Accurate methods for computing rotation-dominated flows

Logbook

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After finishing the literature survey and project outline, I set up the basic infrastructure for the project. All the data, code and written work will be maintained in a private `git`[1] repository at `https://github.com/mbuettner/PHASM201`. I've also set up individual LaTeXprojects for the literature survey, final report and this logbook.

The coming month will be spent on reading papers about existing well-balanced methods in-depth to evaluate which ones are worth implementing during the second phase of the project.

29 October 2014

Read LeVeque [1998] today. He balances the SWEs with source terms by introducing another Riemann problem in the centre of the cell, such that each cell doesn't just have an its average value Q_i but two values $Q_i \pm \delta$, such that the overall cell averages is the same. The Riemann problems at the cell boundaries are then solved between these new values. The trick is that the step inside the cell is chosen such that the resulting flow cancels exactly with the source term - hence this Riemann problem does not even need to be solved. Works up to Courant number 1. Caveat of the paper: the "exact" reference solutions used to compare their method with a fractional step method also use their method, just on a much finer grid. The paper explicitly mentions that this method *only* captures quasi-steady solutions and does not perform well for solutions away from equilibrium. In particular, it is not suited to setting up transcritical flow problems. The paper suggests using other (possibly split) methods to find an approximation to a (quasi-)steady solution and then switch methods.

This method simplifies boundary conditions in the presence of source terms. The approach is also adapted to two dimensions in the paper.

4 November 2014

I found another early paper, Ambrosi [1995], which observes the problems of unbalanced schemes, namely that still-lake equilibria are not preserved.

Rogers et al. [2001] use a different formulation of the SWEs to balance their solver. Instead of writing the SWEs in terms of (h, uh, vh), they use (ζ, uh, vh), where ζ is the disturbance of the water depth from still water level. The advantage is that still-lake equilibria don't contribute any flux terms in this case (while they do in the conventional formulation, due to differing water depth). Note that this method is tailored specifically to source terms due to variable bathymetry. They follow this up in Rogers et al. [2003]. This paper generalises the method to arbitrary source terms and equilibria, but the method does seem to require information about the desired equilibrium to be included in the formulation of the SWEs themselves (meaning that no one solver can be used for several equilibria or will be particularly well suited to systems away from equilibrium). Generally speaking, the core of the method is to rewrite the entire system in variables which are deviations from one particular equilibrium.

7 November 2014

Reading Chertock et al. [submitted 2014]. They focus on the particular case, we're interested in: well-balanced schemes for the SWEs with variable bathymetry and Coriolis force.

[1] `http://git-scm.com/`

They are referencing quite a lot of papers with the same focus, many of which I haven't encountered during the literature survey. It might be worth having a look into those, too. Notably, this paper doesn't use Riemann–problem based methods, but a so-called "central upwind" method and an "evolution Galerkin" method. I'll look into these, and figure out whether they can be implemented with Clawpack as well.

The basis of their scheme is a piecewise *linear* reconstruction, which might indicate that this is not suitable for Godunov-type methods. It might be possible to implement it with Clawpack anyway.

The schemes use Courant numbers 0.5 and 0.6, but the paper makes no mention of what the actual CFL condition is. This might be detailed in the referenced papers.

Another useful takeaway from the paper might be the numerical simulations used for verification, as their area of interest is the same as ours.

Bibliography

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