CS-49: Game Theory

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Problem 16.

A \$10 bill is auctioned to 10 people in the following way:

- Each person chooses a non-negative integer number of dollars to submit as a sealed bid.
- The \$10 bill goes to a uniformly random person who was among the highest bidders, in return for the amount bid.

Find and count all pure Nash equilibria for this game. (You may assume everyone's utility for money is linear in the range 0 to 10.)

If a player bids the single highest bid, then they win the bidding with probability 1. However, if a player is among x players with the highest bid, then the player has a probability $1/n$ of winning the bid. Therefore, whenever possible (except when they make a loss or win nothing by betting \$10), each person would prefer to bid the lowest number that is higher than all other bids. So, for instance, if the second-highest bid is \$5, then the winning bidder would prefer to bid \$6 rather than \$9, since in the first case they win \$4 while in the second case they only win \$1. However, in all cases, each player would prefer their bid to be *at least* equal to the maximum bid; that way, they always have some chance of winning.

In the Nash equilibria, no player has anything to gain by changing their bid. This means:

(i) No player gains an advantage by raising their bid — therefore, their bid must either be the maximum bid that still wins something (\$9) or be \$1 more than all the other bids. Since the condition holds for *all* players, it must be the first case and not the second.

(ii) They do not gain an advantage by lowering their bid — therefore, their bid must not be more than \$1 over the second-highest bid(s).

With all these in mind, we can determine that the Nash equilibrium happens when *every* player's bid is \$9. There is only one Nash equilibrium.