CS-49: Game Theory

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Problem 7.

In WEIGHTED ODDS AND EVENS, Alice and Bob simultaneously put out one or two fingers. If they put out different numbers of fingers, Alice wins from Bob a number of dollars equal to the total number of fingers put out (namely, in this case, 3). If they put out the same number of fingers, Alice pays Bob \$2 or \$4 according to the total number of fingers played.

(a) Find an equilibrium pair of randomized strategies.

$$A = \begin{bmatrix} -2 & +3 \\ +3 & -4 \end{bmatrix}$$

$$B = \begin{bmatrix} +2 & -3 \\ -3 & +4 \end{bmatrix}$$

Alice has two options:

(i) Put out one finger. Half the time, she loses \$2, and half the time she wins \$3, for an expected value of $\$(0.5 \cdot (-2) + 0.5 \cdot (+3)) = \$ + 0.5$ per turn.

(ii) Put out two fingers. Half the time, she loses \$4, and half the time she wins \$3 for an expected value of $\$(0.5 \cdot (-4) + 0.5 \cdot (+3)) = \$ - 0.5$ per turn.

Alice is better off putting out one finger.

Bob has two options:

(i) Put out one finger. Half the time, he wins \$2, and half the time he loses \$3 for an expected value of $\$(0.5 \cdot (+2) + 0.5 \cdot (-3)) = \$ - 0.5$ per turn.

(ii) Put out two fingers. Half the time, he wins \$4, and half the time he loses \$3 for an expected value of $\$(0.5 \cdot (+4) + 0.5 \cdot (-3)) = \$ + 0.5$ per turn.

Bob is better off putting out two fingers.

(b) What is the expected outcome if these strategies are employed?

In the equilibrium pair of strategies, Alice puts out one finger and Bob puts out two fingers. Since the fingers do not match, Alice wins $\$ + 3$ per turn.