Università degli Studi di Milano Corso di studi in Informatica/Matematica

Heuristic algorithms

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Available time: 2 hours and 30 minutes

Note: the answers can be given in Italian or English at will; to avoid penalisations, clarify all assumptions and motivate all computational steps.

Exercise 1 - Given an undirected graph $G = (V, E)$, the *graph colouring problem* consists in assigning to each vertex of V a *colour*, so that adjacent vertices have different colours and the number of used colours is minimum.

Explain why it is a Combinatorial Optimization problem, and propose a possible ground set.

Suggest a procedure to compute the value of the objective for a given solution x and discuss its computational complexity. Is the objective function additive?

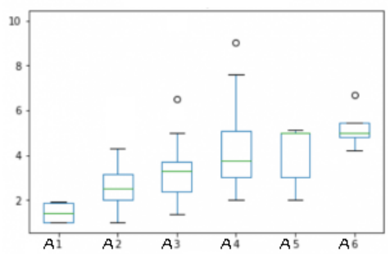
Given a subset x of the ground set, suggest a procedure to evaluate whether x is a feasible solution and discuss its computational complexity.

Does the problem always have a feasible solution? If it does, propose an easy way to compute one.

Exercise 2 - Define the concept of *parameterized complexity* and describe in what it differs from the classical worst-case asymptotic complexity of an algorithm.

Define the concepts of *absolute approximation* and *relative approximation* for a minimization problem.

The following figure represents the performance (percent relative difference δ) of six algorithms on a set of benchmark instances:



What can be deduced on the relative quality of the six algorithms?

Is it possible to draw (at least approximately) the *SQD* diagrams of the six algorithms? If it is, draw one of the diagrams.

Exercise 3 - Describe the concept of *construction graph*, discussing in particular its nodes and arcs, and the information associated to them in constructive heuristics and metaheuristics.

A constructive algorithm for the *Capacitated Minimum Spanning Tree Problem* (*CMSTP*) can be based on the same idea of Kruskal's algorithm for the *MST* problem: the selection criterium is the objective function, the ground set is the edge set E and the search space $\mathcal{F}$ includes all acyclic subsets of edges that (removing the root) form subtrees of weight $\leq W$. Discuss whether the set system $(E, \mathcal{F})$ satisfies the *trivial* axiom and the *hereditarity* axiom.

Apply this algorithm to the instance with root in vertex a, weight function $w_v = 1$ for all $v \in V \setminus \{a\}$ and $w_a = 0$, capacity $W = 2$ and cost function:

Cost	a	b	c	d	e	f
a	0	10	5	4	9	6
b	10	0	9	8	11	7
c	5	9	0	12	15	3
d	4	8	12	0	2	13
e	9	11	15	2	0	7
f	6	7	3	13	7	0

Apply a randomized version of this algorithm in which the edges are selected from a *restricted candidate list* RCL of two elements, the pseudorandom number generator provides the following sequence: 0.7, 0.4, 0.8, 0.2, 0.6, 0.1, ... and lower values correspond to the best candidate, while higher values to the second best.

Exercise 4 - Briefly explain the concept of *very large neighbourhood search.*

Describe the general scheme of the *Variable Neighbourhood Descent.*

Given the following instance of the *Parallel Machine Scheduling Problem* (*PMSP*) with 3 machines:

Task	a	b	c	d	e
d	9	3	4	5	10

and a current solution x that assigns tasks a and b to the first machine, tasks c and d to the second machine and task e to the third one, how many solutions are contained in neighbourhood $N_{\mathcal{T}_1}$ (that is, the transfer of a task to a different machine)? Does this number depend on solution x?

Assuming that x is locally optimal, generate a new starting solution with a *shaking* procedure based on neighbourhood $N_{\mathcal{T}_k}$, setting $k = 3$. While doing that, sort the moves first by increasing task index, then by increasing machine index; assume that the pseudorandom number generator provides the following sequence: 0.1, 0.3, 0.9, 0.4, 0.5, ...; to simplify the computations, do not remove from the neighbourhood the moves already selected: if a move is selected several times, just perform it once.

Exercise 5 - Describe the main variants of the *crossover* operator in genetic algorithms.

Propose an *encoding* for the solutions of the *PMSP*, discussing its possible advantages and disadvantages. Apply the encoding to the solution x described in Exercise 4.

Discuss the role of the two subsets B and D that compose the *reference set* of *Scatter Search* algorithms.

Apply the recombination step of *Scatter Search* to solutions $x = \{\{a, b\}, \{c, d\}, \{e\}\}$ and $x' = \{\{b, e\}, \{a, c\}, \{d\}\}$ to generate a new solution, choosing alternatively from x and from x' in a greedy way.