Generalization of NLIN model for WDM systems to wavelength-dependent Raman gain and attenuation scenarios

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Abstract

Insert article overview

[2] and [] we develop a generalized model for the phenomenon of NLIN (Non Linear Interference Noise) in avelenght-dependent attenuation and Raman gain.

I. Objective and summary of previous results

A. Equation for the field and narrowband approximation

Let us consider the standard NLSE for a fiber with Raman amplification profile $g(z)$

$$\frac{\partial}{\partial z}A = -\frac{\alpha - g(z)}{2}A - \beta_1 \frac{\partial}{\partial t}A - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A + i\gamma|A|^2 A \tag{1}$$

where t is the physical time. Recall that A is proportional to the electric field inside the fiber, in a way such that the dimension of A is $[A^2] = \text{W}$.

This equation holds for a narrowband field, such that $A(z,t)$ is a slowly varying function of t. In a WDM system, this approximation is assumed to be still true, as the usual channel spectral spacing is greater than 12.5GHz in third window (as defined in standard [3] for DWDM architectures). In the presence of hundreds of channels, the total field is still narrowband, as the percentage bandwidth is still low.

In the following, the theoretical framework proposed in [4] is reviewed, and the notation is kept as similar as possible. Then a generalization is developed in the case of two interacting WDM channel fields.

B. Rescaling of fields

Let us define $\psi(z)$ as

$$\frac{d}{dz}\psi(z) = -\frac{\alpha - g(z)}{2}\psi(z) \tag{2}$$

Using such function, define $u(z,t)$ as the normalized field

$$A(z,t) = \psi(z)u(z,t) \tag{3}$$

These definitions, when substituted in 1 give rise to a new equation

$$\frac{\partial}{\partial z}u = -i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}u + i\gamma\psi(z)^2|u|^2 u \tag{4}$$

where $f(z) = \psi(z)^2$. The resulting equation show a new, space-varying, nonlinear coefficien.

II. Coupled NLS equations for WDM channels

Consider two WDM channels named A and B. The following hypotesis are made:

- channels A and B have a spectral separation of Ω,
- both channels have the same nonlinear coefficient,
- the group velocity profile is approximatively linear in the frequency (β_2 is constant) in the whole band of interest
- attenuation and Raman gain depend on the channel choice, but are constant within the band of the same channel.

Following [1] p.

$$\frac{\partial}{\partial z}A_A = -\frac{\alpha_A - g_A(z)}{2}A_A - \beta_1 \frac{\partial}{\partial t}A_A - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A_A + i\gamma(|A_A|^2 + 2|A_B|^2)A_A \tag{5}$$

$$\frac{\partial}{\partial z}A_B = -\frac{\alpha_B - g_B(z)}{2}A_B - \beta_1 \frac{\partial}{\partial t}A_B - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A_B + i\gamma(|A_B|^2 + 2|A_A|^2)A_B \tag{6}$$

Let us consider the WDM channel A as the channel of interest.

We now proceed to normalize the fields A_A, A_B with the respective normalization functions ψ_A, ψ_B. In addition, a moving time reference frame is assumed, taking as a reference the time of arrival of the first pulse in channel A: $T = t - z/v_{gA} = t - \beta_{1A} z$. The resulting coupled equations are

$$\frac{\partial}{\partial z} u_A = -i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2} u_A + i\gamma\left(f_A(z)|u_A|^2 + 2\frac{f_A(z)}{f_B(z)}|u_B|^2\right) u_A \tag{7}$$

$$\frac{\partial}{\partial z} u_B = -\Delta\beta_1 \frac{\partial}{\partial t} u_B - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2} u_B + i\gamma\left(f_B(z)|u_B|^2 + 2\frac{f_B(z)}{f_A(z)}|u_A|^2\right) u_B \tag{8}$$

$$\tag{9}$$

where $\Delta\beta_1 = \beta_{1B} - \beta_{1A} = \beta_2 \Omega$.

III. Generalization of the 0-th order term

The term of the 0-th order is identical as the one derived in [2], and this is clearly a result of superposition property of linear equations. The only exception is due to the notation used: the total field in this case cannot be expressed by a simple sum of terms $u_A^{(0)} + u_B^{(0)}$. There are in fact two caveats:

- $u_A^{(0)}$ and $u_B^{(0)}$ functions represent fields with different normalization constants,
- the functions are derived from *complex amplitudes* of different carrier frequency signals

the equivalent field complex amplitude, with respect to the channel A carrier frequency, is actually

$$\begin{aligned} A_{tot}(z,T) &= A_A(z,T) + A_B(z,T) \\ &= \psi_A(z) u_A(z,T) + \psi_A(z) u_B(z,T) \exp[-i\Omega T] \end{aligned} \tag{10}$$

Let us consider the initial fields as sums of shifted impulses which codify a given message. Let τ be the symbol period:

$$\begin{aligned} u_A(0,T) &= \sum_k a_k g(0, T - k\tau) \\ u_B(0,T) &= \sum_k b_k g(0, T - k\tau) \end{aligned} \tag{11}$$

The solution for the 0-th order field is simply:

$$\begin{aligned} u_A^{(0)}(z,T) &= \sum_k a_k g^{(0)}(z, T - k\tau) \\ u_B^{(0)}(z,T) &= \sum_k b_k g^{(0)}(z, T - k\tau - \beta_2 \Omega z) \end{aligned} \tag{12}$$

because of linearity. As in [2], we define the differential operators

$$\mathbf{U}_A(z) = \exp\left[i\frac{\beta_2}{2} z \frac{\partial^2}{\partial T^2}\right] \tag{13}$$

$$\mathbf{U}_B(z) = \exp\left[i\frac{\beta_2}{2} z \frac{\partial^2}{\partial T^2} - i\Delta\beta_1 z \frac{\partial}{\partial T}\right] \tag{14}$$

So the propagated 0-th order impulses are:

$$g^{(0)}(z, T - k\tau) = \mathbf{U}_j(z) g(z, T - k\tau) \tag{15}$$

with $j \in \{A, B\}$.

IV. Generalization of first order perturbation theory

The separation of fields allow us to analyze separately the effects of SPM and XPM. Let us apply the perturbation method to 7

$$\frac{\partial}{\partial z} u_A^{(1)} = -i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2} u_A^{(1)} + i\gamma\left(f_A(z)|u_A^{(0)}|^2 + 2\frac{f_A(z)}{f_B(z)}|u_B^{(0)}|^2\right) u_A^{(0)} \tag{16}$$

Writing the integral solution to the inhomogeneous linear equation above:

$$u_A^{(1)}(L,T) = i\gamma \int_0^L \mathbf{U}_A(L-z)\left(f_A(z)|u_A^{(0)}|^2 + 2\frac{f_A(z)}{f_B(z)}|u_B^{(0)}|^2\right) u_A^{(0)} dz \tag{17}$$

V. Generalization of estimation error

A. Generic expression under matched filter conditions

Using a matched filter receiver, with matching to the linearly propagated initial pulse waveform $g^{(0)}(z,T)$, we obtain the following equation for the estimation error on the first symbol Δa_0 by expanding the perturbation term

Giustificazione della formulazione di Me

$$\Delta a_0 = \int_{-\infty}^{\infty} u_A^{(1)}(L,T) g^{(0)*}(z,T) dT = \\ = \tag{18}$$

The calculation done in [2, eq. 1] are actually the correct version of the linearly propagated field (superposition of channel of interest and interferer) except for a sign. In the following derivation the calculations are made again.

In order to prove the equation let us recall the linear propagator operator (as defined in [2]):

$$\mathbf{U}[z] = \exp\left[i\frac{\beta_2}{2}z\frac{\partial^2}{\partial t^2}\right] \tag{19}$$

Then, let us focus on the interfering channel field at the *input*

$$u(0,t) = \sum_k b_k g(0,t-\tau k)e^{i\Omega t} \tag{20}$$

and apply the propagator.

This is best done in frequency domain, and, by linearity, it is possible to focus only on the symbol waveform g. Using frequency shifting property

$$g(0,t)e^{i\Omega t} \rightarrow \hat{g}(0,\omega-\Omega) \tag{21}$$

In frequency domain, we have the operator

$$\hat{\mathbf{U}}[z] = \exp\left[-i\frac{\beta_2}{2}z\omega^2\right] \tag{22}$$

Let us focus on the linear propagation of complex envelope *spectrum* of a single impulse

$$\hat{g}^{(0)}(z,\omega) = \exp\left[-i\frac{\beta_2}{2}z\omega^2\right]\hat{g}(0,\omega-\Omega) \tag{23}$$

considering the antitransform, with a square completion argument,

$$\frac{1}{2\pi}\int_{\mathbb{R}} \exp\left[-i\frac{\beta_2}{2}z\omega^2\right]\hat{g}(z,\omega-\Omega)\exp\left[i\omega t\right]d\omega = \tag{24}$$

$$= \frac{1}{2\pi}\int_{\mathbb{R}} \exp\left[-i\frac{\beta_2}{2}z(\omega-\Omega)^2\right]\hat{g}(z,\omega-\Omega)\exp\left[i(\omega-\Omega)t\right] \underbrace{\exp\left[i\Omega t\right]}_{\text{frequency shifting}} \underbrace{\exp\left[-i\beta_2 z\omega\Omega\right]}_{\text{time delay}} \underbrace{\exp\left[i\frac{\beta_2}{2}z\Omega^2\right]}_{\text{constant}} d\omega \tag{25}$$

in the notation of [2], $g^{(0)}(z,t) = \mathbf{U}(z)g(0,t)$ is the pulse propagated as in the channel of interest, so we have the following antitransform relation

$$g^{(0)}(z,t) = \frac{1}{2\pi}\int_{\mathbb{R}} \exp\left[-i\frac{\beta_2}{2}z\omega^2\right]\hat{g}(z,\omega)\exp\left[i\omega t\right]d\omega \tag{26}$$

In conclusion, by using a simple change of variables and the time shifting property:

$$\mathcal{F}^{-1}\left[\exp[-i\omega t_0]\hat{x}(\omega)\right](t) = x(t-t_0) \tag{27}$$

The linearly propagated single impulse of the *interfering* channel is

$$\exp\left[i\Omega t\right]\exp\left[i\frac{\beta_2}{2}\Omega^2 z\right]g^{(0)}(z,t-\beta_2\Omega z) \tag{28}$$

notice that the frequency component $\exp[i\Omega t]$ has opposite sign with respect to [2]. All the other terms are exactly the same. This may be due to a sign error in the usage of the frequency shifting property, which is

$$\mathcal{F}\left[\exp[i\Omega t]x(t)\right](\omega) = \hat{x}(\omega-\Omega) \tag{29}$$

There is still a point to be discussed, regarding the definition of the propagator.

The linear equation to be solved is

$$\frac{\partial}{\partial z}g^{(0)}(z,t) = -i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}g^{(0)}(z,t) \tag{30}$$

By the shift theorem [**Wiener_1926**] it is possible to write it in symbolic form

$$\exp\left[h\frac{\partial}{\partial z}\right]g^{(0)}(z,t) = g^{(0)}(z+h,t) = \exp\left[-i\frac{\beta_2}{2}h\frac{\partial^2}{\partial t^2}\right]g^{(0)}(0,t) \tag{31}$$

In this way we notice that the propagator operator may be defined as

$$\mathbf{U}(h) = \exp\left[-i\frac{\beta_2}{2}h\frac{\partial^2}{\partial t^2}\right] \tag{32}$$

which is in contradiction with respect to [2] in which the sign of the argument is inverted. By calculating again the propagated impulse, the result is

$$\exp\left[i\Omega t - i\frac{\beta_2}{2}\Omega^2 z\right]g^{(0)}(z,t+\beta_2\Omega z) \tag{33}$$

which shows inverted sign on the terms which involve β_2. This aspect will require further investigation, to be justified with physical arguments and to be matched with [2] and [4, eq. 23].

Let us derive the calculations done in [2, eq. 11, 12]. Starting from the highly-dispersed pulse approximation, we get

$$g^{(0)}(z,t) \approx \sqrt{\frac{i}{2\pi\beta_2 z}}\exp\left[-\frac{it}{2\beta_2 z}\right]\hat{g}\left(0,\frac{t}{\beta_2 z}\right) \tag{34}$$

Now, it is possible to compute the coefficient $X_{0,m,m}$ through energy integral in Fourier space by defining $\nu = t/\beta_2 z$

$$X_{0,m,m} = \int_{z_0}^{L} dz f(z)\int_{\mathbb{R}}\frac{d\nu}{4\pi^2\beta_2 z}|\hat{g}(0,\nu)|^2\left|\hat{g}\left(0,\nu-\Omega-\frac{m\tau}{\beta_2 z}\right)\right|^2 \tag{35}$$

The approximation is that the strongest overlap happens at $z_m = -\frac{m\tau}{\beta_2\Omega}$

If the pulse centered at z_m suffers approximately the same attenuation in all of its spatial positions, it is allowed to assume the f function constant and $f(z) = f(z_m)$. A further assumption is made as z_m/z is assumed to be unitary, as most of the overlap happens at $z = z_m$. So the integral becomes

$$X_{0,m,m} = \int_{\mathbb{R}}\frac{d\nu}{2\pi}|\hat{g}(0,\nu)|^2\int_{\mathbb{R}}dz\frac{z_m f(z_m)}{4\pi^2\beta_2 z^2}\left|\hat{g}\left(0,\nu-\Omega-\frac{m\tau}{\beta_2 z}\right)\right|^2 \tag{36}$$

Notice that the integration along all the space allow us to recall an important property of the impulses: they are of unit energy. Using Parseval identity it is possible to eliminate the impulse waveform in the following way. Let us adopt this change of variables:

$$y := -\frac{m\tau}{\beta_2 z} \qquad \Longrightarrow \qquad dy = \frac{m\tau}{\beta_2 z^2} \tag{37}$$

The multiplication by z_m/z creates the term dy along with the other constants:

$$X_{0,m,m} = \int_{\mathbb{R}}\frac{d\nu}{2\pi}|\hat{g}(0,\nu)|^2\int_{\mathbb{R}}\frac{f(z_m)}{2\pi\beta_2\Omega}\Big(\underbrace{-\frac{m\tau}{\beta_2 z^2}dz}_{dy}\Big)\left|\hat{g}\left(0,\nu-\Omega-\frac{m\tau}{\beta_2 z}\right)\right|^2 \tag{38}$$

$$= \frac{f(z_m)}{\beta_2\Omega}\int_{\mathbb{R}}\frac{d\nu}{2\pi}|\hat{g}(0,\nu)|^2\int_{\mathbb{R}}-\frac{dy}{2\pi}\left|\hat{g}\left(0,\nu-\Omega+y\right)\right|^2 \tag{39}$$

If $f(z_m)$ is assumed to be 1 in perfect amplification scenario, the integrals simplify to

$$= \frac{1}{\beta_2\Omega}\int_{\mathbb{R}}\frac{d\nu}{2\pi}|\hat{g}(0,\nu)|^2\int_{\mathbb{R}}\frac{dy}{2\pi}\left|\hat{g}\left(0,\nu-\Omega+y\right)\right|^2 \tag{40}$$

finally, both integrals, by Parseval, sum to 1, so

$$X_{0,m,m} = \frac{1}{\beta_2\Omega} \tag{41}$$

when z_m falls inside the fiber and in the region of high dispersion, 0 otherwise.

The generalization of the above calculation has only one critical point:

- In the overlap region the cumulative pulse attenuation f is assumed to be constant.

If this assumption holds true, the expression generalizes naturally

$$X_{0,m,m} = \frac{f_B(z_m)}{\beta_2\Omega} \tag{42}$$

Otherwise, we may be interested in cases in which this assumption do not hold:

1) walkoff near zero (interaction happens not only near z_m, but in a broader region)

2) very long fibers (pulses fully interact without border effects)

in general when interaction may not be assumed local.

Ipotesi di gaus

suppongano impulsi gaussiani: l'effetto della propagazione lineare è esprimibile in forma chiusa come

$$g(z,t) = \frac{U_0 \exp[\frac{i}{2}\arctan(D(z))]}{(1+D^2(z))^{1/4}} \exp\left[-\frac{t^2}{2T_0^2}\frac{1+iD(z)}{1+D^2(z)}\right] \tag{43}$$

dove $D(z) = z\beta_2/T_0^2$.

Assumendo la normalizzazione dell'energia dell'impulso a 1, i parametri di ampiezza e larghezza devono soddisfare

$$U_0^2 T_0 \sqrt{\pi} = 1 \tag{44}$$

Usando questa scrittura dell'impulso, si sostituisce nella scrittura del coefficiente di XPM.

$$X_{0,m,m} = \int_0^L dz f_B(z) \int_{\mathbb{R}} dt |g^{(0)}(z,t)|^2 |g^{(0)}(z, t - mT - \beta_2 \Omega z)|^2 \tag{45}$$

quindi considerando

$$|g^{(0)}(z,t)|^2 = \frac{U_0^2}{(1+D^2(z))^{1/2}} \exp\left[-\frac{t^2}{T_0^2}\frac{1}{1+D^2(z)}\right]$$

si ha la seguente espressione

$$X_{0,m,m} = \int_0^L dz f_B(z) \int_{\mathbb{R}} dt \frac{U_0^4}{1+D^2(z)} \exp\left[-\frac{1}{T_0^2(1+D^2(z))}\underbrace{\left(t^2 + (t - mT - \beta_2\Omega z)^2\right)}_{\varphi}\right]$$

Per comodità di scrittura, definiamo s come

$$s := mT + \beta_2 \Omega z$$

allora è possibile riscrivere φ come

$$\begin{aligned} \varphi &= 2t^2 - 2ts + s^2 \\ &= \left(\sqrt{2}t - \frac{s}{\sqrt{2}}\right)^2 + \frac{s^2}{2} \end{aligned}$$

a questo punto cambiamo variabile di integrazione: $\eta := \sqrt{2}t - \frac{s}{\sqrt{2}}$ da cui $dz\,dt = dz\,d\eta\frac{1}{\sqrt{2}}$. Perciò $\varphi = \eta^2 + \frac{s^2}{2}$, e si riscrive l'integrale come

$$X_{0,m,m} = \int_0^L dz f_B(z) \int_{\mathbb{R}} \frac{d\eta}{\sqrt{2}} \frac{U_0^4}{1+D^2(z)} \exp\left[-\frac{\eta^2}{T_0^2(1+D^2(z))}\right] \exp\left[-\frac{s^2}{2T_0^2(1+D^2(z))}\right]$$

Possiamo ora assumere che l'integranda contribuisca all'integrale solo *localmente*, ovvero approssimativamente per $z = z_m = -mT/\beta_2\Omega$. Questo significa che le funzioni $f_B(z)$ e $D(z)$ possono essere sostituite con le costanti $f_B(z_m)$ e $D(z_m)$, rispettivamente. Possiamo inoltre estendere l'integrazione spaziale a tutto $\mathbb{R}$, per ogni m tale per cui $z_m \in [0, L]$. Allora è possibile semplificare l'integrale:

$$X_{0,m,m} = f_B(z_m)\frac{U_0^4}{1+D^2(z_m)}\frac{1}{\sqrt{2}} \int_{\mathbb{R}} dz \int_{\mathbb{R}} d\eta \exp\left[-\frac{\eta^2}{T_0^2(1+D^2(z))}\right] \exp\left[-\frac{s^2}{2T_0^2(1+D^2(z))}\right]$$

Restano così due integrali gaussiani si facile soluzione, infatti ricordando

$$\int_{\mathbb{R}} dt \exp\left[-\frac{t^2}{\alpha}\right] = \sqrt{\alpha\pi} \tag{46}$$

si ha la soluzione dell'integrale in η

$$f_B(z_m)\frac{U_0^4}{1+D^2(z_m)}\frac{1}{\sqrt{2}}(T_0^2(1+D^2(z_m)))^{\frac{1}{2}}\sqrt{\pi} \int_{\mathbb{R}} dz \exp\left[-\frac{s^2}{2T_0^2(1+D^2(z))}\right] \tag{47}$$

Infine, utilizzando s come nuova variabile di integrazione

$$s = mT + \beta_2\Omega z \qquad dz = \frac{1}{\beta_2\Omega} ds \tag{48}$$

è possibile risolvere anche l'ultimo integrale, quindi si ha

$$X_{0,m,m} = \frac{f_B(z_m)}{\beta_2 \Omega} \frac{U_0^4}{\cancel{1 + D^2(z_m)}} \frac{1}{\cancel{\sqrt{2}}} \cancel{\sqrt{2}} T_0^2 \cancel{(1 + D^2(z_m))} \pi = \frac{f_B(z_m)}{\beta_2 \Omega} U_0^4 T_0^2 \pi \tag{49}$$

Ora ricordiamo la *condizione di normalizzazione* per l'energia degli impulsi (44), sostituendo si ha una cancellazione dei parametri U_0 e T_0 dell'impulso

$$X_{0,m,m} = \frac{f_B(z_m)}{\beta_2 \Omega} \underbrace{U_0^4 T_0^2 \pi}_{=1} = \frac{f_B(z_m)}{\beta_2 \Omega} \tag{50}$$

Si noti come questa espressione sia molto simile con quella derivata tramite l'approssimazione di Papoulis [2, eq. 10] (in questo caso abbiamo assunto $z_m \in [0, L]$). Inoltre, mentre l'approssimazione originaria è valida solo a partire da una lunghezza di dispersione ($z_0 = \beta_2 / T_0^2$), la (50) è valida *sempre* per impulsi gaussiani.

Quanto ottenuto nella (50) fa sospettare che lo stesso risultato sarebbe stato ottenibile usando l'approssimazione in modo esatto. Infatti un aspetto fondamentale del ragionamento in [2] è che gli impulsi siano proporzionali e scalati rispetto ai loro *spettri*. Questo per un impulso gaussiano è sempre vero.

Verifica di Pa

Verifichiamo se l'approssimazione vale in modo esatto: scriviamo i campi in dominio del tempo e della frequenza e confrontiamoli con la [2, eq. 10]. Secondo l'Appendice 1, nel dominio del tempo abbiamo questa espressione equivalente

$$u(z,t) = U_0 \left(\frac{1 + iD(z)}{1 + D^2(z)} \right)^{\frac{1}{2}} \exp\left[-\frac{t^2}{2T_0^2} \frac{1 + iD(z)}{1 + D^2(z)} \right] \tag{51}$$

Mentre nel dominio della frequenza si ha (trasformata standard)

$$\hat{u}(z,\omega) = U_0 T_0 \exp\left[-\frac{1}{2} \omega^2 (T_0^2 - i\beta_2 z) \right] \tag{52}$$

ora si sostituisce $\omega \leftarrow \frac{t}{\beta_2 z}$ e si ottiene

$$\begin{aligned} \hat{u}(z,\omega) &= U_0 T_0 \exp\left[-\frac{t^2}{2\beta_2^2 z^2} (T_0^2 - i\beta_2 z) \right] \\ &= U_0 T_0 \exp\left[-\frac{t^2}{2T_0^2} \left(\frac{1}{D^2(z)} - i \frac{1}{D(z)} \right) \right] \\ &= U_0 T_0 \exp\left[-\frac{t^2}{2T_0^2} \left(\frac{1 - iD(z)}{D^2(z)} \right) \right] \end{aligned}$$

Osserviamo l'approssimazione

$$u(z,t) \approx \sqrt{\frac{i}{2\pi\beta_2 z}} \underbrace{\exp\left[-i \frac{t^2}{2\beta_2 z} \right] \hat{u}\left(0, \frac{t}{\beta_2 z}\right)}_{A} \tag{53}$$

il termine contrassegnato da A risulta

$$A = U_0 T_0 \exp\left[-i \frac{t^2}{2T_0^2} \frac{1}{D(z)} \right] \exp\left[-\frac{t^2}{2T_0^2} \left(\frac{1 - iD(z)}{D^2(z)} \right) \right] = U_0 T_0 \exp\left[-\frac{t^2}{2T_0^2} \left(\frac{1}{D^2(z)} \right) \right] \tag{54}$$

Quindi dobbiamo verificare la seguente uguaglianza

$$U_0 \sqrt{\frac{1 + iD(z)}{1 + D^2(z)}} \exp\left[-\frac{t^2}{2T_0^2} \frac{1 + iD(z)}{1 + D^2(z)} \right] \stackrel{?}{=} U_0 \sqrt{\frac{i}{2\pi D(z)}} \exp\left[-\frac{t^2}{2T_0^2} \left(\frac{1}{D^2(z)} \right) \right] \tag{55}$$

Queste espressioni non sembrano tuttavia combaciare esattamente. Possiamo approssimare il termine di sinistra, per $D(z) >> 1$ con

$$U_0 \sqrt{\frac{i}{D(z)}} \exp\left[-\frac{t^2}{2T_0^2} \frac{1}{D^2(z)} \right] \exp\left[-\frac{t^2}{2T_0^2} \frac{i}{D(z)} \right] \tag{56}$$

tuttavia si nota che manca un termine 2π a *denominatore*, e l'esponenziale di fase *scompare* dall'espressione.

La conclusione *provvisoria* è che l'approssimazione non vale in maniera esatta, ed anzi la sua validità nel caso gaussiano è da valutare in un'ulteriore analisi.

Espressione del

pressione del campo in (43) contiene un termine di fase del tipo $\exp\left[\frac{i}{2}\arctan(D(z))\right]$. Questo termine viene scritto in modo per evidenziare la divisione del coefficiente della funzione gaussiana tra una componente di modulo ed una di 'espressione è ottenibile tramite antitrasformata di Fourier, da cui si ha l'espressione

$$u(z,t) = U_0 \left(\frac{1+iD(z)}{1+D^2(z)}\right)^{\frac{1}{2}} \exp\left[-\frac{t^2}{2T_0^2}\frac{1+iD(z)}{1+D^2(z)}\right] \tag{57}$$

Può essere utile evidenziare l'algebra del passaggio da questa espressione a quella data. Infatti, usando delle semplici identità goniometriche:

$$\begin{aligned}
&\exp\left[\frac{i}{2}\arctan(D(z))\right] = \exp\left[2i\arctan(D(z))\right]^{\frac{1}{4}} = \\
&= \left[\cos(2\arctan(D(z))) + i\sin(2\arctan(D(z)))\right]^{\frac{1}{4}} = \\
&= \left[\frac{1-t^2}{1+t^2} + i\frac{2t}{1+t^2}\right]^{\frac{1}{4}} = \qquad \text{dove } t = \tan\left(\frac{\cancel{2}\arctan(D(z))}{\cancel{2}}\right) = D(z) \\
&= \left[\frac{(1+iD(z))^2}{1+D^2(z)}\right]^{\frac{1}{4}} = \frac{(1+iD(z))^{\frac{1}{2}}}{(1+D^2(z))^{\frac{1}{4}}}
\end{aligned}$$

References

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