CS-49: Game Theory

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Problem 10.

Suppose you are playing a sum of two games, one of which is just a single NIM stack of size 5. In the other game a move can produce a position of value $*k$ for any $k \in \mathcal{K}$, where $\mathcal{K}$ is a finite set of nonnegative integers. What sets $\mathcal{K}$ result in this sum of games being in the outcome class $\mathbf{P}$?

The sum of the two games is in class $\mathbf{P}$ only if the XOR sum of their values is 0. For this to happen, either;

1. The sum of the values of the stacks in the other game must equal 5 (example: 3 and 2).
2. The XOR sum of the values of the stack(s) in the other game must equal 5 (example: 7 and 2).

Therefore,

$$\mathcal{K} = \left\{ n_1, \ldots, n_k \mid \left(\sum_i n_i = 5 \right) \text{ or } (n_1 \oplus \ldots \oplus n_k = 5) \right\}$$