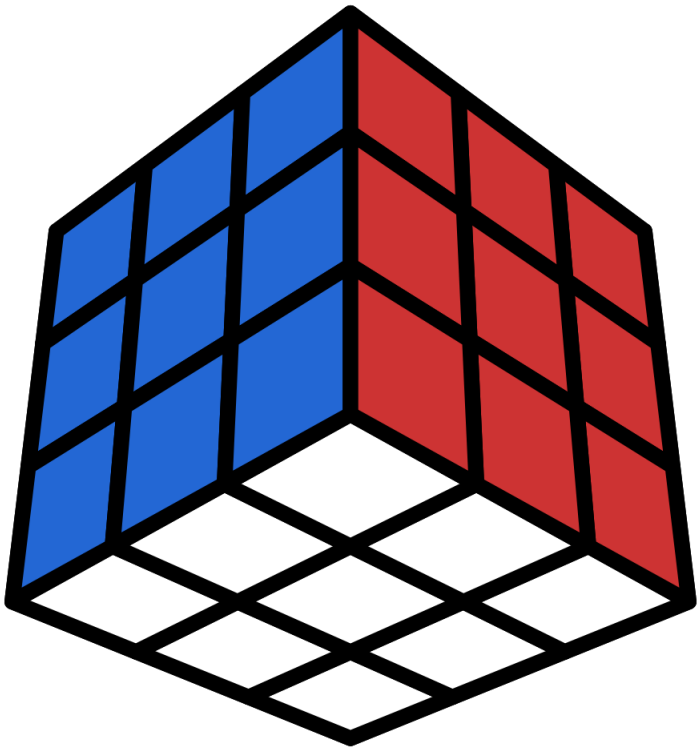
Solving the Rubik's Cube



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1. Introduction

I first learned to solve a Rubik's cube when I was twelve years old. At the time, I followed a series of YouTube videos by online creator Dan Brown. My first attempt took me a few hours, but once it was finally solved, it was exhilarating. Little could describe the feeling of satisfaction that comes with solving a Rubik's cube for the first time, a puzzle that is often portrayed by pop culture as the quintessential brain teaser for geniuses. Spoiler alert: it's not. In only a couple of days of practice, I was able to consistently solve the cube in under 3 minutes. I didn't know it at the time, but for the next 8 years of my life I would become very competitive at strange discipline that is speed solving the cube.

Today, I can solve a Rubik's cube in about ten seconds; I certainly know much more about the puzzle than I did at the time. In this instruction manual, I hope to bring you the same feeling I got when I solved my first cube. Moreover, I hope to convince you that solving a Rubik's cube is not as hard as you may imagine, and that given a methodology, anyone can do it. If you're ready to crack the cube, keep reading.

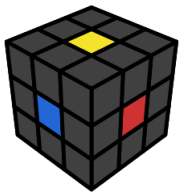
2. Thinking about the Cube

Before we can start solving the cube, there are various concepts which will help us in thinking about the puzzle. It is worth the effort to consider them before we start solving, as it will make the process smoother as we go on.

2.1. The Various Types of Pieces

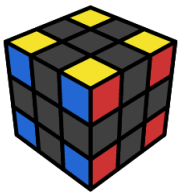
It is common for beginners to think about the Rubik's cube as a six sides, each with nine stickers. However this interpretation is actually quite limiting. In order to solve the cube, it will be better for us to think of the cube not as uncoordinated stickers, but instead as *interconnected types of pieces*.

Rubik's cubes have three types of pieces: centers, edges, and corners.



The centers

The edges



The corners

It's crucial to note that *each piece type will always remain the same.* That is: centers will always be centers, edges will always be edges, and corners will always be corners.

Additionally, no matter what moves you make on the cube, *the centers will never move, relative to each other.* This is extremely important, because it means that we can use the centers of the cube as a reference for how to solve it.

To solve the cube is to put every piece where it belongs. This may sound obvious, but how do we know where any piece needs to go? The answer is actually quite simple: **a piece goes**

between the centers of its colors. This is so important that it is worth considering some examples.

Example 1 — Solving an Edge

Suppose we are solving the cube and we are interested in solving the BLUE, YELLOW edge.

Note: The grayed-out pieces don't matter, they could be anything and we are not interested in them.

To solve this piece, we would need to put the BLUE, YELLOW edge between the BLUE YELLOW centers.

This solves the edge.

Example 2 — Solving a Corner

Suppose we are solving the cube and we are interested in solving the BLUE, YELLOW, RED corner.

To solve it, we would need to put the BLUE, YELLOW, RED corner between the BLUE, YELLOW, RED centers.

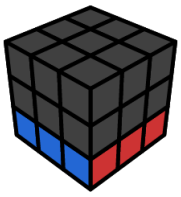
This solves the corner. Notice that a piece can be in the correct *location*, but *oriented* incorrectly. This can happen for both corners and edges.

> ***Note***: The *location* of a piece is where it is on the cube. The *orientation* of a piece is how it is rotated.

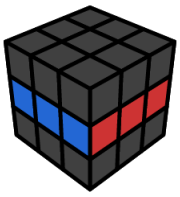
Don't worry if not everything makes complete sense at this stage, we will develop most of this intuition as we solve the cube.

2.2. The Different Layers

Another very helpful type of intuition we use in solving the cube is to think about it in *layers*. A standard 3x3 Rubik's cube has three layers: the first layer, the second layer, and the third layer (most often referred to as the last layer.)



The first layer



The middle layer

The last layer

We differentiate the layers in this way because we will solve each one slightly differently. Notice that the first layer is on the *bottom*: it is common, when solving the cube, to maintain one orientation throughout the solve.

2.3. The Colors Matter

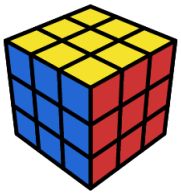
The last important piece of intuition we need before we can start solving our first cube is the notion of *adjacent* and *opposite* colors. This is arguably the most subjective part of this guide, and you don't absolutely need to know it, but it can be helpful to know when solving the cube.

On a *standard* Rubik's cube, the color scheme is as follows:

- White opposite Yellow
- Blue opposite Green
- Red opposite Orange

Note: For people with colorblindness, it is common to change red stickers to pink, or some other color to distinguish it from the green.

The two opposite sides of the cube illustrate which colors are adjacent.



Most cubes you will encounter will follow this color scheme.

3. Notation

In order to manipulate the cube, people have developed a notation, luckily, it is quickly learned, and if you forget it, you can simply refer back to this part of the manual.

There are six different types of moves (one for each side) and three different types of rotation (one for each axis) of the cube, each of which with two variants (clockwise or counter-clockwise.)

In this table, you would be holding the cube with Yellow on top, and Blue in front.

3.1. Moves

U

F

R

D

B

L

U'

F'

R'

D'

B'

L'

3.2. Rotations

x

y

z

x'

y'

z'

4. The First Layer

It's time to actually solve the Rubik's cube! As mentioned above, we will be doing this layer by layer. This means that the first step on our journey is to complete the first layer. There are three major steps to completing the first layer: building the daisy, building the cross, and inserting the corners. We'll be sure to take it one step at a time.

4.1. The Daisy

The first step on our quest to solve the cube is called the daisy. By the end of this step, our cube will look like this.

The goal of this step is to prepare ourselves for the cross, which is often a difficult step for beginners because the way that it is completed highly depends on the state of your cube. To resolve this, its good to split up this step into 2 to make it easier.

To complete the daisy, look for all White edges, and the Yellow center. Our goal will be to move all of these edges on the yellow face.

Notice that both the other centers and the second stickers on the white edges are grayed out, this is because they don't matter at this stage. In other words, you can put any white edge on the yellow side and as long as your stickers match the ones in the image, you're doing the right thing.

Let us look at some examples,

Example 1

(start)

`R`

Example 2

(start)

`U`

`F'`

4.2. The Cross

Now that we have our daisy, finishing the cross should be much easier. Pick one of your White edges, and by rotating only the yellow side, align it with its adjacent center, as shown

Once the edge is aligned, simply rotate the adjacent color's center twice to place the edge in its final location

Now repeat this process with the other 3 edges, and rotate your puzzle around: you should see your cross is fully solved.

4.3. Inserting the Corners

Our goal for this step is to complete the first layer of the cube, all that is left for us to do here is to insert the corners. By the end of this step, our cube will look like this.

Note: We can inspect and rotate the cube at anytime, but keep in mind that when working on it, the white side should always on the bottom.

To do this, we first need to find our four WHITE corners. We'll focus on each one at a time.

There are two possibilities for where these corners might be, either in the top layer or in the bottom layer. We'll look at each case separately to learn how to deal with each of them.

In either case, you'll find that the algorithm that we perform is the same, so this should make no difference.

The Algorithm

```
R U R' U'
```

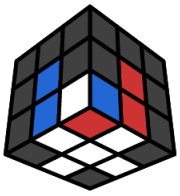
4.3.1. Top layer corners

Our goal will be to insert the WHITE corners in their respective positions. The first step is to *position the corner above where it belongs.*

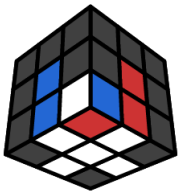
In this case, we picked the BLUE, WHITE, RED corner, and rotated it between the RED, and BLUE centers.

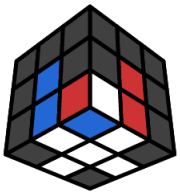
> ***Note***: The corner currently between the BLUE, WHITE, and RED center could be any piece, it does not matter to us since it will be pushed out.

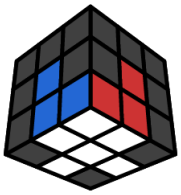
Now perform the algorithm `R U R' U'` . This should insert the corner in the correct location. However there is a problem: the corner might be where it belongs, but it could be *misoriented*, as shown below.



To solve this, simply repeat the algorithm in twos until the corner is oriented correctly. Below is a figure representing the three possible orientations of a corner.







You can repeat this process with any of your top layer corners, keeping in mind that you are only targeting corners with WHITE on them.

4.3.2. Bottom layer corners

If a WHITE corner is in the bottom layer, simply perform the algorithm once, this should turn the corner into a top layer corner. You can then simply follow the instructions for that step.

Example white corner

After `R U R' U'`

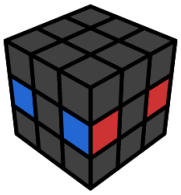
You can repeat this process for each of your bottom layer corners.

Once you have repeated the steps described above, your first layer should be complete! At this stage, you can pat yourself on the back because we are actually closer to being than you may think, and the difficult part is largely behind us. If you are ready to keep going, let's move on to the second layer.

5. The Second Layer

The second layer might seem daunting, but we actually only need to solve 4 pieces, this is because the centers are already done for us (recall that centers are always solved relative to each other)

At this step, the only pieces we are looking for are *edges with neither* WHITE *nor* YELLOW *on them.* We are essentially solving only these four pieces.

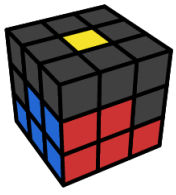


The second layer is very algorithmic, which means that we will mostly performing one algorithm the whole time. To be exact, we will be performing one of two different algorithms: what we will call the *lefty* algorithm and the *righty* algorithm. These algorithms are actually exactly the same, one is simply the mirror of the other.

5.1. The Lefty Algorithm

The lefty algorithm inserts an edge into the left side.

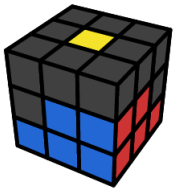
```
(U' L' U L) (U F U' F')
```



5.2. The Righty Algorithm

The righty algorithm inserts an edge into the right side.

```
(U R U' R') (U' F' U F)
```



> ***Note***: We parenthesize algorithms to make them easier to read. This does not change how we actually execute them: we still read them from left to right.

How it works

What these algorithms allow us to do is to *insert* an edge into the middle layer (what we are working on) without disturbing any of our existing work (i.e. The bottom layer.) Specifically, these algorithms take the *top front* edge, and insert it either in the *front right* or *front left* edge

> ***Note***: As a beginner, it is perfectly normal (and in fact expected) if you don't understand how this algorithm works. As you gain more experience with the puzzle, you will learn to understand your algorithms much better.

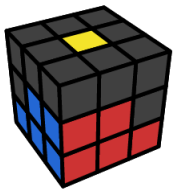
5.3. Solving

In order to solve the second layer, first identify an edge that doesn't have Yellow or White on it. In this case, we'll be solving the Red Blue edge.

Next, align this edge with its center

Now you need to figure out if this edge needs to go to the left, or to the right. To do this, look at the top sticker of the edge, if the matching center is on the left, perform the lefty algorithm. If the center is on the right, perform the righty algorithm. In this case, our edge goes on the left

From here, we perform the lefty algorithm, which inserts our edge into its final location.



5.3.1. Misinserted Edges

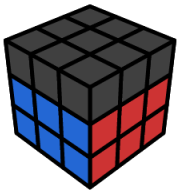
It's possible at some point throughout this process that an edge you want to solve will happen to already be in the second layer, but placed or oriented incorrectly.

Incorrect Placement

Incorrect Orientation

If this is the case, simply pick a random edge in your top layer, and treat it as if it belonged in that spot. Then, using the lefty or the righty algorithm, use it to kick out the edge you want which you can then insert again correctly.

Once you have repeated this process with all four edges, your second layer should be solved.



6. The Last Layer

We've solved two thirds of our cube! This is a big milestone because at this stage, the puzzle is solved enough that, by covering the last layer in the palm of your hand, you could trick somebody into thinking it's completely solved.

If you aren't so interested in these tricks, do not worry as by the end of this section, we will have completely and honestly solved it.

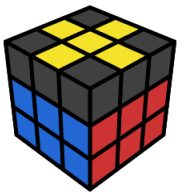
The last layer is unfortunately the most involved part of the solve, and with good reason: more progress means we need be more careful not to undo our progress. To do this, we will introduce only *two* new algorithms, and a couple of variations on things we've seen before.

Solving the last layer is done in four steps: (1) orienting the edges, (2) permuting the edges, (3) permuting the corners, (4) solving the corners. We'll be sure to take it one step at a time and before we know it, our cube will be solved.

6.1. Edge Orientation

We start solving the last layer by the edges. Our goal in this step will be to create a Yellow cross with all four correctly permuted edges. By the end of this stage, our cube will look like this.

What we want to accomplish in this step is to *orient* all edges of the last layer correctly. More concretely, by the end of this step, we will have our last layer edges forming a cross on the top layer, as such



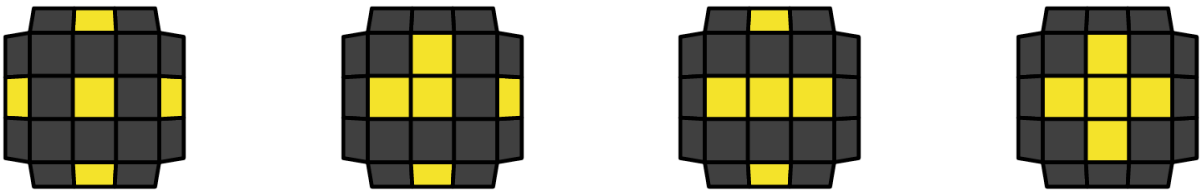
Doing this is surprisingly easy, as we will only be repeating one algorithm until it is done. The algorithm is as follows

```
F (R U R' U') F'
```

Notice the `R U R' U'` . This is the same algorithm that we used all the way back when we were inserting the first layer corners in section 4.3.

Note: This four move algorithm is actually used so often that it has a name: *the sexy move.*

To solve the YELLOW cross, first look at the edges of your last layer. You should see one of the following four patterns.



If you already have a cross on the top layer, you can skip this step. If you don't, first identify which of the following patterns you have in your top layer, and align it to match with its corresponding case. Next, simply perform the algorithm once. This should change the shape on your top layer to match another case shown above. Align it so that it matches the picture, and keep perform the algorithm again. Keep repeating this process until your cross is fully solved.

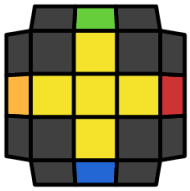
6.2. Edge Permutation

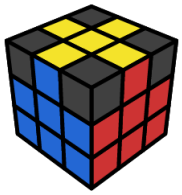
Next up is to permute the edges. To do this, we'll use a simple algorithm: *the sune*. This again is an algorithm that is so common that it has its own name. The sune is a great algorithm for plenty of reasons, but for us now, it's only good for one thing: permuting edges.

The sune algorithm goes as follows

```
(R U R') (U R U2 R')
```

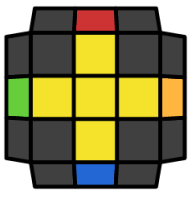
There are 3 possible cases that we can get at this stage of the puzzle. The first is that all of our edges are correctly permuted, in which case you can skip this step! This happens surprisingly often and it's always a nice surprise.



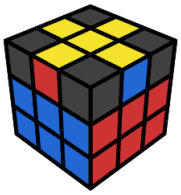


If your last layer edge colors match the picture above, (note that you might need to rotate the layer for your to match) you can skip this step.

The second possible case is **the opposite swap**: when two opposite edges need to swap sides.



The third and final possibility is **adjacent swap**: when two adjacent edges need to swap.



Note: If you can't identify which of these cases you think you have, rotate the top layer and check again, as it my be difficult to spot at first glance.

The Adjacent Swap

If two adjacent edges of your puzzle need to swap, place the two incorrect edges in front and to the left as shown, and perform the following algorithm.

```
[sune] U
```

Note: In the image above, RED is treated as the front face.

The Opposite Swap

If two opposite edges of your puzzle need to swap, place the two incorrect edges in front and in the back (so the side edges of your puzzle should be correct), and perform the following algorithm:

```
[sune] U' [sune]
```

6.3. Corner Permutation

We've finally solved all of our edges, which means that we're down to the last four corners of the cube. You can pat yourself on the back for the work you've done so far; most of the work is behind us. At this stage, there's only one new algorithm that we're going to need to learn, and I promise that it's easy. After that, we'll use our good old sexy move to orient our corners and before we know it, we'll have solved the Rubik's Cube.

To permute the corners, we'll need that new algorithm I was mentioning. This algorithm has no name, and frankly I've never seen it used at the top level, but for the sake of this guide, we'll call it **the Pillar's Algorithm**.

The Pillar's Algorithm

```
U R U' L U R' U' L
```

This might look big and scary but I promise that it's easier than you think. The reason I call this algorithm the pillar's algorithm is because, when you do it, it looks like two pillars getting broken up and put back together. One pattern to notice when learning this algorithm is that the top layer always moves back and forth, and the left and right side alternate.

What this algorithm does is actually very simple: it performs a *clockwise rotation* of the left and back two corners, keeping the front right corner in place. We'll use it to find a systematic method to permute the corners.

At this stage, you can run into one of two scenarios: the first is that all of your corners are permuted correctly (in which case you can skip this step), the second is that one of them is permuted correctly, but the others are not, and the last is that none of them are permuted correctly.

> ***Note***: Be sure to align your edges with their correct sides before evaluating if your corners are correctly permuted.

No correct corners

If none of your corners are correctly permuted, you can simply perform the pillars algorithm and check again.

One correct corner

If one of your corners is permuted correctly, hold the cube so that the corner is on the right hand side, and perform the pillars algorithm. At this stage, one of two things could happen: (1) that correctly permuted all your corners, and you are done with this step, or (2) your other corners are still not correctly permuted. If (2) is the case, simply perform the pillars algorithm again, still with the correct corner in the right hand side, and check again.

By this point, all your corners should be correctly permuted.

6.4. Corner Orientation

We've reached the final step on our journey to solve the cube. At the beginning, you may not have believed that you would solve this thing, but at this stage, I hope that you believe it more.

For this step, we're going to turn the puzzle around completely, so that the side that was once our white cross is now facing up. Now a simple but methodical procedure will follow. If you execute these instructions without fail, I vow that your puzzle will be solved, however if you make a mistake, you may be set back quite far in the solving process. As a kid, this was by far the most stressful step for me, and understandably so: who would want to make a mistake so close to the end? To make this step easier, we'll break down the process as much as possible.

The Procedure

1. Rotate the bottom layer of the cube so that an incorrectly oriented corner in the bottom right hand side.

2. Perform `[sexy]` twice. Recall that `[sexy]` is the algorithm that we used all the way

back when making our first face: `R U R' U'` .

3. If the corner is solved but the cube is not fully solved, go to step 1. If the corner is not solved, repeat step 2.

At this stage, your cube should be fully solved! If this is your first time solving a Rubik's Cube, congratulations! It is an honor to me for this guide to be some people's introduction to the puzzle. I wish you the best in your cubing adventures.

7. Concluding Thoughts

If you were able to solve your first Rubik's Cube using this guide, congratulations, you have achieved something that most people can't say they've done. I hope that you can see that solving the Rubik's Cube is not necessarily as hard as you may have imagined. I hope you see that, using the right procedure, almost anyone can do it. The sense of wonder that comes with solving a Rubik's Cube is something that I hope to have conveyed to you in this guide, in the same way that it was conveyed to me when I was younger. With a little more practice, you should not need to refer to this guide as much and, eventually, you should no longer need it at all. However you'd be foolish to think that this is all there is to the cube: some of the top players today use hundreds of different algorithms to shave off fractions of a second from their times, and the scene is very competitive. Should you chose to go down this route of competitive speedcubing, it has been my pleasure to lay down the first brick on the road of your journey.