

Structural Design Project

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Executive Summary

We present an analysis of a truss designed to meet the specifications of Problem 1S of the Structural Design Project. The analysis shows that our truss is able to meet all the necessary criteria, with no predicted failure at the maximum load, with calculations to verify our prediction. An estimate of the cost for constructing the truss is also given and compared between our truss and the trusses tested. Our truss is much more inefficient than ideal but makes up for it with significant strength. The observed failure modes of the tested truss are also analysed.

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1. Problem Statement

We were assigned to 'Problem 1S: Steel Cantilever', which involves the design of a plane truss, complete with bars, joints and bracing, to meet various performance criteria:

Constraints

- The truss must attach with four pairs of M6 tapped holes at each corner of a rectangular rigid plate. From the front view, the height must be 255 mm and the width (between midpoints of holes) must be 115 mm. The distance between the centres of each pair of holes must be 25 mm.
- At the opposite end of the truss, a spreader bar of length 115 mm must be attached, on which the load will be placed.
- The load must be carried at a distance of 815 mm from the plate.
- Two identical plane faces must be constructed, attached to each side of the plate.

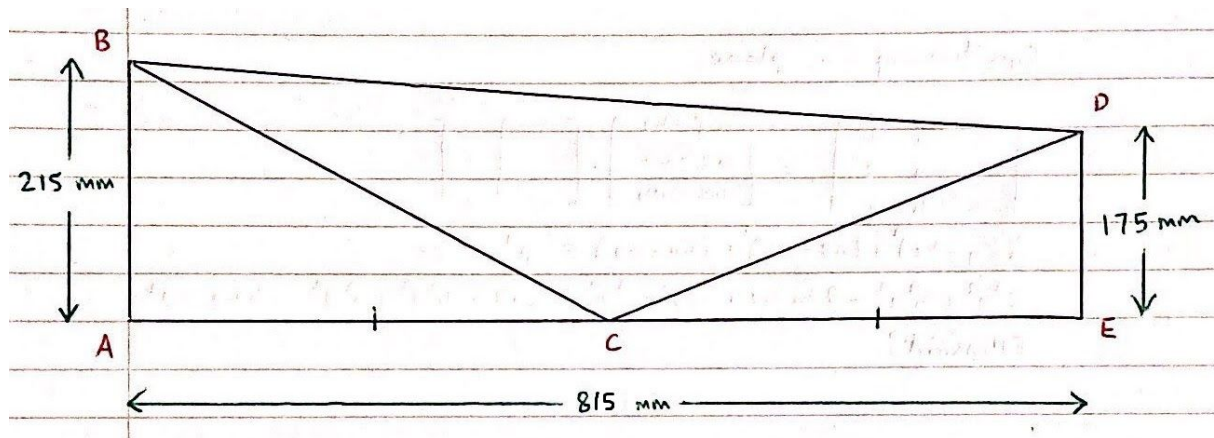
Requirements

- The truss must avoid failure at an initial upwards load of 135 N. This is first applied to verify the lateral stability of the structure.
- The truss must avoid failure at the working downwards load of 1350 N. This is applied through the spreader bar: each half of the truss has an effective load of 675 N.
- There must be no visible deflection at the working load. This is estimated to mean no joint may deflect by more than 1 mm, but may be somewhat subjective.
- The load factor at collapse must be approximately 2: each half of the truss should not fail until at least 1350 N (total load 2700 N), for both upwards and downwards loads.
- The structure should also be lightweight, easy to construct, and supplemented with technical drawings.

2. Alternative Designs

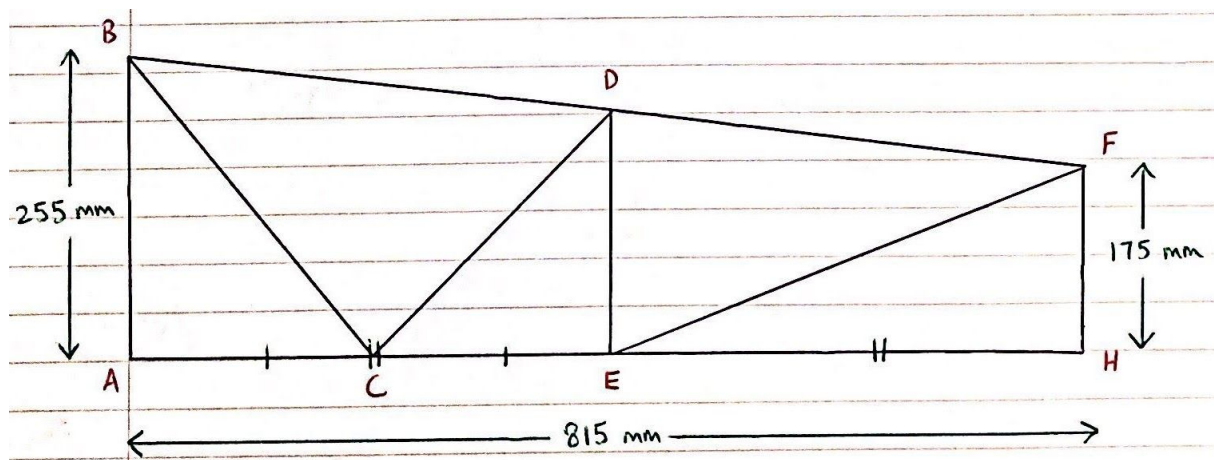
We considered two different truss designs before finding our third design which we predicted to meet the requirements. Each was discarded for different reasons.

First design



- Misread the height of the plate as 215 mm instead of 255 mm: this truss is not physically compatible with the problem.
- Predicted to fail by buckling (mode A) in bar AC (compression = 2.56 kN at load W), which could not be feasibly prevented without significant costs.

Second design



- Fails by buckling (mode A) in bar AC (compression = 2.16 kN at load W).

Only rough sketches were drawn for the first two trusses. In the diagrams, line AB represents the plate to which the truss is attached, not an additional bar.

3. Final Design and Calculations

Scale drawings and the parts list for our final design are given below. As before, line AE represents the plate and not an additional bar on the truss. The truss is symmetrical about CF, giving identical performance regardless of load direction. AEF is an equilateral triangle.

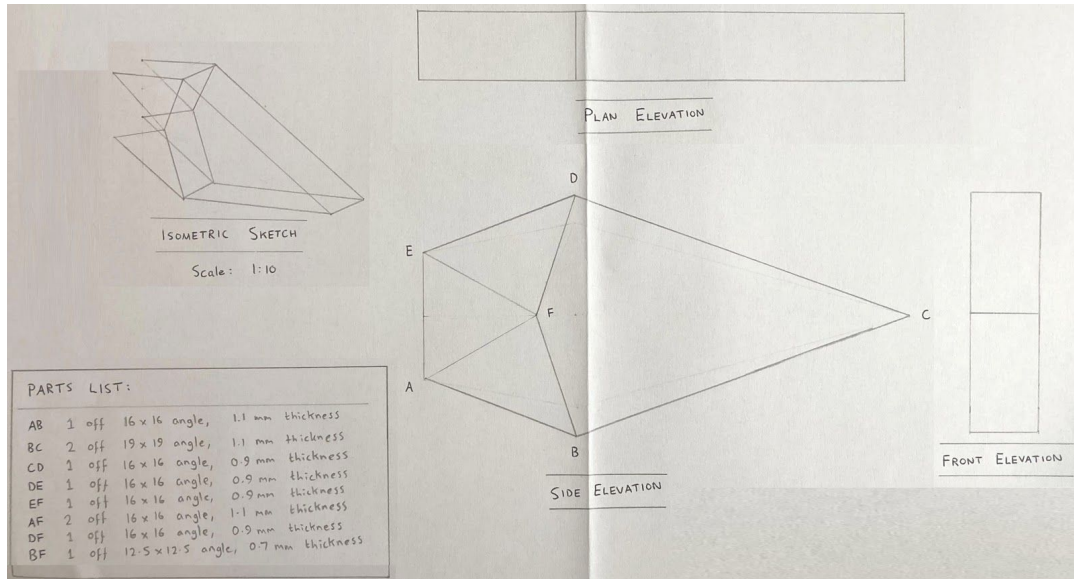


Fig. 1: Drawing of plane section of truss and parts list. Cropped from original drawing.

C

$\theta = 22.5^\circ$

$\uparrow: T_{CD} \sin \theta = W + T_{BC} \sin \theta$

$\leftarrow: T_{CD} \cos \theta + T_{BC} \cos \theta = 0$

$\hookrightarrow T_{CD} = 1.31 W = 882 \text{ N}, T_{BC} = -1.31 W = -882 \text{ N}.$

B

$\alpha = 85.14^\circ, \beta = 55.12^\circ$

$\uparrow: T_{AB} \sin(\theta + \alpha + \beta) + T_{BF} \sin(\theta + \alpha) = -T_{BC} \sin \theta$

$\leftarrow: T_{AB} \cos(\theta + \alpha + \beta) + T_{BF} \cos(\theta + \alpha) = -T_{BC} \cos \theta$

$\hookrightarrow T_{AB} = -1.59 W = -1071 \text{ N}, T_{BF} = 1.02 W = 687 \text{ N}$

D

$\uparrow: T_{DE} \sin(\theta + \alpha + \beta) + T_{DF} \sin(\theta + \alpha) = T_{CD} \sin \theta$

$\leftarrow: -T_{DE} \cos(\theta + \alpha + \beta) + T_{DF} \cos(\theta + \alpha) = -T_{CD} \cos \theta$

$\hookrightarrow T_{DE} = 1.59 W = 1071 \text{ N}, T_{DF} = -1.02 W = -687 \text{ N}$

F

$\gamma = 72.36^\circ, \delta = 30^\circ$

$\uparrow: T_{EF} \sin(180^\circ - \delta) + T_{AF} \sin(180^\circ + \delta) = T_{BF} \sin \gamma - T_{DF} \sin \gamma$

$\leftarrow: T_{EF} \cos(180^\circ - \delta) + T_{AF} \cos(180^\circ + \delta) = -T_{BF} \cos \gamma - T_{DF} \cos \gamma$

$\hookrightarrow T_{EF} = 1.94 W = 1309 \text{ N}, T_{AF} = -1.94 W = -1309 \text{ N}$

Reactions: \hat{i} \hat{j} $R_A = (3.2\hat{i} + 0.5\hat{j})W, R_E = (-3.2\hat{i} + 0.5\hat{j})W$

Fig. 2: Handwritten calculations to find the internal forces of the truss at $W = 675 \text{ N}$.

4. Predictions

4.1. Performance

The forces in the truss were verified by writing a computer program^[1] and tested with a reputable truss calculator^[2]. The results verify the calculations in Fig. 2.

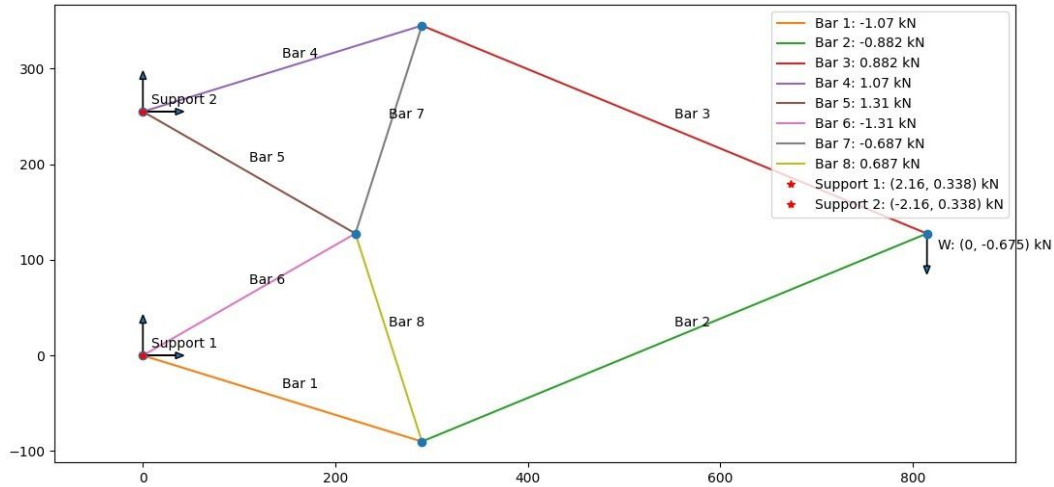


Figure 3: Program output for truss analysis, displayed using Matplotlib.

To improve the resistance of the truss to bending, parallel bracing members were planned to be put at joints B and D. The largest strain was found to be 0.04%, in member 5 (EF). With a length of 255 mm, the extension is approximately 0.1 mm (not a visible deflection).

The behaviour of the truss at the intended collapse load, $2W = 1350$ N, is predicted below. Members closest to their limits are highlighted in orange.

Member	Force at $2W$ / N	Stress at $2W$ / MPa	Limiting stress and predicted failure mode
1 (AB)	-2140	-114	-148, buckling A
2 (BC)	-1760	-75.8	-112, buckling B
3 (CD)	1760	115	216, yielding
4 (DE)	2140	139	216, yielding
5 (EF)	2620	170	216, yielding
6 (AF)	-2620	-139	-200, buckling B
7 (DF)	-1370	-89.3	-165, buckling A
8 (BF)	1370	137	216, yielding

4.2. Costing

All costing is done using the tables and definitions in the Costing Sheet.

- Gusset plates of thickness 0.9 mm were used at all joints. A total area of approximately 0.086 m² of steel sheet was used, giving a sheet mass of 0.605 kg and a sheet cost of 36.3.
- The combined mass of the truss members was calculated to be 0.935 kg, for an angle cost of 56.1.
- 45 steel pop rivets (diameter 3.2 mm) and 10 M5 bolts (diameter 5 mm) were used, for a cost of 29.5.
- The cost of one side of the truss is therefore 121.7.
- Accounting for the other side (identical), the bracing members, and the 8 M6 bolt connections to the plate, we calculated a total cost of **269**.

5. Analysis of the Trusses Tested

The testing of two trusses to their destruction was observed, fabricated from the designs of Group 50 and Group 55. The trusses, alongside our own (with our predictions where necessary) are compared below.

Performance and Failure

Lab Group	Weight / kg	Cost / a.u.	Failure load & mode	Result (working load: 1.35 kN; ultimate load: 2.70 kN \pm 10%)
Group 50	1.64	150.3	1.25 kN, separation from the wall plate	Failed both working load and ultimate load
Group 55	2.06	153.8	2.62 kN, buckling B	Passed both working load and ultimate load
Group 47 (ours)	~ 3.5 (estimated)	269.0	3.43 kN, yielding (predicted)	Pass working load but collapse beyond range of ultimate load (predicted)

Commentary

The chosen trusses excelled in their low cost and similarly low weight, but only one was also able to meet the loading requirements. Both designs had very good drawings.

Group 50's structure failed due to an oversight in the construction process, with a member not being attached properly to the plate. Had this issue been fixed, the structure would have likely easily passed the working load. However, this resulted in a highly efficient yet incompetent structure.

Group 55's structure passed the working load without any visible deflection and got to within 3% of the intended ultimate load, and so this structure passed all across the board.

Our structure was too heavy and expensive to be considered for testing. It was unclear whether the structure should collapse at $2W$ or beyond $2W$; if the target was to collapse at $2W$ then our truss would have theoretically overshot this significantly and therefore would be considered a failure to meet this condition.

6. *Conclusions*

We built our truss with a sole focus on strength without any consideration for cost and weight: these were almost twice that of the only successful tested group, but predicted to collapse higher as well.

We also produced drawings which lacked detail and understanding of the joints and the length dimensions of the members, which would have made it difficult to construct by us or the fabricators. Despite being theoretically very strong, the possibility of errors during the construction process due to not considering the finer details may have caused failure at lower loads. With more planning and attention to these issues however, we believe our truss could have been very successful.

7. References

- [1] Truss calculator; SkyCiv
<https://skyciv.com/free-truss-calculator/>
Accessed 24th November 2020.

- [2] Simple Truss Calculator script
<https://github.com/lorcan2440/SimpleTrussCalculator>
Created 24th November 2020.