CS-49: Game Theory

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Problem 8.

Before last basketball season, WNBA star Vera Similitude's lifetime free-throw percentage was below 80%. After the season, it was above 80%. Must there have been a moment in the season when it was exactly 80%? Justify your answer!

Yes, the ratio must have been exactly 80% at some point in the season. Suppose that Vera has X free throws in total initially, of which Y were successful. X/Y is her lifelong free-throw ratio, which is below 0.8. Let $n = Y - X$ be the difference between her successful and unsuccessful free throws. To show that the percentage must equal 80%:

1. Order n containers. For each free throw, fill up the containers with coins. But consider each container "full" if it has 4 coins in it.
2. For each new free throw that she successfully scores this season, add a coin to the next non-full container.
3. For each new free throw that she misses, order one extra container that needs to be filled.

In this way, each container corresponds to a miss, and each coin corresponds to a successful free throw. Since each container is matched up to four coins, the ration is 80% exactly when all the available containers have been filled with 4 coins each. On the other hand, the ratio is *above* 80% only if there are coins that cannot be fit in a container, since all containers are already filled with 4 coins each. But for this to happen, there must have been a moment in the season when all containers were filled, and no extra coin was yet to be added to a container. Hence, there must have been a moment in the season when the ratio was exactly 80%.