Numbermind

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Problem to solve

Guess a secret code (sequence of digits) by generating guesses with the solutions of a SAT solver.

After each guess we are told how many coincidences there are with the secret code.

There is a coincidence iff both digit and its position are the same in the guess and the secret code.

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- Keep iterating until the secret code is found.

Cardinality constraint

Exactly k

The constraint "exactly k propositional variables in X are true" can be rephrased as the conjunction of "at least k" and "at most k".

$$\bigwedge_{\substack{I\subseteq[n]\\|I|=n-k+1}} \bigvee x_i \wedge \bigwedge_{\substack{I\subseteq[n]\\|I|=k+1}} \bigvee_{i\in I} \neg x_i$$

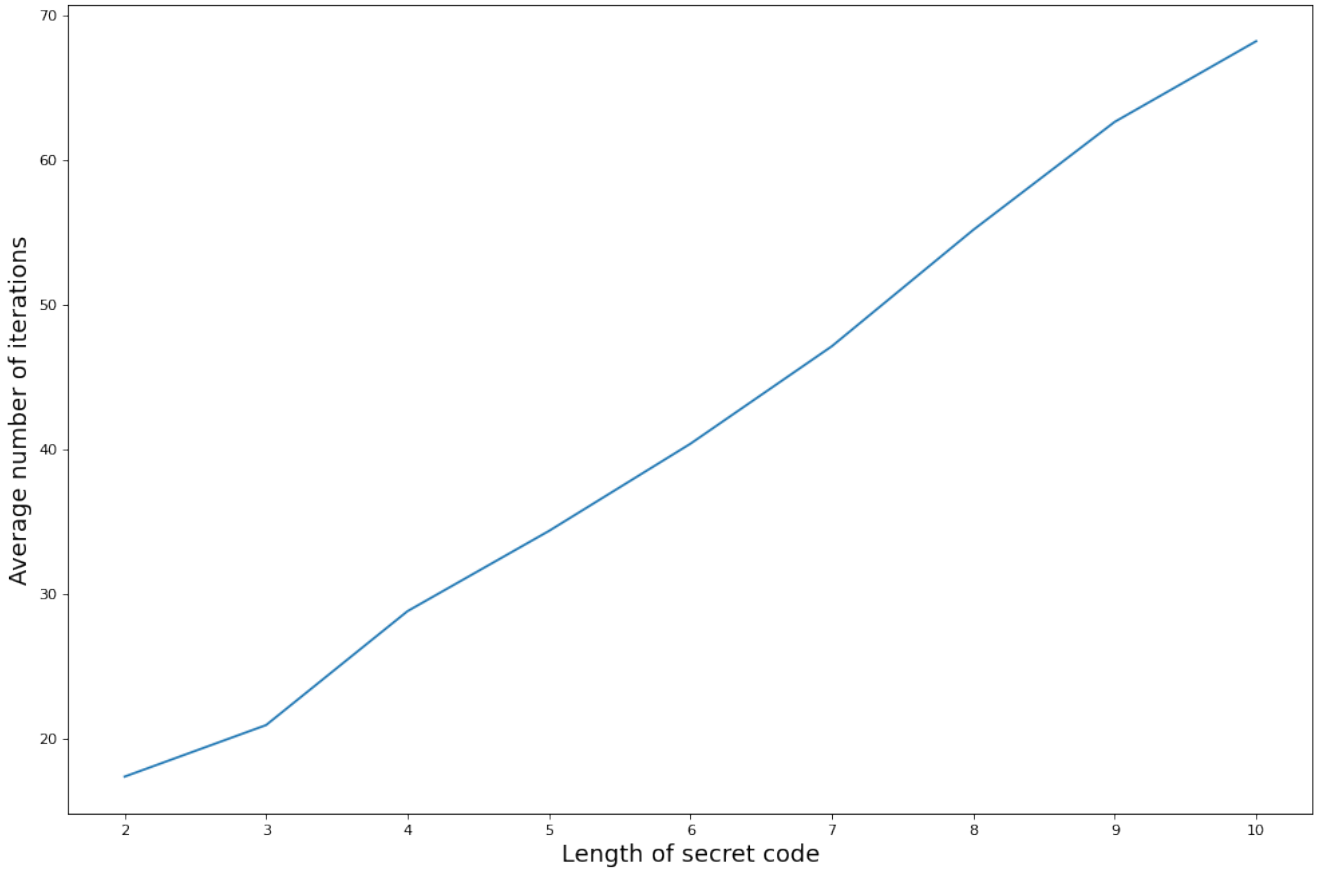
Example (Exactly 2 among X={a, b, c, d})

$$(a \vee b \vee c) \wedge (a \vee b \vee d) \wedge (b \vee c \vee d) \wedge (\neg a \vee \neg b \vee \neg c) \wedge$$

$$(\neg a \vee \neg b \vee \neg d) \wedge (\neg a \vee \neg c \vee \neg d) \wedge (\neg b \vee \neg c \vee \neg d)$$

Results and Conclusions

- Secret code is correctly predicted.
- Average number of iterations with respect to length of secret code:



Thank you for your attention