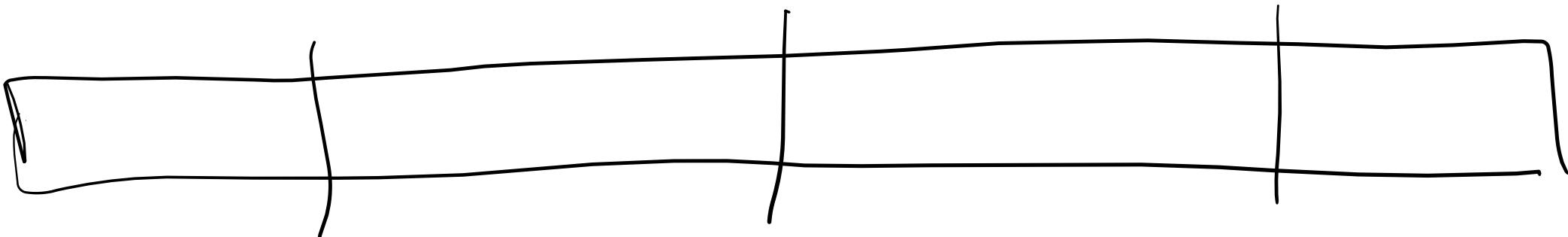


ALGORITMI RICORSIVI DI ORDINAMENTO



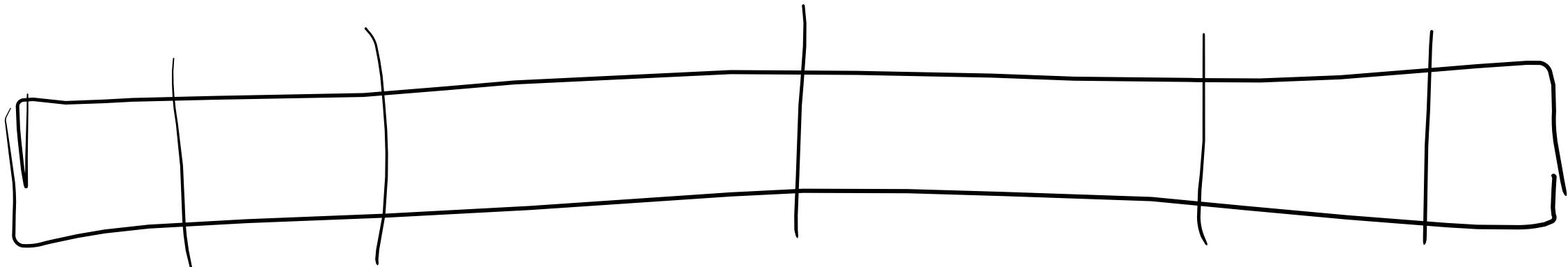
DIVIDE - E^T - IMPERA

COMBINE

- ① SUDDIVIDERSI IL PROBLEMA IN SOTTOPROBLEMI PIÙ PICCOLI

② RISOLVERE | SUBPROBLEMI
IN PANIERA | INDEPENDENTI

③ COMBINARSI IN SOLUZIONI
DEI SUBPROBLEMI



Caso BASE: 1 elemento è sempre ordinato

PASSO 1 : spessere l'array fin dal
ottenerne ~~il~~ problem
di dimensione 1

PASSO 2 : NELL CASO BASE
NON DOVO FARLO NUCCA

PASSO 3 : COMBINARE

MERGE SORT

MERGE () → PASSO ③

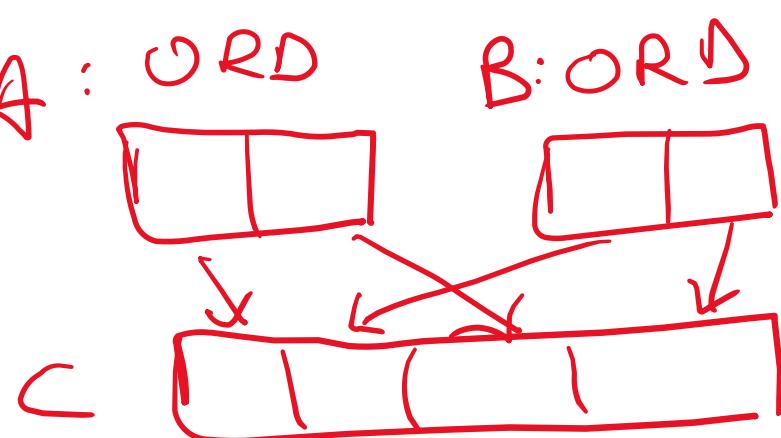
1 PROBLEMA DI DIMENSIONE N
 N SOTTOPROBLEMI DI DIMENSIONE 1

$i - 1$

A: ORD

B: ORD

i



MERGE (A, p, q, r)

$$m_1 = q - p + 1$$

$$m_2 = r - q$$

CREO DUE ARRAY

$L \leftarrow A[p \rightarrow q]$

$R \leftarrow A[q \rightarrow r]$

$L[m_1+1] \leftarrow \infty$

$R[m_2+1] \leftarrow \infty$

for $i = p \dots r$, $i = 1, \dots, m_1$, $j = 1, \dots, m_2$

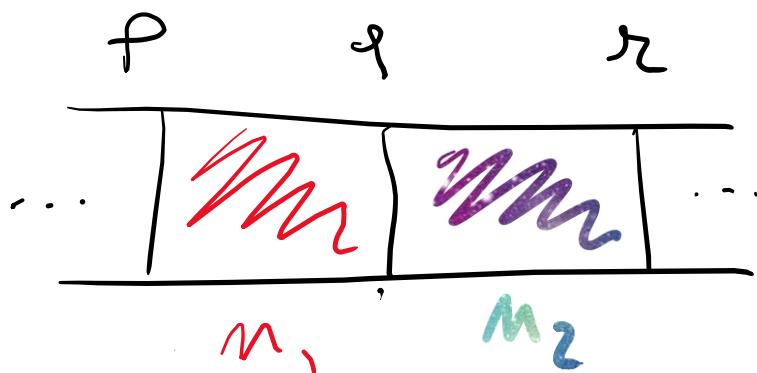
m_1 m_2
L , R

$p = \text{INDICE INIZIO SOTTOARRAY}$

$r = \text{INDICE FINE SOTTOARRAY}$

$q = \text{INDICE C. c.}$

$$p \leq q \leq r$$



for $R = p \dots r$, $i = 1, \dots, m_1$, $j = 1, \dots, m_2$

if $L[i] \leq R[j]$

$A[k] = L[i]$

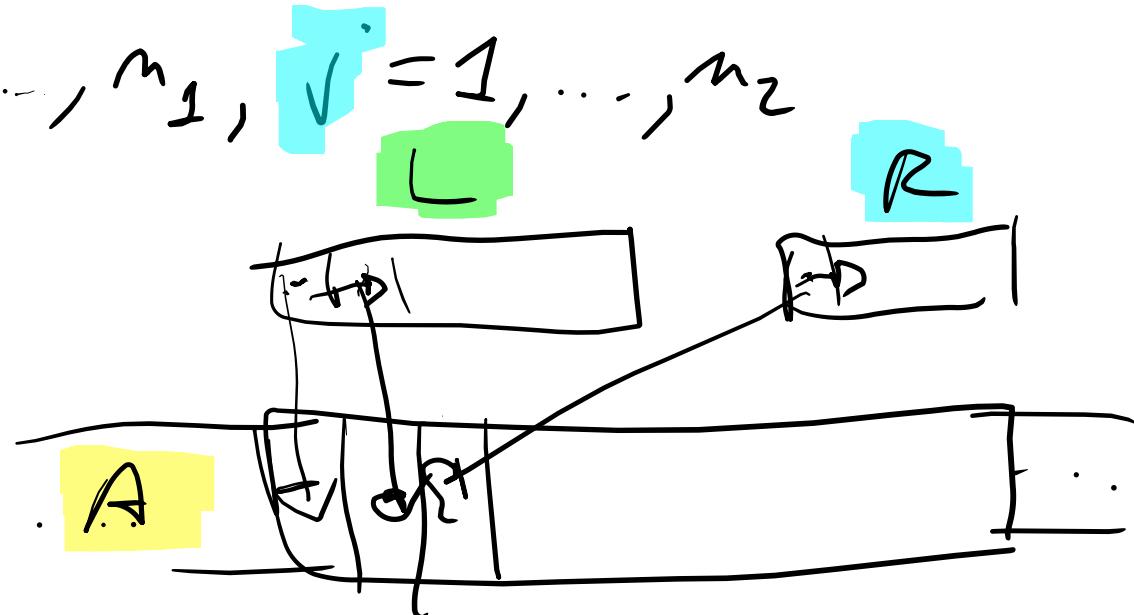
$i++$

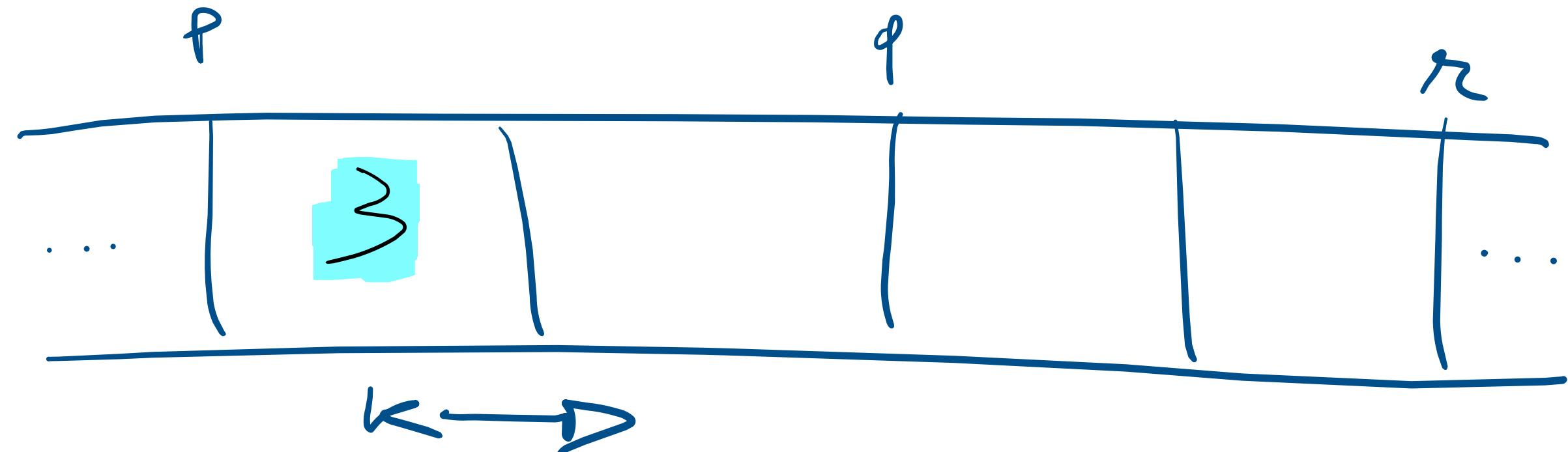
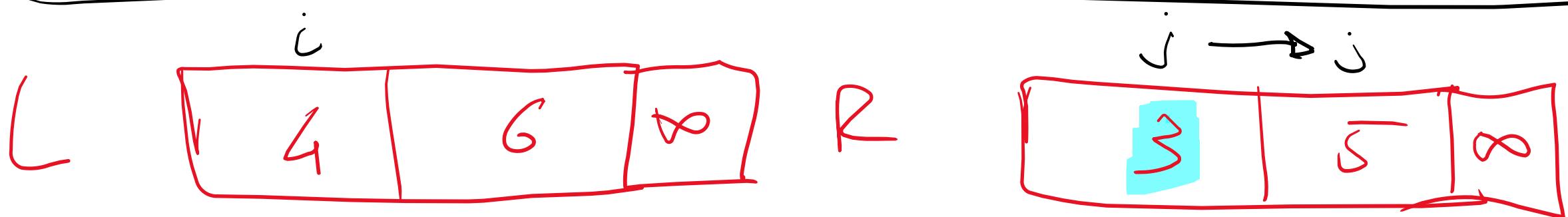
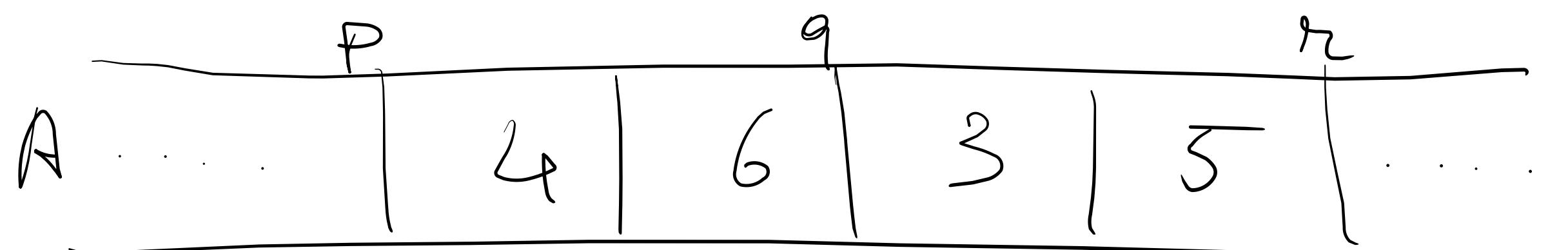
else

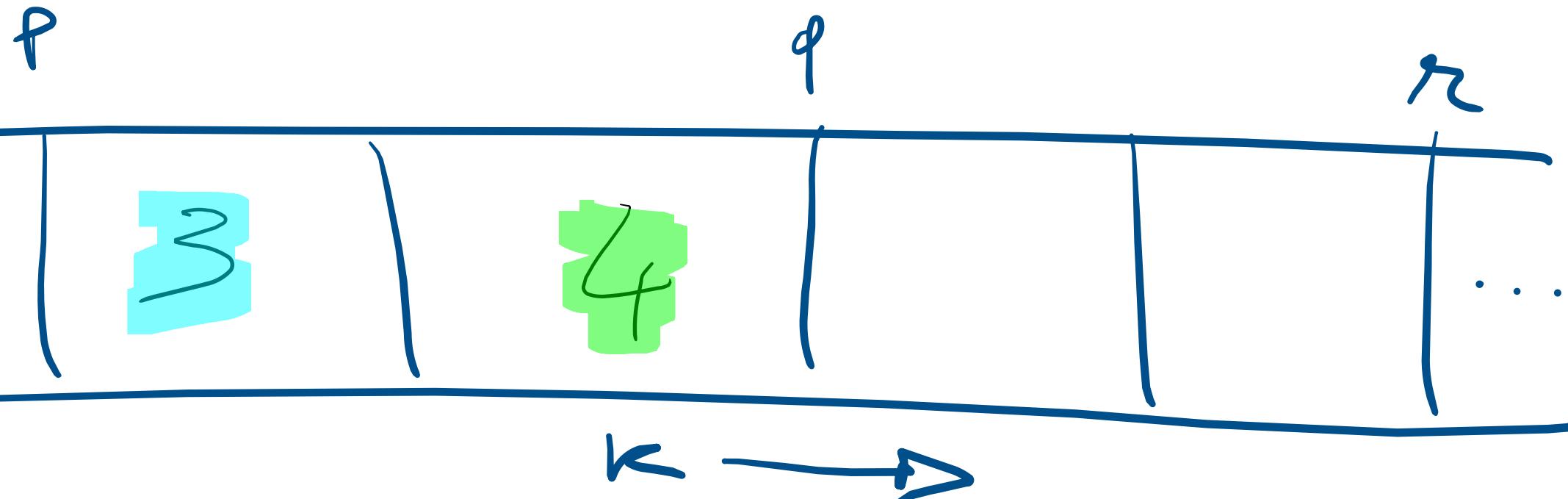
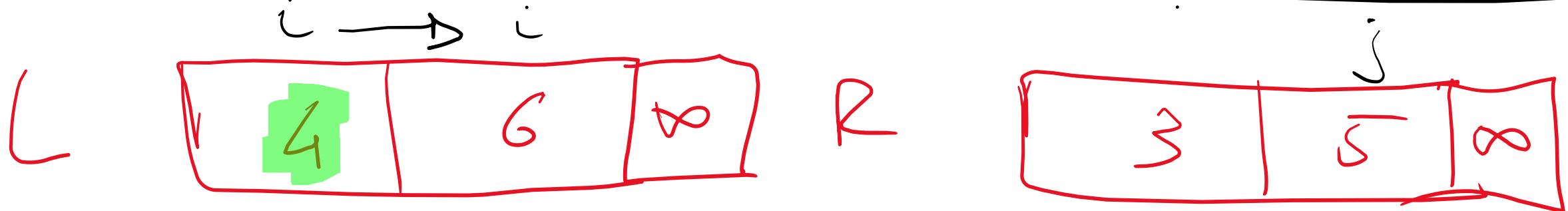
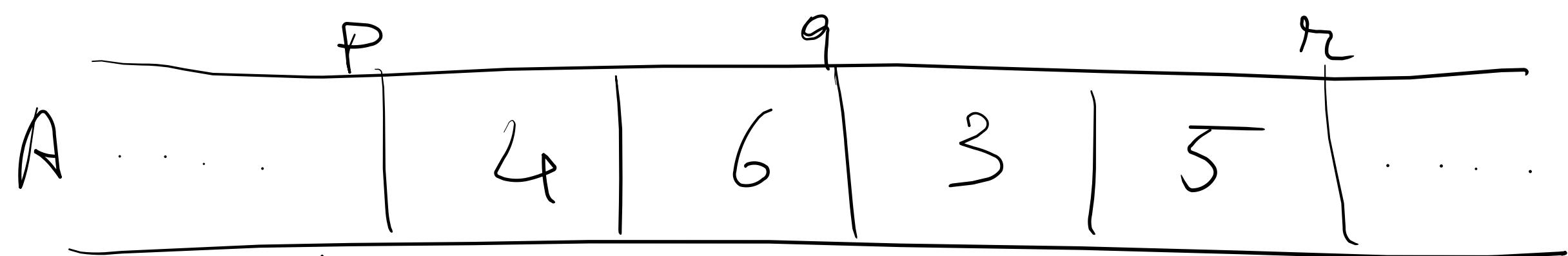
$A[k] = R[j]$

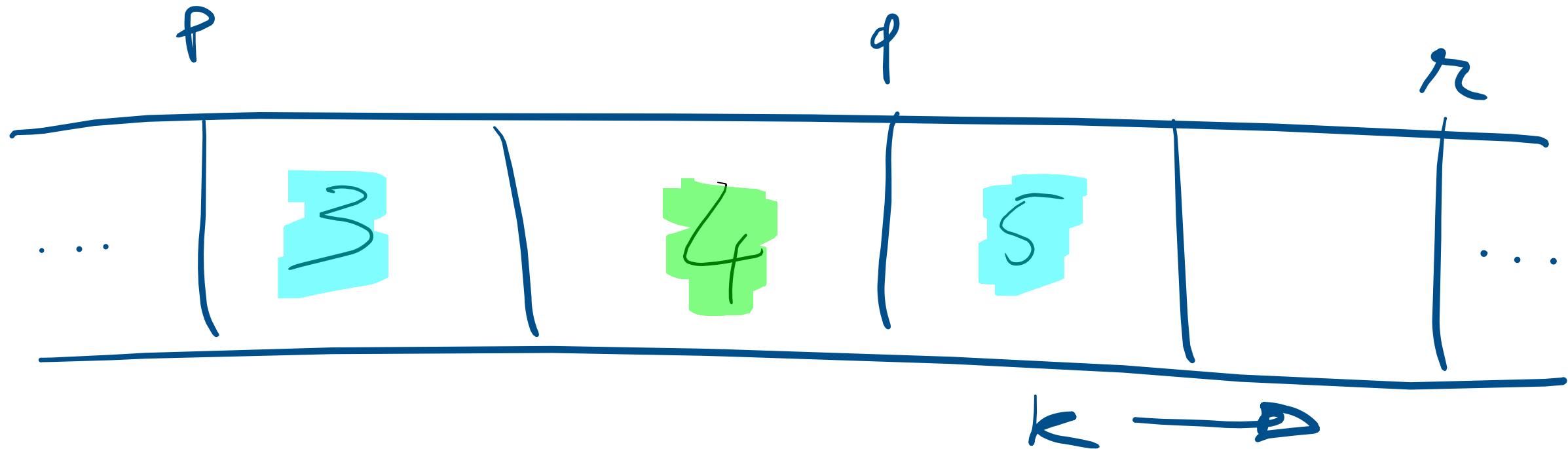
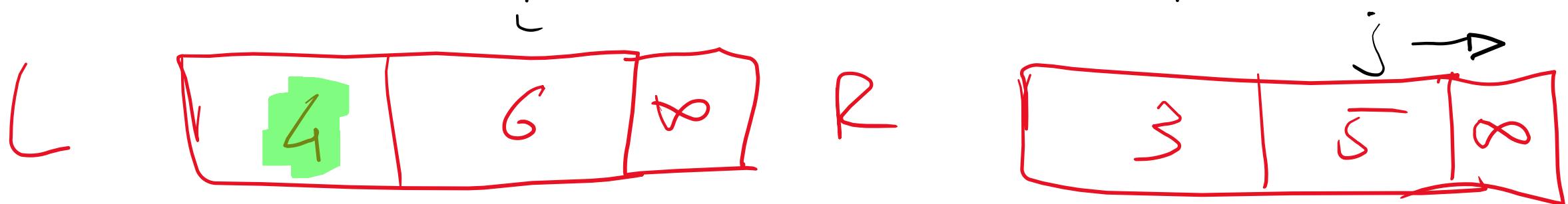
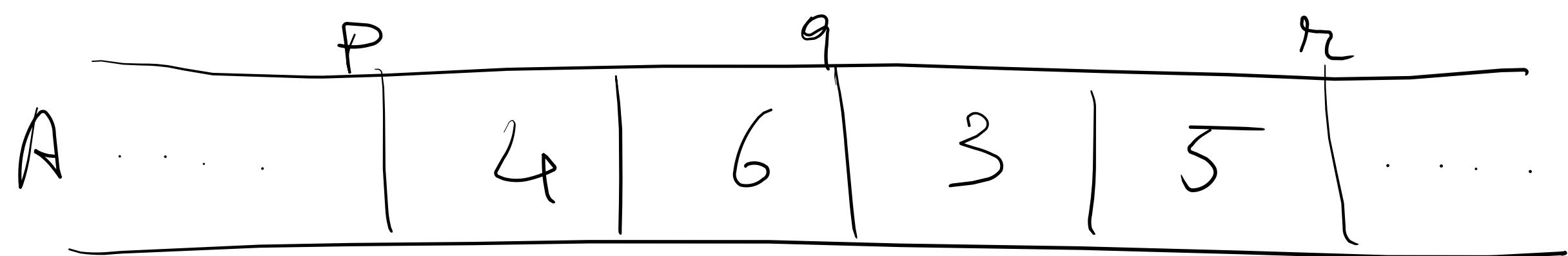
$j++$

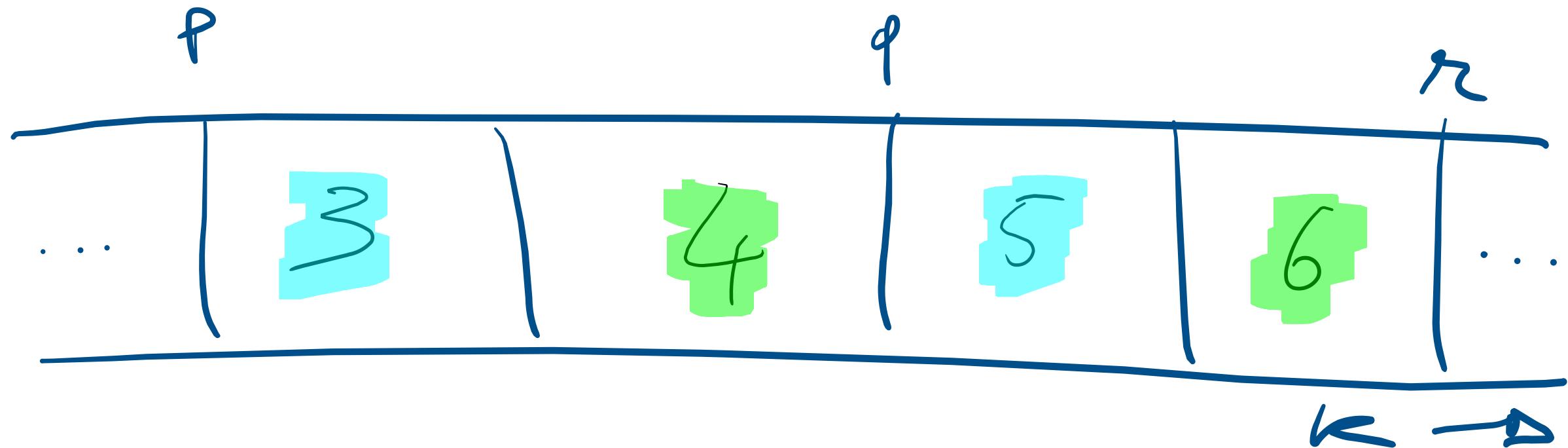
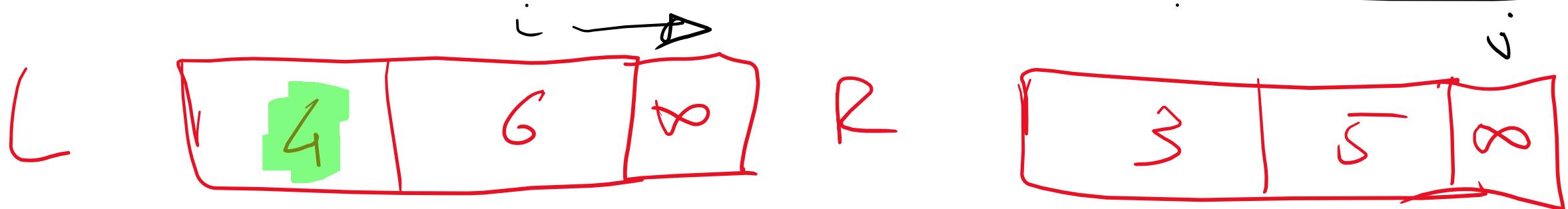
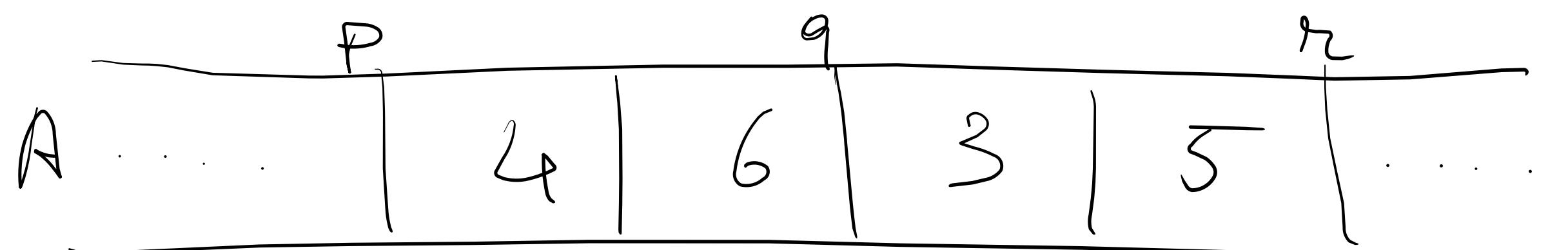
$R++$











~~LA~~

PROCEDURA

MSPBO

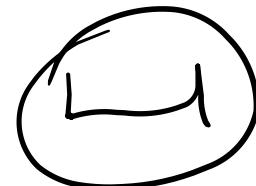
RICHIEDE

TEMPO

m

o

REGIME



(n)



MERGE-SORT(A, P, R)

If ($P < R$)

$$q = \lfloor (P+R)/2 \rfloor$$

MERGE-SORT(A, P, q)

MERGE-SORT(A, q+1, R)

MERGE(A, P, R)

5	2	4	7	1	3	2	6
0	x	xx	x	xx	x	xx	x
2	5	4	7	3	2	6	6
2	5	4	7	3	2	6	7

① MERGESORT($A, 0, 7$) , $q = \lfloor \frac{7+0}{2} \rfloor = 3$

② MERGESORT($A, 0, 3$) , ~~$q = 1$~~

③ MERGESORT($A, 4, 7$) , $q = 5$

MERGE($A, 0, 3, 7$)

③ MERGESORT($A, 0, 1$)
 ③ MERGESORT($A, 2, 3$)
 ③ MERGE($A, 0, 2, 3$)
 → MERGESORT($A, 4, 5$)
 ③ MERGESORT($A, 6, 7$)
 MERGE($A, 1, 5, 7$)

1	2	2	3	4	5	6	7
---	---	---	---	---	---	---	---

0

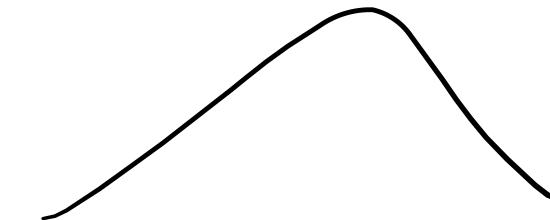
2	4	5	7
---	---	---	---

1	3	3	6
---	---	---	---

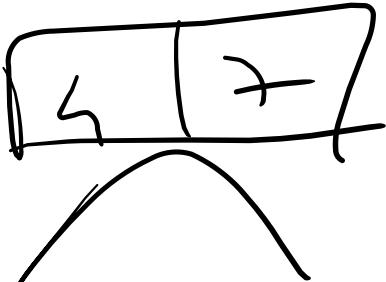
1

$$3 = \log_2 8$$

a



2	5
---	---



1	3
---	---

2	6
---	---

2

con

5	2	4	7	1	3	2	6
---	---	---	---	---	---	---	---

3

QUICKSORT

ORDINA AL ELEMENTI SUL POSIZ
E QUINDI NON MA BISOGNA
DI ARREGGIARSI

2	8	7	13	5	6	4
---	---	---	----	---	---	---

i
p_{ij}

i = p - 1

j = p

PIVOT

FAR SCORRERES j de p e 2 - 1

SE L'ELEMENFO A[j] < PIVOT

Allora FACCIO AVANZARE i

ESISTIBLO A[i] ON A[j]

Alla fine SCAMBIA IL PIVOT GN A[i+1]

2	8	7	$\sqrt{3}$	5	6	4
---	---	---	------------	---	---	---

$i = 1$



$i = p - 1$

$j = p$

PIVOT

FAR CORRERESSO j da $p + 2 - 1$

SE L'ELEMENFO $A[j] < \text{PIVOT}$

Allora FACCIO AVANZARE i

ESISTIBLO $A[i]$ ON $A[j]$

ALLA FINN SCAMBIA IL PIVOT GN $A[i+1]$

2	8	7		13	5	6		4
ϕ_i	j							$i = 1$

$$i = p - 1$$

$$j = p$$

PIVOT

FAR CORRERES j da $p + 2 - 1$

SE L'ELEMENFO $A[j] < \text{PIVOT}$

Allora FACCIO AVANZARE i

ESISTIBLO $A[i]$ ON $A[j]$

ALLA FINI SCAMBIALO PIWOT ON $A[i+1]$

2	8	7	13	5	6	4
ϕ_i		$j \rightarrow$				$i = 1$

$$i = p - 1$$

$$j = p$$

PIVOT

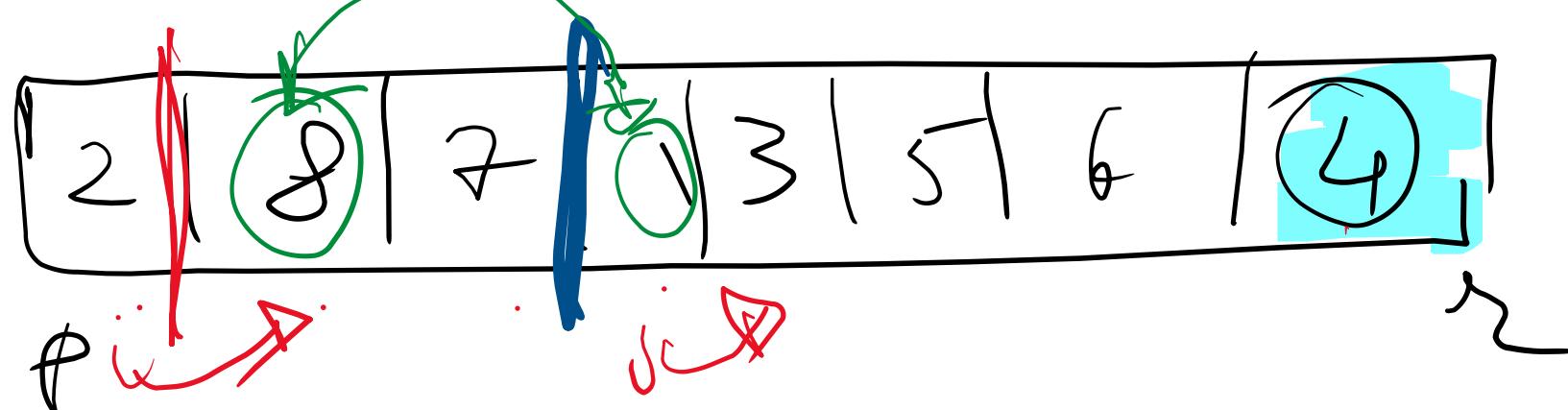
FAR CORRERES j da $p + 2 - 1$

SE L'ELEMENFO $A[j] < PIVOT$

Allora FACCIO AVANZARE i

ESISTIBO $A[i]$ ON $A[j]$

ALLA FINI SCAMBIALO PIWOT ON $A[i+1]$



$$i = p - 1$$

$$j = p$$

PIVOT

FAR SCORRERESSO j da $p + 2 - 1$

SE L'ELEMENSO $A[j] < \text{PIVOT}$

Allora FACCIO AVANZARE i

\Leftarrow SOTTRIBLO $A[i]$ ON $A[j]$

ALLA FINNÉ SCAMBIALO j CON PIVOT $A[i+1]$



$i = p - 1$

$j = p$

PIVOT

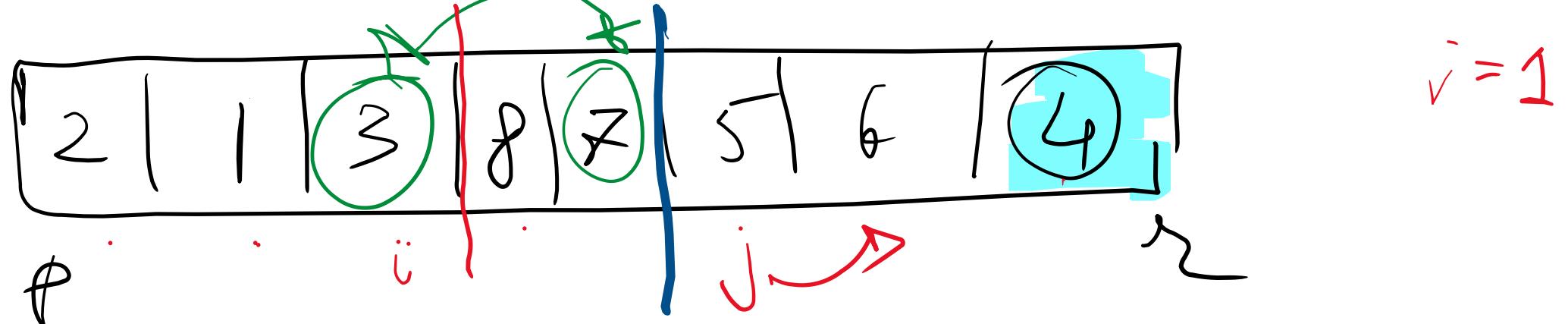
FAR CORRERES j da $p + 2 - 1$

SE L'ELEMENFO $A[j] < \text{PIVOT}$

Allora FACCIO AVANZARE i

ESISTIBLO $A[i]$ ON $A[j]$

ALLA FINN SCAMBIA IL PIVOT GN $A[i+1]$



$$i = p - 1$$

$$j = p$$

PIVOT

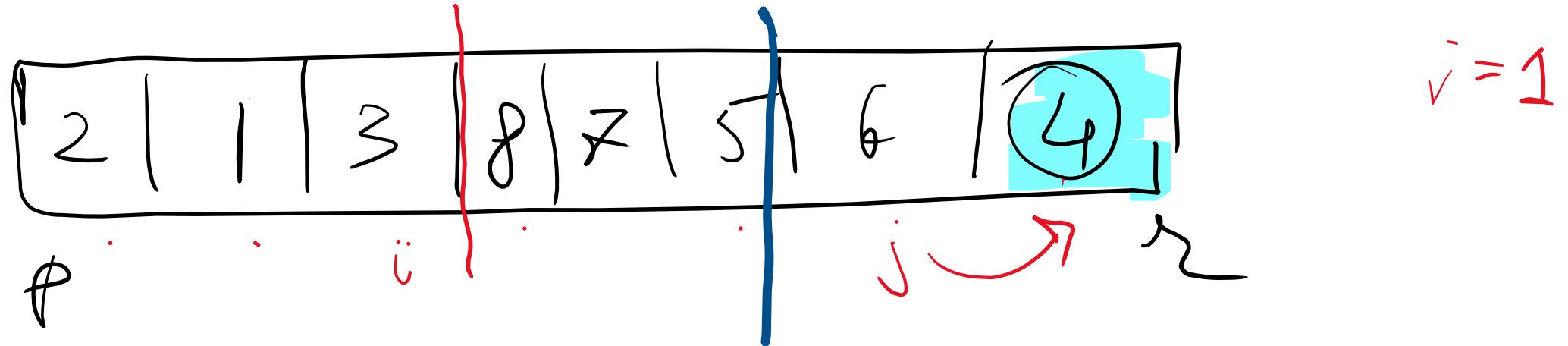
FAR SCORRERES j da $p + 2 - 1$

SE L'ELEMENFO $A[j] < \text{PIVOT}$

Allora FACCIO AVANZARE i

\Leftarrow SOTTRIBLO $A[i]$ ON $A[j]$

ALLA FINN \Rightarrow SCAMBIA IL PIVOT GN $A[i+1]$



$$i = p - 1$$

$$j = p$$

PIVOT

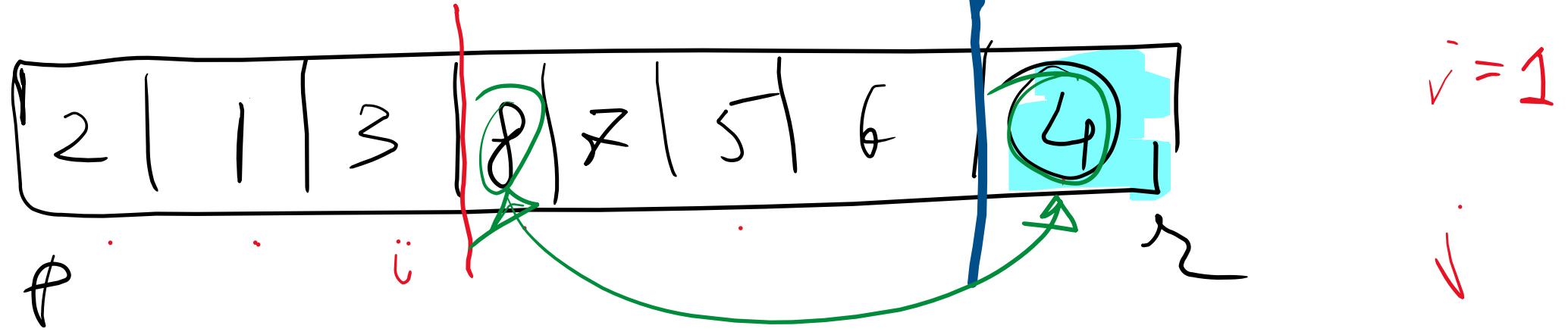
FAR SCORRERES j de p e $2-1$

SE L'ELEMENFO $A[j] < PIVOT$

Allora FACCIO AVANZARE i

\Leftarrow SOTTRIBLO $A[i]$ ON $A[j]$

Alla fine SCAMBIA IL PIVOT GN $A[i+1]$



$i = p - 1$

$j = p$

PIVOT

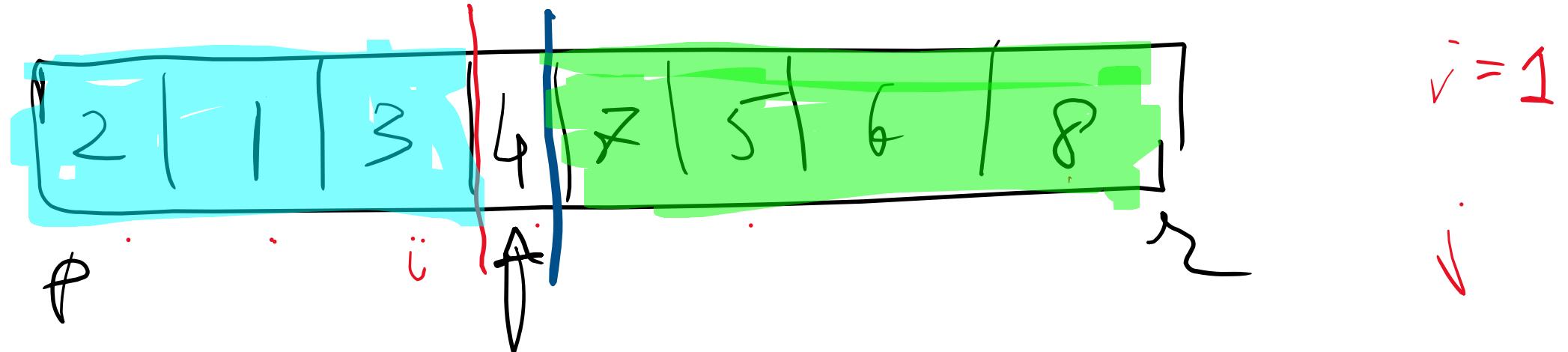
FAR SCORRERES j de p e $2 - 1$

SE L'ELEMENFO $A[j] \leq$ PIVOT

Allora FACCIO AVANZARE i

← SOTTRIBLO $A[i]$ ON $A[j]$

ALLA FINNÉ SCAMBIA IL PIVOT GN $A[i+1]$



$$i = p + 1$$

$$j = p$$

PIVOT

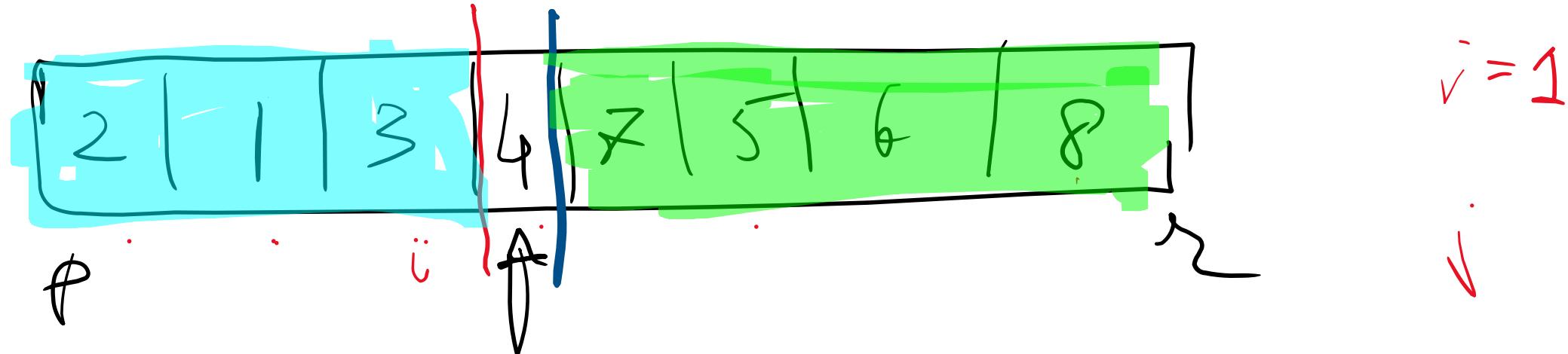
FAR SCORRERES j de p e $\textcircled{2-1}$

SE L'ELEMENFO $A[j] < \text{PIVOT}$

Allora FACCIO AVANZARE i

\Leftarrow SOTTRIBLO $A[i]$ ON $A[j]$

Alla fine SCAMBIA IL PIVOT GN $A[i+1]$



FAR SCORRERETE j òLE P ϵ $\circlearrowleft 2 - 1$

SE L'ELEMENTO $A[j] < \text{PIVOT}$

Allora FACCIO AVANZARE i

\Leftarrow SOTTRIBLO $A[i]$ ON $A[j]$

Alla fine SCAMBIA IL PIVOT GN $A[i+1]$

RESTITUISCO $i + 1$, CHE È LA POSIZIONE
FINALE DEL PIVOT

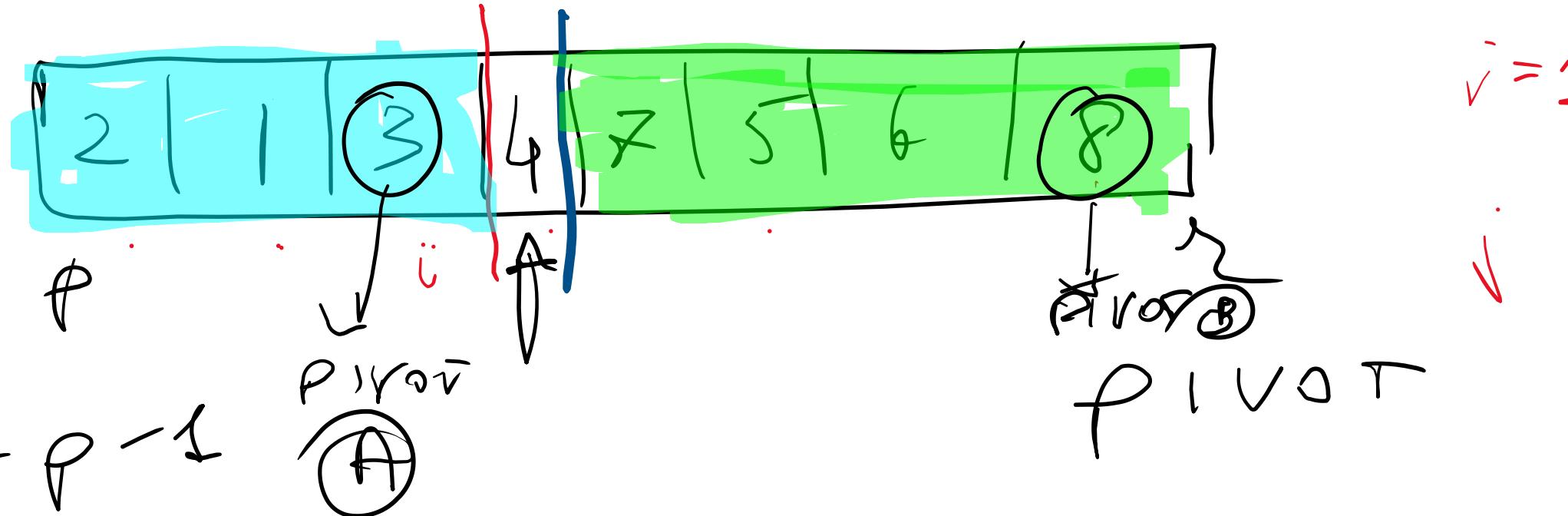
QUICKSORT (A , p , r)

if $p < r$

$q = \text{PARTITION} (A, p, r)$

(A) $\text{QUICKSORT}(A, p, q-1)$

(B) $\text{QUICKSORT}(A, q+1, r)$



$v = p$

FAR SCORRERESS j de p e $\textcircled{2-1}$

SE L'ELEMENSO $A[j] < \text{PIVOT}$

Allora FACCIO AVANZARE i

ESISTIBLO $A[i]$ ON $A[j]$

Alla fine SCAMBIA IL PIVOT GN $A[i+1]$

20.00
100 - 88.00

① LA SCOLTA DEL PIVOT È
CASUALITÀ, CIÒ È NON STIAMO
FA CONDO ALCUNA CONSIDERAZIONE
SUL SUO UAWRÈ EFFETTIVO

$$A = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 \}$$

i *j* →

I

$$i=0, j=2$$

II

$$i=2, j=3$$

III

$$i=1, j=4$$

$$A = \{ 9, 19, 13, 5, 12, 8, 7, 4, 21, 26, 11 \}$$

IV

$$i=2, j=5$$

$$A = \{ 9, 5, 13, 19, 12, 8, 7, 4, 21, 26, 11 \}$$

V

$$i=2, j=6$$

VI

$$i=3, j=2$$

$$A = \{ 9, 5, 13, 19, 12, 8, 7, 4, 21, 26, 11 \}$$

V $i=2, j=6$ VI $i=3, j=2$ $A = \{9, 5, 8, 7, 12, 13, 1, 4, 2, 1, 2, 6, 11\}$

VII $i=4, j=8$ $A = \{9, 5, 8, 7, 12, 13, 1, 4, 2, 1, 2, 6, 11\}$

VIII $i=5, j=9$ $A = \{9, 5, 8, 7, 4, 13, 1, 2, 12, 6, 11\}$

IX $i=5, j=10$ X $i=6, j=11$ $\{9, 5, 8, 7, 12, 13, 1, 2, 11, 10, 12, 2, 13, 6, 11\}$

XI $i=2, j=12$ $A = \{9, 5, 8, 7, 1, 2, 12, 13, 1, 10, 11\}$

SWAP($A[i+1], A[r]$) $\Rightarrow A = \{9, 5, 8, 7, 1, 2, 11, 12, 13, 1, 10, 12\}$