

CS-49: Game Theory

Amittai Siavava

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Problem 5.

Determine (with proof!) the **P** and **N** positions for SUBTRACTION, where the set of subtractibles is $\{3, 4, 5\}$.

Recall that;

1. A position is in class **P** if *all* possible moves end in positions in class **N**.
2. A position is in class **N** if there exists a move that ends in a position in class **P**.

Note that only a single move is needed!

By writing out the possible values for the first 23 moves (see table on next page), we see this general pattern emerge:

$$\mathbf{P} = \{0, 1, 2\} \cup \{8, 9, 10\} \cup \{16, 17, 18\} \cup \dots$$

$$\mathbf{N} = \{3, 4, 5, 6, 7\} \cup \{11, 12, 13, 14, 15\} \cup \{19, 20, 21, 22, 23\} \cup \dots$$

In general, a given position n is in the class **P** iff $n \pmod{8} < 3$, and a given position n is in the class **N** iff $n \pmod{8} \geq 3$.

Current	Next			P or N?
0	-3	-4	-5	P
1	-2	-3	-4	P
2	-1	-2	-3	P
3	0	-1	-2	N
4	1	0	-1	N
5	2	1	0	N
6	3	2	1	N
7	4	3	2	N
8	5	4	3	P
9	6	5	4	P
10	7	6	5	P
11	8	7	6	N
12	9	8	7	N
13	10	9	8	N
14	11	10	9	N
15	12	11	10	N
16	13	12	11	P
17	14	13	12	P
18	15	14	13	P
19	16	15	14	N
20	17	16	15	N
21	18	17	16	N
22	19	18	17	N
23	20	19	18	N
⋮	⋮	⋮	⋮	⋮

TABLE 1. Potential Payoff vs. Outright Money.