CS-49: Game Theory

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Problem 12.

Compute the value of $1 \times n$ CRAM for $n = 7, 8, 9, 10$.

For even n, the next player $\mathbf{N}$ can win as follows:

(i) Let the squares be numbered $1, 2, \ldots, n$ from left to right.

(ii) $\mathbf{N}$ marks the two contiguous squares $n/2, n/2 + 1$.

(iii) This leaves two identical positions, each of dimensions $1 \times \left(\frac{n}{2} - 1\right)$. Since $\mathbf{P}$ moves first in this new situation, $\mathbf{N}$ can win by strategy-stealing.

For odd n, the best strategy for the next player is to leave a position with an odd dimension for the other player, since leaving a position with even dimensions loses by the strategy above. The outcome therefore depends on how many such moves can be made before running out — that is,

$$\mathbf{outcome}(G) = \begin{cases} \mathbf{N} & \text{if} \quad n \pmod 2 \equiv 1 \\ \mathbf{P} & \text{if} \quad n \pmod 2 \equiv 0 \end{cases}$$

Particularly, $(1 \times 7) \in \mathbf{N}$ and $(1 \times 9) \in \mathbf{P}$.

Therefore, the values of the games are:

Dimensions	Class	Value
(1×7)	$\mathbf{N}$	1
(1×8)	$\mathbf{N}$	1
(1×9)	$\mathbf{P}$	0
(1×10)	$\mathbf{N}$	1

TABLE 1. Values of $1 \times n$ CRAM for $n = 7, 8, 9, 10$