRF", or "J2000 frame", $\mathbf{r_{eq}}$:

$$\begin{aligned} x_{eq} &= x_{ecl} \\ y_{eq} &= + \cos\varepsilon \;\; y_{ecl} \; - \sin\varepsilon \;\; z_{ecl} \\ z_{eq} &= + \sin\varepsilon \;\; y_{ecl} \; + \cos\varepsilon \;\; z_{ecl} \end{aligned} \qquad (8-35)$$

   where the obliquity at J2000 is $\varepsilon = 23^{\circ}\!.43928$.

8.10.2 Solution of Kepler's Equation, $M = E - e^* \sin E$

Given the mean anomaly, M, and the eccentricity, e^*, both in degrees, start with

$$E_0 = M + e^* \sin M \tag{8-36}$$

and iterate the following three equations, with $n = 0, 1, 2, ...$, until $|\Delta E| \leq tol$ (noting that e^* is in degrees; e is in radians):

$$\Delta M = M - (E_n - e^* \sin E_n) \quad \Delta E = \Delta M/(1 - e \cos E_n) \ \ ; \ \ E_{n+1} = E_n + \Delta E. \tag{8-37}$$

For the approximate formulae in this present context, $tol = 10^{-6} degrees$ is sufficient.

8.10.3 Approximate Accuracies of the Keplerian Formulae

Table 8.10.1 gives the accuracies that one can expect from the Keplerian formulation given in this section

Table 8.10.1

Approximate errors, in heliocentric longitude, λ, latitude, ϕ, and in distance, ρ, from the Keplerian formulation of the present section.

	1800 – 2050			3000 BC to 3000 AD		
	λ ["]	ϕ ["]	ρ [1000km]	λ ["]	ϕ ["]	ρ [1000km]
Mercury	15	1	1	20	15	1
Venus	20	1	4	40	30	8
EM Bary	20	8	6	40	15	15
Mars	40	2	25	100	40	30
Jupiter	400	10	600	600	100	1000
Saturn	600	25	1500	1000	100	4000
Uranus	50	2	1000	2000	30	8000
Neptune	10	1	200	400	15	4000
Pluto	5	2	300	400	100	2500

Table 8.10.2

Keplerian elements and their rates, with respect to the mean ecliptic and equinox of J2000, valid for the time-interval 1800 AD - 2050 AD.

	a	e	I	L	ϖ	Ω
	$[au, au/cty]$	$[rad, rad/cty]$	$[deg, deg/cty]$	$[deg, deg/cty]$	$[deg, deg/cty]$	$[deg, deg/cty]$
Mercury	0.38709927	0.20563593	7.00497902	252.25032350	77.45779628	48.33076593
	0.00000037	0.00001906	-0.00594749	149472.67411175	0.16047689	-0.12534081
Venus	0.72333566	0.00677672	3.39467605	181.97909950	131.60246718	76.67984255
	0.00000390	-0.00004107	-0.00078890	58517.81538729	0.00268329	-0.27769418
EM Bary	1.00000261	0.01671123	-0.00001531	100.46457166	102.93768193	0.0
	0.00000562	-0.00004392	-0.01294668	35999.37244981	0.32327364	0.0
Mars	1.52371034	0.09339410	1.84969142	-4.55343205	-23.94362959	49.55953891
	0.00001847	0.00007882	-0.00813131	19140.30268499	0.44441088	-0.29257343
Jupiter	5.20288700	0.04838624	1.30439695	34.39644051	14.72847983	100.47390909
	-0.00011607	-0.00013253	-0.00183714	3034.74612775	0.21252668	0.20469106
Saturn	9.53667594	0.05386179	2.48599187	49.95424423	92.59887831	113.66242448
	-0.00125060	-0.00050991	0.00193609	1222.49362201	-0.41897216	-0.28867794

Uranus	19.18916464	0.04725744	0.77263783	313.23810451	170.95427630	74.01692503
	-0.00196176	-0.00004397	-0.00242939	428.48202785	0.40805281	0.04240589
Neptune	30.06992276	0.00859048	1.77004347	-55.12002969	44.96476227	131.78422574
	0.00026291	0.00005105	0.00035372	218.45945325	-0.32241464	-0.00508664
Pluto	39.48211675	0.24882730	17.14001206	238.92903833	224.06891629	110.30393684
	-0.00031596	0.00005170	0.00004818	145.20780515	-0.04062942	-0.01183482

Table 8.10.3

Keplerian elements and their rates, with respect to the mean ecliptic and equinox of J2000, valid for the time-interval 3000 BC – 3000 AD. Further, the computation of M for Jupiter through Pluto *must* be augmented by the additional terms described above and given in Table 8.10.4.

	a	e	I	L	ϖ	Ω
	$[au, au/cty]$	$[rad, rad/cty]$	$[deg, deg/cty]$	$[deg, deg/cty]$	$[deg, deg/cty]$	$[deg, deg/cty]$
Mercury	0.38709843	0.20563661	7.00559432	252.25166724	77.45771895	48.33961819
	0.00000000	0.00002123	-0.00590158	149472.67486623	0.15940013	-0.12214182
Venus	0.72332102	0.00676399	3.39777545	181.97970850	131.76755713	76.67261496
	-0.00000026	-0.00005107	0.00043494	58517.81560260	0.05679648	-0.27274174
EM Bary	1.00000018	0.01673163	-0.00054346	100.46691572	102.93005885	-5.11260389
	-0.00000003	-0.00003661	-0.01337178	35999.37306329	0.31795260	-0.24123856
Mars	1.52371243	0.09336511	1.85181869	-4.56813164	-23.91744784	49.71320984
	0.00000097	0.00009149	-0.00724757	19140.29934243	0.45223625	-0.26852431
Jupiter	5.20248019	0.04853590	1.29861416	34.33479152	14.27495244	100.29282654
	-0.00002864	0.00018026	-0.00322699	3034.90371757	0.18199196	0.13024619
Saturn	9.54149883	0.05550825	2.49424102	50.07571329	92.86136063	113.63998702
	-0.00003065	-0.00032044	0.00451969	1222.11494724	0.54179478	-0.25015002
Uranus	19.18797948	0.04685740	0.77298127	314.20276625	172.43404441	73.96250215
	-0.00020455	-0.00001550	-0.00180155	428.49512595	0.09266985	0.05739699
Neptune	30.06952752	0.00895439	1.77005520	304.22289287	46.68158724	131.78635853
	0.00006447	0.00000818	0.00022400	218.46515314	0.01009938	-0.00606302
Pluto	39.48686035	0.24885238	17.14104260	238.96535011	224.09702598	110.30167986
	0.00449751	0.00006016	0.00000501	145.18042903	-0.00968827	-0.00809981

Table 8.10.4

Additional terms which must be added to the computation of M for Jupiter through Pluto, 3000 BC to 3000 AD, as described above.

	b	c	s	f
	$[^o/cty^2]$	$[^o]$	$[^o]$	$[^o/cty]$
Jupiter	-0.00012452	0.06064060	-0.35635438	38.35125000
Saturn	0.00025899	-0.13434469	0.87320147	38.35125000
Uranus	0.00058331	-0.97731848	0.17689245	7.67025000
Neptune	-0.00041348	0.68346318	-0.10162547	7.67025000
Pluto	-0.01262724			

8.11 The Availability of Ephemerides

The fundamental ephemerides used in the Astronomical Almanac since 1984, DE200 (1984-2002) and DE405(2003-), are both available on a CD-ROM from the publisher, Willmann-Bell. The package allows a professional user to obtain the rectangular coordinates of the Sun, Moon, and nine major planets by means of a simple subroutine written in standard fortran:

URL#3

Willmann-Bell, Inc.
PO Box 35025
Richmond, VA 23235
804-320-7016
804-272-5920 (Fax)

The ephemerides are also available via FTP from URL#4. There are also references to other packages and toolkits for the use of the ephemerides, as well as to the software being available in other programming languages. These are documented in the ”README” on URL#4.

The README on URL#4 also contains a few modifications to the software from the CD-ROM. However, it should be easier for the user to retrieve the software from the website, even when using the data files from the CD-ROM.

One may also use the interactive website, ”Horizons”, URL#5, which uses the JPL ephemerides, giving positional data (in 2003) for over 70 astronomical quantities for the 9 planets, more than 115 natural satellites and approaching 200,000 asteroids and comets; Horizons uses the full precision of the JPL ephemerides.

There are certainly other sources of ephemerides, though not all use DE200 or DE405, the fundamental ephemerides of the Astronomical Almanac. Further, there do exist a number of packages which provide less accurate positions for the solar system bodies.

A description of a number of options may be found at URL#6.

8.12 References

URL#1 http://ssd.jpl.nasa.gov/plan-eph-data/index.html

URL#2 http://maia.usno.navy.mil/conv2000.html

URL#3 http://www.willbell.com/software/jpl.htm

URL#4 http://ssd.jpl.nasa.gov/eph.info.html

URL#5 http://ssd.jpl.nasa.gov/

URL#6 http://ssd.jpl.nasa.gov/iau-comm4/access2ephs.html

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