Generalization of NLIN model - Approximation for time integrals

Francesco Lorenzi
Dipartimento di Ingegneria dell'Informazione
Università degli studi di Padova

The approximation in equation (20) (corrected in errata, eq.(3)) relates the pulse time evolution with its spectrum:

$$g^{(0)}(z,t) \approx \sqrt{\frac{i}{2\pi\beta_2 z}} \exp\left[-\frac{it^2}{2\beta_2 z}\right] \hat{g}\left(0, \frac{t}{\beta_2 z}\right) \tag{1}$$

the computation of this approximation is done in Papoulis [1]. To clarify the expression, the passages are derived below.

We know that both coupled NLS equations have linear propagators $\mathbf{U}_A(z)$ and $\mathbf{U}_B(z)$. Both propagators have a chromatic dispersion part, whereas $\mathbf{U}_B(z)$ contains also a group velocity term. Focusing on dispersion, we use the symbol $\mathbf{U}(z)$ to represent the dispersion propagator

$$\mathbf{U}(z) = \exp\left[-i\frac{\beta_2}{2} z \frac{\partial^2}{\partial t^2}\right] \tag{2}$$

which corresponds to the following frequency response

$$\hat{\mathbf{U}}(z) = \exp\left[i\frac{\beta_2}{2} z\omega^2\right]. \tag{3}$$

The last frequency response is a *Quadratic Phase Filter* (QPF): an all-pass filter with quadratic phase relationship. Remember the Gaussian function Fourier transform and antitranform pair

$$\exp(-st^2) \xrightarrow{\mathcal{F}} \sqrt{\frac{\pi}{s}} \exp\left(-\frac{\omega^2}{4s}\right). \tag{4}$$

The multiplication by the propagator in the frequency domain corresponds to a convolution in the time domain by a Gaussian

$$g^{(0)}(z,t) = \sqrt{\frac{i}{2\pi\beta_2 z}} \int_{-\infty}^{\infty} g(0,\tau) \exp\left[-i\frac{(t-\tau)^2}{2\beta_2 z}\right] dz \tag{5}$$

This particular convolution is a *Fresnel transform*, of the form

$$g^{(0)}(z,t) = \sqrt{\frac{\alpha}{i\pi}} \int_{-\infty}^{\infty} g(0,\tau) \exp\left[-i\alpha(t-\tau)^2\right] dz \tag{6}$$

with

$$\alpha = \frac{1}{2\beta_2 z} \tag{7}$$

By expanding the square in the exponential,

$$g^{(0)}(z,t) = \sqrt{\frac{i}{2\pi\beta_2 z}} \int_{-\infty}^{\infty} g(0,\tau) \exp\left[-i\frac{t^2 - 2t\tau + \tau^2}{2\beta_2 z}\right] dz \tag{8}$$

we can select terms which

The approximation written in 1 is possible when the following expression is

In terms of fiber length and dispersion length, for a Gaussian pulse we have

A general theory for In presence of small dispersion, the approximation is accurate if the pulse shape is an eigenfunction of the chirp operator.

References

[1] Athanasios Papoulis. "Pulse compression, fiber communications, and diffraction: a unified approach". In: *Journal of the Optical Society of America A* 11.1 (Jan. 1994), p. 3. DOI: 10.1364/josaa.11.000003. URL: https://doi.org/10.1364%2Fjosaa.11.000003.