CS222 Homework 6

Algorithm Analysis & Deadline: 2020-10-20 Firday 24:00

Exercises for Algorithm Design and Analysis by Li Jiang, 2020 Autumn Semester

1. An adventure game uses a graph G consisting of N rooms (numbered from 1 to N) to represent the places need to be explored. These rooms can only be linked in one-way, meaning that a person passing through this path can only move from one room to another but cannot return to the room they left/explored. It's worth noting that there is no circle in this graph. People taking part in the game appear randomly in any of the rooms connected via corridors, and we can explore rooms in the same corridor. You should note that the path taken by two players may contain some of the same rooms.

   How many players are the minimum needed to explore all the rooms?

   Describe your design first and write down your algorithm in the form of pseudo-code.

2. Suppose there are $M \times N$ rooms, each of which holds a different number of treasures. Please select several rooms so that the selected rooms have no common sides (i.e., the selected rooms cannot be adjacent), and the selected rooms' treasures add up to the greatest value. Describe your design first and write down your algorithm in the form of pseudo-code.

   Note: The value of the treasure is definitely not negative.

3. There are a large number of servers in the data center. Each server has different performance, the program is executed on different servers with different efficiency, and now there is a set of the new programs that need to run. Please assign these programs to the appropriate server based on the historical test results to make this batch of programs run most efficiently. Describe your design first and write down your algorithm in the form of pseudo-code.

4. Given a weighted directed graph $G(V, E)$ and its corresponding weight matrix $W = (w_{ij})_{n \times n}$ and shortest path matrix $D = (d_{ij})_{n \times n}$, where w_{ij} is the weight of edge (v_i, v_j) and d_{ij} is the weight of a shortest path from pairwise vertex v_i to v_j. Now, assume the weight of a particular edge (v_a, v_b) is decreased from w_{ab} to w'_{ab}. Design an algorithm to update matrix D with respect to this change, whose time complexity should be no larger than $O(n^2)$. Describe your design first and write down your algorithm in the form of pseudo-code.

5. Please review the papers to find an algorithmic problem related to the non-obvious network flow or circulation problem, summarize its problem formulation, and transform it into the network flow problem.

   Remark: You need to upload your .pdf file and write the pseudocode.