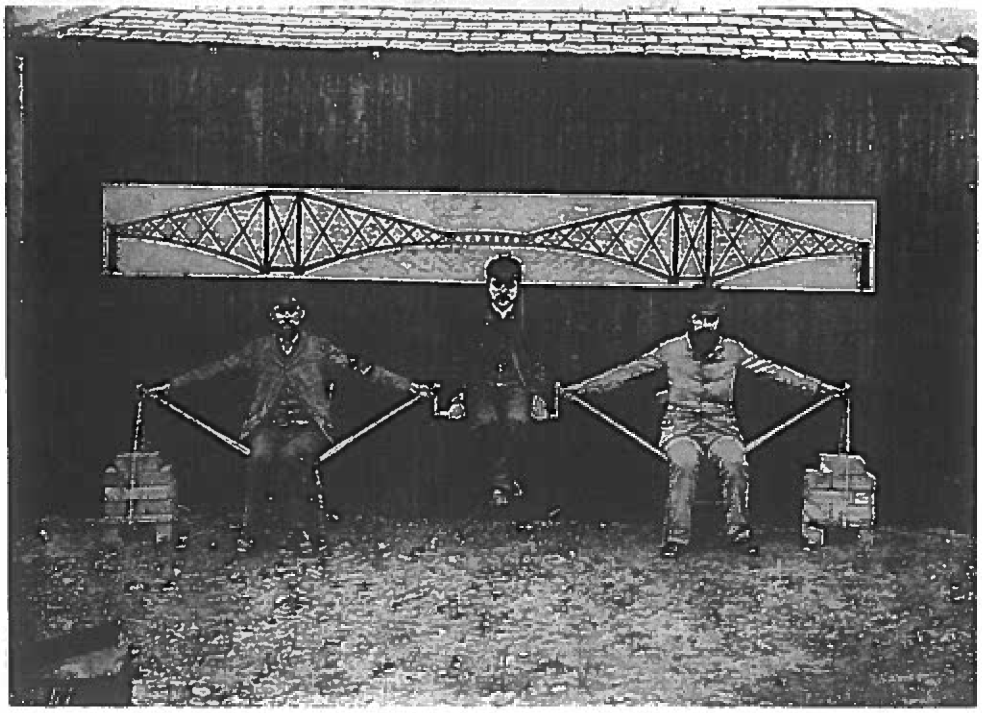
Part IA Structural Design Course 2020/21

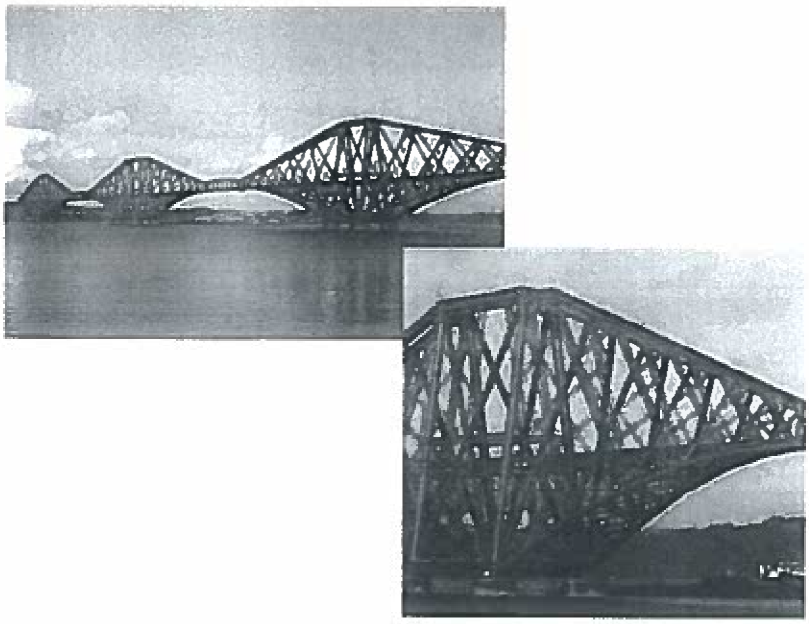
Course Handout

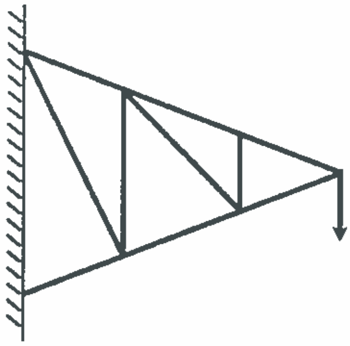
Supplementary Information to the Design Handout



1. INTRODUCTION

The aim of the Structural Design Course is to design, build and test a light-weight truss. There are many examples of truss structures, ranging from the micro-scale of modern, engineered materials to the familiar structures of Victorian Engineering.





In this course you will design and manufacture an ~~aluminium~~ Steel cantilevered truss, using a combination of angle sections and sheet material.

The process involves the following stages:

- Design — not just analysis
- Draw — Engineers must communicate designs to a third party
- Build
- Cost
- Test
- Report

2. DESIGN BRIEF: PROBLEM 1S

The design brief is given in your Design Handout, as summarised below.

A structure is required to carry a vertical load W at a horizontal distance of 815 mm from a rigid vertical plate as shown. The plate has four pairs of M6 tapped holes to which the structure may be attached. The load is applied to the structure through a spreader bar length of 115 mm, thus dividing the applied load into two loads, W/2.

The working loads are: W = +1350 N and W = -135 N; at both these loads, there must be no visible deformation. The load factor at collapse equals 2, and is applicable to both positive and negative values of W.

It is essential to confirm the loading arrangements before detailed drawings are made; the designers are responsible for making any required measurements.

The objective is to design a structure with the given material that will satisfy the loading conditions; importantly, the structure must be lightweight and simply made.

- 4 pairs of M6 holes at anchor plate
- Load applied through spreader bar
- Initial upwards load of 135 N applied through 'freely hanging' weights to assess lateral stability
- Working load W = 1350 N

  No visible deflection at working load
- Collapse load factor = 2

3. PROGRAMME

- Design – first two morning sessions
- Draw – last two morning sessions
- Build – five afternoon sessions (Dyson Centre)
- Cost – costing sheet
- Test – Structures Teaching Lab
  - Most expensive first
  - Brief (1 min) talk prior to test
- Report

4. DESIGN

Objective: to design the optimal structure,

i.e. the most economical structure that just carries the maximum design load. (2W)

Process: devise configuration of bars

calculate bar forces

design/size members

design joints

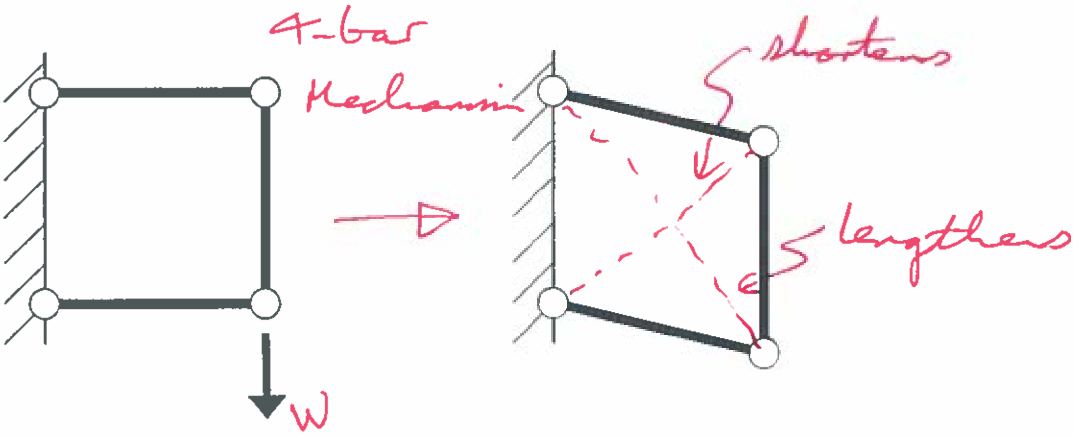
Note that design is an *Iterative* process.

Assume pin-jointed

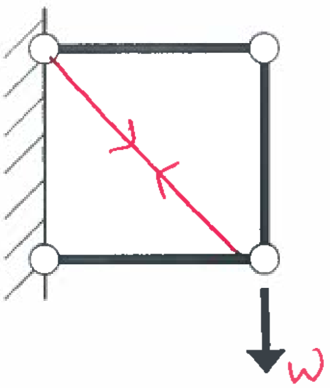
In a truss structure, the bar forces are carried by axial stresses in all members. The stresses may be tensile (pulling) or compressive (pushing), and must not exceed the limits beyond which a member will fail. Three main types of failure are considered: stability, tensile failure and compressive failure (by squashing or buckling).

4.1 Configuration – Ensuring Stability

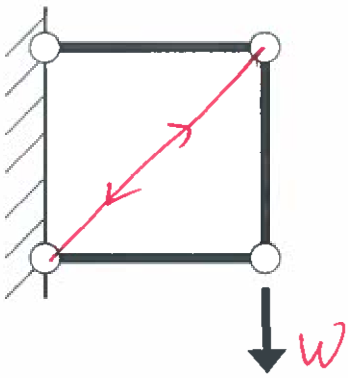
We aim to devise a configuration of bars that connects the load to the support(s) via the shortest load path. A triangulated arrangement of bars must be used; otherwise, the structure can fail as a mechanism under small loads.



Here, the distance between corner points can freely increase or decrease, if the joints are assumed to behave as pins. A triangulating bar is therefore used (either in tension or in compression) in order to resist these changes in length: tension is preferable, since compressive members are generally weaker due to buckling (see later).

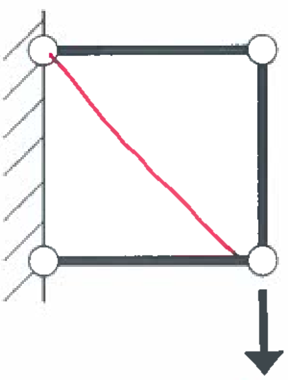


Tension – good!

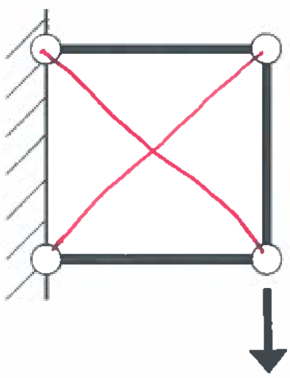


Compression – take care

Take care to use just enough triangulation.

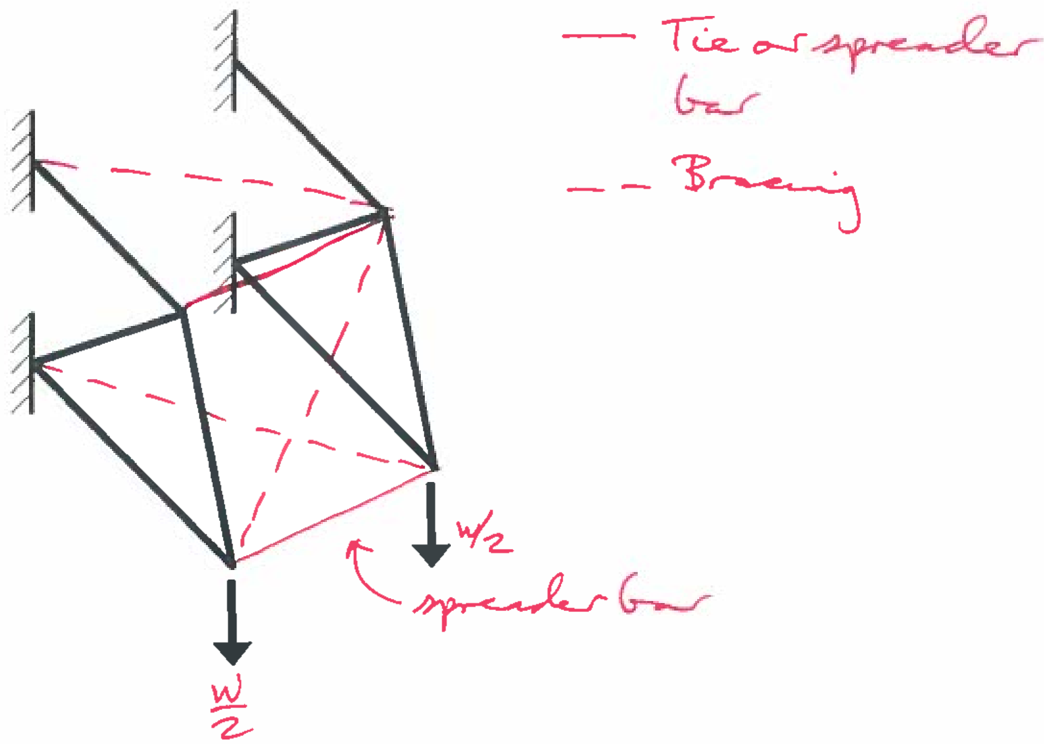


'Just stiff'
(determinate – no redundancy)



'Over stiff'
(indet. – redundant)

A single, vertical plane structure will normally move out of plane under very small vertical loads. For out-of-plane stability, the structure must be either truly three-dimensional or two 2-D planes connected together. For the latter, two aspects must be considered. First, the planes are joined by perpendicular bars or *ties* at corner joints: second, all planes on the outside of the new structure, which are potentially mechanistic, need to be triangulated using *bracing*.



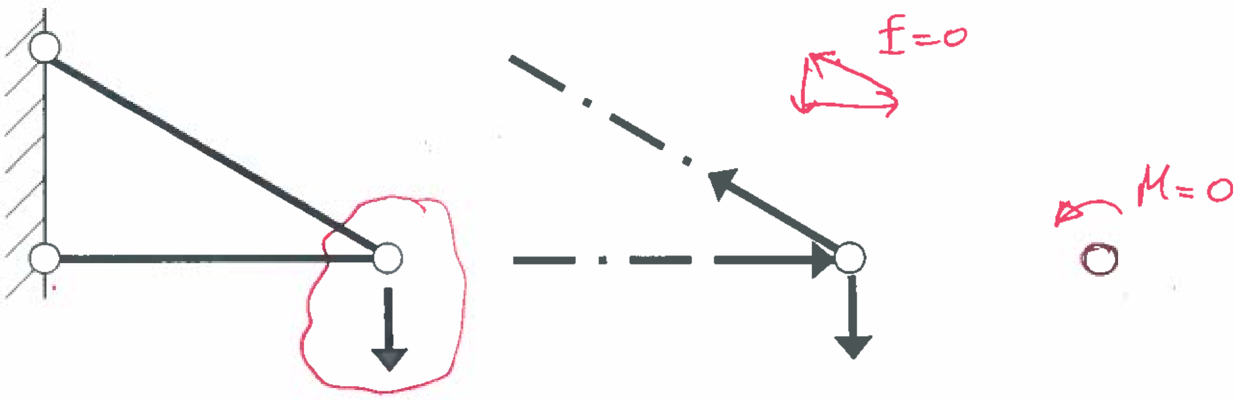
The bracing is so-called for it carries no load, theoretically, since there is no component of load acting within each plane of triangulation. The loading plate or spreader bar, depending on the design case, also ties the planes together.

The distinction between the load-bearing, primary structure of the bars and the secondary structure of the ties and bracing is critical: it is very helpful in the design process to remember which is which!

Once we have a satisfactory configuration, we are in a position to calculate the bar forces.

4.2 Bar Forces

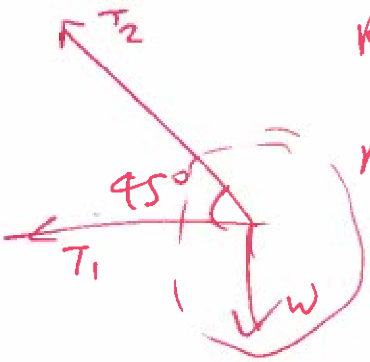
In a triangulated structure, provided the *centroidal axes* of the bars intersect at the joints, so do the lines of action of the bar forces. There can therefore be no moments acting at the joints and we may assume they behave as *pinned*. N.B. only load at joints



The bar forces may be calculated by statics alone, by resolving forces at the joints using *free-body diagrams*.

Remember to be consistent with the sign convention (e.g. +ve tensile)

Example

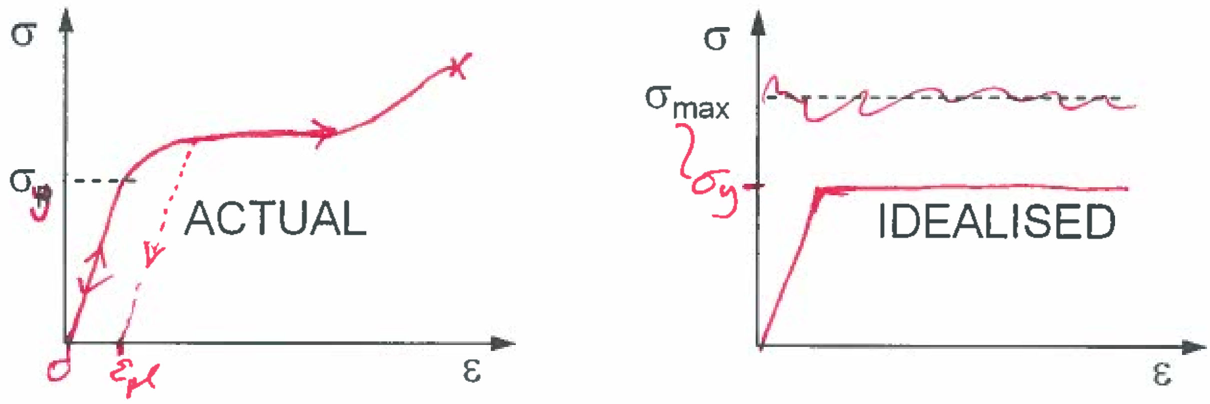


$R(\uparrow)$ $T_2 = \sqrt{2}W$

$R(\rightarrow)$ $T_1 = -W$

4.3 Designing Members for Tension

A typical tension test is given in terms of stress vs strain.

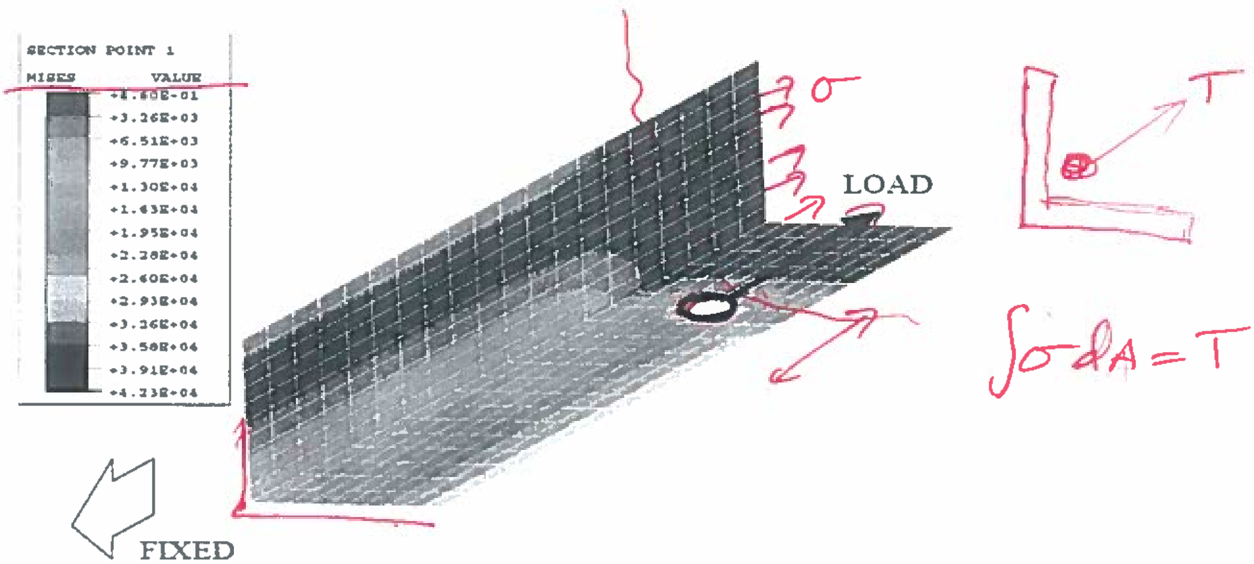


The behaviour is simplified by means of an idealised form, with a distinct maximum stress σ_{max}.

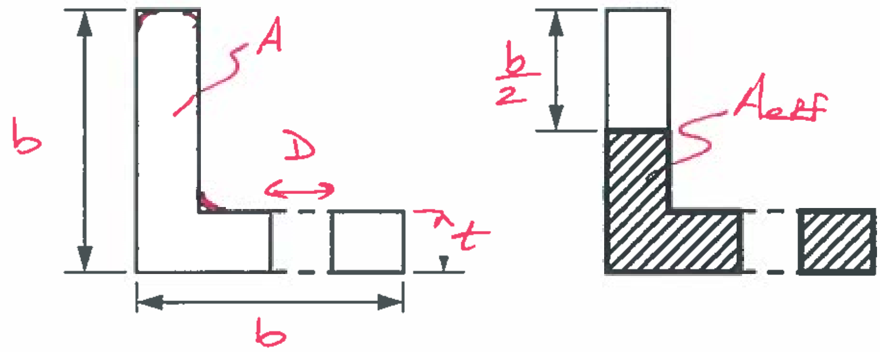
For mild steel, $\sigma_{max} = \sigma_y = 216\ N/mm^2$ (distinct yield stress)

For aluminium alloy, $\sigma_{max} = 255\ N/mm^2$ ($\equiv \sigma_y$)

If the stress is uniformly carried over the cross-section, then its required area must simply be greater than the (force in bar)/σy. In practice, end effects arise from the presence of a hole for connecting members by rivets or bolts. Predicted levels of von Mises stress (a measure of the resultant stress magnitude that indicates the onset of yielding when equal to the material yield stress) from a detailed computer simulation (finite-element analysis) are shown below – with levels varying from practically zero (blue) to high (red) stress. Note the latter behind the load at the hole.



A smaller area is available near the end to carry the load; the stresses here must therefore locally increase, and this is the most critical part under tension. The following effective area is therefore used:



If ~~b«t,~~ $b \gg t$ $A_{eff} = (2bt - Dt - \frac{b}{2}t) \times 0.9$

A hole ("ineffective upstand")

So the actual stress = $\frac{T}{A_{eff}}$

Note that the "0.9" factor allows for corners which are rounded in practice rather than being perfectly right-angled. ie. rolling margin.

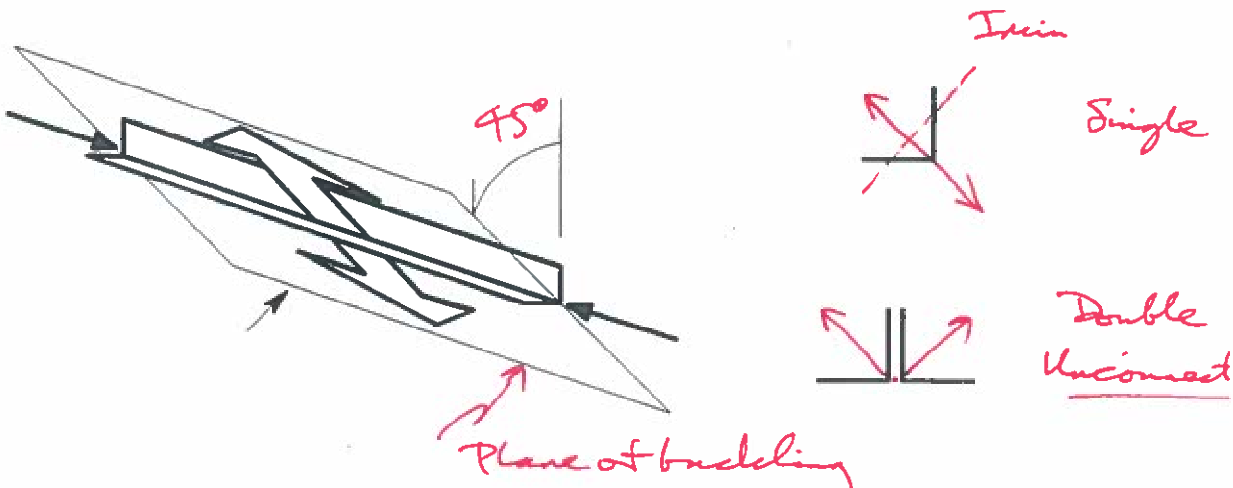
4.4 Designing Members for Compression

If the member is very short, it fails by squashing at σ_{max}. Otherwise, it fails by buckling, which is a geometric effect. The critical buckling force depends on:

- the full cross-sectional area (end effects are not crucial); (don't worry about holes)
- the longest unsupported length between a pair of joints or connected points in a given member;
- the cross-sectional shape. (I)

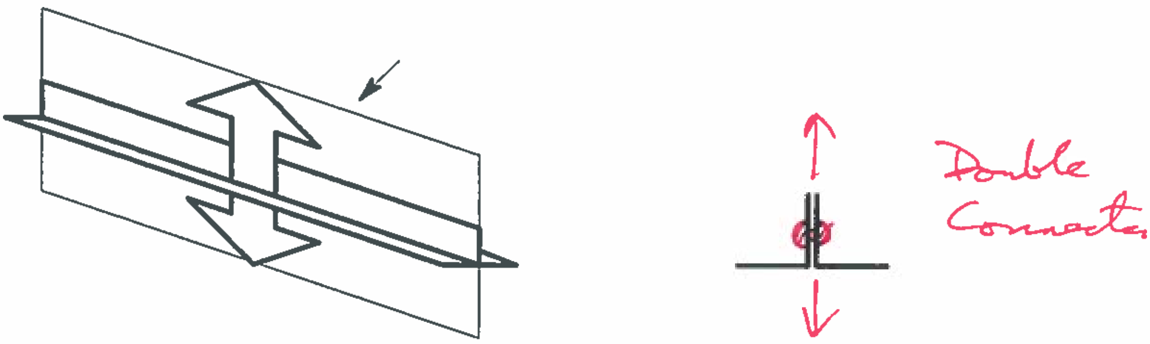
Buckling design curves are provided in the Design Handout, and show the critical buckling stress, σ_{CR}, for a given slenderness ratio, L/b. For a single member, or a double back-to-back but unconnected member, the (weakest) buckling direction is at 45°, as shown. This is known as buckling in mode A.

Mode A

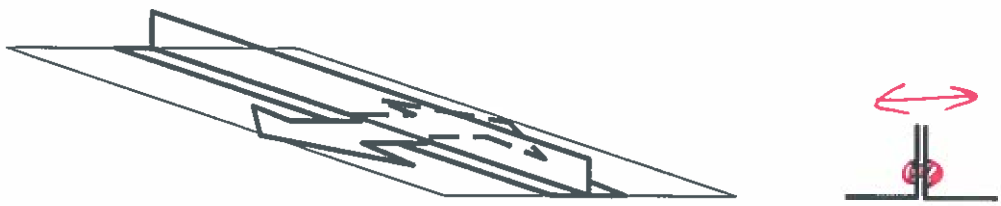


Double members, connected together periodically along their length by rivets (or bolts), behave differently.

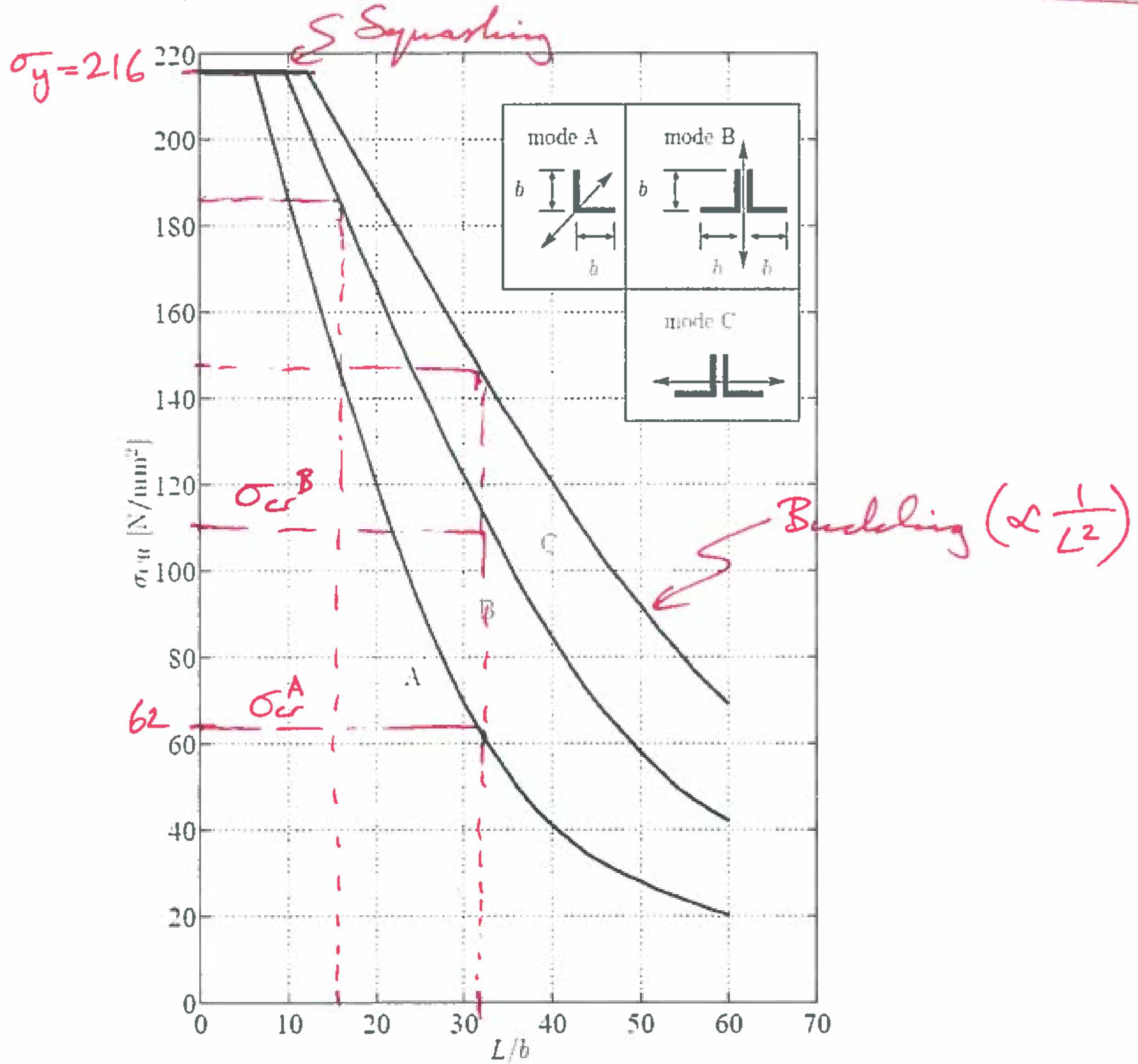
Mode B



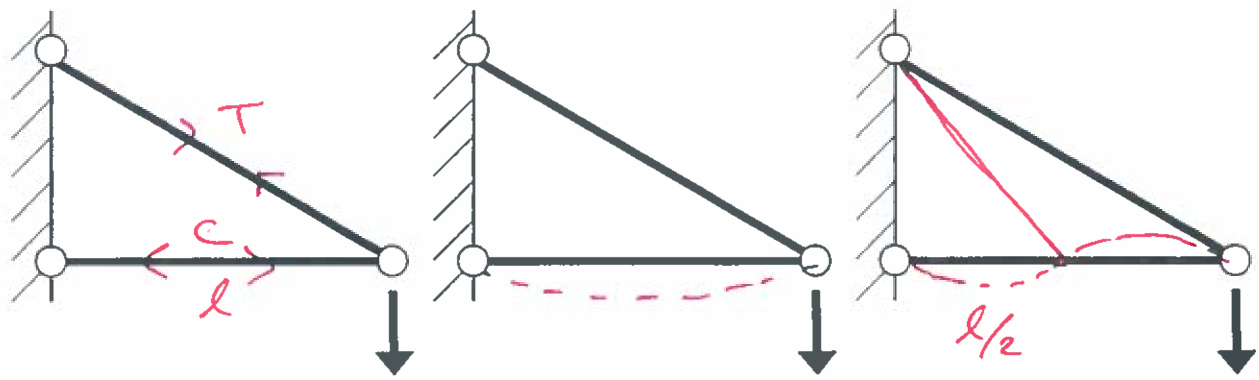
Mode C (if Mode prevented)



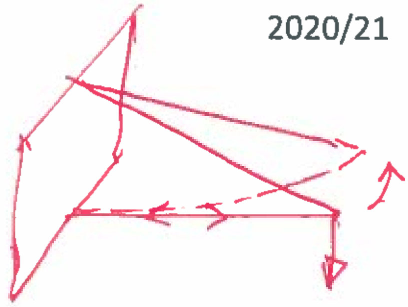
Example 1: determine σ_{CR} for a slenderness ratio, L/b = 40, using the design curves for mild steel angles.

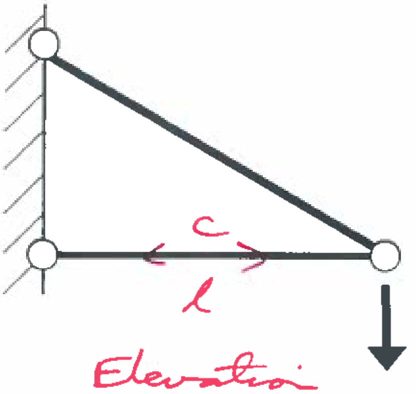


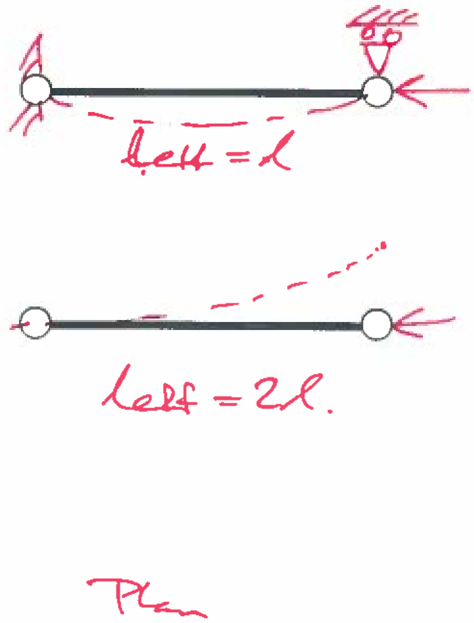
Bracing members are used to reduce the length and, hence, slenderness, to increase σ_{CR}. But take care: to realise the design strength indicated by the curves, the compression member must be fully constrained against translation at both ends.

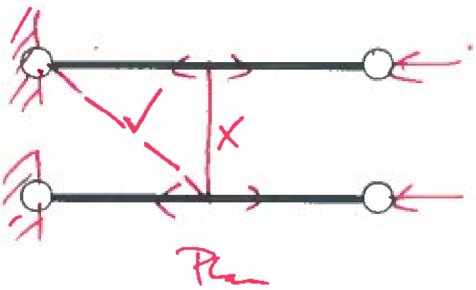


And take care what you brace against, and don't forget to think in 3D!



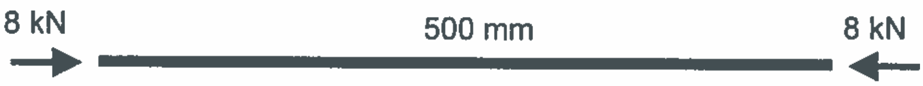






As for stability, the bracing members are assumed to carry no force. The smallest available section may therefore be used.

Example 2: design a member to carry the load shown below, using the previous buckling curves for steel angle struts.



i) For a single section (which would buckle in Mode A).

Member stress = $\sigma = \dfrac{8000\ N}{32\ mm^2} = 250\ N/mm^2$

L/b = $\dfrac{500}{16} \simeq 32$ (rounding up)

Mode A buckling gives:

$\sigma_{cr}^{A} = 62\ N/mm^2$

$\sigma \gg \sigma_{cr}$ ∴ fails

ii) Try two members, back to back (which perform as two 'Mode A' members).

New member stress = $\frac{8000\,N}{2 \times 32\,mm^2} = 125\ N/mm^2$

L/b = the same

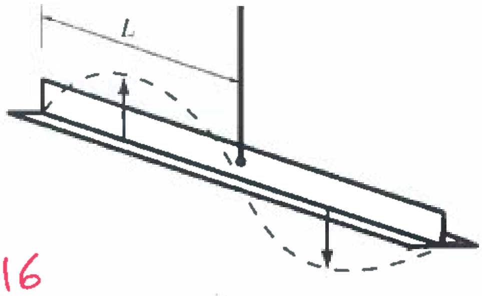
so σ_{cr}^{A} = the same $\therefore\ \sigma > \sigma_{cr}$ fails

iii) Try riveting the sections together (e.g. three rivets) along their length.

L/b = the same

Mode B buckling gives: $\sigma_{cr}^{B} = 114\ N/mm^2 \ \therefore\ \sigma > \sigma_{cr}$ fails but only just

iv) Try adding a central bracing member to halve the effective length for Mode B.

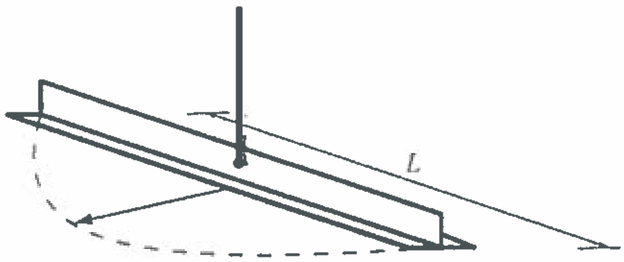


Now L/b = $\frac{250}{16} \simeq 16$

Mode B buckling now gives: $\sigma_{cr}^{B} = 183\ N/mm^2 > \sigma\ \therefore$ safe!

v) Final checks.

(a) The effective length for Mode C buckling is still 500 mm, so might it fail in this way?



L/b = $\frac{500}{16} \simeq 32$

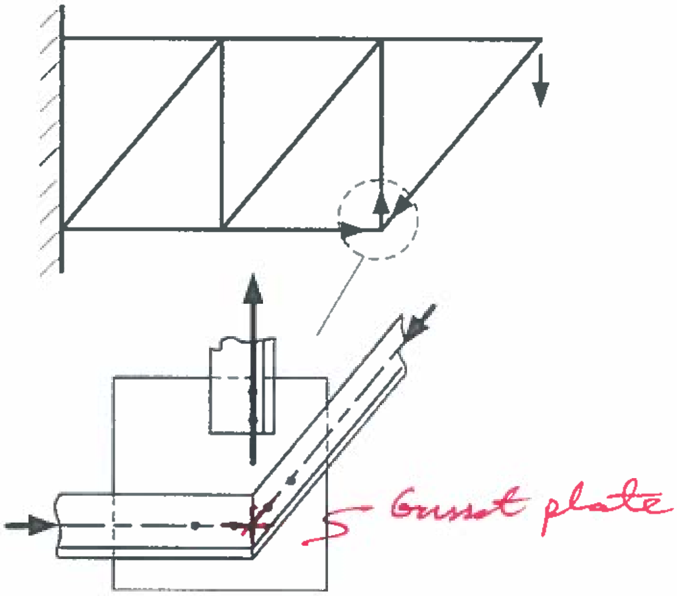
Mode ~~B~~ C buckling gives: $\sigma_{cr}^{C} = 146\ N/mm^2\ \therefore$ safe (phew!)

(b) Might Mode A buckling happen between the bracing and the endpoints?

eg. between rivets as in iv).

4.5 Joint Design

We must now take our 'space-frame', in which each line represents the centroidal axis of a bar, and design the joints to hold it together. A load-bearing joint in a 'pin-jointed' truss looks something like the figure below.



In a truss structure, there is only one mechanism by which any member can fail, depending on whether it is in tension or compression. However, there are three main ways by which such a joint can fail:

1. the lines of action of the bar forces (centroidal axes of the members) do not meet at a point, causing the joint to twist and disintegrate;

2. some part of the load path through the joint is not strong enough;

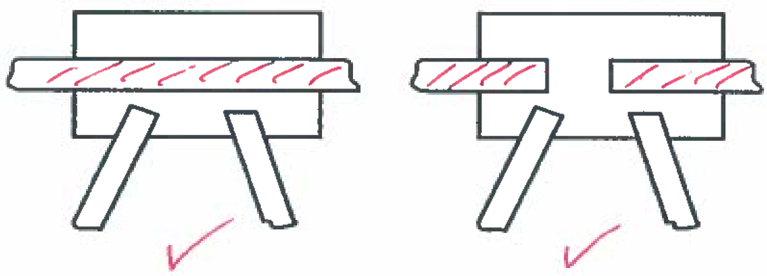
3. the gusset plate buckles under compression.

Remember that bracing members are small and only carry very light loads, so often do not need a gusset plate – they can simply be riveted to the members of the primary structure.

We shall now address each of these failure modes in turn. But first, the first rule in joint design is to avoid them!

Continuity

Try to design with continuous members.

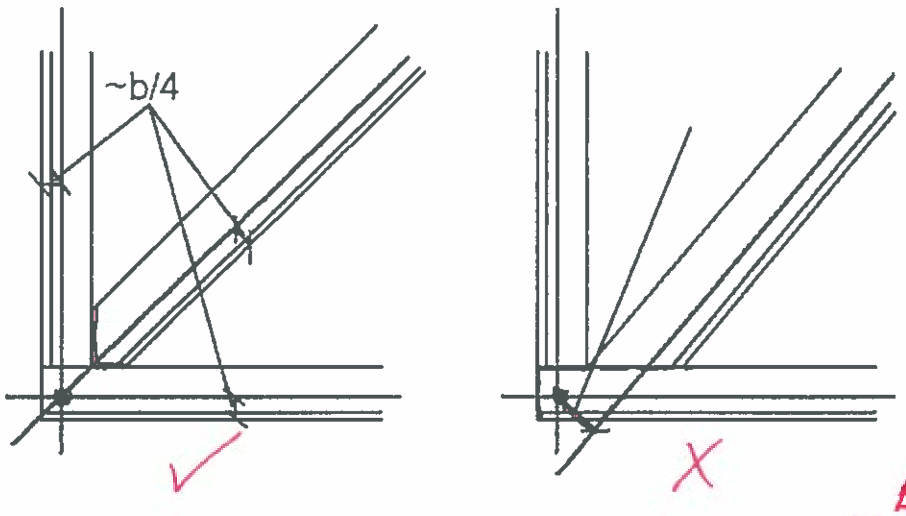


Lines of Action

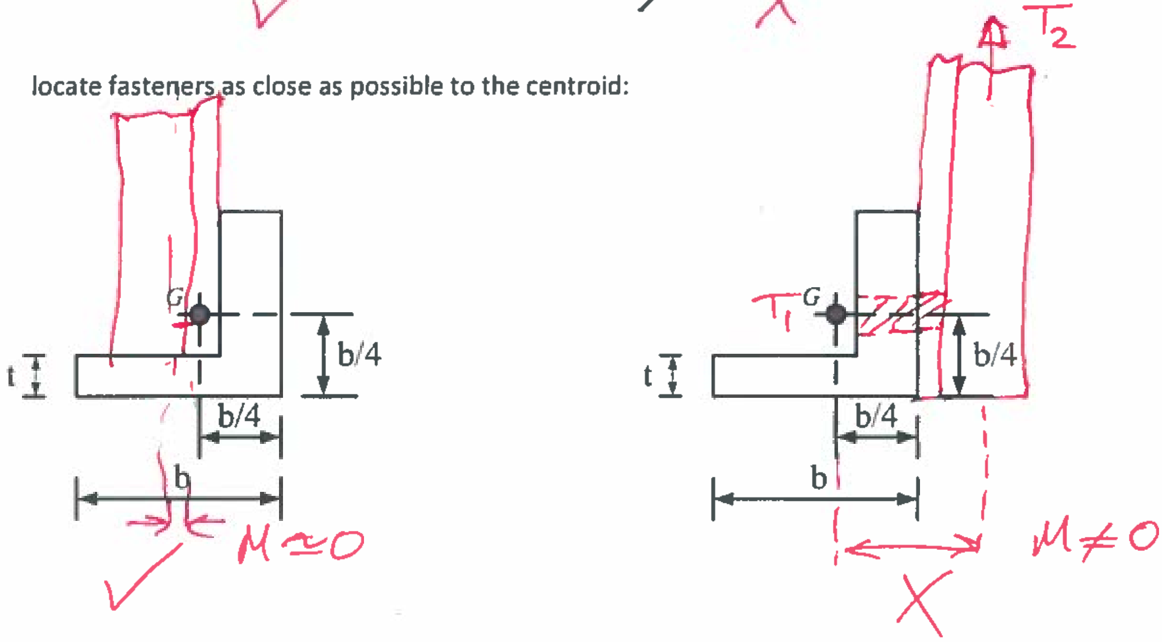
Along most of its length, the stress in a member is uniformly distributed over its cross-sectional area. The line of action of the resultant force is therefore assumed to lie along the centroidal axis of the section, as shown below. When designing joints:

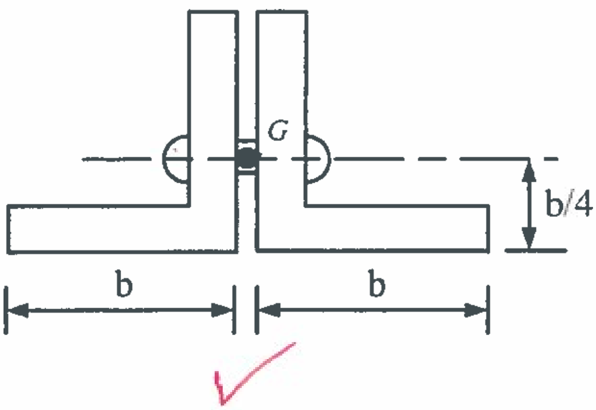
- ensure lines of action intersect at a single point:

Quick to fabricate but;
violates pin-joi
assumption.
→ rotation
→ large deflect
→ extra membe
stresses



- locate fasteners as close as possible to the centroid:

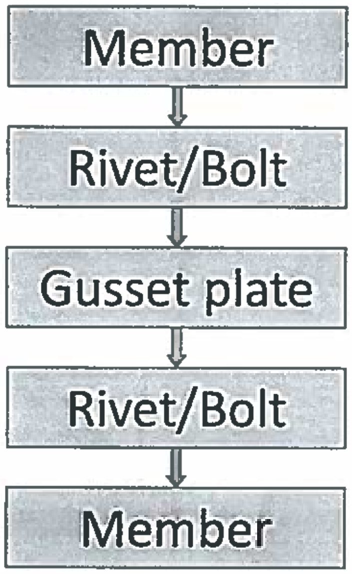




Load Paths

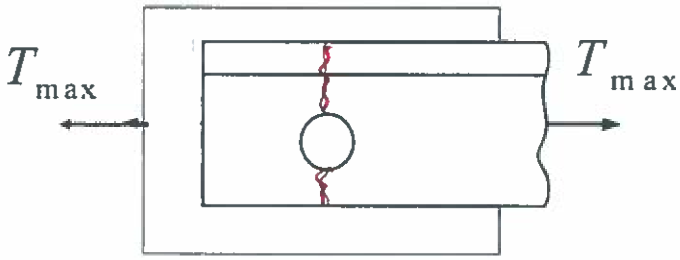
To transfer from one member to another, the load usually needs to pass through three more elements: the rivets (or bolts) fixed to the first member, the gusset plate, and the rivets or bolts fixed to the second member. Each of these five elements must be strong enough for its task, or the joint will fail at the weakest link.

- Think about the *load path*
- All elements on the path must transmit the load
- Strength is governed by the weakest link
- Assume load is split equally amongst fasteners



The members themselves can fail in tension, shear or bearing.

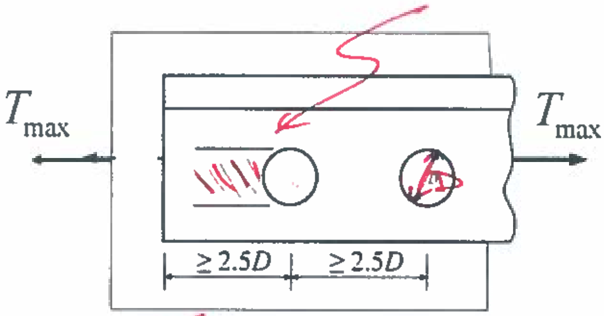
(i) Tensile Failure: $T_{\max} = \sigma_y A_{eff}$



(ii) Shear Failure (in tension *and* compression)

Local shearing or 'tearing out'

Empirical 'rules of thumb'.

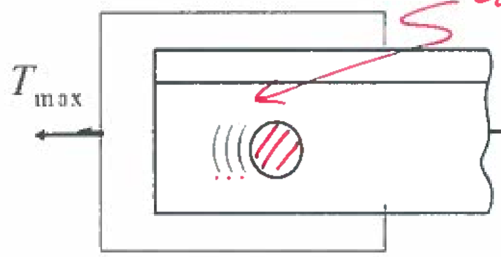


Keep apart

Keep away from edges

(iii) Bearing Failure (in tension *and* compression): $T_{max} = \sigma_y Dt$ or use T_{max} from the Design Handout.

local compression ⇒ local yielding



T_{max}

Fasteners

t_{plate}

t_{member}

Pop-rivets or bolts may be used for the fasteners but:

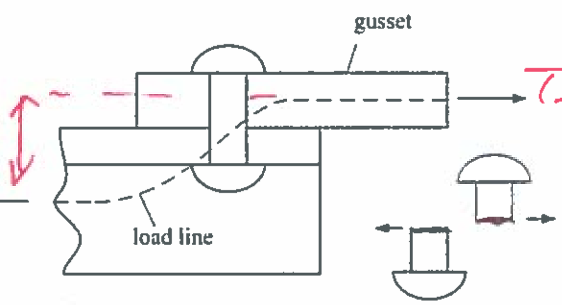
- do not mix rivets and bolts at the same joint; – differ in stiffness
- ensure even load share: fastener spacing < 32t – $t = \min\{t_{plate}, t_{member}\}$

  $S_o \geq 2.5D$ and $< 32t$ ≤ 4 fasteners in a row;
- bolts stronger but rivets cheaper, and only single-side access required.

Critical loads for the different fasteners are tabulated in the Design Handout. The following failure modes should be considered.

(i) Single Shear

Couple

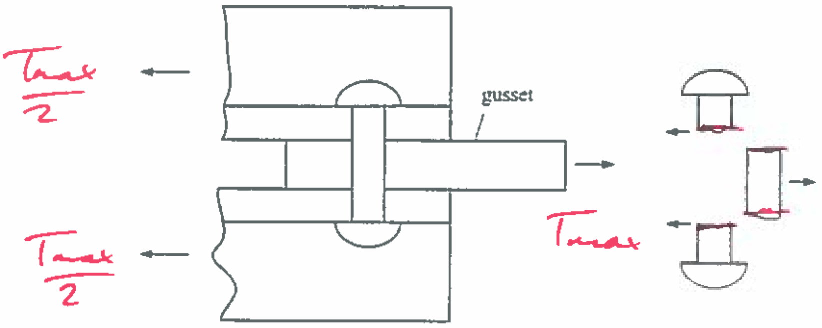


T_{max} T_{max}

$T_{max} = 1.26$ kN (if little tension)

Since pop-rivets have little tensile strength, single shear should be avoided.

(ii) Double Shear

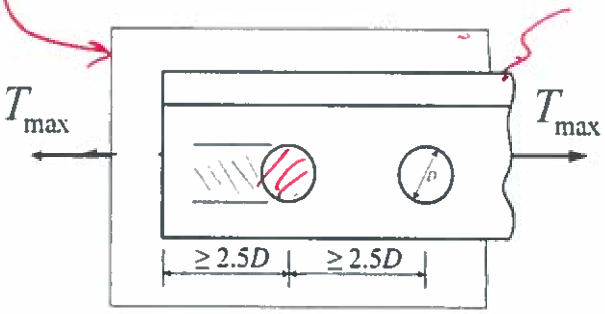


$T_{max} = 2.52$ kN

Twice as strong as single shear. – two shear planes and no rotation

Gusset Plates

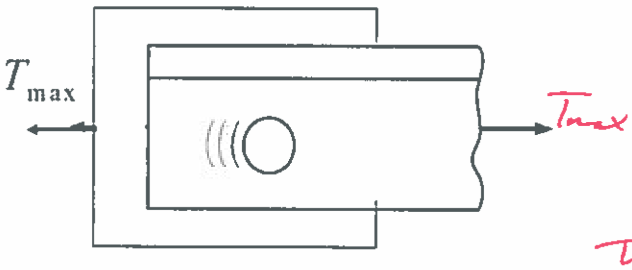
(i) Shear Failure (in tension *and* compression)

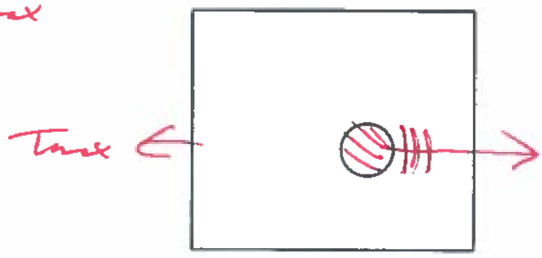


As for members

(ii) Bearing Failure (in tension *and* compression)

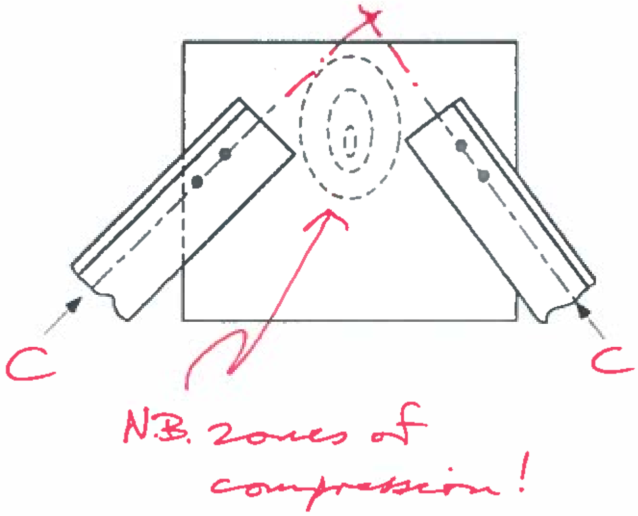
T_{max} from Design Handout (as member check)

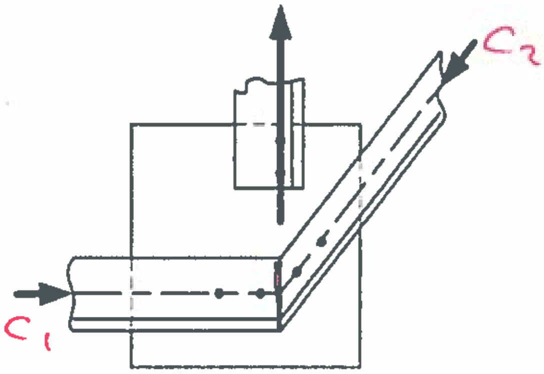




cf. loading plates
Design for bearing
and shear.

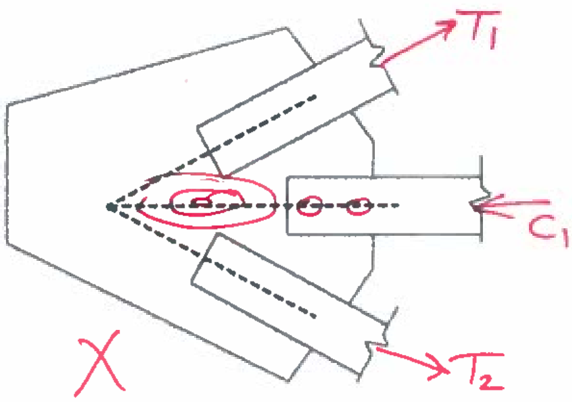
(iii) Plate Buckling

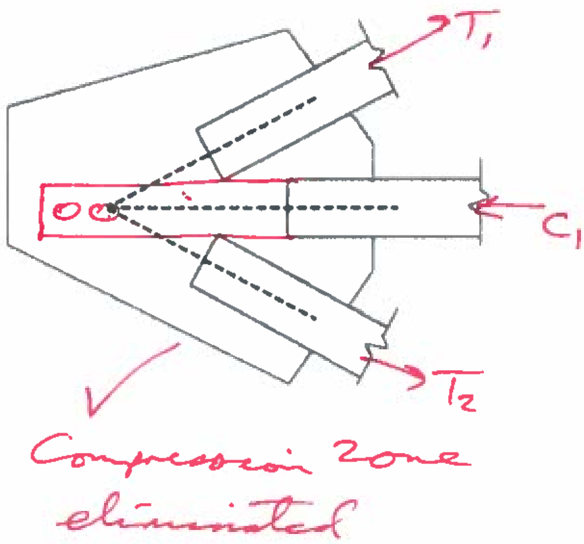


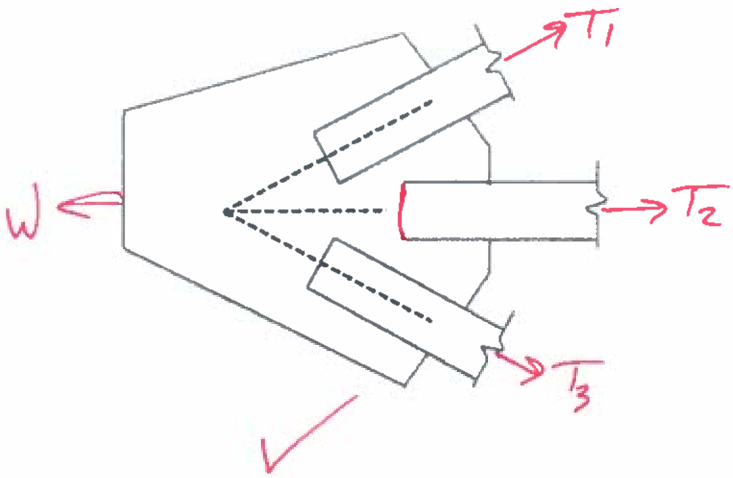


Avoid plate compression: abut compression members

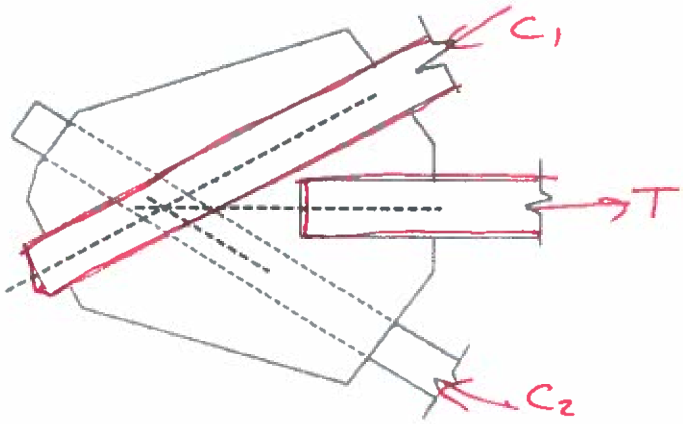
use plate thicker than angle

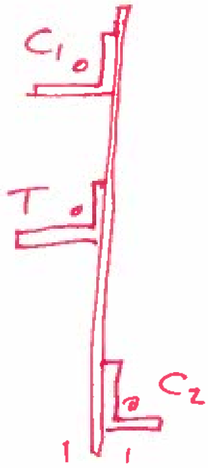






Not easy if two compression members





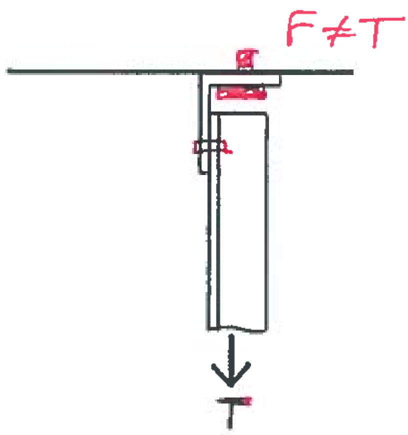
Good for plate tension

but... $M \neq 0$

... brace or redesign

Base Plate Connection

(i) Prying Action



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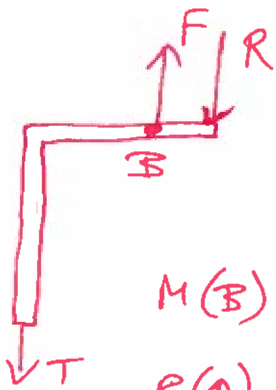
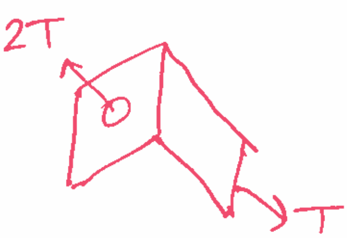


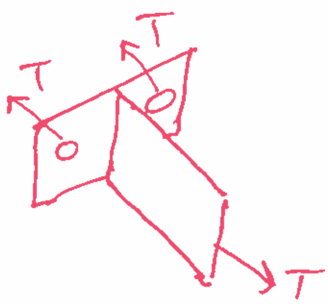
M(B) $R = T$

R(↑) $F = 2T$

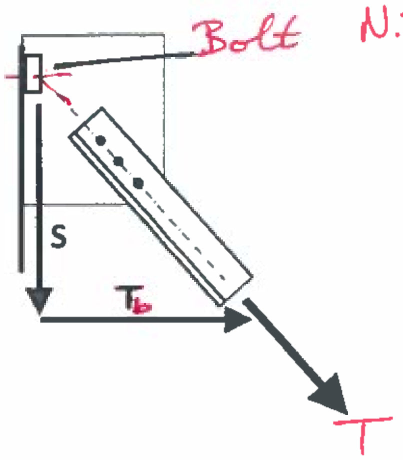
Think about the *load path:*

- Bolt load = 2T
- Ensure 2T < bolt failure load
- Use small connecting bracket
- Reinforce (e.g. use large washer)
- Use both base-plate holes





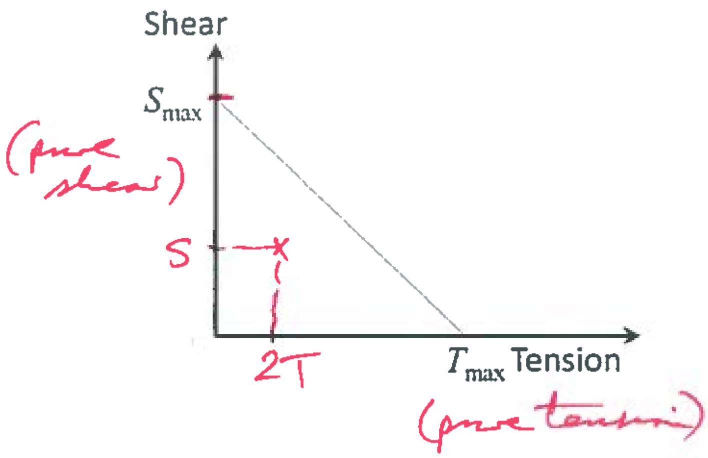
(ii) Combined Tension and Shear



N.B. line of action of T intersects ℄ of bolt

$T_b = 2T$ (prying)

Use an *interaction diagram*



- Assume straight line
- Ensure 'operating point' (2T, S) lies below line

5. DRAWING

Engineers must be able to communicate their ideas, and drawings remain an essential component of any design. The objective is to produce design drawings with sufficient detail to enable manufacture by a third party.

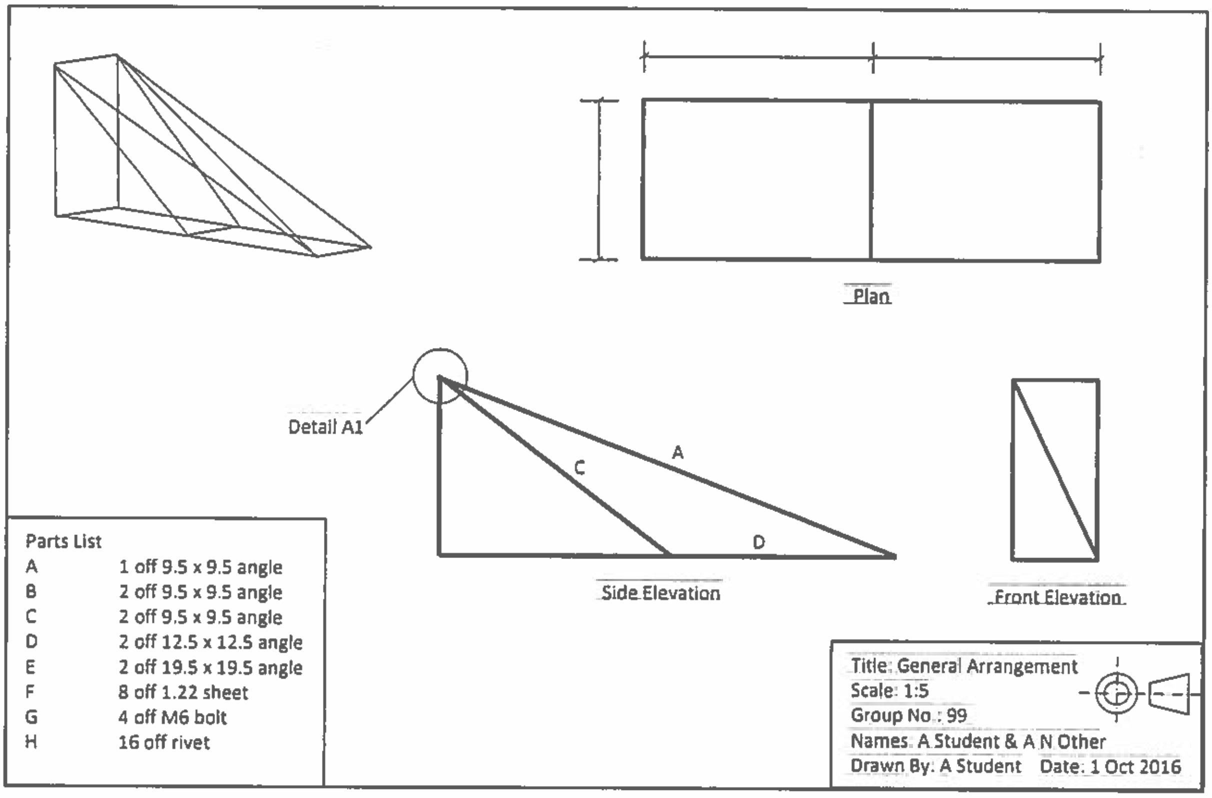
Drawings should be produced to scale using a consistent projection method (first angle or third angle). All dimensions should be explicitly marked but without including redundant information.

5.1 The General Arrangement

A scale drawing on a single A3 page, showing:

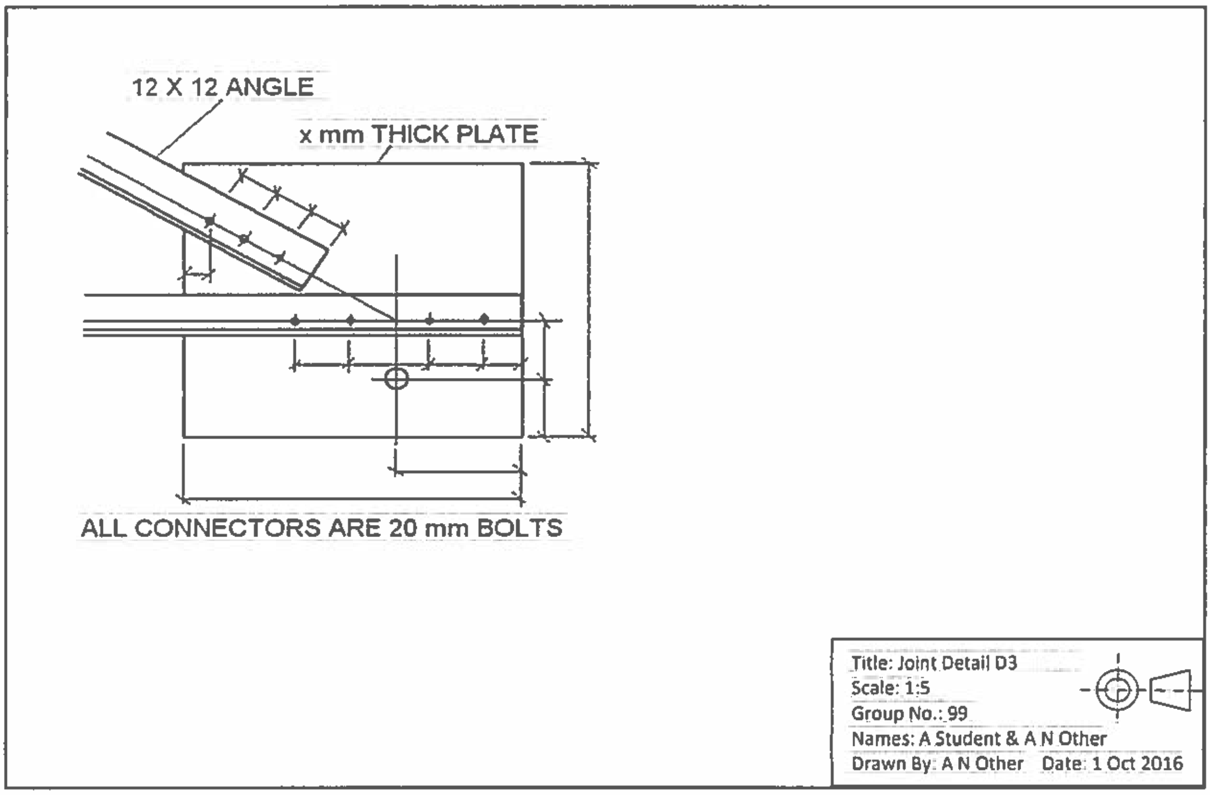
- The overall configuration of members and bracing
- Plan, side and end views
- Overall dimensions
- Parts list by item no. (section, size, length, gusset thickness, etc.)

It is often helpful to include a simple isometric sketch to aid interpretation.



5.2 Detailing

The detailing is given on additional scale drawings, which, together with the GA, provide all the information required for manufacture.



- Show clearly how all joints work
- Mark centroidal lines (chained)
- Include bolt/rivet holes (as circles, with centres crossed)
- Include a parts list by item no. (gusset thickness, bolts, rivets, washers, etc.)

6. REPORTING

As part of communicating their designs, Engineers are often required to produce some form of design substantiation report. The objective is to produce a concise technical report that summarises the basis of the final design and how it is expected to perform.

6.1 Report Structure

In no more than 10 pages you must provide:

- Summary
- Original design brief, supplemented with any additional data or assumptions
- Alternative design concepts, summarising their advantages and disadvantages
- Full set of calculations to substantiate your final design
- Costing sheet (this may be stuck into your report), with a brief commentary on how your design compared with the lightest.
- List of deviations (as-built compared to as-designed)
- Test results & observations (failure load and mode of failure; comparisons with other designs)
- Modifications (recommended improvements, given deviations, lessons learnt, etc.)
- Conclusions

Acknowledgements

The Cambridge Structural Design Course is over 60 years old. It was created by Mr John Dwight, soon after his appointment as a lecturer in the Department in 1961. Since then, many have contributed to its gradual evolution but it remains largely unchanged, with generations of students recognising the material covered here.