Achieving General Points in the 2-User Gaussian MAC Without Time-Sharing or Rate-Splitting by Means of Iterative Coding

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Abstract — **In this paper, we analyze iterative coding for the frame-asynchronous binary-input discrete-time Gaussian multiple access channel. Using the method of density evolution, we optimize degree distributions to exhibit code ensembles whose iterative decoding thresholds are only few tenths of dB away from the Shannon limit.**

I. Introduction

Let us consider a two-user multiple access channel (MAC) with inputs X_1 and X_2 and output Y. The achievable rate region (see [1]) is described by $\mathcal{R} = \{(R_1, R_2) \mid R_1 \leq I(X_1;Y|X_2),\ R_2 \leq I(X_2;Y|X_1)$ and $R_1 + R_2 \leq I(X_1, X_2;Y)\}$. Time-sharing between corner points and rate-splitting (see [4]) have been proposed as low-complexity schemes to achieve all points inside this region, relying on "single-user coding" and "successive cancellation". *Joint* decoding can also be performed, but has long been thought to be of high complexity and therefore was not considered to be practical.

It is tempting to investigate how the recent advances in the design and analysis of iterative coding systems can be harnessed to find low complexity coding schemes for multiple access channels. It is worth noting that the literature on this subject is quite large and due to space constraints, we will only mention the ones closest to ours.

II. Factor Graph and Decoder

Consider the binary-input discrete-time two user Gaussian MAC defined by

$$Y_i = X_i^1 + X_i^2 + Z_i$$

where the two inputs X_i^1 and X_i^2 are assumed to be elements of $\{\pm 1\}$ and where $\{Z_i\}_i$ is an i.i.d. sequence of zero mean Gaussian random variables with variance σ^2. Assuming equal priors on the inputs, the optimal estimator for bit X_i^1 is given by

$$\text{argmax}_{X_i^1 \in \{\pm 1\}} p(X_i^1|Y)$$

Following the by now standard methodology of factor graphs [2], we can derive the graph describing the decoder and the messages passed between its nodes. In order to completely specify the decoder, we still have to chose a message-passing schedule.

III. Density Evolution and Optimization

Given a proper message passing schedule, it is quickly verified that density evolution can be derived similarly to the single user case. Nevertheless, it is worth noting that, contrary to most cases investigated so far, one can not assume both codewords to be equal to the all-one codeword. However, because of the symmetries of the problem, an implementation with reasonable complexity is still possible. Recently Palanki et. al. [3] investigated the noiseless case for which, they showed that density evolution can be simply expressed as a one-dimensional recursion. They also proposed capacity achieving degree sequences based on single-user degree sequences and a construction they called graph-splitting that required frame-synchronism. In this work, we extend this analysis to the frame-asynchronous case. They also investigated the Gaussian MAC capacity region and showed that one could get within 1dB of the corner points.They suggested time-sharing to achieve general points in the region. Chayat et. al. [5] also investigated the Gaussian MAC. They showed that for the equal-power case one can get within 2dB of capacity assuming the sum-rate is smaller than one. For larger sum-rates, they proposed the use of different power levels for the users, enabling successive cancellation followed by rate-splitting.

In the current work, we concentrate on the optimization of the LDPC degree distributions in the joint iterative decoder. Our results suggest that any point in the capacity region can be achieved without requiring time-sharing or rate-splitting, just by choosing the appropriate codes and applying the joint iterative decoder. This also suggests that MACs can be addressed with the same techniques as single user channels, techniques whose complexity is linear in the block-length and approach capacity. An extension to a larger number of users is possible, but in that case, the optimization of the degree distributions becomes more complicated.

As an example, for a 2-user Gaussian MAC and the rate couple $(R_1, R_2) = (0.5, 0.5)$, our optimized code ensembles have an iterative coding threshold equal to $\sigma^* = 0.778$, which is only $0.18dB$ away from Shannon limit $\sigma_S^* = 0.7945$.

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