Logical Semantics

Introduction

Shu-Kai Hsieh

October 29, 2019

Graduate Institute of Linguistics
National Taiwan University

Table of contents

Introduction

Semantic Representation

Converting NATURAL LANGUAGE into a **Meaning Representation**, which should be[1]

- unambiguous;
- link language to external knowledge, observations, and actions;
- support computational inference;
- expressive enough

1

[1] Esenstein, 2019

Formal Semantics

Formal Semantics

Assumptions

Linguistics can be described as an (interpreted) formal system.

- Mathematical logic serves as a metalanguage to indicate the signification of signs unambiguously.
- Propositional logic and Predicate logic.

Propositional Logic

- Propositional logic concerns declarative sentences that have a unique TRUTH VALUE (true or false). Sentences are the smallest logical unit that cannot be further broken down.
- Logical connectives and TRUTH TABLE 眞値表.
- A **well-formed formula** (*wff*) is a logical expression that has a meaning. A simple variable that represents a proposition P is the simplest form of a *wff*.

Deduction

Rules of Inference

Inference rule	Form of implication
RI$_1$: **Addition** P $\therefore P \lor Q$	$P \rightarrow (P \lor Q)$
RI$_2$: **Conjunction** P Q $\therefore P \land Q$	$P \land Q \rightarrow P \land Q$
RI$_3$: **Simplification** $P \land Q$ $\therefore P$	$(P \land Q) \rightarrow P$
RI$_4$: **Modus ponens** P $P \rightarrow Q$ $\therefore Q$	$(P \land (P \rightarrow Q)) \rightarrow Q$
RI$_5$: **Modus tollens** $\neg Q$ $P \rightarrow Q$ $\therefore \neg P$	$(\neg Q \land (P \rightarrow Q)) \rightarrow \neg Q$
RI$_6$: **Hypothetical syllogism** $P \rightarrow Q$ $Q \rightarrow R$ $\therefore P \rightarrow R$	$((P \rightarrow Q) \land (Q \rightarrow R)) \rightarrow (P \rightarrow R)$

Exercise

see notebook.

Limits

- insufficient

First order logic

Assumption

The world contains **terms, predicates** and **quantifier**.

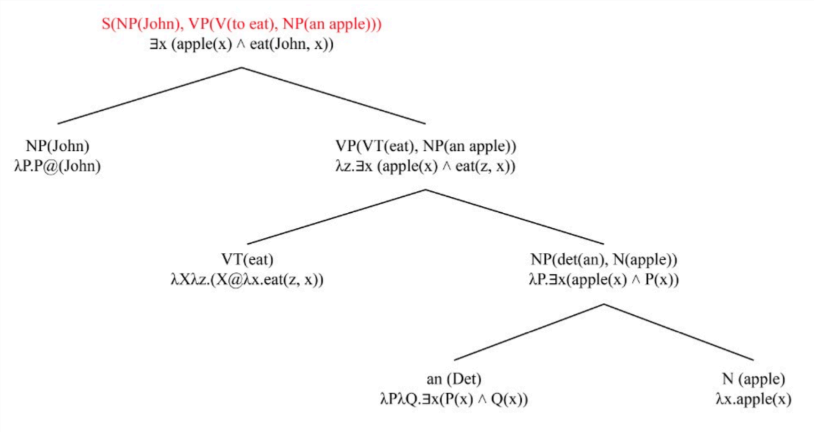
Lambda Calculus

Lambda Calculus

The smallest universal programming language of the world

- Invented in 1936 by Alonzo Church.
- Often considered to be an extension of first-order logic to include the operator lambda (λ) that makes it possible to connect variables.
- Also popular in functional programming languages (LISP, HASKELL), even PYTHON.

Analysis of a simple sentence with a transitive verb



Event and State Representation

- **States** are conditions, or properties, that remain unchanged over an extended period of time.
- **Events** denote changes in some *state-of-affairs*.[1]

The representation of both states and events may involve a host of *participant*, *props*, *times* and *locations*.

1

[1]事態：事件的發生或存在狀態

Event and State Representation

- FOL (wrongly) assumes the predicates have fixed arity (i.e., they take a fixed number of arguments).
- **neo-Davidsonian event representation**: introduce the notion of **event variable** to allow us to make assertions about particular events.

Representing Time

Temporal logic

Time flows

Other types of Logic

Possibility and Necessity: An Introduction to Modality

Modality

Modal claims in our daily life

'He might be able to..'; 'This must happen'.....

Definition (Palmer, 1986)

'Semantic information associated with the speaker's attitude or opinion about what is said.' Modality concerns the mode or way in which a claim is true or false, and how something exists or does not exist.[1]

[1] https://1000wordphilosophy.com/2018/12/08/possibility-and-necessity-an-introduction-to-modality/

Modality

- Modality includes: **necessity, possibility, actuality, validity** and **believability** of a PROPOSITION; involving *objective* measures of factual status as well as *subjective* attitudes and orientations towards a proposition.
- In philosophical terms, modal expressions convey a **propositional attitude** –an indication by the speaker of their commitment toward the content of their statement.

Worlds

- expressed world (EW, asserted by the proposition) and reference world (RW, normally the actual world of speech).
- Where the RW corresponds with the EW, we have actual modality, or realis. When the RW does not coincide with the EW, we have non-actual modality, or irrealis.
- The modal status of a proposition depends on the extent to which the two modal deictic points diverge. This degree of coincidence translates as necessity, possibility, obligation, commitment, and so on.

Mood and Modality

- Modality is a semantic feature that captures the speaker's attitude toward a proposition.
- Mood is a grammatical feature that refers to the inflections for a subset of modal denotations.

Example

The subjunctive mood is the grammatical device a language uses to express hypotheticality or uncertainty.

Modal expressions

Language provides various means for signaling the modality.

- (semi-) modal auxiliaries (`shall`, `should`, `can`, `could`, `may`, `might`, `must`, `would`, `has to`, `ought to`)
- modal adverbs (`possibly`, `maybe`, `perhaps`, `necessarily`, `honestly`, `actually`, `allegedly`)
- other modal constructions (`Kyle is required/allowed to go`)

THREE types of Modality

- **Logical modality**: examines the possible truth or necessity of a proposition according to logic.

Example

(a). A square must have 4 sides. logical necessity

(b). Socrates might have been a woman. logical possibility

THREE types of Modality

- **Epistemic modality**: evaluates propositional truth according to our knowledge of the real world, i.e., it analyzes the necessity or possibility of a proposition's truth in light of what we know about the reality.

Example

(a). The dinosaurs must have died out suddenly. epistemic necessity

(b). There might/could be intelligent life in deep space.

epistemic possibility

Linguistic Cues

- English used to mark the distinction between logical AND epistemic possibility. ‘May‘ only marked epistemic modality while ‘might‘ could be used for EITHER logical OR epistemic modality.

THREE types of Modality

- **Deontic modality**: is concerned with obligation, duty, or normative action, which expresses the imposition of an expressed world on a reference world. As the way that languages express the restriction of possible future states of affairs to a single choice.
- Deontic necessity expresses what someone is required to do (**obligation**). Deontic possibility expresses what behavior is permissible (**permission**).

Example

(a). You must be home by midnight. deontic necessity

(b). Visitors may use the downstairs sitting room after 6 p.m.

deontic possibility

漢語例子

Modal Logic

Modality :

- (Grammar): of denoting the *mood* of a verb; relating to a modal verb.
- (Logic): of a proposition in which the predicate is affirmed of the subject with some qualification, or which involves the affirmation of possibility, impossibility, necessity, or contingency.

Modal Operators

- Two new logical constants, $\Box$ (N, necessity) and $\Diamond$ (P, possibility), are introduced.
- Use $\Box$ as the symbol for logical necessity, and $\Diamond$ for logical possibility.
- Logical, Epistemic, Deontic modality.
- Modality needs to work with **possible worlds**.

Modality and Possible Worlds

- The idea of possible worlds was *invented* to provide a concrete way of evaluating modal expressions in logic.
- We can use the concept of possible worlds to distinguish the different types of modality we have just reviewed.
- Logical modality uses the full set of possible worlds.
  - A logically necessary proposition is true in all possible worlds.
  - A logically possible proposition is true in at least one possible world.

Formalization

Logical necessity

$\Box$ (S) $\equiv \forall w$ (S is true in w), where w ranges over the entire set of possible worlds.

Logical possibility

$\Diamond$ (S) $\equiv \exists w$ (S is true in w), where w ranges over the entire set of possible worlds.

Formalization

Epistemic modality would be evaluated over a more restricted set of possible worlds —those worlds that are compatible with the rules of logic and physics.

Epistemic necessity

$\Box_e$ (S) $\equiv \forall w_e$ (S is true in w_e), where w_e ranges over epistemically possible worlds.

Epistemic possibility

$\Diamond_e$ (S) $\equiv \exists_w$ (S is true in w_e), where w_e ranges over epistemically possible worlds.

Formalization

Deontic modality is evaluated to the most restrictive set of possible worlds. In addition to being compatible with the rules of logic, deontically possible worlds must satisfy some code of behavior.

Deontic necessity

$\Box_{po}$ (S) $\equiv \forall w_{po}$ (S is true in w_{po}), where w_{po} ranges over deontically possible worlds (a.k.a. perfect obedience worlds).

Deontic possibility

$\Diamond_{po}$ (S) $\equiv \exists w_{po}$ (S is true in w_{po}), where w_{po} ranges over deontically possible worlds (a.k.a. perfect obedience worlds).

Interdefinability with Negation

Example

A proposition is necessarily true if there is no possibility of its being false.

A proposition is possibly true if it is not necessarily false.

- Necessity ($\Box$) and possibility ($\Diamond$) can be paraphrased by each other in combination with negation.

(a) $\Box p \equiv \neg\Diamond\neg p$

(b) $\Diamond p \equiv \neg\Box\neg p$

(c) $\Box\neg p \equiv \neg\Diamond p$

(d) $\Diamond\neg p \equiv \neg\Box p$

Math

$$e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$$