Generalization of NLIN model for WDM systems to wavelength-dependent Raman gain and attenuation scenarios

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***Abstract*—Starting from [1] and [2], a generalized model for the phenomenon of NLIN (Non Linear Interference Noise) in the presence of wavelenght-dependent attenuation and Raman gain is developed.**

I. Objective and summary of previous results

A. *Equation for the field and narrowband approximation*

Let us consider the standard NLSE for a fiber with Raman amplification profile $g(z)$

$$\frac{\partial}{\partial z}A = -\frac{\alpha - g(z)}{2}A - \beta_1\frac{\partial}{\partial t}A - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A + i\gamma|A|^2 A \quad (1)$$

where t is the physical time. Recall that A is proportional to the complex envelope of the electric field inside the fiber, in a way such that the dimension of A is $[A^2]$ = W.

This equation holds for a narrowband field, such that $A(z,t)$ is a slowly varying function of t. In a WDM system, this approximation is assumed to be still valid, as the usual channel spectral spacing is usually in the range [12.5, 100]GHz in third window (as defined in standard [3] for DWDM architectures). As for CWDM applications, the spacing is $20nm$ which corresponds to $\approx$ 2.5THz in third window [4]. In the presence of tens of channels for DWDM case, the total field is still narrowband, as the percentage bandwidth is still low.

However, while this assumption allow the dispersion parameter to be simplified in the writing of β_2 (using a second order expansion of the phase constant), the narrowband assumption is more delicate for the attenuation terms. In fact, a constant attenuation and Raman gain over the signal bandwidth may be an assumption too strong to make, especially when interested in tilting Raman amplification profile for multi-pump schemes. This is the case of the study in [5].

In the following, some of the most critical aspect of the theoretical model for NLIN, proposed in [2], are reviewed, and the notation is kept as similar as possible. Then a generalization is developed in the case of two interacting WDM channel fields.

B. *Rescaling of fields*

Let us define $\psi(z)$ as

$$\frac{d}{dz}\psi(z) = -\frac{\alpha - g(z)}{2}\psi(z) \quad (2)$$

Using such function, define $u(z,t)$ as the normalized field

$$A(z,t) = \psi(z)u(z,t) \quad (3)$$

These definitions, when substituted in 1 give rise to a new equation

$$\frac{\partial}{\partial z}u = -i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}u + i\gamma f(z)|u|^2 u \quad (4)$$

where $f(z) = \psi(z)^2$. The resulting equation show a new, space-varying, nonlinear coefficien.

II. Coupled NLS equations for WDM channels

Consider two WDM channels named A and B. The following hypotesis are made:

- channels A and B have a spectral separation of Ω,
- both channels have the same nonlinear coefficient,
- the group velocity profile is approximatively linear in the frequency (β_2 is constant) in the whole band of interest
- attenuation and Raman gain depend on the channel choice, but are constant within the band of the same channel.

Following [6]

$$\begin{aligned}\frac{\partial}{\partial z}A_A = &-\frac{\alpha_A - g_A(z)}{2}A_A - \beta_1\frac{\partial}{\partial t}A_A - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A_A \\ &+ i\gamma(|A_A|^2 + 2|A_B|^2)A_A\end{aligned} \quad (5)$$

$$\begin{aligned}\frac{\partial}{\partial z}A_B = &-\frac{\alpha_B - g_B(z)}{2}A_B - \beta_1\frac{\partial}{\partial t}A_B - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A_B \\ &+ i\gamma(|A_B|^2 + 2|A_A|^2)A_B\end{aligned} \quad (6)$$

Let us consider the WDM channel A as the channel of interest.

We now proceed to normalize the fields A_A, A_B with the respective normalization functions ψ_A, ψ_B. In addition, a moving time reference frame is assumed, taking as a reference the time of arrival of the first pulse in channel A: $T = t - z/v_{gA} = t - \beta_{1A}z$. The resulting coupled equations are

$$\begin{aligned}\frac{\partial}{\partial z}u_A = &-i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}u_A \\ &+ i\gamma\left(f_A(z)|u_A|^2 + 2f_A(z)|u_B|^2\right)u_A\end{aligned} \quad (7)$$

$$\begin{aligned}\frac{\partial}{\partial z}u_B = &-\Delta\beta_1\frac{\partial}{\partial t}u_B - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}u_B \\ &+ i\gamma\left(f_B(z)|u_B|^2 + 2f_A(z)|u_A|^2\right)u_B\end{aligned} \quad (8)$$

where $\Delta\beta_1 = \beta_{1B} - \beta_{1A} = \beta_2\Omega$.

III. Generalization of the 0-th order term

The term of the 0-th order is identical as the one derived in [1], and this is clearly a result of superposition property of linear equations. The only exception is due to the notation used: the total field in this case cannot be expressed by a simple sum of terms $u_A^{(0)} + u_B^{(0)}$. There are in fact two caveats:

- $u_A^{(0)}$ and $u_B^{(0)}$ functions represent fields with different normalization constants,
- the functions are derived from *complex amplitudes* of different carrier frequency signals

the equivalent field complex amplitude, with respect to the channel A carrier frequency, is actually

$$\begin{aligned} A_{tot}(z,T) &= A_A(z,T) + A_B(z,T) \\ &= \psi_A(z)u_A(z,T) + \psi_A(z)u_B(z,T)\exp[-i\Omega T] \end{aligned} \tag{9}$$

Let us consider the initial fields as sums of shifted impulses which codify a given message. Let τ be the symbol period:

$$\begin{aligned} u_A(0,T) &= \sum_k a_k g(0, T - k\tau) \\ u_B(0,T) &= \sum_k b_k g(0, T - k\tau) \end{aligned} \tag{10}$$

The solution for the 0-th order field is simply:

$$\begin{aligned} u_A^{(0)}(z,T) &= \sum_k a_k g^{(0)}(z, T - k\tau) \\ u_B^{(0)}(z,T) &= \sum_k b_k g^{(0)}(z, T - k\tau - \beta_2\Omega z) \end{aligned} \tag{11}$$

because of linearity. As in [1], we define the differential operators $\mathbf{U}_A(z) = \exp[-i\frac{\beta_2}{2}z\frac{\partial^2}{\partial T^2}]$ and $\mathbf{U}_B(z) = \exp[-i\frac{\beta_2}{2}z\frac{\partial^2}{\partial T^2} - i\Delta\beta_1 z\frac{\partial}{\partial T}]$. So the propagated 0-th order impulses are:

$$g^{(0)}(z, T - k\tau) = \mathbf{U}_j(z)g(z, T - k\tau) \tag{12}$$

with $j \in \{A, B\}$.

IV. Generalization of first order perturbation theory

The separation of fields allow us to analyze separately the effects of SPM and XPM. Let us apply the perturbation method to 7

$$\begin{aligned} \frac{\partial}{\partial z}u_A^{(1)} &= -i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}u_A^{(1)} \\ &+ i\gamma\left(f_A(z)|u_A^{(0)}|^2 + 2f_B(z)|u_B^{(0)}|^2\right)u_A^{(0)} \end{aligned} \tag{13}$$

Writing the integral solution to the inhomogeneous linear equation above:

$$\begin{aligned} &u_A^{(1)}(L,T) = \\ &i\gamma\int_0^L \mathbf{U}_A(L-z)\left(f_A(z)|u_A^{(0)}|^2 + 2f_B(z)|u_B^{(0)}|^2\right)u_A^{(0)}dz \end{aligned} \tag{14}$$

V. Generalization of estimation error

A. Generic expression under matched filter conditions

Using a matched filter receiver, with matching to the linearly propagated initial pulse waveform $g^{(0)}(z,T)$, we obtain the following equation for the estimation error on the first symbol Δa_0

$$\Delta a_0 = \int_{-\infty}^{\infty} u_A^{(1)}(L,T)g^{(0)*}(L,T)dT \tag{15}$$

this expression is of course not influenced by the choice of representation of fields using the two channel complex amplitudes.

B. Expanded expression for given modulation format

By using the first order perturbation as solved in IV we get an expression ready to be solvedwith respect to a given modulation format

$$\begin{aligned} &\Delta a_0 = \\ &i\gamma\int_{-\infty}^{\infty}\int_0^L g^{(0)*}(z,T)\left(f_A(z)|u_A^{(0)}|^2 + 2f_B(z)|u_B^{(0)}|^2\right)u_A^{(0)}dzdT \end{aligned} \tag{16}$$

In this passage the fact that $\mathbf{U}_A(L-z)g^{(0)*}(L,T) = g^{(0)*}(z,T)$ was exploited. Notice that, being that the two fiber linear propagation terms are different, the substitution as done in [1] is valid only because matching is done by considering the propagated symbol waveform over channel A. Finally, substituting the modulation and using the same notation as [1, eq. 5, 6, 7], the resulting expression is

$$\Delta a_0 = i\gamma\sum_{h,k,m}\left(a_h a_k^* a_m S_{h,k,m} + 2a_h b_k^* b_m X_{h,k,m}\right) \tag{17}$$

in order to mantain the notation uniform, the 17 is kept as the original, whereas the generalized interaction terms are

$$\begin{aligned} S_{h,k,m} = \int_0^L f_A(z)\int_{-\infty}^{\infty} &g^{(0)*}(z,T) \\ &g^{(0)}(z,T-h\tau)g^{(0)*}(z,T-k\tau) \\ &g^{(0)}(z,T-m\tau)dTdz \end{aligned} \tag{18}$$

for the SPM

$$\begin{aligned} X_{h,k,m} = \int_0^L f_B(z)\int_{-\infty}^{\infty} &g^{(0)*}(z,T) \\ &g^{(0)}(z,T-h\tau)g^{(0)*}(z,T-k\tau-\beta_2\Omega z) \\ &g^{(0)}(z,T-m\tau-\beta_2\Omega z)dTdz \end{aligned} \tag{19}$$

for the XPM.

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