Analysis and comparison of matrix vector multiplication codes

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May 2019

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Introduction

- Kernel: result[i] = result[i] + A[i,j]∗vector[j]
- Matrix size changing: $n_1 = 40$, $n_2 = 400$, $n_3 = 1000$
- Different optimization flags
- What is Eigen? How to change the kernel to match its performance?

Tools

- Stream $\rightarrow$ Bandwidth 6000 MB/s
- LIKWID: topology and perfctr
- Hardware: Interlagos 12-core Opteron processor.

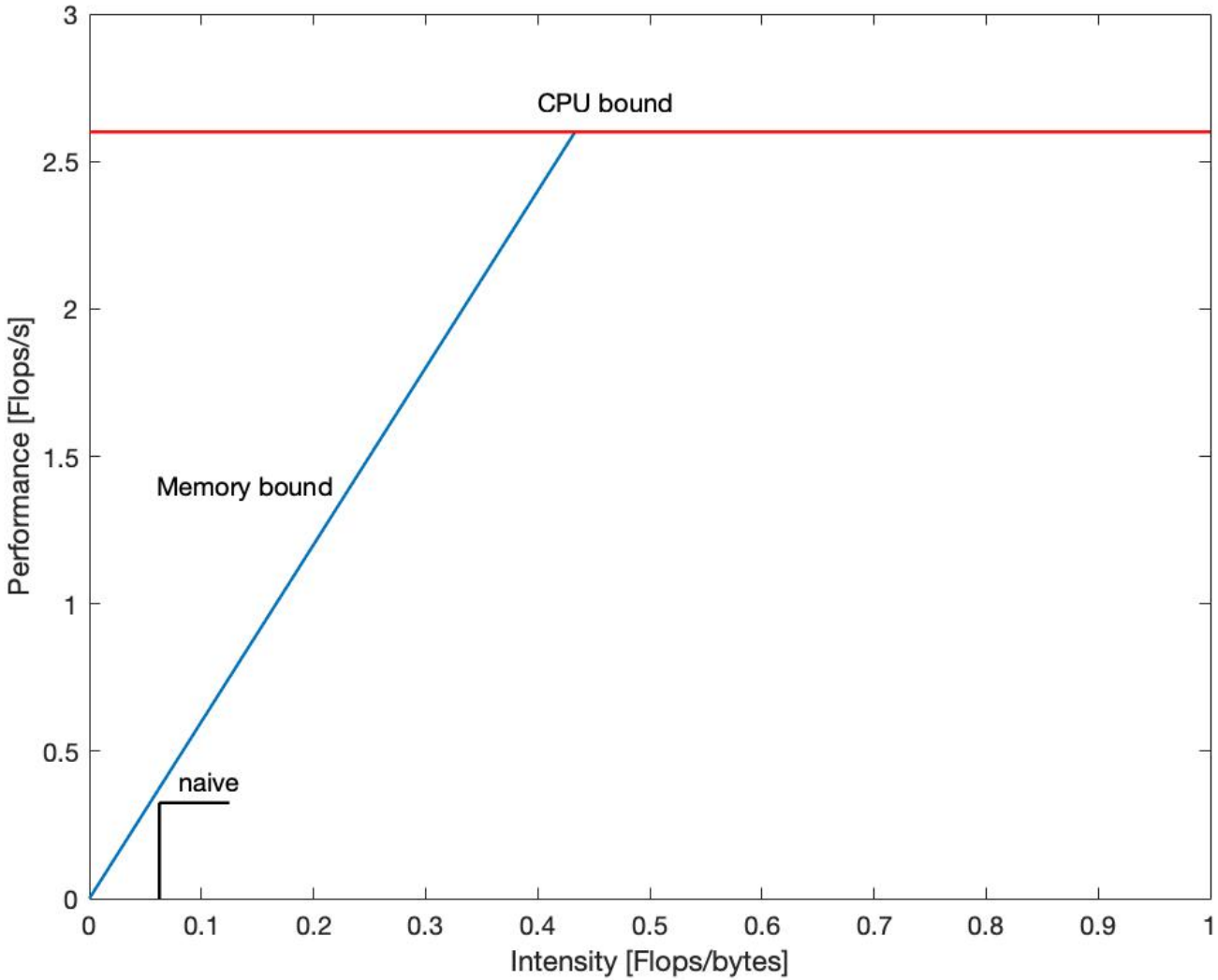
Roofline model

- Bottlenecks: CPU bound vs memory bound
- $P_{max} = f_{max} * \frac{n_{operations}}{n_{cycles}} * v$
- $P = min(P_{max}, \text{Bandwidth} * \text{Computational intensity})$

Naive

```
for i = 1:n
 for j = 1:n
   result[i] = result[i] + A[i,j]*v[j]
```

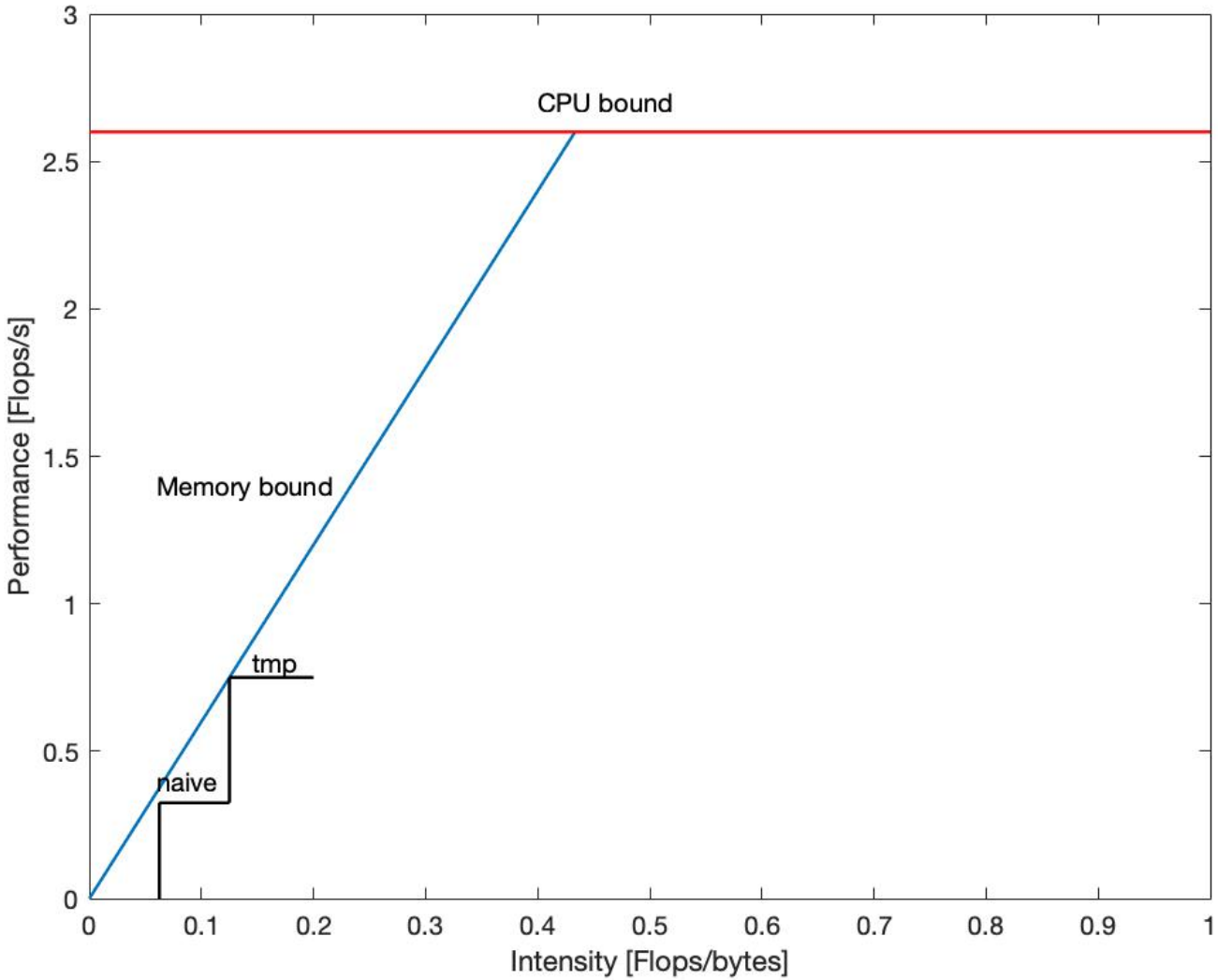
- Simplest possible code, useful for benchmark.
- $P_{max} = \frac{2.6}{8} \frac{Gflops}{s}$, not exploiting pipelines of sum and product
- Computational intensity $= \frac{2}{32} \frac{Flops}{Bytes}$


CPU bound
Memory bound
naive
Performance [Flops/s]
Intensity [Flops/bytes]
0
0.5
1
1.5
2
2.5
3
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Temporary variable

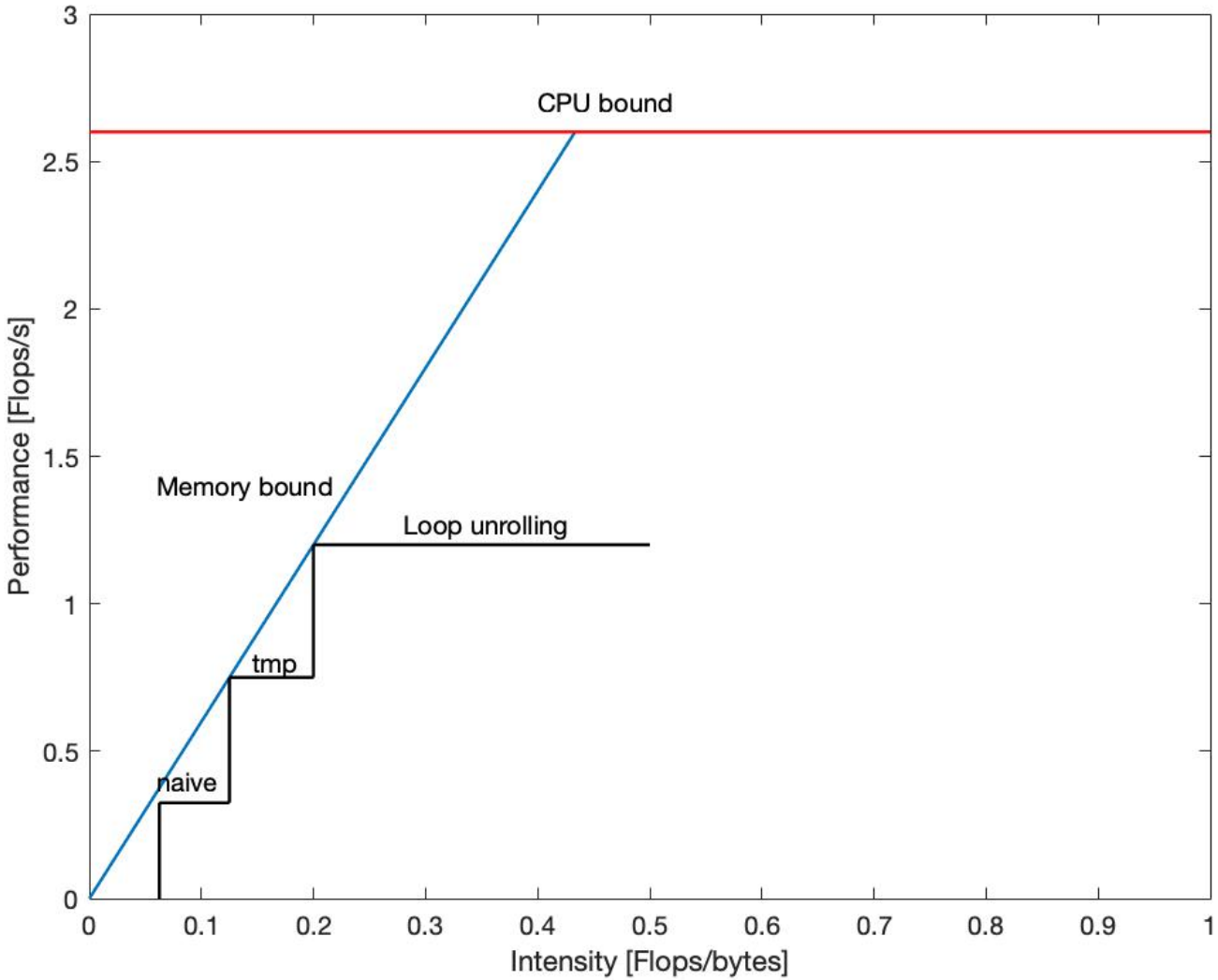
```
for i = 1:n
tmp = result[i]
 for j = 1:n
    tmp = tmp + A[i,j]*v[j]
 result[i] = tmp
```

- Additional temporary variable in outer loop to speed up cache lookup
- $P_{max} = \frac{2.6}{8} \frac{Gflops}{s}$
- Computational intensity $= \frac{2}{16} \frac{Flops}{Bytes}$


CPU bound
Memory bound
tmp
naive
Performance [Flops/s]
Intensity [Flops/bytes]
0
0.5
1
1.5
2
2.5
3
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

Loop unrolling

- Manually split two loops four by four to exploit pipelines of sum and product
- $P_{max} = 2.6 \frac{Gflops}{s}$
- Computational intensity $= \frac{32}{160} \frac{Flops}{Bytes}$


CPU bound
Memory bound
Loop unrolling
tmp
naive
Performance [Flops/s]
Intensity [Flops/bytes]
0
0.5
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0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

Comparing the Mflops/s

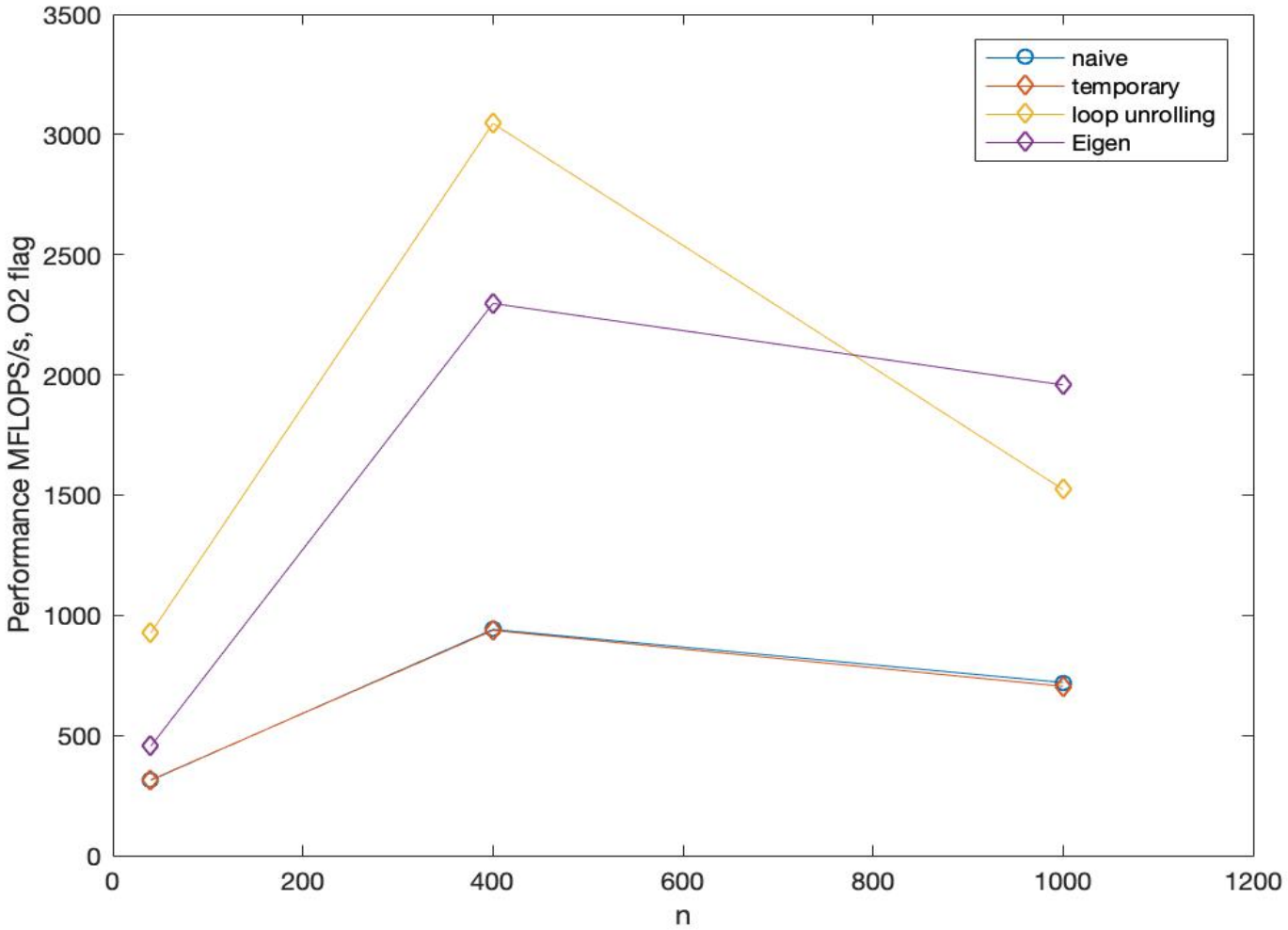
O0	naive	with_tmp	loop unrolling	eigen
n = 40	195	204	313	59
n = 400	328	332	453	70
n = 1000	327	328	445	94

O1	naive	with_tmp	loop unrolling	eigen
n = 40	302	255	882	464
n = 400	948	345	2346	2339
n = 1000	740	344	1544	1922

O2	naive	with_tmp	loop unrolling	eigen
n = 40	313	315	926	455
n = 400	941	937	3046	2297
n = 1000	720	704	1523	1958

O3	naive	with_tmp	loop unrolling	eigen
n = 40	315	321	915	383
n = 400	936	961	2935	3429
n = 1000	761	766	1538	2024

Data visualization

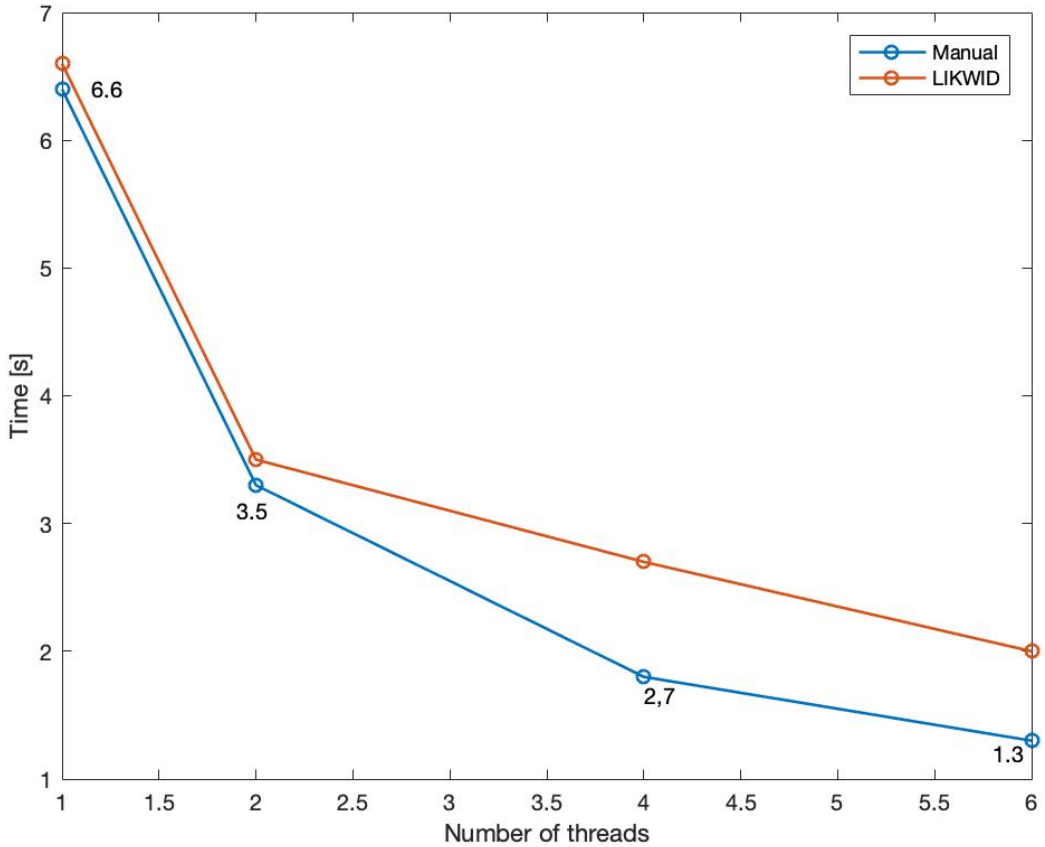


What is OMP?

- API that supports multi-platform shared memory multiprocessing.
- Used for parallelism within a multi-core node.
- Based on threads: processes that can share memory.

Case study

With $n = 50000$, $O3$ flag and different number of threads:



Achievements

- Outperformed Eigen when $n = 40$
- Outperformed Eigen when $n = 400$ and $O2$ flag.

Future directions

- Cache aware roof line model
- SIMD vectorization
- Understanding GCC behaviour when matrix is saved in L1