Università degli Studi di Milano Corso di studi in Informatica/Matematica

Heuristic algorithms
(Prof. Roberto Cordone)
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Available time: Available time: 2 hours and 30 minutes

Note: the answers can be given in Italian or English at will; to avoid penalisations, clarify all assumptions and motivate all computational steps.

Exercise 1 - Given a directed graph $G = (N, A)$, a length function $c : A \to \mathbb{N}$ on the arcs of the graph and two nodes $s, t \in N$, the *longest path problem* requires to find a simple path (including each node at most once) of maximum total length from node s to node t.

Explain why it is a Combinatorial Optimization problem, and propose a possible ground set.

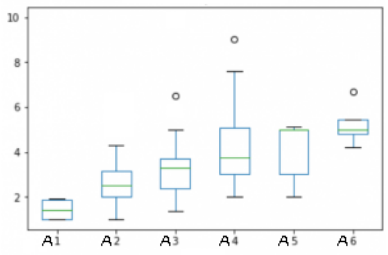
Suggest a procedure to compute the value of the objective for a given solution x and discuss its computational complexity. Is the objective function additive?

Given a subset x of the ground set, suggest a procedure to evaluate whether x is a feasible solution and discuss its computational complexity.

Do feasible solutions always exist? If they do, propose an easy way to compute one.

Exercise 2 - Define the concept of *parameterized complexity* and describe how it differs from the classical worst-case asymptotic complexity of an algorithm.

The following figure represents the performance (percent relative difference δ) of six algorithms on a set of benchmark instances:



What is the meaning of the circles appearing in some of the plots?

What is the meaning of the rectangular boxes and of the extremal horizontal lines (also known as *whiskers*)?

What can be deduced on the relative quality of the six algorithms?

Exercise 3 - Describe the general scheme of the *Ant System*.

Given the following instance of the *Bin Packing problem* (*BPP*):

Items	a	b	c	d	e	f
Volumes	3	18	15	45	51	6

Bin capacity
60

apply the First-Fit and the Decreasing First-Fit algorithms to solve the problem.

Apply the following randomised version of the Decreasing First-Fit algorithm to the instance above reported: the container remains the first fit, but the object is chosen at random in a *restricted candidate list RCL* of two elements. Assume that the pseudorandom number generator provide the following sequence: 0.2, 0.8, 0.6, 0.4, 0.6, 0.3, ... and that lower values correspond to the best candidate, higher values to the second best.

Exercise 4 - Given the following instance of the *Parallel Machine Scheduling Problem* (*PMSP*) with 3 machines:

Task	a	b	c	d	e
d	8	13	7	12	6

and a current solution x that assigns tasks a and b to the first machine, tasks c and d to the second machine and task e to the third one, how many solutions are contained in neighbourhood $N_{\mathcal{T}_1}$ (that is, the transfer of a task to a different machine)? Does this number depend on solution x?

Starting from that solution, consider the following *Tabu Search* algorithm: neighbourhood $N_{\mathcal{T}_1}$ is visited with the global-best strategy (in case of ties, select the task with the smallest index; in case of further ties, select the machine with the smallest index); set the tenure to $L = 2$ and define the attribute as the index of the moved task. Apply two iterations of the method, showing the moves performed and the update of the data structure to manage the tabu.

Describe the typical disadvantages of the basic *Tabu Search* (forbidding solutions) with respect to the attribute-based version.

Exercise 5 - Describe the *tournament selection* mechanism, explaining the role of the parameter that sets the tournament size.

Given the following instance of the *Knapsack Problem* with capacity $V = 5$:

Objects	a	b	c	d	e	f	g	h	i
Prize ϕ	11	6	4	2	8	5	12	7	10
Volume v	1	1	1	1	1	1	1	1	1

apply a deterministic recombination operator (such as the ones used in Scatter Search) to solutions $x = \{a, b, c, d, e\}$ and $x' = \{b, e, g, h, i\}$ using the basic greedy algorithm to complete the recombination.

What are the main differences between this operator and the classical *crossover* operator used in genetic algorithms?

Propose an encoding for the solutions of the *Knapsack Problem* and briefly discuss its advantages and disadvantages.