

CS-49: Game Theory

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Problem 3.

Your utility for owning $\$x$ is $\log x$. (The base of the logarithm doesn't matter; for this problem I recommend the natural logarithm.) You have $\$a$ and are permitted to make one bet on the flip of a coin that comes up "heads" with probability p which is known to you and greater than $1/2$. If you bet $\$b$ and win, you win $\$b$; otherwise you lose the amount bet. What fraction of $\$a$ should you bet, to maximize your expected utility?

Suppose you bet $\$b$ which is a fraction m of your total wealth $\$a$. Then:

$$\text{Current utility} = \log a$$

$$\text{Utility if your bet wins} = \log(a + b) = \log((1 + m)a) = \log(1 + m) + \log a$$

$$\text{Utility if you bet \$b and lose} = \log(a - b) = \log((1 - m)a) = \log(1 - m) + \log a$$

$$\begin{aligned} \text{Expected utility} &= p(\log(1 + m) + \log a) + (1 - p)(\log(1 - m) + \log a) \\ &= p\log a + (1 - p)\log a + p\log(1 + m) + (1 - p)\log(1 - m) \\ &= \log a + p\log(1 + m) + (1 - p)\log(1 - m) \end{aligned}$$

To improve utility, we want to maximize the change

$$p\log(1 + m) + (1 - p)\log(1 - m).$$

Via a little guess-work (see plots), it seems $m = 2p - 1$ is a good choice to maximize the change in utility when the probability of a favorable outcome is known to be p .

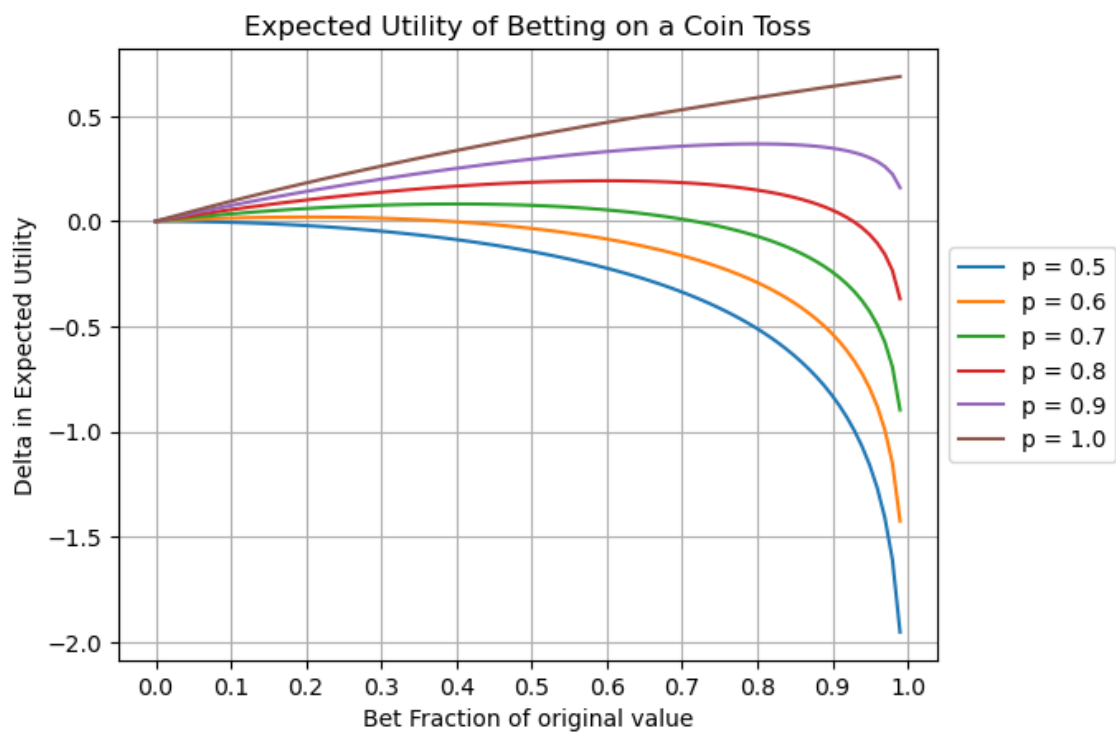


FIGURE 1. The change in utility with various p and various m .

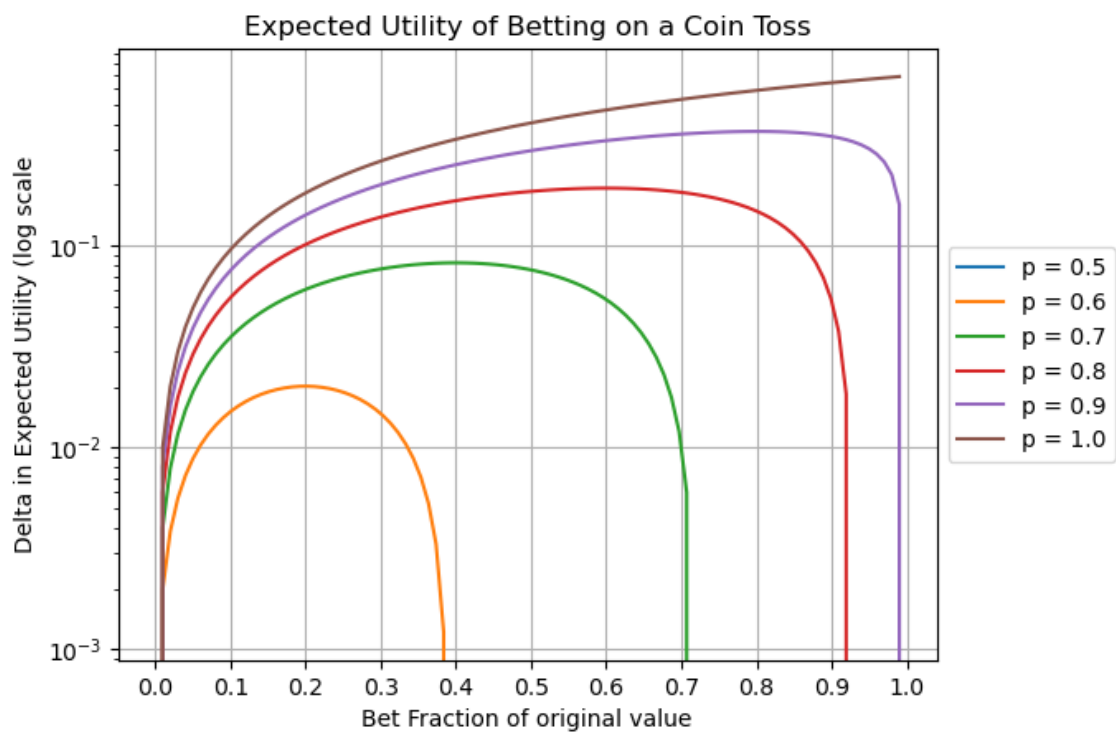


FIGURE 2. The change in utility with various p and various m (log scale).