Tables for balanced Dyck words with restricted run-length

For $\ell \in [2,6]$ and $n \in [18]$, we explicitly constructed all Dyck words of length $2n$ with run-lengths at most ℓ and counted them. The author's source code in Python is available for download at: https://github.com/magdevska/nono-codes. The implementation used stores the codewords; thus, fails for longer codewords due to insufficient memory. Eventually, we only needed the values for $n \leq 14$ to obtain the results regarding non-overlapping codes in the main text of the dissertation, and we did not bother to improve the implementation.

n	$\mathcal{D}_{2n,2}$	$\mathcal{D}_{2n,3}$	$\mathcal{D}_{2n,4}$	$\mathcal{D}_{2n,5}$	$\mathcal{D}_{2n,6}$
1	1	1	1	1	1
2	2	2	2	2	2
3	4	5	5	5	5
4	8	13	14	14	14
5	17	34	41	42	42
6	37	93	122	131	132
7	82	262	374	417	428
8	185	753	1,173	1,357	1,416
9	423	2,198	3,745	4,495	4,769
10	978	6,502	12,126	15,107	16,297
11	2,283	19,449	39,718	51,386	56,369
12	5,373	58,724	131,372	176,558	196,972
13	12,735	178,739	438,186	611,861	694,310
14	30,372	547,836	1,472,196	2,136,157	2,465,863
15	72,832	1,689,407	4,977,666	7,506,191	8,815,182
16	175,502	5,237,939	16,924,333	26,526,310	31,695,477
17	424,748	16,318,137	57,829,759	94,216,344	114,546,703
18	1,032,004	51,056,027	198,480,074	336,150,486	415,861,577

Table 1: The number of balanced Dyck words of length $2n$ having no runs longer than ℓ for $\ell \in [2,6]$ and $n \in [18]$.

n	$\hat{\mathcal{D}}_{2n,2}$	$\hat{\mathcal{D}}_{2n,3}$	$\hat{\mathcal{D}}_{2n,4}$	$\hat{\mathcal{D}}_{2n,5}$	$\hat{\mathcal{D}}_{2n,6}$
1	1	1	1	1	1
2	1	2	2	2	2
3	1	4	5	5	5
4	2	8	13	14	14
5	4	20	34	41	42
6	8	53	98	122	131
7	17	145	294	381	417
8	37	406	907	1,223	1,366
9	82	1,162	2,856	4,009	4,565
10	185	3,383	9,139	13,361	15,503
11	423	9,874	29,643	45,129	53,349
12	978	29,808	97,245	154,131	185,620
13	2,283	89,849	322,088	531,393	651,893
14	5,373	273,078	1,075,583	1,846,958	2,307,830
15	12,735	835,930	3,617,414	6,464,715	8,227,145
16	30,372	2,574,969	12,241,986	22,767,434	29,508,002
17	72,832	7,975,761	41,656,955	80,619,182	106,406,536
18	175,502	24,825,784	142,442,817	286,854,320	385,547,441

Table 2: The number of balanced Dyck words of length $2n$ starting with a sequence of at most $\ell-1$ ones, ending in a sequence of at most $\ell-1$ zeros and having no runs longer than ℓ in between for $\ell \in [2,6]$ and $n \in [18]$.