Optimization demo

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1 Introduction

This runs through a number of optimization routines using the **Optim** and **NLopt** packages for **Julia**. It also outputs the code using the **listings** package and the system true-type font Consolas, which requires the document be compiled w/ X$_{\text{E}}$L^{A}T$_{\text{E}}$X

2 Code

Listing 1: Main Call

```julia
using DataFrames
using Distributions
using NLopt
using Optim
using Dates

# projectDir  = "~/OneDrive/Rice/Class/econ 515 - Labor/NLopt/"
# projectDir  = "C:/mja3/SkyDrive/Rice/Class/econ 515 - Labor/NLopt/"
projectDir  = "C:/Users/magerton/OneDrive/Rice/Class/econ 515 - Labor/NLopt/"
cd(projectDir)

# -------------------------------------------------------------
# ----------------------- Logging     -------------------------
# -------------------------------------------------------------

doLog = false
originalSTDOUT = STDOUT

if doLog == true
  (outRead, outWrite) = redirect_stdout()
  println("----------------------------------------------------------------")
  println("Results for Homework 5\nStarted run at " * string(now()))
  println("----------------------------------------------------------------")
end

# -------------------------------------------------------------
# ----------------------- Probit DGP -----------------------
# -------------------------------------------------------------


N      = 10^5
k      = 4
u      = rand(Normal(),N)
X      = [ones(N) reshape( rand(Uniform(-1,1), N*(k-1)), (N,k-1) )]
βTrue  = [2.0, 3, 5, 10]
y      = X*βTrue + u
d      = 1.0 * (y .>= 0 )


include("Probit-Functions.jl")

# Being declared in main prog makes these global
maxit = 100
count = 0

# -------------------------------------------------------------
# -------------- Optim Optimization -----------------------
# -------------------------------------------------------------
```

```julia
for meth in [:bfgs :l_bfgs :nelder_mead :newton :simulated_annealing]
  println("\n------------- doing method $meth --------------\n\n")
  tic();  global count = 0
  res = Optim.optimize(LL, g!, h!, zeros(k), method=meth, iterations=maxit )
  println("\n$(count) evaluations over $(toq()) seconds \n\n $(res)")
end

res   = Optim.optimize(LL, g!, h!, zeros(k), method=:newton, iterations=maxit )
vcv   = vcov(res, h!)
myse  = se(res, h!)
tstat = coef(res) ./ se(res, h!)
pval  = cdf(Normal(), -abs(tstat))

# ---------------------------------------------------------------
# -------------- NLopt Optimization -----------------------
# ---------------------------------------------------------------

# Note that GlobalocalOpt routines require a local optimzation routine
# Also, global optimizers (usually) require lower and upper bounds
# The sets ending in "Constr" seem to be okay w/ nonlinear constraints

# Global routines
meth_Global        = [ :GN_DIRECT, :GN_DIRECT_L, :GN_DIRECT_L_RAND, :GN_DIRECT_NOSCAL,
                        :GN_DIRECT_L_NOSCAL, :GN_DIRECT_L_RAND_NOSCAL, :GN_ORIG_DIRECT,
                        :GN_ORIG_DIRECT_L, :GN_ISRES, :GN_ESCH, :GN_CRS2_LM  ]
meth_GlobalBroken   = [:GD_STOGO, :GD_STOGO_RAND]
meth_GlobalLocalOpt = [:GN_MLSL_LDS, :GN_MLSL, :G_MLSL, :G_MLSL_LDS, :GD_MLSL_LDS,
                        :GD_MLSL, :AUGLAG, :AUGLAG_EQ]

# Derivative-based routines
methDerivConstr = [:LD_MMA , :LD_SLSQP, :LD_CCSAQ ]
methDeriv       = [:LD_LBFGS, :LD_VAR1, :LD_VAR2, :LD_TNEWTON, :LD_TNEWTON_RESTART,
                    :LD_TNEWTON_PRECOND, :LD_TNEWTON_PRECOND_RESTART ]
methDerivBroken = [:LD_LBFGS_NOCEDAL]

# Non derivative-based routines
methNonDerivBroken = [:LN_NEWUOA]
methNonDeriv       = [:LN_PRAXIS, :LN_NEWUOA_BOUND, :LN_NELDERMEAD, :LN_SBPLX, :LN_BOBYQA]
methNonDerivConstr = [:LN_COBYLA, :LN_AUGLAG, :LD_AUGLAG, :LN_AUGLAG_EQ, :LD_AUGLAG_EQ ]


for meth in [meth_Global, methNonDeriv, methNonDerivConstr, methDeriv, methDerivConstr]

  println("\n------------- doing method $meth --------------\n\n")
  tic();  global count = 0

  opt = NLopt.Opt(meth, k)

  NLopt.lower_bounds!(opt, -100*ones(k))
  NLopt.upper_bounds!(opt,  100*ones(k))
  NLopt.xtol_rel!(opt,1e-12)
  NLopt.maxeval!(opt, maxit)

  # Configure local optimization if needed
  if any( meth .== meth_GlobalLocalOpt )
    opt_local = Opt(:LN_NELDERMEAD, 2)
    local_optimizer!(opt, opt_local)
  end

  NLopt.min_objective!(opt, LL)

  ### this is not used but would be good to figure out
  # inequality_constraint!(opt, (x,g) -> myconstraint(x,g,2,0), 1e-8)
  # inequality_constraint!(opt, (x,g) -> myconstraint(x,g,-1,1), 1e-8)

  (minf,minx,ret) = NLopt.optimize(opt, zeros(k))
  println("\n$(count) evaluations over $(toq()) seconds. Returned $ret")
  println("\t\tvalue = $(round(minf,5)) at $(round(minx,3))")
end

# ---------------------------------------------------------------
# ----------------------- Logging   -------------------------
# ---------------------------------------------------------------

if doLog == true

  println("-------------------------------------------------------------------")
  println("Finished run at " * string(now()))
  println("-------------------------------------------------------------------")
```

```julia
  close(outWrite)
  stringOut = readavailable(outRead)
  close(outRead)
  redirect_stdout(originalSTDOUT)

  f = open("Probit-Results.txt", "w")
  write(f, stringOut )
  close(f)

  println(stringOut)
end
```

Listing 2: Functions

```julia
# -------------- Normal distribution functions prevent 0.0 and 1.0 ----------------------

function normcdf(x::Vector{Float64})
  out = Distributions.cdf(Distributions.Normal(), x)
  out + (out .== 0.0)*eps(1.0) - (out .== 1.0)*eps(1.0)
end


function normpdf(x::Vector{Float64})
  Distributions.pdf(Distributions.Normal(), x)
end

# -------------- Likelihood, gradietn and hessian ----------------------
# see q/lambda trick in Greene chapter on Models for discrete choice!

function λ(β::Vector{Float64})
        q =     2d-1
        q .* normpdf(q .* X*β) ./ normcdf(q.*X*β)
end


function LL(β::Vector{Float64})
        out = - sum( log( normcdf( (2d-1) .* X*β) ) )
        countPlus!(out)
        return(out)
end


function LL(β::Vector{Float64}, grad::Vector{Float64})
        out = - sum( log( normcdf( (2d-1) .* X*β) ) )
        if length(grad) > 0
                grad[:] = - sum( λ(β) .* X,  1 )
        end
        countPlus!(out)
        return(out)
end



function g!(β::Vector{Float64}, grad::Vector{Float64})
  grad[:] = - sum( λ(β) .* X,  1 )
end


function h!(β::Vector{Float64}, hess::Matrix{Float64})
  hh = zeros(size(hess))
  A = λ(β) .* ( λ(β) + X*β )

  for i in 1:size(X)[1]
    hh += A[i] * X[i,:]'*X[i,:]
  end

  hess[:] = hh
end


# -------------- Results ----------------------

function vcov(obj::Optim.OptimizationResults, h!)
        β     = obj.minimum
        k     = length(β)
        hess = zeros((k,k))
        h!(β, hess)
        hess \ eye(k)
end
```

```julia
function se(obj::Optim.OptimizationResults, h!)
        sqrt(diag(vcov(obj, h!)))
end

function coef(obj::Optim.OptimizationResults)
        obj.minimum
end

# -------------- Convenience ----------------------

function printCounter(count)
        if count <= 5
                denom = 1
        elseif count <= 50
                denom = 10
        elseif count <= 200
                denom = 25
        elseif count <= 500
                denom = 50
        elseif count <= 2000
                denom = 100
        else
                denom = 500
        end
        mod(count, denom) == 0
end


function countPlus!()
  global count += 1
  if printCounter(count)
    println("Eval $(count)")
  end
end


function countPlus!(out::Float64)
  global count += 1
  if printCounter(count)
    println("Eval $(count): value = $(round(out,5))")
  end
    return count
end
```

```
-------------------------------------------------------------------
Results for Homework 5
Started run at 2015-02-20T17:54:54
-------------------------------------------------------------------

------------- doing method bfgs ---------------


Eval 1: value = 69314.71806
Eval 2: value = 522499.94328
Eval 3: value = 522499.36799
Eval 4: value = 604556.58837
Eval 5: value = 605038.81929
Eval 10: value = 1.24310289294e6
Eval 20: value = 53054.23414
Eval 30: value = 12063.4039
Eval 40: value = 9433.57492
Eval 50: value = 9297.41048

63 evaluations over 2.194429952 seconds

 Results of Optimization Algorithm
 * Algorithm: BFGS
 * Starting Point: [0.0,0.0,0.0,0.0]
 * Minimum: [1.9941319641020363,2.9734023667622855,4.966838258399663,9.948946629220615]
 * Value of Function at Minimum: 9005.585251
 * Iterations: 11
 * Convergence: true
   * |x - x'| < 1.0e-32: false
   * |f(x) - f(x')| / |f(x)| < 1.0e-08: true
   * |g(x)| < 1.0e-08: false
   * Exceeded Maximum Number of Iterations: false
 * Objective Function Calls: 63
 * Gradient Call: 63

------------- doing method l_bfgs ---------------


Eval 1: value = 69314.71806
Eval 2: value = 522499.94328
Eval 3: value = 522499.36799
Eval 4: value = 604556.58837
Eval 5: value = 605038.81929
Eval 10: value = 1.24310289294e6
Eval 20: value = 53054.23414
Eval 30: value = 12063.4039
Eval 40: value = 9433.57492
Eval 50: value = 9297.41048

74 evaluations over 2.56712781 seconds

 Results of Optimization Algorithm
 * Algorithm: L-BFGS
 * Starting Point: [0.0,0.0,0.0,0.0]
 * Minimum: [1.9941318188612838,2.9734022058745952,4.966836664222613,9.94894551480561]
 * Value of Function at Minimum: 9005.585251
 * Iterations: 14
 * Convergence: true
   * |x - x'| < 1.0e-32: false
   * |f(x) - f(x')| / |f(x)| < 1.0e-08: false
   * |g(x)| < 1.0e-08: true
   * Exceeded Maximum Number of Iterations: false
 * Objective Function Calls: 74
 * Gradient Call: 74

------------- doing method nelder_mead ---------------


Eval 1: value = 83812.57347
Eval 2: value = 71567.64066
Eval 3: value = 66245.48813
Eval 4: value = 45622.50667
Eval 5: value = 69314.71806
Eval 10: value = 42582.02377
Eval 20: value = 21968.70994
Eval 30: value = 10620.66202
Eval 40: value = 9684.70017
Eval 50: value = 9579.63565
Eval 75: value = 9055.45141
```

```
Eval 100: value = 9005.73391
Eval 125: value = 9005.58942
Eval 150: value = 9005.58547
Eval 175: value = 9005.58526

182 evaluations over 2.580744347 seconds

 Results of Optimization Algorithm
 * Algorithm: Nelder-Mead
 * Starting Point: [0.0,0.0,0.0,0.0]
 * Minimum: [1.994091985908281,2.9733706842878846,4.966727349214394,9.948782352480595]
 * Value of Function at Minimum: 9005.585254
 * Iterations: 100
 * Convergence: false
   * |x - x'| < NaN: false
   * |f(x) - f(x')| / |f(x)| < 1.0e-08: false
   * |g(x)| < NaN: false
   * Exceeded Maximum Number of Iterations: true
 * Objective Function Calls: 181
 * Gradient Call: 0

------------- doing method newton ---------------


Eval 1: value = 69314.71806
Eval 2: value = 28730.92381
Eval 3: value = 11417.86096
Eval 4: value = 51328.45191
Eval 5: value = 11419.5006
Eval 10: value = 9006.15142

18 evaluations over 1.798683836 seconds

 Results of Optimization Algorithm
 * Algorithm: Newton's Method
 * Starting Point: [0.0,0.0,0.0,0.0]
 * Minimum: [1.9941318187095596,2.973402205634398,4.966836663816231,9.948945514260659]
 * Value of Function at Minimum: 9005.585251
 * Iterations: 5
 * Convergence: true
   * |x - x'| < 1.0e-32: false
   * |f(x) - f(x')| / |f(x)| < 1.0e-08: true
   * |g(x)| < 1.0e-08: false
   * Exceeded Maximum Number of Iterations: false
 * Objective Function Calls: 18
 * Gradient Call: 18

------------- doing method simulated_annealing ---------------


Eval 1: value = 69314.71806
Eval 2: value = 123188.93136
Eval 3: value = 275696.24803
Eval 4: value = 148564.55174
Eval 5: value = 100940.93763
Eval 10: value = 73937.55541
Eval 20: value = 79985.70438
Eval 30: value = 29580.24635
Eval 40: value = 21475.82243
Eval 50: value = 22716.73745
Eval 75: value = 14961.12582
Eval 100: value = 15146.27875

101 evaluations over 1.459202674 seconds

 Results of Optimization Algorithm
 * Algorithm: Simulated Annealing
 * Starting Point: [0.0,0.0,0.0,0.0]
 * Minimum: [0.8223543620693234,1.160224982618416,2.483347488993278,5.324613469479287]
 * Value of Function at Minimum: 12121.635263
 * Iterations: 100
 * Convergence: false
   * |x - x'| < NaN: false
   * |f(x) - f(x')| / |f(x)| < NaN: false
   * |g(x)| < NaN: false
   * Exceeded Maximum Number of Iterations: true
 * Objective Function Calls: 101
 * Gradient Call: 0

------------- doing method GN_DIRECT ---------------
```

```
Eval 1: value = 69314.71806
Eval 2: value = 2.16680026714e6
Eval 3: value = 1.43756507177e6
Eval 4: value = 9.07917607571e6
Eval 5: value = 6.87730707388e6
Eval 10: value = 1.508524611779e7
Eval 20: value = 1.82594668366e6
Eval 30: value = 3.06148513208e6
Eval 40: value = 225937.84331
Eval 50: value = 203967.52174
Eval 75: value = 34186.0066
Eval 100: value = 1.43756507177e6

100 evaluations over 1.357789567 seconds. Returned MAXEVAL_REACHED
                value = 27727.59006 at [0.0,0.0,1.646,2.469]

------------- doing method GN_DIRECT_L ---------------


Eval 1: value = 69314.71806
Eval 2: value = 2.16680026714e6
Eval 3: value = 1.43756507177e6
Eval 4: value = 9.07917607571e6
Eval 5: value = 6.87730707388e6
Eval 10: value = 1.508524611779e7
Eval 20: value = 3.06148513208e6
Eval 30: value = 1.06087528667e6
Eval 40: value = 169531.57744
Eval 50: value = 8.87354318267e6
Eval 75: value = 948653.61225
Eval 100: value = 7.8059045317e6

100 evaluations over 1.389880935 seconds. Returned MAXEVAL_REACHED
                value = 27727.59006 at [0.0,0.0,1.646,2.469]

------------- doing method GN_DIRECT_L_RAND ---------------


Eval 1: value = 69314.71806
Eval 2: value = 2.16680026714e6
Eval 3: value = 1.43756507177e6
Eval 4: value = 9.07917607571e6
Eval 5: value = 6.87730707388e6
Eval 10: value = 5.22872737518e6
Eval 20: value = 3.06148513208e6
Eval 30: value = 690051.88976
Eval 40: value = 5.64204297879e6
Eval 50: value = 300548.0422
Eval 75: value = 18630.22385
Eval 100: value = 12597.36265

100 evaluations over 1.374154098 seconds. Returned MAXEVAL_REACHED
                value = 12597.36265 at [1.646,2.469,2.469,6.584]

------------- doing method GN_DIRECT_NOSCAL ---------------


Eval 1: value = 69314.71806
Eval 2: value = 2.16680026714e6
Eval 3: value = 1.43756507177e6
Eval 4: value = 9.07917607571e6
Eval 5: value = 6.87730707388e6
Eval 10: value = 1.508524611779e7
Eval 20: value = 3.06148513208e6
Eval 30: value = 1.06087528667e6
Eval 40: value = 169531.57744
Eval 50: value = 8.87354318267e6
Eval 75: value = 948653.61225
Eval 100: value = 7.8059045317e6

100 evaluations over 1.388710395 seconds. Returned MAXEVAL_REACHED
                value = 27727.59006 at [0.0,0.0,1.646,2.469]

------------- doing method GN_DIRECT_L_NOSCAL ---------------


Eval 1: value = 69314.71806
Eval 2: value = 2.16680026714e6
Eval 3: value = 1.43756507177e6
Eval 4: value = 9.07917607571e6
```

```
Eval 5: value = 6.87730707388e6
Eval 10: value = 1.508524611779e7
Eval 20: value = 3.06148513208e6
Eval 30: value = 1.06087528667e6
Eval 40: value = 169531.57744
Eval 50: value = 8.87354318267e6
Eval 75: value = 948653.61225
Eval 100: value = 7.8059045317e6


100 evaluations over 1.368685293 seconds. Returned MAXEVAL_REACHED
                value = 27727.59006 at [0.0,0.0,1.646,2.469]

------------- doing method GN_DIRECT_L_RAND_NOSCAL ---------------


Eval 1: value = 69314.71806
Eval 2: value = 2.16680026714e6
Eval 3: value = 1.43756507177e6
Eval 4: value = 9.07917607571e6
Eval 5: value = 6.87730707388e6
Eval 10: value = 1.508524611779e7
Eval 20: value = 3.06148513208e6
Eval 30: value = 1.06087528667e6
Eval 40: value = 169531.57744
Eval 50: value = 8.87354318267e6
Eval 75: value = 948653.61225
Eval 100: value = 7.8059045317e6


100 evaluations over 1.3908795 seconds. Returned MAXEVAL_REACHED
                value = 27727.59006 at [0.0,0.0,1.646,2.469]

------------- doing method GN_ORIG_DIRECT ---------------


Eval 1: value = 69314.71806
Eval 2: value = 1.43756507177e6
Eval 3: value = 2.16680026714e6
Eval 4: value = 6.87730707388e6
Eval 5: value = 9.07917607571e6
Eval 10: value = 4.37951996629e6
Eval 20: value = 2.70728728215e6
Eval 30: value = 4.80721762063e6
Eval 40: value = 1.21142553615e6
Eval 50: value = 2.39527430227e6
Eval 75: value = 4.67868281134e6
Eval 100: value = 387015.09297


105 evaluations over 1.430492278 seconds. Returned MAXEVAL_REACHED
                value = 34186.0066 at [0.0,0.0,2.469,2.469]

------------- doing method GN_ORIG_DIRECT_L ---------------


Eval 1: value = 69314.71806
Eval 2: value = 1.43756507177e6
Eval 3: value = 2.16680026714e6
Eval 4: value = 6.87730707388e6
Eval 5: value = 9.07917607571e6
Eval 10: value = 4.37951996629e6
Eval 20: value = 2.70728728215e6
Eval 30: value = 445080.19773
Eval 40: value = 8.25335989789e6
Eval 50: value = 153150.8917
Eval 75: value = 203967.52174
Eval 100: value = 28273.74818


101 evaluations over 1.401331697 seconds. Returned MAXEVAL_REACHED
                value = 27727.59006 at [0.0,0.0,1.646,2.469]

------------- doing method GN_ISRES ---------------


Eval 1: value = 69314.71806
Eval 2: value = 6.33309622016e6
Eval 3: value = 7.97319507019e6
Eval 4: value = 5.39487770026e6
Eval 5: value = 1.53877230653e6
Eval 10: value = 6.01474930398e6
Eval 20: value = 1.6064061933e6
Eval 30: value = 5.70825521586e6
Eval 40: value = 8.41366791293e6
```

```
Eval 50: value = 7.71963075999e6
Eval 75: value = 6.47055723949e6
Eval 100: value = 5.85907090221e6

100 evaluations over 1.328751864 seconds. Returned MAXEVAL_REACHED
                value = 69314.71806 at [0.0,0.0,0.0,0.0]

------------- doing method GN_ESCH ---------------


Eval 1: value = 69314.71806
Eval 2: value = 6.66623627726e6
Eval 3: value = 1.257011465698e7
Eval 4: value = 6.30274696195e6
Eval 5: value = 5.50749985057e6
Eval 10: value = 5.02129834807e6
Eval 20: value = 5.12038714806e6
Eval 30: value = 5.93691208342e6
Eval 40: value = 4.03066558347e6
Eval 50: value = 7.68722851681e6
Eval 75: value = 5.1595252253e6
Eval 100: value = 4.86038237138e6

100 evaluations over 1.318100508 seconds. Returned MAXEVAL_REACHED
                value = 69314.71806 at [0.0,0.0,0.0,0.0]

------------- doing method GN_CRS2_LM ---------------


Eval 1: value = 69314.71806
Eval 2: value = 3.10618712483e6
Eval 3: value = 9.55058869539e6
Eval 4: value = 5.52424866117e6
Eval 5: value = 6.26545311827e6
Eval 10: value = 4.42474104635e6
Eval 20: value = 8.81559134224e6
Eval 30: value = 7.5760368093e6
Eval 40: value = 6.43832739236e6
Eval 50: value = 3.61918050853e6
Eval 75: value = 3.73275295143e6
Eval 100: value = 5.65052483568e6

100 evaluations over 1.336540113 seconds. Returned MAXEVAL_REACHED
                value = 69314.71806 at [0.0,0.0,0.0,0.0]

------------- doing method LN_PRAXIS ---------------


Eval 1: value = 69314.71806
Eval 2: value = 69314.71806
Eval 3: value = 69314.71806
Eval 4: value = 2.16680026714e6
Eval 5: value = 1.903509465181e7
Eval 10: value = 6.57530526999e6
Eval 20: value = 1.018705196886e7
Eval 30: value = 34089.64305
Eval 40: value = 14203.41003
Eval 50: value = 11462.75753
Eval 75: value = 9009.52763
Eval 100: value = 9005.58629

103 evaluations over 1.442396278 seconds. Returned MAXEVAL_REACHED
                value = 9005.58627 at [1.993,2.972,4.965,9.945]

------------- doing method LN_NEWUOA_BOUND ---------------


Eval 1: value = 69314.71806
Eval 2: value = 1.43756507177e6
Eval 3: value = 8.26079361645e6
Eval 4: value = 7.12815094667e6
Eval 5: value = 1.87995177839e6
Eval 10: value = 29060.54578
Eval 20: value = 14422.04996
Eval 30: value = 10904.48767
Eval 40: value = 9213.04678
Eval 50: value = 9025.93952
Eval 75: value = 9005.58642
Eval 100: value = 9005.58525

100 evaluations over 1.410054436 seconds. Returned MAXEVAL_REACHED
```

```
                  value = 9005.58525 at [1.994,2.973,4.967,9.949]

------------- doing method LN_NELDERMEAD ---------------


Eval 1: value = 69314.71806
Eval 2: value = 1.43756507177e6
Eval 3: value = 8.26079361645e6
Eval 4: value = 7.12815094667e6
Eval 5: value = 1.87995177839e6
Eval 10: value = 5.26532434548e6
Eval 20: value = 756417.61043
Eval 30: value = 874299.16741
Eval 40: value = 23636.69577
Eval 50: value = 11373.09042
Eval 75: value = 9136.83738
Eval 100: value = 9010.13958

100 evaluations over 1.414881108 seconds. Returned MAXEVAL_REACHED
                  value = 9009.99922 at [1.99,3.007,4.999,10.06]

------------- doing method LN_SBPLX ---------------


Eval 1: value = 69314.71806
Eval 2: value = 1.43756507177e6
Eval 3: value = 8.26079361645e6
Eval 4: value = 4.20139462005e6
Eval 5: value = 7.38053337393e6
Eval 10: value = 3.11192080809e6
Eval 20: value = 4.72966150811e6
Eval 30: value = 840595.17021
Eval 40: value = 57086.37429
Eval 50: value = 25639.16066
Eval 75: value = 17272.32354
Eval 100: value = 13144.15564

100 evaluations over 1.428803872 seconds. Returned MAXEVAL_REACHED
                  value = 13135.65238 at [0.995,0.78,2.223,4.983]

------------- doing method LN_BOBYQA ---------------


Eval 1: value = 69314.71806
Eval 2: value = 1.43756507177e6
Eval 3: value = 8.26079361645e6
Eval 4: value = 7.12815094667e6
Eval 5: value = 1.87995177839e6
Eval 10: value = 28843.75889
Eval 20: value = 18986.33861
Eval 30: value = 9837.7847
Eval 40: value = 9803.33434
Eval 50: value = 9449.95731
Eval 75: value = 9010.89399
Eval 100: value = 9005.58797

100 evaluations over 1.375777321 seconds. Returned MAXEVAL_REACHED
                  value = 9005.58609 at [1.994,2.973,4.967,9.948]

------------- doing method LN_COBYLA ---------------


Eval 1: value = 69314.71806
Eval 2: value = 1.43756507177e6
Eval 3: value = 8.26079361645e6
Eval 4: value = 7.12815094667e6
Eval 5: value = 1.87995177839e6
Eval 10: value = 2.08716024819e6
Eval 20: value = 54394.37105
Eval 30: value = 18254.1135
Eval 40: value = 14369.86927
Eval 50: value = 12676.77918
Eval 75: value = 10805.11852
Eval 100: value = 10090.1181

100 evaluations over 1.439452731 seconds. Returned MAXEVAL_REACHED
                  value = 10082.14579 at [1.198,1.849,3.193,6.419]

------------- doing method LN_AUGLAG ---------------
```

```
Eval 1: value = 69314.71806
Eval 2: value = 1.43756507177e6
Eval 3: value = 8.26079361645e6
Eval 4: value = 7.12815094667e6
Eval 5: value = 1.87995177839e6
Eval 10: value = 2.08716024819e6
Eval 20: value = 54394.37105
Eval 30: value = 18254.1135
Eval 40: value = 14369.86927
Eval 50: value = 12676.77918
Eval 75: value = 10805.11852
Eval 100: value = 10090.1181

101 evaluations over 1.449915192 seconds. Returned MAXEVAL_REACHED
                value = 10082.14579 at [1.198,1.849,3.193,6.419]

------------- doing method LD_AUGLAG ---------------


Eval 1: value = 69314.71806
Eval 2: value = 2.98134358826e6
Eval 3: value = 2.98134358826e6
Eval 4: value = 2.98134358826e6
Eval 5: value = 2.98134358826e6
Eval 10: value = 54164.92581
Eval 20: value = 9034.92166
Eval 30: value = 9022.89868
Eval 40: value = 179051.54685
Eval 50: value = 9007.38505
Eval 75: value = 9005.77784
Eval 100: value = 9005.62595

101 evaluations over 3.716285617 seconds. Returned MAXEVAL_REACHED
                value = 9005.6187 at [1.989,2.966,4.954,9.925]

------------- doing method LN_AUGLAG_EQ ---------------


Eval 1: value = 69314.71806
Eval 2: value = 1.43756507177e6
Eval 3: value = 8.26079361645e6
Eval 4: value = 7.12815094667e6
Eval 5: value = 1.87995177839e6
Eval 10: value = 2.08716024819e6
Eval 20: value = 54394.37105
Eval 30: value = 18254.1135
Eval 40: value = 14369.86927
Eval 50: value = 12676.77918
Eval 75: value = 10805.11852
Eval 100: value = 10090.1181

101 evaluations over 1.468672394 seconds. Returned MAXEVAL_REACHED
                value = 10082.14579 at [1.198,1.849,3.193,6.419]

------------- doing method LD_AUGLAG_EQ ---------------


Eval 1: value = 69314.71806
Eval 2: value = 2.98134358826e6
Eval 3: value = 2.98134358826e6
Eval 4: value = 2.98134358826e6
Eval 5: value = 2.98134358826e6
Eval 10: value = 54164.92581
Eval 20: value = 9034.92166
Eval 30: value = 9022.89868
Eval 40: value = 179051.54685
Eval 50: value = 9007.38505
Eval 75: value = 9005.77784
Eval 100: value = 9005.62595

101 evaluations over 3.675072361 seconds. Returned MAXEVAL_REACHED
                value = 9005.6187 at [1.989,2.966,4.954,9.925]

------------- doing method LD_LBFGS ---------------


Eval 1: value = 69314.71806
Eval 2: value = 2.29975991034e6
Eval 3: value = 197612.8752
Eval 4: value = 39122.13348
Eval 5: value = 16532.28355
```

```
Eval 10: value = 9038.56623

19 evaluations over 0.677324401 seconds. Returned FTOL_REACHED
                value = 9005.58525 at [1.994,2.973,4.967,9.949]

------------- doing method LD_VAR1 ---------------


Eval 1: value = 69314.71806
Eval 2: value = 2.29975991034e6
Eval 3: value = 197612.8752
Eval 4: value = 39122.13348
Eval 5: value = 16532.28355
Eval 10: value = 9070.38039
Eval 20: value = 9005.58526

29 evaluations over 1.076960758 seconds. Returned FTOL_REACHED
                value = 9005.58525 at [1.994,2.973,4.967,9.949]

------------- doing method LD_VAR2 ---------------


Eval 1: value = 69314.71806
Eval 2: value = 2.29975991034e6
Eval 3: value = 197612.8752
Eval 4: value = 39122.13348
Eval 5: value = 16532.28355
Eval 10: value = 9070.38039
Eval 20: value = 9005.58526

29 evaluations over 1.073800024 seconds. Returned FTOL_REACHED
                value = 9005.58525 at [1.994,2.973,4.967,9.949]

------------- doing method LD_TNEWTON ---------------


Eval 1: value = 69314.71806
Eval 2: value = 69314.71679
Eval 3: value = 35254.90899
Eval 4: value = 35254.90836
Eval 5: value = 35254.90864
Eval 10: value = 13970.53419
Eval 20: value = 9420.79002
Eval 30: value = 9005.86593
Eval 40: value = 9005.58525

41 evaluations over 1.497510662 seconds. Returned SUCCESS
                value = 9005.58525 at [1.994,2.973,4.967,9.949]

------------- doing method LD_TNEWTON_RESTART ---------------


Eval 1: value = 69314.71806
Eval 2: value = 2.29975991034e6
Eval 3: value = 197612.8752
Eval 4: value = 39122.13348
Eval 5: value = 39122.13254
Eval 10: value = 9788.93891
Eval 20: value = 9005.77523
Eval 30: value = 9005.58525

35 evaluations over 1.32096833 seconds. Returned SUCCESS
                value = 9005.58525 at [1.994,2.973,4.967,9.949]

------------- doing method LD_TNEWTON_PRECOND ---------------


Eval 1: value = 69314.71806
Eval 2: value = 69314.71679
Eval 3: value = 35254.90899
Eval 4: value = 35254.90838
Eval 5: value = 35254.90874
Eval 10: value = 13934.91277
Eval 20: value = 9037.61532
Eval 30: value = 9005.58528
Eval 40: value = 9005.58525

40 evaluations over 1.44803151 seconds. Returned SUCCESS
                value = 9005.58525 at [1.994,2.973,4.967,9.949]

------------- doing method LD_TNEWTON_PRECOND_RESTART ---------------
```

```
Eval 1: value = 69314.71806
Eval 2: value = 2.29975991034e6
Eval 3: value = 197612.8752
Eval 4: value = 39122.13348
Eval 5: value = 39122.13268
Eval 10: value = 9503.91542
Eval 20: value = 9005.58701

28 evaluations over 1.028676833 seconds. Returned SUCCESS
                value = 9005.58525 at [1.994,2.973,4.967,9.949]

------------- doing method LD_MMA ---------------


Eval 1: value = 69314.71806
Eval 2: value = 2.98134358826e6
Eval 3: value = 2.98134358826e6
Eval 4: value = 2.98134358826e6
Eval 5: value = 2.98134358826e6
Eval 10: value = 54164.92581
Eval 20: value = 9034.92166
Eval 30: value = 9022.89868
Eval 40: value = 179051.54685
Eval 50: value = 9007.38505
Eval 75: value = 9005.77784
Eval 100: value = 9005.62595

100 evaluations over 3.715190525 seconds. Returned MAXEVAL_REACHED
                value = 9005.6187 at [1.989,2.966,4.954,9.925]

------------- doing method LD_SLSQP ---------------


Eval 1: value = 69314.71806
Eval 2: value = 2.78517499227e6
Eval 3: value = 4.29831151909e6
Eval 4: value = 334437.10118
Eval 5: value = 69000.01016
Eval 10: value = 90846.84306
Eval 20: value = 21156.73618
Eval 30: value = 9007.79238
Eval 40: value = 9005.58525
Eval 50: value = 9005.58525

52 evaluations over 1.410140701 seconds. Returned XTOL_REACHED
                value = 9005.58525 at [1.994,2.973,4.967,9.949]

------------- doing method LD_CCSAQ ---------------


Eval 1: value = 69314.71806
Eval 2: value = 2.78497464593e6
Eval 3: value = 2.78497464593e6
Eval 4: value = 2.78497464593e6
Eval 5: value = 2.78497464593e6
Eval 10: value = 137073.07198
Eval 20: value = 9136.97316
Eval 30: value = 11282.31738
Eval 40: value = 28894.90182
Eval 50: value = 9019.90213
Eval 75: value = 9010.94843
Eval 100: value = 9006.33905

100 evaluations over 3.742855201 seconds. Returned MAXEVAL_REACHED
                value = 9006.33905 at [1.971,2.938,4.907,9.842]
--------------------------------------------------------------------
Finished run at 2015-02-20T17:55:58
--------------------------------------------------------------------
```