A Guided Tour of CML, the Coded Modulation Library

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*Outline

1. CML overview
   - What is it? How to set it up and get started?

2. Uncoded Modulation
   - Simulate uncoded BPSK and QAM in AWGN and Rayleigh fading

3. Coded Modulation
   - Simulate a turbo code from UMTS 25.212
   - *Specify a custom eIRA LDPC parity check matrix

4. Ergodic (Shannon) capacity analysis
   - Determine the modulation constrained capacity of BPSK and QAM

5. EXIT analysis
   - Determine the EXIT characteristic for BPSK in AWGN for a particular LDPC degree distribution

**Outline

6. Outage analysis
   - Determine the outage probability over block fading channels
   - Determine the outage probability of finite-length codes

7. Digital Network Coding
   - Describe CML implementation of digital network coding
   - Simulate error rate of digital network coding for non-channel-coded 4-FSK in Rayleigh fading

8. The internals of CML

9. Throughput calculation
   - Convert BLER to throughput for hybrid-ARQ

What is CML?

- CML is an open source toolbox for simulating capacity approaching codes in MATLAB.
- Available for free at the **Iterative Solutions** website:
    - www.iterativesolutions.com
- Runs in MATLAB, but uses c-mex for efficiency.
- First release was in Oct. 2005.
    - Used code that has been developed since 1996.

Features

- Simulation of BICM (bit interleaved coded modulation)
  - Turbo, LDPC, or convolutional codes.
  - User-specified eIRA LDPC parity check matrices.
  - PSK, QAM, FSK modulation.
  - BICM-ID: Iterative demodulation and decoding.
- Generation of ergodic capacity curves
  - BICM/CM constrained modulation.
- **EXIT analysis
  - Generate LDPC EXIT characteristics for all CML detector types.
- Information outage probability
  - Block fading channels.
  - Blocklength-constrained channels (AWGN or fading)

Features

- ■ **Digital network coding
  - – Error-rate of digital network coding in the two-way relay channel
  - – Noncoherent FSK modulation
- ■ Calculation of throughput of hybrid-ARQ.

Supported Standards

- **Binary turbo codes:**
  - UMTS/3GPP, including HSDPA and LTE.
  - cdma2000/3GPP2.
  - CCSDS.

- **Duobinary turbo codes:**
  - DVB-RCS.
  - WiMAX IEEE 802.16.

- **LDPC codes:**
  - DVB-S2.
  - Mobile WiMAX IEEE 802.16e.

Simulation Data is Valuable

- CML saves simulation state frequently
  - parameter called "save_rate" can be tuned to desired value.
- CML can be stopped at any time.
  - Intentionally: Hit CTRL-C within matlab.
  - Unintentionally: Power failure, reboot, etc.
- CML automatically resumes simulation
  - If a simulation is run again, it will pickup where it left off.
  - Can reset simulation by setting "reset=1".
  - SNR points can be added or deleted prior to restarting.
- Simulations can be made more confident by requesting additional trials prior to restarting.
  - The new results will be added to the old ones.

Compiled Mode

- A flag called "compiled_mode" can be used to run CML independently of matlab.

- CML must first be compiled using the matlab compiler.

- Advantages:
  - Can run on machines without matlab.
  - Can run on a grid computer.

WebCML

- WebCML is a new initiative sponsored by NASA and NSF.
- Idea is to upload simulation parameters to a website and hit a "simulate" button.
  - Simulation begins on the webserver.
  - The webserver will divide the simulation into multiple jobs which are sent to a grid computer.
- Results can be retrieved while simulation is running and once it has completed.
- The grid is comprised of ordinary desktop computers.
  - The grid compute engine is a screen saver.
    - Kicks in only when computer is idle.
  - Users of WebCML are encouraged to donate their organizations computers to the grid.

Getting Started with CML

- **Download**
  - www.iterativesolutions.com/download.htm
  - Decompress downloaded archive
    - Root directory will be ./cml

- **About simulation databases**
  - A large database of previous simulation results is available.
  - Unzip each database and place each extracted directory into the ./cml/output directory

- **About C-mex files.**
  - C-mex files are compiled for PC computers.
  - For unix and mac computers, must compile.
    - Within matlab, cd to ./cml/source and type "make".

Starting and Interacting with CML

- Launch MATLAB
- Cd to the <CMLROOT> directory
- Type "CmlStartup"
  - This sets up paths and determines the version of MATLAB.
- To run CML, only two functions are needed:
  - CmlSimulate
    - Runs one or more simulations.
    - Simulation parameters are stored in .m scripts.
    - Input arguments tell CML which simulation(s) to run.
  - CmlPlot
    - Plots the results of one or more simulations.

Scenario Files and the SimParam Structure

- **The parameters associated with a set of simulations is stored in a scenario file.**
  - Located in one of two directories
    - ./cml/scenarios for publicly available scenarios
    - ./cml/localscenarios for personal user scenarios
    - Other directories could be used if they are on the matlab path.
  - .m extension.
- **Exercise**
  - Edit the example scenario file: UncodedScenarios.m
- **The main content of the scenario file is a structure called sim_param**
  - Sim_param is an array.
  - Each element of the array is called a *record* and corresponds to a single distinct simulation.

Common Parameters

- List of all parameters can be found in:
  - ./cml/mat/DefineStructures.m
  - ./cml/documentation/readme.pdf
- Default values are in the DefineStructures.m file
- Some parameters can be changed between runs, others cannot.
  - sim_param_changeable
  - sim_param_unchangeable

Dissecting the SimParam Structure: The simulation type

- **sim_param(record).sim_type =**
  - 'uncoded'
    - BER and SER of uncoded modulation
  - 'coded'
    - BER and FER of coded modulation
  - 'capacity'
    - The Shannon capacity under modulation constraints.
  - **'exit'
    - EXIT characteristic of selected detector and parameterized LDPC code
  - 'outage'
    - The information outage probability of block fading channels
    - Assumes codewords are infinite in length
  - 'bloutage'
    - Information outage probability in AWGN or ergodic/block fading channels
    - Takes into account lenth of the code.

Dissecting the SimParam Structure: The simulation type

- **sim_param(record).sim_type** =
  - 'throughput'
    - By using FER curves, determines throughput of hybrid ARQ
    - This is an example of an *analysis* function … no simulation involved.

Lesser Used Simulation Types

- **sim_param(record).sim_type** =
  - 'bwcapacity'
    - Shannon capacity of CPFSK under bandwidth constraints.
  - 'minSNRvsB'
    - Capacity limit of CPFSK as a function of bandwidth

Parameters Common to All Simulations

- **sim_param(record).**
  - comment = {string}
    - Text, can be anything.
  - legend = {string}
    - What to put in figure caption
  - linetype = {string}
    - Color, type, and marker of line. Uses syntax from matlab "plot".
  - filename = {string}
    - Where to save the results of the simulation
    - Once filename is changed, any parameter can be changed.
  - reset = {0,1} with default of 0
    - Indication to resume "0" or restart "1" simulation when run again.
    - If reset = 1, any parameter may be changed.

Specifying the Simulation

■ sim_param(record).

- SNR = {vector}
  - Vector containing SNR points in dB
  - Can add or remove SNR points between runs
- SNR_type = {'Eb/No in dB' or 'Es/No in dB'}
  - For some simulation types, only one option is supported.
  - E.g. for *capacity* simulations, it must be Es/No
- save_rate = {scalar integer}
  - An integer specifying how often the state of the simulation is saved
  - Number of trials between saves.
  - Simulation echoes a period '.' every time it saves.

Specifying the Simulation (cont'd)

■ sim_param(record).

- max_trials = {vector}
  - A vector of integers, one for each SNR point
  - Tells simulation maximum number of trials to run per point.
- max_frame_errors = {vector}
  - Also a vector of integers, one for each SNR point.
  - Tells simulation maximum number of frame errors to log per point.
  - Simulation echoes a 'x' every time it logs a frame error.
- minBER = {scalar}
  - Simulation halts once this BER is reached
- minFER = {scalar}
  - Simulation halts once this FER is reached.

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     degree distribution

Specifying Modulation

- **sim_param(record).**
  - modulation = {string}
    - Specifies the modulation type
    - May be 'BPSK', 'QPSK', 'QAM', 'PSK', 'APSK', 'HEX', or 'FSK'
    - 'HSDPA' used to indicate QPSK and QAM used in HSDPA.
    - All but FSK are 2 dimensional modulations
      - Uses a complex scalar value for each symbol.
    - Default is 'BPSK'
    - New (version 1.9 and above): Can also be set to "custom".
  - mod_order = {integer scalar}
    - Number of points in the constellation.
    - Power of 2.
    - Default is 2.
    - In some cases, M=0 is used to indicate an unconstrained Gaussian input.
  - S_matrix = {complex vector}
    - Only used for "custom" modulation type.
    - A vector of length "mod_order" containing the values of the symbols in the signal set S.

Specifying Modulation

- **sim_param(record).**
  - mapping = {integer vector}
    - A vector of length M specifying how data bits are mapped to symbols.
    - Vector contains the integers 0 through M-1 exactly once.
    - ith element of vector is the set of bits associated with the ith symbol.
    - Alternatively, can be a string describing the modulation, like 'gray' or 'sp'
    - Default is 'gray'
  - framesize = {integer scalar}
    - The number of symbols per Monte Carlo trial
    - For coded systems, this is number of bits per codeword
  - demod_type = {integer scalar}
    - A flag indicating how to implement the demodulator
      - 0 = log-MAP (approximated linearly)
      - 1 = max-log-MAP
      - 2 = constant-log-MAP
      - 3 and 4 other implementations of log-MAP
    - Max-log-MAP is fastest.
    - Does not effect the uncoded error rate.
      - However, effects coded performance

M-ary Complex Modulation

- $\mu = \log_2 M$ bits are mapped to the symbol $\mathbf{x}_k$, which is chosen from the set $S = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_M\}$
  - The symbol is multidimensional.
  - 2-D Examples: QPSK, M-PSK, QAM, APSK, HEX
    - These 2-D signals take on complex values.
  - M-D Example: FSK
    - FSK signals are represented by the M-dimensional complex vector $\mathbf{X}$.
- The signal $y = hx_k + n$ is received
  - h is a complex fading coefficient (scalar valued).
  - n is complex-valued AWGN noise sample
  - More generally (FSK), $\mathbf{Y} = h\,\mathbf{X} + \mathbf{N}$
    - Flat-fading: All FSK tones multiplied by the same fading coefficient h.
- Modulation implementation in CML
  - The complex signal set S is created with the **CreateConstellation** function.
  - Modulation is performed using the **Modulate** function.

Log-likelihood of Received Symbols

- Let $p(\mathbf{x}_k|\mathbf{y})$ denote the probability that signal $\mathbf{x}_k \in S$ was transmitted given that $\mathbf{y}$ was received.

- Let $f(\mathbf{x}_k|\mathbf{y}) = K\, p(\mathbf{x}_k|\mathbf{y})$, where K is any multiplicative term that is constant for all $\mathbf{x}_k$.

- When all symbols are equally likely, $f(\mathbf{x}_k|\mathbf{y}) \propto f(\mathbf{y}|\mathbf{x}_k)$

- For each signal in S, the receiver computes $f(\mathbf{y}|\mathbf{x}_k)$
  - This function depends on the modulation, channel, and receiver.
  - Implemented by the **Demod2D** and **DemodFSK** functions, which actually computes $\log f(\mathbf{y}|\mathbf{x}_k)$.

- Assuming that all symbols are equally likely, the most likely symbol $\mathbf{x}_k$ is found by making a hard decision on $f(\mathbf{y}|\mathbf{x}_k)$ or $\log f(\mathbf{y}|\mathbf{x}_k)$.

Example: QAM over AWGN.

- Let y = x + n, where n is complex i.i.d. $N(0, N_0/2)$ and the average energy per symbol is $E[|x|^2] = E_s$
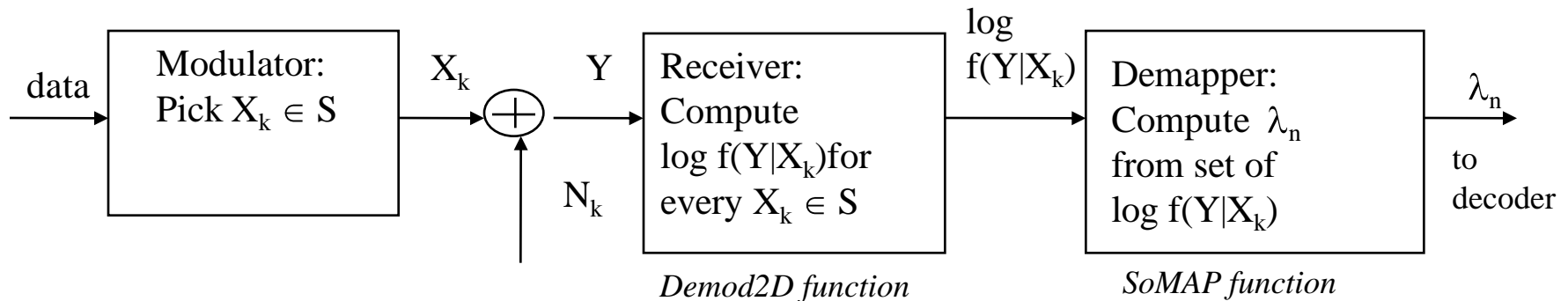
$$p(y \mid x_k) = \frac{1}{2\sigma^2} \exp\left\{ \frac{-\left| y - x_k \right|^2}{2\sigma^2} \right\}$$

$$f(y \mid x_k) = \exp\left\{ \frac{-\left| y - x_k \right|^2}{2\sigma^2} \right\}$$

$$\log f(y \mid x_k) = \frac{-\left| y - x_k \right|^2}{2\sigma^2}$$

$$= \frac{-E_s \left| y - x_k \right|^2}{N_o}$$

Converting symbol liklihoods to bit LLR

Demod2D function   *SoMAP function*

- ■ The symbol likelihoods must be transformed into bit log-likelihood ratios (LLRs):

$$\lambda_n = \log \frac{P[d_n = 1]}{P[d_n = 0]} = \log \frac{\sum\limits_{X_k \in S_n^{(1)}} f(Y / X_k)}{\sum\limits_{X_k \in S_n^{(0)}} f(Y / X_k)}$$
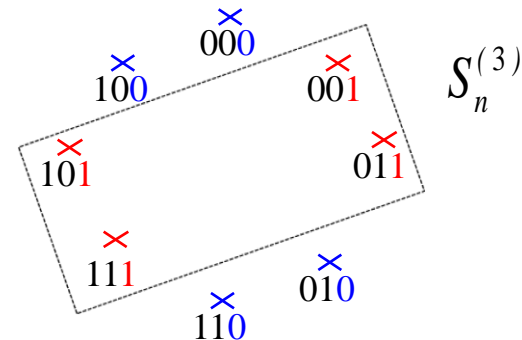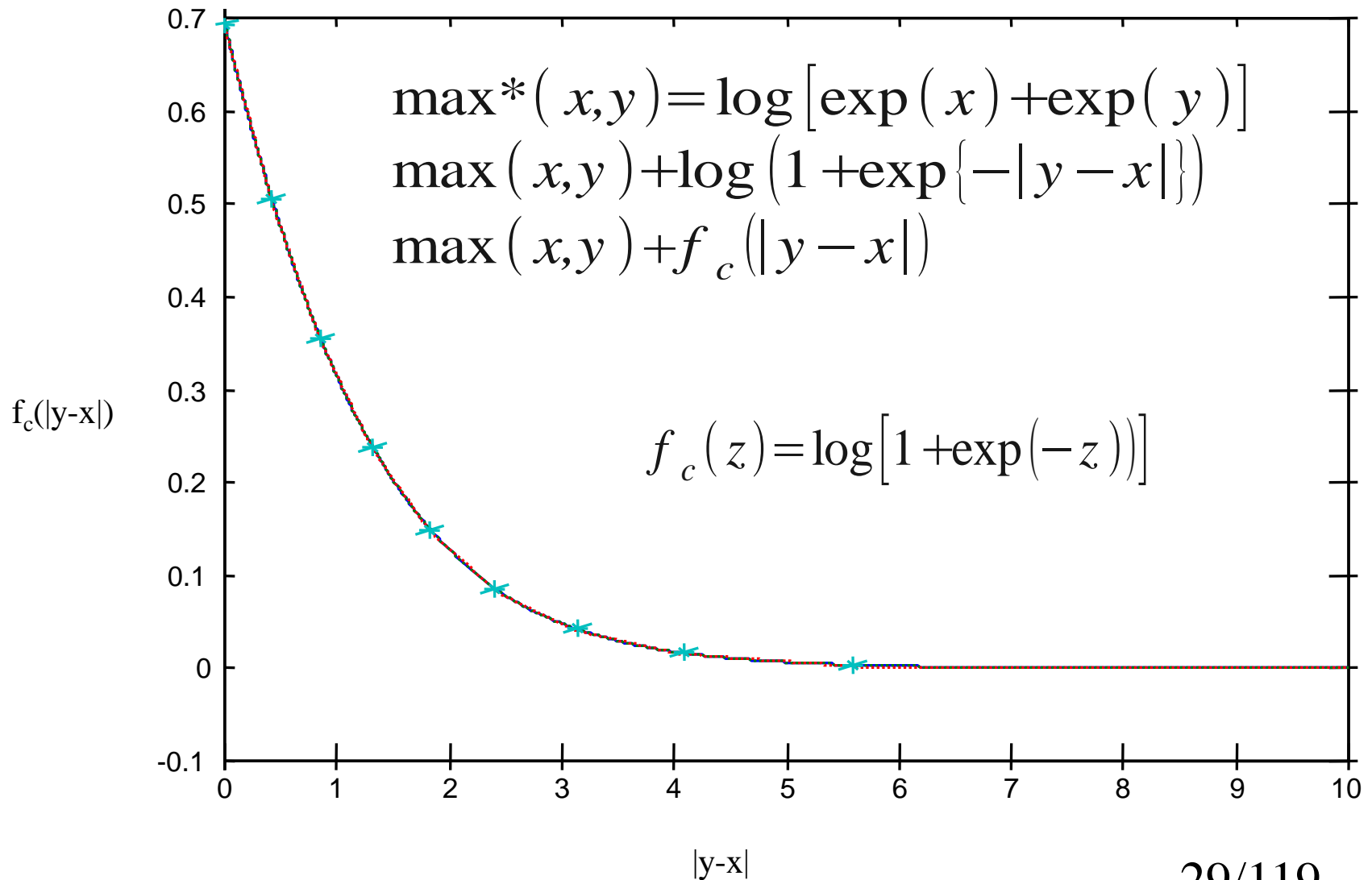
- – Where $S_n^{(1)}$ represents the set of symbols whose nth bit is a 1.
- – and $S_n^{(0)}$ is the set of symbols whose nth bit is a 0.

Log-domain Implementation

$$\lambda_n = \log \frac{\displaystyle\sum_{X_k \in S_n^{(1)}} f(Y \mid X_k)}{\displaystyle\sum_{X_k \in S_n^{(0)}} f(Y \mid X_k)}$$

$$= \log \sum_{X_k \in S_n^{(1)}} f(Y \mid X_k) - \log \sum_{X_k \in S_n^{(0)}} f(Y \mid X_k)$$

$$= \max_{X_k \in S_n^{(1)}}{}^* \{\log f(Y \mid X_k)\} - \max_{X_k \in S_n^{(0)}}{}^* \{\log f(Y \mid X_k)\} \quad \text{log-MAP} \atop \text{demod\_type} = 0$$

$$\approx \max_{X_k \in S_n^{(1)}} \{\log f(Y \mid X_k)\} - \max_{X_k \in S_n^{(0)}} \{\log f(Y \mid X_k)\} \quad \text{max-log-MAP} \atop \text{demod\_type} = 1$$

The max* function

$$\max{}^*(x,y) = \log\left[\exp(x) + \exp(y)\right]$$
$$\max(x,y) + \log\left(1 + \exp\{-|y-x|\}\right)$$
$$\max(x,y) + f_c(|y-x|)$$

$$f_c(z) = \log\left[1 + \exp(-z)\right]$$

FSK-Specific Parameters

- **sim_param(record).**
  - h = {scalar}
    - The modulation index
    - h=1 is orthogonal
  - csi_flag = {integer scalar}
    - 0 = coherent (only available when h=1)
    - 1 = noncoherent w/ perfect amplitudes
    - 2 = noncoherent without amplitude estimates

Specifying the Channel

- **sim_param(record).**
    - channel = {'AWGN', 'Rayleigh', 'block'}
        - 'Rayleigh' is "fully-interleaved" Rayleigh fading
        - 'block' is for coded simulation type only
    - blocks_per_frame = {scalar integer}
        - For block channel only.
        - Number of independent blocks per frame.
        - Block length is framesize/blocks_per_frame
    - bicm = {integer scalar}
        - 0 do not interleave bits prior to modulation
        - 1 interleave bits prior to modulation (default)
        - 2 interleave and perform iterative demodulation/decoding
        - This option is irrelevant unless a channel code is used

Exercises

- Create and run the following simulations:
  - BPSK in AWGN
  - 64QAM with gray labeling in AWGN
  - 64QAM with gray labeling in Rayleigh fading
- Choices that need to be made?
  - Framesize?
  - Save_rate?
  - Min_BER?
  - Min_frame_errors?
  - Demod_type?
- Plot all the results on the same figure.

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   - ***Specify a custom eIRA LDPC parity check matrix**

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Coded Systems:
Code Configuration

■ Only for sim_param(record).sim_type = 'coded'

■ sim_param(record).code_configuration = {scalar int}

- 0 = Convolutional
- 1 = binary turbo code (PCCC)
- 2 = LDPC
- 3 = HSDPA turbo code
- 4 = UMTS turbo code with rate matching
- 5 = WiMAX duobinary tailbiting turbo code (CTC)
- 6 = DVB-RCS duobinary tailbiting turbo code

Convolutional Codes

- **Only rate 1/n mother codes supported.**
  - Can puncture to higher rate.
- **Code is always terminated by a tail.**
  - Can puncture out the tail.
- **sim_param(record).**
  - g1 = {binary matrix}
    - Example: (133,171) code from Proakis
      - g1 = [1 0 1 1 0 1 1
              1 1 1 1 0 0 1];
    - Constraint length = number of columns
    - Rate 1/n where n is number of rows.
  - nsc_flag1 = {scalar integer}
    - 0 for RSC
    - 1 for NSC
- **Can handle cyclic block codes as a rate 1 terminated RSC code**

Convolutional Codes: Decoding Algorithms

- sim_param(record).decoder_type = {integer scalar}

  negative value for Viterbi algorithm

  0 = log-MAP (approximated linearly)

  1 = max-log-MAP

  2 = constant-log-MAP

  3 and 4 other implementations of log-MAP

- Decodes over entire trellis (no sliding window traceback)

Punctured Convolutional Codes

- **sim_param(record).**
  - pun_pattern1 = {binary matrix}
    - Puncturing pattern
    - n rows
    - arbitrary number of columns (depends on puncture period)
    - 1 means keep bit, 0 puncture it.
    - number greater than 1 is number of times to repeat bit.
  - tail_pattern1 = {binary matrix}
    - tail can have its own puncturing pattern.

Turbo Codes

- **sim_param(record).**
  - Parameters for first constituent code
    - g1
    - nsc_flag1
    - pun_pattern1
    - tail_pattern1
  - Parameters for second constituent code
    - g2
    - nsc_flag2
    - pun_pattern2
    - tail_pattern2

Turbo Codes (cont'd)

- **sim_param(record).**
  - code_interleaver = {string}
    - A string containing the command used to generate the interleaver.
    - Examples include:
      - "CreateUmtsInterleaver(5114)" % UMTS interleaver.
      - "CreateLTEInterleaver(6144)" % LTS interleaver.
      - "CreateCCSDSInterleaver(8920)" % CCSDS interleaver.
      - "randperm(40)-1" % a random interleaver of length 40.
      - Can replace above lengths with other valid lengths.
  - decoder_type = {integer scalar}
    - Same options as for convolutional codes (except no Viterbi allowed).
  - max_iterations = {integer scalar}
    - Number of decoder iterations.
    - Decoder will automatically halt once codeword is correct.
  - plot_iterations = {integer scalar}
    - Which iterations to plot, in addition to max_iterations

UMTS Rate Matching

- sim_param(record)
  - framesize = {integer scalar}
    - number of data bits
  - code_bits_per_frame = {integer scalar}
    - number of code bits
- When code_configuration = 4, automatically determines rate matching parameters according to UMTS (25.212)

HSDPA Specific Parameters

- **sim_param(record).**
  - N_IR = {integer scalar}
    - Size of the virtual IR buffer
  - X_set = {integer vector}
    - Sequence of redundancy versions (one value per ARQ transmission)
  - P = {integer scalar}
    - Number of physical channels per turbo codeword
- **Examples from HSET-6 TS 25.101**
  - N_IR = 9600
  - QPSK
    - framesize = 6438
    - X_set = [0 2 5 6]
    - P = 5 (i.e. 10 physical channels used for 2 turbo codewords)
  - 16-QAM
    - framesze = 9377
    - X_set = [6 2 1 5]
    - P = 4 (i.e. 8 physical channels used for 2 turbo codewords)

LDPC

- **sim_param(record).**
  - parity_check_matrix = {string}
    - A string used to generate the parity check matrix
  - decoder_type
    - 0 Sum-product (default)
    - 1 Min-sum
    - 2 Approximate-min-star
  - max_iterations
    - Number of decoder iterations.
    - Decoder will automatically halt once codeword is correct.
  - plot_iterations
    - Which iterations to plot, in addition to max_iterations

**LDPC

- **Specifying a Parity Check Matrix**
  - Several methods available
    - DVB-S2 type via CML function 'InitializeDVBS2()'
    - Flat-text alist file [1]
    - CML-native H_rows, H_cols format stored as .mat file

  - All parity check matrix data files must be stored in
    <CMLROOT>/data/ldpc

    Example:
     <CMLROOT>/data/ldpc/test_ldpc_hmat.alist

[1] http://www.inference.phy.cam.ac.uk/mackay/codes/alist.html

**LDPC

- Specifying a Parity Check Matrix
  Function InitializeDVBS2()
  - Generates parity check matrix conforming to DVB-S2 standard

  - Example

  CML Scenario: LdpcHmat

  Record: 1

**LDPC

- Specifying a Parity Check Matrix
  File-based specification
    - Consider an eIRA LDPC code having rate K/N
    - The parity check matrix has form

$$
H = \begin{bmatrix}
 & & 1\ 1 & \\
 & & \ \ 1\ 1 & \\
 & (1) & \quad\ \cdot & \\
 & & \quad\ \ \cdot & \\
 & & \quad\ \ \ \cdot & \\
 & & \quad\quad 1\ 1 & \\
 & & \quad\quad\ \ \ 1 &
\end{bmatrix}
$$

    - User-specified parity check matrices define the N-K x K sub-matrix denoted by (1)

**LDPC

- **Specifying a Parity Check Matrix**
  H_rows, H_cols format
  - Parity check matrix stored as .mat file
  - File contains two variables, H_rows and H_cols
    - H_rows: N-K x R double matrix
    - H_cols: K x C double matrix

    where R denotes the maximum weight of all rows and C denotes the maximum weight of all columns
  - First dimension of H_rows and H_cols denotes a particular parity check matrix row or column, respectively.
  - Each entry of H_rows and H_cols denotes the location of a "1" for row or column specified by the first dimension with entry "0" denoting the absence of a "1".

**LDPC

- Specifying a Parity Check Matrix
  H_rows, H_cols format
  - Trivial Example

$$H = \begin{bmatrix} 1 & 1 & & 1 & 1 & 1 & \\ 1 & & & 1 & & 1 & 1 \\ & & 1 & 1 & & & 1 & 1 \\ & & & 1 & & & & 1 \end{bmatrix}$$

$$H\_rows = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 4 & 0 \\ 2 & 3 & 0 \\ 3 & 0 & 0 \end{bmatrix} \qquad H\_cols = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 3 & 4 \\ 1 & 2 \end{bmatrix}$$

**LDPC

- Specifying a Parity Check Matrix
  alist format

  - Parity check matrix stored as .alist file in
    - <CMLROOT>/data/ldpc

  - Parity check matrix must obey eIRA constraint

  - For a full description of the alist format, see

  http://www.inference.phy.cam.ac.uk/mackay/codes/alist.html

**LDPC

- **Specifying a Parity Check Matrix**
  Examples

CML Scenario: LdpcHmat

- Record 1: InitializeDVBS2()

- Record 2: alist
  - Data file: <CMLROOT>/data/ldpc/test_ldpc_hmat.alist

- Record 3: H_rows, H_cols
  - Data file: <CMLROOT>/data/ldpc/test_ldpc_hmat.mat

Block Fading

- For coded simulations, block fading is supported.
- Sim_param(record).channel = 'block'
- Sim_param(record).blocks_per_frame
  - The number of independent blocks per frame
- Example, HSDPA with independent retransmissions
  - blocks_per_frame = length(X_set);

Exercises

- ■ Simulate
  - – A convolutional code with g=(7,5) over AWGN with BPSK
  - – The same convolutional code punctured to rate 3/4.
  - – The UMTS turbo code with 16-QAM
    - • Unpunctured w/ 640 input bits
    - • Punctured to force the rate to be 1/2.
    - • Compare log-MAP and max-log-MAP
  - – HSDPA
    - • HSET-6
    - • Quasi-static block fading

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Noisy Channel Coding Theorem (Shannon 1948)

■ Consider a memoryless channel with input X and output Y
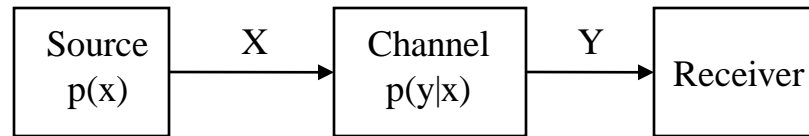
  – The channel is completely characterized by p(x,y)

■ The *capacity* C of the channel is

$$C = \max_{p(x)} I(X;Y) = \max_{p(x)} \left\{ \iint p(x, y) \log \frac{p(x, y)}{p(x)p(y)} \, dxdy \right\}$$

  – where $I(X,Y)$ is the (average) *mutual information* between X and Y.

■ The channel capacity is an upper bound on *information rate* r.

  – There exists a code of rate r < C that achieves reliable communications.
  – "Reliable" means an arbitrarily small error probability.

Capacity of the AWGN Channel with Unconstrained Input

■ **Consider the one-dimensional AWGN channel**

The input X is drawn from *any* distribution with average energy $E[X^2] = E_s$
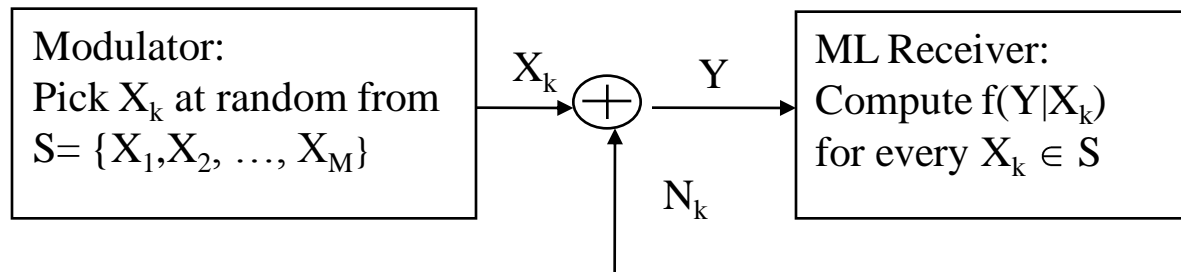
$$X \xrightarrow{} \oplus \xrightarrow{} Y = X+N$$

N~zero-mean white Gaussian with energy $E[N^2] = N_0/2$
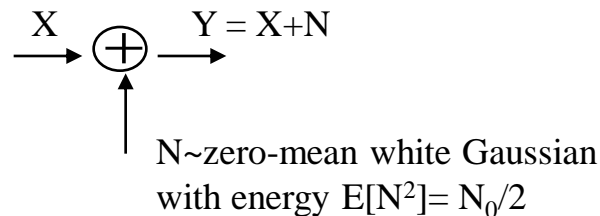
■ **The capacity is**

$$I(X; Y) = \frac{1}{2} \log_2 \left(\frac{2E_s}{N_o} + 1 \right) \qquad \text{bits per channel use}$$

■ **The X that attains capacity is Gaussian distributed.**
  – Strictly speaking, Gaussian X is not practical.

Capacity of the AWGN Channel with a Modulation-Constrained Input

- Suppose X is drawn with equal probability from the finite set $S = \{X_1, X_2, \ldots, X_M\}$

| Modulator: | | ML Receiver: |
| Pick X_k at random from $S = \{X_1, X_2, \ldots, X_M\}$ | $X_k \rightarrow \oplus \rightarrow Y$ with N_k | Compute $f(Y|X_k)$ for every $X_k \in S$ |

  – where $f(Y|X_k) = \kappa\, p(Y|X_k)$ for any κ common to all X_k

- Since p(x) is now fixed

$$\lfloor I(X;Y) \rceil = I(X;Y)$$

  – i.e. calculating capacity boils down to calculating mutual info.

Entropy and Conditional Entropy

- Mutual information can be expressed as:

$$I(X;Y) = H(X) - H(X/Y)$$

- Where the ***entropy*** of X is

$$H(X) = E[h(X)] = \int p(x)h(x)dx$$

where $\quad \underbrace{h(x) = \log \dfrac{1}{p(x)} = -\log p(x)}_{\text{self-information}}$

- And the ***conditional entropy*** of X given Y is

$$H(X/Y) = E[h(X/Y)] = \iint p(x,y)h(x/y)dxdy$$

where $\quad h(x|y) = -\log p(x|y)$

Calculating Modulation-Constrained Capacity

■ To calculate:

$$I(X;Y) = H(X) - H(X/Y)$$

■ We first need to compute H(X)

$$H(X) = E[h(X)]$$

$$= E\left[\log\frac{1}{p(X)}\right]$$

$$= E[\log M] \qquad p(X) = \frac{1}{M}$$

$$= \log M$$

■ Next, we need to compute H(X|Y)=E[h(X|Y)]

– This is the "hard" part.

– In some cases, it can be done through numerical integration.

– Instead, let's use Monte Carlo simulation to compute it.
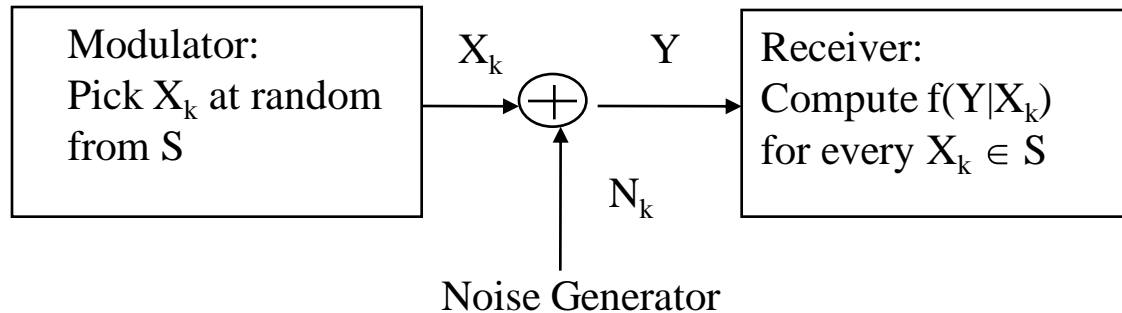
Step 1: Obtain p(x|y) from f(y|x)

Modulator:
Pick X_k at random
from S

X_k

Y

Receiver:
Compute $f(Y|X_k)$
for every $X_k \in S$

N_k

Noise Generator
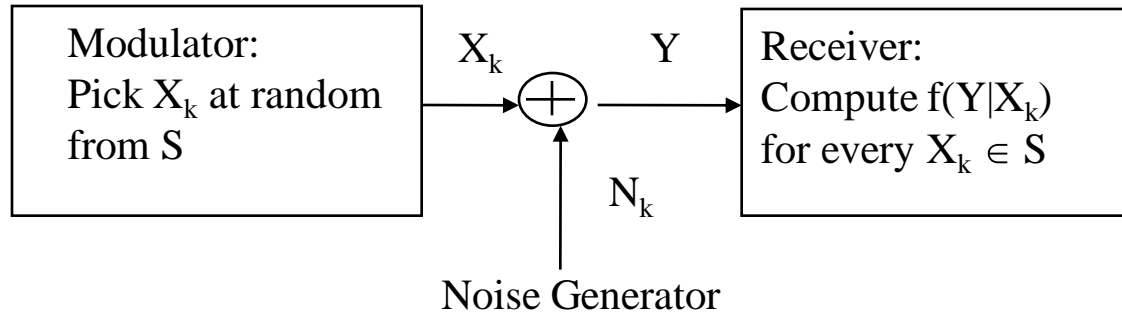
■ Since

$$\sum_{x' \in S} p(x'|y) = 1$$

■ We can get p(x|y) from

$$p(x|y) = \frac{p(x|y)}{\sum_{x' \in S} p(x'|y)} = \frac{\dfrac{p(y|x)p(x)}{p(y)}}{\sum_{x' \in S} \dfrac{p(y|x')p(x')}{p(y)}} = \frac{f(y|x)}{\sum_{x' \in S} f(y|x')}$$
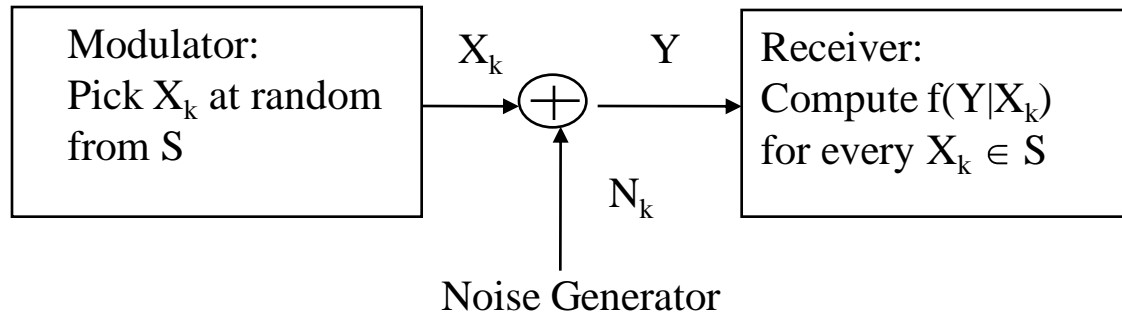
Step 2: Calculate h(x|y)

Modulator: Pick X_k at random from S → X_k → ⊕ ← N_k (Noise Generator) → Y → Receiver: Compute f(Y|X_k) for every $X_k \in S$

- Given a value of x and y (from the simulation) compute

$$p(x|y) = \frac{f(y|x)}{\displaystyle\sum_{x' \in S} f(y|x')}$$

- Then compute

$$h(x/y) = -\log p(x/y) = -\log f(y/x) + \log \sum_{x' \in S} f(y/x')$$
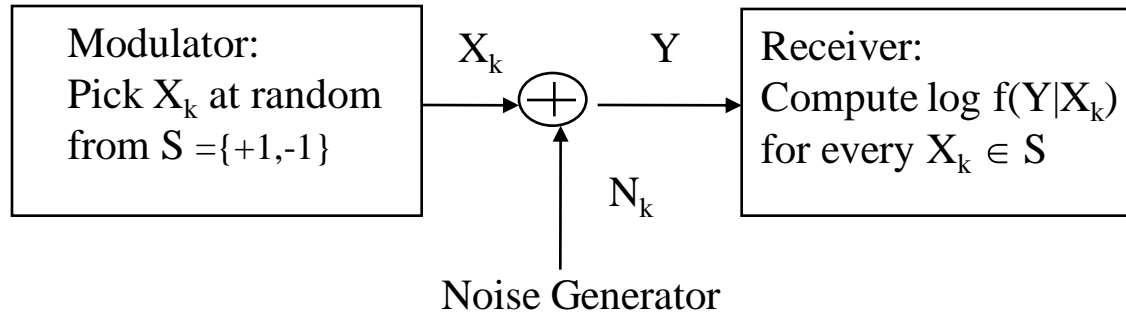
Step 3: Calculating H(X|Y)

■ Since:

$$H(X/Y) = E[h(X/Y)] = \iint p(x, y) h(x/y) \, dx \, dy$$

■ Because the simulation is ergodic, H(X|Y) can be found by taking the sample mean:

$$\frac{1}{N} \sum_{n=1}^{N} h(X^{(n)}/Y^{(n)})$$

■ where $(X^{(n)}, Y^{(n)})$ is the nth realization of the random pair (X,Y).

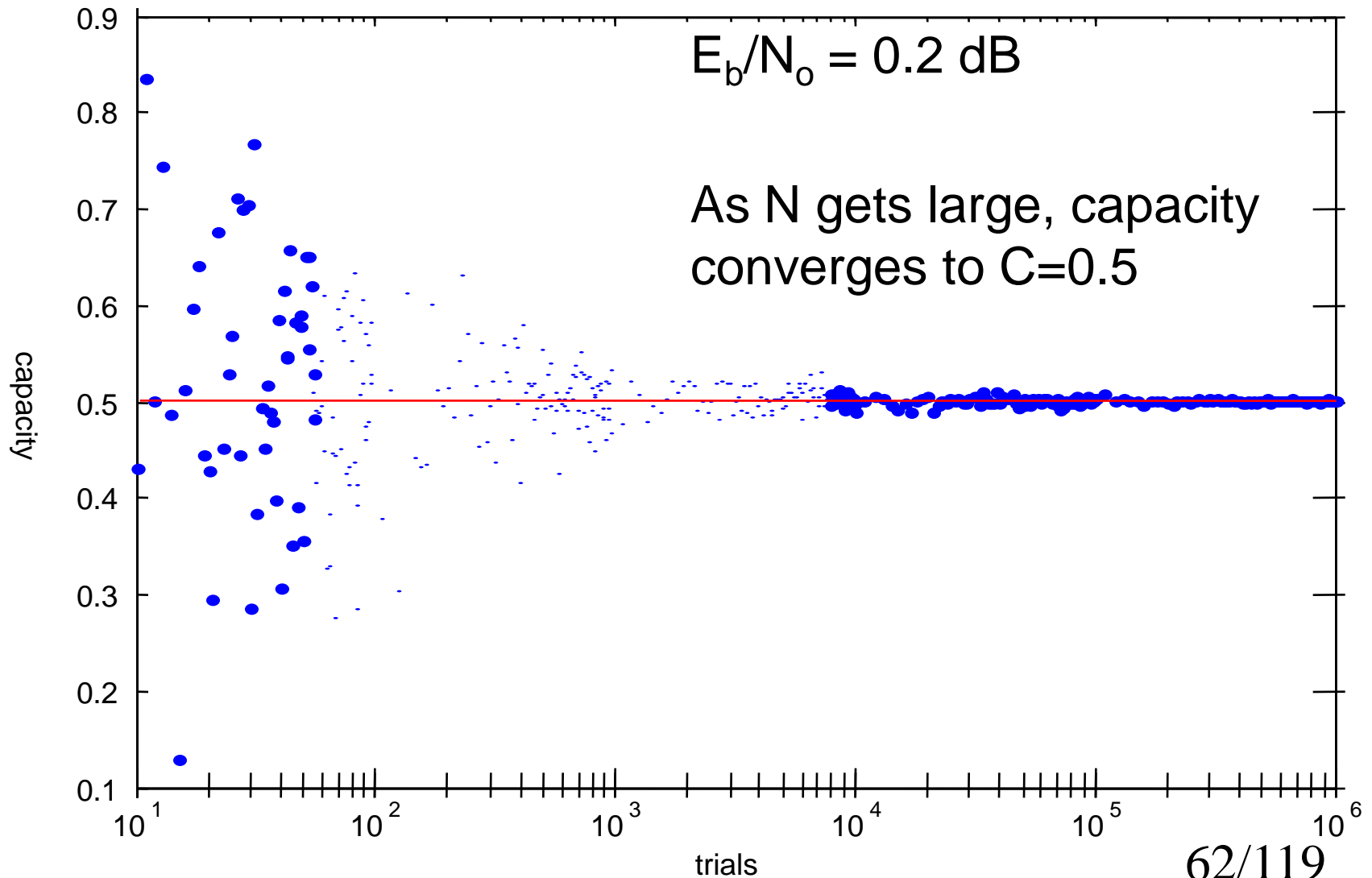 – i.e. the result of the nth simulation trial.

Example: BPSK

Modulator:
Pick X_k at random
from $S = \{+1, -1\}$

X_k Y

Receiver:
Compute $\log f(Y|X_k)$
for every $X_k \in S$

N_k
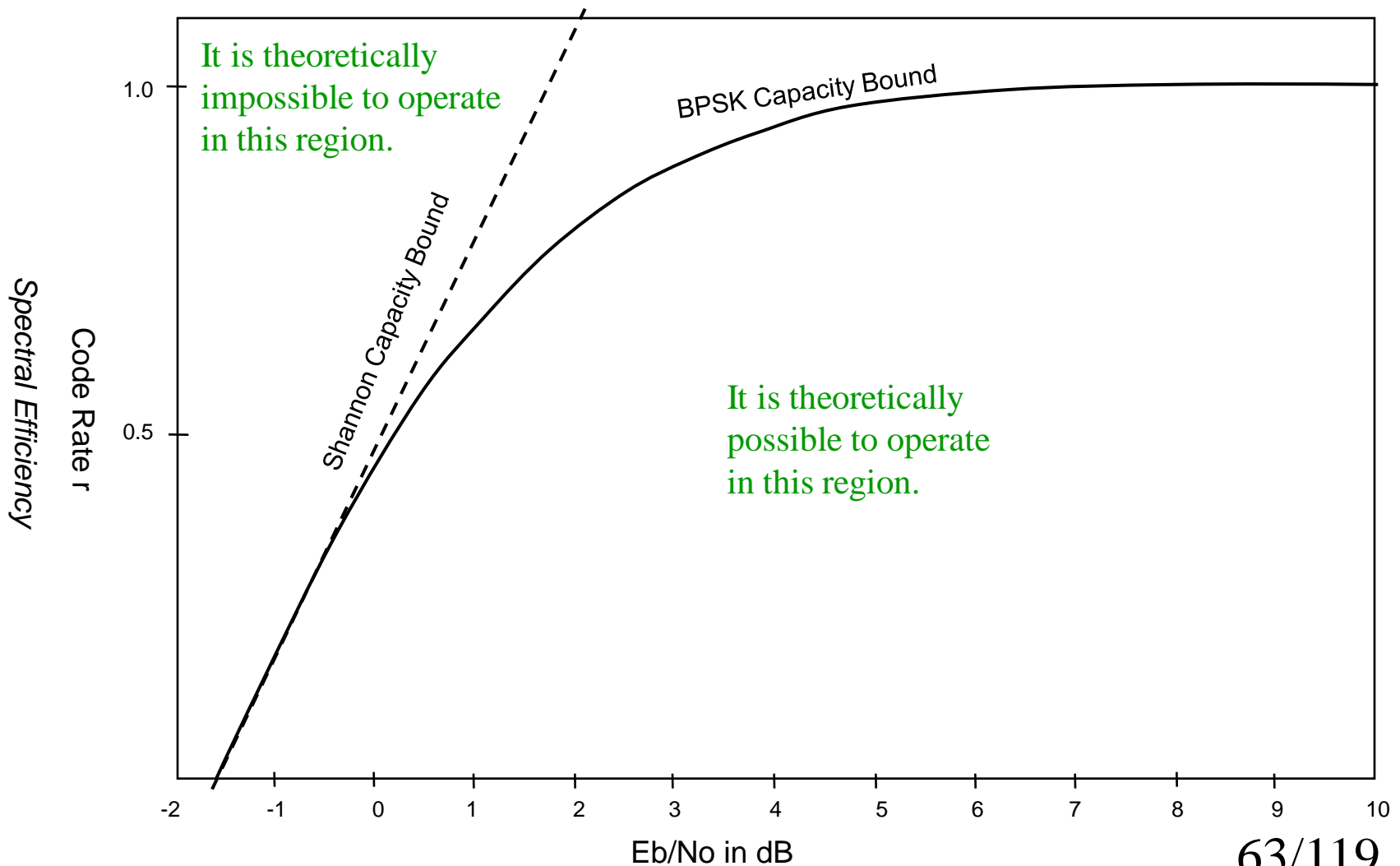
Noise Generator

■ Suppose that $S = \{+1, -1\}$ and N has variance $N_0/2E_s$

■ Then:

$$\log f(y|x) = -\frac{E_s}{N_o}\|y - x\|^2$$

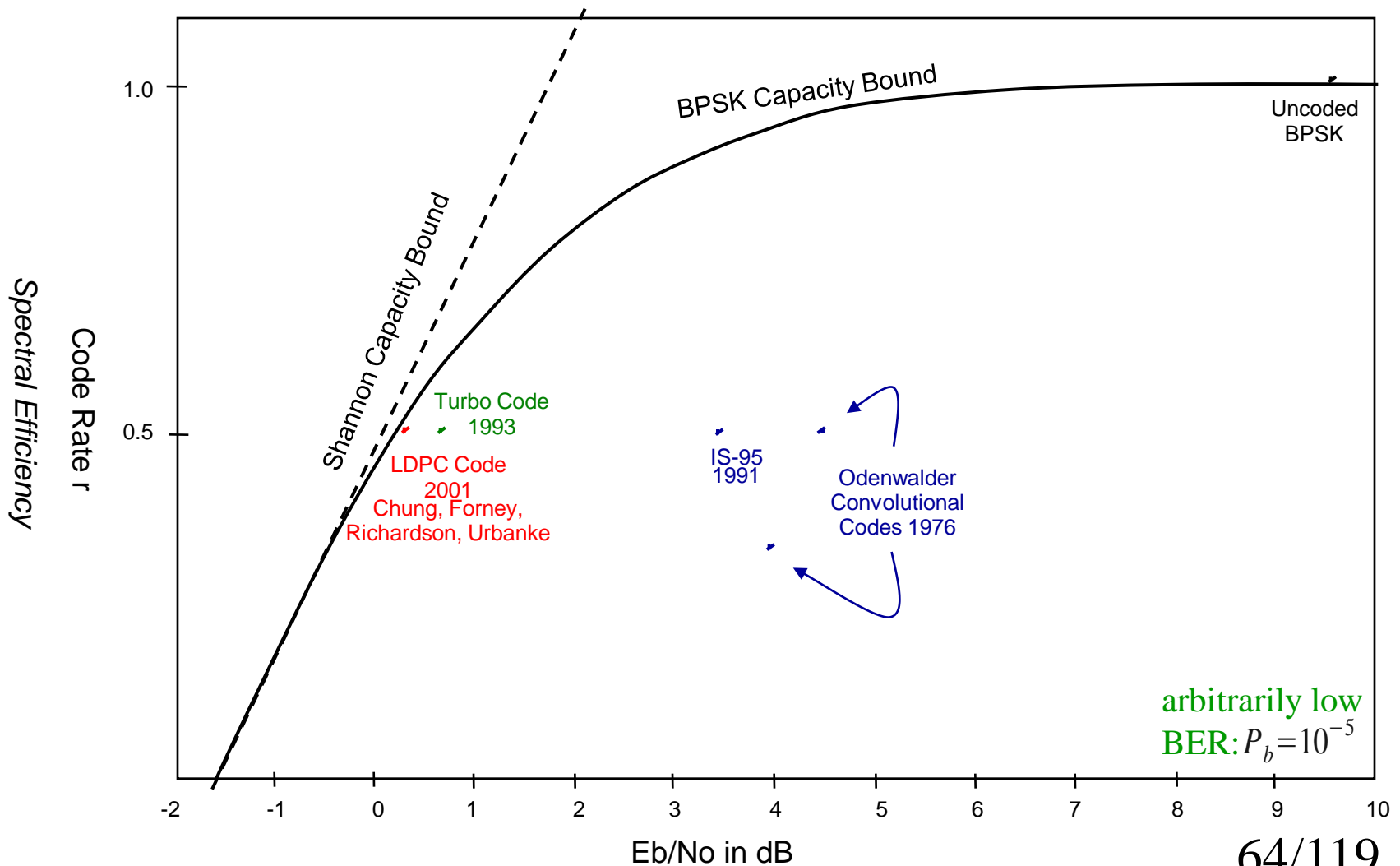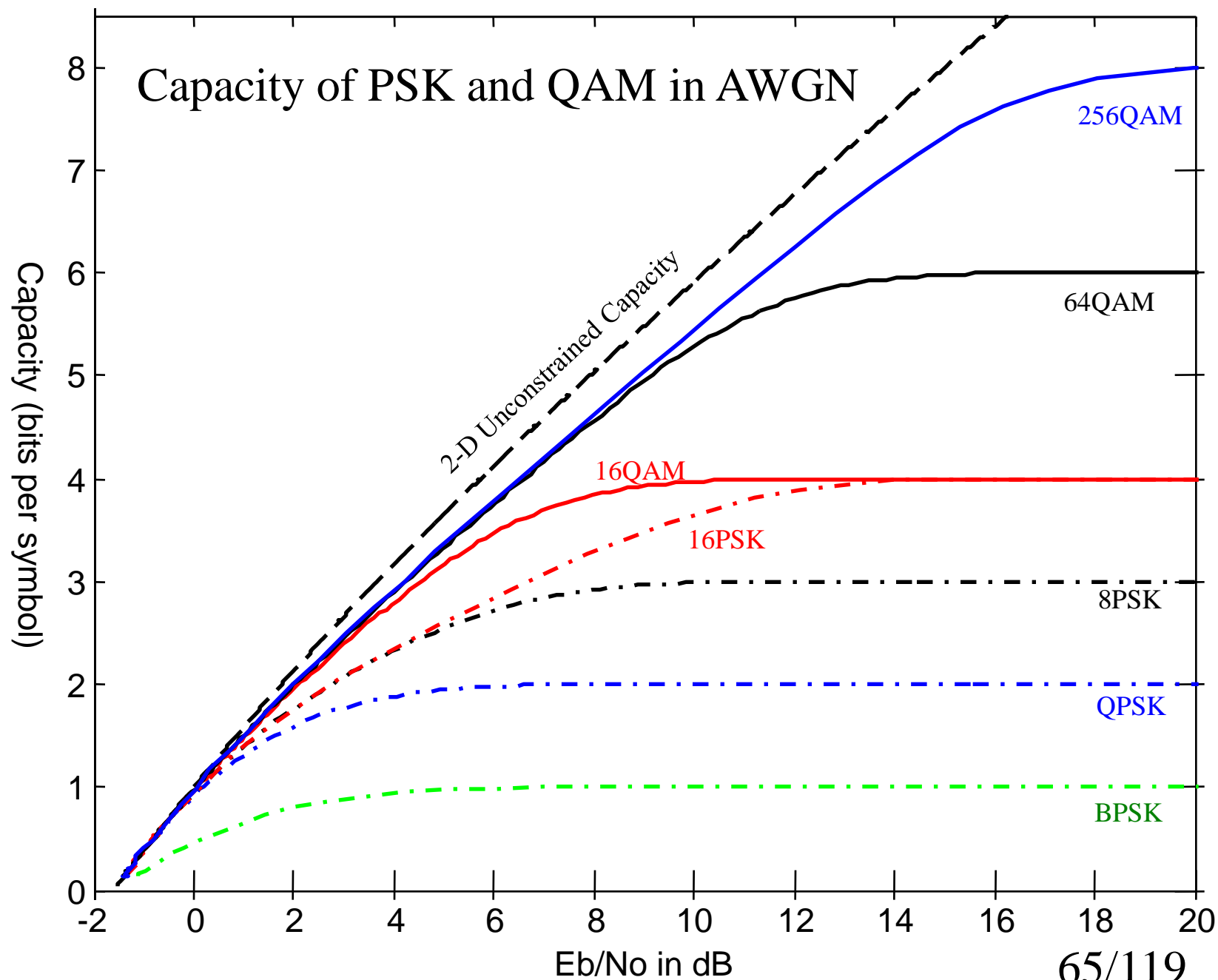# BPSK Capacity as a Function of Number of Simulation Trials

$E_b/N_o = 0.2$ dB

As N gets large, capacity converges to C=0.5

capacity

trials

Unconstrained vs.
BPSK Constrained Capacity

It is theoretically impossible to operate in this region.

BPSK Capacity Bound

Shannon Capacity Bound

It is theoretically possible to operate in this region.

Spectral Efficiency

Code Rate r

1.0

0.5

Eb/No in dB

-2   -1   0   1   2   3   4   5   6   7   8   9   10

Power Efficiency of Standard Binary Channel Codes

Spectral Efficiency

Code Rate r

1.0

0.5

Shannon Capacity Bound

BPSK Capacity Bound

Uncoded BPSK

Turbo Code
1993

LDPC Code
2001
Chung, Forney,
Richardson, Urbanke

IS-95
1991

Odenwalder
Convolutional
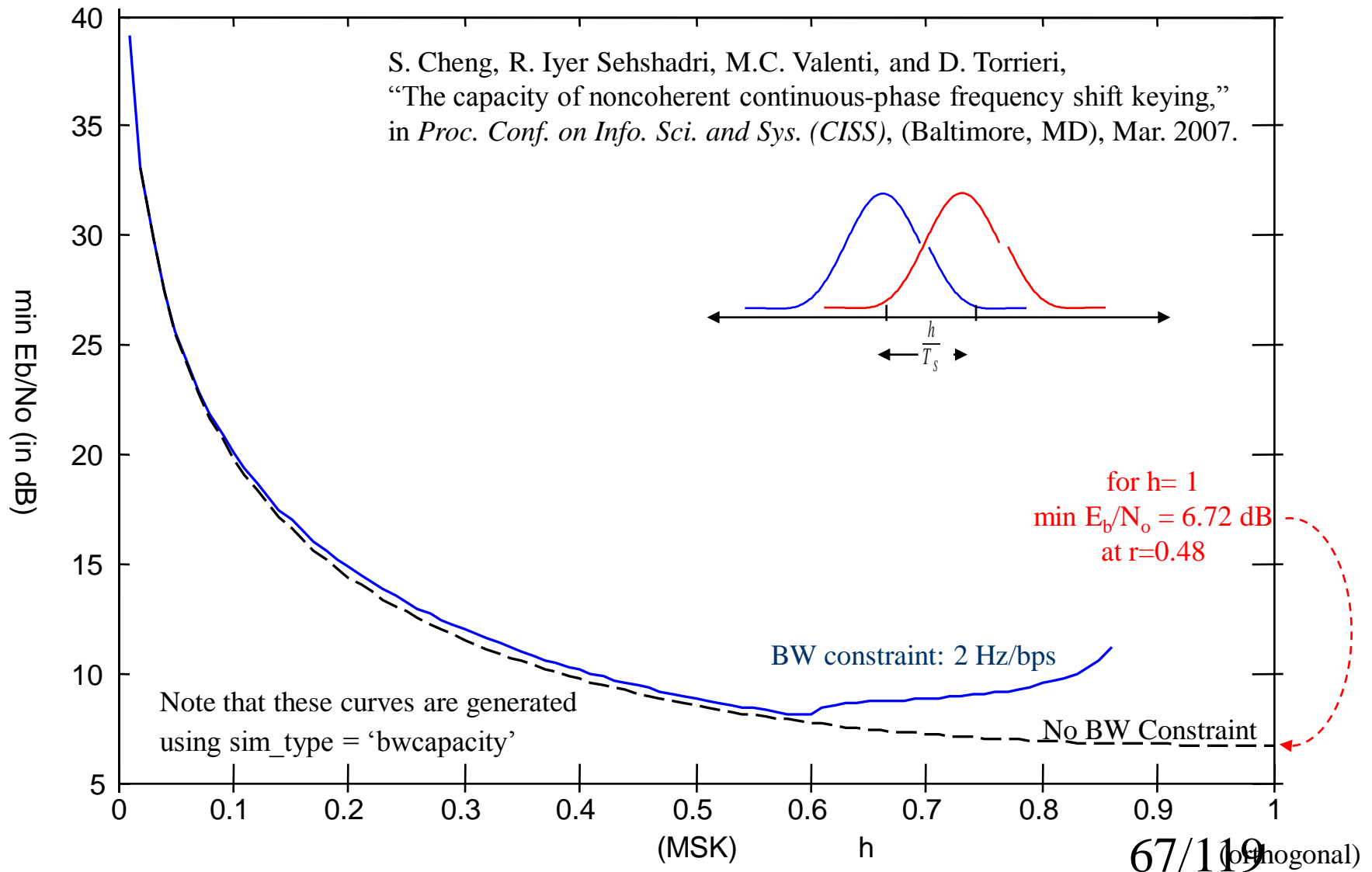Codes 1976

arbitrarily low
BER: $P_b = 10^{-5}$

Eb/No in dB

Capacity of PSK and QAM in AWGN

Capacity of Noncoherent Orthogonal FSK in AWGN

W. E. Stark, "Capacity and cutoff rate of noncoherent FSK with nonselective Rician fading," *IEEE Trans. Commun.*, Nov. 1985.

M.C. Valenti and S. Cheng, "Iterative demodulation and decoding of turbo coded M-ary noncoherent orthogonal modulation," *IEEE JSAC*, 2005.
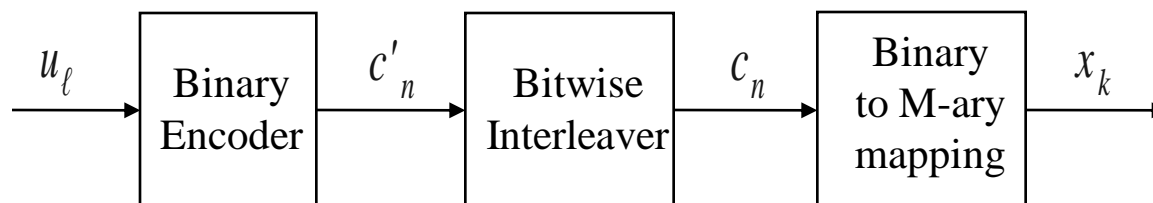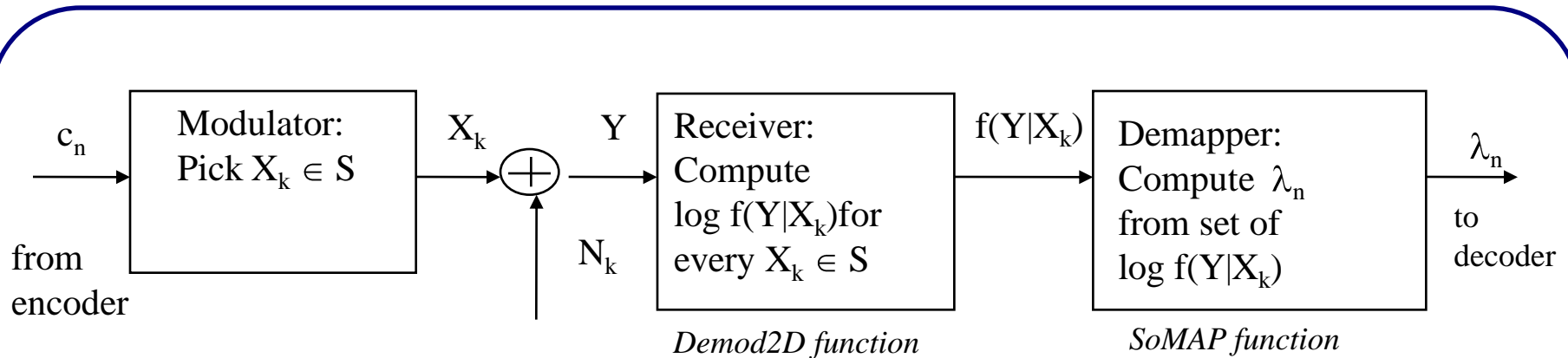
Noncoherent combining penalty

min E_b/N_o = 6.72 dB at r=0.48

M=2

M=4

M=16

M=64

Minimum Eb/No (in dB)

Rate R (symbol per channel use)
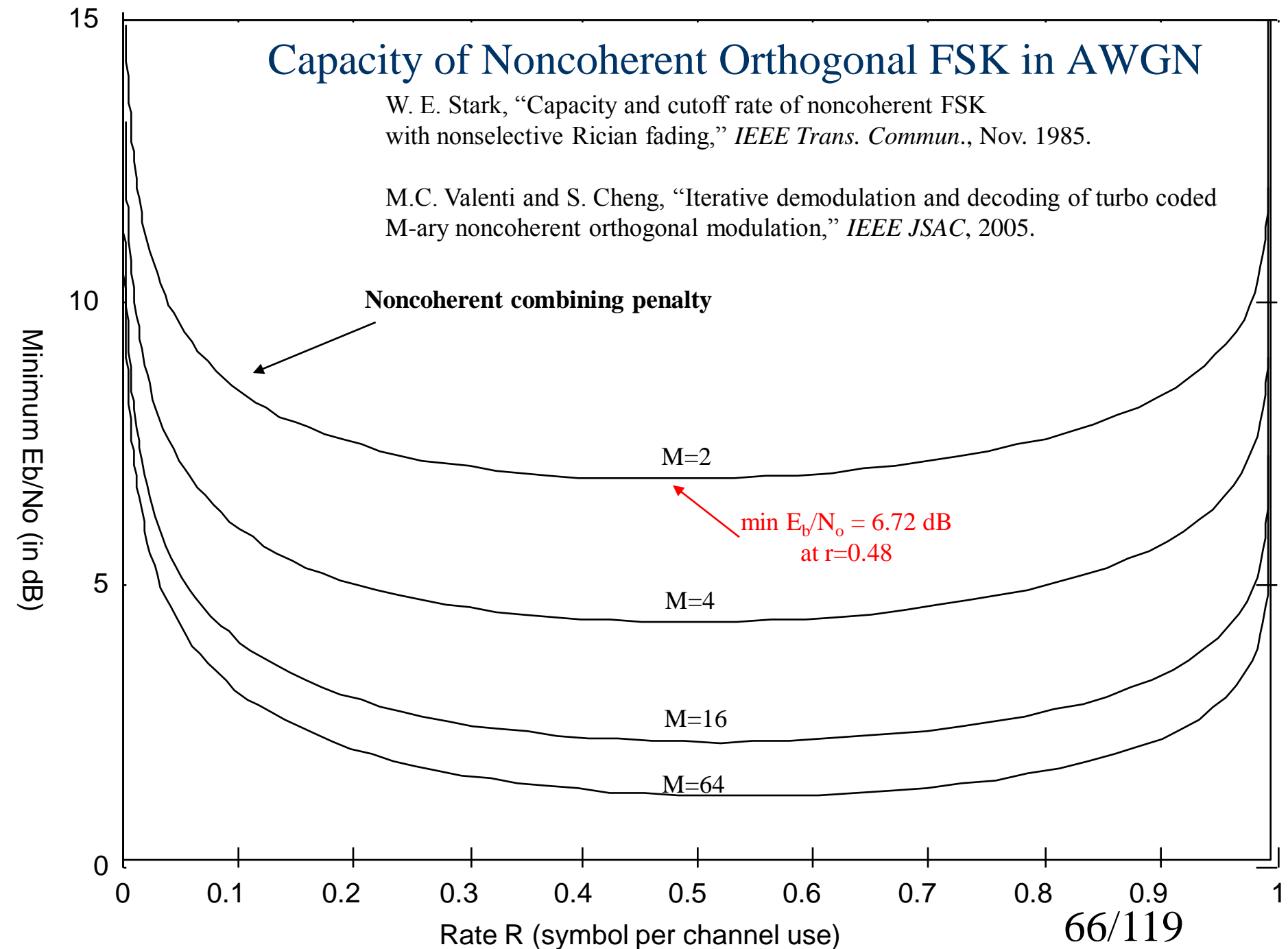
Capacity of Nonorthogonal CPFSK

S. Cheng, R. Iyer Sehshadri, M.C. Valenti, and D. Torrieri,
"The capacity of noncoherent continuous-phase frequency shift keying,"
in *Proc. Conf. on Info. Sci. and Sys. (CISS)*, (Baltimore, MD), Mar. 2007.

for h= 1
min E_b/N_o = 6.72 dB
at r=0.48

BW constraint: 2 Hz/bps

No BW Constraint

Note that these curves are generated
using sim_type = 'bwcapacity'

min Eb/No (in dB)

(MSK)   h   (orthogonal)

BICM
(Caire 1998)

■ Coded modulation (CM) is required to attain the aforementioned capacity.

  – Channel coding and modulation handled jointly.

  – Alphabets of code and modulation are matched.

  – e.g. trellis coded modulation (Ungerboeck); coset codes (Forney)

■ Most off-the-shelf capacity approaching codes are binary.

■ A pragmatic system would use a binary code followed by a bitwise interleaver and an M-ary modulator.

  – Bit Interleaved Coded Modulation (BICM).

u_ℓ → | Binary Encoder | → c'_n → | Bitwise Interleaver | → c_n → | Binary to M-ary mapping | → x_k

BICM Receiver

Demod2D function   *SoMAP function*

- ■ The symbol likelihoods must be transformed into bit log-likelihood ratios (LLRs):

$$\lambda_n = \log \frac{\sum\limits_{X_k \in S_n^{(1)}} f(Y \mid X_k)}{\sum\limits_{X_k \in S_n^{(0)}} f(Y / X_k)}$$

- – where $S_n^{(1)}$ represents the set of symbols whose nth bit is a 1.
- – and $S_n^{(0)}$ is the set of symbols whose nth bit is a 0.
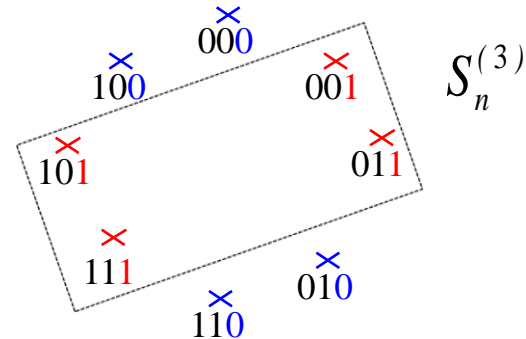
BICM Capacity
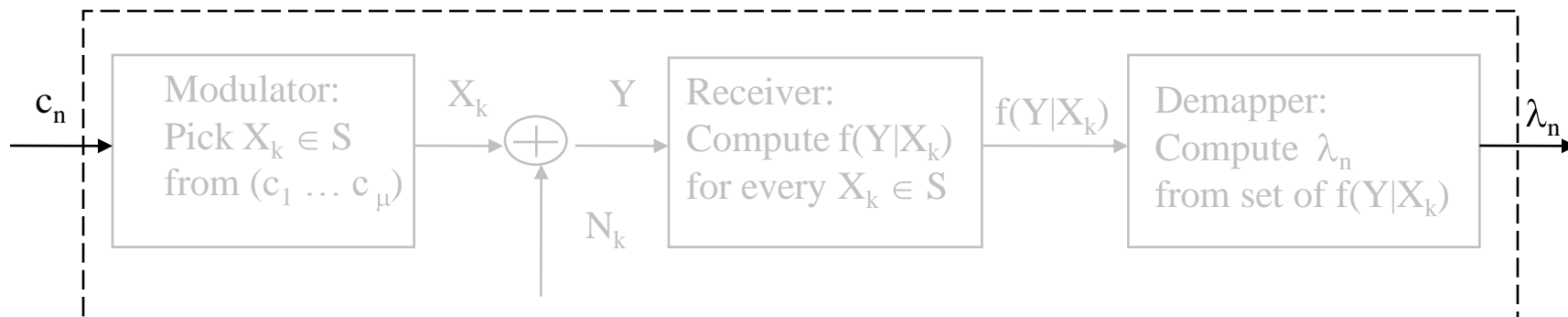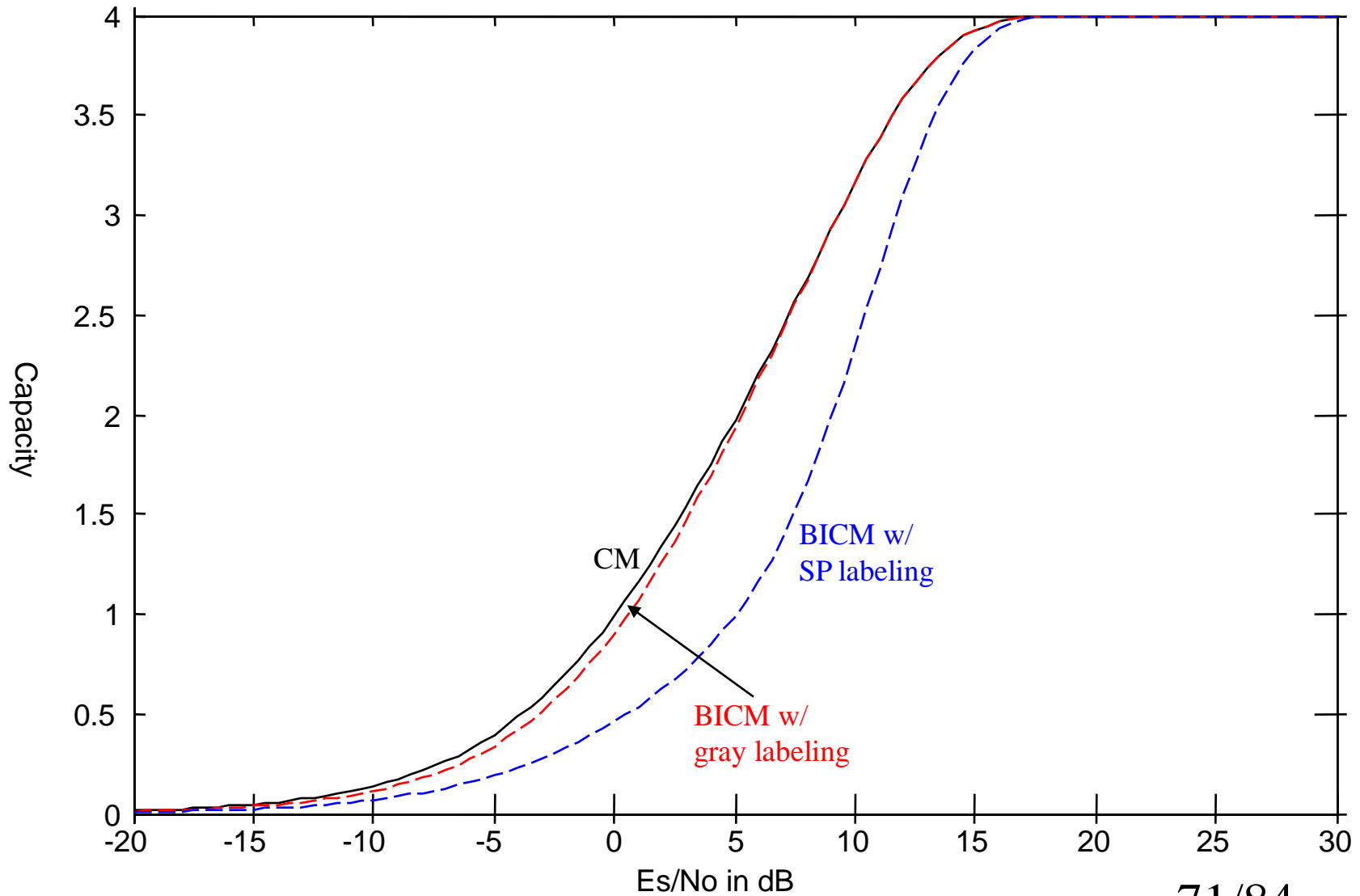

- Can be viewed as $\mu=\log_2 M$ binary parallel channels, each with capacity

$$C_n = I\left(c_n, \lambda_n\right)$$

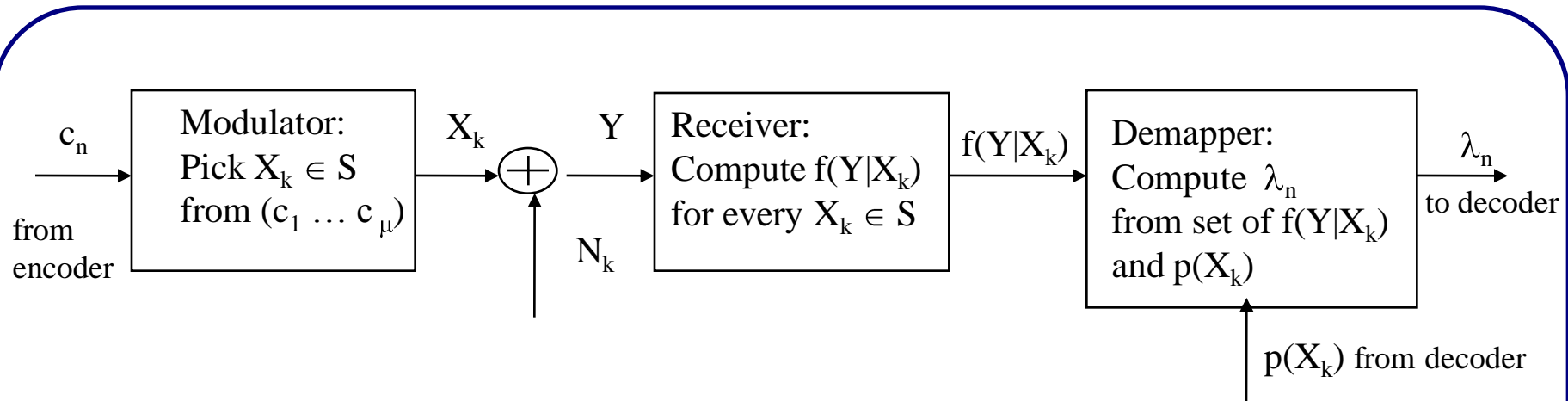- Capacity over parallel channels adds:

$$C = \sum_{n=1}^{\mu} C_n$$

- As with the CM case, Monte Carlo integration may be used.

CM vs. BICM Capacity for 16QAM

CM

BICM w/
SP labeling

BICM w/
gray labeling

Capacity

Es/No in dB

BICM-ID
(Li & Ritcey 1997)

Diagram flow: from encoder c_n → Modulator: Pick $X_k \in S$ from $(c_1 \ldots c_\mu)$ → X_k → $\oplus$ (with N_k) → Y → Receiver: Compute $f(Y|X_k)$ for every $X_k \in S$ → $f(Y|X_k)$ → Demapper: Compute λ_n from set of $f(Y|X_k)$ and $p(X_k)$ → λ_n to decoder. $p(X_k)$ from decoder.

- **A SISO decoder can provide side information to the demapper in the form of a priori symbol likelihoods.**
  - BICM with Iterative Detection The demapper's output then becomes

$$\lambda_n = \log \frac{\sum_{X_k \in S_n^{(1)}} f(Y / X_k) p(X_k)}{\sum_{X_k \in S_n^{(0)}} f(Y / X_k) p(X_k)}$$

Capacity Simulations in CML

- sim_param(record).sim_type = 'capacity'
- Exact same parameters as for uncoded simulations
  - SNR
  - SNR_type = 'Es/No in dB'
  - framesize
  - modulation
  - mod_order
  - channel
  - bicm
  - demod_type
  - max_trials

Exercises

- **Determine the capacity for**
  - BPSK in AWGN
  - 64QAM with gray labeling in AWGN
  - 64QAM with gray labeling in Rayleigh fading
- **Setup BICM-ID for**
  - 16-QAM with SP mapping in AWGN and (7,5) CC.

Outline

1. CML overview
   - What is it? How to set it up and get started?

2. Uncoded Modulation
   - Simulate uncoded BPSK and QAM in AWGN and Rayleigh fading

3. Coded Modulation
   - Simulate a turbo code from UMTS 25.212
   - *Specify a custom eIRA LDPC parity check matrix

4. Ergodic (Shannon) capacity analysis
   - Determine the modulation constrained capacity of BPSK and QAM

5. **EXIT analysis**
   - **Determine the EXIT characteristic for BPSK in AWGN for a particular LDPC degree distribution**

**EXIT Analysis

- EXIT analysis may be applied to design LDPC code degree distributions which optimize decoding performance

Variable Nodes

$d_{v,1} = 3$ $d_{v,2} = 2$
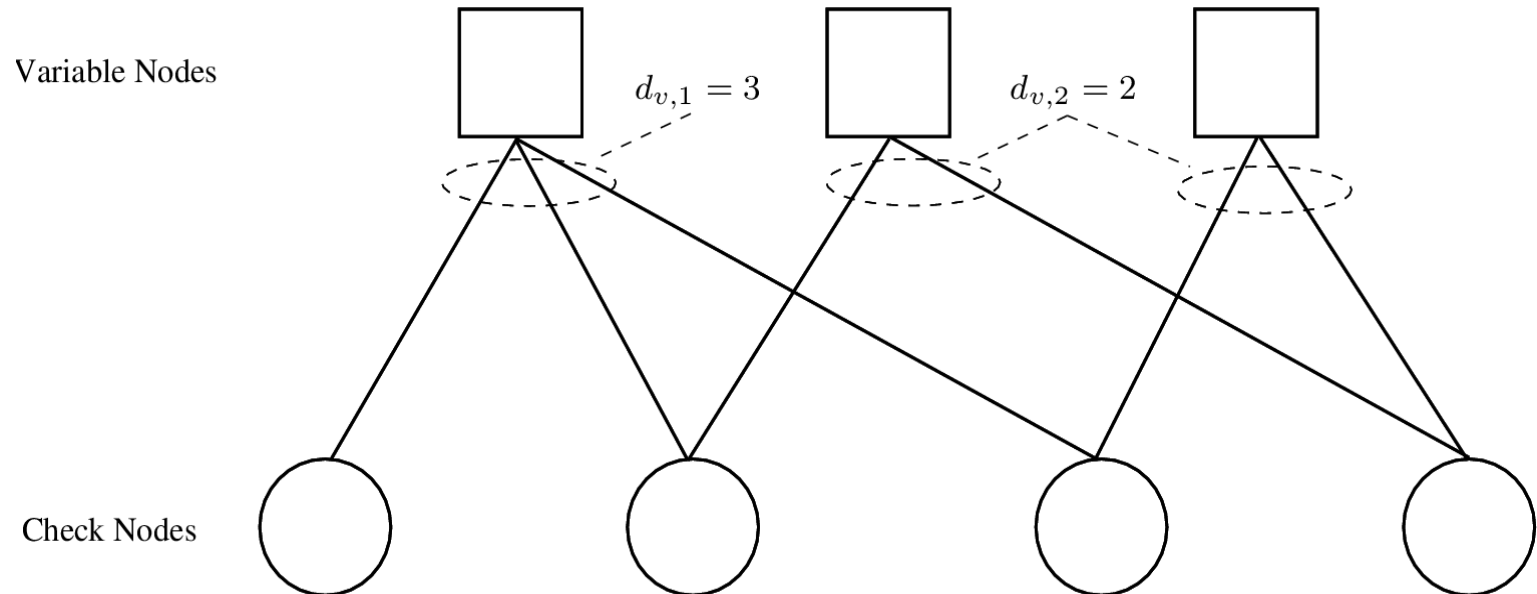
Check Nodes

**EXIT Analysis

- Design goal: match mutual information transfer characteristics of LDPC decoders through selection of variable node degrees
  - VND - variable node decoder
  - CND - check node decoder

- VND decoder captures characteristic of detector and variable node decoder

- CND curve depends only on channel code parameters

**EXIT Analysis

– First step of generating VND transfer characteristic is to compute the mutual information at the input and output of the detector

$$I_{E,DET}(I_A, E_S / N_0)$$

– where

I_A mutual information of a-prior LLRs at detector input

– Any detector supported by CML may be analyzed.

**EXIT Analysis

– The extrinsic information at the output of the combined variable node decoder/detector is

$$I_{E,VND}(I_A, d_v, E_S/N_0) = \ldots$$

$$J\left(\sqrt{(d_v - 1)[J^{-1}(I_A)^2] + [J^{-1}(I_{E,DET}(I_A, E_S/N_0))]^2}\right)$$

– where

d_v variable node degree

$J()$ function computing the mutual information $I(X, \Lambda(Y))$ [1]

and $Y = X + N$ is an AWGN channel where X is drawn from a BPSK constellation, and $\Lambda(Y)$ is the LLR of the channel output.

[1] S. ten Brink, G. Kramer, and A. Ashikhmin, "Design of low-density parity-check codes for modulation and detection," IEEE Trans. Commun., vol. 52, pp. 670–678, Apr. 2004.

**EXIT Analysis

– Considering an LDPC decoder having multiple variable node degrees (irregular)

$$I_{E,\ VND}(I_A, E_S / N_0) = \sum b_i \cdot I_{E,\ VND}(I_A, d_{v,i}, E_S / N_0)$$

– where

D number of distinct variable node degrees

b_i fraction of edges incident on variable nodes of degree $d_{v,i}$

$d_{v,i}$ i-th variable node degree

**EXIT Analysis - LDPC

- – The extrinsic information at the output of the CND is

$$I_{E,CND}(I_A, d_c) = 1 - J(\sqrt{d_c - 1} \cdot J^{-1}(1 - I_A))$$

- – where

d_c check node degree (check regular)

- – Note that the CND transfer characteristic depends only on the code parameters.

**EXIT Simulation Example

- First step: Simulate detector characteristic
  CML Scenario: ExitP2P

Record: 1

- sim_param(record).sim_type = 'exit'
- sim_param(record).exit_param.exit_phase = 'detector'
- sim_param(record).exit_param.exit_type = 'ldpc'
- sim_param(record).exit_param.requested_IA = 0 : 0.01 : 0.9
- sim_param(record).channel = 'awgn'
- sim_param(record).SNR = [0 : 0.5 : 10]
- sim_param(record).modulation = 'psk'
- sim_param(record).mod_order = 4

**EXIT Simulation Example

■ Second step: Simulate decoder characteristic
  CML Scenario: ExitP2P

Record: 2

- sim_param(record).sim_type = 'exit'
- sim_param(record).exit_param.exit_phase = 'decoder'
- sim_param(record).exit_param.exit_type = 'ldpc'
- sim_param(record).exit_param.requested_IA = 0 : 0.01 : 0.9
- sim_param(record).channel = 'awgn'
- sim_param(record).SNR = [0 : 0.5 : 10]
- sim_param(record).modulation = 'psk'
- sim_param(record).mod_order = 4

**EXIT Simulation Example

■ Second step: Simulate decoder characteristic
   CML Scenario: ExitP2P

Record: 2

- sim_param(record).exit_param.rate = 0.6
- sim_param(record).exit_param.dv = [2   4   19]
- sim_param(record).exit_param.dv_dist = [0.4   0.52 0.08]
- sim_param(record).exit_param.dc = 11
- sim_param(record).exit_param.det_scenario = 'ExitP2P'
- sim_param(record).exit_param.det_record = 1

**EXIT Simulation Example

User Simulation Steps

Start MATLAB and initialize CML

>> CmlStartup

Execute detector simulation

>> CmlSimulate('ExitP2P', 1)

Execute decoder simulation

>> CmlSimulate('ExitP2P', 2)

Plot results

>> CmlPlot('ExitP2P', 2,1)   % Red argument: SNR to plot

Outline

6. Outage analysis

   - Determine the outage probability over block fading channels
   - Determine the outage probability of finite-length codes

7. Digital Network Coding

   - Describe CML implementation of digital network coding

   - Simulate error rate of digital network coding for non-channel-coded 4-FSK in Rayleigh fading

8. The internals of CML

9. Throughput calculation

   - Convert BLER to throughput for hybrid-ARQ

Ergodicity
vs. Block Fading

- Up until now, we have assumed that the channel is *ergodic*.
  - The observation window is large enough that the time-average converges to the statistical average.
- Often, the system might be *nonergodic.*
- Example: *Block fading*

b=1	b=2	b=3	b=4	b=5
γ_1	γ_2	γ_3	γ_4	γ_5

The codeword is broken into B equal length blocks
The SNR changes randomly from block-to-block
The channel is conditionally Gaussian
The instantaneous Es/No for block b is γ_b

Accumulating Mutual Information

- The SNR γ_b of block b is a random.
- Therefore, the mutual information I_b for the block is also random.
  - With a complex Gaussian input, $I_b = \log(1+\gamma_b)$
  - Otherwise the modulation constrained capacity can be used for I_b

b=1	b=2	b=3	b=4	b=5
$I_1 = \log(1+\gamma_1)$	I_2	I_3	I_4	I_5

The mutual information of each block is $I_b = \log(1+\gamma_b)$
Blocks are conditionally Gaussian
The entire codeword's mutual info is the sum of the blocks'

$$I_1^B = \sum_{b=1}^{B} I_b \qquad \text{(Code combining)}$$

Information Outage

- An ***information outage*** occurs after B blocks if

$$I_1^B < R$$

  – where R≤log$_2$M is the rate of the coded modulation

- An outage implies that no code can be reliable for the particular channel instantiation

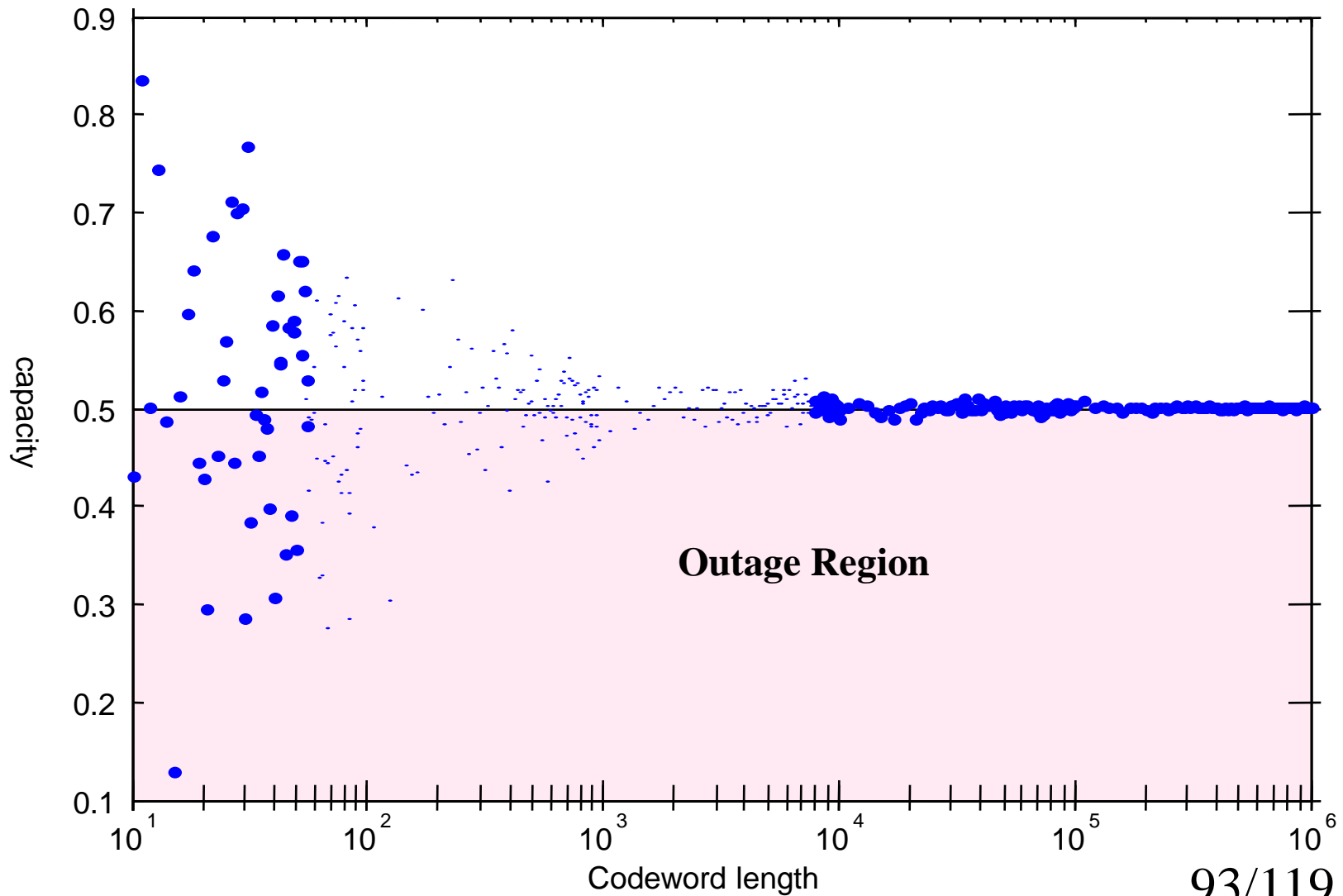- The information outage probability is

$$P_0 = P\left[I_1^B < R\right]$$
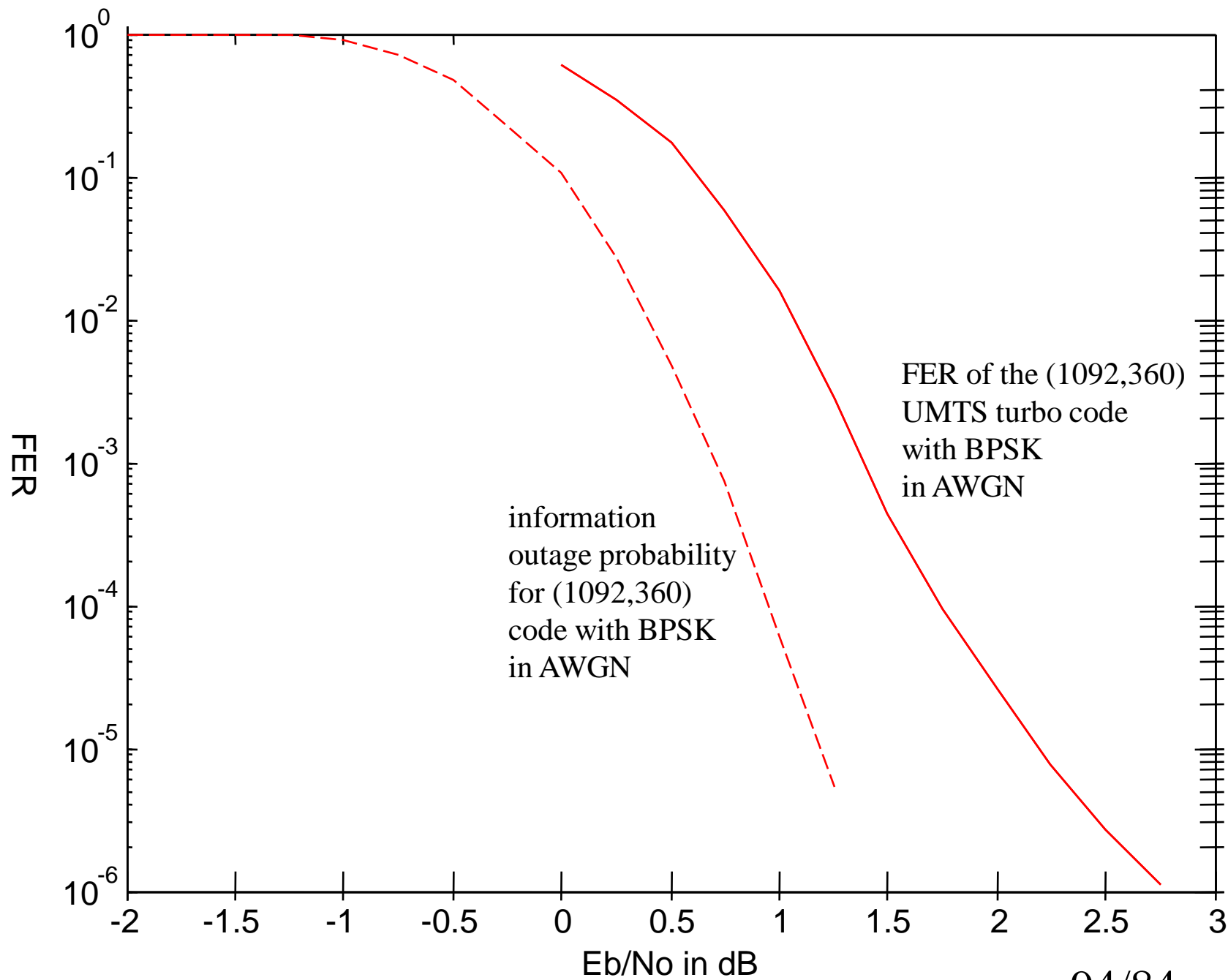
  – This is a practical bound on FER for the actual system.

Information Outage Probability vs. Es/No in dB

16-QAM
R=2
Rayleigh Block Fading

Modulation Constrained Input
Unconstrained Gaussian Input

B=1

B=10   B=4   B=3   B=2

as B →∞, the curve becomes vertical at the ergodic Rayleigh fading capacity bound

Notice the loss of diversity
(see Guillén i Fàbrebas and Caire 2006)
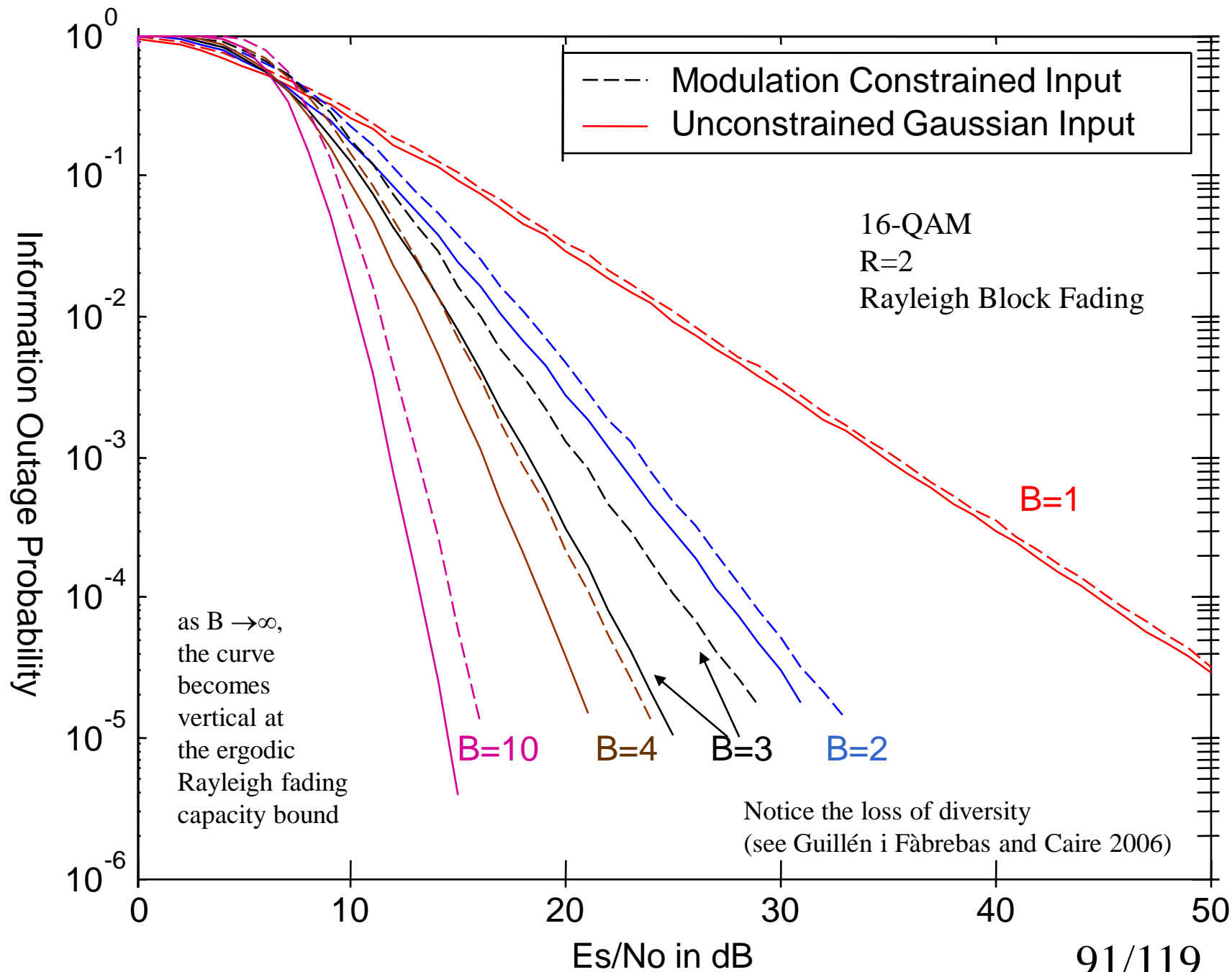
Outage Simulation Type

- **sim_param(record).**
  - blocks_per_frame
    - Assumes block fading channel
  - mod_order
    - 0 for Gaussian input case
  - rate
    - Code rate.
    - Outage whenever MI < rate
  - combining_type = {'code', 'diversity'}
  - input_filename
    - Required if mod_order > 0
    - Contains results of a capacity simulation.
    - Used for a table look-up operation

Finite Length Codeword Effects

FER of the (1092,360)
UMTS turbo code
with BPSK
in AWGN

information
outage probability
for (1092,360)
code with BPSK
in AWGN

FER

Eb/No in dB

94/84

Bloutage Simulation Type

■ Set up like an uncoded simulation

  – framesize

  – specify the modulation

    • Set mod_order = 0 for unconstrained Gaussian input

  – specify the channel (AWGN, Rayleigh, etc.)

■ Also requires the rate

■ Saves FER, not BER

Outline

6. Outage analysis
    - Determine the outage probability over block fading channels
    - Determine the outage probability of finite-length codes

7. Digital Network Coding
    - **Describe CML implementation of digital network coding**
    - **Simulate error rate of digital network coding for non-channel-coded 4-FSK in Rayleigh fading**

8. The internals of CML

9. Throughput calculation
    - Convert BLER to throughput for hybrid-ARQ

**Digital Network Coding

- Digital network coding (DNC) is a relaying scheme which improves the capacity of multiple-access relay channels.

- Two or more nodes transmit in the same time and band to a relay, deliberately interfering.

- Relay receives interferes signal, encodes and broadcasts such that nodes may decode desired information.

Tx Phase 1 – Node to Relay

$g(b_1)$ $g(b_2)$

N_1 R N_2

Tx Phase 2 – Relay to Nodes

$g(b_1 \oplus b_2)$ $g(b_1 \oplus b_2)$

N_1 R N_2
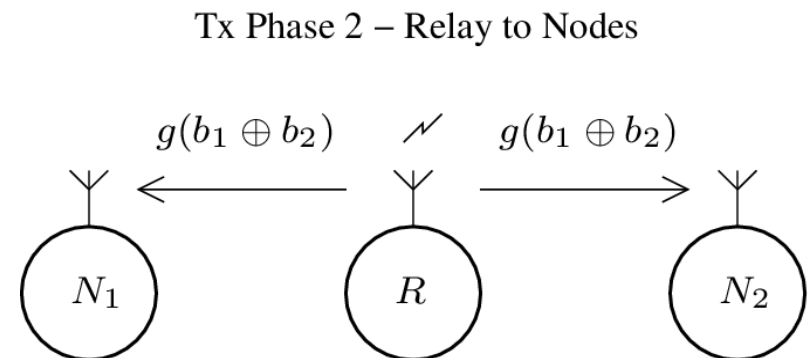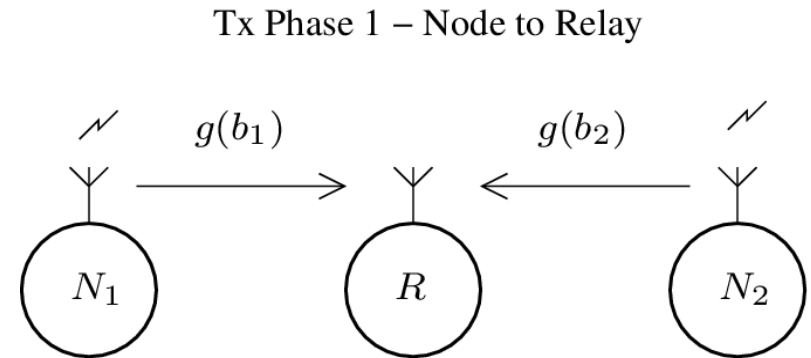
Digital Network Coding Example
Two-way Relay Channel

**Digital Network Coding

Technical Requirements

– Network encoding scheme at relay.

– Demodulation scheme at relay which performs network encoding.

– Demodulator producing soft-output channel observations suitable for capacity-approaching iterative decoding.

**Digital Network Coding
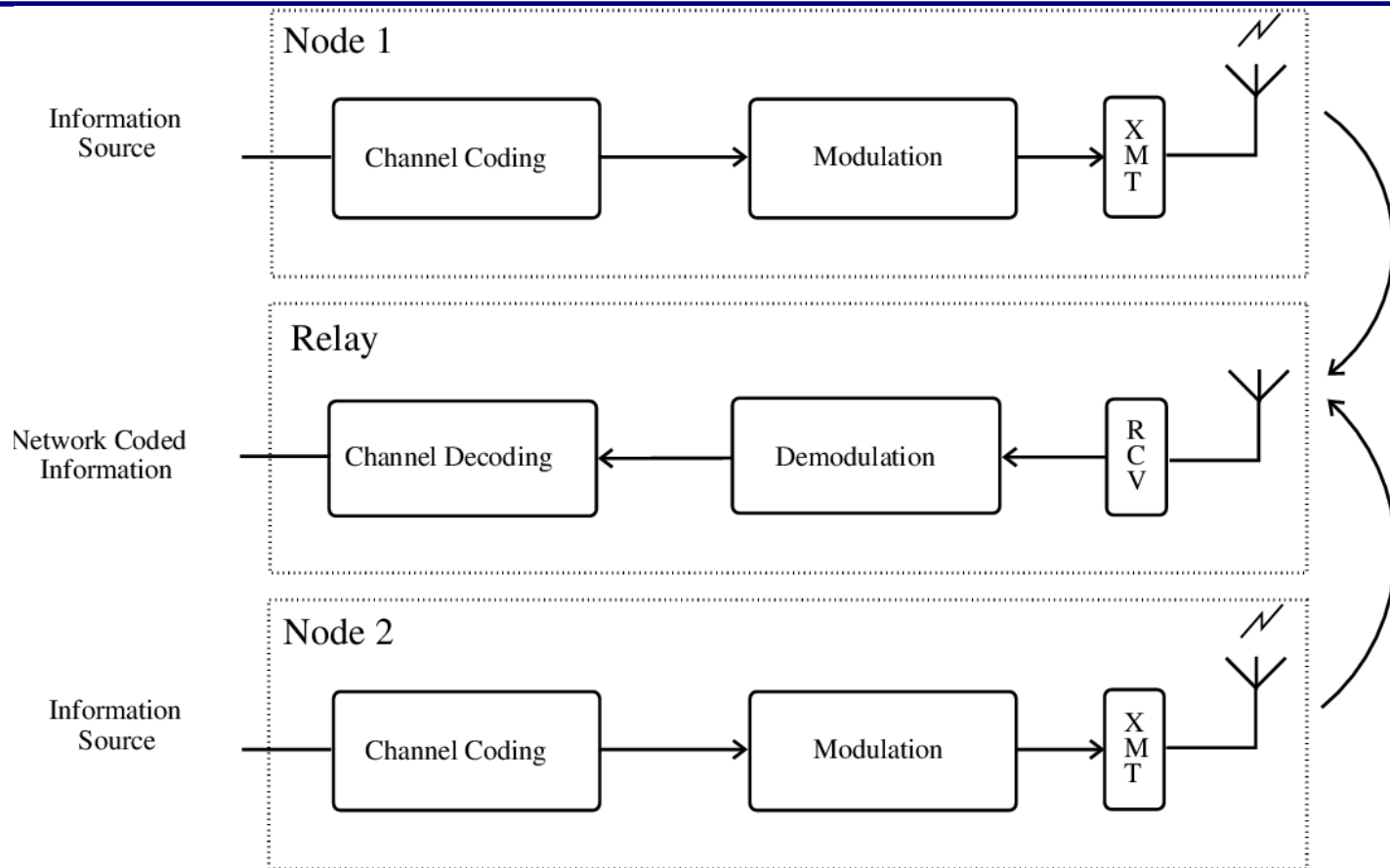
Tx Phase 1 - Block Diagram

**Digital Network Coding
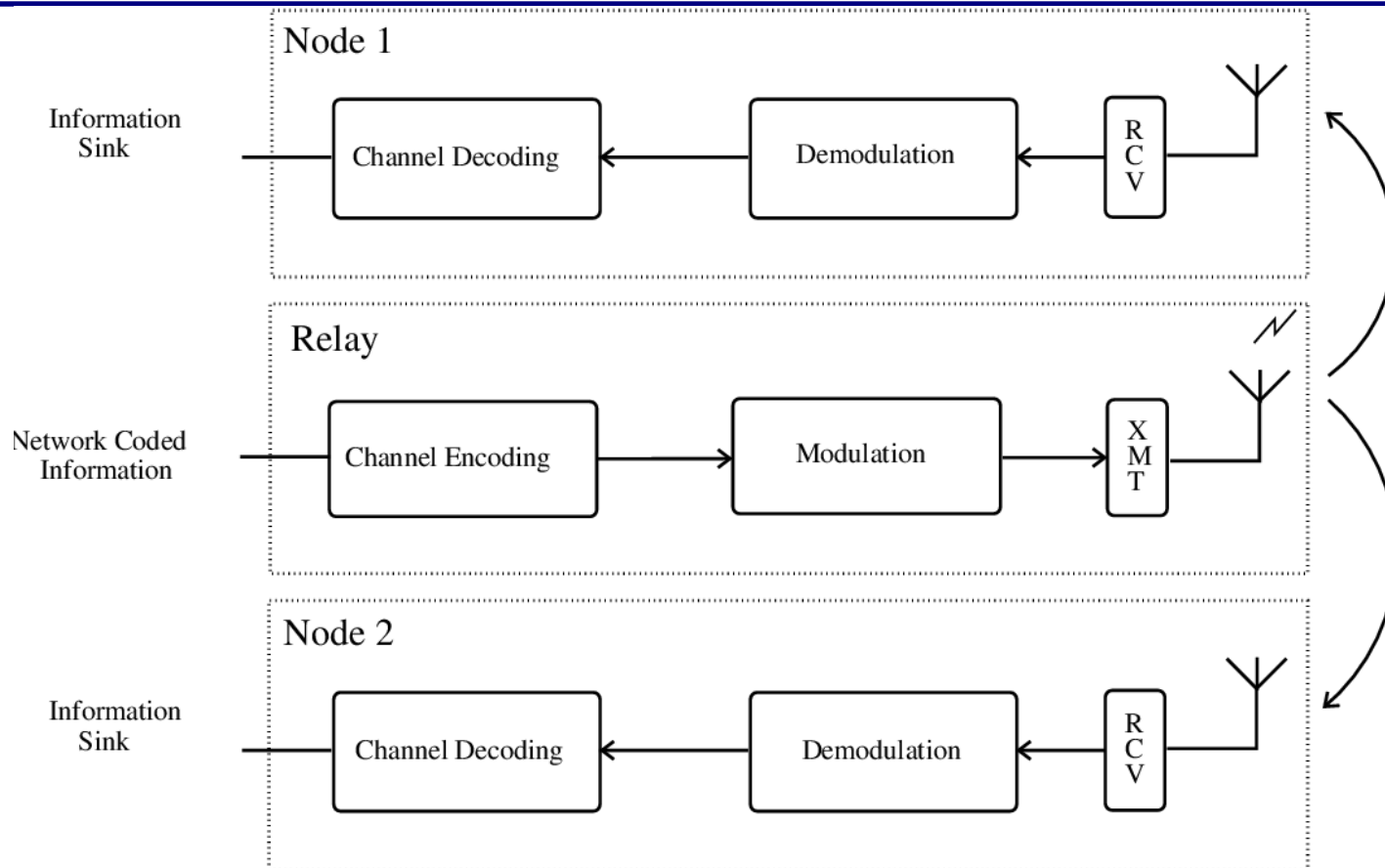
Tx Phase 2 - Block Diagram

**Digital Network Coding

CML DNC Implementation [2]

- Topology: Two-way Relay Channel

- Network encoding scheme at relay.
  - Exclusive-or $b = b_1 \oplus b_2$

- Demodulation scheme at relay which performs network encoding.
  - Noncoherent FSK

- Demodulator producing soft-output channel observations suitable for capacity-approaching iterative decoding.
  - Codes supported: Turbo and LDPC

[2] M. C. Valenti, D. Torrieri, and T. Ferrett, "Noncoherent physical-layer network coding using binary CPFSK modulation," Proc. IEEE Military Commun. Conf., Oct. 2009.

**Digital Network Coding

CML DNC Performance Simulation

- – Compute relay error rate as a function of
  - Modulation order {2, 4, 8, 16 …}
  - Channel Type {Rayleigh, AWGN}
  - Channel State Information {full, partial, none}
  - Channel code {Turbo, LDPC}

**DNC Simulation Example

■ CML Scenario: DncTwrc
  Record: 1

- sim_type = 'uncoded'

- topology = 'twrc'

- twrc_param.protocol = 'dnc'

- twrc_param.energy_ratio = 1   % sources use same Tx energy

- modulation = 'FSK'

- mod_order = 4

- channel = 'Rayleigh'

- csi_flag = 1 % Partial CSI

- SNR = [0 : 0.5 : 50]

**DNC Simulation Example

User Simulation Steps
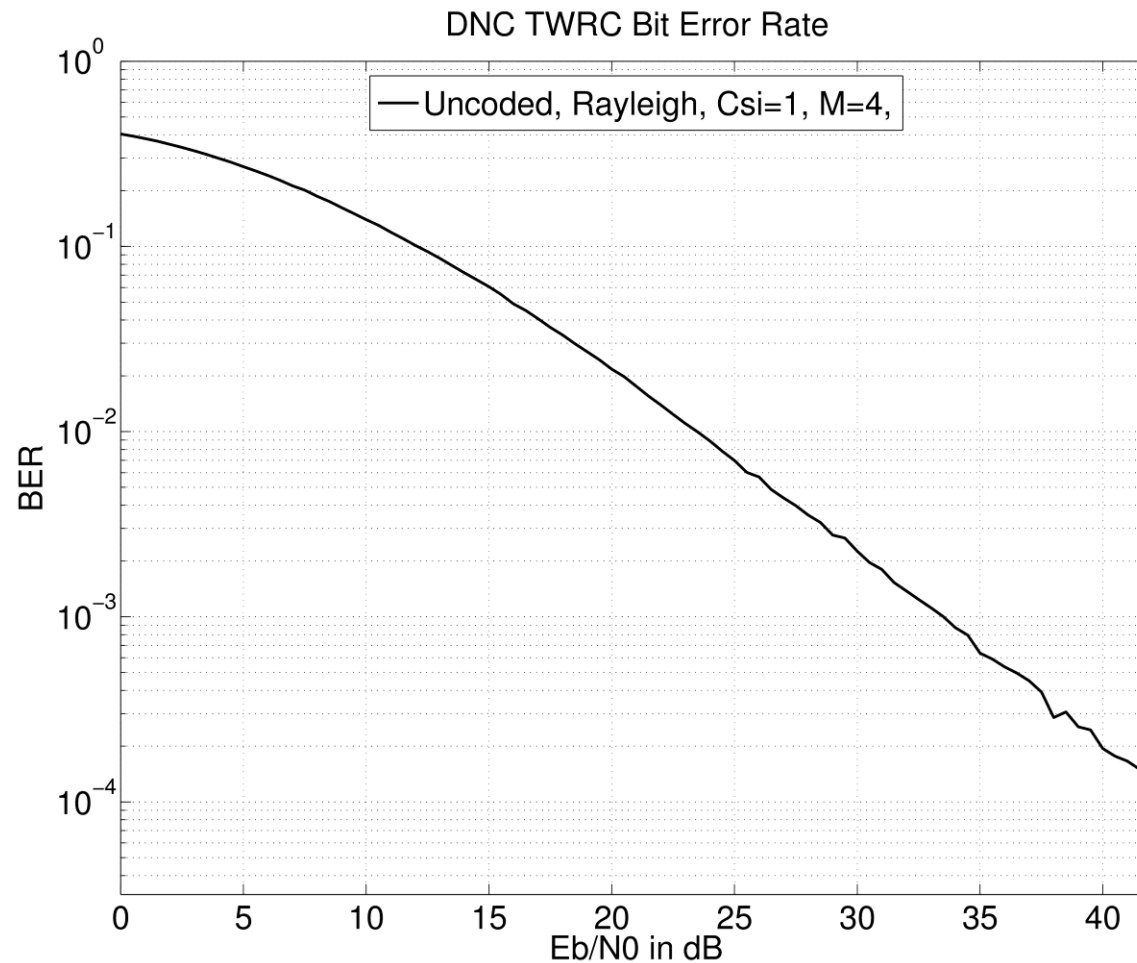
Start MATLAB and initialize CML
>> CmlStartup

Execute DNC simulation
>> CmlSimulate('DncTwrc', 1)

Plot results
>> CmlPlot('DncTwrc', 1)

**DNC Simulation Example

DNC TWRC Bit Error Rate

Legend: Uncoded, Rayleigh, Csi=1, M=4,

x-axis: Eb/N0 in dB

y-axis: BER

Outline

Main Program Flow

- **CmlSimulate**
  - ReadScenario
    - Runs SingleRead for each record
    - Performs sanity check on sim_param structure
    - Initializes or restores the sim_state structure
  - For each record
    - SingleSimulate if a simulation
    - Otherwise, runs one of the analysis functions:
      - CalculateThroughput
      - CalculateMinSNR
      - CalculateMinSNRvsB

SingleSimulate

- **Seeds random number generator**
- **Branches into**
  - SimulateMod
    - For uncoded, coded, and bloutage point-to-point channel
  - SimulateTwrc
    - For uncoded, coded, two-way relay channel
  - SimulateUGI
    - For a blocklength-constrained outage simulation with unconstrained Gaussian input.
  - SimulateCapacity
  - SimulateCapacityTwrc
  - SimulateOutage

SimulateMod

- Main subfunctions (coded/uncoded) cases:
  - CmlEncode
  - CmlChannel
  - CmlDecode
- For bloutage, replace CmlDecode with
  - Somap
  - capacity

SimulateTwrc

- Main subfunctions (coded/uncoded) cases:
  - CmlEncode
  - CmlTwrcRelayChannel
  - CmlTwrcRelayComputeSymbolLh
  - CmlInitSomap
  - CmlTwrcRelaySomap
  - CmlTwrcRelayDecode

SimulateCapacity

- Operates like SimulateMod with sim_type = 'bloutage'
    - However, instead of comparing MI of each codeword against the rate, keeps a running average of MI.

- In case of exit simulation, computes detector and decoder mutual information characteristics

SimulateCapacityTwrc

- Similar to SimulateTwrc but computes mutual information at output of detector and Somap rather than error rates

- In case of exit simulation, computes detector and decoder mutual information characteristic

SimulateOutage

- Randomly generates SNR for each block

- Performs table lookup to get MI from SNR
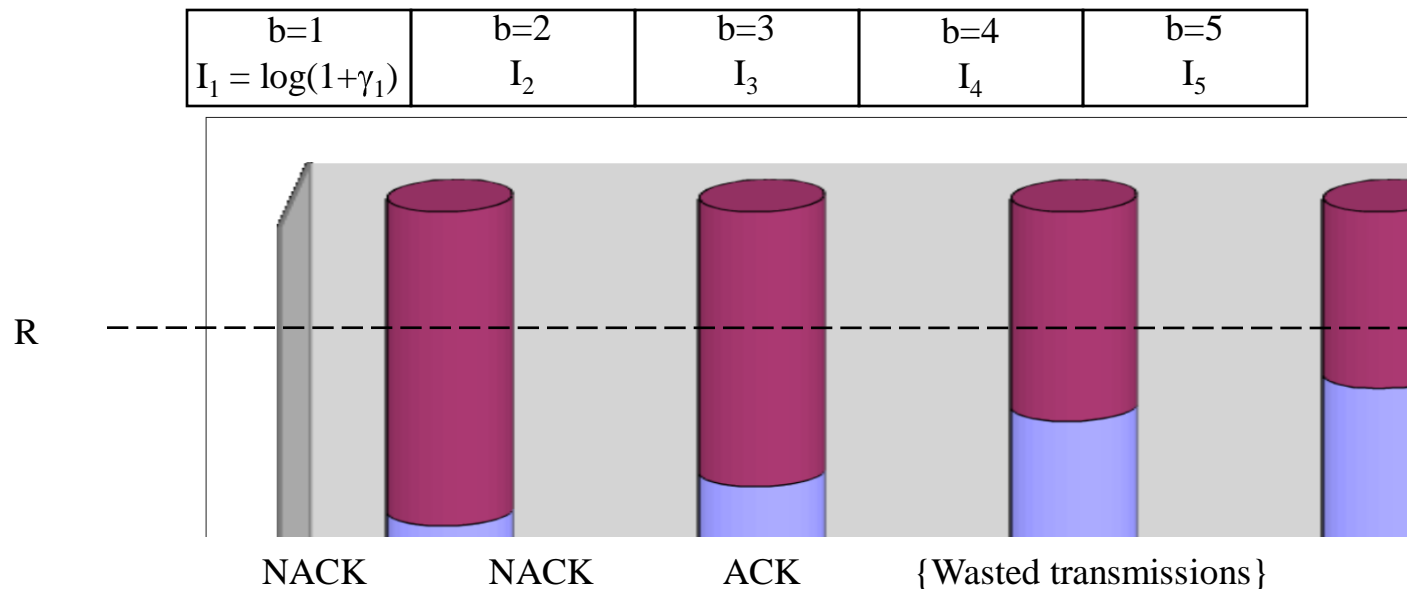
- Compares MI against threshold

Outline

Hybrid-ARQ
(Caire and Tunnineti 2001)

- Once $I_1^B > R$ the codeword can be decoded with high reliability.
- Therefore, why continue to transmit any more blocks?
- With hybrid-ARQ, the idea is to request retransmissions until $I_1^B > R$
  - With hybrid-ARQ, outages can be avoided.
  - The issue then becomes one of latency and throughput.

b=1 $I_1 = \log(1+\gamma_1)$	b=2 I_2	b=3 I_3	b=4 I_4	b=5 I_5

R

NACK   NACK   ACK   {Wasted transmissions}

Outline

6. Outage analysis
    - Determine the outage probability over block fading channels
    - Determine the outage probability of finite-length codes

7. Digital Network Coding
    - Describe CML implementation of digital network coding
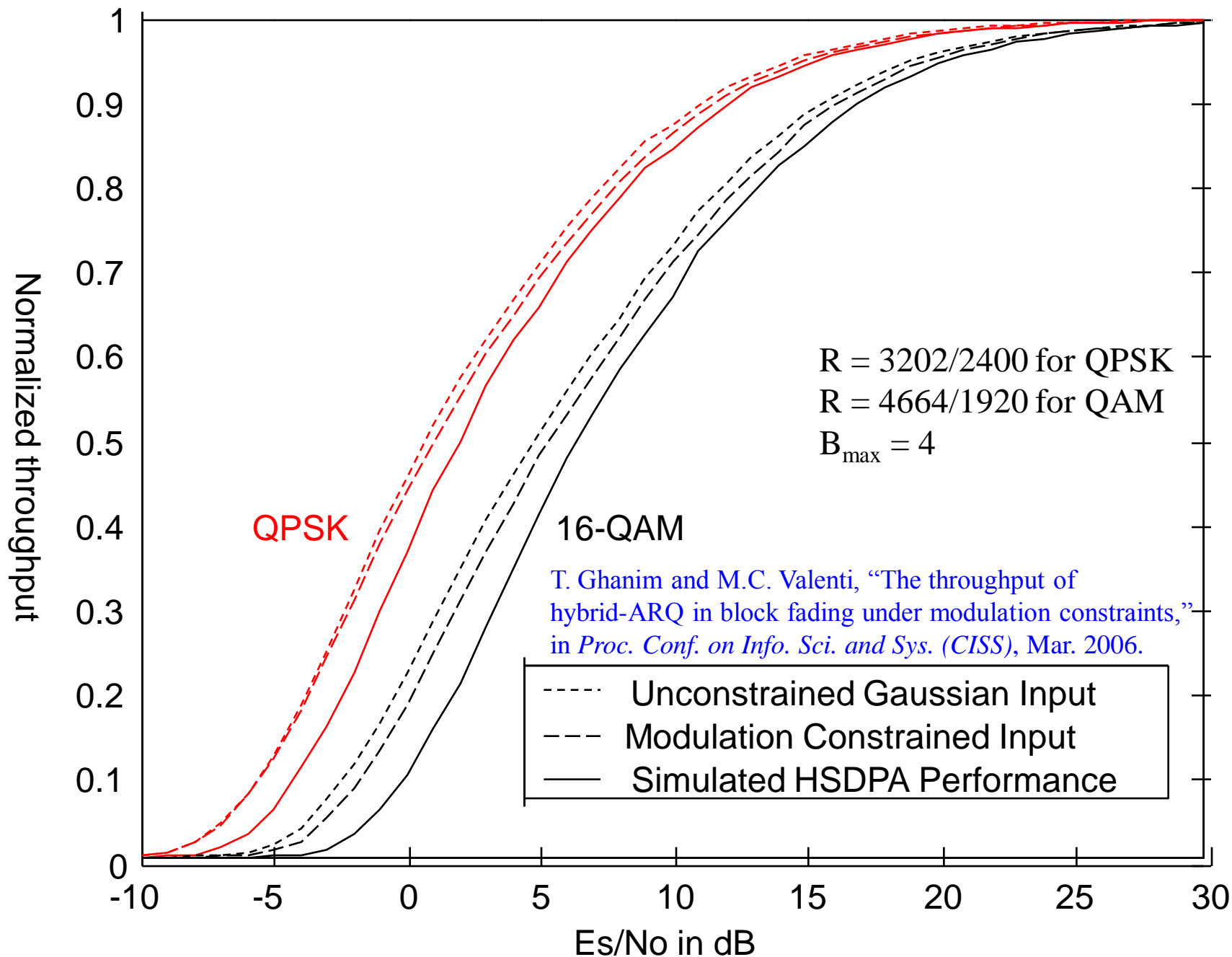   - Simulate error rate of digital network coding for non-channel-coded 4-FSK in Rayleigh fading

8. The internals of CML

9. **Throughput calculation**
    - **Convert BLER to throughput for hybrid-ARQ**

Latency and Throughput of Hybrid-ARQ

- With hybrid-ARQ B is now a random variable.
  - The average *latency* is proportional to E[B].
  - The average *throughput* is inversely proportional to E[B].
- Often, there is a practical upper limit on B
  - Rateless coding (e.g. Raptor codes) can allow $B_{max} \rightarrow \infty$
- An example
  - HSDPA: High-speed downlink packet access
  - 16-QAM and QPSK modulation
  - UMTS turbo code
  - HSET-1/2/3 from TS 25.101
  - $B_{max} = 4$

R = 3202/2400 for QPSK
R = 4664/1920 for QAM
$B_{max} = 4$

QPSK

16-QAM

T. Ghanim and M.C. Valenti, "The throughput of hybrid-ARQ in block fading under modulation constraints," in *Proc. Conf. on Info. Sci. and Sys. (CISS)*, Mar. 2006.

- - - - Unconstrained Gaussian Input
- - - Modulation Constrained Input
——— Simulated HSDPA Performance

Conclusions: Design Flow with CML

- **When designing a system, first determine its capacity.**
  - Only requires a slight modification of the modulation simulation.
  - Does not require the code to be simulated.
  - Allows for optimization with respect to free parameters.
- **After optimizing with respect to capacity, design the code.**
  - BICM with a good off-the-shelf code.
  - Optimize code with respect to the EXIT curve of the modulation.
- **Information outage analysis can be used to characterize:**
  - Performance in slow fading channels.
  - Delay and throughput of hybrid-ARQ retransmission protocols.
  - Finite codeword lengths.