

## UC Berkeley's CS61A – Lecture 13 – Trees

### Your Smart Light Can Tell Amazon and Google When You Go to Bed

[www.bloombergquint.com/pursuits/your-smart-light-can-tell-amazon-and-google-when-you-go-to-bed](http://www.bloombergquint.com/pursuits/your-smart-light-can-tell-amazon-and-google-when-you-go-to-bed)



"Amazon and Google want smart home device manufacturers to have smart appliances transmit a continuous stream of customer information to data hubs by modifying their code. The companies said they collect the data to make it easier for consumers to manage home electronics, although some device makers say such data collection doesn't give users enough control over what data they share, or how it can be used. Park Associates' Brad Russell said, "You can learn the behaviors of a household based on their patterns." One smart device maker, Logitech, is trying to partly fulfill this mandate: instead of telling smart speakers what each device connected to Logitech's Harmony remote controls are doing, the company reports back with broad descriptions, specifying that a user is watching TV, rather than passing on information about their choice of channel, for example."

## Announcements

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Optional Hog strategy contest ends TODAY Friday 2/22.

Hog Composition Scores have been released

- You can find your score and any comments on your Hog submission on [Ok](#)
- Submit revisions (via `python3 ok --revise`) by Tuesday, February 26th to recover any lost points!

Maps (SOLO – no partners) Project released and due Thursday 2/28 (8 days!)

- Submit a day early (Wednesday 2/27) to receive 1 extra credit point

## Limitations on Dictionaries (from last time)

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Dictionaries are **unordered** collections of key-value pairs

Dictionary keys do have two restrictions:

- A key of a dictionary **cannot be** a list or a dictionary (or any *mutable type*)
- Two **keys cannot be equal**; There can be at most one value for a given key

This first restriction is tied to Python's underlying implementation of dictionaries

The second restriction is part of the dictionary abstraction

If you want to associate multiple values with a key, store them all in a sequence value

# Box-and-Pointer Notation

# The Closure Property of Data Types

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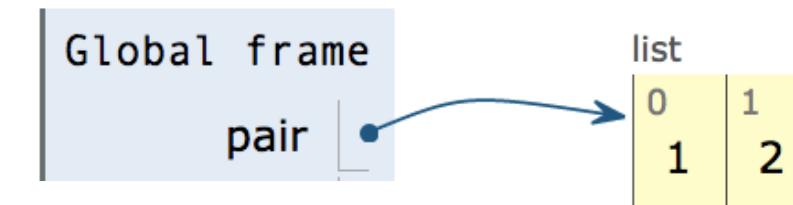
- A method for combining data values satisfies the *closure property* if:  
The result of combination can itself be combined using the same method
- Closure is powerful because it permits us to create hierarchical structures
- Hierarchical structures are made up of parts, which themselves are made up of parts, and so on

Lists can contain lists as elements (in addition to anything else)

# Box-and-Pointer Notation in Environment Diagrams

Lists are represented as a row of index-labeled adjacent boxes, one per element

Each box either contains a primitive value or points to a compound value

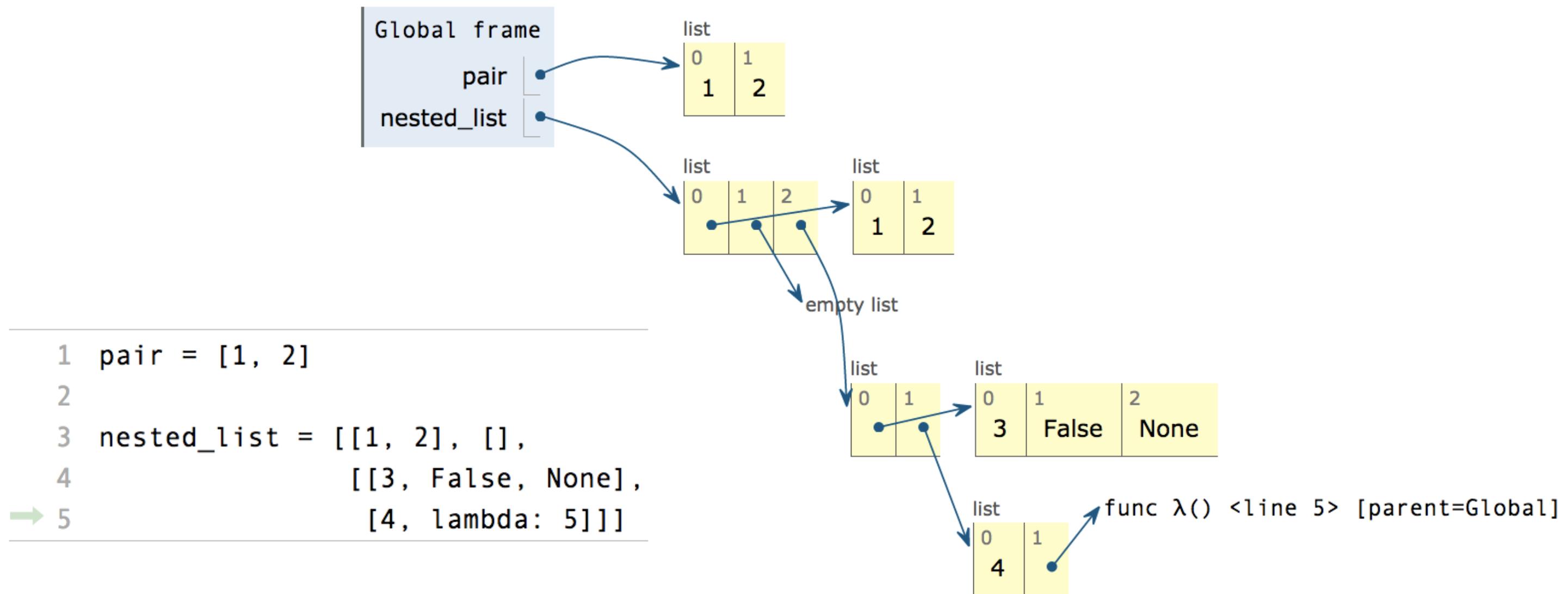


```
pair = [1, 2]
```

# Box-and-Pointer Notation in Environment Diagrams

Lists are represented as a row of index-labeled adjacent boxes, one per element

Each box either contains a primitive value or points to a compound value

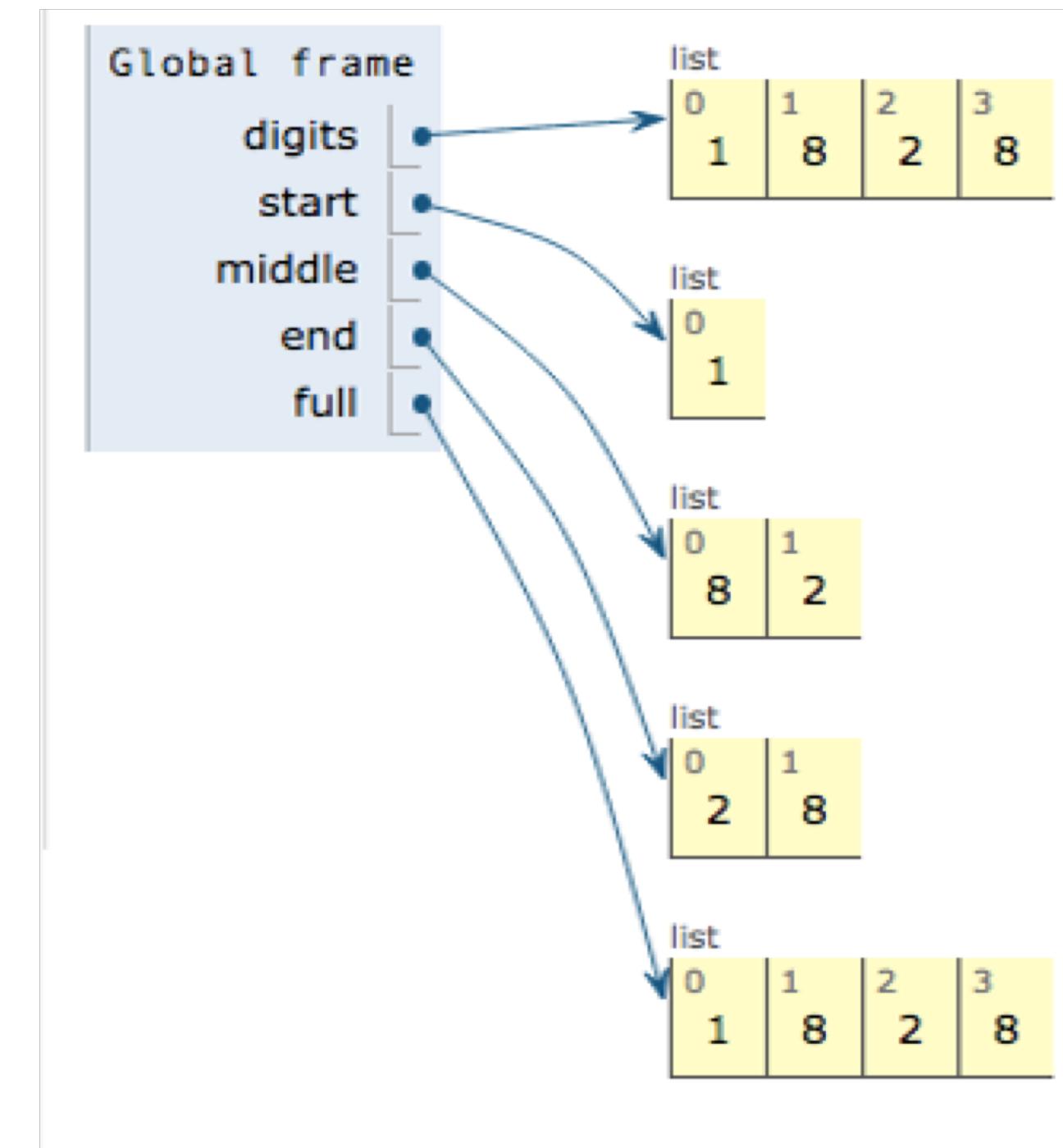


# Slicing

(Demo1)

## Slicing Creates New Values

```
1 digits = [1, 8, 2, 8]
2 start = digits[:1]
3 middle = digits[1:3]
4 end = digits[2:]
5 full = digits[:]
```



# Processing Container Values

## Sequence Aggregation (Demo2 after each one)

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Several built-in functions take iterable arguments and aggregate them into a value

- **sum(iterable[, start])** → value

Return the sum of an iterable (not of strings) plus the value of parameter 'start' (which defaults to 0). When the iterable is empty, return start.

- **max(iterable[, key=func])** → value  
**max(a, b, c, ..., [key=func])** → value

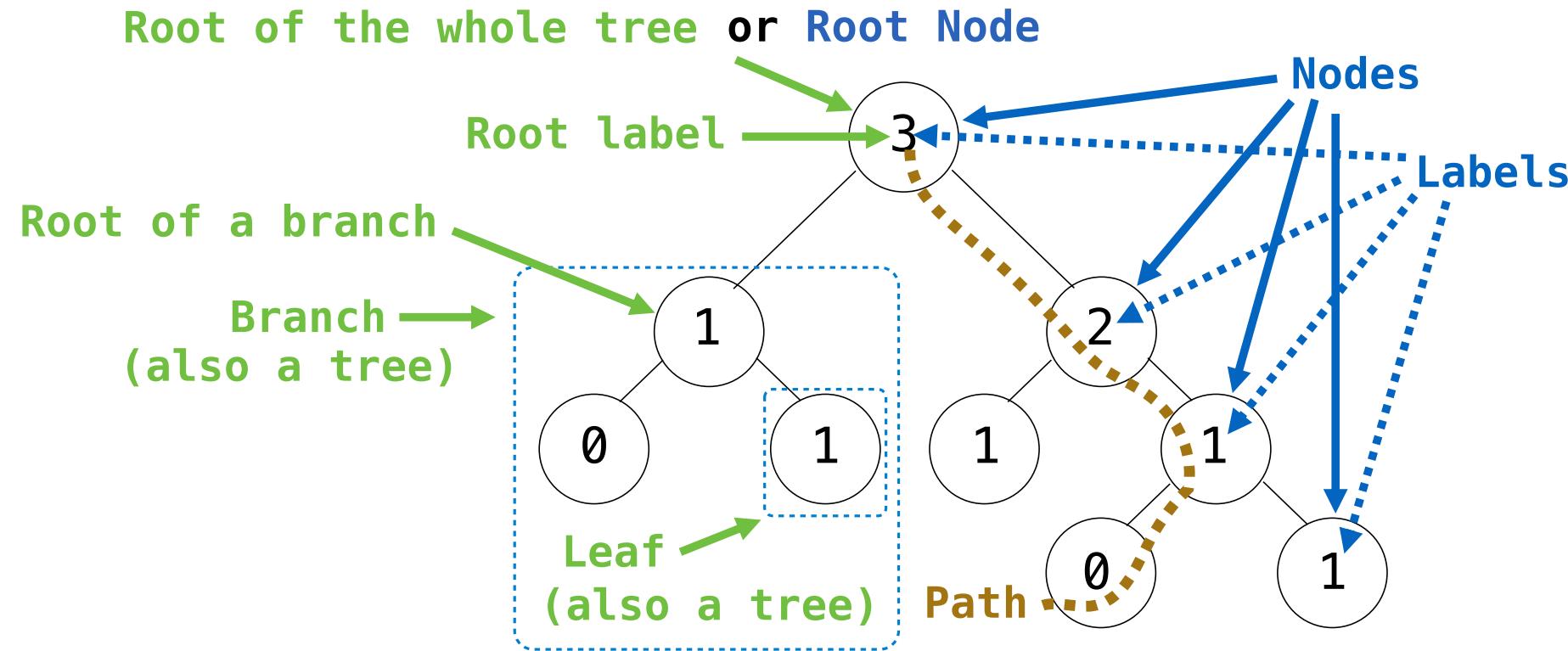
With a single iterable argument, return its largest item.  
With two or more arguments, return the largest argument.

- **all(iterable)** → bool

Return True if `bool(x)` is True for all values x in the iterable.  
If the iterable is empty, return True.

# Trees

# Tree Abstraction



## Recursive description (wooden trees):

A **tree** has a **root label** and a list of **branches**

Each **branch** is a **tree**

A **tree** with zero **branches** is called a **leaf**

A **tree** starts at the **root**

## Relative description (family trees):

Each location in a tree is called a **node**

Each **node** has a **label** that can be any value

One node can be the **parent/child** of another

The top node is the **root node**

*People often refer to labels by their locations: "each parent is the sum of its children"*

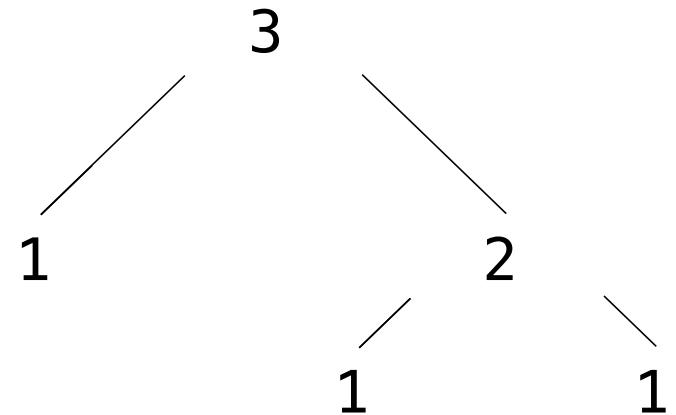
# Implementing the Tree Abstraction

```
def tree(label, branches=[]):
    return [label] + branches

def label(tree):
    return tree[0]

def branches(tree):
    return tree[1:]
```

- A **tree** has a root **label** and a list of **branches**
- Each branch is a tree



```
>>> tree(3, [tree(1),
...             tree(2, [tree(1),
...                         tree(1)]))]
[3, [1], [2, [1], [1]]]
```

# Implementing the Tree Abstraction

```
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)

def label(tree):
    return tree[0]

def branches(tree):
    return tree[1:]

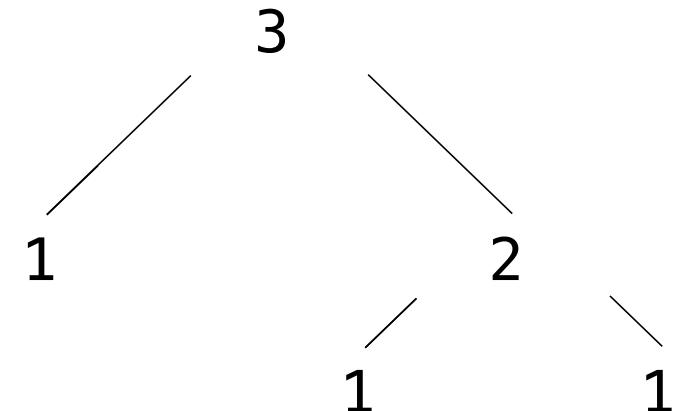
def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
            return False
    return True
```

Verifies the tree definition

Creates a list from a sequence of branches

Verifies that tree is bound to a list

- A **tree** has a root **label** and a list of **branches**
- Each branch is a tree



```
>>> tree(3, [tree(1),
...             tree(2, [tree(1),
...                         tree(1)]))]
...             [3, [1], [2, [1], [1]]]
```

```
def is_leaf(tree):
    return not branches(tree)
```

(Demo3)

## Example: Printing Trees

(Demo4a)

# Tree Processing

(Demo4)

## Tree Processing Uses Recursion

Processing a leaf is often the base case of a tree processing function

The recursive case typically makes a recursive call on each branch, then aggregates

```
def count_leaves(t):
    """Count the leaves of a tree."""
    if is_leaf(t):
        return 1
    else:
        branch_counts = [count_leaves(b) for b in branches(t)]
        return sum(branch_counts)
```

(Demo5)

## Discussion Question

Implement `leaves`, which returns a list of the leaf labels of a tree

*Hint:* If you `sum` a list of lists, you get a list containing the elements of those lists

```
>>> sum([ [1], [2, 3], [4] ], [])
[1, 2, 3, 4]
>>> sum([ [1] ], [])
[1]
>>> sum([ [[1]], [2] ], [])
[[1], 2]
>>> def leaves(tree):
        """Return a list containing the leaf labels of tree.
        >>> leaves(fib_tree(5))
        [1, 0, 1, 0, 1, 1, 0, 1]
        """
        if is_leaf(tree):
            return [label(tree)]
        else:
            return sum(List of leaf labels for each branch, [])
```

`branches(tree)`

`[b for b in branches(tree)]`

`leaves(tree)`

`[s for s in leaves(tree)]`

`[branches(b) for b in branches(tree)]`

`[branches(s) for s in leaves(tree)]`

`[leaves(b) for b in branches(tree)]`

`[leaves(s) for s in leaves(tree)]`

## Creating Trees

A function that creates a tree from another tree is typically also recursive

```
def increment_leaves(t):
    """Return a tree like t but with leaf labels incremented."""
    if is_leaf(t):
        return tree(label(t) + 1)
    else:
        bs = [increment_leaves(b) for b in branches(t)]
        return tree(label(t), bs)

def increment(t):
    """Return a tree like t but with all labels incremented."""
    return tree(label(t) + 1, [increment(b) for b in branches(t)])

def tree_map(t,f):
    """Return a tree like t but with all labels having f applied to them."""
    return tree(f(label(t)), [tree_map(b,f) for b in branches(t)])
```