Remarks on CONSTRUCTION OF THE CIRCLE IN *UniMath* by M Bezem, U Buchholz, and D Grayson

Following Math Reviews [ref], homotopy type theory is an 'important . . . research topic' which 'mixes two apparently distant disciplines: type theory (a branch of logic) and homotopy theory (a branch of algebraic topology)'.

This note is not about the mathematics of the paper in question (which I believe is unexceptionable) but rather about its place in the mathematical literature. The subject is quite new, and much of its literature is to be found in computer science journals; in fact six of the nine references in this paper are on

`https : //en.wikipedia.org/wiki/GitHub` .

For that reason I have used Math Reviews' discussion of the founding document of the subject (ref [8] of the paper, a collaborative introduction to the field):

> . . . the writing of the text has the explicit objective of finding an informal style of expressing formal type theory, emulating the way working mathematicians write their papers without paying attention to foundations.

as a basis for the following comments.

In their introduction BBG say

> As a prerequisite we require, of the reader, a working knowledge of homotopy type theory as described in, for example, the first four chapters of [8],

and I believe that, under this assumption, they write conscientiously in the style described above. Moreover, as I will try to explain below, I believe their paper is an important contribution to the subject, of interest to non-specialists; but I worry that, because so much relevant background literature may not be familiar to even quite sophisticated algebraists, the paper will not receive the attention it deserves.

I am in fact not well-informed about type theory, and I may misunderstand the issues the paper addresses; but the MathSciNet review says

> . . . These gaps make the choice of the subtitle of the book ("uni-

> valent foundations of mathematics") debatable, although it is true that HoTT is presented frequently in other publications as a new foundation for mathematics. The incomplete specification of higher inductive types, the lack of a constructive semantics and the excessive reliance on types and terms whose existence is imposed by axiom, and not by construction, make it premature to claim that new foundations of mathematics are described in the book. Several current lines of research are intended to overcome these shortcomings.

However the paper's abstract says

> We show that the type $T\mathbb{Z}$ of $\mathbb{Z}$-torsors has the dependent universal property of the circle, which characterizes it up to a unique homotopy equivalence. The construction uses Voevodskys Univalence Axiom and propositional truncation, yielding a standalone construction of the circle **not using higher inductive types** (my emphasis).

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If I understand correctly, this issue is discussed at the end of the introduction (p 5). I don't think it's fair for a reviewer to suggest the authors of a paper should write some other paper than the paper they wrote, but I suspect the significance of the paper might be easier for more 'working mathematicians' to grasp if this point were clearer.

Reference MR3204653 The Univalent Foundations Program : Homotopy type theory and the univalent foundations of mathematics. The Univalent Foundations Program, Princeton, NJ; Institute for Advanced Study (IAS), Princeton, NJ, 2013.