In [18]:

```
import matplotlib.pyplot as plt
import numpy as np
```

In [19]:

```
V = 39
N = 7
```

In [20]:

```
X = np.array([V + 13, V + 16, V + 19, V + 23, V + 26, V + 30, V + 42])
Y = np.array([V + 3, V + 5, V + 4, V + 6, V + 6, V + 9, V + 8])
X
```

Out[20]:

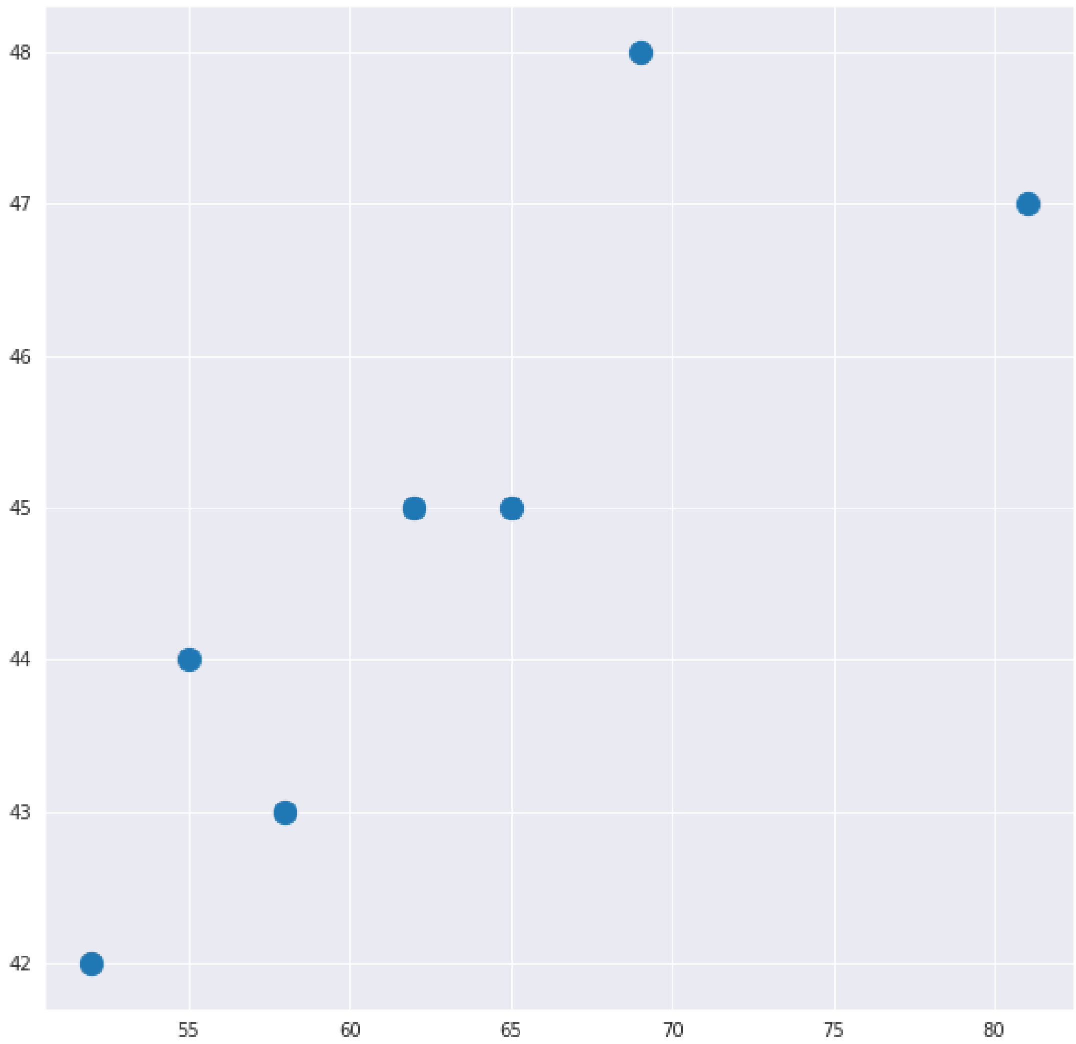
```
array([52, 55, 58, 62, 65, 69, 81])
```

In [21]:

```
plt.figure(figsize=(10, 10))
plt.scatter(X, Y, s=150)
```

Out[21]:

```
<matplotlib.collections.PathCollection at 0x7fae08be25e0>
```



Linear regression dependence

$$Y = X \cdot B$$

$$X = \begin{pmatrix} 1 & 52 \\ 1 & 55 \\ 1 & 58 \\ 1 & 62 \\ 1 & 65 \\ 1 & 69 \\ 1 & 81 \end{pmatrix}$$

$$B = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}$$

$$B = \left(X^T X\right)^{-1} X^T \cdot Y$$

In [22]:

```
X_ = np.stack((np.ones_like(X), X), axis=1)
X_
```

Out[22]:

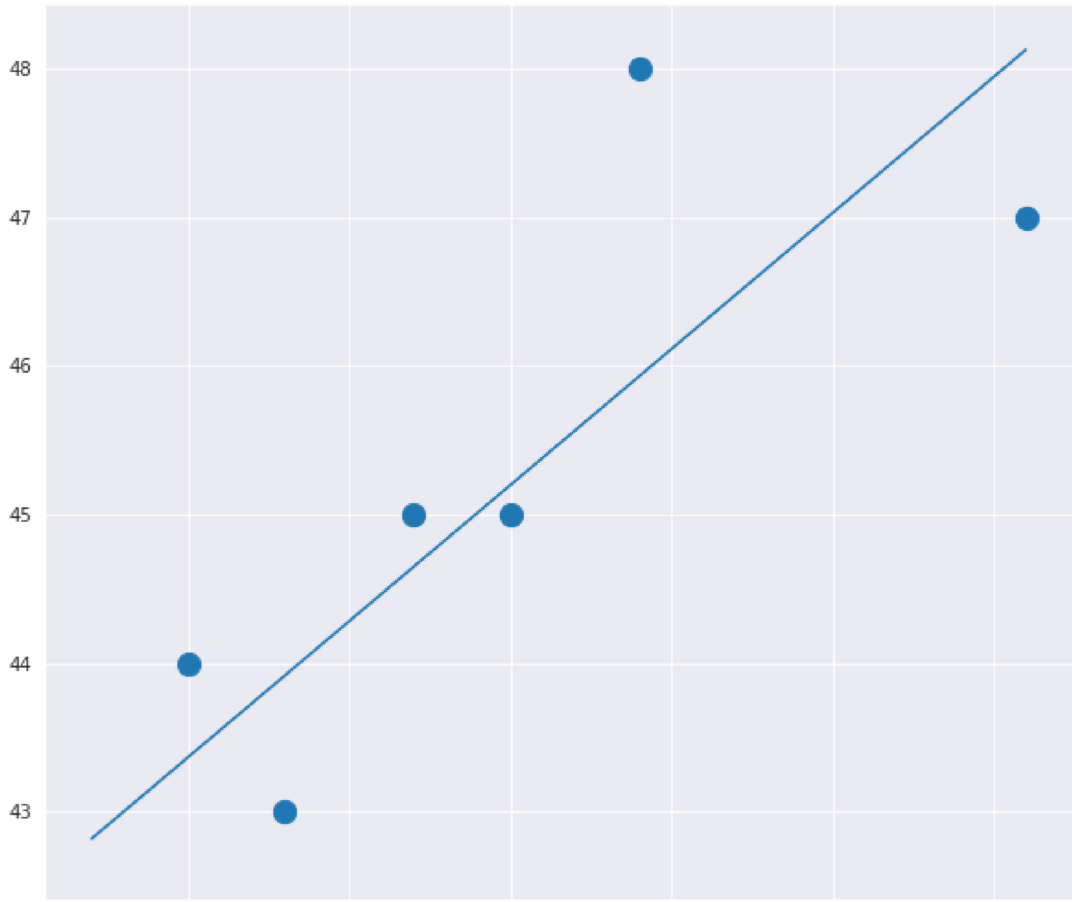
```
array([[ 1, 52],
       [ 1, 55],
       [ 1, 58],
       [ 1, 62],
       [ 1, 65],
       [ 1, 69],
       [ 1, 81]])
```

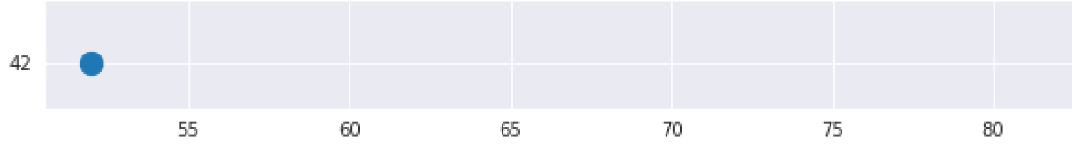
In [23]:

```
[b_0, b_1] = (np.linalg.inv(X_.T @ X_) @ X_.T) @ Y
plt.figure(figsize=(10, 10))
plt.scatter(X, Y, s=150)
xrange = np.linspace(min(X), max(X), 10)
plt.plot(xrange, xrange * b_1 + b_0)
```

Out[23]:

```
[<matplotlib.lines.Line2D at 0x7fae08b50ca0>]
```





Pearson Correlation

$$r_{xy} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n\sum x_i^2 - \left(\sum x_i\right)^2}\sqrt{n\sum y_i^2 - \left(\sum y_i\right)^2}}$$

In [24]:

```
nominator = (N * np.dot(X, Y) - np.sum(X) * np.sum(Y))
denominator = np.sqrt((N * np.sum(X ** 2) - (np.sum(X) ** 2))) * np.sqrt((N * np.sum(Y *
* 2) - np.sum(Y) ** 2))
cor = nominator / denominator
print(f"The Pearson correlation coefficient is {cor}")
```

```
The Pearson correlation coefficient is 0.8461939260999832
```

Determination coef

$$R^2 = \frac{RSS}{TSS}$$
$$RSS = \sum (y_i - \hat{y}_i)^2$$
$$TSS = \frac{1}{n} \sum (y_i - \bar{y})^2$$
$$\hat{y} = \frac{1}{n} \sum y_i$$

In [25]:

```
RSS = np.sum((Y - b_0 - b_1 * X) ** 2)
TSS = np.sum((Y - Y.mean()) ** 2)
s = np.sqrt(RSS / (N - 1 - 1))
R2 = 1 - RSS / TSS
R2
```

Out[25]:

```
0.716044160568504
```

R^2 is not low, so The model reveals a correlation, but does not correspond to the data

In [26]:

```
X_test = 10
y_preds = b_0 + b_1 * X_test
print(f"The predicted value for X = 10 is {y_preds}")
```

```
The predicted value for X = 10 is 35.13717693836953
```

Проверьте гипотезу о значимости параметра по критерию Стьдента

In [27]:

```
t_cr = 2.571
Q = np.array([np.linalg.inv((X_.T @ X_))[i][i] for i in range(2)])
S = s * Q
```

In [28]:

```
Q = np.sqrt(Q)
```

In [29]:

```
np.array((Q, S))
```

Out[29]:

```
array([[2.66055047e+00, 4.17080643e-02],
       [8.74205047e+00, 2.14837640e-03]])
```

In [30]:

```
np.abs([b_0,b_1] / S) > t_cr
```

Out[30]:

```
array([ True,  True])
```

Вывод:

По критерию Стьюдента оба параметра значимы

Проверьте гипотезу о значимости модели по критерию Фишера

In [31]:

```
F = R2 / (1 - R2) * (N - 1 - 1)
F_cr = 6.6079
```

In [32]:

```
F > F_cr
```

Out[32]:

```
True
```

Вывод:

По критерию Фишера модель значима