CS-49: Game Theory

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Problem 11.

Replace each of the first five letters of your official Dartmouth email address by its position in the alphabet (a number between 1 and 26), and consider the resulting 5-stack NIM position. Find all winning moves (if any) from that position.

letters:`amitt`

Converting to binary:

A = 1 = 00001

M = 13 = 01101

I = 9 = 01001

T = 20 = 10100

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0 0 0 0 1	A
0 1 1 0 1	M
0 1 0 0 1	I
1 0 1 0 0	T
1 0 1 0 0	T
0 0 1 0 1	

FIGURE 1. Initial game value.

A winning move needs to change the game's value to 0, meaning it must make position 1 and 3 have an even number of 1s across the five stacks. With the limitation to deduction, only one possible move achieves that:

1. Remove 5 stones from stack 2 (for letter M). This changes the value of the stack from 01101 (equal to 13, or M) to 01000 (equal to 8, or H) with this new value;

0 0 0 0 1	A
0 1 0 0 0	H
0 1 0 0 1	I
1 0 1 0 0	T
1 0 1 0 0	T
0 0 0 0 0	

FIGURE 2. Game value after move.

This move goes from AMITT to AHITT. Since the new game state has value 0, it is in class P and is therefore a winning position for the player who just moved.