## Findr User Manual

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## 1 Tutorial

See [1].

## 2 Introduction

#### 2.1 Basic information

Findr (Fast Inference of Networks from Directed Regulations) is a C library for the inference of gene regulatory networks. Interfaces for binary and python2/3, as well as a standalone package for R are provided for major functions. Findr performs causal inferences of pairwise gene regulations based on

genotype and gene expression datasets or from gene expression dataset alone. Findr then bases on the causal inference results as prior edge information to construct a directed acyclic graph. A sparse Bayesian network can then be inferred with the R package lassopv to produce the p-value of every possible regulation [2]. Methodologies can be found in publication [3, 4]. The latest version of this documentation can be downloaded here.

#### 2.2 Highlights

Findr provides accurate yet fast inferences of regulatory networks. In pairwise inference, Findr addresses the pitfalls of traditional causal inference ([3]) and is nearly 1,000,000 times faster than existing causal inference software (CIT [5]). This unprecedented speed allows for whole-transcriptome causal network reconstruction, with a tutorial in [1]. Findr is also orders of magnitude faster than other network reconstruction methods, such as bnlearn, with the resulting Bayesian network having comparable or better performances in terms of predictive power, inference accuracy, and FDR control.

For pairwise inference, Findr follows the inference strategy of Trigger [6], and performs multiple likelihood ratio tests with false positive rate estimation. The major difference is that Findr addressed the statistical and computational drawbacks of traditional causal inference, whose performance tends to get worse as the number of samples increase due to elevated false negative rate [3]. Findr has been demonstrated to outperform all other methods on the DREAM 5 Systems Genetics Challenge datasets. Findr also outperformed other inference methods in speed and accuracy in miRNA target prediction, include Pearson and Spearman correlations, mutual information, Lasso and elastic-net regressions, GENIE3 [7], CIT [5], etc. In addition, Findr performs meaningful false positive rate estimation, without being overly conservative as in traditional causal inference.

For network reconstruction, Findr uses prior edge information, such as Findr's own output from pairwise inference, to build a maximal directed acyclic graph with efficient algorithm [8]. The maximal directed acyclic graph can then be reduced to a sparse Bayesian network with proper variable selection [2, 4].

The acceleration of Findr originated from the statistical advances in [3] which have enabled analytical computation of null distributions, as well as multithreading with OpenMP and C language implementation with GSL (GNU Scientific Library).

#### 2.3 License

Findr is licensed under GNU Affero General Public License, Version 3 (AGPL-3), which can be downloaded from <a href="https://www.gnu.org/licenses/agpl-3.0">https://www.gnu.org/licenses/agpl-3.0</a>. GSL is separated licensed by its authors under GPL.

#### 2.4 Contact

The author welcomes feedbacks in any form and perspective. No matter you have enquiries, suggestions, or critics, in installation, usage, future development, methodology, or collaboration, you can reach me at: Lingfei.Wang.github@outlook.com.

# 3 Findr library

Findr library is the core of computation for the software. The binary and python interfaces require Findr to be pre-installed, and call Findr during computation. The Findr R package contains a copy of Findr library by itself and does not need to install Findr library separately. Users of Findr R package can jump to Section 4.3.

#### 3.1 Installation

The Findr library and its interfaces can function without installation after some adjustments. However I recommend installation following the steps below. If you encounter any errors during installation that can't be resolved independently, please contact us with detailed error messages.

- **Prerequisites:** Findr is written in C and by nature supports Windows, Linux, and Mac OS X platforms. The user also needs the following to build Findr library:
  - A recent GCC compiler that supports OpenMP interface.
  - An up-to-date GNU make utility.
  - GNU Scientific Library (GSL). GSL must be installed or placed in a searchable location for include and link.
- **Download:** Findr can be downloaded with git from github with the command:

git clone https://github.com/lingfeiwang/findr.git

Alternatively it can also be downloaded from the webpage https://github.com/lingfeiwang/findr.

- Customize makefile (optional): You can customize how Findr is built and installed by editing its Makefile. This is useful for example when the user provides custom GSL location. Another possibility is the user does not have administrative priviledge on the computer, so a local installation is desired. In the latter case, the user must make sure the custom installation can be correctly located by binary and/or python interfaces, e.g. with environmental variables.
- **Build:** The source can be built with one line of command after downloading, in the directory it is downloaded or unzipped:

make

• Install: If you can gain superuser privilege, a normal install is recommended. This is also one line of command:

sudo make install

Otherwise, the user is suggested to change install location by changing the 'PREFIX'. Then use the following command to install without superuser privilege:

make install

The same PREFIX should then be set for binary interface. For details, see FAQ 3 in Section 7. I strongly advise against keeping multiple versions of Findr on the same machine, including installed and not installed versions, as this may confuse interfaces of which library to use.

- **Update:** To update Findr, simply download a newer version and install it to the same location. This will replace the older version.
- Uninstall: If you ever need to remove Findr from your machine permanently, this can be done with the command in the directory containing Findr source:

sudo make uninstall

The 'PREFIX' variable should be adjusted beforehand if you want to uninstall from a custom location. Preferrably, the source code should have the same version as the installed library. Sudo may be needed depending on the location of installation.

#### 3.2 General concepts

- Library initialization: The library must be initialized before called. During initialization, Findr initializes GSL, and processes the following initialization parameters:
  - logly: The logging level of Findr. For details, see Logging below.

- rs: Initial seed for random number generator. By default, uses the current time.
- nth: Maximum number of parallel threads for computation. For best performance, it should not exceed the number of CPU cores. Its default value is the number of cores automatically detected, which is not always accurate.

#### • Logging: Findr has 13 logging levels:

- Critical (0): The calculation has to halt due to critical error(s). Output is invalid.
- Error (1-3): Part of function is lost in calculation due to uncritical error(s). Part of output is invalid.
- Warning (4-6): Calculation is successful but Findr has encountered abnormal data. Findr attempted to correct anomalies but errors may have been introduced in output. User is advised to inspect their input.
- Info (7-9): Running information of Findr.
- Debug (10-12): More verbose running information.

During initialization, the user is requested to input the logging level (loglv), so that only message levels not greater than loglv would printed. Findr library does not have a default log level, but in binary, python interfaces and R package, the default log level is 6.

#### 3.3 Using Findr library

Findr library is well documented in its header files and source code. Developers are encouraged to build programs that call Findr, and to modify Findr freely for specific needs under current license.

## 4 Interfaces and packages

I provide binary and python interfaces for the major functions of Findr. Incompatible versions between interfaces and the library can create potential errors and limit reproducibility in data analysis. For this reason, users are advised to install/update interface(s) immediately after the library, and ensure they have the same version. Version differences are warned during interface execution. The R package of Findr includes the library of the same version, so the issue is absent.

The binary interface, python interface, and R package reveals the same set of functionality of Findr, under different naming and language conventions. User of a specific interface or package can proceed to installation and usage instructions of the respective section below, and then look for desired functions in Section 5.

#### 4.1 Binary interface

The binary interface follows the same build, install, and uninstall steps as Findr library itself, after downloading with the command:

git clone https://github.com/lingfeiwang/findr-bin.git

or from the webpage https://github.com/lingfeiwang/findr-bin.

After installation, type 'findr' would execute the binary. However, if Findr library is installed in a custom location, user should change the Makefile of binary interface correspondingly, such as the 'PREFIX' variable. If the binary interface is installed in a custom location, the user needs to add the install location to 'PATH' environmental variable.

The binary interface only provides minimal validity checks on input data, although errors in calculation are notified in logs (see Section 3.2). Binary interface should be invoked as:

findr loglv rs nth method method\_args...

The first three parameters are Findr library initialization parameters, as explained in Section 3.2 and

Section 5.1. For automatic configuration, input zeros for all of them. method stands for the name of data analysis routine the user want to call, and method\_args are its arguments which will be passed on to the method function. Available methods are listed in Section 5 in red.

Binary interface receives bulk data (vectors and row-major matrices) as input from files, and exports bulk output data to files. Both raw and tsv (tab separated values) input formats are accepted. By default, raw format is used where float type data are in 32-bit raw format in native endianness, and genotype data are in 8-bit unsigned integer format. To use tsv format for all input and output files, append '\_tsv' to the method name. Tsv format files should contain one row in each line, use tab (or space) as column separator, and not contain row or column names/indices.

#### 4.2 Python2/3 interface

The python(2/3) interface requires numpy, as it uses numpy.ndarray as input and output format. It also requires pre-installed Findr library. To install Findr's python interface, execute the following from command line (with proper privilege):

```
pip install findr
```

or download and install it from github with

```
git clone https://github.com/lingfeiwang/findr-python.git cd findr-python sudo python setup.py install
```

Alternatively, the python interface can be downloaded from <a href="https://github.com/lingfeiwang/findr-python">https://github.com/lingfeiwang/findr-python</a>.

To use Findr, first obtain an initialized library object with:

```
import findr
l=findr.lib(path=None,loglv=6,rs=0,nth=0)
```

path gives the exact location to search for Findr shared library in addition to default locations. Other three optional parameters are for library initialization and can be omitted for ordinary use. After above lines, Findr routines can be called as I's method. Exposed python functions of Findr are listed in Section 5, in which python method names are in green.

Bulk input and output data are formatted in numpy.ndarray for vectors and matrices, with 32-bit float for expression data and 8-bit unsigned integer for genotype data by default. Function returns are in dictionary type, in which each key contains an independent returned object. All functions share a returned key 'ret', which indicates a success in calculation with 0 and failure otherwise.

#### 4.3 R package

Findr's R package is a standalone package which includes Findr library and GSL. It is locally compiled by \*nix toolset, such as GCC and GNU make. On Mac OS and Linux, the R package can be downloaded and installed with

```
git clone https://github.com/lingfeiwang/findr-R.git cd findr-R R CMD INSTALL findr
```

Alternative, it can be downloaded at <a href="https://github.com/lingfeiwang/findr-R">https://github.com/lingfeiwang/findr-R</a>. To use Findr, load the library with:

```
library(findr)
```

An optional intialization step can follow:

```
| findr.lib(loglv=6,rs=0,nth=0) |
```

After above lines, Findr routines can then be called. Exposed R functions of Findr are listed in Section 5, in which R function names are in blue.

Bulk input and output data are formatted in R matrix or vector format, with double type for expression data and integer for genotype data. Function returns are in list type, in which each element contains an independent returned object.

## 5 Major functions

#### 5.1 Library initialization

• Library initialization: Initializes the library and provides log level, initial random seed, and maximum thread count.

(automatic through three initial parameters: loglv, rs, and nth, whose 0 indicates to use default values)

```
\begin{array}{l} l{=}findr.lib(path{=}None,loglv{=}6,rs{=}0,nth{=}0) \\ findr.lib(loglv{=}6,rs{=}0,nth{=}0) \end{array}
```

path	Extra location to search for Findr shared library in addition to default search paths.
	Only accepted in python interface.
loglv	Log level of library Findr. Only messages whose levels are not greater than the des-
	ignated log level will be printed. Level 0: critical, 1-3: errors, 4-6: warnings, 7-9:
	information, 10-12: debug. Default value is level 6.
rs	Initial random seed. Default value means to use current time.
nth	Maximum number of parrallel threads in calculation. For best performance, this should
	not exceed the number of CPU cores. Default value indicates using the number of cores
	automatically detected, which is not always accurate

#### 5.2 Inference of pairwise regulations

Findr provides multiple methods to infer pairwise regulations.

Depending on the statistic, Findr provides either the posterior probability (i.e. local precision FDR) of the alternative hypothesis (i.e. having pairwise regulation) based on log likelihood ratio in Section 5.2.1, or p-values of the log likelihood ratio under the null hypothesis in Section 5.2.2. By default, the posterior probability is computed, but p-values can be obtained using the function name suffix '\_pv'. P-values can be helpful when posterior probabilities are less effective at small secondary target sizes, or when downstream p-value based analysis is needed.

Findr infers regulations differently depending on the data availability. With only pairwise expression data alone, methods pij\_rank can be performed. With additional discrete causal anchor information (e.g. eQTLs), Findr provides pij\_gassist, pij\_gassist\_trad, and pijs\_gassist. Similarly for continuous causal anchors, the user can apply pij\_cassist, pij\_cassist\_trad, and pijs\_cassist. Every function computes the posterior probability of alternative hypothesis, but also has a version for p-values, as mentioned above.

Below, assume we want to infer pairwise regulation  $A \to B$ , with optional causal anchor E. E should strongly associate with A, so for every A we choose a different E, represented as E(A).

#### 5.2.1 Inference of pairwise regulation posterior probabilities

For best accuracy, it is advised to include as many secondary targets of the same type (e.g. gene expression levels) as possible during inference, and then pick the ones of interest from Findr's output. Avoiding preselection of the minimal set of secondary targets of interest could improve the accuracy of conversion from log likelihood ratios to probabilities.

The exposed functions for pairwise regulation probability inference are listed below:

• With only pairwise expression data, the inference of pairwise regulation is based on the correlation  $A \cdots B$ . Its posterior probability can be computed with [3]: pij\_rank ft ft2 nt nt2 ns fp nodiag memlimit

ans=l.pij\_rank(dt,dt2,nodiag=False,memlimit=0) ans=findr.pij\_rank(dt,dt2,nodiag=FALSE)

$\operatorname{ft}$	Input matrix of expression levels of $A$ , in file path, numpy.ndarray, or matrix
$\mathrm{dt}$	format. Element [i,j] is the expression level of gene i of sample j. The matrix has
$\mathrm{dt}$	dimension (nt,ns).
ft2	Input matrix of expression levels of B, in file path, numpy.ndarray, or matrix
dt2	format. Element [i,j] is the expression level of gene i of sample j. The matrix has
$\mathrm{dt}2$	dimension $(nt2,ns)$ .
nt	Number of genes for $A$ .
nt2	Number of genes for $B$ .
ns	Number of samples.
nodiag	When $A$ and $B$ are the same, log likelihood ratio between alternative and null
	hypotheses gives infinity. To avoid its contamination in the conversion from log
	likelihood ratios into probabilities, users need to arrange data accordingly, when
	$\{A\}$ and $\{B\}$ are the same or when $\{A\}$ is a subset of $\{B\}$ . The top submatrix of
	B's expression data must be identical with $A$ , and nodiag must be set to 1, True,
	or TRUE. Otherwise, in the default configuration, $\{A\}$ and $\{B\}$ should not have
	any intersection and nodiag $= 0$ , False, or FALSE.
memlimit	The approximate memory usage limit in bytes for the library. For datasets require
$\operatorname{memlimit}$	a larger memory, calculation will be split into smaller chunks. If the memory limit
	is smaller than minimum required, calculation can fail with an error message.
	Use the default value 0 to indicate unlimited memory usage. R version does not
	provide this limit and only uses unlimited memory, because R itself uses extra
	memory.
fp	Output matrix of inferred probability of correlation $A \cdots B$ versus no correla-
ans['p']	tion A B, in file path, numpy.ndarray, or matrix format. Element [i,j] is the
ans	probability of gene i being correlated with gene j. The matrix has dimension
	(nt,nt2).

• With discrete causal anchor data available, Findr performs 5 tests for causal inference  $A \to B$ , which are then combined to form a single posterior probability. The choice of combination is either the novel one [3] or the traditional one [6].

(Recommended:) For the novel combination in [3] of posterior probabilities of each test: pij\_gassist fg ft ft2 nt nt2 ns fp na nodiag memlimit ans=l.pij\_gassist(dg,dt,dt2,na=None,nodiag=False,memlimit=0) ans=findr.pij\_gassist(dg,dt,dt2,na=NULL,nodiag=FALSE)

For the traditional combination in [6] of posterior probabilities of each test<sup>1</sup>:

pij\_gassist\_trad fg ft ft2 nt nt2 ns fp na nodiag memlimit ans=l.pij\_gassist\_trad(dg,dt,dt2,na=None,nodiag=False,memlimit=0) ans=findr.pij\_gassist\_trad(dg,dt,dt2,na=NULL,nodiag=FALSE)

fg	Input matrix of best eQTL genotype data $E(A)$ , each row of which is the best
$d\mathbf{g}$	eQTL of the corresponding row of ft,dt. Data is in file path, numpy.ndarray, or
$d\mathbf{g}$	matrix format. Element [i,j] is the genotype value of the best eQTL of gene i
	of sample j, and should be among values $0, 1, \ldots, na$ . The matrix has dimension
	(nt,ns).

<sup>&</sup>lt;sup>1</sup> Note: this is not intended as a loyal reimplementation of Trigger. Instead, it simply performs the tests suggested in Trigger, but all preprocessing, postprocessing, and exact hypotheses can be different. A significant overlap of the top predictions of this method and Trigger has been observed in existing studies.

ft Input matrix of expression levels of A, in file path, numpy.ndarray dt format. Element [i,j] is the expression level of gene i of sample j. The dt dimension (nt,ns).  ft2 Input matrix of expression levels of B, in file path, numpy.ndarray dt2 format. Element [i,j] is the expression level of gene i of sample j. The dt2 dimension (nt2,ns).  nt Number of genes for A.  nt2 Number of genes for B.  Number of samples.  Number of alleles for the species considered. This constrains every general constraints and the sample is the sample of the species considered. This constrains every general constraints are samples.	y, or matrix
dt dimension (nt,ns).  ft2 Input matrix of expression levels of B, in file path, numpy.ndarray dt2 format. Element [i,j] is the expression level of gene i of sample j. The dt2 dimension (nt2,ns).  nt Number of genes for A.  nt2 Number of genes for B.  Number of samples.	y, or matrix
ft2 Input matrix of expression levels of $B$ , in file path, numpy.ndarray dt2 format. Element [i,j] is the expression level of gene i of sample j. The dimension (nt2,ns).  nt Number of genes for $A$ .  nt2 Number of genes for $B$ .  Number of samples.	*
$\begin{array}{lll} \text{dt2} & \text{format. Element [i,j] is the expression level of gene i of sample j. The} \\ \text{dt2} & \text{dimension (nt2,ns).} \\ \text{nt} & \text{Number of genes for } A. \\ \text{nt2} & \text{Number of genes for } B. \\ \text{ns} & \text{Number of samples.} \\ \end{array}$	*
$\begin{array}{cccc} \operatorname{dt2} & \operatorname{dimension} \ (\operatorname{nt2,ns}). \\ & \operatorname{nt} & \operatorname{Number} \ \operatorname{of} \ \operatorname{genes} \ \operatorname{for} \ A. \\ & \operatorname{nt2} & \operatorname{Number} \ \operatorname{of} \ \operatorname{genes} \ \operatorname{for} \ B. \\ & \operatorname{ns} & \operatorname{Number} \ \operatorname{of} \ \operatorname{samples}. \end{array}$	e matrix has
ns Number of samples.	
*	
na Number of alleles for the species considered. This constrains every ge	
	notype data
to be among $0, 1, \ldots, na$ . If unspecified $(0, None, or NULL)$ , na is an	utomatically
determined as the maximum value of fg, dg, or dg.	
nodiag When $A$ and $B$ are the same, log likelihood ratio between alternation	ive and null
hypotheses can give infinity. To avoid its contamination in the convers	sion from log
likelihood ratios into probabilities, users need to arrange data accord	dingly, when
$\{A\}$ and $\{B\}$ are the same or when $\{A\}$ is a subset of $\{B\}$ . The top s	submatrix of
B's expression data must be identical with $A$ , and nodiag must be set	t to 1, True,
or TRUE. Otherwise, in the default configuration, $\{A\}$ and $\{B\}$ show	uld not have
any intersection and nodiag $= 0$ , False, or FALSE.	
memlimit The approximate memory usage limit in bytes for the library. For data	asets require
memlimit a larger memory, calculation will be split into smaller chunks. If the m	nemory limit
is smaller than minimum required, calculation can fail with an err	or message.
Use the default value 0 to indicate unlimited memory usage. R versi	ion does not
provide this limit and only uses unlimited memory, because R itsel	f uses extra
memory.	
fp Output matrix of inferred probability after combination of the five	tests, in file
ans['p'] path, numpy.ndarray, or matrix format. Element [i,j] is the probabil	lity of alter-
ans native hypothesis for $A = \text{gene i}$ and $B = \text{gene j}$ . The matrix has	s dimension
(nt,nt2).	

• With discrete causal anchor data available, Findr can also output all 5 test results for causal inference  $A \to B$ , to allow for arbitray combination by the user: pijs\_gassist fg ft ft2 nt nt2 ns fp1 fp2 fp3 fp4 fp5 na nodiag memlimit ans=l.pijs\_gassist(dg,dt,dt2,na=None,nodiag=False,memlimit=0)

 $ans=findr.pijs\_gassist(dg,dt,dt2,na=NULL,nodiag=FALSE)$ 

fg	Input matrix of best eQTL genotype data $E(A)$ , each row of which is the best
$d\mathbf{g}$	eQTL of the corresponding row of ft,dt. Data is in file path, numpy.ndarray, or
$d\mathbf{g}$	matrix format. Element [i,j] is the genotype value of the best eQTL of gene i
	of sample j, and should be among values $0, 1, \ldots, na$ . The matrix has dimension
	(nt,ns).
ft	Input matrix of expression levels of $A$ , in file path, numpy.ndarray, or matrix
dt	format. Element [i,j] is the expression level of gene i of sample j. The matrix has
dt	dimension (nt,ns).
ft2	Input matrix of expression levels of B, in file path, numpy.ndarray, or matrix
dt2	format. Element [i,j] is the expression level of gene i of sample j. The matrix has
$\mathrm{dt}2$	dimension $(nt2,ns)$ .
nt	Number of genes for $A$ .
nt2	Number of genes for $B$ .
ns	Number of samples.

na	Number of alleles for the species considered. This constrains every genotype data to be among $0, 1, \ldots, na$ . If unspecified $(0, \text{None}, \text{ or NULL})$ , na is automatically determined as the maximum value of $fg$ , $dg$ , or $dg$ .
nodiag	When $A$ and $B$ are the same, log likelihood ratio between alternative and null hypotheses can give infinity. To avoid its contamination in the conversion from log likelihood ratios into probabilities, users need to arrange data accordingly, when $\{A\}$ and $\{B\}$ are the same or when $\{A\}$ is a subset of $\{B\}$ . The top submatrix of $B$ 's expression data must be identical with $A$ , and nodiag must be set to $A$ , True, or TRUE. Otherwise, in the default configuration, $A$ and $A$ should not have any intersection and nodiag $A$ should not have
memlimit memlimit	The approximate memory usage limit in bytes for the library. For datasets require a larger memory, calculation will be split into smaller chunks. If the memory limit is smaller than minimum required, calculation can fail with an error message. Use the default value 0 to indicate unlimited memory usage. R version does not provide this limit and only uses unlimited memory, because R itself uses extra memory.
fp1 ans['p1'] ans\$p1	Output vector of inferred probability of test 1, $E(A) \to A$ (alternative) versus $E(A) = A$ (null), in file path, numpy.ndarray, or array format. Element [i] is the probability of best eQTL of gene i regulates gene i. The vector has dimension (nt). For nodiag=0, False, FALSE, because the function expects significant eQTLs, p1 always return 1. For nodiag=1, True, TRUE, uses diagonal elements of p2. Consider replacing p1 with your own (1-FDR) from eQTL discovery.
fp2 ans['p2'] ans\$p2	Output matrix of inferred probability of test 2, $E(A) \to A \cdots B$ with $E(A) \to B$ (alternative) versus $E(A) \to A \leftarrow B$ (null), in file path, numpy ndarray, or matrix format. Element [i,j] is the probability of alternative hypothesis for A = gene i and B = gene j. The matrix has dimension (nt,nt2).
fp3 ans['p3'] ans\$p3	Output matrix of inferred probability of test 3, $E(A) \to A \to B$ (null) versus $E(A) \to A \cdots B$ with $E(A) \to B$ (alternative), in file path, numpy.ndarray, or matrix format. Element [i,j] is the probability of null hypothesis for A = gene i and B = gene j. The matrix has dimension (nt,nt2).
fp4 ans['p4'] ans\$p4	Output matrix of inferred probability of test 4, $B \leftarrow E(A) \rightarrow A$ with $A \cdots B$ (alternative) versus $E(A) \rightarrow A$ $B$ (null), in file path, numpy.ndarray, or matrix format. Element [i,j] is the probability of alternative hypothesis for $A = \text{gene i}$ and $B = \text{gene j}$ . The matrix has dimension (nt,nt2).
fp5 ans['p5'] ans\$p5	Output matrix of inferred probability of test 5, $B \leftarrow E(A) \rightarrow A$ with $A \cdots B$ (alternative) versus $B \leftarrow E(A) \rightarrow A$ (null), in file path, numpy.ndarray, or matrix format. Element [i,j] is the probability of alternative hypothesis for $A = \text{gene i}$ and $B = \text{gene j}$ . The matrix has dimension (nt,nt2).

With continuous causal anchor data available, Findr performs the same 5 tests for causal inference A → B, which are adapted for continuous anchors. The 5 tests are then combined to form a single posterior probability. The choice of combination is either the novel one [3] or the traditional one [6]. The method names and parameters are similar with the discrete causal anchor case. The differences are the input of causal anchor data and the removal of the number of alleles parameter.

```
(Recommended:) For the novel combination in [3] of posterior probabilities of each test: pij_cassist fc ft ft2 nt nt2 ns fp nodiag memlimit ans=l.pij_cassist(dc,dt,dt2,nodiag=False,memlimit=0) ans=findr.pij_cassist(dc,dt,dt2,nodiag=FALSE)
```

For the traditional combination in [6] of posterior probabilities of each test: pij\_cassist\_trad fc ft ft2 nt nt2 ns fp nodiag memlimit ans=l.pij\_cassist\_trad(dc,dt,dt2,nodiag=False,memlimit=0)

## $ans=findr.pij\_cassist\_trad(dc,dt,dt2,nodiag=FALSE)$

fc	Input matrix of continuous anchor data $E(A)$ , each row of which is the anchor
dc	of the corresponding row of ft,dt. Data is in file path, numpy.ndarray, or matrix
m dc	format. Element [i,j] is the continuous value of anchor of gene i of sample j. The
ac	matrix has dimension (nt,ns).
ft	Input matrix of expression levels of $A$ , in file path, numpy.ndarray, or matrix
$\mathrm{dt}$	format. Element [i,j] is the expression level of gene i of sample j. The matrix has
${ m dt}$	dimension (nt,ns).
ft2	Input matrix of expression levels of B, in file path, numpy.ndarray, or matrix
dt2	format. Element [i,j] is the expression level of gene i of sample j. The matrix has
<u>dt2</u>	dimension (nt2,ns).
nt	Number of genes for $A$ .
nt2	Number of genes for $B$ .
ns	Number of samples.
nodiag	When $A$ and $B$ are the same, log likelihood ratio between alternative and null
	hypotheses can give infinity. To avoid its contamination in the conversion from log
	likelihood ratios into probabilities, users need to arrange data accordingly, when
	$\{A\}$ and $\{B\}$ are the same or when $\{A\}$ is a subset of $\{B\}$ . The top submatrix of
	B's expression data must be identical with $A$ , and nodiag must be set to 1, True,
	or TRUE. Otherwise, in the default configuration, $\{A\}$ and $\{B\}$ should not have
	any intersection and nodiag $= 0$ , False, or FALSE.
memlimit	The approximate memory usage limit in bytes for the library. For datasets require
$\operatorname{memlimit}$	a larger memory, calculation will be split into smaller chunks. If the memory limit
	is smaller than minimum required, calculation can fail with an error message.
	Use the default value 0 to indicate unlimited memory usage. R version does not
	provide this limit and only uses unlimited memory, because R itself uses extra
	memory.
fp	Output matrix of inferred probability after combination of the five tests, in file
ans['p']	path, numpy.ndarray, or matrix format. Element [i,j] is the probability of alter-
ans	native hypothesis for $A = gene i$ and $B = gene j$ . The matrix has dimension
	(nt,nt2).

• With continuous causal anchor data available, Findr can also output all 5 test results for causal inference  $A \to B$ , to allow for arbitray combination by the user: pijs\_cassist fc ft ft2 nt nt2 ns fp1 fp2 fp3 fp4 fp5 nodiag memlimit ans=l.pijs\_cassist(dc,dt,dt2,nodiag=False,memlimit=0) ans=findr.pijs\_cassist(dc,dt,dt2,nodiag=FALSE)

fc	Input matrix of continuous causal anchor data $E(A)$ , each row of which is the
dc	anchor of the corresponding row of ft,dt. Data is in file path, numpy.ndarray, or
dc	matrix format. Element [i,j] is the continuous anchor of gene i of sample j. The
	matrix has dimension (nt,ns).
ft	Input matrix of expression levels of A, in file path, numpy.ndarray, or matrix
$\mathrm{dt}$	format. Element [i,j] is the expression level of gene i of sample j. The matrix has
$\mathrm{dt}$	dimension (nt,ns).
ft2	Input matrix of expression levels of B, in file path, numpy.ndarray, or matrix
dt2	format. Element [i,j] is the expression level of gene i of sample j. The matrix has
dt2	dimension (nt2,ns).
nt	Number of genes for $A$ .
nt2	Number of genes for $B$ .
ns	Number of samples.

nodiag	When $A$ and $B$ are the same, log likelihood ratio between alternative and null
	hypotheses can give infinity. To avoid its contamination in the conversion from log
	likelihood ratios into probabilities, users need to arrange data accordingly, when
	$\{A\}$ and $\{B\}$ are the same or when $\{A\}$ is a subset of $\{B\}$ . The top submatrix of
	B's expression data must be identical with $A$ , and nodiag must be set to 1, True,
	or TRUE. Otherwise, in the default configuration, $\{A\}$ and $\{B\}$ should not have
	any intersection and nodiag $= 0$ , False, or FALSE.
memlimit	The approximate memory usage limit in bytes for the library. For datasets require
memlimit	a larger memory, calculation will be split into smaller chunks. If the memory limit
	is smaller than minimum required, calculation can fail with an error message.
	Use the default value 0 to indicate unlimited memory usage. R version does not
	provide this limit and only uses unlimited memory, because R itself uses extra
	memory.
fp1	Output vector of inferred probability of test 1, $E(A) \rightarrow A$ (alternative) versus
ans['p1']	E(A) A (null), in file path, numpy ndarray, or array format. Element [i] is
ans\$p1	the probability of continuous anchor of gene i regulates gene i. The vector has
	dimension (nt). For nodiag=0, False, FALSE, because the function expects sig-
	nificant continuous anchors, p1 always return 1. For nodiag=1, True, TRUE, uses
	diagonal elements of p2.
fp2	Output matrix of inferred probability of test 2, $E(A) \to A \cdots B$ with $E(A) \to B$
$\operatorname{ans}[\mathrm{`p2'}]$	(alternative) versus $E(A) \to A \leftarrow B$ (null), in file path, numpy.ndarray, or matrix
ans p2	format. Element [i,j] is the probability of alternative hypothesis for A = gene i
	and $B = \text{gene j}$ . The matrix has dimension (nt,nt2).
fp3	Output matrix of inferred probability of test 3, $E(A) \rightarrow A \rightarrow B$ (null) versus
ans['p3']	$E(A) \to A \cdots B$ with $E(A) \to B$ (alternative), in file path, numpy.ndarray, or
ans p3	matrix format. Element [i,j] is the probability of null hypothesis for A = gene i
	and $B = \text{gene j}$ . The matrix has dimension (nt,nt2).
fp4	Output matrix of inferred probability of test 4, $B \leftarrow E(A) \rightarrow A$ with $A \cdots B$
ans['p4']	(alternative) versus $E(A) \to A$ $B$ (null), in file path, numpy.ndarray, or matrix
ans\$p4	format. Element [i,j] is the probability of alternative hypothesis for A = gene i
	and $B = \text{gene j}$ . The matrix has dimension (nt,nt2).
fp5	Output matrix of inferred probability of test 5, $B \leftarrow E(A) \rightarrow A$ with $A \cdots B$
ans['p5']	(alternative) versus $B \leftarrow E(A) \rightarrow A$ (null), in file path, numpy.ndarray, or matrix
ans p5	format. Element [i,j] is the probability of alternative hypothesis for A = gene i
	and $B = \text{gene j}$ . The matrix has dimension (nt,nt2).

## 5.2.2 Inference of pairwise regulation p-values

Findr also computes p-values alone. The exposed functions for pairwise regulation p-value inference are listed below:

• With only pairwise expression data, the inference of pairwise regulation is based on the correlation  $A \cdots B$ . Its p-value can be computed with [3]: pij\_rank\_pv ft ft2 nt nt2 ns fp memlimit ans=l.pij\_rank\_pv(dt,dt2,memlimit=0) ans=findr.pij\_rank\_pv(dt,dt2)

ft	Input matrix of expression levels of A, in file path, numpy.ndarray, or matrix
$\mathrm{dt}$	format. Element [i,j] is the expression level of gene i of sample j. The matrix has
$\mathrm{dt}$	dimension (nt,ns).
ft2	Input matrix of expression levels of B, in file path, numpy.ndarray, or matrix
dt2	format. Element [i,j] is the expression level of gene i of sample j. The matrix has
dt2	dimension (nt2,ns).

$\operatorname{nt}$	Number of genes for $A$ .
nt2	Number of genes for $B$ .
ns	Number of samples.
memlimit	The approximate memory usage limit in bytes for the library. For datasets require
memlimit	a larger memory, calculation will be split into smaller chunks. If the memory limit
	is smaller than minimum required, calculation can fail with an error message.
	Use the default value 0 to indicate unlimited memory usage. R version does not
	provide this limit and only uses unlimited memory, because R itself uses extra
	memory.
fp	Output matrix of inferred p-values of no correlation A B, in file path,
ans['p']	numpy.ndarray, or matrix format. Element [i,j] is the p-value of gene i being
ans	not correlated with gene j. The matrix has dimension (nt,nt2).

With discrete causal anchor data available, Findr performs 5 tests for causal inference A → B.
 The 5 p-values then allow arbitray combination by the user:
 pijs\_gassist\_pv fg ft ft2 nt nt2 ns fp1 fp2 fp3 fp4 fp5 na memlimit
 ans=l.pijs\_gassist\_pv(dg,dt,dt2,na=None,memlimit=0)
 ans=findr.pijs\_gassist\_pv(dg,dt,dt2,na=NULL)

<ul> <li>eQTL of the corresponding row of ft,dt. Data is in file path, numpy.ndarray matrix format. Element [i,j] is the genotype value of the best eQTL of ger of sample j, and should be among values 0, 1,,na. The matrix has dimens (nt,ns).</li> <li>ft Input matrix of expression levels of A, in file path, numpy.ndarray, or madt format. Element [i,j] is the expression level of gene i of sample j. The matrix dimension (nt,ns).</li> <li>ft2 Input matrix of expression levels of B, in file path, numpy.ndarray, or madt2 format. Element [i,j] is the expression level of gene i of sample j. The matrix dimension (nt2,ns).</li> <li>nt Number of genes for A.</li> <li>nt2 Number of genes for B.</li> <li>ns Number of samples.</li> <li>na Number of alleles for the species considered. This constrains every genotype of to be among 0, 1,,na. If unspecified (0, None, or NULL), na is automatic determined as the maximum value of fg, dg, or dg.</li> <li>memlimit The approximate memory usage limit in bytes for the library. For datasets required a larger memory, calculation will be split into smaller chunks. If the memory list is smaller than minimum required, calculation can fail with an error messare Use the default value 0 to indicate unlimited memory usage. R version does provide this limit and only uses unlimited memory, because R itself uses expression to the provide tive), in file path, numpy.ndarray, or array format. Element [i] is the p-value ans['p1'] output vector of p-values of test 1, E(A) A (null) versus E(A) → A (alternass['p2'] output matrix of p-values of test 2, E(A) → A ← B (null) versus E(A) → A ans['p2'] with E(A) → B (alternative), in file path, numpy.ndarray, or matrix formatic file formatic formatic formatic formatic formatic file formatic file formatic formatic for</li></ul>	fg	Input matrix of best eQTL genotype data $E(A)$ , each row of which is the best
of sample j, and should be among values $0,1,\dots,na$ . The matrix has dimens (nt,ns).  ft Input matrix of expression levels of $A$ , in file path, numpy.ndarray, or madt format. Element [i,j] is the expression level of gene i of sample j. The matrix dimension (nt,ns).  ft2 Input matrix of expression levels of $B$ , in file path, numpy.ndarray, or madt2 format. Element [i,j] is the expression level of gene i of sample j. The matrix dimension (nt2,ns).  nt Number of genes for $A$ .  nt2 Number of genes for $B$ .  ns Number of samples.  na Number of alleles for the species considered. This constrains every genotype of to be among $0,1,\dots,na$ . If unspecified $(0, \text{None}, \text{ or NULL})$ , na is automatic determined as the maximum value of fg, dg, or dg.  memlimit memlimit a larger memory, calculation will be split into smaller chunks. If the memory list is smaller than minimum required, calculation can fail with an error messe. Use the default value 0 to indicate unlimited memory usage. R version does provide this limit and only uses unlimited memory, because R itself uses expression that the proposition of the path, numpy.ndarray, or array format. Element [i] is the p-value ans ['p1'] Output vector of p-values of test $1, E(A) = A$ (null) versus $E(A) \rightarrow A$ (alternative), in file path, numpy.ndarray, or matrix formats $E(A) \rightarrow A$ (alternative) output matrix of p-values of test $E(A) \rightarrow A$ (null) versus $E(A) \rightarrow A$ with $E(A) \rightarrow B$ (alternative), in file path, numpy.ndarray, or matrix formats.		eQTL of the corresponding row of ft,dt. Data is in file path, numpy.ndarray, or
<ul> <li>(nt,ns).</li> <li>ft Input matrix of expression levels of A, in file path, numpy.ndarray, or made to format. Element [i,j] is the expression level of gene i of sample j. The matrix dimension (nt,ns).</li> <li>ft2 Input matrix of expression levels of B, in file path, numpy.ndarray, or made to format. Element [i,j] is the expression level of gene i of sample j. The matrix dimension (nt2,ns).</li> <li>nt Number of genes for A.</li> <li>nt2 Number of genes for B.</li> <li>ns Number of samples.</li> <li>na Number of alleles for the species considered. This constrains every genotype of to be among 0,1,,na. If unspecified (0, None, or NULL), na is automatic determined as the maximum value of fg, dg, or dg.</li> <li>memlimit memlimit a larger memory, calculation will be split into smaller chunks. If the memory is smaller than minimum required, calculation can fail with an error messe. Use the default value 0 to indicate unlimited memory usage. R version does provide this limit and only uses unlimited memory, because R itself uses expression to the path, numpy.ndarray, or array format. Element [i] is the p-value ans ['p1'] output vector of p-values of test 1, E(A) A (null) versus E(A) → A (alternative), in file path, numpy.ndarray, or matrix formats ['p2'] with E(A) → B (alternative), in file path, numpy.ndarray, or matrix formats ['p2']</li> </ul>	dg	matrix format. Element [i,j] is the genotype value of the best eQTL of gene i
ft Input matrix of expression levels of $A$ , in file path, numpy.ndarray, or madt format. Element $[i,j]$ is the expression level of gene i of sample j. The matrix dimension $(nt,ns)$ .  ft2 Input matrix of expression levels of $B$ , in file path, numpy.ndarray, or madt2 format. Element $[i,j]$ is the expression level of gene i of sample j. The matrix dimension $(nt2,ns)$ .  nt Number of genes for $A$ .  nt2 Number of genes for $B$ .  ns Number of samples.  na Number of alleles for the species considered. This constrains every genotype of to be among $0,1,\ldots,na$ . If unspecified $(0, \text{None}, \text{ or NULL})$ , na is automatic determined as the maximum value of $fg$ , dg, or dg.  memlimit The approximate memory usage limit in bytes for the library. For datasets require a larger memory, calculation will be split into smaller chunks. If the memory list is smaller than minimum required, calculation can fail with an error messare use the default value 0 to indicate unlimited memory usage. R version does provide this limit and only uses unlimited memory, because R itself uses expression to the properties of the position of the properties of the		of sample j, and should be among values $0, 1, \ldots, na$ . The matrix has dimension
format. Element [i,j] is the expression level of gene i of sample j. The matrix dimension (nt,ns).  ft2		(nt,ns).
dt dimension (nt,ns).  ft2 Input matrix of expression levels of B, in file path, numpy.ndarray, or match dt2 format. Element [i,j] is the expression level of gene i of sample j. The matrix dt2 dimension (nt2,ns).  nt Number of genes for A.  nt2 Number of samples.  na Number of samples.  na Number of alleles for the species considered. This constrains every genotype of the beamong 0, 1,, na. If unspecified (0, None, or NULL), na is automatice determined as the maximum value of fg, dg, or dg.  memlimit memlimit a larger memory, calculation will be split into smaller chunks. If the memory list is smaller than minimum required, calculation can fail with an error messare Use the default value 0 to indicate unlimited memory usage. R version does provide this limit and only uses unlimited memory, because R itself uses expressionally in file path, numpy.ndarray, or array format. Element [i] is the p-value ans ['p1'] output matrix of p-values of test 1, E(A) → A (null) versus E(A) → A (alternative), in file path, numpy.ndarray, or matrix formatrix formatrix of p-values of test 2, E(A) → A ← B (null) versus E(A) → A··  ans ['p2'] with E(A) → B (alternative), in file path, numpy.ndarray, or matrix formatrix forma	ft	Input matrix of expression levels of A, in file path, numpy.ndarray, or matrix
ft2 Input matrix of expression levels of $B$ , in file path, numpy.ndarray, or matrix dt2 format. Element $[i,j]$ is the expression level of gene i of sample j. The matrix dt2 dimension (nt2,ns).  nt Number of genes for $A$ .  nt2 Number of genes for $B$ .  ns Number of alleles for the species considered. This constrains every genotype of to be among $0,1,\ldots,na$ . If unspecified $(0, \text{None}, \text{ or NULL})$ , na is automatic determined as the maximum value of fg, dg, or dg.  memlimit memlimit The approximate memory usage limit in bytes for the library. For datasets required a larger memory, calculation will be split into smaller chunks. If the memory list is smaller than minimum required, calculation can fail with an error messed Use the default value 0 to indicate unlimited memory usage. R version does provide this limit and only uses unlimited memory, because R itself uses expression to the default value of the first tive, in file path, numpy.ndarray, or array format. Element [i] is the p-value ans $[p1]$ Output vector of p-values of test $[p1]$ The vector has dimension $[p2]$ Output matrix of p-values of test $[p1]$ The path, numpy.ndarray, or matrix formatrix $[p2]$ Output matrix of p-values of test $[p1]$ The approximate $[p2]$ The matrix of p-values of test $[p2]$ The matrix $[p2]$ Output matrix of p-values of test $[p3]$ The path, numpy.ndarray, or matrix formatrix $[p2]$ The matrix $[p3]$ Th	$\mathrm{dt}$	format. Element [i,j] is the expression level of gene i of sample j. The matrix has
dt2 format. Element [i,j] is the expression level of gene i of sample j. The matrix dt2 dimension (nt2,ns).  nt Number of genes for $A$ .  nt2 Number of genes for $B$ .  ns Number of alleles for the species considered. This constrains every genotype of to be among $0, 1, \ldots, na$ . If unspecified $(0, \text{None}, \text{ or NULL})$ , na is automatic determined as the maximum value of fg, dg, or dg.  memlimit memlimit a larger memory, calculation will be split into smaller chunks. If the memory li is smaller than minimum required, calculation can fail with an error messare use the default value 0 to indicate unlimited memory usage. R version does provide this limit and only uses unlimited memory, because R itself uses expression to the library.  fp1 Output vector of p-values of test 1, $E(A) = A$ (null) versus $E(A) \to A$ (alternative), in file path, numpy.ndarray, or array format. Element [i] is the p-value ans['p1'] ans['p2'] with $E(A) \to B$ (alternative), in file path, numpy.ndarray, or matrix formatrix formatrix of p-values of test 2, $E(A) \to A \leftarrow B$ (null) versus $E(A) \to A \leftarrow B$ and $E(A) \to A \leftarrow B$ (null) versus $E(A) \to A \leftarrow B$ and $E(A) \to A \leftarrow B$ and $E(A) \to B$ (alternative), in file path, numpy.ndarray, or matrix formatrix format	$\mathrm{dt}$	dimension (nt,ns).
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nt Number of genes for $A$ .  nt2 Number of genes for $B$ .  ns Number of samples.  na Number of alleles for the species considered. This constrains every genotype of to be among $0, 1, \ldots, na$ . If unspecified $(0, \text{None}, \text{ or NULL})$ , na is automatic determined as the maximum value of $fg$ , $dg$ , or $dg$ .  memlimit memlimit a larger memory usage limit in bytes for the library. For datasets require a larger memory, calculation will be split into smaller chunks. If the memory list is smaller than minimum required, calculation can fail with an error messed. Use the default value 0 to indicate unlimited memory usage. R version does provide this limit and only uses unlimited memory, because R itself uses expressed between two provides and the provided ans $[p1]$ output vector of p-values of test $1, E(A) = A$ (null) versus $E(A) \to A$ (alternative), in file path, numpy.ndarray, or array format. Element [i] is the p-value ans $[p2]$ output matrix of p-values of test $2, E(A) \to A \leftarrow B$ (null) versus $E(A) \to A \hookrightarrow A \hookrightarrow B$ and $E(A) \to B$ (alternative), in file path, numpy.ndarray, or matrix form	$\mathrm{dt}2$	format. Element [i,j] is the expression level of gene i of sample j. The matrix has
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Number of samples.  Number of alleles for the species considered. This constrains every genotype of to be among $0, 1, \ldots, na$ . If unspecified $(0, \text{None}, \text{ or NULL})$ , na is automatic determined as the maximum value of fg, dg, or dg.  memlimit  The approximate memory usage limit in bytes for the library. For datasets required a larger memory, calculation will be split into smaller chunks. If the memory list is smaller than minimum required, calculation can fail with an error messed Use the default value 0 to indicate unlimited memory usage. R version does provide this limit and only uses unlimited memory, because R itself uses expressed tive), in file path, numpy.ndarray, or array format. Element [i] is the p-value ans ['p1'] ans best eQTL of gene i does not regulate gene i. The vector has dimension (nt).  fp2  Output matrix of p-values of test $(a, b) \in (a, b)$ and $(a, b) \in (a, b)$ of $(a, b) $	nt	Number of genes for $A$ .
Number of alleles for the species considered. This constrains every genotype of to be among $0, 1, \ldots, na$ . If unspecified $(0, \text{None}, \text{ or NULL})$ , na is automatic determined as the maximum value of fg, dg, or dg.  The approximate memory usage limit in bytes for the library. For datasets required a larger memory, calculation will be split into smaller chunks. If the memory limits is smaller than minimum required, calculation can fail with an error message use the default value 0 to indicate unlimited memory usage. R version does provide this limit and only uses unlimited memory, because R itself uses expended tive), in file path, numpy.ndarray, or array format. Element [i] is the p-value ans['p1'] anssp1 best eQTL of gene i does not regulate gene i. The vector has dimension (nt). Output matrix of p-values of test $2, E(A) \rightarrow A \leftarrow B$ (null) versus $E(A) \rightarrow A \cdots$ with $E(A) \rightarrow B$ (alternative), in file path, numpy.ndarray, or matrix form	nt2	Number of genes for $B$ .
to be among $0, 1, \ldots, na$ . If unspecified $(0, \text{None}, \text{ or NULL})$ , na is automatic determined as the maximum value of fg, dg, or dg.  The approximate memory usage limit in bytes for the library. For datasets required a larger memory, calculation will be split into smaller chunks. If the memory list is smaller than minimum required, calculation can fail with an error messare Use the default value 0 to indicate unlimited memory usage. R version does provide this limit and only uses unlimited memory, because R itself uses expensely memory.  The approximate memory usage limit in bytes for the library. For datasets required a larger memory limit in the smaller chunks. If the memory limit is smaller than minimum required, calculation can fail with an error messare usage. R version does provide this limit and only uses unlimited memory, because R itself uses expensely in the smaller than minimum required, calculation can fail with an error messare usage. R version does provide this limit and only uses unlimited memory usage. R version does provide this limit and only uses unlimited memory and the smaller chunks. If the memory limit is a larger memory limit is a larger memory usage limit in bytes for the library. For datasets required to a larger memory limit is a	ns	Number of samples.
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memlimit memlimit The approximate memory usage limit in bytes for the library. For datasets required a larger memory, calculation will be split into smaller chunks. If the memory limits is smaller than minimum required, calculation can fail with an error messare Use the default value 0 to indicate unlimited memory usage. R version does provide this limit and only uses unlimited memory, because R itself uses expending the provided this limit and only uses unlimited memory.  [p1] Output vector of p-values of test 1, $E(A) = A$ (null) versus $E(A) \to A$ (alternative), in file path, numpy.ndarray, or array format. Element [i] is the p-value best eQTL of gene i does not regulate gene i. The vector has dimension (nt).  [p2] Output matrix of p-values of test 2, $E(A) \to A \leftarrow B$ (null) versus $E(A) \to A \cdots$ ans ['p2'] with $E(A) \to B$ (alternative), in file path, numpy.ndarray, or matrix form		to be among $0, 1, \ldots, na$ . If unspecified (0, None, or NULL), na is automatically
memlimit a larger memory, calculation will be split into smaller chunks. If the memory li is smaller than minimum required, calculation can fail with an error messar Use the default value 0 to indicate unlimited memory usage. R version does provide this limit and only uses unlimited memory, because R itself uses expensively.  fp1 Output vector of p-values of test 1, $E(A)$ $A$ (null) versus $E(A) \rightarrow A$ (alternative), in file path, numpy.ndarray, or array format. Element [i] is the p-value best eQTL of gene i does not regulate gene i. The vector has dimension (nt).  fp2 Output matrix of p-values of test 2, $E(A) \rightarrow A \leftarrow B$ (null) versus $E(A) \rightarrow A \cdots$ with $E(A) \rightarrow B$ (alternative), in file path, numpy.ndarray, or matrix form		<u> </u>
is smaller than minimum required, calculation can fail with an error messare Use the default value 0 to indicate unlimited memory usage. R version does provide this limit and only uses unlimited memory, because R itself uses expression memory.  [p1] Output vector of p-values of test 1, $E(A) = A$ (null) versus $E(A) \to A$ (alternative), in file path, numpy.ndarray, or array format. Element [i] is the p-value best eQTL of gene i does not regulate gene i. The vector has dimension (nt).  [p2] Output matrix of p-values of test 2, $E(A) \to A \leftarrow B$ (null) versus $E(A) \to A \cdots$ ans ['p2'] with $E(A) \to B$ (alternative), in file path, numpy.ndarray, or matrix form	$\operatorname{memlimit}$	The approximate memory usage limit in bytes for the library. For datasets require
Use the default value 0 to indicate unlimited memory usage. R version does provide this limit and only uses unlimited memory, because R itself uses expending memory.  fp1 Output vector of p-values of test 1, $E(A) = A$ (null) versus $E(A) \to A$ (alternative), in file path, numpy.ndarray, or array format. Element [i] is the p-value best eQTL of gene i does not regulate gene i. The vector has dimension (nt).  fp2 Output matrix of p-values of test 2, $E(A) \to A \leftarrow B$ (null) versus $E(A) \to A \cdots$ with $E(A) \to B$ (alternative), in file path, numpy.ndarray, or matrix form	$\operatorname{memlimit}$	a larger memory, calculation will be split into smaller chunks. If the memory limit
provide this limit and only uses unlimited memory, because R itself uses expensions.  fp1 Output vector of p-values of test 1, $E(A)$ $A$ (null) versus $E(A) \rightarrow A$ (alternative), in file path, numpy.ndarray, or array format. Element [i] is the p-value ans\$p1 best eQTL of gene i does not regulate gene i. The vector has dimension (nt).  fp2 Output matrix of p-values of test 2, $E(A) \rightarrow A \leftarrow B$ (null) versus $E(A) \rightarrow A \cdots$ ans['p2'] with $E(A) \rightarrow B$ (alternative), in file path, numpy.ndarray, or matrix form		is smaller than minimum required, calculation can fail with an error message.
fp1 Output vector of p-values of test 1, $E(A)$ $A$ (null) versus $E(A) \rightarrow A$ (alter ans['p1'] tive), in file path, numpy.ndarray, or array format. Element [i] is the p-value ans\$p1 best eQTL of gene i does not regulate gene i. The vector has dimension (nt). Output matrix of p-values of test 2, $E(A) \rightarrow A \leftarrow B$ (null) versus $E(A) \rightarrow A \cdots$ ans['p2'] with $E(A) \rightarrow B$ (alternative), in file path, numpy.ndarray, or matrix form		
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ans\$p1 best eQTL of gene i does not regulate gene i. The vector has dimension (nt).  fp2 Output matrix of p-values of test 2, $E(A) \to A \leftarrow B$ (null) versus $E(A) \to A \cdots$ ans['p2'] with $E(A) \to B$ (alternative), in file path, numpy.ndarray, or matrix form	-	
fp2 Output matrix of p-values of test 2, $E(A) \to A \leftarrow B$ (null) versus $E(A) \to A \cdots$ ans['p2'] with $E(A) \to B$ (alternative), in file path, numpy.ndarray, or matrix form		
ans['p2'] with $E(A) \rightarrow B$ (alternative), in file path, numpy.ndarray, or matrix form		
and $\mathbb{P}_{\mathbf{p}}$ Element if it is the problem of null hypothesis for $\Lambda = \text{cons}$ and $\mathbf{p} = \text{cons}$ if		
	ans p2	Element [i,j] is the p-value of null hypothesis for A = gene i and B = gene j. The
matrix has dimension (nt,nt2).		matrix has dimension (nt,nt2).

fp3	Output matrix of p-values of test 3, $E(A) \to A \to B$ (null) versus $E(A) \to A \cdots B$
ans['p3']	with $E(A) \to B$ (alternative), in file path, numpy.ndarray, or matrix format.
ans p3	Element $[i,j]$ is the p-value of null hypothesis for $A = \text{gene } i$ and $B = \text{gene } j$ . The
	matrix has dimension (nt,nt2).
fp4	Output matrix of p-values of test 4, $B \leftarrow E(A) \rightarrow A$ with $A \cdots B$ (alternative)
ans['p4']	versus $E(A) \to A$ B (null), in file path, numpy.ndarray, or matrix format.
ans p4	Element $[i,j]$ is the p-value of null hypothesis for $A = \text{gene } i$ and $B = \text{gene } j$ . The
	matrix has dimension (nt,nt2).
fp5	Output matrix of p-values of test 5, $B \leftarrow E(A) \rightarrow A$ with $A \cdots B$ (alternative)
ans['p5']	versus $B \leftarrow E(A) \rightarrow A$ (null), in file path, numpy.ndarray, or matrix format.
ans p5	Element $[i,j]$ is the p-value of null hypothesis for $A = \text{gene } i$ and $B = \text{gene } j$ . The
	matrix has dimension (nt,nt2).

• With continuous causal anchor data available, Findr performs the same 5 tests for causal inference  $A \to B$ , which are adapted for continuous anchors. The method names and parameters are similar with the discrete causal anchor case. The differences are the input of causal anchor data and the removal of the number of alleles parameter.

```
pijs_cassist_pv fc ft ft2 nt nt2 ns fp1 fp2 fp3 fp4 fp5 memlimit ans=l.pijs_cassist_pv(dc,dt,dt2,memlimit=0) ans=findr.pijs_cassist_pv(dc,dt,dt2)
```

fc Input matrix of continuous causal anchor data E(A), each row of dc anchor of the corresponding row of ft,dt. Data is in file path, numpy matrix format. Element [i,j] is the continuous anchor of gene i of samatrix has dimension (nt,ns).  ft Input matrix of expression levels of A, in file path, numpy.ndarray format. Element [i,j] is the expression level of gene i of sample j. The dimension (nt,ns).  ft2 Input matrix of expression levels of B, in file path, numpy.ndarray format. Element [i,j] is the expression level of gene i of sample j. The dt2 dimension (nt2,ns).  nt Number of genes for A.  nt2 Number of genes for B.  ns Number of samples.  memlimit The approximate memory usage limit in bytes for the library. For dat a larger memory, calculation will be split into smaller chunks. If the n is smaller than minimum required, calculation can fail with an end use the default value 0 to indicate unlimited memory usage. R vers provide this limit and only uses unlimited memory, because R itse memory.  fp1 Output vector of p-values of test 1, E(A) A (null) versus E(A) — ans['p1'] tive), in file path, numpy.ndarray, or array format. Element [i] is the continuous anchor of gene i does not regulate gene i. The vector has (nt).  fp2 Output matrix of p-values of test 2, E(A) → A ← B (null) versus E(A ans['p2'] with E(A) → B (alternative), in file path, numpy.ndarray, or means part is the p-value of null hypothesis for A = gene i and B = Element [i,j] is the p-value of null hypothesis for A = gene i and B = Element [i,j] is the p-value of null hypothesis for A = gene i and B = Element [i,j] is the p-value of null hypothesis for A = gene i and B = Element [i,j] is the p-value of null hypothesis for A = gene i and B = Element [i,j] is the p-value of null hypothesis for A = gene i and B = Element [i,j] is the p-value of null hypothesis for A = gene i and B = Element [i,j] is the p-value of null hypothesis for A = gene i and B = Element [i,j] is the p-value of null hypothesis for A = gene i and B = Element [i,j] is the p-valu	
dc matrix format. Element [i,j] is the continuous anchor of gene i of samatrix has dimension (nt,ns).  ft Input matrix of expression levels of A, in file path, numpy.ndarray dt format. Element [i,j] is the expression level of gene i of sample j. The dt dimension (nt,ns).  ft2 Input matrix of expression levels of B, in file path, numpy.ndarray dt2 format. Element [i,j] is the expression level of gene i of sample j. The dt2 dimension (nt2,ns).  nt Number of genes for A.  nt2 Number of genes for B.  ns Number of samples.  memlimit The approximate memory usage limit in bytes for the library. For dat memlimit a larger memory, calculation will be split into smaller chunks. If the n is smaller than minimum required, calculation can fail with an enterouse the default value 0 to indicate unlimited memory usage. R verse provide this limit and only uses unlimited memory, because R itse memory.  fp1 Output vector of p-values of test 1, E(A) A (null) versus E(A) — ans['p1'] tive), in file path, numpy.ndarray, or array format. Element [i] is the continuous anchor of gene i does not regulate gene i. The vector has (nt).  fp2 Output matrix of p-values of test 2, E(A) → A ← B (null) versus E(A ans['p2'] with E(A) → B (alternative), in file path, numpy.ndarray, or material path, nump	which is the
matrix has dimension (nt,ns).  ft Input matrix of expression levels of $A$ , in file path, numpy.ndarra dt format. Element [i,j] is the expression level of gene i of sample j. The dimension (nt,ns).  ft2 Input matrix of expression levels of $B$ , in file path, numpy.ndarra dt2 format. Element [i,j] is the expression level of gene i of sample j. The dt2 dimension (nt2,ns).  nt Number of genes for $A$ .  nt2 Number of genes for $B$ .  ns Number of samples.  memlimit The approximate memory usage limit in bytes for the library. For dat memlimit a larger memory, calculation will be split into smaller chunks. If the n is smaller than minimum required, calculation can fail with an entry usage in the default value 0 to indicate unlimited memory usage. R verse provide this limit and only uses unlimited memory, because R itse memory.  fp1 Output vector of p-values of test 1, $E(A) = A$ (null) versus $E(A) = A$ ans['p1'] tive), in file path, numpy.ndarray, or array format. Element [i] is the continuous anchor of gene i does not regulate gene i. The vector has (nt).  fp2 Output matrix of p-values of test 2, $E(A) \to A \leftarrow B$ (null) versus $E(A) \to A$ ans['p2'] with $E(A) \to B$ (alternative), in file path, numpy.ndarray, or matrix of p-values of test 2, $E(A) \to A \leftarrow B$ (null) versus $E(A) \to A$ and $E(A) \to A$ and $E(A) \to A$ and $E(A) \to B$ (alternative), in file path, numpy.ndarray, or matrix of p-values of test 2, $E(A) \to A \leftarrow B$ (null) versus $E(A) \to A$ and $E(A) \to A$ and $E(A) \to B$ (alternative), in file path, numpy.ndarray, or matrix of p-values of test 2, $E(A) \to A \leftarrow B$ (null) versus $E(A) \to A$ and $E(A)$	ndarray, or
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dt dimension (nt,ns).  ft2 Input matrix of expression levels of $B$ , in file path, numpy.ndarragedt2 format. Element [i,j] is the expression level of gene i of sample j. The dt2 dimension (nt2,ns).  nt Number of genes for $A$ .  nt2 Number of genes for $B$ .  ns Number of samples.  memlimit The approximate memory usage limit in bytes for the library. For date a larger memory, calculation will be split into smaller chunks. If the number is smaller than minimum required, calculation can fail with an end use the default value 0 to indicate unlimited memory usage. Revers provide this limit and only uses unlimited memory, because Revers provide this limit and only uses unlimited memory.  fp1 Output vector of p-values of test 1, $E(A) = A$ (null) versus $E(A) = A$ ans['p1'] tive), in file path, numpy.ndarray, or array format. Element [i] is the continuous anchor of gene i does not regulate gene i. The vector has (nt).  fp2 Output matrix of p-values of test 2, $E(A) \rightarrow A \leftarrow B$ (null) versus $E(A) \rightarrow A$ ans['p2'] with $E(A) \rightarrow B$ (alternative), in file path, numpy.ndarray, or matrix of p-values of test 2, $E(A) \rightarrow A \leftarrow B$ (null) versus $E(A) \rightarrow A$ ans['p2'] with $E(A) \rightarrow B$ (alternative), in file path, numpy.ndarray, or matrix of p-values of test 2, $E(A) \rightarrow A \leftarrow B$ (null) versus $E(A) \rightarrow A$ and $E(A) \rightarrow B$ (alternative), in file path, numpy.ndarray, or matrix of p-values of test 2, $E(A) \rightarrow A \leftarrow B$ (null) versus $E(A) \rightarrow A$ and $E(A) \rightarrow B$ (alternative), in file path, numpy.ndarray, or matrix of p-values of test 2, $E(A) \rightarrow A \leftarrow B$ (null) versus $E(A) \rightarrow$	2 0
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(nt). <b>fp2</b> Output matrix of p-values of test 2, $E(A) \rightarrow A \leftarrow B$ (null) versus $E(A) \rightarrow B$ (alternative), in file path, numpy.ndarray, or matrix	-
fp2 Output matrix of p-values of test 2, $E(A) \to A \leftarrow B$ (null) versus $E(A) \to B$ (alternative), in file path, numpy.ndarray, or matrix of p-values of test 2, $E(A) \to A \leftarrow B$ (null) versus $E(A) \to B$ (alternative), in file path, numpy.ndarray, or matrix of p-values of test 2, $E(A) \to A \leftarrow B$ (null) versus $E(A) \to B$ (null	
ans['p2'] with $E(A) \rightarrow B$ (alternative), in file path, numpy.ndarray, or ma	$\overline{A) \rightarrow A \cdots B}$
	•
ans $p2$ Element $[i,j]$ is the p-value of null hypothesis for $A = gene i$ and $B = gene i$	
matrix has dimension (nt,nt2).	
fp3 Output matrix of p-values of test 3, $E(A) \to A \to B$ (null) versus $E(A) \to A \to B$	$\overline{A) \rightarrow A \cdots B}$
ans ['p3'] with $E(A) \to B$ (alternative), in file path, numpy.ndarray, or magnetic states and the states of the path in the path is a state of the path in the path is a state of the path in the path is a state of the path in the path is a state of the path in the path is a state of the path in the path is a state of the path in the path is a state of the path in the path is a state of the path in the path is a state of the path in the path is a state of the path in the path is a state of the path in the path is a state of the path in the path is a state of the path in the path is a state of the path in the path is a state of the path in the path is a state of the path in the path in the path is a state of the path in the path in the path in the path is a state of the path in	
ans\$p3 Element [i,j] is the p-value of null hypothesis for A = gene i and B =	
matrix has dimension (nt,nt2).	

fp4	Output matrix of p-values of test 4, $E(A) \to A$ $B$ (null) versus $B \leftarrow E(A) \to A$
ans['p4']	with $A \cdots B$ (alternative), in file path, numpy.ndarray, or matrix format. Element
ans p4	[i,j] is the p-value of null hypothesis for $A = gene  i$ and $B = gene  j$ . The matrix
	has dimension (nt,nt2).
fp5	Output matrix of p-values of test 5, $B \leftarrow E(A) \rightarrow A$ (null) versus $B \leftarrow E(A) \rightarrow A$
ans['p5']	with $A \cdots B$ (alternative), in file path, numpy.ndarray, or matrix format. Element
ans p5	[i,j] is the p-value of null hypothesis for $A = gene  i$ and $B = gene  j$ . The matrix
	has dimension (nt,nt2).

#### 5.3 Reconstruction of directed acyclic graphs

This section provides efficient reconstruction of directed acyclic graphs from prior information of edge significance. As an example, the edge significance may come from the output of the inference of pairwise regulations, e.g. pij\_gassist or pij\_rank.

The exposed functions for reconstruction of directed acyclic graphs are listed below:

• Reconstruction of a single directed acyclic graph from the given edge significance prior by adding the most significant edge one at a time, whilst avoiding cycles. Methodology can be found in [8, 4].

```
netr_one_greedy fprior nt fnet namax nimax nomax
ans=l.netr.one_greedy(prior,namax=None,nimax=None,nomax=None)
ans=findr.netr_one_greedy(prior,namax=NULL,nimax=NULL,nomax=NULL)
```

fprior	Input matrix of significance levels of all edges, in file path, numpy.ndarray, or
prior	matrix format. Element [i,j] is the significance level of edge i to j. The matrix has
prior	dimension (nt,nt).
$\overline{\mathrm{nt}}$	Number of nodes to reconstruct network for.
namax	Maximum total number of edges in the reconstructed network. Defaults (0 for
	binary interface) to unlimited, i.e. nt*(nt-1)/2.
nimax	Maximum number of incoming edges for each node in the reconstructed network.
	Defaults (0 for binary interface) to unlimited.
nomax	Maximum number of outgoing edges for each node in the reconstructed network.
	Defaults (0 for binary interface) to unlimited.
fnet	Output boolean matrix of reconstructed network, in file path (8-bit integer for raw
ans[`net']	format), numpy.ndarray(dtype=bool), or matrix format. Element [i,j]=1, True,
ans	TRUE if an edge exists from node i to j on the reconstructed network, and 0,
	False, FALSE if not. The matrix has dimension (nt,nt).

## 6 Examples

Part of GEUVADIS Consortium dataset is provided within the binary and python interfaces, as well as the R package. Usage of examples is provided in every distribution as below:

- Binary interface: see file EXAMPLES.
- Python interface: see module findr.examples.
- R package: see documentation of every function or Findr package. A tutorial is provided for R in [1].

## 7 Frequently asked questions

1. I have GCC on my Mac but when compiling Findr still asks me to download GCC.

On Mac, Apple installs its own C compiler which tries to pretend to be GCC, although many functionalities of GCC are lacked in Apple's compiler. Findr needs some of these functionalities (such as OpenMP). You can download the source code of GCC from <a href="https://gcc.gnu.org">https://gcc.gnu.org</a>. Because of the complications in building GCC, some Mac users prefer to download unofficial binary copies of GCC from third parties, such as Homebrew. The unofficial binary GCC may be installed under a name other than 'gcc'. Under such circumstances, consult the question 'How do I change C compiler name?'

#### 2. How do I change C compiler name?

For Findr library or binary interface, open Makefile with any text editor. Find the lines 'CC=gcc' and 'LD=gcc', and change gcc into your C compiler name. After that, run 'make distclean' before jumping back at the Build phase of installation in Section 3.1. For Findr R package, go to src/lib and perform the same operation. For python interface, no C code is included so no change is needed.

3. Can I use Findr if I don't have admin rights?

Yes. Suppose you would like to install Findr to /path/to/install. You can install both Findr library and binary as the following:

```
make PREFIX=/path/to/install make PREFIX=/path/to/install install
```

After that, you should also include /path/to/install in the include, library, binary, and pkg-config paths of the system. This can be done by adding the following lines in the .bashrc file in your home folder (or accordingly if you use other shells):

```
export PATH=/path/to/install/bin:$PATH
export LD_LIBRARY_PATH=/path/to/install/lib:$LD_LIBRARY_PATH
export LIBRARY_PATH=/path/to/install/lib:$LIBRARY_PATH
export CPATH=/path/to/install/include:$CPATH
export PKG_CONFIG_PATH=/path/to/install/pkgconfig:$PKG_CONFIG_PATH
```

For python, you can add the "--user" option to pip for user mode installation.

#### 4. Does Findr support Windows?

Findr can run on Windows in a number of ways but it is no longer tested or supported. The computation code itself should be platform independent but installation of GSL can be complicated on Windows. In previous testing versions, Findr compiled and ran smoothly on \*nix port platforms on Windows, such as Cygwin or MSYS2.

Ever since Windows released support for Ubuntu Bash recently, I have decided to drop Windows native support. Findr and all its interfaces should integrate better with Bash on Windows. (https://msdn.microsoft.com/commandline/wsl)

#### References

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