10.09.2019г / Задача 7

$$\int_0^2 \ln(x^2+4)\,dx =$$

$$\int fg' = fg - \int f'g$$
$$f = \ln(x^2+4) \rightarrow f' = \frac{1}{x^2+4}(x^2+4)' = \frac{2x}{x^2+4}$$
$$g' = 1 \rightarrow g = x$$

$$= \underbrace{\ln(x^2+4)\cdot x\Big|_0^2}_{A} - \underbrace{\int_0^2 \frac{2x}{x^2+4}\cdot x\,dx}_{B} = 2\ln(8) - (4-\pi) =$$
$$= 2\ln(8) + \pi - 4$$
$$(= \ln(64) + \pi - 4)$$

$$A = \ln(x^2+4)\,x\Big|_0^2 = 2\ln(8) - 0\cdot\ln(4) = 2\ln(8)$$

$$B = \int_0^2 \frac{2x^2}{x^2+4}\,dx = 2\int_0^2 \frac{x^2}{x^2+4}\,dx = 2\int_0^2 \frac{x^2+4-4}{x^2+4}\,dx = 2\int_0^2 \frac{x^2+4\,dx}{x^2+4} - 2\int_0^2 \frac{4}{x^2+4}\,dx =$$

$$= 2\int_0^2 dx - 2\cdot 4\int_0^2 \frac{1}{x^2+4}\,dx = 2\cdot x\Big|_0^2 - 8\int_0^2 \frac{1}{x^2+4}\,dx = 4 - 8\int_0^2 \frac{1}{x^2+4}\,dx =$$

$$\underline{\underline{du = \frac{dx}{2}}} \quad 4 - 8\int_0^1 \frac{2}{4u^2+4}\,du = 4 - \frac{8}{2}\int_0^1 \frac{1}{u^2+1}\,du = 4 - 4\int_0^1 \frac{1}{u^2+1}\,du =$$

$$= 4 - 4\arctan(u)\Big|_0^1 = 4 - 4\arctan\left(\frac{x}{2}\right)\Big|_0^2 = 4 - 4(\arctan(1) - \arctan(0)) =$$

$$= 4 - 4\left(\frac{\pi}{4} - 0\right) = 4 - \pi$$