

1. All horses are the same color; we can prove this by induction on the number of horses in a given set. Here's how: "If there's just one horse then it's the same color as itself, so the basis is trivial. For the induction step, assume that there are n horses numbered 1 to n . By the induction hypothesis, horses 1 through $n - 1$ are the same color, and similarly horses 2 through n are the same color. But the middle horses, 2 through $n - 1$, can't change color when they're in different groups; these are horses, not chameleons. So horses 1 and n must be the same color as well, by transitivity. Thus all n horses are the same color; QED." What, if anything, is wrong with this reasoning?

所有的马是同一个颜色的。我们可以使用数学归纳法来证明。如果只有一匹马，那么自然颜色相同。下面是归纳部分，马匹 1 到马匹 $n-1$ 的颜色是相同的。类似的马匹 2 到马匹 n 的颜色也是相同的。注意到马匹 2 到马匹 $n-1$ 的颜色，在两个组合里是一个颜色。那么根据传递性， N 个马的颜色都是一样的。

请问上述证明过程有什么问题？

A: 当 $N=2$ 的时候， $2..n-1$ 是空集。上述的传递性不存在。因此无法成立。