Anova

- J = N. OBS x SAMPLE
- I = N. TREATMENTS / POPULATION

Single Factor

COMPARISON OF MORE THAN TWO POPULATIONS (TREATMENTS) WITH SAME SAMPLES

$H_0: \mu_1 \dots \mu_N, \ \mu_1 = \mu_2 = \dots = \mu_N$

ASSUME: POPULATIONS ARE ALL N.D WITH VAR $= \sigma^2$ | $E(X_{ij}) = \mu_i, \ VAR(X_{ij}) = \sigma^2$

POP. SAMPLE MEAN: $\bar{X}_i = \dfrac{\sum_1^J X_{ij}}{J}$

GRAND MEAN: $\bar{X}_{..} = \dfrac{\sum^I \sum^J X_{ij}}{IJ}$

POP. SAMPLE VARIANCE: $S_i^2 = \dfrac{\sum^J (X_{ij} - \bar{X}_i)^2}{J-1}$

IDEA: COMPARE DIFF BETWEEN SAMPLE TO DIFF WITHIN SAMPLE

$MSTR = \dfrac{J}{I-1} \sum_i (\bar{X}_i - \bar{X}_{..})^2$ (BETWEEN SAMPLES) $\quad MSE = \dfrac{S_1^2 + \dots + S_I^2}{I}$ (WITHIN SAMPLES)

TEST STATISTIC: $F = \dfrac{MSTR}{MSE}$. H_0 TRUE $\rightarrow E(MSTR) = E(MSE)$

REJECT WHEN: $f \geq c = \alpha$, or $f \geq F_{\alpha, I-1, I(J-1)}$

- **Sums of Squares**

TOTAL $\quad SST = \sum^I \sum^J (X_{ij} - \bar{X}_{..})^2 = \sum_i \sum_j X_{ij}^2 - \dfrac{1}{IJ} X_{..}^2$

$X_{i.} = \sum^J X_{ij}, \quad X_{..} = \sum^I \sum^J X_{ij}$

TREATMENT $\quad SSTR = \sum^I \sum^J (\bar{X}_{i.} - \bar{X}_{..})^2 = \dfrac{1}{J} \sum^I X_i^2 - \dfrac{1}{IJ} X_{..}^2$

$SST = SSTR + SSE$

VARIANCE BIAS

ERROR $\quad SSE = \sum^I \sum^J (X_{ij} - \bar{X}_{i.})^2 =$

MSTR $\dfrac{SSTR}{I-1}$

MSE $\dfrac{SSE}{I(J-1)}$

$F = \dfrac{MSTR}{MSE}$

Tukey's Procedure

USES STUDENTIZED RANGE DISTRIBUTION $Q_{\alpha, m, n}$

IDENTIFIES SIGNIFICANTLY DIFFERENT μ_i WHEN ANOVA IS INCONCLUSIVE

- SORT μ_i's IN INCREASING ORDER
- COMPUTE $w = Q_{\alpha, I, I(J-1)} \sqrt{MSE/J}$
- UNDERLINE CONNECT THOSE DIFFERING BY LESS THAN w
- NO SAME LINE $\rightarrow$ SIGNIFICANT DIFFERENCE

FUNCTION IS NOT $\mu_i - \mu_j$

CONSTANTS c_i

$$\sum c_i \bar{x}_i \pm t_{\alpha/2, I(J-1)} \sqrt{\frac{MSE \sum c_i^2}{J}}$$

- POSSIBLE TO ADAPT TO DIFF. N. SAMPLES x POP
- POSSIBLE TO TRANSFORM R.V.S TO EQUALIZE VARIANCES

Anova Model Equations

$X_{ij} = \mu_i + \epsilon_{ij}$, ϵ IS RANDOM DEVIATION FROM POP.

REFORMULATE

$X_{ij} = \mu_i + \epsilon$ TO $\mu = \dfrac{1}{I} \sum^I \mu_i, \ \alpha_1 \dots \alpha_I = \mu_i - \mu \rightarrow \mu_i = \mu + \underline{\alpha_i}$ EFFECT DUE TO iTH TREATMENT

$\rightarrow X_{ij} = \mu + \alpha_i + \epsilon_{ij}, \ H_0: \alpha_1 = \alpha_2 = \alpha_I = 0$ (FIXED EFFECTS MODEL)

- CAN BE MADE INTO RANDOM EFFECTS MODEL

$\alpha_i \rightarrow A_i$, A_i IS RV NORMALLY INDEPENDENTLY

$E(A_i) = E(\epsilon_{ij}) = 0$

TWO FACTOR ANOVA

MODEL EQUATIONS

$X_{ij} = \mu_{ij} + \epsilon_{ij}$, ϵ_{ij} NORMAL WITH COMMON VARIANCE σ^2 , JOINT EFFECTS, PROBLEMATIC

$\rightarrow X_{ij} = \alpha_i + \beta_j + \epsilon_{ij}$, $\mu_{ij} = \alpha_i + \beta_j$ **ADDITIVE MODEL**, SUM OF EFFECT DUE TO FACTOR A AND EFFECT DUE TO B

$\sum^{I} \alpha_i = 0$, $\sum^{J} \beta_j = 0$, COMMON VARIANCE σ^2

HYPOTHESES

$H_{0A}: \alpha_1 = \alpha_2 = \dots \alpha_I = 0$ NO EFFECT DUE TO A

$H_{0B}: \beta_1 = \beta_2 = \dots = \beta_J$ NO EFFECT DUE TO B

- $\hat{\mu} = \bar{X}_{..}$, $\hat{\alpha}_i = \bar{X}_{i.} - \bar{X}_{..}$, $\hat{\beta}_j = \bar{X}_{.j} - \bar{X}_{..}$
- $SST = SSA + SSB + SSE$

STATISTICS

H_{0A} vs $H_{aA} \longrightarrow f = \dfrac{MSA}{MSE} \xrightarrow{\text{REJECT}} f_A \geq F_{\alpha, I-1, (I-1)(J-1)}$

H_{0B} vs $H_{aB} \longrightarrow f = \dfrac{MSB}{MSE} \xrightarrow{\text{REJECT}} f_B \geq F_{\alpha, J-1, (I-1)(J-1)}$

TUKEY: JUST LIKE SINGLE FACTOR, BUT SPLIT

NON ADDITIVE STRUCTURE

INTERACTION BETWEEN FACTORS

- $\alpha_i = \mu_{i.} - \mu$
- $\beta_j = \mu_{.j} - \mu$
- $\gamma = \mu_{ij} - (\mu + \alpha_i + \beta_j)$

- $\mu_{ij} = \mu + \alpha_i + \beta_j + \gamma_{ij}$

$\longrightarrow X_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ijk}$

- $SST = SSA + SSB + SSAB + SSE$

HYPOTHESES: H_{0A}, H_{0B}, H_{0AB}

STATISTICS

f_A, f_B

$f_{AB} = \dfrac{MSAB}{MSE}$

RANDOM EFFECTS: UFC IT COULD HAVE THEM $\gamma \rightarrow G$

THREE FACTOR ANOVA

$\mu_{ijk} = \mu + \alpha_i + \beta_j + \delta_k + \gamma_{ij}^{AB} + \gamma_{ik}^{AC} + \gamma_{jk}^{BC} + \gamma_{ijk}$, f_A, f_B, f_{ABC} **SAME STUFF**

2^I FACTORIAL EXPERIMENTS: FULL GENERALIZATION