

Response to the reviewers

We thank the reviewers for their critical assessment of our work. In the following we address their concerns point by point.

1 Reviewer 1

1.1 Terminology

Point P 1.1.1 — First, the authors refer to the neurons as “free” or “quiet” and refer to the “free phase” and “quiet phase”. They say that they follow Bose and Booth in doing so. But I have read (and written) MANY papers on bursting/CPGs/multi-phase solutions, and they always refer to “active” and “silent” rather than “free” and “quiet”. The authors should switch their word use as well to avoid confusing the field

Reply: TODO

Point P 1.1.2 — Second, the authors use ISI and IBI incorrectly. The ISI is the inter-spike interval. This is the period BETWEEN spikes, NOT the entire period T of a spiking event (see e.g., line 181). The IBI is the inter-burst interval. In this paper, that would be the silent phase duration for one neuron. It is NOT the entire period of a bursting event, nor is it the delay from the end of one neuron’s active phase to the start of the other neuron’s active phase (e.g., line 266). These need to be corrected throughout the paper.

Reply: TODO

Point P 1.1.3 — Third, in the Results, the authors state the signs of various derivatives without proof. Some are obvious (e.g., eqn. (30)) but others, such as eqn. (31), are not. Some brief justification is needed.

Reply: TODO

Point P 1.1.4 — Fourth, Fig. 11A seems to contrast with Fig. 4A. Fig. 11A seems to show massive multistability of solutions for different n , whereas Fig. 4A only has bistability. Some clarification is needed

Reply: TODO

Point P 1.1.5 — And fifth, the notation $n-n$ is non-ideal, as it looks like n minus n . In my opinion, $n:n$ is more standard and clearer.

Reply: TODO

1.2 Major points

Point P 1.2.1 — I don't see the justification for assuming $d'=(1-d)/\tau_{\text{aud}}$ when a neuron is ACTIVE (see below for "active" meaning). Can the authors provide some biological justification that the available synaptic resources actually increase during a spike? If not, it seems critical that they give some other justification that this assumption is OK.

Reply: We agree with the reviewer on this important point. This is what we did to fix it. Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Point P 1.2.2 — Reviewer 1's second point.

Reply: And our reply to it.

1.3 Minor Points

Point P 1.3.1 — Line 31: It's a CPG composed of reciprocally inhibitory neurons; referring to a "reciprocally inhibitory CPG" is misleading.

Reply: Changed to "CPG composed of reciprocally inhibitory neurons".

Point P 1.3.2 — The par. starting on line 38 is unclear: The 1-d conditions for n:n solutions in [6] are stated to be for $n \leq 2$ (line 43); the authors should clarify if n:n in line 48 also refers to $n \leq 2$ or not.

Reply: Added clarification of $n \leq 2$.

Point P 1.3.3 — Results line 130: It is important to note that both neurons inhibit each other at all times. S may get small but it's nonzero. Thus, "inhibited" cell is not really well-defined.

Reply: Removed "inhibited cell" notation.

Point P 1.3.4 — Line 137: Wang & Rinzel, 1992 (and perhaps Skinner et al. from 1993 or 1994) should be cited in reference to release.

Reply: Added Wang & Rinzel, 1992 and Skinner et al. 1994 reference when "release" is mentioned.

Point P 1.3.5 — I don't understand Fig. 4B. The ISI is less than the IBI, so how can their ratio be bigger than 1? Or perhaps the axis label "ISI/IBI" does not refer to ISI divided by IBI? Clarification is needed.

Reply: TODO

Point P 1.3.6 — Line 200: "revolve" should be replaced by "evolve".

Reply: Reworked the whole paragraph formerly starting at line 194.

Point P 1.3.7 — Line 213 is incorrect: It's not the decay of d that matters but the decay of s before the next spike occurs. Correction needed – except now I realize that lines 207-219 can be cut, as they add nothing relative to the previous paragraph.

Reply: Definition of g^* was changed due to the change in model, consequently the paragraph starting in line 207 was removed.

Point P 1.3.8 — It seems like the authors should be able to analytically compute or at least approximate g^* and should give some explanation for the delay in Fig. 6A, left branch. This must relate to the silent cell spending too short of a time in the silent phase. Fig. 4 is certainly relevant.

Reply: TODO

Point P 1.3.9 — Lines 246-7: The authors should revise because s is determined by d and by τ .

Reply: TODO

Point P 1.3.10 — Line 270: It seems like the active phase ends at time $(n-1)T + \delta T$, yet the authors say it ends as $(n-1)T$. This may be more convenient for their analysis but it's not correct usage of the phase terminology, so clarification is needed.

Reply: TODO

Point P 1.3.11 — It's disorienting to see $\delta_n(d^*)$ in eqn. (20) when just above (line 320) the same quantity is called $d_n = d(t_n^-)$. The authors should pick one notation for both places.

Reply: As suggested, we have removed the d_n notation completely.

Point P 1.3.12 — Cut line 339 and equation (27). These add a bit of confusion and nothing else.

Reply: TODO

Point P 1.3.13 — Fig. 9 caption: mention where the Q_n curves intersect.

Reply: Added intersection to caption in Figure F_n and Q_n figure.

Point P 1.3.14 — Eqn. (29): remind the reader that $d_n(d^*)$ comes from eqn. (20) and that g^* is obtained numerically.

Reply: TODO

Point P 1.3.15 — Lines 364-6 including eqn. (36) should be cut – they are neither new nor helpful here.

Reply: TODO

Point P 1.3.16 — Eqs. (37),(38) can and should be combined.

Reply: TODO

Point P 1.3.17 — Eqn. (44) is confusing: here the authors want $d_f^* = \phi_h(\bar{g})$, so it's strange to express it as a function of d_f^* again.

Reply: TODO

Point P 1.3.18 — Fig. 11B needs more explanation, such as a reminder that the orange curves are only computed over small intervals of \bar{g} where the solution branch is stable.

Reply: TODO

Point P 1.3.19 — The notation $\bar{g}_+(n) < \bar{g}_-(n)$ seems strange – seems like these should be reversed (the math is fine, I'm questioning the choice of notation).

Reply: TODO

Point P 1.3.20 — Line 415: [21] is cited twice.

Reply: TODO

Point P 1.3.21 — Rather than stating eqn. (54), it would be clearer just to reference eqn. (47) there.

Reply: TODO

Point P 1.3.22 — A better explanation of what is shown in Fig. 12 is needed. Is this $\bar{g}_+(n)$ and $\bar{g}_-(n)$ for various n , computed numerically from (56) and (58)?

Reply: TODO

Point P 1.3.23 — Discussion lines 459-462: Please explain what such “light” has been shed here. If none, as it appears, that’s OK for this math paper, but then this comment doesn’t really belong in the discussion.

Reply: TODO

Point P 1.3.24 — I’m confused about lines 534-6. I have never heard of learning in a CPG. Please clarify what is meant.

Reply: TODO