Hypothesis Testing

- **PARAMETRIC** → ABOUT DISTRIBUTION PARAMETERS
- **NON-PARAMETRIC** → ON, IE, ABOUT TYPE OF DISTRIBUTION

- **TYPE I**: H_0 REJECTED WHEN TRUE, FALSE HIT, "FALSE POSITIVE", $\alpha = P(H_1 | H_0)$
- **TYPE II**: H_0 ACCEPTED WHEN FALSE, MISS, "FALSE NEGATIVE", $\beta = P(H_0 | H_1)$

- **SIGNIFICANCE**, MAX EXPOSURE TO WRONGLY REJECT H_0, α
  MAX RISK OF TYPE I, TRUST LEVEL $1-\alpha$

- **POWER**, PROB OF CORRECTLY REJECTING H_0, $1-\beta$

1 - DEFINE DISTRIBUTION
2 - DEFINE NULL HYPOTHESIS H_0
3 - DEFINE ALTERNATIVE HYPOTHESIS H_1
4 - DEFINE TEST STATISTIC WITH DISTRIBUTION KNOWN WHEN H_0 IS TRUE, G
5 - DEFINE ACCEPT / REFUSAL REGIONS, CHOOSE α
6 - ACCEPT H_0 IF G IS IN ACCEPTANCE REGION

GENERAL TEST STATISTIC $T = \dfrac{\hat{\theta} - \theta_0}{\sigma_\theta}$

Z-TEST

NORMAL POPULATION
KNOWN σ

$H_0: \bar{\mu} = \mu$

$Z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{N}}$, Z-SCORE, N° OF STDDEVS AWAY

- $\mu > \mu_0 \longrightarrow z \geq z_\alpha$
- $\mu < \mu_0 \longrightarrow z \leq z_\alpha$
- $\mu \neq \mu_0 \longrightarrow z \geq z_{\alpha/2}$, $z \leq -z_{\alpha/2}$

FOR A TEST OF SIGNIFICANCE α

- CAN COMPUTE β, IE $\phi\left(z_\alpha + \dfrac{\mu_0 - \mu'}{\sigma / \sqrt{N}}\right)$

- **LARGE SAMPLES** ($>> 40$)

$Z = \dfrac{\bar{X} - \mu_0}{S / \sqrt{N}}$ UNKNOWN σ

- **POPULATION PROPORTION**

$H_0: p = p_0$

$Z = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/N}}$ PROPORTION OF PPL WITH CHAR.

TODO: NYMAN - PEARSON

- OK WHEN $N p_0 \geq 10$, $N(1-p_0) \geq 10$ → SO STUFF IS N.D.
  ELSE DIRECTLY USE BINOMIAL

ONE SAMPLE T-TEST

SAMPLES FROM NORMAL DISTRIBUTION; NONNORMAL

$T = \dfrac{\bar{X} - \mu}{S / \sqrt{N}}$; T DISTRIBUTION $N-1$ DOF

- COME Z-TEST MA CON T-QUANTILI
- VARIANCE UNKNOWN, $N << 30 - 40$

$H_0: \bar{\mu} = \mu$ UNKNOWN VARIANCE

P-VALUE

- PROBABILITY, ASSUMING H_0 TRUE, OF OBTAINING TEST STATISTIC AT LEAST AS CONTRADICTORY TO H_0 AS THE ONE CALCULATED (OUT OF ALL POSSIBLE TEST STATISTIC VALUES)
- THE SMALLER THE P-VALUE → MORE EVIDENCE AGAINST NULL HYPOTHESIS, SMALLEST α AT WHICH I REJECT H_0
- REJECT H_0 IF $P \leq \alpha$, DO NOT IF $P > \alpha$, IDENTICAL TO REJECTION REGION METHOD

- **FOR Z-TEST** → AREA UNDER CURVE

$$P \begin{cases} 1 - \Phi(z) & \text{UPPER TAILED} \\ \Phi(z) & \text{LOWER TAILED} \\ 2[1 - \Phi(z)] & \text{TWO TAILED} \end{cases}$$

LIKELIHOOD RATIO PRINCIPLE (NYMAN-PEARSON LEMMA)

$$\lambda(x_1 \dots x_N) = \frac{ML(\theta, \Omega_0)}{ML(\theta, \Omega_1)}$$, FIND MLES FOR ALL, REJECT H_0 WHEN $\lambda \leq K$

GIVES CRITICAL REGION OF MAX POWER FOR THAT GIVEN SIZE | GENERAL FRAMEWORK FOR DERIVING TESTS

TWO POPULATIONS

Z-TEST, DIFF OF 2 POPULATION, NORMAL POPULATIONS, KNOWN VARIANCE

$H_0: \mu_1 - \mu_2 = \Delta_0$

$\bar{x} - \bar{y}$, $\sigma_{\bar{x}-\bar{y}} = \sqrt{\frac{\sigma_1^2}{M} + \frac{\sigma_2^2}{N}}$; $H_0 = \mu_1 - \mu_2 = \Delta_0$

$$z = \frac{\bar{x} - \bar{y} - \Delta_0}{\sqrt{\frac{\sigma_1^2}{M} + \frac{\sigma_2^2}{N}}}$$

- **LARGE SAMPLES** . UNKNOWN DISTRIBUTIONS, LARGE SAMPLES

$H_0: \mu_1 - \mu_2 = \Delta_0$

CLT ON

$M, N > 40$

$$z = \frac{\bar{x} - \bar{y} - \Delta_0}{\sqrt{\frac{S_1^2}{M} + \frac{S_2^2}{N}}} \rightarrow \text{C.I.} \quad \bar{x} - \bar{y} \pm z_{\alpha/2} \cdot \sqrt{\frac{S_1^2}{M} + \frac{S_2^2}{N}}$$

- **NON-NORMAL, SMALL SAMPLES, T-TEST**

UNKNOWN VARIANCES

$\bar{x} - \bar{y} \pm t_{\alpha/2} \sqrt{\frac{S_1^2}{M} + \frac{S_2^2}{N}}$

$H_0: \mu_1 - \mu_2 = \Delta_0$

$$t = \frac{\bar{x} - \bar{y} - \Delta_0}{\sqrt{\frac{S_1^2}{M} + \frac{S_2^2}{N}}}$$

- **IF SAME VARIANCES**

$$z = \frac{\bar{x} - \bar{y} - (\mu_1 - \mu_2)}{\sqrt{\sigma^2\left(\frac{1}{M} + \frac{1}{N}\right)}}$$

PAIRS

- **PAIRED t-TEST**

$\mu_D = \Delta_0$

$D = X - Y$

$\mu_D = \mu_1 - \mu_2$

$$t = \frac{\bar{d} - \Delta_0}{S_D / \sqrt{N}}$$

DIFFERENT INDEPENDENT PAIRS

$\bar{d} =$ AVG OF ALL D'S

C.I. $\bar{d} \pm t_{\alpha/2, N-1} \cdot \frac{Sd}{\sqrt{N}}$

PAIRS ARE NON-INDEPENDENT.

- **INCORRELATION TEST** $H_0: \rho(x,y) = 0$

$$R_N = \frac{\mathrm{COV}(X,Y)}{S_{N,X}, S_{N,Y}}$$

$$T_N = R_N \cdot \sqrt{\frac{N-2}{1-R_N^2}}$$

IS T-STUDENT

N−2 DOF

IF H_0 TRUE

$H_0: \rho(x,y) = 0$

NORMAL DISTRIBUTIONS

- **DIFF. BETWEEN POP. PROPORTIONS**

$\hat{P}_1 = \frac{X}{M}$, $\hat{P}_2 = \frac{Y}{N}$

$H_0: P_1 - P_2 = 0$

$X \sim BIN(M, P_1)$, $Y \sim BIN(N, P_2)$

$$z = \frac{\hat{P}_1 - \hat{P}_2 - (P_1 - P_2)}{\sqrt{\frac{P_1 Q_1}{M} + \frac{P_2 Q_2}{N}}}$$

$Q_i = 1 - P_i$

F-TEST, TWO DISTRIBUTIONS

NORMAL DISTRIBUTION, KNOWN VARIANCES

H_0 $\sigma_1^2 = \sigma_2^2$, REJECT IF RATIO DIFFS FROM 1

$X_1 \dots X_M$; ND, σ_1^2

$Y_1 \dots Y_N$; ND, σ_2^2

$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$ IS F-DISTRIBUTION $V_1 = M-1$, $V_2 = N-1$ B.C. $\frac{S_i^2}{\sigma_i^2}$

$f = \frac{S_1^2}{S_2^2}$

UNKNOWN $\mu \rightarrow f_{M-1,N-1}$

KNOWN $\mu \rightarrow f_{M,N}$

- $f \geq F_{\alpha; M-1, N-1}$, $\sigma_1^2 > \sigma_2^2$

A BIT TRICKY TO FIND P-VALUES

- $f \leq F_{1-\alpha, M-1, N-1}$; $\sigma_1^2 < \sigma_2^2$
- $f \geq F_{\alpha/2, M-1, N-1}$ || $f \leq F_{1-\alpha/2, M-1, N-1}$ $\quad \sigma_1^2 \neq \sigma_2^2$

ANOVA

χ^2-TEST, VARIANCE (SINGLE POP), NORMAL DISTRIBUTION, KNOWN VARIANCE

$$(N-1)\frac{\bar{S}^2}{\sigma^2} = \chi^2(N-1) = \chi^2$$

$H_0: \sigma^2 = \bar{S}^2$, $\sigma^2 \gtrless \bar{S}^2$

USE χ^2 QUANTILES

- **LARGE SAMPLES**

$$Z_N = \frac{(S_N - \sigma_0)\sqrt{2N}}{\sigma_0}, \quad N > 30$$

COMPARE SAMPLE VARIANCE TO KNOWN VARIANCE