

Analysis and comparison of matrix vector multiplication codes

Ludovico Bessi

Matematica per l'ingegneria
Politecnico di Torino

May 2019

Table of Contents

- 1 Introduction and tools used
- 2 Different implementations
- 3 Comparison with Eigen
- 4 OpenMP
- 5 Conclusions

Introduction

- Kernel: $\text{result}[i] = \text{result}[i] + A[i,j] * \text{vector}[j]$
- Matrix size changing: $n_1 = 40$, $n_2 = 400$, $n_3 = 1000$
- Different optimization flags
- What is Eigen? How to change the kernel to match its performance?

Tools

- Stream → Bandwidth 6000 MB/s
- LIKWID: topology and perfctr
- Hardware: Interlagos 12-core Opteron processor.

Roofline model

- Bottlenecks: CPU bound vs memory bound
- $P_{max} = f_{max} * \frac{n_{operations}}{n_{cycles}} * v$
- $P = \min(P_{max}, \text{Bandwidth} * \text{Computational intensity})$

Naive

```
for i = 1:n
  for j = 1:n
    result[i] = result[i] + A[i,j]*v[j]
```

- Simplest possible code, useful for benchmark.
- $P_{max} = \frac{2.6}{8} \frac{Gflops}{s}$, not exploiting pipelines of sum and product
- Computational intensity = $\frac{2}{32} \frac{Flops}{Bytes}$

Temporary variable

```
for i = 1:n
  tmp = result[i]
  for j = 1:n
    tmp = tmp + A[i,j]*v[j]
  result[i] = tmp
```

- Additional temporary variable in outer loop to speed up cache lookup
- $P_{max} = \frac{2.6}{8} \frac{Gflops}{s}$
- Computational intensity = $\frac{2}{16} \frac{Flops}{Bytes}$

Loop unrolling

- Manually split two loops four by four to exploit pipelines of sum and product
- $P_{max} = 2.6 \frac{Gflops}{s}$
- Computational intensity = $\frac{32}{160} \frac{Flops}{Bytes}$

Comparing the Mflops/s

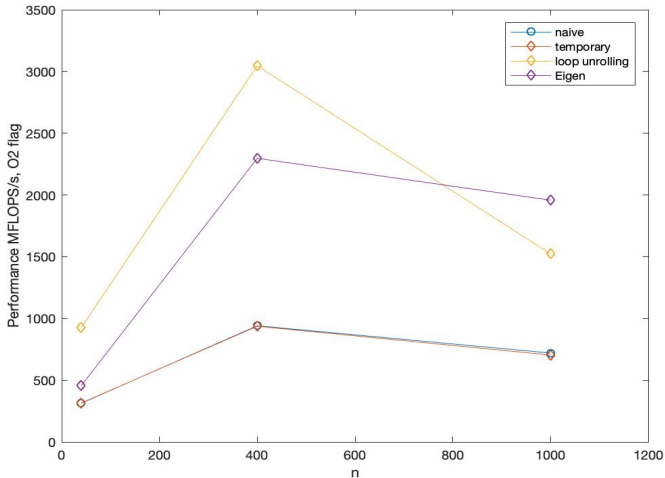
O0	naive	with_tmp	loop unrolling	eigen
n = 40	195	204	313	59
n = 400	328	332	453	70
n = 1000	327	328	445	94

O1	naive	with_tmp	loop unrolling	eigen
n = 40	302	255	882	464
n = 400	948	345	2346	2339
n = 1000	740	344	1544	1922

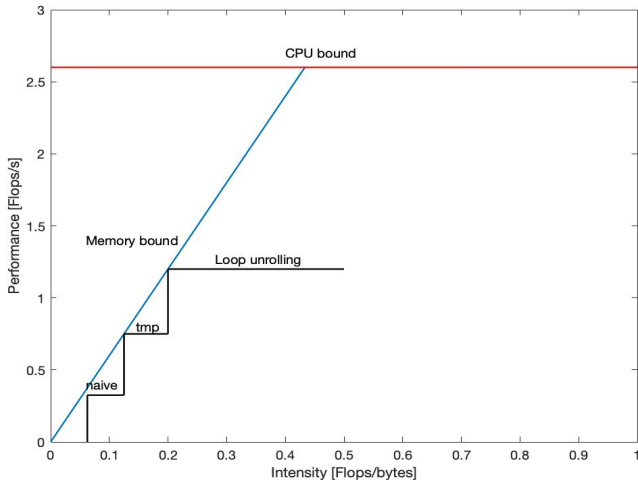
O2	naive	with_tmp	loop unrolling	eigen
n = 40	313	315	926	455
n = 400	941	937	3046	2297
n = 1000	720	704	1523	1958

O3	naive	with_tmp	loop unrolling	eigen
n = 40	315	321	915	383
n = 400	936	961	2935	3429
n = 1000	761	766	1538	2024

Data visualization



Roofline graph

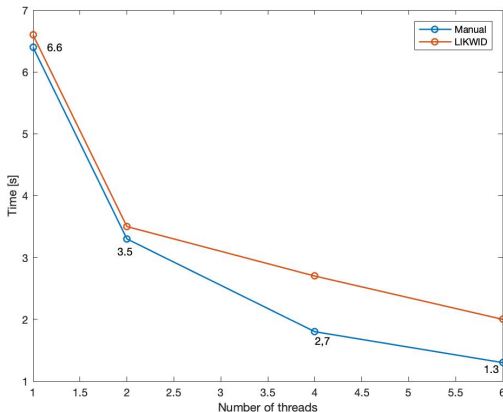


What is OMP?

- API that supports multi-platform shared memory multiprocessing.
- Used for parallelism within a multi-core node.
- Based on threads: processes that can share memory.

Case study

With $n = 50000$, *O3* flag and different number of threads:



Achievements

- Outperformed Eigen when $n = 40$
- Outperformed Eigen when $n = 400$ and O2 flag.

Future directions

- Cache aware roof line model
- SIMD vectorization
- Understanding GCC behaviour when matrix is saved in L1