Person code: hqy408
Subject: Physics
Word Count: 4000

How can the physical build of a drone be changed to increase its velocity?

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1. Introduction

A favourite hobby of mine is to hike in the alps, which comes with risks of injury due to unadapted footwear. When hiking alone, a small injury can rapidly become a large problem, as it's difficult to get emergency assistance. Yearly, an average of 110 people die while hiking in Austria[1]. Rapid aerial intervention could help injured hikers by providing assistance or medication. Helicopters and large drones require space to land, and are also expensive to build and fly. It is therefore necessary to build a small and fast drone which could be deployed in a matter of minutes. Therefore, the purpose of this essay is to answer the question:

How can the physical build of such a drone be changed to increase its velocity?

More specifically, how one can make informed engineering decisions when designing a drone based on the underlying physics of drone flight. The type of drone investigated is a quadcopter, which possesses 4 rotors, for greater stability and control than a standard helicopter or jet-propelled aircraft, as it can move in all directions, as seen in Figure 1, meaning it is more fit to answer problems of rapid intervention in cluttered environments[2].

Figure 1 - First Person View racing quadcopter [3]

[1] Gatterer, Hannes, et al. "Mortality in Different Mountain Sports Activities Primarily Practiced in the Summer Season-A Narrative Review." *International Journal of Environmental Research and Public Health*, MDPI, 15 Oct. 2019, www.ncbi.nlm.nih.gov/pmc/article

[2] Satam, Ihab Abdulrahman. "Studying a Flying Robot for Path Planning and Scanning", *Northern Technical University*, Jun. 2015, https://www.researchgate.net/publication/342419637_Studying_a_Flying_Robot_for_Path_Planning_and_Scanning

[3] "[Hot Item] High Quality Hot Selling 220mm Carbon Fiber Qav220 Fpv Racing Quadcopter Brushless Motor 20A ESC for Sale." *Made*, www.made-in-china.com/showroom/ztrdrone/product-detailWBcECUbulOVo/China-High-Quality-Hot-Selling-220mm-Carbon-Fiber-Qav220-Fpv-Racing-Quadcopter-Brushless-Motor-20A-ESC-for-Sale.html.

The best material, in terms of its strength to mass ratio, will be determined experimentally by calculating the shear modulus of 3 materials: balsa wood, aluminium and a carbon-fiber-reinforced-polymer composite, or CFRC.

Propeller choice is an important aspect of maximising forward thrust, as different propeller shapes and sizes result in different air flow, which in turn changes the lift force, and the energy usage of the motors. More energy usage necessitates heavier batteries to ensure sufficient flight time, which increases the mass of the drone, and therefore slows it down[4]. The best trade-off between energy consumption and velocity, by changing the shape and size of the propellers, will also be determined.

1.1. Why must a drone be made lighter?

To increase the horizontal velocity of the drone, its forward thrust force must increase. As an object travels faster in the air, the drag forces increase, due to more frequent collisions with air particles[5]. When travelling at a constant velocity, the forward force and drag force on the drone are equal, according to Newton's 1st law[6]. Therefore, for the velocity of the drone to increase, which also increases drag forces, the thrust force of the drone must also increase, achieved by decreasing the mass of the drone. To understand why, it is important to understand how drones fly.

To hover, the downwards force that gravity exerts on the drone must be balanced by the upwards force that the motors exert on the air. This is due to Newton's 3rd law[7], where the first body is the drone, and the second the air. As the propellers push down the air, the air pushes the drone up by the same force. If this upwards force is equal to the weight of the drone, which is equal to its mass multiplied by the acceleration due to gravity, such that:

$$W = m \times g,$$

where m is the mass (kg) and g the acceleration due to gravity[8], $9.81\, ms^{-2}$, the drone is in translational equilibrium and therefore hovers[9]. A smaller mass means a smaller weight, and so a smaller needed upwards force.

[4] "Factors to Consider When Selecting Drone Propeller Blades." *Unmanned Systems Technology*, 29 Apr. 2020, www.unmannedsystemstechnology.com/2017/12/selecting-drone-propeller-blades-considerations/.
[5] "Free Fall and Air Resistance." *The Physics Classroom*, www.physicsclassroom.com/class/newtlaws/Lesson-3/Free-Fall-and-Air-Resistance.
[6] "Newton's Laws of Motion." *Encyclopædia Britannica*, Encyclopædia Britannica, Inc., www.britannica.com/science/Newtons-laws-of-motion.
[7] "Newton's Laws of Motion." *Encyclopædia Britannica*, Encyclopædia Britannica, Inc., www.britannica.com/science/Newtons-laws-of-motion.
[8] "The Acceleration of Gravity." *The Physics Classroom*, www.physicsclassroom.com/class/1DKin/Lesson-5/Acceleration-of-Gravity.
[9] Tippens, Paul E., "Chapter 4A, Transnational Equilibrium", *Southern Polytechnic State University,* 2007.

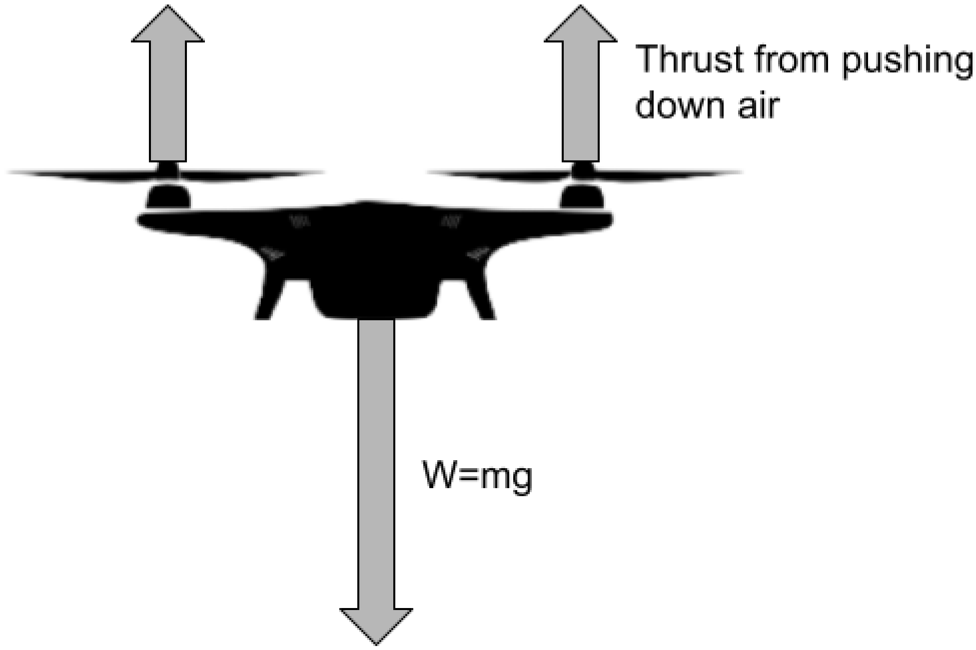

Figure 2 - Free body diagram of a drone hovering [10]

To move forwards, the drone must tilt at an angle θ, as seen in Figure 3, so that the thrust is split into a horizontal and a vertical component, allowing it to fly horizontally, as seen by the thin black lines on the diagram.

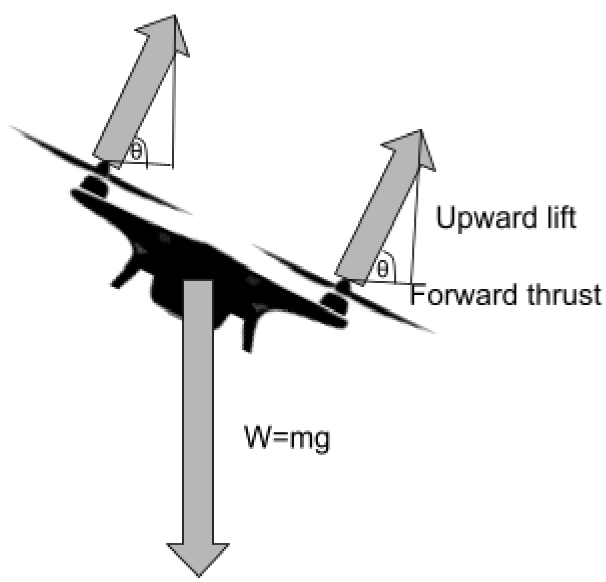

Figure 3 - free body diagram of a drone when it travels forward

[10] “Drone Clipart Clip Art - Transparent Background Drone Clipart, HD Png Download - Vhv.” *Vhv.rs*, www.vhv.rs/viewpic/TToihiJ_drone-clipart-clip-art-transparent-background-drone-clipart/.

The vertical component must remain greater than the weight, however, to ensure the drone remains airborne[11]. Tilting the drone more forward results in greater forward force, which in turn results in a greater velocity, as explained previously. However, if the overall force that the propellers exert on the air remains constant, the downward force will decrease, since the vector sum of both forces remained constant. If the drone becomes vertical, it will travel at maximum horizontal velocity, due to thrust becoming the horizontal component entirely. Without any downward force on the air, it will no longer fly[12].

Therefore, the smaller the necessary downward force is (due to smaller mass), the larger the horizontal component of thrust can be, and the larger the forward velocity.

To be able to fly, drones require a number of components which result in a minimum mass, even when optimized. Table 1 breaks down all necessary components and their minimum mass for a small drone. There are few opportunities to decrease the mass. One of the central and heaviest components of the drone is the frame[13]. Better material choice can result in a stronger and lighter frame, allowing for a faster and more reliable drone.

Table 1 - Breakdown of all necessary components and their mass

Component	Mass (g)
Camera	3
GPU	18
ESCs	15
Power bank (battery)	300
Flight controller	20
Inertial measurement unit	9
Connectors	5
Frame	To be determined

The total minimum mass is therefore around 370g, and the mass of the frame should be decreased to a maximum.

[11] Reid, John S., "Drone flight – what does basic physics say?", *Basic Drone Physics*, 2018.
[12] Reid, John S., "Drone flight – what does basic physics say?", *Basic Drone Physics*, 2018.
[13] Rodman, Dan. "How Much Do FPV Drones Weigh?" *Drone Jay | A Blog Containing Both Informational Tips and Interesting Articles about Drones*, 20 May 2020, dronejay.com/how-much-do-fpv-drones-weigh/.

2. Choice of material to build a light but sturdy frame between balsa wood, aluminium and CFRC.

The simplest possible quadcopter structure consists of two rigid tubes forming a cross, with the electronics at its center, and motors at its ends. This minimises material use and good maneuverability, since the drone is symmetrical on two axes[14], but does not maximize velocity. When drone width equals length, it can travel in any direction at equal velocity, providing an equal amount of forward thrust. However, it is most important it can travel forwards the fastest, to ensure maximal velocity in straight lines[15]. To achieve this, it is important to have the drone be as vertical as possible, to have it push as much air behind it as possible, while making sure enough air is pushed downwards ensuring the drone remains airborne. To increase this, the vertical distance between the front and back propellers must be increased, achievable by increasing drone width. With the same angle θ to the horizontal, a longer hypotenuse will result in a longer triangle base, as illustrated in Figure 5, where an increase in z results in an increase in the distance between the back and front propellers, y.

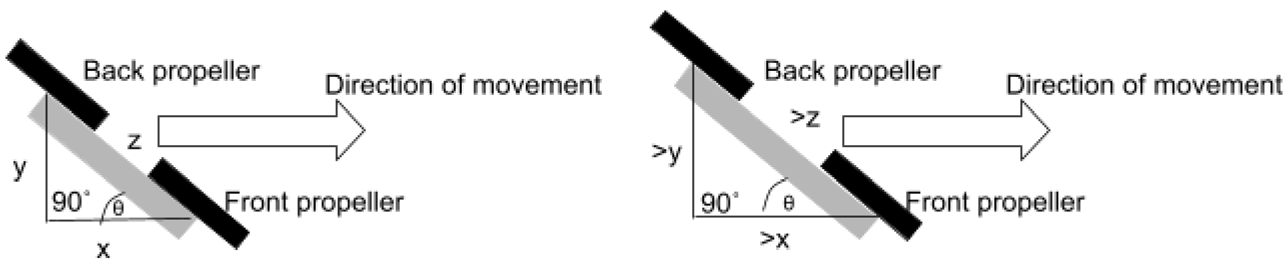

Figure 4 - How does changing the length z affect the height y with constant angle θ

Increasing drone length increases size and therefore mass. This extra mass then means more weight has to be overcome for take off, and the angle θhas to be decreased, resulting in y remaining the same even with an increase in z. However, this assumes the width of the drone is kept constant. High thrust is not necessary when the drone flies sideways, and so the width of the drone can be decreased. This loss of sideways velocity allows for a lighter drone, and so allows for more forward thrust and therefore velocity.

The only component that can be made lighter is the frame (see Section 1.1). Its material can be chosen to allow for more strength and less mass. The material must fulfill two criteria: it must be strong enough to not deform as the drone applies downward force, as well as resist strong winds and potential crashes. It must also not be completely rigid, for it to be able to absorb vibrations caused by the motors without making the entire drone vibrate.

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[14] Brown, Written by Jack, and Jack Brown. “How to Build Your Own Drone: Step-by-Step DIY Homemade Project.” *My Drone Lab*, 6 May 2021, www.mydronelab.com/blog/how-to-build-a-drone.html.
[15] PBS Terra, “The Extreme Physics Behind Drone Racing”, *PBS Terra*, 28 Jan. 2021, https://www.youtube.com/watch?v=Pgj95EntvW0

A number of materials fulfill the rigidity criteria including aluminium, carbon fibre reinforced composite (CFRC), different types of wood and thermoplastics. However, materials such as PVC or plywood quickly lose their tensile strength as the required pieces get larger[16]. Therefore, aluminium, CFRC and balsa wood were selected, and the shear modulus for each was determined experimentally, to find which material had the highest strength to mass ratio while remaining somewhat flexible.

2.1. What is the shear modulus of a material?

The shear modulus of a material refers to its stiffness, or how easy it is to bend or stretch the material. The higher the modulus, the harder it is to bend the material. It is found by dividing the stress on the material by the strain, such that the shear modulus, in Pa, is:

$$Shear\ modulus\ =\ \frac{Stress}{Strain}$$ [17]

Stress is the force, in N, applied on the material divided by the cross sectional area of the material, in m^2, such that:

$$Stress = \frac{Force}{Area}$$

Strain is the deflection, in m, perpendicular to the orientation of the material, divided by the length of the material, in m, such that:

$$Strain = \frac{deflection}{length}$$

Figure 5 details how deflection is found.

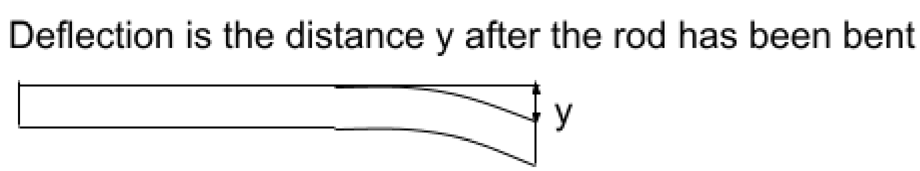

Figure 5 - Details of how deflection is measured

Therefore, overall the shear modulus is found in Pa such that:

$$Shear\ modulus\ =\ \frac{\frac{Force}{Area}}{\frac{deflection}{length}}$$

2.2. Methodology

To find the shear modulus of the three selected materials, a simple experiment was designed. Masses were attached to a rod of each material, which was allowed to bend on a length of 0.1 m. This length was

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[16] coderman, Nikola Jovanovic -. "What Are Drones Made Of? - Materials and Engineering Resources." *Matmatch*, 11 Sept. 2021, matmatch.com/blog/what-are-drones-made-of/.

[17] *Young's Modulus*, depts.washington.edu/matseed/mse_resources/Webpage/Biomaterials/young's_modulus.htm.

chosen due to it being the typical length of drone arms. The vertical distance between the original height of the rod and its new height were measured using a caliper. The length, mass, outer diameter and inner diameter (the aluminium and CFRC rods were hollow of each rod was measured. The mass was measured using a conical flask to hold the rod vertically on a weighing scale. The experiment was set up as seen in Figure 6.

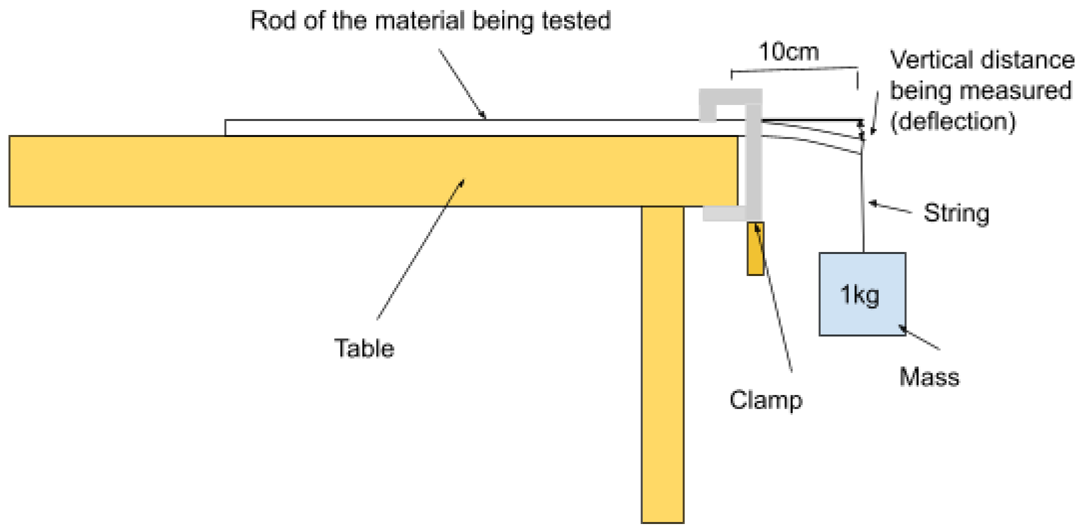


Figure 6 - Setup of the experiment to determine the shear modulus

The experiment was repeated using 7 different masses, going from 0.5kg to 1.1kg, with increments of 0.1kg. For each mass, 3 trials were conducted, and an arithmetic mean of the deflection was taken between the 3 trials. The shear modulus was then calculated, before it was divided by the mass per each 0.1 m of the material, giving the shear modulus per kg of material.

2.3. Experimental data

Table 2 - Deflection in mm for each mass in range 0.5 to 1.1 kg for a 0.1 m long rod of balsa wood with cross sectional area $3.14 \times 10^{-4} m^2$

	Deflection (mm)			
Mass used (kg)	Trial 1	Trial 2	Trial 3	Arithmetic mean
0.5	2.5	2.3	2.3	2.4
0.6	4.0	3.9	4.3	4.1
0.7	5.0	5.0	5.0	5.0
0.8	6.0	6.0	6.0	6.0
0.9	6.2	6.5	6.3	6.3
1.0	7.0	7.0	7.0	7.0
1.1	7.5	7.5	7.4	7.5

Table 3 - Distance from horizontal for each mass in range 0.5 to 1.1 kg for a 0.1 m long rod of aluminium with cross sectional area $1.38 \times 10^{-4} m^2$

	Deflection, mm			
Mass used (g)	Trial 1	Trial 2	Trial 3	Arithmetic mean
0.5	1.5	1.5	1.5	1.5
0.6	2.0	2.0	2.0	2
0.7	2.5	2.5	2.5	2.5
0.8	3.1	3.0	3.0	3.0
0.9	3.5	3.6	3.5	3.5
1.0	4.1	3.9	4.0	4.0
1.1	4.4	4.5	4.4	4.4

Table 4 - Distance from horizontal for each mass in range 0.5 to 1.1 kg for a 0.1 m long rod of CFRC with cross sectional area $1.13 \times 10^{-4} m^2$

	Deflection, mm			
Mass used (g)	Trial 1	Trial 2	Trial 3	Arithmetic mean
0.5	0.8	0.8	0.8	0.8
0.6	1.0	0.9	0.8	0.9
0.7	1.0	1.0	1.0	1.0
0.8	1.2	1.1	1.1	1.1
0.9	1.3	1.2	1.1	1.2
1.0	1.1	1.2	1.2	1.2
1.1	1.3	1.2	1.3	1.3

Table 5 - Stress and strain values for each material, at each mass in the range 0.5 to 1.1kg with increments of 0.1kg, calculated using the arithmetic mean of the perpendicular distance from the horizontal position of the rod, and a g value of $9.80665 ms^{-2}$.

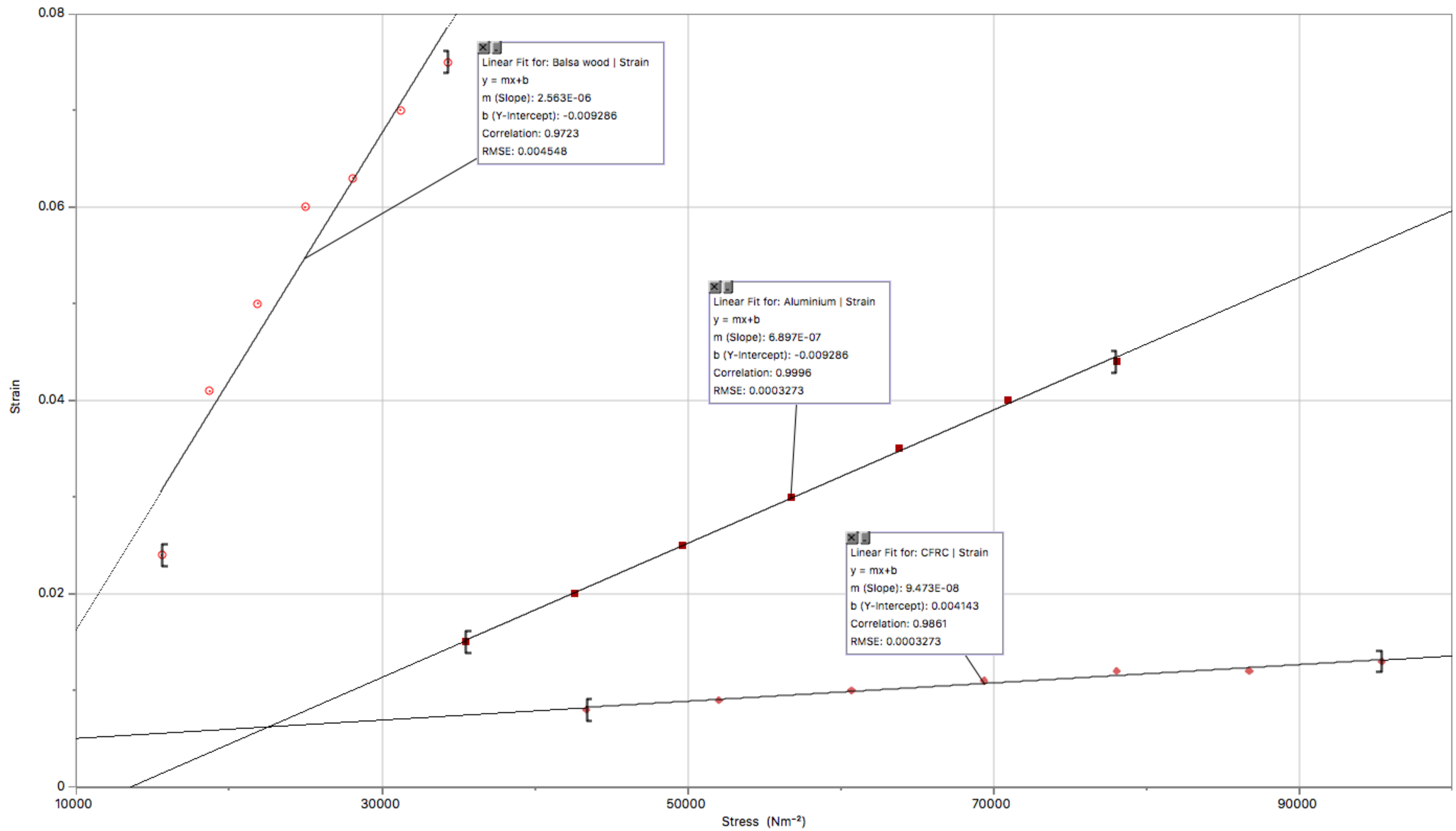
Note: strain has no units since it is simply the ratio between lengths, or $\frac{m}{m}$, which is simply equal to 1.

	Data for a 0.1 m long rod of balsa wood with cross sectional area $3.14 \times 10^{-4} m^2$		Data for a 0.1 m long rod of aluminium with cross sectional area $1.38 \times 10^{-4} m^2$		Data for a 0.1 m long rod of CFRC with cross sectional area $1.13 \times 10^{-4} m^2$	
Mass used (kg)	Stress (Nm^{-2})	Strain	Stress (Nm^{-2})	Strain	Stress (Nm^{-2})	Strain
0.5	15600	0.024	35500	0.015	43400	0.008
0.6	18700	0.041	42600	0.020	52000	0.009
0.7	21900	0.050	49700	0.025	60700	0.010
0.8	25009	0.060	56800	0.030	69400	0.011
0.9	28100	0.063	63800	0.035	78000	0.012
1.0	31200	0.070	70900	0.040	86700	0.012
1.1	34300	0.075	78000	0.044	95400	0.013

Table 6 - Example calculations for balsa wood when 0.5kg was used.

Action and formula	Result
$Stress = \frac{Force}{Area} = \frac{m \times 9.81}{cross\ sectional\ area}$	$\frac{0.5 \times 9.81}{3.14 \times 10^{-4}} = 15621 Nm^{-2}$
$Strain = \frac{deflection}{length} = \frac{\Delta l}{l}$	$\frac{2.4 \times 10^{-3}}{0.1} = 0.024$

Figure 7 - Strain experienced by a 0.1 m rod of balsa wood, aluminium and CFRC when varying amounts of stress is applied (through the use of masses ranging from 0.5kg to 1.1kg, in increments of 0.1kg)



2.4. Results analysis and conclusion

As can be seen on the graph in Figure 7, there is a clear linear trend between stress and strain. This is expected, as the higher the mass used, the more the rod was seen to bend. The gradient of each trend line, denoted by m on the graph, can be used to calculate the shear modulus for each material. In fact, it is calculated such that:

$$Gradient = \frac{Strain}{Stress}$$

This can be written as

$$Gradient = \frac{1}{Shear\ modulus},$$

since the the shear modulus in Pa is found as

$$Shear\ modulus = \frac{Stress}{Strain}.$$

Therefore, the shear modulus in is simply the inverse of the gradient, or

$$Shear\ modulus = \frac{1}{gradient}.$$

Using this, the shear modulus for each material can then be calculated.

Table 7 - Gradient of the trend line of the graph, shear modulus and shear modulus per mass obtained when plotting strain against stress for balsa wood, aluminium and CFRC

Material	Gradient of trend line (m^2N^{-1})	Shear modulus (*Pa*)	Shear modulus per mass (*Pa.kg*$^{-1}$)
Balsa wood	2.56×10^{-6}	3.90×10^{5}	8.17×10^{7}
Aluminium	6.90×10^{-7}	1.45×10^{6}	1.66×10^{8}
CFRC	9.47×10^{-8}	1.06×10^{7}	2.59×10^{9}

From this data it is clear that CFRC has the higher shear modulus at 1.06×10^{7}Pa, while also being the lightest material, the 0.1 m only weighing in at 4.1×10^{-3}kg. Its shear modulus per mass ended up at 2.59×10^{9}, more than 15 times the shear modulus per mass of aluminium, the next best material tested. 1kg of aluminium is twice as rigid as 1kg of balsa wood. Although the aluminium was bent significantly less than the balsa wood rod during the experiment, its higher mass explains this smaller difference in shear modulus per mass, while CFRC both saw the smallest deflection and was the lightest material.

The percent discrepancy between the experiment shear modulus found for each material and the accepted theoretical values was calculated such that:

$$Discrepancy\ (\%) = \frac{theoretical - experimental}{theoretical} \times 100$$

For balsa wood, the theoretical value[18] is at 3.5×10^{6}, giving a discrepancy of 9.89%. For aluminium, however, the theoretical value[19] is at 2.5×10^{8}, giving a discrepancy of 9.99%. Finally, for CFRC, the theoretical value is at[20] 2.3×10^{9}, giving a discrepancy of 9.53%. This means that the experiment consistently found a shear modulus much smaller than the theoretical value, however, the amount by which it was undershot remained proportionally constant, giving a discrepancy of around 10% for all materials, suggesting that the experiment had a systematic error which increased the perceived deflection.

One possible explanation for this error is the way measurements were taken, using a caliper to measure small changes (of a few millimeters), with a precision of 1 mm. This means that small changes were difficult to record, and while performing the experiment, these small changes were rounded up, to the nearest 0.1 of a mm, giving the impression that the materials were half as stiff as they actually are.

Despite this inaccuracy in the experiment, the results can be used to formulate a conclusion, since the overall trend of CFRC having the highest shear modulus, followed by aluminium and then balsa wood can be seen in both the experiment and the theoretical values.

Therefore, using the shear modulus per mass values, it can be determined that CFRC is the most advantageous material to use for building a drone, due to its high rigidity but small mass. However, as seen in the experiment, it is still able to bend slightly, allowing for it to effectively absorb any vibrations in the flight. CFRC's rigidity can be explained by its carbon fiber structure, which is simply a long ladder of carbon atoms which are tightly bonded together, aligned along the axis of the fiber[21]. This carbon fiber is then coated with epoxy, which is itself a polymer which can fill in gaps and allow for good adhesion[22], which is used to protect the fiber. This protection is necessary when the fiber is weaved into cylinders, to give them the desired strength. This strength comes from the 4 covalent bonds that each carbon atom is able to form with others, similarly to diamond. However, here a ladder of carbon atoms is formed instead of a lattice, allowing for the fiber to be wound to increase strength[23], while keeping its mass low due the composite having a low density.

The optimal structure for velocity would therefore be built out of CFRC.

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[18] Mechanics of balsa (Ochroma pyramidale) wood - DSpace@MIT

[19] CMS, From Extrel, et al. "Properties: Aluminum - Advantages and Properties of Aluminum." *AZoM.com*, 10 Nov. 2021, www.azom.com/properties.aspx?ArticleID=1446.

[20] "DragonPlate: Engineered Carbon Fiber Composite Sheets, Tubes and Structural Components: Made in USA." *Dragon Plate*, dragonplate.com/what-is-carbon-fiber.

[21] carbon2. "carbon2." *Material Science Research India*, 25 June 2017, www.materialsciencejournal.org/vol14no1/carbon-fibres-production-properties-and-potential-use/.

[22] Stradley, Marlo. "What Is Epoxy, and What Is It Made Of? Powerblanket Epoxy Curing Solutions." *Powerblanket*, Powerblanket, 27 Nov. 2019, www.powerblanket.com/blog/what-epoxy-is-made-of/.

[23] International Union of Pure and Applied Chemistry. "Nomenclature of Regular Double-Strand Organic Polymers (RulesApproved 1993)". *Pure Appl. Chem.* **48,** 373-385 (1993)

3. Choice of propeller size and pitch for maximum thrust and minimum energy consumption

For the drone to fly forwards faster, for a longer time, it needs energy-efficient propellers built for velocity. There are 2 main variables that can be changed in a propeller to increase velocity or decrease energy consumption: size and pitch[24]. Size simply refers to the length of the propeller, or the diameter of the circle it creates when rotating[25]. Pitch, on the other hand, refers to the angle of each blade on the propeller[26], as explained in Figure 8.

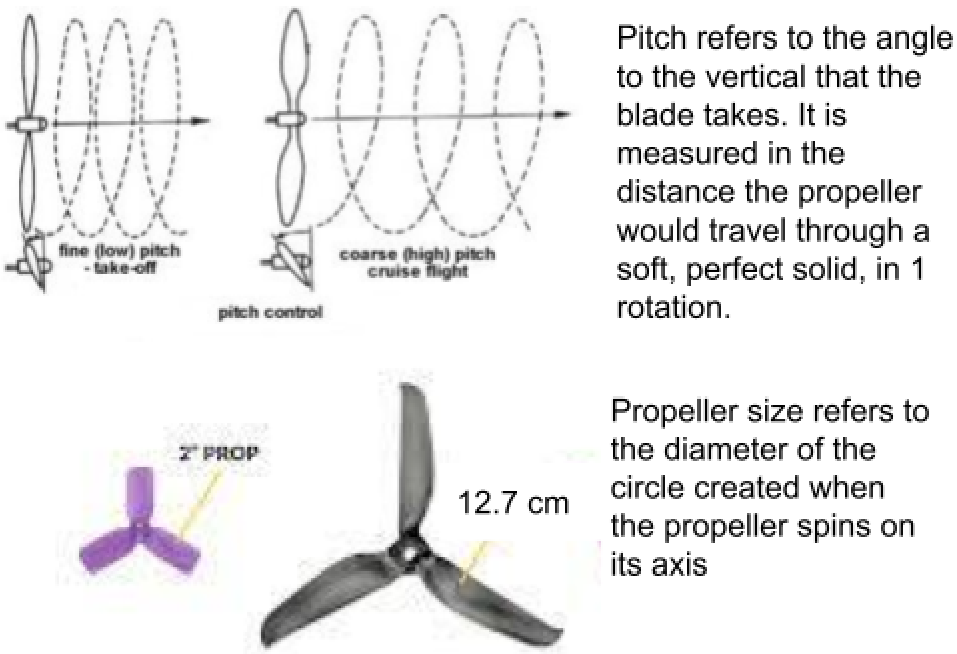

Figure 8 - Explanation of propeller pitch, size and number of blades

5.1 cm [27]

Propeller size and shape largely depends on the size of the drone itself. The propeller cannot be longer than the arms of the drone, or so small that it cannot provide enough lift. Therefore, first the maximum available space for the propellers must be determined.

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[24] GetFPV, et al. "All about Multirotor FPV Drone Propellers." *GetFPV Learn*, 23 Mar. 2018, www.getfpv.com/learn/new-to-fpv/all-about-multirotor-fpv-drone-propellers/.
[25] Henry, et al. "How to Choose Propeller for Mini Quad." *Oscar Liang*, 16 July 2020, oscarliang.com/choose-propellers-mini-quad/.
[26] DNDrone Nodes is an online communication platform that brings together experts and enthusiasts in drone research. "Quadcopter Props Main Characteristics & Best Picks." *Drone Nodes*, dronenodes.com/quadcopter-props-best-picks-characteristics/.
[27] James. "Miniquad Propeller Buyers Guide." *Propwashed*, www.propwashed.com/miniquad-propeller-buyers-guide/.

The dimensions of the onboard computer are $69.9 \times 45\, mm$, or 0.07 m by 0.045 m[28]. Therefore, such a minimum space must be available at the center of the drone, since other components take up less space. The propellers must be attached as close as possible to the center of mass of the drone, to ensure the moment of inertia is as small as possible. The moment of inertia refers to force needed to rotate an object[29], and so in the case of the drone, a smaller moment of inertia would allow for more maneuverability. Having the propellers closer to the center of mass also has the advantage of shorter arms, which are lighter. However, the distance between the front and back propellers must also be as large as possible, to allow for maximum forward thrust. Knowing this information, a sketch of the drone can be made, as seen in Figure 9.

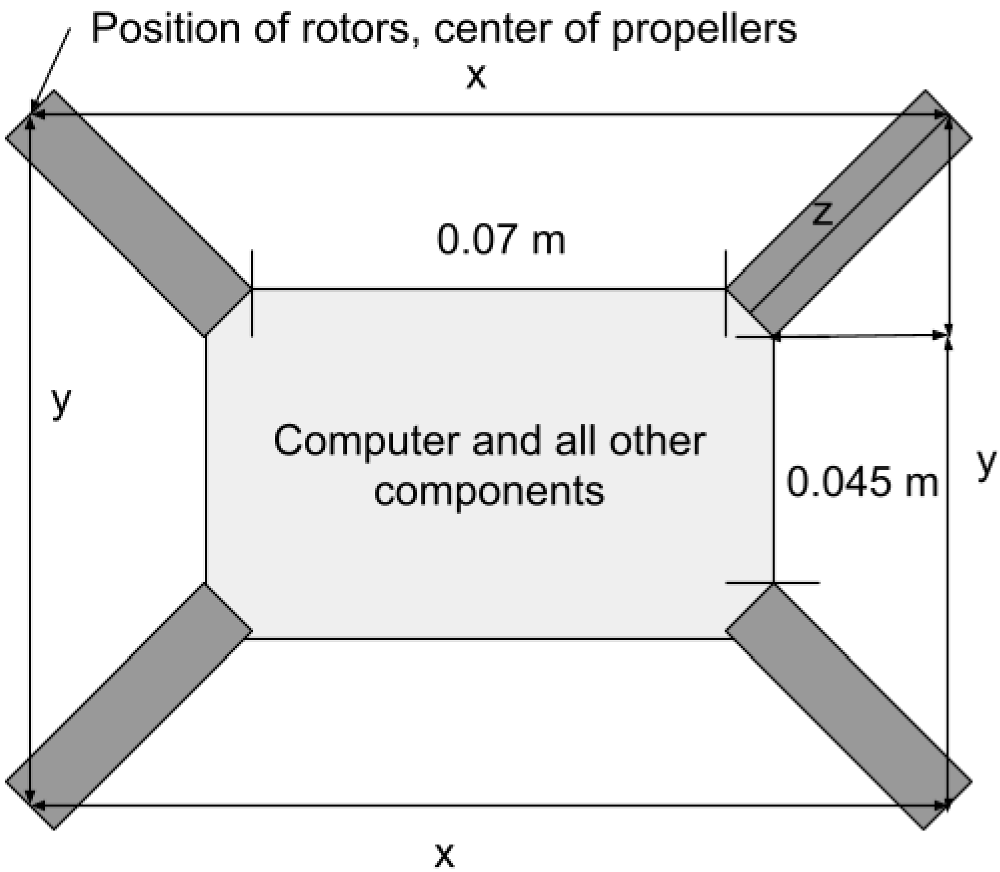


Figure 9 - Sketch of the drone structure

Here, z approximately corresponds to half the size of the propellers (the actual z distance will be slightly longer to ensure proper airflow). x and y correspond to the distance between the rotors. From this, it is clear that the goal is to minimize z while maximizing x, to ensure quicker changes of direction, maximal velocity and minimal mass.

[28] "NVIDIA Jetson Nano." *NVIDIA*, www.nvidia.com/en-us/autonomous-machines/embedded-systems/jetson-nano/product-development/.
[29] "Rotational-Linear Parallels." *Moment of Inertia*, hyperphysics.phy-astr.gsu.edu/hbase/mi.html#mix.

All 3 variables can be related in one equation in terms of each other, using Pythagoras' theorem, such that:

$$\left(\frac{y-0.045}{2}\right)^2 + \left(\frac{x-0.07}{2}\right)^2 = z^2$$

From this equation it is clear that z is proportional to x, that is as x increases, z also increases. This means that one cannot be minimized while the other is maximized, and one must be prioritised. Decreasing the z value also has the advantage of decreasing the overall size of the drone, which decreases the size and mass of the frame, which means the shortest possible z value must be found.

At its minimum, we can therefore see that z has to be greater than 0.045 m to ensure that the two propellers are free to move without interfering with each other, since this is the minimal y distance, as seen on Figure 10.

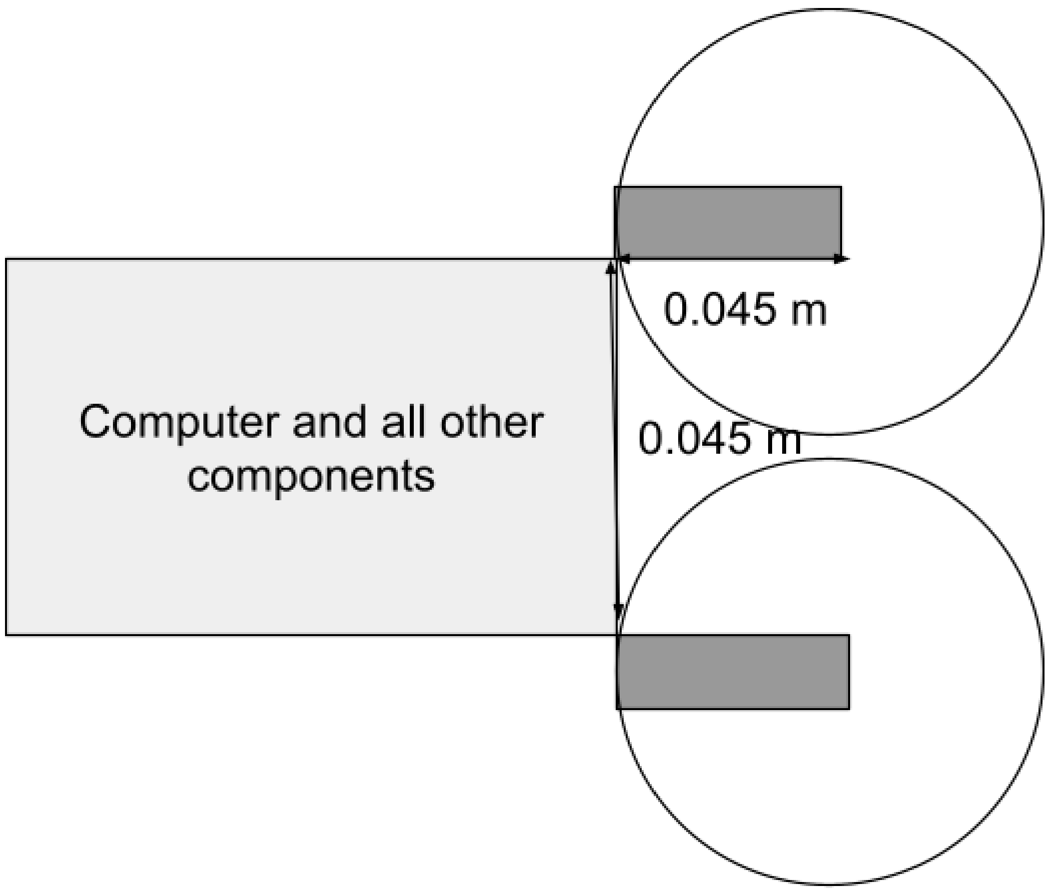


Figure 10 - Sketch of the drone structure

3.1. Finding the optimal propeller size

For propeller size, there is a trade off between velocity and energy consumption, assuming constant rotations per minute (RPM)[30]. A simple rule can be established: a greater propeller size will give more thrust, but will also consume more energy, reducing flight time, or requiring larger batteries, in turn increasing the mass of the drone. To understand why this rule works, it is first important to understand how thrust is calculated. From Newton's second law of motion, we know that force is equal to the rate of change of momentum, such that:

$$F = \frac{\Delta p}{\Delta t} = \frac{m(v_2 - v_1)}{t_2 - t_1},$$

Where F is the force exerted by the motor in N,
m is the mass of air being displaced in kg,
and v_1, v_2 correspond to the velocity of air before and after passing through the motor, in in ms^{-1}, at times t_1, t_2, in s.
To account for the ever changing mass of the air, mass flow rate can be used, which is found such that[31]:

$$\mu = \frac{m}{t} = \rho \times A \times v,$$

Where ρ is the density (kgm^{-3}) of the air, A is the area of the propeller circle (m^2) and v is the velocity of the air particles through the propeller, in ms^{-1}. We can rewrite Newton's law as:

$$F = \mu \times (v_2 - v_1)$$

From this, it can be seen that there are two ways to increase thrust: to increase the area on which the propeller applies a downward force, which is found such that:

$$A = \pi \times r^2,$$

Where r is half the size of the propeller, or by increasing the velocity v_2 of the particles as they leave the propeller, which is done by increasing the rotation velocity of the propeller.

Therefore,

$$F = d \times \pi \times r^2 \times v \times (v_2 - v_1)$$

A larger propeller increases the mass flow rate, meaning kinetic energy has to be supplied to more particles, and so the overall work done by the motors increases, meaning that more energy has to be supplied by the battery, decreasing flight time. A larger propeller is also less responsive to changes in direction, since it is moving a greater volume of air.

[30] Durst, Gloria. "How to Choose Motor and Propeller for Quadcopter." *Drones*, 17 Mar. 2021, top-10-drones.com/choose-motor-propeller-quadcopter/.
[31] "General Thrust Equation." *NASA*, NASA, www.grc.nasa.gov/www/k-12/VirtualAero/BottleRocket/airplane/thrsteq.html.

Increasing the velocity of the particles as they leave the propeller is another solution, which allows for quick velocity and direction changes due to the smaller volume of air that is displaced. To increase the velocity, the rotational velocity of the propeller can be increased, which also increases the energy consumption. However, it also increases the mass flow rate, since more air is sucked in by the low pressure of the propeller area, and more air is outputted. This has the advantage of increasing the thrust twice, as well as quicker acceleration. For this reason, a smaller propeller should be used for greater velocities.

Typically, propeller sizes range from 0.076 m to 0.20 m. Knowing the minimal space requirements determined in the previous section, a 0.076 m propeller in length is the closest to the minimal length of 0.045 m, and so would be the smallest fitting with the frame[32].

x and y values can now be found, knowing that y is slightly more than the length of the propeller, giving 0.08 for y, and so:

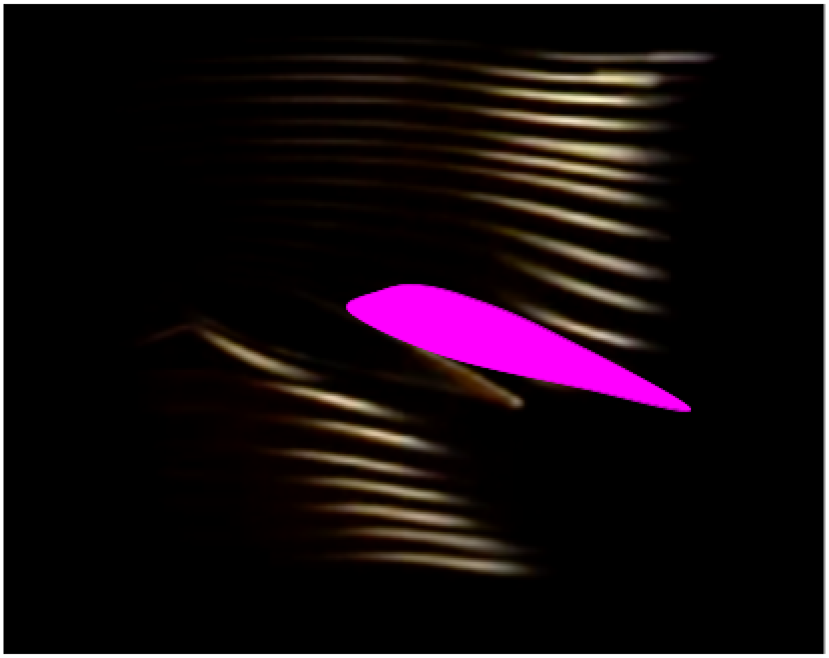
$$(\frac{0.08-0.045}{2})^2 + (\frac{x-0.07}{2})^2 = 0.076^2$$

Giving an x value of 0.20 m.

3.2. Finding the optimal pitch size

Increasing the velocity of the air particles increases thrust. To understand how this can be done, we must first understand how exactly lift is being generated by the propeller. A propeller can be seen as a spinning airplane wing, generating lift in the same way the wings of an airplane.

[32] DNDrone Nodes is an online communication platform that brings together experts and enthusiasts in drone research. “Quadcopter Props Main Characteristics & Best Picks.” *Drone Nodes*, dronenodes.com/quadcopter-props-best-picks-characteristics/.



Here, we can see that as smoke (white lines) travels past the pink wing, it takes more time for the smoke below to travel through (it ends up being delayed), accumulating, and creating an upwards force.

Figure 11 - Effect of shape and angle of attack of wing on air flow [33]

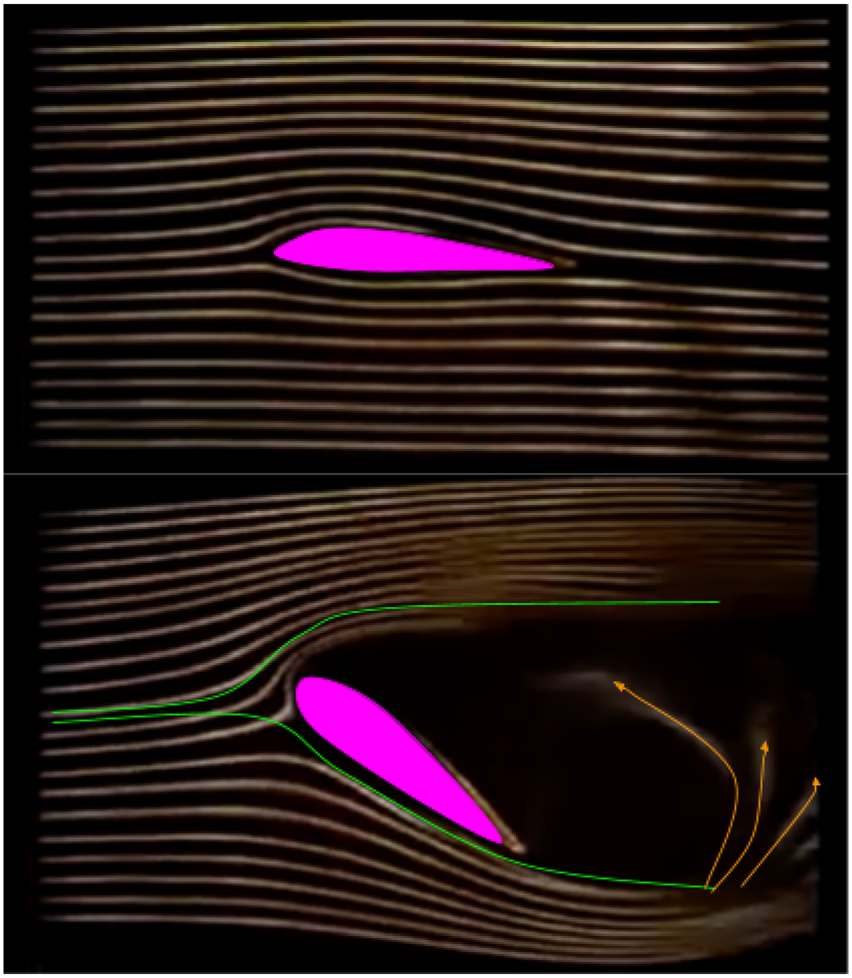
In Figure 11, due to the shape of the wing, air particles travel slower below the wing creating a low pressure area above the wing, and high pressure below[34]. This generates lift, due to the principle of diffusion, which states that a substance will tend to spread from an area of high concentration to low concentration, for the concentration to become equal across all spaces[35]. This means that air particles will attempt to flow from the high pressure area below the propeller to the low pressure area above the propeller, and so there will be an upward force on the propeller, which is the lift.

Increasing the pitch of the propeller increases the difference of pressure above and below the propeller, as seen in Figure 12.

[33] https://www.youtube.com/watch?v=6UlsArvbTeo

[34] Extra, AeroTime. "Why Do Airplanes Fly?" *AeroTime Hub*, 18 July 2019, www.aerotime.aero/23033-why-do-planes-fly.

[35] Bartee, Lisa, et al. "Passive Transport: Diffusion." *Principles of Biology*, Open Oregon Educational Resources, openoregon.pressbooks.pub/mhccmajorsbio/chapter/passive-transport-diffusion/.



Here, the pitch is very low, and the wing (in pink), interferes little with airflow. There is also no noticeable difference in pressure and so no lift.

Here, the pitch is very high, and we can see that air below the wing has to travel a much greater distance, highlighted in green, creating a high pressure area.

The air's desire to move upwards can be seen by the smoke particles travelling upwards after they have gone around the wing, in orange.

Figure 12 - The effect of changing pitch on airflow [36]

Therefore, air particles flow at a faster rate through the propeller area when pitch is greater, increasing the mass flow rate μ, and so increasing the upward thrust[37]. However, this also increases the number of collisions between the air and propeller, and so more work has to be done to rotate the propeller, increasing the energy needed for the drone to fly.

Propeller manufacturers measure pitch in terms of the distance that the propeller would travel if it was rotated once in a soft solid, relating to how the propeller's shape makes it act like a screw that moves forwards when rotated.

To find the optimal pitch, secondary data was collected, from an experiment which compared thrust to power usage for different pitches of a 0.0762 m size propeller, using the same motor and RPM[38].

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[36] Department of Engineering, University of Cambridge, "how wings work? Smoke streamlines around an airfoil", 17 Feb. 2008, https://www.youtube.com/watch?v=6UlsArvbTeo
[37] Department of Engineering, University of Cambridge, "how wings work? Smoke streamlines around an airfoil", 17 Feb. 2008, https://www.youtube.com/watch?v=6UlsArvbTeo
[38] SiieeFPV, "3 Inch Prop Roundup - static test", 5 May. 2016, https://www.youtube.com/watch?v=Hs7CC0ePI_E

Table 8 - Thrust in N per power usage in W for propellers of pitch 0.102, 0.076 and 0.114 m.

Propeller pitch (m)	Thrust per power usage (NW^{-1})
0.102	2.38
0.076	2.51
0.114	2.11

From this data, we can see that the propeller with a pitch of 0.076 m performs better than the other pitches, in terms of how much thrust it can generate per watt of power. This suggests that the thrust that higher pitches provide does not outweigh the energy loss from them. Therefore, a propeller of pitch 0.076 m should be used in this drone.

4. Conclusion and evaluation

After designing and understanding how a faster drone can be made, a final solution can be presented. A lighter frame material, which also provides enough strength to support the downward force the motors apply, allows for more forward thrust, and so a higher velocity. It was determined experimentally that a carbon fiber reinforced polymer composite had the best shear modulus to mass ratio, this being more than 15 times the ratio for aluminium, another popular material.

Approximate dimensions of 0.20 m in length and 0.08 m in width were determined, which allow for propellers of size 0.076 cm to be used. These would have a pitch of 0.076 m, which provides the highest thrust to power usage ratio. Decreasing energy consumption has the advantage of decreasing the need for heavy batteries, which would greatly increase the mass of the drone, decreasing its velocity. The final model of the drone can be seen in Figure 13.

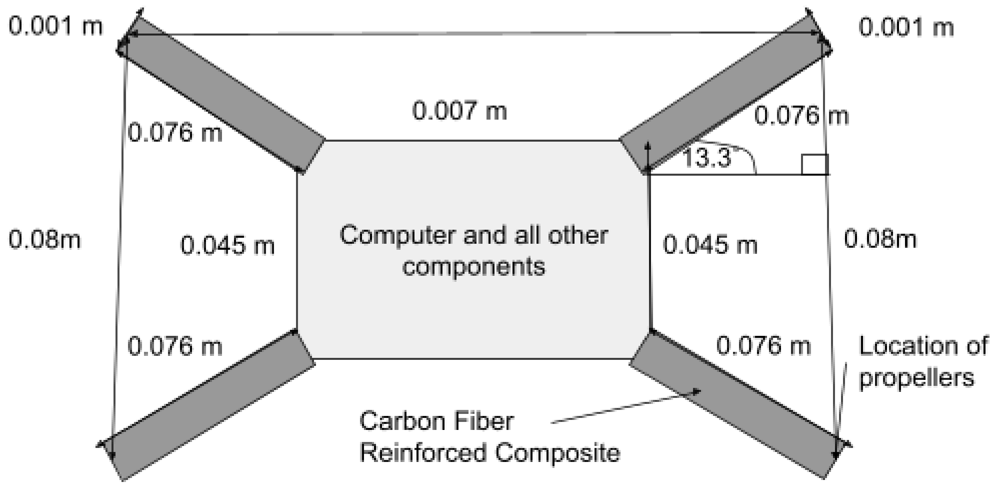

Figure 13 - Final sketch of the drone structure

Other factors could be explored to increase the velocity, such as different propeller shapes, mass distribution, numbers of rotors, and overall drone sizes. Moreover, the most efficient motors should be explored to determine the right size of battery.

The final drone also needs to be built and tested in a real life setting to determine the extent of the advantages that each selected solution offers, and determine a final maximum velocity, which would provide information on the feasibility of a quadcopter drone when in the context of rapid deployment in order to provide assistance for the injured.

4.1 Evaluation

Sources used throughout the essay had varying degrees of reputability, varying from drone enthusiast blog articles to published physics papers. However, the less credible sources were always supported by physics theory, which suggests their arguments were logical and reasonable. The secondary data used came from a single source, a video describing the experiment conducted, and its strengths and weaknesses, indicating the results were conclusive.

A number of factors which affect the velocity of drones were explored, while also considering how changing these factors affect energy consumption, using both primary and secondary data, as well as relevant physics theory.

One large limitation is that there are few sources on the physics of drones, since drone enthusiasts often use a trial and error method to develop the fastest drones, and so many factors such as the number of blades on the propeller could not be investigated in this essay due to this lack of scientific backing, even though they could have a large impact on velocity.

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