Generalization of NLIN model for WDM systems considering fiber attenuation and Raman gain

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Abstract

Starting from [2], in which a model for NLIN (Nonlinear Interference Noise) is provided in the assumption of perfect amplification, a generalized model is developed for this phenomenon, in the presence of wavelength and position-dependent attenuation and Raman gain. This analysis aims to open the possibility to address the optimization problem with respect to noise performance in multi-pump Raman amplifiers, which was a topic studied in [4].

I. Introduction

NLIN is a phenomenon which is a crucial limit to the performances of modern WDM (Wavelength Division Multiplexing) links. In systems that implement Raman amplification, along with Amplified Stimulated Emission noise (ASE), NLIN is a major subject of interest in determining the noise performance of amplifiers. A time domain model for NLIN is proposed in [2]: the model defines a coefficient of intrachannel interaction which can be used with general modulation schemes. However, in this work, the nonlinear interaction is modeled assuming a *perfect amplification*, which means that the coefficient of Raman amplification $g(z)$ is identically equal to α, and this relationship holds for *every* WDM channel. In order to drop this assumption, it is required to develop a slight modification of the model.

The aim of the article is to generalize the proposed model to generic Raman gain and attenuation scenarios: in particular the goal is to derive a convenient expression for the XPM (Cross Phase Modulation) coefficient $X_{0,m,m}$ similar to the one in [2], studying the similarities and the differences of the two approaches.

In section II a review of past work is given along with some comments on notation and methods that are useful for the rest of the article. In section III we develop a new framework in which it is possible to generalize the analysis done in past works to scenarios in which wavelength and position dependent Raman gain and attenuation are present. In order to compute the XPM coefficient of interest, in section IV the approximation developed in [2] is considered, and its limits are described. To circumvent these limitations, in section V the computation are done in the Gaussian pulses hypotesis, which allow a simplification of original expression of the coefficient. Finally, in section VI some comments are give on the numerical computation of the expression in the Gaussian pulses scenario. An appendix is given to clarify mathematical aspects of the generalization.

II. Summary of previous results

In the model developed in [2], a single nonlinear Schrödinger Equation (NLSE) is adopted to model the field inside the fiber. Let us examine the structure of the equation.

A. Equation for the field and narrowband approximation

Let us consider the standard NLSE for a fiber with Raman amplification profile $g(z)$

$$\frac{\partial}{\partial z}A = -\frac{\alpha - g(z)}{2}A - \beta_1\frac{\partial}{\partial t}A - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A + i\gamma|A|^2A \tag{1}$$

where t is the physical time. Recall that A is proportional to the electric field inside the fiber, in a way such that the dimension of A is $[A^2] = \mathrm{W}$.

This equation holds for a narrowband field, such that $A(z,t)$ is a slowly varying function of t. In a WDM system, this approximation is assumed to be still true, as the usual channel spectral spacing is greater than 12.5GHz in third window (as defined in standard [3] for DWDM architectures). In the presence of tens of channels, the total field may still be assumed as narrowband.

However, while this assumption can be used for model dispersion in a simple way, using β_2 (second order derivative of the phase constant with respect to frequency), it is not sufficient in assuming constant attenuation-gain terms along all the wavelengths. In fact, a constant attenuation and Raman gain over the signal bandwidth may be an assumption too strong to make, especially when interested in multiple pump Raman amplification. This is the case of the study in [4], from which the present further analysis stemmed. So the model with a single NLSE equation holds for *perfect amplification* scenarios, but needs to be modified in the generalization.

Let us comment now a mathematical procedure that simplify the notation in the NLSE, in presence of attenuation.

B. Rescaling of fields

Let $\psi(z)$ be defined as a solution of the following differential equation

$$\frac{d}{dz}\psi(z) = -\frac{\alpha - g(z)}{2}\psi(z) \tag{2}$$

using such function, let $u(z,t)$ be defined as the *normalized field*

$$A(z,t) = \psi(z)u(z,t) \tag{3}$$

These definitions, when substituted in 1 give the following equation:

$$\frac{\partial}{\partial z}u = -i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}u + i\gamma f(z)|u|^2 u \tag{4}$$

where $f(z) = \psi(z)^2$. The advantage in using the 4 is that the attenuation dynamics can be described with a space-dependent nonlinear term: $\gamma f(z)$.
In the model from [2], using the *perfect amplification* assumption, $\gamma f(z) = \gamma$, since $f(z) \equiv 1$.

III. Coupled NLS equations for WDM channels

By including attenuation and gain it is required to drop the narrowband approximation: in doing so a solution model is the one of the coupled NLSE.
Consider two WDM channels named A and B. The following hypotesis are made:

- channels A and B have a spectral separation of Ω (a multiple of the WDM spectral spacing),
- both channels have the same nonlinear coefficient (it depends from modal field distribution in the core which is assumed to be the same for all channels),
- the group velocity profile is approximately linear in the frequency (β_2 is constant) in the whole band of interest,
- attenuation and Raman gain depend on the channel choice, but are approximately constant within the band of a given channel.

Following Agrawal [1, p.263], a system of *coupled NLSE* is given:

$$\frac{\partial}{\partial z}A_A = -\frac{\alpha_A - g_A(z)}{2}A_A - \beta_{1A}\frac{\partial}{\partial t}A_A - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A_A + i\gamma(|A_A|^2 + 2|A_B|^2)A_A \tag{5}$$

$$\frac{\partial}{\partial z}A_B = -\frac{\alpha_B - g_B(z)}{2}A_B - \beta_{1B}\frac{\partial}{\partial t}A_B - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A_B + i\gamma(|A_B|^2 + 2|A_A|^2)A_B \tag{6}$$

Let A be the WDM channel of interest, and B the interfering one.

Let us now proceed in normalizing the fields A_A, A_B with the respective normalization functions ψ_A, ψ_B, as described in the rescaling equation (II-B). In addition, a moving time reference frame is assumed, taking as a reference the time of arrival of the first pulse in channel A: *from this point, until the end of the article, the variable t indicates the normalized time.*
These passages lead to:

$$\frac{\partial}{\partial z}u_A = -i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}u_A + i\gamma\left(f_A(z)|u_A|^2 + 2\frac{f_A(z)}{f_B(z)}|u_B|^2\right)u_A \tag{7}$$

$$\frac{\partial}{\partial z}u_B = -\Delta\beta_1\frac{\partial}{\partial t}u_B - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}u_B + i\gamma\left(f_B(z)|u_B|^2 + 2\frac{f_B(z)}{f_A(z)}|u_A|^2\right)u_B \tag{8}$$

where $\Delta\beta_1 = \beta_{1B} - \beta_{1A} = \beta_2\Omega$.
Following [2], a first order perturbation analysis is proposed for these equations.

A. Generalization of the 0^{th} order term

Since the attenuation only affects the nonlinear term, the normalized field of the 0^{th} order must be identical as the one derived in [2, eq. 1]. The only exception is due to the notation used: the total field in this case can not be expressed by a simple sum of terms $u_A^{(0)} + u_B^{(0)}$. There are in fact two notational caveats:

- $u_A^{(0)}$ and $u_B^{(0)}$ functions represent normalized fields with different normalization constants,
- the functions are derived from *complex amplitudes* (see for example [5, pp. 523-525]) of *different* carrier frequency signals

Let us consider the initial fields as sums of shifted impulses which encode a given message. Let T be the symbol period:

$$\begin{aligned} u_A(0,t) &= \sum_k a_k g(0, t - kT) \\ u_B(0,t) &= \sum_k b_k g(0, t - kT) \end{aligned} \tag{9}$$

Let $g^{(0)}(z,t)$ be the linearly propagated field in channel A. The solution for the 0^{th} order field is:

$$\begin{aligned} u_A^{(0)}(z,t) &= \sum_k a_k g^{(0)}(z, t - kT) \\ u_B^{(0)}(z,t) &= \sum_k b_k g^{(0)}(z, t - kT - \beta_2 \Omega z) \end{aligned} \tag{10}$$

because of linearity, and definition of $g^{(0)}$.

As in [2], we define the operator of linear propagation for channel A

$$\mathbf{U}_A(z) = \exp\left[i \frac{\beta_2}{2} z \frac{\partial^2}{\partial t^2} \right]. \tag{11}$$

B. Generalization of first order perturbation theory

The splitting in two of the equation allow us to analyze separately the effects of SPM and XPM in a natural way. Let us apply the perturbation method to equation (7) for channel A:

$$\frac{\partial}{\partial z} u_A^{(1)} = -i \frac{\beta_2}{2} \frac{\partial^2}{\partial t^2} u_A^{(1)} + i\gamma \left(f_A(z) |u_A^{(0)}|^2 + 2 \frac{f_A(z)}{f_B(z)} |u_B^{(0)}|^2 \right) u_A^{(0)} \tag{12}$$

Notice that the normalized fields $u_A^{(0)}$ and $u_B^{(0)}$ can not be summed together because they are complex amplitudes with respect to different carriers. However, their squared absolute value is the power of the wave, so the summation of these terms makes physical sense.

Writing the integral solution to the inhomogeneous linear equation above gives

$$u_A^{(1)}(L,t) = i\gamma \int_0^L \mathbf{U}_A(L-z) \left(f_A(z) |u_A^{(0)}|^2 + 2 \frac{f_A(z)}{f_B(z)} |u_B^{(0)}|^2 \right) u_A^{(0)} dz. \tag{13}$$

Using this result it is possible to obtain the estimation error of a receiver.

C. Generalization of estimation error

Using a matched filter receiver, with matching to the linearly propagated initial pulse waveform $g^{(0)}(L,T)$, we obtain the following equation for the estimation error on the first symbol Δa_0

$$\Delta a_0 = \int_{-\infty}^{\infty} u_A^{(1)}(L,t) g^{(0)*}(L,t) dt \tag{14}$$

Recall that $\mathbf{U}_A$ is unitary, so it holds

$$\mathbf{U}_A(L-z) g^{(0)*}(L,t) = g^{(0)*}(z,t) \tag{15}$$

Using this identity, the expression for the error can be written as[1]:

$$\Delta a_0 = i\gamma \int_{z_0}^L dz \int_{-\infty}^{\infty} dt g^{(0)*}(z,t) \left(f_A(z) |u_A^{(0)}|^2 + 2 \frac{f_A(z)}{f_B(z)} |u_B^{(0)}|^2 \right) u_A^{(0)} \tag{16}$$

By substituting the modulation of choice as in (10), we get an expression ready to be computed with respect to a given modulation format. Finally, substituting the modulation and using the same notation as [2, eq. 5, 6, 7], the resulting expression is

$$\Delta a_0 = i\gamma \sum_{h,k,m} (a_h a_k^* a_m S_{h,k,m} + 2 a_h b_k^* b_m X_{h,k,m}) \tag{17}$$

In conclusion the terms that generalize the model, including attenuation and gain, are written in the interaction terms, as follows

$$S_{h,k,m} = \int_{z_0}^L dz f_A(z) \int_{-\infty}^{\infty} dt g^{(0)}(z,t) g^{(0)*}(z, t-hT) g^{(0)}(z, t-kT) g^{(0)*}(z, t-mT) \tag{18}$$

for the SPM, and

$$X_{h,k,m} = \int_{z_0}^L dz f_B(z) \int_{-\infty}^{\infty} dt g^{(0)}(z,t) g^{(0)*}(z, t-hT) g^{(0)}(z, t-kT-\beta_2\Omega z) g^{(0)*}(z, t-mT-\beta_2\Omega z) \tag{19}$$

for the XPM.

This argument proves that the only modification to the original model is to include the terms $f_A(z)$, $f_B(z)$, which represent the power exchanged with the medium, into nonlinear interaction terms $S_{h,k,m}$, $X_{h,k,m}$.

[1]Notice that, being that the two fibers linear propagation terms are different, the substitution described in equation (15) is only valid because filter matching is done considering the propagated symbol waveform over channel A.

IV. Computation of $X_{0,m,m}$

A. High dispersion approximation

Let us first comment the calculations done in [2, eq. 11, 12]. Starting from the highly-dispersed pulse approximation, we get

$$g^{(0)}(z,t) \approx \sqrt{\frac{i}{2\pi\beta_2 z}} \exp\left[-\frac{it}{2\beta_2 z}\right] \hat{g}\left(0, \frac{t}{\beta_2 z}\right) \tag{20}$$

Now, it is possible to compute the coefficient $X_{0,m,m}$ through energy integral in Fourier space by defining $\nu = t/\beta_2 z$

$$X_{0,m,m} = \int_{z_0}^{L} dz f(z) \int_{\mathbb{R}} \frac{d\nu}{4\pi^2\beta_2 z} |\hat{g}(0,\nu)|^2 \left|\hat{g}\left(0, \nu - \Omega - \frac{mT}{\beta_2 z}\right)\right|^2 \tag{21}$$

The approximation is that the strongest overlap happens at $z_m = -\frac{mT}{\beta_2\Omega}$. By the *perfect amplification* hypotesis, it is assumed that $f(z) \equiv 1$. A further assumption is made claiming that most of the overlap happens at $z = z_m$, so the integrand is non negligible only when $z_m/z \approx 1$. The integral becomes

$$X_{0,m,m} = \int_{\mathbb{R}} \frac{d\nu}{2\pi} |\hat{g}(0,\nu)|^2 \int_{\mathbb{R}} dz \frac{z_m f(z_m)}{4\pi^2\beta_2 z^2} \left|\hat{g}\left(0, \nu - \Omega - \frac{mT}{\beta_2 z}\right)\right|^2 \tag{22}$$

Notice that the integration along all the space allow us to recall an important property of the impulses: they are of unit energy. Using Parseval identity it is possible to eliminate the impulse waveform in the following way. Let us adopt this change of variables:

$$y := -\frac{mT}{\beta_2 z} \tag{23}$$

The multiplication by z_m/z creates the term dy along with the other constants:

$$X_{0,m,m} = \int_{\mathbb{R}} \frac{d\nu}{2\pi} |\hat{g}(0,\nu)|^2 \int_{\mathbb{R}} \frac{f(z_m)}{2\pi\beta_2\Omega} \Big(\underbrace{-\frac{mT}{\beta_2 z^2} dz}_{dy} \Big) \left|\hat{g}\left(0, \nu - \Omega - \frac{mT}{\beta_2 z}\right)\right|^2 \tag{24}$$

$$= \frac{f(z_m)}{\beta_2\Omega} \int_{\mathbb{R}} \frac{d\nu}{2\pi} |\hat{g}(0,\nu)|^2 \int_{\mathbb{R}} -\frac{dy}{2\pi} \left|\hat{g}\left(0, \nu - \Omega + y\right)\right|^2 \tag{25}$$

If $f(z_m)$ is assumed to be 1 in perfect amplification scenario, the integrals simplify to

$$= \frac{1}{\beta_2\Omega} \int_{\mathbb{R}} \frac{d\nu}{2\pi} |\hat{g}(0,\nu)|^2 \int_{\mathbb{R}} \frac{dy}{2\pi} \left|\hat{g}\left(0, \nu - \Omega + y\right)\right|^2 \tag{26}$$

finally, both integrals, by Parseval theorem, sum to 1, so

$$X_{0,m,m} = \frac{1}{\beta_2\Omega} \tag{27}$$

when z_m falls inside the fiber and in the region of high dispersion, 0 otherwise.

B. High dispersion approximation is unfit for generalized scenario

The *generalization* of the above calculation has one major difficulty: in the overlap region the cumulative pulse attenuation f is assumed to be constant. If this assumption holds true, the expression generalizes naturally

$$X_{0,m,m} = \frac{f_B(z_m)}{\beta_2\Omega} \tag{28}$$

Otherwise, we may be interested in cases in which interaction can not be assumed as local.

Furthermore, the approximation used for highly-dispersed pulses (20) is valid for lengths higher than the dispersion length, and in the region of interest for Raman amplifiers the approximation may not suffice in calculating the XPM coefficient, as they employ relatively short fibers. The order of on the order of 10^2km, whereas, for $\beta_2 = -4\text{ps}^2\text{km}^{-1}$, and $T_0 = 100$ps, the dispersion length is $L_D = 2500$km.

Dropping the high dispersion approximation, the integral to consider is the one stemming from time domain analysis (19):

$$X_{0,m,m} = \int_0^L dz f_B(z) \int_{-\infty}^{\infty} dt |g^{(0)}(z,t)|^2 |g^{(0)}(z, t - mT - \beta_2\Omega z)|^2. \tag{29}$$

In order to find a suitable computational simplification, let us assume that the pulses are Gaussian.

V. Gaussian pulses

The effect of linear propagation for Gaussian pulses has a closed form expression:

$$g(z,t) = \frac{U_0 \exp[\frac{i}{2}\arctan(D(z))]}{(1+D^2(z))^{1/4}} \exp\left[-\frac{t^2}{2T_0^2}\frac{1+iD(z)}{1+D^2(z)}\right] \tag{30}$$

where $D(z) = z\beta_2/T_0^2$. Some comments on this representation are given in Appendix A.

Assuming pulse energy normalization to 1, amplitude and width parameters need to satisfy the following equation

$$U_0^2 T_0 \sqrt{\pi} = 1 \tag{31}$$

substituting the relative expressions in (29), the following expression is given

$$X_{0,m,m} = \int_0^L dz f_B(z) \int_{\mathbb{R}} dt \frac{U_0^4}{1+D^2(z)} \exp\left[-\frac{1}{T_0^2(1+D^2(z))} \underbrace{\left(t^2 + (t - mT - \beta_2\Omega z)^2\right)}_{\varphi}\right] \tag{32}$$

Let s be defined as

$$s := mT + \beta_2 \Omega z \tag{33}$$

then, the term φ can be rewritten as:

$$\varphi = 2t^2 - 2ts + s^2 \tag{34}$$

$$= \left(\sqrt{2}t - \frac{s}{\sqrt{2}}\right)^2 + \frac{s^2}{2} \tag{35}$$

let us use a change of variable of integration: $\eta := \sqrt{2}t - \frac{s}{\sqrt{2}}$, from which $dz\,dt = dz\,d\eta\frac{1}{\sqrt{2}}$. So we have the expression $\varphi = \eta^2 + \frac{s^2}{2}$, and the integral admits this expression

$$X_{0,m,m} = \int_0^L dz f_B(z) \int_{\mathbb{R}} \frac{d\eta}{\sqrt{2}} \frac{U_0^4}{1+D^2(z)} \exp\left[-\frac{\eta^2}{T_0^2(1+D^2(z))}\right] \exp\left[-\frac{s^2}{2T_0^2(1+D^2(z))}\right] \tag{36}$$

A. Local interaction approximation

The last step in the simplification procedure is the assumption of local interaction for $z = z_m = -mT/\beta_2\Omega$. This means that the function $D(z)$ can be substituted with constant $D(z_m)$. This is justified also by numerical simulations with usual Raman amplifiers parameters: they show that this approximation usually gives integrals with relative error $< 1\%$. We can also extend integration to $\mathbb{R}$, for every m such that $z_m \in [0, L]$, if we assume to neglect partial collisions at the borders. Than it is possible to simplify the integral

$$\frac{1}{\sqrt{2}} \frac{U_0^4}{1+D^2(z_m)} \int_0^L dz f_B(z) \int_{\mathbb{R}} d\eta \exp\left[-\frac{\eta^2}{T_0^2(1+D^2(z_m))}\right] \exp\left[-\frac{s^2}{2T_0^2(1+D^2(z_m))}\right]. \tag{37}$$

The remaining equation contains two Gaussian integrals

$$\int_{\mathbb{R}} dt \exp\left[-\frac{t^2}{\alpha}\right] = \sqrt{\alpha\pi} \tag{38}$$

integrating with respect to η we obtain

$$\sqrt{\frac{\pi}{2}} \frac{U_0^4 T_0}{(1+D^2(z_m))^{\frac{1}{2}}} \int_0^L dz f_B(z) \exp\left[-\frac{s^2}{2T_0^2(1+D^2(z_m))}\right] \tag{39}$$

and finally, expressing the dependency of s from z,

$$X_{0,m,m} = \sqrt{\frac{\pi}{2}} \frac{U_0^4 T_0}{(1+D^2(z_m))^{\frac{1}{2}}} \int_0^L dz f_B(z) \exp\left[-\frac{(mT + \beta_2\Omega z)^2}{2T_0^2(1+D^2(z_m))}\right] \tag{40}$$

At this point, there is no possibility to find a general analytical solution to the integral. In fact, different Raman amplification schemes give different functions f_B, and the analysis must be done separately on each of them. Still, let us explore the case in which there is *perfect amplification*: the final result is interesting as the result is the same as the one given by the high dispersion approximation. In this hypotesis, $f_B \equiv 1$, and the coefficient is computed as

$$\sqrt{\frac{\pi}{2}} \frac{U_0^4 T_0}{(1+D^2(z_m))^{\frac{1}{2}}} \int_0^L dz \exp\left[-\frac{s^2}{2T_0^2(1+D^2(z_m))}\right]. \tag{41}$$

which contains a truncated Gaussian integral. If we further assume that the contribution to the integral is negligible out of the range $[0, L]$, this integral has simple analytical solution. Using s as a new integration variable it is possible to solve as

$$\sqrt{\frac{\pi}{2}}\frac{U_0^4 T_0}{(1+D^2(z_m))^{\frac{1}{2}}}\int_{\mathbb{R}}\frac{ds}{\beta_2\Omega}\exp\left[-\frac{s^2}{2T_0^2(1+D^2(z_m))}\right]=\frac{1}{\beta_2\Omega}U_0^4T_0^2\pi. \tag{42}$$

Recall the normalization condition for the pulse energy in (31): the substitution cancels both parameters U_0 and T_0 from the final expression, so

$$X_{0,m,m}=\frac{1}{\beta_2\Omega}\underbrace{U_0^4T_0^2\pi}_{=1}=\frac{1}{\beta_2\Omega}. \tag{43}$$

The procedure in equations (41 - 43) assumes that borders effects and non-perfect amplification profiles are neglected, which in Raman amplifiers are not reasonable assumptions. Nonetheless the expression in (43) is interesting in which it sheds some light into the approximation [2, eq. 10]. In fact, Gaussian pulses have Gaussian spectrum, and the approximation works because of this property.

VI. Computation of spatial integral

The generalization procedure, in the Gaussian pulse approximatiton lead to the simplified integral in equation (40). In order to setup the integration technique, it is possible to notice that only some indexes m are to be considered as valid collision in a short fiber such as the one of interest for the amplifier. By using a threshold on the pulse collision energy (normalized to 1), it is possible to select only the desired m, and to filter out all the m that give place to weak interaction. Let $0 < \xi < 1$ be such threshold. Let M be the set of the m values to be considered. The computation of M is complicated, because of the very definition of ξ, which involves the actual computation of the integral

$$M=\left\{m \;\middle|\; \sqrt{\frac{\pi}{2}}\frac{U_0^4T_0\beta_2\Omega}{(1+D^2(z_m))^{\frac{1}{2}}}\int_0^L dz f_B(z)\exp\left[-\frac{(mT+\beta_2\Omega z)^2}{2T_0^2(1+D^2(z_m))}\right]\geq\xi\right\} \tag{44}$$

Since we are only concerned about the selection of the m, one possible approximation is to discard the contribution of $f_B(z)$. This assumption is coherent if inter-collision spacing is sufficiently small. By normalizing the Gaussian and computing the upper and lower extreme of integration with respect to the Gaussian peak, and considering a new normalized threshold, which is related to the pulse energy by

$$\epsilon=\sqrt{\frac{2}{\pi}}\frac{(1+D^2(z_m))^{\frac{1}{2}}}{U_0^4T_0\beta_2\Omega}\xi \tag{45}$$

it is possible to obtain a trascendental condition on m to be solved simply using some solver or by table lookup. The alternative, simplified condition is

$$\frac{1}{2}\left[\Phi\left(\frac{mT+\beta_2\Omega L}{\sqrt{2}T_0(1+D^2(z_m))^{\frac{1}{2}}}\right)-\Phi\left(\frac{mT}{\sqrt{2}T_0(1+D^2(z_m))^{\frac{1}{2}}}\right)\right]>\epsilon \tag{46}$$

where Φ is the Gaussian error function. This approximation must be validated, but it is a good starting point to understand the behaviour of the interactions.

VII. Conclusion

In order to generalize the NLIN model, a coupled NLSE model was adopted, by redefining signal and interferent field. This formulation, when applied to the analysis of noise on a symbol, gives SPM and XPM coefficients of interaction which are very similar to the one in [2]. In order to compute the coefficient for XPM, the approximation proposed in [2] was studied, but it is unfit in the case of Raman amplifiers. So, in order to find a suitable simplification of time domain non approximated integrals, the assumption of Gaussian pulses was examinated. This last analysis, along with local interaction approximation, lead to the direct analytical computation of one of the two integrals. The last integral can be computed numerically including partial collisions at the borders, and different Raman gain and attenuation profiles.

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Appendix A

The field expression in (30) contains a phase term in the form $\exp\left[\frac{i}{2}\arctan(D(z))\right]$. This term highlights the phase and modulus representation of the coefficient of the exponentials. However, it may not seem so straightforward. From Fourier antitransform, the field has the following expression

$$u(z,t) = U_0 \left(\frac{1+iD(z)}{1+D^2(z)}\right)^{\frac{1}{2}} \exp\left[-\frac{t^2}{2T_0^2}\frac{1+iD(z)}{1+D^2(z)}\right] \tag{47}$$

the two field expression are connected by this calculations which uses fundamental goniometric identities

$$\exp\left[\frac{i}{2}\arctan(D(z))\right] = \exp\left[2i\arctan(D(z))\right]^{\frac{1}{4}} = \tag{48}$$

$$= \left[\cos(2\arctan(D(z))) + i\sin(2\arctan(D(z)))\right]^{\frac{1}{4}} = \tag{49}$$

$$= \left[\frac{1-t^2}{1+t^2} + i\frac{2t}{1+t^2}\right]^{\frac{1}{4}} = \qquad \text{where } t = \tan\left(\frac{\cancel{2}\arctan(D(z))}{\cancel{2}}\right) = D(z) \tag{50}$$

$$= \left[\frac{(1+iD(z))^2}{1+D^2(z)}\right]^{\frac{1}{4}} = \frac{(1+iD(z))^{\frac{1}{2}}}{(1+D^2(z))^{\frac{1}{4}}} \tag{51}$$