

Scotch 7.0 Maintainer's Guide

(version 7.0.7)

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Abstract

This document describes some internals of the LIBSCOTCH library.

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1 Introduction

This document is a starting point for the persons interested in using SCOTCH as a testbed for their new partitioning methods, and/or willing to contribute to it by making these methods available to the rest of the scientific community.

Much information is missing. If you need specific information, please send an e-mail, so that relevant additional information can be added to this document.

2 Coding style

The Scotch coding style is now well established. Hence, potential contributors are requested to abide by it, to provide a global ease of reading while browsing the code, and to ease the work of their followers.

In this section, the numbering of the characters of each line is assumed to start from zero.

2.1 Typing

2.1.1 Spacing

Expressions are like sentences, where words are separated by spaces. Hence, an expression like "if (n == NULL) {" reads: "if n is-equal-to NULL then", with words separated by single spaces.

As in standard typesetting, there is no space after an opening parenthesis, nor before a closing one, because they are not words.

When it follows a keyword, an opening brace is always on the same line as the keyword (save for special cases, e.g. preprocessing macros between the keyword and the opening brace). This is meant to maximize the number of "useful readable lines" on the screen. However, closing braces are on a separate line, aligned with the indent of the line that contains the matching opening brace. This is meant to find in a glance the line that contains this opening brace.

Brackets are not considered as words: they are stuck both to the word on their left and the word on their right.

Reference and dereference operators "&" and "*" are stuck to the word on their right. However, the multiplication operator "*" counts as a word in arithmetic expressions.

Semicolons are always stuck to the word on their left, except when they follow an empty instruction, e.g., an empty loop body or an empty for field. Empty instructions are materialized by a single space character, which makes the semicolon separated from the preceding word. For instance: "for (;;);".

Ternary operator elements "?" and ":" are considered as words and are surrounded by spaces. When the ternary construct spans across multiple lines, they

are placed at the beginning of each line, before the expression they condition, and not at the end of the previous line.

2.1.2 Aligning

When several consecutive lines contain similar expressions that are strongly connected, e.g. arguments of a memAllocGroup() routine, or assignments of multiple fields of the same structure(s), extra spaces can be added to align parts of the expressions. This is a matter of style and opportunity.

For instance, when consecutive lines contain function calls where opening parentheses are close to each other and their arguments overlap, open parentheses have to be aligned. However, when arguments do not overlap, alignment is not required (e.g., for return statements with small parameters).

2.1.3 Idiomatic specificities

While, in C, return is a keyword which does not need parentheses around its argument, the SCOTCH coding style treats it as if it were a function call, thus requiring parentheses around its argument when it has one.

2.2 Indenting

Indenting is subject to the following rules:

- All indents are of two characters. Hence, starting from column zero, all lines start at even column numbers.
- Tabs are never used in the source code. If your text editor replaces chunks of spaces by tabs, it is your duty to disable this feature or to make sure to replace all tabs by spaces before the files are committed. Unwanted tabs are shown in red when performing a "git diff" prior to committing.

Condition bodies are always indented on the line below the condition statement. "if" statements are always placed at the beginning of a new line, except when used as an "else if" construct, in which the two keywords appear on the same line, separated by a single space.

Loop bodies are always indented on the line below the loop statement, except when the loop body is an empty instruction. In this case, the terminating semicolon is placed on the same line as the loop statement, after a single space.

2.3 Comments

All comments are C-style, that is, of the form "/*...*/". C++-style comments should never be used.

There are three categories of comments: file comments, function/data structure comments, and line comments. Commenting is subject to the following rules:

- File comments are standard header blocks that must be copied as is. Hence, there is little to say about them. On top of each file should be placed a license header, which depends on the origin of the file.
- Block comments start with "/*" and end with "*/" on a separate subsequent line. Intermediate lines start with "**". All these comment markers are placed at column zero. Comment text is separated from the comment markers

by a single space character. Text in block comments is made of titles or of full sentences, that are terminated with a punctuation sign (most often a final dot).

• Line comments are of two types: structure definition line comments in header files, and code line comments.

Structure definition line comments in header files start with "/*+" and end with "+*/". This is an old Doxygen syntax, which has been preserved over time. Code line comments start classically with "/*" and end with "*/".

All these comments start at least at character 50. If the C code line is longer, comment lines start one character after the end of the line, after a single space. End comment markers are placed at least one character after the end of the comment text. When several line comments are present on consecutive lines, comment terminators are aligned to the farthest comment terminator.

Comment text always starts with an uppercase letter, and have no terminating punctuation sign. They are written in the imperative mode, and a positive form (no question asked).

Line comments for C pre-processing conditional macros (e.g. "#else" or "#endif") are not subject to indentation rules. They start one character after the keyword, and are not subject to end marker alignment, except when consecutive lines bear the same keyword (i.e., a "#endif" statement).

3 Naming conventions

Data types, variables, structure fields and function names follow strict naming conventions. These conventions strongly facilitate the understanding of the meaning of the expressions, and prevent from coding mistakes. For instance, "verttax[edgenum]" would clearly be an invalid expression, as a vertex array cannot be indexed by an edge number. Hence, potential contributors are required to follow them strictly.

3.1 File inclusion markers

File inclusion markers are #define's which indicate that a given source file (either a ".c" source code file or a ".h" header file) has been already encountered.

To minimize risks of collisions with symbols of external libraries, file inclusion markers start with a prefix that represents the name of the project, followed by the name of the file in question (without its type suffix). While filenames can be long, this is not an issue since the length of the significant part of C preprocessor symbols is at least 63 characters¹, thus longer than that of C identifiers, which is 32 characters. Header file marker identifiers are suffixed with "_H", while C source file markers have no suffix.

In order to further minimize risks of collisions, file inclusion markers should be placed in a file only when needed, that is, when effectively used as the parameter of a conditional inclusion statement within another source file.

The current project prefixes are:

• SCOTCH_: the SCOTCH project itself;

¹See e.g. https://gcc.gnu.org/onlinedocs/cpp/Implementation-limits.html

• ESMUMPS: the ESMUMPS library, which is treated as a separate project to avoid conflicts with data structures and files that exist in both libraries, such as Graph's.

3.2 Variables and fields

Variables and fields of the sequential SCOTCH software are commonly built from a radical and a suffix. When contextualization is required, e.g., the same kind of variable appear in two different objects, a prefix is added. In PT-SCOTCH, a second radical is commonly used, to inform on variable locality or duplication across processes.

Common radicals are:

- vert: vertex.
- velo: vertex load.
- vnoh: non-halo vertex, as used in the Hgraph structure.
- vnum: vertex number, used as an index to access another vertex structure. This radical typically relates to an array that contains the vertex indices, in some original graph, corresponding to the vertices of a derived graph (e.g., an induced graph).
- vlbl: user-defined vertex label (at the user API level).
- edge: edge (i.e., arcs, in fact).
- edlo: edge (arc) load.
- enoh: non-halo edge (i.e., arcs, in fact).
- arch: target architecture.
- graf: graph.
- mesh: mesh.

Common suffices are:

- bas: start "based" value for a number range; see the "nnd" suffix below. For number basing and array indexing, see Section 3.4.
- end: vertex end index of an edge (e.g., vertend, wrt. vertnum). The end suffix is a sub-category of the num suffix.
- nbr: number of instances of objects of a given radical type (e.g., vertnbr, edgenbr). They are commonly used within "un-based" loop constructs, such as: "for (vertnum = 0; vertnum < vertnbr; vertnum ++) ...".
- nnd: end based value for a number range, commonly used for loop boundaries. Usually, *nnd = *nbr+baseval. For instance, vertnnd = vertnbr+baseval. They are commonly used in based loop constructs, such as: "for (vertnum = baseval; vertnum < vertnnd; vertnum ++) ...". For local vertex ranges, e.g., within a thread that manages only a partial vertex range, the loop construct would be: "for (vertnum = vertbas; vertnum < vertnnd; vertnum ++) ...".

- num: based or un-based number (index) of some instance of an object of a given radical type. For instance, vertnum is the index of some (graph) vertex, that can be used to access adjacency (verttab) or vertex load (velotab) arrays. $0 \le \text{vertnum} < \text{vertnbr}$ if the vertex index is un-based, and baseval $\le \text{vertnum} < \text{vertnnd}$ if the index is based, that it, counted starting from baseval.
- ptr: pointer to an instance of an item of some radical type (e.g., grafptr).
- sum: sum of several values of the same radical type (e.g., velosum, edlosum).
- tab: reference to the first memory element of an array. Such a reference is returned by a memory allocation routine (e.g., memAlloc) or allocated from the stack.
- tax (for "table access"): reference to an array that will be accessed using based indices. See Section 3.4.
- tnd: pointer to the based after-end of an array of items of radix type (e.g. velotnd). Variables of this suffix are mostly used as bounds in loops.
- val: value of an item. For instance, baseval is the indexing base value, and veloval is the load of some vertex, that may have been read from a file.

Common prefixes are:

- src: source, wrt. active. For instance, a source graph is a plain Graph structure that contains only graph topology, compared to enriched graph data structures that are used for specific computations such as bipartitioning.
- act: active, wrt. source. An active graph is a data structure enriched with information required for specific computations, e.g. a Bgraph, a Kgraph or a Vgraph compared to a Graph.
- ind: induced, wrt. original.
- src: source, wrt. active or target.
- org: original, wrt. induced. An original graph is a graph from which a derived graph will be computed, e.g. an induced subgraph.
- tqt: target.
- coar: coarse, wrt. fine (e.g. coarvertnum, as a variable that holds the number of a coarse vertex, within some coarsening algorithm).
- fine: fine, wrt. coarse.
- mult: multinode, for coarsening.

3.3 Functions

Like variables, routines of the SCOTCH software package follow a strict naming scheme, in an object-oriented fashion. Routines are always prefixed by the name of the data structure on which they operate, then by the name of the method that is applied to the said data structure. Some method names are standard for each class.

Standard method names are:

- Alloc: dynamically allocate an object of the given class. Not always available, as many objects are allocated on the stack as local variables.
- Init: initialization of the object passed as parameter.
- Free: freeing of the external structures of the object, to save space. The object may still be used, but it is considered as "empty" (e.g., an empty graph). The object may be re-used after it is initialized again.
- Exit: freeing of the internal structures of the object. The object must not be passed to other routines after the Exit method has been called.
- Copy: make a fully operational, independent, copy of the object, like a "clone" function in object-oriented languages.
- Load: load object data from stream.
- Save: save object data to stream.
- View: display internal structures and statistics, for debugging purposes.
- Check: check internal consistency of the object data, for debugging purposes.
 A Check method must be created for any new class, and any function that creates or updates an instance of some class must call the appropriate Check method, when compiled in debug mode.

3.4 Array index basing

The LIBSCOTCH library can accept data structures that come both from FORTRAN, where array indices start at 1, and C, where they start at 0. The start index for arrays is called the "base value", commonly stored in a variable (or field) called baseval.

In order to manage based indices elegantly, most references to arrays are based as well. The "table access" reference, suffixed as "tax" (see Section 3.2), is defined as the reference to the beginning of an array in memory, minus the base value (with respect to pointer arithmetic, that is, in terms of bytes, times the size of the array cell data type). Consequently, for any array whose beginning is pointed to by *tab, we have *tax = *tab - baseval. Consequently *tax[baseval always represents the first cell in the array, whatever the base value is. Of course, memory allocation and freeing operations must always operate on *tab pointers only.

In terms of indices, if the size of the array is xxxxnbr, then xxxxnnd = xxxxnbr + baseval, so that valid indices xxxxnum always belong to the range [baseval; vertnnd]. Consequently, loops often take the form:

```
for (xxxxnum = baseval; xxxxnum < xxxxnnd; xxxxnum ++) {
    xxxxtax[xxxxnum] = ...;
}</pre>
```

4 Structure of the libScotch library

As seen in Section 3.3, all of the routines that comprise the LIBSCOTCH project are named with a prefix that defines the type of data structure onto which they apply and a prefix that describes their purpose. This naming scheme allows one to categorize functions as methods of classes, in an object-oriented manner.

This organization is reflected in the naming and contents of the various source files.

The main modules of the LIBSCOTCH library are the following:

- arch: target architectures used by the static mapping methods.
- bgraph: graph edge bipartitioning methods, hence the initial.
- graph: basic (source) graph handling methods.
- hgraph: graph ordering methods. These are based on an extended "halo" graph structure, thus for the initial.
- hmesh: mesh ordering methods.
- kgraph: k-way graph partitioning methods.
- library: API routines for the LIBSCOTCH library.
- mapping: definition of the mapping structure.
- mesh: basic mesh handling methods.
- order: definition of the ordering structure.
- parser: strategy parsing routines, based on the FLEX and BISON parsers.
- vgraph: graph vertex bipartitioning methods, hence the initial.
- vmesh: mesh node bipartitioning methods.

Every source file name is made of the name of the module to which it belongs, followed by one or two words, separated by an underscore, that describe the type of action performed by the routines of the file. For instance, for module bgraph:

- bgraph.h is the header file that defines the Bgraph data structure,
- bgraph_bipart_fm.[ch] are the files that contain the Fiduccia-Mattheyses-like graph bipartitioning method,
- bgraph_check.c is the file that contains the consistency checking routine bgraphCheck for Bgraph structures,

and so on. Every source file has a comments header briefly describing the purpose of the code it contains.

5 Files and data structures

User-manageable file formats are described in the SCOTCH user's guide. This section contains information that are relevant only to developers and maintainers.

For the sake of portability, readability, and reduction of storage space, all the data files shared by the different programs of the Scotch project are coded in plain ASCII text exclusively. Although one may speak of "lines" when describing file formats, text-formatting characters such as newlines or tabulations are not mandatory, and are not taken into account when files are read. They are only used to provide better readability and understanding. Whenever numbers are used to label objects, and unless explicitly stated, **numberings always start from zero**, not one.

5.1 Decomposition-defined architecture files

Decomposition-defined architecture files are the way to describe irregular target architectures that cannot be represented as algorithmically-coded architectures.

Two main file formats coexist: the "deco 0" and "deco 2" formats. "deco" stands for "decomposition-defined architecture", followed by the format number. The "deco 1" format is a compiled form of the "deco 0" format. We will describe it here.

The "deco 1" file format results from an $O(p^2)$ preprocessing of the "deco 0" target architecture format. While the "deco 0" format contains a distance matrix between all pairs of terminal domains, which is consequently in in $\Theta(p^2/2)$, the "deco 1" format contains the distance matrix between any pair of domains, whether they are terminal or not. Since there are roughly 2p non-terminal domains in a target architecture with p terminal domains, because all domains form a binary tree whose leaves are the terminal domains, the distance matrix of a "deco 1" format is in $\Theta(2p^2)$, that is, four times that of the corresponding "deco 0" file.

Also, while the "deco 0" format lists only the characteristics of terminal domains (in terms of weights and labels), the "deco 1" format provides these for all domains, so as to speed-up the retrieval of the size, weight and label of any domain, whether it is terminal or not.

The "deco 1" header is followed by two integer numbers, which are the number of processors and the largest terminal number used in the decomposition, respectively (just as for "deco 0" files). Two arrays follow.

The first array has as many lines as there are domains (and not only terminal domains as in the case of "deco 0" files). Each of these lines holds three numbers: the label of the terminal domain that is associated with this domain (which is the label of the terminal domain of smallest number contained in this domain), the size of the domain, and the weight of the domain. The first domain in the array is the initial domain holding all the processors, that is, domain 1. The other domains in the array are the resulting subdomains, in ascending domain number order, such that the two subdomains of a given domain of number i are numbered 2i and 2i+1.

The second array is a lower triangular diagonal-less matrix that gives the distance between all pairs of domains.

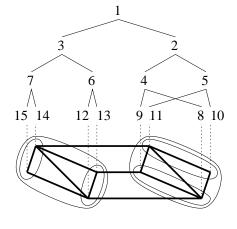
For instance, Figure 1 and Figure 2 show the contents of the "deco 0" and "deco 1" architecture decomposition files for UB(2,3), the binary de Bruijn graph of dimension 3, as computed by the amk_grf program.

6 Data structure explanations

This section explains some of the data structures implemented in SCOTCH and PT-SCOTCH.

6.1 Dorder

Distributed orderings are data structures used in PT-SCOTCH to represent orderings distributed on a set of processing elements. Like for the centralized ordering of type Order (see Section 6.6), a distributed ordering consists of an inverse permutation, which provides the old indices of the reordered vertices, and a column block decomposition of the reordered matrix, to help perform more efficient block computations at the solve stage. The column block decomposition is defined as a tree structure, the nodes of which, of type DorderCblk, represent column blocks



```
deco 0
8 15
0 1 15
  1 14
    13
  1
  1
    9
  1
    8
  1 10
2
  1
2
  1 2
  1 1
      2
  2
      1 2
 2 2 1
        1 1
 2 3 1 2 2 1
```

Figure 1: "deco 0" target decomposition file for UB(2,3). The terminal numbers associated with every processor define a unique recursive bipartitioning of the target graph.

tree nodes containing consecutive, reordered vertices. A tree node may have children nodes, which represent the decomposition of a column block into sub-column blocks, e.g., when a subdomain is decomposed into two separated subdomains and a separator.

Because its column blocks are distributed across multiple processing elements, the Dorder data type is much more complex than the Order data type. Hence, it is important to fully understand the Order data type before delving into the meanders of the Dorder data type and its ancillary data types. The main difference between the two is that, since graphs are distributed across multiple processing elements, column block information has to be duplicated on all the processing elements which contain a piece of a given graph. In order to reconciliate this information, to provide a centralized block column ordering, all distributed column block tree node structures are identified by a DorderIndex data structure.

The distributed column block tree data structure, created by way of parallel graph separation algorithms, always ends up in leaves, when nested dissection no longer succeeds or when the distributed subgraphs are folded onto single processing elements. As soon as the latter happens, a purely sequential graph ordering process can take place on each of them. This leads to the creation of a leaf DorderCblk node, into which the resulting locally-computed, centralized column block sub-tree is compacted as an array of DorderNode data structures. From the above, at the time being, the distributed column block tree structure contains only either nested dissection nodes, of type DORDERCBLKNEDI, and leaf nodes, of type DORDERCBLK LEAF. In order to facilitate the integration of centralized column block sub-trees into a global distributed column block tree, the values of the type flags are the same for the Dorder and Order data types.

The fields of the Dorder data structure are the following:

baseval

Base value for the inverse permutation.

vnodglbnbr

deco	2	2	2	2	1	2	2	1
1	3	1	2	2	1	2	2	2
8 15	3	1	2	2	1	2	1	2
0 8 8	1	1	2	2	3	2	3	1
3 4 4	2	2	3	1	3	2	3	2
0 4 4	1	3	3	1	2	2	1	2
5 2 2	1	1	2	2	1	1	1	2
3 2 2	2	1	2	2	1	1	1	2
2 2 2	2	2	3	3	2	2	3	1
0 2 2	2	2	1	3	2	1	2	2
6 1 1	1	2	2	1	1	2	2	2
5 1 1	1	1	1	3	3	2	3	3
7 1 1	2	1	2	3	3	2	1	2
3 1 1	1							
4 1 1								
2 1 1								
1 1 1								
0 1 1								

Figure 2: "deco 1" target decomposition file for UB(2,3), compiled with the acpl tool from the "deco 0" file displayed in Figure 1.

Overall number of node vertices to order across all processing elements. For graph orderings, this number is equal to the number of non-halo vertices in the initial graph.

cblklocnbr

Local number of locally-rooted column blocks. This number is the sum of the number of centralized column blocks, of type DorderNode, held by the current processing element, plus the number of distributed column block tree nodes, of type DorderCblk, the proclocnum index of which is equal to the rank of the processing element. This allows one to count only once each distributed column block tree node, when summing the cblklocnbr fields over all processing elements.

linkdat

Start of the doubly-linked list of distributed column block tree nodes, of type DorderCblk, on the given processing element. This list is circular, to allow for the insertion of new nodes at the end of the list in constant time.

proccomm

MPI communicator for managing the distributed ordering. It should be the same as that of the initial distributed graph to be ordered.

proclocnum

Rank of the given processing element within the communicator.

mutelocdat

When multi-threading is activated, allows one to create critical sections to update the ordering data in a thread-safe manner.

6.1.1 DorderIndex

Since the ordering data structure is distributed, pointers cannot be used to refer to parent or children column block tree node data structures across processing elements. The <code>DorderIndex</code> data type defines an identifier for column block tree nodes. These identifiers are unique, in the sense that, on each processing element, no two <code>DorderCblk</code> structures will have the same identifier. However, several <code>DorderCblk</code> structures may bear the same <code>DorderIndex</code> values on different processing elements, in the case when they are siblings which maintain the local information about the same distributed column block tree node.

The fields of the DorderIndex data type are the following:

proclocnum

Smallest rank among the processing elements on which a copy of the column block tree node resides.

cblklocnum

Local number of the column block tree node data structure on the processing element of aforementioned rank.

6.1.2 DorderLink

Since distributed column block tree nodes, of type <code>DorderCblk</code>, are created on the fly on each processing element, are in small numbers, and are heavy structures, they are not stored in a single resizable array, but as individual cells which are allocated when needed. Consequently, these structures have to be linked together, for proper management.

The DorderLink data type aims at chaining all DorderCblk structures in a circular, doubly-linked list. New nodes are inserted at the end of the list, such that a simple traversal yields nodes in ascending creation order, which is essential for locally-rooted nodes when gathering them to create a centralized ordering. The DorderLink structure is the first field of the DorderCblk structure, so that a simple pointer cast allows one to retrieve the tree node structure from the current link.

The fields of the DorderLink data structure are the following:

nextptr

Pointer to the next distributed column block tree node created on the given processing element.

prevptr

Pointer to the previous distributed column block tree node created on the given processing element.

6.1.3 DorderNode

The distributed column block tree data structure ends up in leaves, when either the parallel nested dissection stops, or when distributed subgraphs are located on single processing elements. In the first case, the distributed subgraph is centralized, after which, in both cases, a centralized ordering strategy is applied to the centralized subgraph, and a centralized block ordering is computed. This centralized block ordering is represented as an Order data structure, containing an inverse permutation and a tree of OrderCblk nodes. Since the distributed ordering will eventually have to be centralized, the local, centralized orderings will have to be compacted and sent to the root processing element. In order to anticipate this and to save space, once a centralized ordering is computed on some processing element, the resulting column block tree is compacted into a single array of Dorder Node cells.

The fields of the DorderNode data type are the following:

fathnum

Un-based index of the father node of the given node in the node array, or -1 if the given node is a local root and has to be connected to the father of the local leaf of the distributed column block tree.

typeval

Type of centralized column block tree node. The admissible values are constants of the kind ORDERCBLK*.

vnodnbr

Number of node vertices in the column block.

cblknum

Rank of the tree node among the children of its father, starting from zero.

Like for the DorderCblk data type, there are no references from a node to its children, but a reference from each node to its father, with all information needed to rebuild a global centralized column block tree when all node information is centralized on a single processing element.

6.1.4 DorderCblk

The DorderCblk data type represents distributed column block tree nodes within distributed orderings. A tree node may be a leaf node, or have children nodes which describe the decomposition of a column block into sub-column blocks, e.g., when a graph is decomposed into two separated subgraphs and a separator.

Since, by nature, every distributed column block tree node concerns a set of vertices distributed across a set of processing elements, each of the latter holds a copy of the tree node, the identifier of which, of type <code>DorderIndex</code>, contains identical information: the smallest rank among the involved processing elements within the communicator used to manage the distributed ordering, and an index incrementally generated on this processing element. Unlike for the <code>Order</code> data type, there are no pointers from a tree node to its child nodes; on the opposite, the <code>DorderCblk</code> node contains a <code>DorderIndex</code> referring to its father node. The only information a tree node will hold about its children is their number.

The fields of the DorderCblk data type are the following:

linkdat

Doubly-linked list structure to chain together all the DorderCblk structures on a given processing element.

ordelocptr

Pointer to the distributed ordering to which the given distributed column block tree node belongs.

typeval

Type of tree node; at the time being, it is either DORDERCBLKNEDI for a nested dissection node, or DORDERCBLKLEAF for a leaf node.

fat.hnum

Identifier of the father of the given column block tree node. If the given tree node is a root, the value of the father index is $\{0, -1\}$.

cblknum

Identifier of the given column block tree node. The process number is the smallest rank among all the processing elements sharing node vertices, and the local number is provided incrementally on this processing element.

ordeglbval

Un-based global start index of the node vertices in the distributed column block tree node.

vnodglbnbr

Number of node vertices contained in the distributed column block tree node, over all the involved processing elements. If the column block has sub-column blocks, the sum of all the vnodglbnbr values of the sub-column blocks must be equal to the vnodglbnbr of the column block.

cblkfthnum

Index of the given column block tree node among its siblings, starting from zero.

data

Union field holding the information concerning either the leaf node or the nested dissection node. This field has two sub-fields:

leaf

Leaf field, which has the following sub-fields:

ordelocval

Un-based start index in the global inverse permutation array for the local vertices.

vnodlocnbr

Number of node vertices in the given permutation fragment.

periloctab

Pointer to the local, un-based, inverse permutation fragment array, of size vnodlocnbr. The values of the periloctab array are based according to the baseval field of the Dorder data type.

nodelocnbr

Number of local column block tree nodes associated with the permutation fragment.

nodeloctab

Pointer to the local, un-based, array of local column block tree nodes, of size nodelocnbr.

cblklocnum

Un-based index, in nodeloctab, of the root local column block tree node.

nedi

Nested dissection field. This field has a single sub-field:

cblkglbnbr

Number of sub-column blocks within this column block. For nested dissection, this number is either 2 (two separated parts and no separator) or 3 (two separated parts and a separator).

6.2 Graph

Graphs are the fundamental underlying data structures of all the algorithms implemented in Scotch. The Graph structure is the foundational data structure, from which subclasses will be derived, according to the specific needs of the Scotch modules. It is sometimes referred to as the *source graph* structure, with respect to the *target architecture* Arch onto which source graphs are to be mapped.

The Graph structure, being a foundational data structure, does not possess any variable fields related to actual computations, e.g., partition state variables or an execution context. Such fields will be found in *active* graphs, e.g., Bgraph, Kgraph, Vgraph.

A Graph is described by means of adjacency lists. These data are stored in arrays and scalars of type SCOTCH_Num, as shown in Figures 3 and 4. The Graph fields have the following meaning:

baseval

Base value for all array indexing.

vertnbr

Number of vertices in graph.

edgenbr

Number of arcs in graph. Since edges are represented by both of their ends, the number of edge data in the graph is twice the number of graph edges.

verttax

Based array of start indices in edgetax of vertex adjacency sub-arrays.

vendtax

Based array of after-last indices in edgetax of vertex adjacency sub-arrays. For any vertex i, with baseval $\leq i < (\mathtt{vertnbr} + \mathtt{baseval})$, $(\mathtt{vendtax}[i] - \mathtt{verttax}[i])$ is the degree of vertex i, and the indices of the neighbors of i are stored in edgetax from edgetax[verttax[i]] to edgetax[vendtax[i] - 1], inclusive.

When all vertex adjacency lists are stored in order in edgetax, it is possible to save memory by not allocating the physical memory for vendtax. In this case, illustrated in Figure 3, verttax is of size vertnbr +1 and vendtax points to verttax +1. This case is referred to as the "compact edge array" case, such that verttax is sorted in ascending order, verttax[baseval] = baseval and verttax[baseval + vertnbr] = (baseval + edgenbr).

velotax

Optional based array, of size **vertnbr**, holding the integer load associated with every vertex.

vnumtax

When the current graph is a subgraph of some initial graph, this based array, of size vertnbr, holds the initial vertex indices of the subgraph vertices. This array is not defined (*i.e.*, vnumtax = NULL) when the graph is the initial graph.

edgetax

Based array, of a size equal at least to $(\max_i(\text{vendtax}[i]) - \text{baseval})$, holding the adjacency array of every vertex.

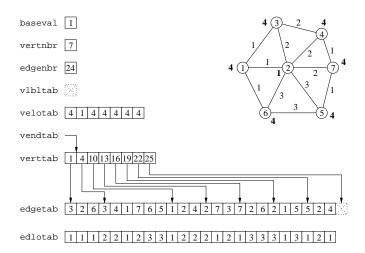


Figure 3: Sample graph and its description using a compact edge array. Numbers within vertices are vertex indices, bold numbers close to vertices are vertex loads, and numbers close to edges are edge loads. Since the edge array is compact, verttax is of size vertnbr + 1 and vendtax points to verttax + 1.

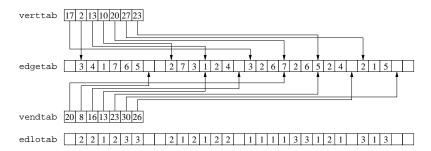


Figure 4: Adjacency structure of the sample graph of Figure 3 with disjoint edge and edge load arrays. Both verttax and vendtax are of size vertnbr. This allows for the handling of dynamic graphs, the structure of which can evolve with time.

edlotax

Optional based array, of a size equal at least to $(\max_i(\mathtt{vendtax}[i]) - \mathtt{baseval})$, holding the integer load associated with every arc. Matching arcs should always have identical loads.

Dynamic graphs can be handled elegantly by using the vendtax array. In order to dynamically manage graphs, one just has to allocate verttax, vendtax and edgetax arrays that are large enough to contain all of the expected new vertex and edge data. Original vertices are labeled starting from baseval, leaving free space at the end of the arrays. To remove some vertex i, one just has to replace verttax[i] and vendtax[i] with the values of verttax[vertnbr-1] and vendtax[vertnbr-1], respectively, and browse the adjacencies of all neighbors of former vertex vertnbr-1 such that all (vertnbr-1) indices are turned into is. Then, vertnbr must be decremented.

To add a new vertex, one has to fill verttax[vertnbr-1] and vendtax[vertnbr-1] with the starting and end indices of the adjacency sub-array of the new vertex. Then, the adjacencies of its neighbor vertices must also be updated to account for it. If free space had been reserved at the end of each of the neighbors, one just has to increment the vendtax[i] values of every neighbor i, and add the index of the new

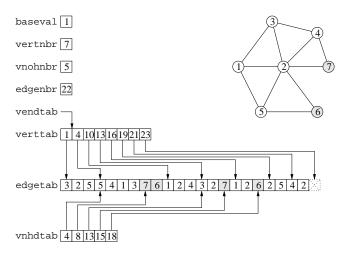


Figure 5: Sample halo graph and its description using a compact edge array. Numbers within vertices are vertex indices. Greyed values are indices of halo vertices. Halo vertices have the highest indices in the graph, and are placed last in the adjacency sub-arrays of each non-halo vertex.

vertex at the end of the adjacency sub-array. If the sub-array cannot be extended, then it has to be copied elsewhere in the edge array, and both $\mathtt{verttax}[i]$ and $\mathtt{vendtax}[i]$ must be updated accordingly. With simple housekeeping of free areas of the edge array, dynamic arrays can be updated with as little data movement as possible.

6.3 Hgraph

The Hgraph structure holds all the information necessary to represent and perform computations on a halo graph. This term refers to graphs some vertices of which are kept to preserve accurate topological information, but are usually not subject to actual computations. These halo vertices are collectively referred to as the halo of the graph. Halo graphs are notably used in sparse matrix reordering, where, in the process of nested dissection, a graph is cut into three pieces: a vertex separator, and two separated parts. Each of these parts must preserve the real degree information attached to all their vertices, including those next to the separator. If halo graphs were not used, the degrees of these vertices would appear smaller than what they really are in the whole graph. Preserving accurate degree information is essential for algorithms such as the minimum degree vertex ordering method. Some vertex separation algorithms also aim at balancing halo vertices; in this case, separators will be computed on halos, but this information will not be preserved once a separator has been computed on the regular vertices.

Halo graphs exhibit specific structural and topological properties, illustrated in Figure 5. In order to distinguish easily halo vertices from regular vertices and write efficient algorithms, halo vertices have the highest vertex indices in the graph. Because the degrees of halo vertices need not be preserved, no edges connect two halo vertices; the adjacency of halo vertices is only made of regular vertices. Also, in the adjacency arrays of regular vertices, all non-halo vertices are placed before halo vertices. All these properties allow one to easily induce the non-halo graph from some halo graph, without having to create new adjacency arrays. An additional vertex index array is present just for this purpose.

Halo graph fields have the following meaning:

Underlying source graph that contains all regular and halo vertices. This is where to search for fields such as baseval, vertnbr, vertnnd, verttax, vendtax, etc.

vnohnbr

Number of non-halo vertices in graph. Hence, $0 \le \text{vnohnbr} \le \text{s.vertnbr}$.

vnhdtax

Array of after-last indices in s.edgetax of non-halo vertex adjacency subarrays. Since this information only concerns non-halo vertices, vnhdtax is of size vnohnbr, not vertnbr. For any non-halo vertex i, with baseval $\leq i < (\text{vnohnbr} + \text{baseval})$, the indices of the non-halo neighbors of i are stored in s.edgetax from s.edgetax [s.verttax[i]] to s.edgetax [vnhdtax[i] - 1], inclusive, and its halo neighbors are stored from s.edgetax [vnhdtax[i]] to s.edgetax [s.vendtax[i]], inclusive.

vnlosum

Sum of non-halo vertex loads. Hence, $0 \le vnlosum \le s.velosum$.

enohnbr

Number of non-halo arcs in graph. Hence, $0 \le \text{enohnbr} \le \text{s.edgenbr}$.

6.4 Kgraph

The Kgraph structure holds all the information necessary to compute a k-way (re)mapping of some graph onto a target architecture. Consequently, it contains a Graph, defined as field s, and a reference to an Arch, through the field m.archptr, as well as two Mapping structures: one for the current mapping to compute, and one to store the old mapping from which to remap. Additional information comprise data to model the cost of remapping, and data associated with the state and cost of the current mapping: list of frontier vertices, load of each partition domain, plus the execution context for multi-threading execution.

The Graph structure is internal to the Kgraph because every new Kgraph contains a different graph topology (e.g., a band graph or a coarsened graph). The Arch is accessed by reference because it is constant data which can be shared by many Kgraphs. For the sake of consistency, the grafptr fields of each mapping m and r.m must point to &s, while their two archptr fields must point to the same target architecture. This redundency is the price to pay for lighter memory management.

6.4.1 Mappings

The domnorg field, which must contain a valid domain in the architecture m.archptr, is the starting point for the k-way mapping. This domain may be smaller than the full architecture when parallel partitioning is performed: in this case, each process may receive a separate subgraph and sub-architecture to work on.

Each of the two mappings has its own specificities. The current mapping, defined as field m, is never incomplete: all the cells of its m.parttax array are non-negative values that index a valid domain in the domain array m.domntab. These domains are all subdomains of the architecture referenced through field m.archptr.

More restrictively, the domains attached to non-fixed vertices must be included in domnorg, which may be smaller.

The current mapping evolves with time, according to the various algorithms that the user can activate in the strategy string. These algorithms will create derived Kgraphs (e.g., band graphs or coarsened graphs), to which mapping methods will be applied, before the result is ported back to their parent Kgraph. Depending on the kind of the derived graph, the m.parttax array may be specific, but the m.domntab array will always be ported back as is. Consequently, in order to save memory copying, the policy which is implemented is that the derived Kgraph gets the pointer to the m.domntab of its parent, while the latter is set to NULL. The derived graph can therefore reallocate the array whenever needed, without the risk of an old, invalid, pointer being kept elsewhere. Then, when the processing of the derived Kgraph ends, the most recent pointer is copied back to the m.domntab field of the parent graph, and the m.parttax array is updated accordingly, after which the derived Kgraph can be destroyed without freeing the pointer.

The old mapping, defined as field r.m, may contain incomplete mapping information: some of the cells of its r.m.parttax array may be equal to -1, to indicate that no prior mapping information is available (e.g., when the vertex did not exist in the previous mapping). Since old mappings do not change, the r.m.domntab field can be shared among all derived Kgraphs. It is protected from double memory freeing by not setting the MAPPINGFREEDOMN flag in field r.m.flagval.

6.5 Mapping

The Mapping structure defines how individual vertices of a Graph are mapped individually onto (parts of) an Arch. A mapping is said *complete* if all source graph vertices are assigned to terminal target domains, *i.e.*, individual vertices of the target architecture, or *partial* if at least one of the source graph vertices is assigned to a target domain that comprises more than one vertex. In the course of the graph mapping process, the destination of source vertices are progressively refined, from an initial target domain that usually describes the whole of the target architecture, to terminal domains.

Since ArchDom, the data structure that describes target architecture domains, is big and costly to handle (e.g., to compare if two ArchDoms are identical), the handling of domains in mapping is indirect: in the part array parttax, each vertex is assigned an integer domain index that refers to a domain located in the domain array domntab. Hence, when two graph vertices have the same index in parttax, they belong to the same domain and induce no communication cost. However, the opposite is false: two vertices may have a different index in parttax and yet belong to the same target domain. This is for instance the case when one of the vertices is a fixed vertex that has been set to a specific terminal domain at initialization time, and one of its neighbors is successively mapped to smaller and smaller subdomains that eventually amount to the same terminal domain.

In the case of a remapping, the mapping information regarding the former placement of the vertices may be incomplete, e.g., because the vertex did not exist before. Such a mapping is said to be *incomplete*. It is characterized by the fact that some cells of the parttax array are equal to -1, to indicate an unknown terminal domain number. To allow for this, the mapping must have the MAPPING INCOMPLETE flag set. Incomplete mappings are only valid when holding remapping information; new mappings being computed must have all their parttax cells set with non-negative values that point to valid domains in the domntab array. New

mappings can therefore only be partial or complete.

When a mapping is initialized, all parttax values for non-fixed vertices are set to index 0, and domntab[0] is set to the root domain for the mapping. In the general case for centralized mapping, the initial domain is equal to archDomFrst(archptr). However, when a centralized mapping process is launched as a part of a distributed mapping process, the initial domain may be a subset of the whole target architecture.

There is no obligation for the domntab array to contain only one instance of some target domain. On the contrary, as described above, the same domain may appear at least twice: once for fixed vertices, and once for non-fixed vertices on which mapping algorithms are applied. However, for efficiency reasons (e.g., avoiding to compute vertex distances that are equal to zero), it is preferable that duplicate domains are avoided in the domntab array. This is the case by nature with recursive bipartitioning, as the domains associated with branches of the biparitioning tree are all distinct.

Making the distinction between fixed and non-fixed vertices, which is relevant to mapping algorithms, is not in the scope of the Mapping data structure, which only represents a global state. This is why no data related to fixed vertices is explicitly present in the mapping itself (it may be found, e.g., in the Kgraph data structure). However, for handling fixed vertices in an efficient way, the semantics of the Mapping data structure is that all domains that are associated with fixed vertices must be placed first in the domntab array. The purpose of this separation is because, when the imbalance of a mapping is computed, the loads of non-fixed vertices that belong to some (partial) domain and of fixed vertices that belong to domains that are subdomains of this domain have to be aggregated. This aggregation procedure is made easier if both types of domains are kept separate. For efficiency reasons, fixed domains should appear only once in the fixed part of domntab.

The Mapping structure is mainly used within the Kgraph structure, which contains two instances of it: one for the current mapping to be computed, and one for the old mapping, in the case of remapping. The building of a Kgraph from another one (e.g., when creating a band graph or a coarsened graph) may lead to situations in which some Mapping arrays may be re-used, and thus should not be freed when the derived Mapping is freed. This is why the Mapping structure contains flags to record whether its arrays should be freed or not. These flags are the following:

MAPPINGFREEDOMN

Set if the domain array has to be freed when the mapping is freed. A common case for sharing the domain array is when a coarser Kgraph is computed: the domain array of the coarse old mapping can re-use that of the fine old mapping.

MAPPINGFREEPART

Set if the part array has to be freed when the mapping is freed. A common case for sharing the part array is when the user part array is kept as the part array for the initial Kgraph current mapping structure.

The main fields of the Mapping data structure are the following:

flagval

Set of flags indicating whether the parttax and domntab have to be freed on exit.

grafptr

Pointer to the Graph associated with the mapping, that gives access to the base value grafptr->baseval and the number of source vertices grafptr->vertnbr.

archptr

Pointer to the Arch associated with the mapping, that is necessary to perform all distance computations on the mapping.

parttax

Based array of Anums, of size grafptr->vertnbr, that provides the index of the target domains onto which all graph vertices are currently mapped. Indices are un-based.

domnt.ab

Un-based array of ArchDoms, of size domnmax, that stores the target domains to which source graph vertices are indirectly associated through the parttax array.

domnnbr

Number of target domain slots currently used in domntab. After a mapping is initialized, $1 \le \text{domnnbr} < \text{domnmax}$, because source graph vertices must be associated to some domain, hence domntab should at least contain one domain.

domnnbr

Number of target domain slots currently used in domntab.

domnmax

Size of the domntab array.

mutedat

When multi-threading is activated, allows to create critical sections to update the mapping data in a thread-safe manner.

6.6 Order

Orderings are data structures used in Scotch to represent fill-minimizing block orderings of adjacency matrices represented as graphs. A block ordering, contained in the Order data structure, is defined by an inverse permutation, which provides the old indices of the reordered vertices, and a column block decomposition of the reordered matrix, to help performing more efficient block computations at the solving stage.

Inverse permutations are used, instead of direct permutations, because their processing is more local: when ordering some subgraph, the only ordering information to provide is the un-based start index, usually called ordenum, in the inverse permutation vector, usually called peritab, while the vnumtax array of the Graph structure holds the values of the vertex indices to write in the sub-array of peritab starting at index ordenum, of a size equal to the number of concerned vertices in the Graph. Once an ordering is computed, it is straightforward to compute the direct permutation permtab from the inverse permutation peritab, in case it is needed.

The column block decomposition is defined as a tree structure, whose nodes, of type OrderCblk, represent column blocks containing consecutive, reordered

vertices. A tree node may have ordered children nodes, which represent the decomposition of a column block into sub-column blocks, e.g., when a subdomain is decomposed into two separated subdomains and a separator.

The main fields of the Order data structure are the following:

flagval

Flag that indicates whether the peritab inverse permutation array has to be freed on exit.

baseval

Base value for inverse permutation values.

vnodnbr

Number of vertex nodes in the ordering. When the associated graph structure is a Graph, this number is equal to its vertnbr field; when it is a Mesh, it is equal to its vnodnbr field.

treenbr

Number of tree nodes in the ordering. This number is equal to 1 when only the root tree node is present, and is incremented each time a new tree node is added to the tree structure.

cblknbr

Number of column blocks in the ordering. This number is equal to 1 when only the root tree node is present. When some column block is decomposed into c sub-column blocks, it is increased by (c-1), since this represents the number of additional column blocks in the structure.

rootdat

Root column block of the ordering. This structure, of type OrderCblk, is initialized to contain all Graph vertices, or Mesh vertex nodes, in a single column block, after which reordering algorithms are applied and lead to the creation of sub-column blocks, e.g., in the case of nested dissection.

peritab

Pointer to the inverse permutation array.

mutedat

Mutual exclusion lock. When multi-threading is activated, it allows to create critical sections to update the ordering data in a thread-safe manner.

6.6.1 OrderCblk

Column blocks are sets of reordered unknowns which are likely to be processed efficiently together when solving the linear system, e.g., using BLAS block computation routines. The column block decomposition of the reordered matrix is represented as a tree whose nodes are instances of the OrderCblk data structure. The column block decomposition tree will be used to create the block elimination tree of the unknowns of the linear system, which amounts to linking each column block to a father block. This building is performed by the orderTree() routine.

A column block tree node OrderCblk is defined by its type (i.e., whether it is a leaf, a nested dissection node, etc.), its width (i.e., the number of node vertices it contains), and, if it is not a leaf, the description of the sub-column blocks it contains. The main fields of the OrderCblk data structure are the following:

typeval

Set of flags that define the nature of the column block tree node. They must be the same as the distributed column block tree node flags of the Dorder Cblk distributed column block data structure. Consequently, these flags must be separate bits, so that values can be or-ed (especially, concerning ORDER CBLKLEAF in hdgraphOrderNd()). These flags are the following:

ORDERCBLKLEAF

Leaf column block (before it is subdivided into sub-column blocks, or definitely). In this case, the other fields of the column block tree node are such that $\mathtt{cblknbr} = 0$ and $\mathtt{cblktab} = \mathtt{NULL}$.

ORDERCBLKNEDI

Nested-dissection separator tree node. The separator is always the last sub-column block. Hence, if the separator is not empty, the node has three sub-column blocks (hence $\mathtt{cblknbr} = 3$), while, if the separator is empty, the column block tree node has only two sub-column blocks (hence $\mathtt{cblknbr} = 2$). None of the separated parts can be empty (else, the tree node would be of type ORDERCBLKSEQU).

ORDERCBLKDICO

Disconnected components tree node. It contains an arbitrary number (always strictly greater than 1) of sub-column blocks, which represent disconnected components to be ordered independently. Since the sub-column blocks are not connected, their father in the elimination tree will not be the column block tree node itself, but its father (or none if the column block is the root column block, *i.e.*, the rootdat field of the Order data structure).

ORDERCBLKSEOU

Sequential tree node. It contains an arbitrary number (always strictly greater than 1) of sub-column blocks, which represent mutually dependent blocks. Consequently, the father of each sub-column block in the elimination tree will be the next sub-column block, except for the last sub-column block, whose father will be the column block itself.

vnodnbr

Number of nodes (*i.e.*, vertices) contained in the column block. If the column block has sub-column blocks, the sum of all the vnodnbr values of the sub-column blocks must be equal to the vnodnbr of the column block.

cblknbr

Number of sub-column blocks. If the column block is a leaf, cblknbr = 0 and cblktab = NULL.

cblktab

Array of cblknbr structures of type DorderCblk, which hold the data about the sub-column blocks if the column block is not a leaf. cblktab has to be freed on exit.

7 Code explanations

This section explains some of the most complex algorithms implemented in SCOTCH and PT-SCOTCH.

7.1 dgraphCoarsenBuild()

The dgraphCoarsenBuild() routine creates a coarse distributed graph from a fine distributed graph, using the result of a distributed matching. The result of the matching is available on all MPI processes as follows:

coardat.multlocnbr

The number of local coarse vertices to be created.

coardat.multloctab

The local multinode array. For each local coarse vertex to be created, it contains two values. The first one is always positive, and represents the global number of the first local fine vertex to be mated. The second number can be either positive or negative. If it is positive, it represents the global number of the second local fine vertex to be mated. If it is negative, its opposite, minus two, represents the local edge number pointing to the remote vertex to be mated.

coardat.procqsttax

Array (restricted to ghost vertices only) that records on which process is located each ghost fine vertex.

7.1.1 Creating the fine-to-coarse vertex array

In order to build the coarse graph, one should create the array that provides the coarse global vertex number for all fine vertex ends (local and ghost). This information will be stored in the coardat.coargsttax array.

Hence, a loop on local multinode data fills coardat.coargsttax. The first local multinode vertex index is always local, by nature of the matching algorithm. If the second vertex is local too, coardat.coargsttax is filled instantly. Else, a request for the global coarse vertex number of the remote vertex is forged, in the vsnddattab array, indexed by the current index coarsndidx extracted from the neighbor process send index table nsndidxtab. Each request comprises two numbers: the global fine number of the remote vertex for which the coarse number is seeked, and the global number of the coarse multinode vertex into which it will be merged.

Then, an all-to-all-v data exchange by communication takes place, using either the dgraphCoarsenBuildPtop() or dgraphCoarsenBuildColl() routines. Apart from the type of communication they implement (either point-to-point or collective), these routines do the same task: they process the pairs of values sent from the vsnddattab array. For each pair (the order of processing is irrelevant), the coargsttax array of the receiving process is filled-in with the global multinode value of the remotely mated vertex. Hence, at the end of this phase, all processes have a fully valid local part of the coargsttax array; no value should remain negative (as set by default). Also, the nrcvidxtab array is filled, for each neighbor process, of the number of data it has sent. This number is preserved, as it will serve to determine the number of adjacency data to be sent back to each neighbor process.

Then, data arrays for sending edge adjacency are filled-in. The ercvdsptab and ercvcnttab arrays, of size procglbnbr, are computed according to the data stored in coardat.dcntglbtab, regarding the number of vertex- and edge-related data to exchange.

By way of a call to dgraphHaloSync(), the ghost data of the coargsttax array are exchanged.

Then, edgelocnbr, an upper bound on the number of local edges, as well as ercvdatsiz and esnddatsiz, the edge receive and send array sizes, respectively.

Then, all data arrays for the coarse graph are allocated, plus the main adjacency send array esnddsptab, its receive counterpart ercvdattab, and the index send arrays esnddsptab and esndcnttab, among others.

Then, adjacency send arrays are filled-in. This is done by performing a loop on all processes, within which only neighbor processes are actually considered, while index data in esnddsptab and esndcnttab is set to 0 for non-neighbor processes. For each neighbor process, and for each vertex local which was remotely mated by this neighbor process, the vertex degree is written in the esnddsptab array, plus optionally its load, plus the edge data for each of its neighbor vertices: the coarse number of its end, obtained through the coargsttax array, plus optionally the edge load. At this stage, two edges linking to the same coarse multinode will not be merged together, because this would have required a hash table on the send side. The actual merging will be performed once, on the receive side, in the next stage of the algorithm.

7.2 dgraphFold() and dgraphFoldDup()

The dgraphFold() routine creates a "folded" distributed graph from the input distributed graph. The folded graph is such that it spans across only one half of the processing elements of the initial graph (either the first half, or the second half). The purpose of this folding operation is to preserve a minimum average number of vertices per processing element, so that communication cost is not dominated by message start-up time. In case of an odd number of input processing elements, the first half of them is always bigger that the second.

The dgraphFoldDup() routine creates two folded graphs: one for each half. Hence, each processing element hosting the initial graph will always participate in hosting a new graph, which will depend on the rank of the processing element. When the MPI implementation supports multi-threading, and multi-threading is activated in Scotch, both folded graphs are created concurrently.

The folding routines are based on the computation of a set of (supposedly efficient) point-to-point communications between the sender processes, which will not retain any graph data, and the receiver processes, which will host the folded graph. However, in case of unbalanced vertex distributions, overloaded receiver processes (called sender receiver processes) may also have to send their extra vertices to underloaded receiver processes. A receiver process may receive several chunks of vertex data (including their adjacency) from several sender processes. Hence, folding amounts to a redistribution of vertex indices across all receiver processes. In particular, end vertex indices have to be renumbered according to the global order in which the chunks of data are exchanged. This is why the computation of these exchanges, by way of the dgraphFoldComm() routine, has to be fully deterministic and reproducible across all processing elements, to yield consistent communication data. The result of this computation is a list of point-to-point communications (either all sends or receives) to be performed by the calling process, and an array of sorted global vertex indices, associated with vertex index adjustment values, to convert global vertex indices in the adjacency of the initial graph into global vertex indices in the adjacency of the folded graph. This array can be used, by way of dichotomy search, to find the proper adjustment value for any end vertex number.

To date, the dgraphRedist() routine is not based on a set of point-to-point communications, but collectives. It could well be redesigned to re-use the mecha-

nisms implemented here, with relevant code factorization.

7.2.1 dgraphFoldComm()

The dgraphFoldComm() routine is at the heart of the folding operation. It computes the sets of point-to-point communications required to move vertices from the sending half of processing elements to the receiving half, trying to balance the folded graph as much as possible in terms of number of vertices. For receiver processes, it also computes the data needed for the renumbering of the adjacency arrays of the graph chunks received from sender (or sender receiver) processes.

It is to be noted that the end user and the SCOTCH algorithms may have divergent objectives regarding balancing: in the case of a weighted graph representing a computation, where some vertices bear a higher load than others, the user may want to balance the load of its computations, even if it results in some processing elements having less vertices than others, provided the sums of the loads of these vertices are balanced across processing elements. On the opposite, the algorithms implemented in SCOTCH operate on the vertices themselves, irrespective of the load values that is attached to them (save for taking them into account for computing balanced partitions). Hence, what matters to SCOTCH is that the number of vertices is balanced across processing elements. Whenever SCOTCH is provided with an unbalanced graph, it will try to rebalance it in subsequent computations (e.g., folding). However, the bulk of the work, on the initial graph, will be unbalanced according to the user's distribution.

During a folding onto one half of the processing elements, the processing elements of the other half will be pure senders, that need to dispose of all of their vertices and adjacency. Processing elements of the first half will likely be receivers, that will take care of the vertices sent to them by processing elements of the other half. However, when a processing element in the first half is overloaded, it may behave as a sender rather than a receiver, to dispose of its extra vertices and send it to an underloaded peer.

The essential data that is produced by the dgraphFoldComm() routine for the calling processing element is the following:

commmax

The maximum number of point-to-point communications that can be performed by any processing element. The higher this value, the higher the probability to spread the load of a highly overloaded processing element to (underloaded) receivers. In the extreme case where all the vertices are located on a single processing element, (procglbnbr - 1) communications would be necessary. To prevent such a situation, the number of communications is bounded by a small number, and receiver processing elements can be overloaded by an incoming communication. The algorithm strives to provide a feasible communication scheme, where the current maximum number of communications per processing element suffices to send the load of all sender processing elements. When the number of receivers is smaller than the number of senders (in practice, only by one, in case of folding from an odd number of processing elements), at least two communications have to take place on some receiver, to absorb the vertices sent. The initial maximum number of communications is defined by DGRAPHFOLDCOMMNBR;

commtypval

The type of communication and processing that the processing element will

have to perform: either as a sender, a receiver, or a sender receiver. Sender receivers will keep some of their vertex data, but have to send the rest to other receivers. Sender receivers do send operations only, and never receive data from a sender:

commdattab

A set of slots, of type DgraphFoldCommData, that describe the point-topoint communications that the processing element will initiate on its side. Each slot contains the number of vertices to send or receive, and the target or source process index, respectively;

commvrttab

A set of values associated to each slot in commdattab, each of which contains the global index number of the first vertex of the graph chunk that will be transmitted:

procenttab

For receiver processes only, the count array of same name of the folded distributed graph structure;

vertadjnbr

For receiver processes only, the number of elements in the dichotomy array vertadjtab;

vertadjtab

A sorted array of global vertex indices. Each value represent the global start index of a graph chunk that will been exchanged (or which will remain in place on a receiver processing element);

vertdlttab

The value which has to be added to the indices of the vertices in the corresponding chunk represented in vertadjtab. This array and the latter serve to find, by dichotomy, to which chunk an end vertex belongs, and modify its global vertex index in the edge array in the receiver processing element. Although vertadjtab and vertdlttab contain strongly related information, they are separate arrays, for the sake of memory locality. Indeed, vertadjtab will be subject to a dichotomy search, involving many memory reads, before the proper index is found and a single value is retrieved from the vert dlttab array.

The first stage of the algorithm consists in sorting a global process load array in ascending order, in two parts: the sending half, and the receiving half. These two sorted arrays will contain the source information which the redistribution algorithm will use. Because the receiver part of the sort array can be modified by the algorithm, it is recomputed whenever commmax is incremented. It is the same for sort sndbas, the index of the first non-empty sender in the sort array.

In a second stage, the algorithm will try to compute a valid communication scheme for vertex redistribution, using as many as commmax communications (either sends or receives) per processing element. During this outermost loop, if a valid communication scheme cannot be created, then commmax is incremented and the communication scheme creation algorithm is restarted. The initial value for commmax is DGRAPHFOLDCOMMNBR.

The construction of a valid communication scheme is performed within an intermediate loop. At each step, a candidate sender process is searched for: either a sender process which has to dispose of all of its vertices, or an overloaded receiver process, depending on which has the biggest number of vertices to send. If candidate senders can no longer be found, the stage has succeeded with the current value of commmax; if a candidate sender has been found but a candidate receiver has not, the outermost loop is restarted with an incremented commmax value, so as to balance loads better.

Every time a sender has been found and one or more candidate receivers exist, an inner loop creates as many point-to-point communications as to spread the vertices in chunks, across one or more available receivers, depending on their capacity (i.e., the number of vertices they can accept). If the selected sender is a sender receiver, the inner loop will try to interleave small communications from pure senders with communications of vertex chunks from the selected sender receiver. The purpose of this interleaving is to reduce the number of messages per process: a big message from a sender receiver is likely to span across several receivers, which will then perform only a single receive communication. By interleaving a small communication on each of the receivers involved, the latter will only have to perform one more communication (i.e., two communications only), and the interleaved small senders will be removed off the list, reducing the probability that afterwards many small messages will sent to the same (possibly eventually underloaded) receiver.

In a third stage, all the data related to chunk exchange, which was recorded in a temporary form in the vertadjtab, vertdlttab and slotsndtab arrays, is compacted to remove empty slots and to form the final vertadjtab and vertdlttab arrays to be used for dichotomy search.

The data structures that are used during the computation of vertex global index update arrays are the following:

vertadjtab and vertdlttab

These two arrays have been presented above. They are created only for receiver processes, and will be filled concurrently. They are of size ((commmax+1)*orgprocnbr), because in case a process is a sender receiver, it has to use a first slot to record the vertices it will keep locally, plus commmax for outbound communications. During the second stage of the algorithm, for some slot i, vertadjtab[i] holds the start global index of the chunk of vertices that will be kept, sent or received, and vertdlttab[i] holds the number of vertices that will be sent or received. During the third stage of the algorithm, all this data will be compacted, to remove empty slots. After this, vertadjtab will be an array of global indices used for dichotomy search in dgraphFold(), and vertdlttab[i] will hold the adjustment value to apply to vertices whose global indices are comprised between vertadjtab[i] and vertadjtab[i].

slotsndtab

This array only has cells for receiver-slide slots, hence a size of ((commmax + 1) * procfldnbr) items. During the second stage of the algorithm, it is filled so that, for any non-empty communication slot i in vertadjtab and vertdlttab, representing a receive operation, slotsndtab[i] is the slot index of the corresponding send operation. During the third stage of the algo-

rithm, it is used to compute the accumulated vertex indices across processes.

Here are some examples of redistributions that are computed by the dgraph FoldComm() routine.

```
orgvertcnttab = { 20, 20, 20, 20, 20, 20, 20, 1908 }
partval = 1
vertglbmax = 1908

Proc [0] (SND) 20 -> 0 : { [4] <- 20 }
Proc [1] (SND) 20 -> 0 : { [5] <- 20 }
Proc [2] (SND) 20 -> 0 : { [6] <- 20 }
Proc [3] (SND) 20 -> 0 : { [6] <- 20 }
Proc [4] (RCV) 20 -> 512 : { [0] -> 20 }, { [7] -> 472 }
Proc [5] (RCV) 20 -> 512 : { [1] -> 20 }, { [7] -> 472 }
Proc [6] (RCV) 20 -> 512 : { [2] -> 20 }, { [7] -> 452 }, { [3] -> 20 }
Proc [7] (RSD) 1908 -> 512 : { [4] <- 472 }, { [5] <- 472 }, { [6] <- 452 }
commmax = 4
commsum = 14</pre>
```

We can see in the listing above that some interleaving took place on the first receiver (proc. 4) before the sender receiver (proc. 7) did its first communication towards it.

```
orgvertcnttab = { 0, 0, 0, 20, 40, 40, 40, 100 }
   partval = 1
   vertglbmax = 100
   Proc [0] (SND) 0 -> 0:
   Proc [1] (SND) 0 -> 0:
   Proc [2] (SND) 0 -> 0:
   Proc [3] (SND) 20 -> 0 : { [4] <- 20 }
   Proc [4] (RCV) 40 -> 60 : { [3] -> 20 }
                 40 -> 60 : [7] -> 20
   Proc [5]
            (RCV)
   Proc [6] (RCV) 40 -> 60 : { [7] -> 20 }
   Proc [7] (RSD) 100 -> 60 : { [5] <- 20 }, { [6] <- 20 }
11
   commmax = 4
12
   commsum = 6
```

In the latter case, one can see that the pure sender that has been interleaved (proc. 3) sufficed to fill-in the first receiver (proc. 4), so the first communication of the sender receiver (proc. 7) was towards the next receiver (proc. 5).

8 Procedures for new developments and release

8.1 Adding methods to the libScotch library

The LIBSCOTCH has been carefully designed to allow external contributors to add their new partitioning or ordering methods, and to use Scotch as a testbed for them.

8.1.1 What to add

There are currently seven types of methods which can be added:

- k-way graph mapping methods, in module kgraph;
- graph bipartitioning methods by means of edge separators, in module bgraph, used by the mapping method by dual recursive bipartitioning, implemented in kgraph_map_rb.[ch];
- graph ordering methods, in module hgraph;

- graph separation methods by means of vertex separators, in module vgraph, used by the nested dissection ordering method implemented in hgraph_order_nd.[ch];
- mesh ordering methods, in module hmesh;
- mesh separation methods with vertex separators, in module vmesh, used by the nested dissection ordering method implemented in hmesh_order_nd.[ch];
- graph partitioning methods with separator overlap, in module wgraph.

Every method of these seven types operates on instances of augmented graph structures that contain, in addition to the graph topology, data related to the current state of the partition or of the ordering. For instance, all of the graph bipartitioning methods operate on an instance of a Bgraph, defined in bgraph.h, and which contains fields such as compload0, the current load sum of the vertices assigned to the first part, commload, the load sum of the cut edges, etc.

In order to understand better the meaning of each of the fields used by some augmented graph or mesh structure, contributors can read the code of the consistency checking routines, located in files ending in <code>_check.c</code>, such as <code>bgraph_check.c</code> for a <code>Bgraph</code> structure. These routines are regularly called during the execution of the debug version of SCOTCH to ease bug tracking. They are time-consuming but proved very helpful in the development and testing of new methods.

8.1.2 Where to add

Let us assume that you want to code a new graph separation routine. Your routine will operate on a Vgraph structure, and thus will be stored in files called vgraph_separate_xy.[ch], where xy is a two-letter reminder of the name of your algorithm. Look into the LIBSCOTCH source directory for already used codenames, and pick a free one. In case you have more that one single source file, use extended names, such as vgraph_separate_xy_subname.[ch].

In order to ease your coding, copy the files of a simple and already existing method and use them as a pattern for the interface of your new method. Some methods have an optional parameter data structure, others do not. Browse through all existing methods to find the one that looks closest to what you want.

Some methods can be passed parameters at run time from the strategy string parser. These parameters can be of fixed types only. These types are:

- an integer (int) type,
- a floating-point (double) type,
- an enumerated (char) type: this type is used to make a choice among a list of single character values, such as "yn". It is more readable than giving integer numerical values to method option flags,
- a strategy (SCOTCH Strat type): a method can be passed a sub-strategy of a given type, which can be run on an augmented graph of the proper type. For instance, the nested dissection method in hgraph_order_nd.c uses a graph separation strategy to compute its vertex separators.

8.1.3 Declaring the new method to the parser

Once the new method has been coded, its interface must be known to the parser, so that it can be used in strategy strings. All of this is done in the module strategy method files, the name of which always end in _st.[ch], that is, vgraph_separate_st.[ch] for the vgraph module. Both files are to be updated.

In the header file *_st.h, a new identifier must be created for the new method in the StMethodType enumeration type, preferrably placed in alphabetical order.

In file *_st.c, there are several places to update. First, in the beginning of the module file, the header file of the new method, vgraph_separate_xy.h in this example, must be added in alphabetical order to the list of included method header files.

Then, if the new method has parameters, an instance of the method parameter structure must be created, which will hold the default values for the method. This is in fact a union structure, of the following form:

```
static union {
   VgraphSeparateXyParam param;
   StratNodeMethodData padding;
} vgraphseparatedefaultxy = { { ... } };
```

where the dots should be replaced by the list of default values of the fields of the VgraphSeparateXyParam structure. Note that the size of the StratNode MethodData structure, which is used as a generic padding structure, must always be greater than or equal to the size of each of the parameter structures. If your new parameter structure is larger, you will have to update the size of the StratNodeMethodData type in file parser.h. The size of the StratNodeMethod Data type does not depend directly on the size of the parameter structures (as could have been done by making it an union of all of them) so as to to reduce the dependencies between the files of the library. In most cases, the default size is sufficient, and a test is added in the beginning of all method routines to ensure it is the case in practice.

Finally, the first two method tables must be filled accordingly. In the first one, of type StratMethodTab, one must add a new line linking the method identifier to the character code used to name the method in strategy strings (which must be chosen among all of the yet unused letters), the pointer to the routine, and the pointer to the above default parameter structure if it exists (else, a NULL pointer must be provided). In the second one, of type StratParamTab, one must add one line per method parameter, giving the identifier of the method, the type of the parameter, the name of the parameter in the strategy string, the base address of the default parameter structure, the actual address of the field in the parameter structure (both fields are required because the relative offset of the field with respect to the starting address of the structure cannot be computed at compile-time), and an optional pointer that references either the strategy table to be used to parse the strategy parameter (for strategy parameters) or a string holding all of the values of the character flags (for an enumerated type), this pointer being set to NULL for all of the other parameter types (integer and floating point).

8.1.4 Adding the new method to the Make compilation environment

In order to be compiled with the MAKE environment, the new method must files be added to the Makefile of the src/libscotch source directory. There are several places to update.

First, you have to create the entry for the new method source files themselves. The best way to proceed is to search for the one of an already existing method, such as vgraph_separate_fm, and copy it to the right neighboring place, preferrably following the alphabetical order.

Then, you have to add the new header file to the dependency list of the module strategy method, that is, vgraph_separate_st for graph separation methods. Here again, search for the occurences of string vgraph_separate_fm to see where it is done.

Finally, add the new object file to the component list of the libscotch library file.

8.1.5 Adding the new method to the CMake compilation environment

In order to be compiled with the MAKE environment, the new method files must be added to the CMakeLists.txt of the src/libscotch source directory.

Once all of the above is done, you can recompile SCOTCH and be able to use your new method in strategy strings.

8.2 Adding routines to the public API

The public API of SCOTCH exposes a set of routines which can be called by user programs. These public symbols are subject to specific procedures, in particular to rename them according to the users' needs.

- Implement the C interface of the routine in a src/libscotch/library_*.c source file. This source file must include src/libscotch/module.h, so that function renaming can take place whenever necessary;
- Whether appropriate, implement the Fortran interface of the routine within a src/libscotch/library_*_f.c source file. The routines in this file will call the former ones. This source file must include src/libscotch/module.h as well, so that function renaming can take place whenever necessary;
- Create a SCOTCH_NAME_PUBLIC macro in src/libscotch/module.h , so
 that function renaming can take place, for instance when symbols have to be
 suffixed;
- Create the relevant manual pages in the SCOTCH or PT-SCOTCH user's manual.

8.3 Release procedure

This section describes the procedure for releasing a new version of SCOTCH on the Inria GitLab repository: https://gitlab.inria.fr/scotch/scotch. All commands are relative to the "./src" directory of the SCOTCH distribution.

8.3.1 Removal of debugging flags

Verification of the absence of level-3 debugging flags (such definitions should contain the keyword "BROL" in the associated comment, as an alternate search keyword):

```
grep 'DEBUG_.\+3' *c | grep define
```

8.3.2 Symbol renaming

After compiling with "SCOTCH_RENAME", verification of the absence of unprotected names:

```
make scotch ptscotch esmumps ptesmumps
nm ../lib/libscotch.a | grep -v -e SCOTCH -e ESMUMPS -e " b " -e " d " -e \,
    t " -e " U " -e "scotchf" -e "fprintf" -e "fscanf" -e "malloc" -e "
    realloc" -e "free" -e "memset" -e "random" -e "get_pc_thunk." -e " r .
    LC" | (sed -z 's/\n/\#/g'; echo -n '\#') | sed 's/[a-zA-Z0-9_]\+[.]o
    [:]##//g' | sed 's/#/\n/g' > /tmp/brol.txt
nm ../lib/libptscotch.a | grep -v -e SCOTCH -e ESMUMPS -e " b " -e " d " -e
     "t " -e "U " -e "scotchf" -e "fprintf" -e "fscanf" -e "malloc" -e "
    realloc" -e "free" -e "memset" -e "random" -e "get_pc_thunk." -e " r .
    LC" | (sed -z 's/\n/\#/g'; echo -n '\#') | sed 's/[a-zA-Z0-9_]\+[.]o
    [:] ##//g' \mid sed 's/#/\n/g' >> /tmp/brol.txt
nm ../lib/libesmumps.a | grep -v -e SCOTCH -e ESMUMPS -e " b " -e " d " -e
    " t " -e " U " -e "scotchf" -e "fprintf" -e "fscanf" -e "malloc" -e "
    realloc" -e "free" -e "memset" -e "random" -e "get_pc_thunk." -e " r
    LC" | (sed -z 's/\n/\#/q'; echo -n '\#') | sed 's/[a-zA-Z0-9_]\+[.]o
    [:]##//g' | sed 's/#/\n/g' >> /tmp/brol.txt
nm ../lib/libptesmumps.a | grep -v -e SCOTCH -e ESMUMPS -e " b " -e " d " -
    e " t " -e " U " -e "scotchf" -e "fprintf" -e "fscanf" -e "malloc" -e
    realloc" -e "free" -e "memset" -e "random" -e "get_pc_thunk." -e " r .
    LC" | (sed -z 's/\n/\#/g' ; echo -n '\#') | sed 's/[a-zA-Z0-9_]\+[.]o
    [:] ##//g' | sed 's/#/\n/g' >> /tmp/brol.txt
more /tmp/brol.txt
```

8.3.3 Update of copyright year

Verification and possible modification of the copyright year, defined in the "SCOTCH_COPYRIGHT_STRING" macro:

```
emacs libscotch/module.h
```

In case the year is updated, create a commit with the message:

Set year to 20XX in copyright string

8.3.4 Update of version number

For minor revisions:

```
emacs Makefile ../doc/src/version.tex ../CMakeLists.txt
```

Then, for major releases:

```
emacs ../doc/src/scotch/Makefile ../doc/src/ptscotch/Makefile ../doc/src/maint/Makefile ../INSTALL.txt ../README.txt
```

Create a commit with the message:

Set revision marks to vX.Y.Z

8.3.5 Generation of documentation

Once the version number is changed in relevant files, generate the documentation:

```
cd ../doc/src/maint
make
make install
cd ../ptscotch
```

```
make
make install
cd ../scotch
make
make install
cd ../../src
```

In case of change of major or minor version, remove the old documentation files and add the new ones:

Create a commit with the message:

Generate documentation for vX.Y.Z

8.3.6 Creation of the local tag

On the local master branch:

```
git tag vX.Y.Z
git push --tags origin
```

8.3.7 Merging

Check that the continuous integration went well.

In fpellegr/scotch, on the left, in tab "Merge request", clich on "New merge request", to scotch/scotch. Put version number as title (e.g.: "v7.0.7"), and, in the message, a summary and itemized list of the improvements brought by the release. Then, click to generate the merge request.

In scotch/scotch, on the left, in tab "Merge request", click on the new merge request. Click on "Approve", then on "Merge". A new (short) pipe-line is launched, this time in scotch/scotch.

If the pipe-line succeeds, click to finalize the merging.

8.3.8 Creation of the public tag

In the scotch/scotch home page, click on the icon with a label ("Tag"), then on "New tag". Put the version number as the tag name (e.g., "v7.0.7").

8.3.9 Generation of the asset

In the scotch/scotch home page, click on the icon with a rocket ("Releases"), then on "New release". Select the newly created tag. Put as comment the text of the merge request.