PROBABILITÀ

DEFINIZIONE

- $P(E) \geq 0 \quad \forall E \in F$
- $P(\Omega) = 1$
- SE EVENTI DISGIUNTI, $(E_1 \cap E_2 = \emptyset) \rightarrow P(\bigcup_1^N E) = \sum_1^N P(E)$

PROPRIETÀ

- $P(E^c) = 1 - P(E)$
- $P(E) \leq 1$
- $F \subset E \rightarrow P(E|F) = P(E) - P(F)$
- $F \subset E \rightarrow P(F) \leq P(E)$
- $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
- PRINCIPIO INCL-EXCL POINCARÉ: $P(\bigcup_1^N A_i) = \sum_1^N P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \dots + (-1)^{N-1} P(\bigcap_1^N A_i)$

CAMPIONAMENTO NO REIMMISSIONE

$\Omega_1 =$ DISPOSIZIONI TOTALI NO RIPETIZIONE

$|\Omega_1| = M(M-1)(M-2)\dots 1$

$\Omega_2 =$ N° N-UPLE

$|\Omega_2| = \binom{M}{m}$

CAMPIONAMENTO CON REIMMISSIONE

$\Omega_3 =$ DISPOSIZIONI CON REIMMISSIONE

$|\Omega_3| = M^m$

PROBABILITÀ CONDIZIONATA

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

F. PROB. TOTALI

$$P(E) = \sum_{n=1}^{\infty} P(E \cap F_n) = \sum_{n=1}^{N} P(E|F_n) \cdot F(K)$$

F. BAYES

$$P(F_N|E) = \frac{P(E|F_N)P(F_N)}{\underbrace{\sum_1^N P(E|F_n)P(F_n)}_{P(E)}}$$

F. MOLTIPLICAZIONE

$$P(E_1 \cap E_2 \cap E_3 \dots E_N) = P(E_1) \cdot P(E_2|E_1) \cdot P(E_3|E_2 \cap E_1$$

EVENTI INDIPENDENTI

IFF $P(E \cap F) = P(E)P(F) \longrightarrow P(E|F) = P(E),\ P(F|E) = P(F)$ | CONDITIONAL INDEPENDENCE: $X \perp Y | Z \longleftrightarrow P(X,Y|Z) = P(X|Z)P(Y|Z)$

N EVENTI INDIPENDENTI IFF OGNI SOTTOINSIEME DI ESSI È INDIPENDENTE

VARIABILE ALEATORIA

V.A. È FUNZIONE SU SPAZIO EVENTI ELEMENTARI Ω, $[X: \Omega \rightarrow \mathbb{R}]$ T.C. $x \in \mathbb{R}$ $\{X \leq x\} := \{\omega \in \Omega : X(\omega) \leq x\} \in \mathbb{F}$

FdR: $F_X(x) = P(X \leq m)$, $[0,1]$

- MONOTONA NON DECRESCENTE
- CONTINUA DA DX
- $\lim_{x \to -\infty} F_X(x) = 0$, $\lim_{x \to +\infty} F_X(x) = 1$

MASS/DENSITY: $P(x) = P(X = x)$

- $0 \leq P_X(x) \leq 1$
- $P_X(k) = F_X(k) - F_X(k-1)$
- $\sum P_X(x) = 1$
- $F_X(x) = \sum P_X(x)$

UNIFORM

$U\{a, b\}$ $\quad P(n) = \frac{1}{N}$

$N = b - a + 1$

$E(X) = \frac{n+1}{2}$ $\quad VAR(X) = \frac{n^2 - 1}{12}$

CONTINUOUS

- $f(x)$ INTEGRABILE, $f(x) > 0$
- $\int_{\mathbb{R}} f(x)\, dx = 1$

BERNOULLI

$X \sim Be(p)$

BINOMIALE CON $N=1$

$E(X) = p$

$VAR = p(1-p)$

BINOMIALE

$X \sim Bi(n, p)$

$$P_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$E(X) = np \quad VAR(X) = np(1-p)$

- SOMMA DI N VA. BERNOULLI INDIPENDENTI
- N^o SUCCESSI IN $n \{0/1$ ESPERIMENTI$\}$ BERNOULLI

GEOMETRICA

$X \sim GEOM(p)$

$$P_X(k) = \begin{cases} p(1-p)^{k-1}, & k = 1, 2, \dots \\ 0 & \text{ELSE} \end{cases}$$

PROB. PRIMO SUCCESSO DOPO K PROVE DI BERNOULLI

$$E(X) = \frac{1}{p}$$

$$VAR(X) = \frac{1-p}{p^2}$$

ASSENZA DI MEMORIA

NEGATIVE BINOMIAL $(1, 1-p)$

POISSON

$X \sim P(\lambda)$, $\lambda = N \cdot p$

$N >>$, $p <<$

LIMITE DI $X \sim Bi(n, p)$ IN QUESTO CASO

$$P(k) = \frac{e^{-\lambda} \lambda^k}{k!}$$

$E(X) = \lambda \quad VAR(X) = \lambda$

N^o EVENTI IN INTERVALLO FISSO SPAZIO/TEMPO SE NOTO AVG. RATE E INDIPENDENTI DA ULTIMO TEMPO

HYPERGEOMETRIC

$X \sim HYPERGEOM(k, \underline{N}, \underline{K}, n)$

SAMPLING W/O REPLACEMENT

$$P_X(k) = \frac{\binom{\underline{K}}{k} \binom{\underline{N} - \underline{K}}{n-k}}{\binom{\underline{N}}{n}}$$

$\underline{K}$ = SUCCESS STATES IN POP.

k = SUCCESSES

n = DRAWS

$\underline{N}$ = POP. SIZES

$$\max(0, n + \underline{K} - \underline{N}) \le k \le \min(\underline{K}, n)$$

$$E(X) = n \frac{K}{N} \qquad VAR(X) = n \frac{K}{N} \frac{N-K}{N} \frac{N-n}{N-1}$$

NEGATIVE BINOMIAL

$X \sim NB(R, p)$

R: N^o FALLIMENTI AL ESPERIMENTO STOPPATO

$$P_X(k) = \binom{k + R - 1}{k} p^k (1-p)^R$$

$NB(1, 1-p) = GEOM(p)$

$$E(X) = \frac{p \cdot R}{1-p} \qquad VAR(X) = \frac{pR}{(1-p)^2}$$

Esponenziale

ANALOGO CONTINUO
DENSITÀ GEOMETRICA

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-\mu x} & x \geq 0 \end{cases}$$

$$f(x) = \begin{cases} 0 & t < 0 \\ \mu e^{-\mu x} & t \geq 0 \end{cases}$$

$X \sim \mathcal{E}(\mu)$

$$E(x) = \frac{1}{\mu} \qquad VAR(x) = \frac{2}{\mu^2}$$

ASSENZA DI MEMORIA
'CONSTANT PROBABILITY x UNIT LENGTH'

Uniforme

$$F(x) = \begin{cases} 0 & x < 0 \\ x & 0 < x < 1 \\ 1 & x \geq 1 \end{cases}$$

$$f(x) = \begin{cases} 0 & x < 0 \\ 1 & 0 < x < 1 \\ 0 & x \geq 1 \end{cases}$$

$$E(x) = 1/2 \qquad VAR(x) = \frac{(b-a)^2}{12}$$

Gaussiana

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{x^2}{2}} dx$$

$$\varphi(z) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} \quad \Big| \quad \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$Z \sim N(0,1)$

$$E(x) = 0 \quad VAR(x) = 1 \quad | \quad \mu, \sigma$$

Chi-Squared

$Z_1 .. Z_n \sim N(0,1)$

$Q = \sum_1^n Z_i^2 \sim \chi^2(n)$ SOMMA DI K IID DI GAUSSIANE SQUARED

$$F(x) = \frac{1}{\Gamma(\frac{n}{2})} \gamma\left(\frac{n}{2}, \frac{x}{2}\right)$$

$$f(x) = \frac{1}{2^{n/2}\Gamma(\frac{n}{2})} x^{\frac{n}{2}-1} \cdot e^{-\frac{x}{2}}$$

$$E(x) = k \qquad VAR(x) = 2k$$

POSITIVE SKEWED
MORE SIMMETRIC AS DOF $\to \infty$

Gamma

EXP E χ^2 CASI SPECIALI DI Γ ~~ESPONENZIALE~~

$\Gamma(k, \theta)$, $k > 0$ SHAPE, $\theta > 0$ SCALE — SOMMA DI K VAR ESPONENZIALI IID

$$f(x) = \frac{1}{\Gamma(k)\theta^k} \cdot x^{k-1} \cdot e^{-x/\theta}$$

$$F(x) = \frac{1}{\Gamma(k)} \gamma\left(k, \frac{x}{\theta}\right)$$

$$E(x) = k\theta \qquad VAR(x) = k\theta^2$$

OBS!

$$\Gamma(x) = (x-1)!$$

STUDENT'S T

ESTIMATING MEAN OF NORMALLY DISTRIBUTED POPULATION

WHERE $n \ll$ E $\sigma^2 = ??$

MOAR SAMPLES $\rightarrow$ MOAR LIKE NORMAL

SAMPLE OF SIZE $n \longmapsto v = n-1$

$$f(t) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(1+\frac{t^2}{v}\right)^{-\frac{v+1}{2}}, \quad T = \frac{Z}{\sqrt{Y/N}}, \quad Y = \chi^2(N)$$

$F(t)$ = TROPPO UN CASINO

$E(x) = 0 \qquad VAR(x) = \frac{v}{v-2} \;\Big|\; E(x) = \mu \quad VAR(x) = \frac{v\sigma^2}{v-2}$

SE $N=1$, HO CAUCHY DISTRIBUTION | SE $N = \infty$, HO NORMAL

CAUCHY

RATIO OF 2 $N(0,1)$ (STD CAUCHY), USEFUL IN STUFF WITH RESONANCE

• IS STUDENTS' T FOR $v = 1$

$$f(x;0,1) = \frac{1}{\pi(1+x^2)} \;\Big|\; \frac{1}{\pi\gamma\left(1+\left(\frac{x-x_0}{\gamma}\right)^2\right)}$$

$$F(x;0,1) = \frac{1}{\pi}\arctan(x) + \frac{1}{2} \;\Big|\; \frac{1}{\pi}\arctan\left(\frac{x-x_0}{\gamma}\right)+\frac{1}{2}$$

$E(x)$, $VAR(x)$ UNDEFINED

BETA

IN INTERVAL $(0,1)$ $\quad B(\mu|a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\mu^{a-1}(1-\mu)^{b-1}$

$\beta(\alpha,\beta) \qquad E[\mu] = \frac{a}{a+b} \qquad VAR[\mu] = \frac{ab}{(a+b)^2(a+b+1)}$

• DIRICHLET DISTRIBUTION: MULTIVARIATE BETA

$$DIR(\mu|\alpha) = \frac{\Gamma(\alpha_0)}{\Gamma(\alpha_1)\ldots\Gamma(\alpha_k)}\prod_{k}^{K}\mu_k^{\alpha_k-1}, \quad \alpha_0 = \sum_i^K \alpha_i$$

GIVES A MULTINOMIAL

CONSTRAINT SUM OF μ = 1, α_k MEANING?

WEIBULL

'GENERALIZZA ESPONENZIALE'

'TIME TO FAILURE', 'LIFETIME OF STUFF'

$X \sim N(\mu,\sigma^2)$

$\exp(x) \sim \ln N(\mu,\sigma^2)$

$X \sim \ln N(\mu,\sigma^2) \Rightarrow \ln(x) \sim N(x)$

F

NULL-DISTRIBUTION OF F-STATISTIC, F-TEST

$$F(d_1,d_2) = X = \frac{U_1/d_1}{U_2/d_2}$$

$U_1, U_2 \sim \chi^2$ DI PARAM $= d_1, d_2$ E INDIPENDENTI

$$E(x) = \frac{d_2}{d_2-2} \qquad VAR(x) = \frac{2d_2^2(d_1+d_2-2)}{d_1(d_2-2)^2(d_2-4)}$$

LAPLACE

DOPPIA EXP MIRRORATA A $x=0$

DIFFERENZA DI DUE ESPONENZIALI IID

$L(\mu, b)$ $\quad \mu$ = LOCATION, b = SCALE >0 $\quad \Big|$ STD $\mu=0$, $b=1$

$$f(x) = \frac{1}{2b}e^{-\frac{|x-\mu|}{b}}$$

$$F(x) = \begin{cases} \frac{1}{2}e^{\frac{x-\mu}{b}} & x < \mu \\ 1-\frac{1}{2}e^{-\frac{x-\mu}{b}} & x \geq \mu \end{cases}$$

$E(x) = \mu \qquad VAR(x) = 2b^2$

PARETO

POWER LAW DISTRIBUTION, PARAMETRI $x_m, \alpha > 0$

$$f(x) = \frac{\alpha x_m^\alpha}{x^{\alpha+1}}, \quad x \geq x_m$$

$$F(x) = 1 - \left(\frac{x_m}{x}\right)^\alpha \quad x \geq x_m$$

$$E(x) = \frac{\alpha \cdot x_m}{\alpha-1} \quad \alpha > 1 \qquad VAR(x) = \frac{x_m^2\alpha}{(\alpha-1)^2(\alpha-2)}$$

SE X PARETO $Y = \log\left(\frac{X}{x_m}\right)$ E' ESPONENZIALE α

LOGNORMALE

$\ln N(\mu,\sigma^2) \qquad X = e^{\mu+\sigma Z}, \quad Z \sim N(0,1)$

μ, σ DEL LOG DELLA R.V. N.

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}}e^{\frac{-(\ln(x-\mu))^2}{2\sigma^2}}$$

LOGARITMO OF VARIABLE IS NORMALLY DISTRIBUTED

$$E(x) = e^{\mu+\sigma/2} \qquad VAR(x) = (e^{\sigma^2}-1)e^{2\mu+\sigma^2}$$