Università degli Studi di Milano — Corso di studi in Informatica/Matematica

Heuristic algorithms

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Available time: Available time: 2 hours and 30 minutes

Note: the answers can be given in Italian or English at will; to avoid penalisations, clarify all assumptions and motivate all computational steps.

Exercise 1 - Given an undirected graph $G = (V, E)$ and a cost function $c : E \to \mathbb{N}$ defined on its edges, the *Maximum Perfect Matching Problem* requires to find a collection of edges of maximum total cost, such that no two edges are adjacent to each other and all vertices belong to one of the collected edges.

Explain why it is a Combinatorial Optimization problem, and propose a possible ground set.

Suggest a procedure to compute the value of the objective for a given solution x and discuss its computational complexity. Is the objective function additive?

Given a subset x of the ground set, suggest a procedure to evaluate whether x is a feasible solution and discuss its computational complexity.

Do feasible solutions always exist? If they do, propose an easy way to compute one.

Exercise 2 - Define the concept of average-case asymptotic complexity of an algorithm.

Define the concepts of *absolute approximation* and *relative approximation* for a minimization problem.

Describe the concept of *Solution Quality Distribution* diagram (what it is, not how to build it) and how it is used to evaluate a heuristic algorithm.

The following table reports the result obtained by algorithms A_1 and A_2 on 8 different instances.

	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8
f_{A_1}	107	103	111	104	120	104	100	111
f_{A_2}	102	103	110	107	122	100	100	101

Assume (for the sake of simplicity) that all instances have optimum equal to 100.

Apply the first steps of Wilcoxon's test to the benchmark, up to the computation of statistics W^+ and W^-, that is skipping only the computation of the final p-value.

Exercise 3 - Describe the general scheme of the *Adaptive Research Technique*.

Given the following instance of the *Travelling Salesman Problem* (*TSP*)

Cost	a	b	c	d	e
a	0	10	6	3	7
b	8	0	7	4	6
c	7	5	0	8	7
d	4	2	10	0	2
e	7	8	5	1	0

apply the *Nearest Neighbour* constructive heuristic starting from node b.

Does it provide always the optimal solution? Why, or why not?
(You do not need to prove it, just to state the reason)

Given the above reported instance of the *TSP*, apply a randomized version of the *Nearest Neighbour* heuristic starting from node b, with a restricted candidate list of 2 elements. Assume that the pseudorandom number generator provide the following sequence: 0.6, 0.7, 0.2, 0.1, 0.4, 0.8, ... and that lower values correspond to the best candidate, while higher values to the second best (break ties between nodes using the index order).

Exercise 4 - Describe the general scheme of the *Iterated Local Search.*

Given the following instance of the *Knapsack Problem* with capacity $V = 10$:

Objects	a	b	c	d	e	f	g
Prize ϕ	10	3	5	12	7	6	8
Volume v	5	1	2	4	3	1	6

and the starting solution $x^{(0)} = (b, e, g)$, how many solutions are contained in neighbourhood $N_{\mathcal{H}_1}$ (that is, Hamming distance not larger than 1) and how many in neighbourhood $N_{\mathcal{S}_1}$ (that is, swap an element in the solution with one out of it)?

Consider the neighbourhood $N_{\mathcal{S}_1}$ and perform a single step of *steepest descent* with the *global best strategy.*

Now consider neighbourhood $N_{\mathcal{H}_1}$ and assume that the current iteration index is $t = 16$ and that the current solution is $x^{(5)} = (b, e, g)$. List the tabu and the nontabu neighbour solutions if the *tenure* is $L = 2$ both for adding and dropping elements and the tabu list is represented by vector:

Objects	a	b	c	d	e	f	g
T_o	13	10	14	7	5	15	9

Exercise 5 - Describe the *roulette-wheel selection* mechanism for genetic algorithms, and its potential disadvantages.

Describe the *crossover* mechanism of the genetic algorithm, and its main variants.

Given the following instance of the *Parallel Machine Scheduling Problem* (*PMSP*):

Tasks	a	b	c	d	e	f	g
Durations	9	7	8	13	6	14	11

propose an encoding for a genetic algorithm and a mutation procedure to generate modified encodings (avoid binary encodings, that are likely to correspond to unfeasible solutions). Provide an example

Apply the *Scatter Search* random recombination mechanism to solutions $x = \{\{a, b\}, \{c, d, e\}, \{f, g\}\}$ and $x' = \{\{a, d, g\}, \{b, f\}, \{c, e\}\}$ to generate a new solution with the following pseudorandom number sequence: 0.3, 0.2, 0.7, 0.9, 0.4, 0.3, ...