Exercises

1. • Yes

$$\sigma_{p_1}(R_1 \bowtie_{p_2} R_2) \equiv \{t \mid t \in (R_1 \bowtie_{p_2} R_2) \wedge p_1(t)\}$$

$\hookrightarrow [\mathcal{F}(p_1) \subseteq \mathcal{A}(R_1)]$

$$\equiv \{x \circ y \mid (x \circ y) \in (R_1 \bowtie_{p_2} R_2) \wedge p_1(x)\}$$
$$\equiv \{x \circ y \mid x \in R_1 \wedge p_1(x) \wedge y \in R_2 \wedge p_2(x \circ y)\}$$
$$\equiv \{x \circ y \mid x \in \sigma_{p_1}(R_1) \wedge y \in R_2 \wedge p_2(x \circ y)\}$$
$$\equiv \sigma_{p_1}(R_1) \bowtie_{p_2} R_2$$

• Yes
• No

$$\sigma_{x=6}(\{[x:5]\} ⟖_{a=b} \{[y:5]\}) \not\equiv \sigma_{x=6}(\{[x:5]\}) ⟖_{a=b} \{[y:5]\}$$
$$\sigma_{x=6}(\{[x:5, y:5]\}) \not\equiv \{\} ⟖_{a=b} \{[y:5]\}$$
$$\{\} \not\equiv \{[x:\text{null}, y:5]\}$$

• No
• Yes
• Yes

(filtrar primeiro e depois fazer outer-join pode levar a entries com NULL values e o filtro só vai remover itens do lado que não "comando" → levando a possíveis NULL values
Por outro lado, filtrar depois pode levar à remoção completa da linha em questão.)

2. $R = 1K$ $\quad S = 100K$ $\quad$ tuples per page $= 50$
S in disk $\quad$ access_time $= 10ms$ $\quad t_{speed} = 10K/s$

Nested Loop Join :

$$t_{page\_s} = 10\ ms + \frac{100\ 000}{10\ 000} \times 10^3\ ms$$

$$t_{NL} = \underbrace{1000}_{\text{number pages } R} \times (10 \times 10^{-3} + 10^4) \times 50 = 10^3 \times (10^2 + 10^4) \times 50$$
$$= 5 \times (10^6 + 10^8)$$

Block Nested Loop Join: $\quad t_{block} = 10\,ms + \frac{100\,000}{10\,000} \times 10^3$

$$t_{BNL} = \frac{1000}{100} \cdot \frac{50}{50} \times (10 \times 10^{-3} + 10^4)$$
$$= 10^{-1} + 10^5 = 100000.1 \text{ ms} = 100.0001 \text{ s}$$

3. $(t, R(t))$ $\qquad \operatorname{dom}(R) = \{ t \mid R(t) > 0 \}$

$$L \cap R = \{ (t, \min(L(t), R(t))) \mid t \in \operatorname{dom}(L) \cap t \in \operatorname{dom}(R) \}$$
$$L \cup R = \{ (t, L(t) + R(t)) \mid t \in \operatorname{doms} \}$$
$$L - R = \{ (t, \max(L(t) - R(t), 0)) \mid t \in \operatorname{dom}(L) \}$$

$$\begin{aligned}
L \cap R &= \min(L(t), R(t)) \\
&= \min(L(t) - 0,\ L(t) - (L(t) - R(t))) \\
&= L(t) - \max(0, L(t) - R(t)) \quad [\min(L(t), R(t)) \geq 0]^{*} \\
&= \max(L(t) - \max(0, L(t) - R(t)), 0) \\
&= L - (L - R)
\end{aligned}$$

* For $L(t) \geq R(t)$:

$$\max(L(t) - \max(0, L(t) - R(t)), 0) = \max(L(t) - L(t) + R(t)) = R(t)$$

For $L(t) < R(t)$

$$\max(L(t) - 0) = L(t) \longrightarrow \quad = \min(L(t), R(t))$$

Let $x \in \operatorname{dom}(L) \cup \{null\}$ and $y \in \operatorname{dom}(R) \cup \{null\}$

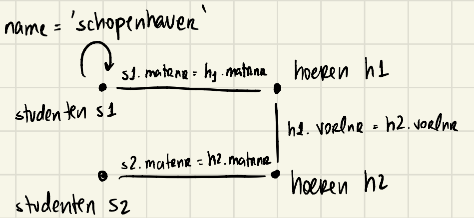
$$\begin{aligned}
(L ⟗_p R)(x \circ y) = {} & L(x) \cdot R(y) \cdot 1_{p(x \circ y)} + \\
& + L(x) \cdot 1_{y \text{ is null} \wedge (\nexists y_2 \in \operatorname{dom}(R) : x \circ y_2)} \\
& + 1_{x \text{ is null} \wedge (\nexists x_2 \in \operatorname{dom}(L) : x_2 \circ y)} \cdot R(y)
\end{aligned}$$

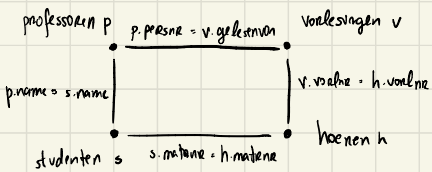
4\. 1. $\sigma_{R1.x=c}$ where $R1.x$ is key → every value is distinct

- selectivity $= \frac{1}{|R|}$
- if not key $= \frac{1}{|R_1.x|}$ where $|R_1.x| = |D_{R1.x}|$

2\.
- $\frac{1}{\max(|R_1|, |R_2|)} \Rightarrow \min\left(\frac{1}{|R_1|}, \frac{1}{|R_2|}\right)$
- $\frac{1}{|R_1|}$
- $\frac{1}{\max(|R_1.x|, |R_2.y|)}$

5\.





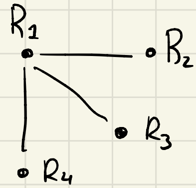
—— 11 ——

$$|\sigma(R_i)| = f_i \cdot R_i$$

$$|R_1 \bowtie R_2| = f_{1,2} \cdot |R_1| \cdot |R_2|$$

$$|T| = \begin{cases} f_i \cdot |R_i| & \text{if } T_i \text{ is a leaf} \\ \left(\prod_{R_i \in T_1, R_j \in T_2} f_{i,j}\right) |T_i| |T_j| & \end{cases}$$

6. bushy w/ cross product → optimal



$|R_1| = \ldots = |R_4| = 10$

$s = 0.5$

$C_{bushy} = 100 + 50 + 0.25 \times 100 \times 50 = 1250 + 100 + 50 = 1400$ ✓

$C_{l.d} = 50 + 250 + 0.5 \times 250 \times 10 = 1250 + 300 = 1550$

$C_{bushy} = c^2 + 0.5c^2 + 0.5^2 \cdot 0.5c^2 \cdot c^2 = c^2 + \cancel{0.5c^2} + 0.5^3 c^4$

$C_{l.d} = \cancel{0.5c^2} + 0.5^2 c^3 + 0.5^3 c^4$

$C_{bushy} < C_{l.d} \Leftrightarrow c^2 + \cancel{0.5^3 c^4} < 0.5^2 c^3 + \cancel{0.5^3 c^4}$

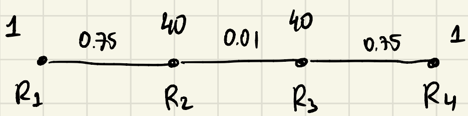
$\Leftrightarrow c^2 < 0.5^2 c^3$

$\Leftrightarrow -0.5c^3 + c^2 < 0 \quad [c \neq 0]$

$\Leftrightarrow -0.5c + 1 < 0$

$\Leftrightarrow c > 2$

Another example that looks nice:



↙ very small

$R_1 \times R_4 = 1$

$R_2 \bowtie R_3 = \underline{0.01} \times 40^2$

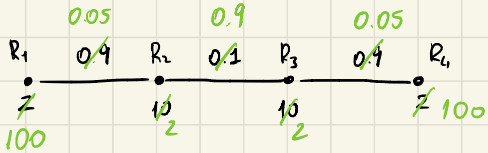
↳ very small so this is preferred over $R_2 \bowtie (R_1 \times R_4)$ R_3

tip:
- cardinality of R_1 and R_4 very small
- relations connected with both R_1 and R_4 have low selectivity <u>and</u> connection between them both have very high selectivity

7. <u>GOO</u>: chooses pair with minimum cost

tip: for no optimality → make algorithm choose best option at first, leading to not optimal solutions after

↳ sometimes, a not as good first pick leads to better solutions



$|R_1 \bowtie R_2| = \cancel{18}^{\,10}$ 3.6
$|R_2 \bowtie R_3| = \cancel{10}$
$|R_3 \bowtie R_4| = \cancel{18}\ 10$

$(R_2 \bowtie R_3) \bowtie R_1 = \cancel{18}^{\,18}$
$(R_2 \bowtie R_3) \bowtie R_4 = 18$
$(R_3 \times R_4)$ ×

$\rightarrow ((R_2 \bowtie R_3) \bowtie R_4) \bowtie R_1 = \cancel{0.9}^{\,0.05} \times 18 \times \cancel{7}^{\,100}$
$= \cancel{32.4}\ 90$

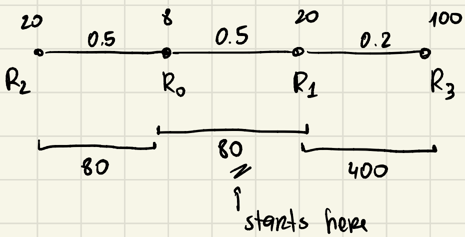
Cout = ~~60.4~~ 111.6

$|R_1 \bowtie R_2| = \cancel{18}\ 10$
$|R_3 \bowtie R_4| = \cancel{18}\ 10$
$(R_1 \bowtie R_2) \bowtie (R_3 \bowtie R_4) = \cancel{0.1}^{\,0.9} \times \cancel{18^2}^{\,10^2} = \cancel{64.4}^{\,90}$ ×

Cout = 100

tip: bigger relationships in the outside and make algorithm start in the middle so the larger relations are appended iteratively and scale the Cout.

Another example: (tutorial)



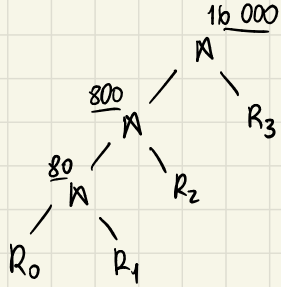
GJO-1 : pick smaller $|R_i|$

GJO-2 : pick relation with makes best cost when joined with set
$\min(\{R_j\} \bowtie R_i)$

GJO-3 : same as GJO-3 but tries with every join at the beginning

GOO : best intermediate cost for every pair of relations

GJO-1 :
GJO-2 :
GJO-3 :



$C_{out} = 16.880$

8. • Bijective function $f_n : [0, 2^{n-1}) \mapsto \Upsilon$
(left-deep join tree for relations $R_0 - R_1 - \dots - R_{n-1}$

• $f_n(i) = h_n(g_n(i))$

• $g_n : [0, 2^{n-1}) \mapsto \{l, r\}^{n-1}$: map $\{0,1\}$ to $\{l, r\}$

• • • TODO

9.

T	i	N	Y	D	B
T	i	N	Y	D	B
0	0	0	0	0	0

→

T	i	N	Y	D	B
T	i	N	Y	B	B
0	0	0	0	0	1

→

T	i	N	Y	D	B
i	i	N	Y	B	B
0	1	0	0	0	1

↓

T	i	N	Y	D	B
i	i	i	Y	B	B
0	1	0	0	0	1

←

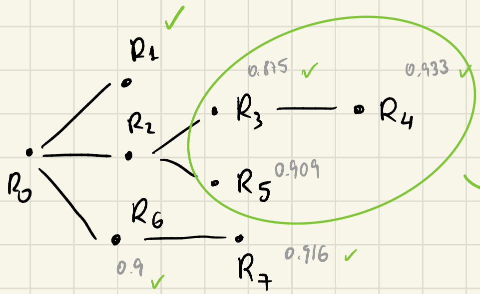
T	i	N	Y	D	B
i	i	i	i	B	B
0	1	0	0	0	1

tip: Union(A,B) if rank(A) = rank(B), B is parent.

10.

R. name	n	s	C	T	rank
R_1	10	0.1	1	1	0
R_2	50	0.2	10	10	0.9
R_3	40	0.2	8	8	0.875
R_4	50	0.3	15	15	0.933
R_5	55	0.2	11	11	0.909
R_6	50	0.2	10	10	0.9
R_7	60	0.2	12	12	0.916
R_2R_3	–	–	90	80	0.877

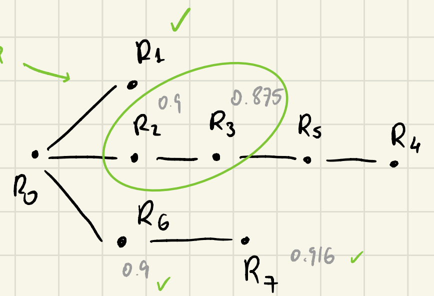
Rank = $\underline{T-1}$



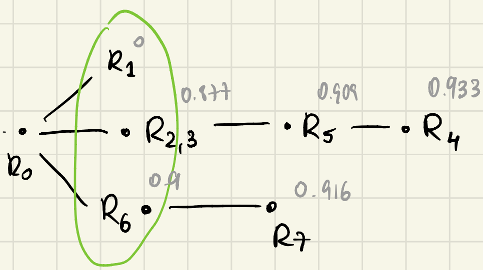
rank (R_3) < rank (R_4) ✓

rank (R_6) < rank (R_7) ✓

merge



$\underline{\text{rank} (R_2) > \text{rank}(R_3)}$



• $C(R_2R_3) = C(R_2) + T(R_2)\ C(R_3)$

$= 10 + 10 \times 8$

$= 90$

rank ($R_{2,3}$) < rank (R_5) ✓

• $T(R_2R_3) = \prod_i s_i \cdot n_i$

$= 0.2^2 \cdot 50 \cdot 40$

$= 10 \cdot 8 = 80$

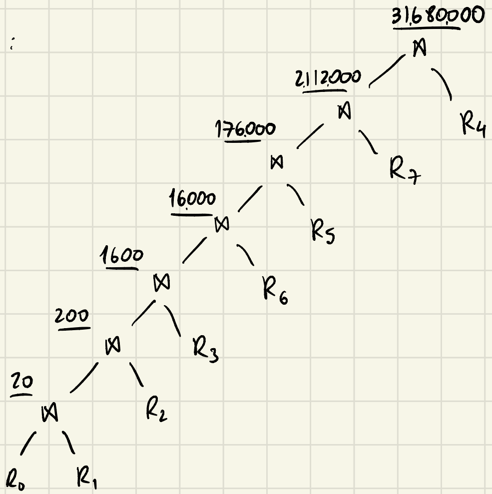
merge and linearize:

$R_0 - R_1 - R_{2,3} - R_6 - R_5 - R_7 - R_4$

$\downarrow$

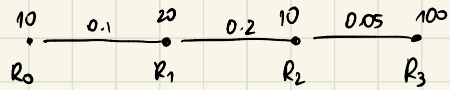
$R_0 - R_1 - R_2 - R_3 - R_6 - R_5 - R_7 - R_4$

Tree :



$C_{out} = 33\ 985\ 820$

11.



root R_0 :

R.name	n	s	C	T	rank
R_1	20	0.1	2	2	0.5
R_2	10	0.2	2	2	0.5
R_3	100	0.05	5	5	0.8

$rank(R_1) \leq rank(R_2) < rank(R_3)$ $\rightarrow$ already ordered

$C_{out} = 20 + 40 + 200 = 260$

root R_1 :

R.name	n	s	C	T	rank
R_0	10	0.1	1	1	0
R_2	10	0.2	2	2	0.5
R_3	100	0.05	5	5	0.8

Linearization: $R_1 - R_0 - R_2 - R_3$

$C_{out} = 20 + 40 + 200 = 260$

root R_2:

R.name	n	s	C	T	rank
R_0	10	0.1	1	1	0
R_1	20	0.1	2	2	0.5
R_3	100	0.05	5	5	0.8

$R_2 - R_{0,1} - R_3 \rightarrow R_2 - R_1 - R_0 - R_3$

$C_{out} = 40 + 40 + 200 = 280$

root R_3:

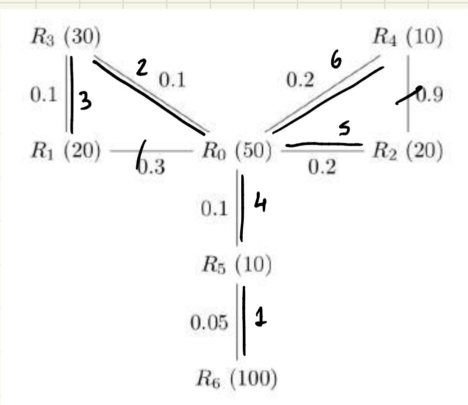
R.name	n	s	C	T	rank
R_0	10	0.1	1	1	0
R_1	20	0.1	2	2	0.5
R_2	10	0.2	2	2	0.5

After denormalization: $R_3 - R_2 - R_1 - R_0$

$C_{out} = 50 + 200 + 200 = 450$

- As IKKBZ solution is optimal (there is no better bushy tree for this query) MVP cannot find a better join tree.

12.



- IKKBZ produces the optimal left-deep tree for the MST
- There could be a better bushy tree for the MST. There could also be a better left-deep tree for the original graph query.

- Using the most selective edges is likely to result in plans with smaller cardinalities

13. **DPsize:** level by level

• fazer de 1 até $\left\lfloor \frac{level}{2} \right\rfloor$ para evitar dups

1: 0, 1, 2, 3, 4 — 5

2: 0|1, 0|3, 1|2, 1|4, 2|3 — 5

3: 01|2, 01|3, 01|4, 03|1, 03|2, 12|0, 12|3, 14|0, 14|2, 23|0, 23|1 — 11

4: 0|123, 0|124, 1|023, 2|013, 2|014, 3|012, 3|014, 3|124, 4|012, 4|013, 4|123, 01|23, 03|12, 03|14, ~~12|03~~, 14|03, 14|23, ~~23|01~~, ~~23|14~~ — 16

5: 0|1234, 2|0134, 3|0124, 4|0123, 03|124, 14|023, 23|014, ~~023|14~~ — 7

Total: 44

binary helps to enum subsets

DPsub:

0, 1, 01, 2, 02, 12, 012, 3, 03, 13, 013, 23, ...

1: 0

10: 1

11: 0|1

100: 2

101: 0|2

110: 1|2

111: 0|12, 1|02, 01|2

1000: 3

1001: 0|3

1010: 1|3

1011: 0|13, 1|03, 01|3

1100: 2|3

1101: 0|23, 2|03, 02|3

1110: 1|23, 2|13, 12|3

1111: 0|123, 1|023, 01|23, 2|013, 02|13, 12|03, 3|012

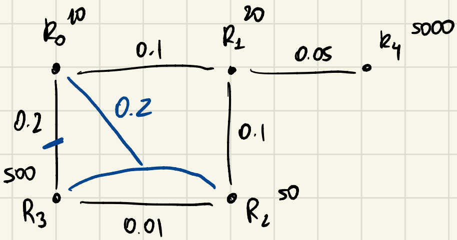
...

15.

4: —
3: —
2: {3}
23: —
1: {4}, {2}, {2,3}
12: {4}, {3}
123: {4}
14: {2}, {2,3}
124: {3}
1234: —

0: {3}, {3,2}, {1}, {1,2}, {1,2,3}, {1,4}, {1,2,4}, {1,2,3,4}
01: {4}, {3}, {2}, {3,2}
012: {4}, {3}
014: {3}, {2}, {2,3}
0124: {3}
03: {2}, {1}, {1,2}, {1,4}, {1,2,4}
013: {4}, {2}
0132: {4}
0134: {2}
01234: —

16.



5000

- $b(0\bowtie 1, 0\bowtie 3) = \dfrac{c((0\bowtie 1)\bowtie 3)}{c((0\bowtie 3)\bowtie 1)} = \dfrac{20+2000}{1000+2000} = \dfrac{202\cancel{0}}{300\cancel{0}}$

- $b(0\bowtie 3, 0\bowtie 1) = \dfrac{300}{202}$

- $b(1\bowtie 0, 1\bowtie 2) = \dfrac{20+100}{100+100} = \dfrac{12}{20} = \dfrac{4}{5}$ • $b(1\bowtie 2, 1\bowtie 0) = 5/4$

- $b(2\bowtie 1, 2\bowtie 3) = \dfrac{100+500}{250+500} = \dfrac{60}{75} = \dfrac{12}{15}$ • $b(2\bowtie 3, 2\bowtie 1) = 15/12$

- $b(3\bowtie 0, 3\bowtie 2) = \dfrac{1000+500}{250+500} = \dfrac{1500}{750} = 2$ • $b(3\bowtie 2, 3\bowtie 0) = 1/2$

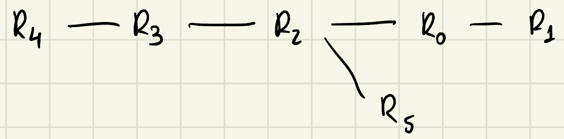
- $b(0\bowtie 1, 1\bowtie 4)$; $b(1\bowtie 2, 1\bowtie 4)$

Replace $\{R_0\} - \{R_3\}$ for $\{R_0\} - \{R_2, R_3\}$ relations $0-3$ and $0-2$ cease to exist

Compute new benefits for

$b(\, 0 \bowtie (2,3),\ 0 \bowtie 1)$
$b(\, (2,3) \bowtie 0,\ 2 \bowtie 1$

17.



LinDP: IKKBZ linearization
study every subtree
find optimal sub-solution
generate optimal bushy-tree
$O(n^3)$

R_0: $n = 20$, $s = 0.2$, $C = 4$, $T = 4$, rank $= 0.75$
R_1: $n = 10$, $s = 0.1$, $C = 1$, $T = 1$, rank $= 0$
R_5: $n = 55$, $s = 0.2$, $C = 11$, $T = 11$, rank $= \frac{10}{11}$

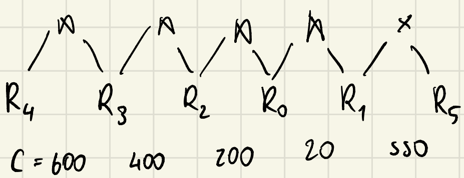
$\text{rank}(R_0) > \text{rank}(R_1) \longrightarrow R_{0,1}$

$R_{0,1}$: $n = -$, $s = -$, $C = C(R_0) + T(R_0)\,C(R_1) = 4 + 4 \times 1 = 8$, $T = 0.1 \times 0.2 \times 20 \times 10 = 4$

rank $= \frac{3}{8}$

After denormalization: $R_4 - R_3 - R_2 - R_0 - R_1 - R_5$

	R_4	R_3	R_2	R_0	R_1	R_5
R_4	0	600	6400	24800	24820	267.020
R_3		0	400	1800	1820	19120
R_2			0	200	220	2420
R_0				0	20	1120
R_1					0	550
R_5						0



R_4 R_3 R_2 R_0 R_1 R_5

- $C((R_4 \bowtie R_3) \bowtie R_2) = 600 + 6000 = 6600$
  $C(R_4 \bowtie (R_3 \bowtie R_2)) = 400 + 6000 = 6400$

18.

SES	TES
{A, B}	{A, B, C, D, E}
{B, C}	{B, C, D, E}
{C, E}	{C, E, D}
{C, D}	{C, D}
{E, F}	{E, F}

edges:

A	BCDE
B	CDE
CD	E
C	D
E	F

19.

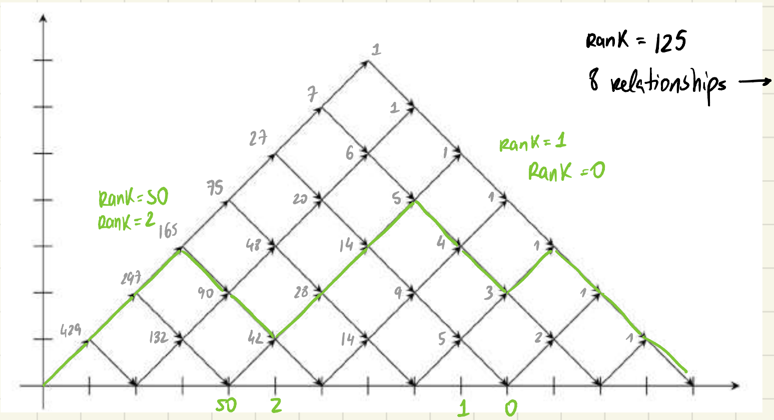
Rank = 64

$\pi[i] = i$
$\text{swap}(\pi[i-1], \pi[R \bmod i])$
$R = \lfloor R/i \rfloor$

- $i = 8$ $\quad R \bmod i = 0 \quad R = 8 \quad$ swap (7, 0)
  [8 2 3 4 5 6 7 1]
- $i = 7$ $\quad R \bmod i = 1 \quad R = 1 \quad$ swap (6, 1)
  [8 7 3 4 5 6 2 1]
- $i = 6$ $\quad R \bmod i = 1 \quad R = 0 \quad$ swap (5, 1)
  [8 6 3 4 5 7 2 1]

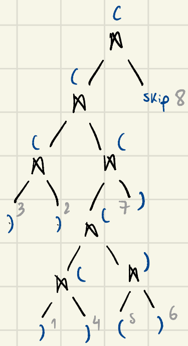
• • •

20.



Rank = 125
8 relationships → 7 inner nodes

- Dyck word: ((( ))((( ))( )))



Permutation: 3 2 1 4 5 6 7 8

(((3 2)((1 4)(5 6)) 7))8)

21. $(A \bowtie B) \bowtie C$

- Memo structure

Class	Initialization	Transformation	Step
$\{A, B, C\}$	$\{A, B\} \bowtie_{111} C$	$C \bowtie_{000} \{A, B\}$	2
		$A \bowtie_{100} \{B, C\}$	3 -
		$B \bowtie_{100} \{A, C\}$	4 -
		$\{B, C\} \bowtie_{000} A$	6
		$\{A, C\} \bowtie_{000} B$	8
$\{A, B\}$	$A \bowtie_{111} B$	$B \bowtie_{000} A$	1
$\{B, C\}$		$B \bowtie_{111} C$	3 -
		$C \bowtie_{000} B$	5
$\{A, C\}$		$A \bowtie_{111} C$	4 -
		$C \bowtie_{000} A$	7

Right associativity

left associativity

when add here, add one class as well

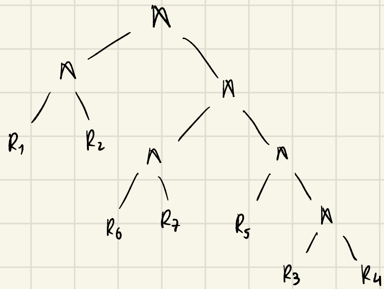
22.

$T_1 : (< R_1 \bowtie R_2 , R_3 >, R_4)$ $\qquad \alpha = [1, 0, 1]$

$T_2 : (< R_6 \bowtie R_7 , R_5 >, R_4)$

Merged anchored list : $(< R_1 \bowtie R_2 , R_6 \bowtie R_7 , R_5 , R_3 >, R_4)$

Resulting tree:



23. • $(R_4 \bowtie_{p_1} (R_3 \bowtie_{p_2} R_5)) \bowtie_{p_3} (R_1 \bowtie_{p_4} R_2)$ ← bushy

- Ordered list: 2143
- Ordinal number: 35 41 23 12

$(1,2,3,4,5) \rightarrow (3 \bowtie 5, 1, 2, 4) \rightarrow (4 \bowtie (3 \bowtie 5), 1, 2) \rightarrow (4 \bowtie (3 \bowtie 5), 1 \bowtie 2)$

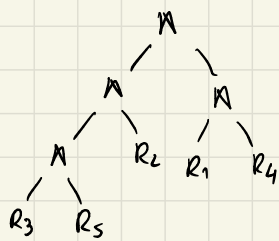
• $(((R_4 \bowtie R_1) \bowtie R_3) \bowtie R_2)$ ← left deep

- ordered list: 4132
- ordinal number: 4121

$(1,2,3,4) \rightarrow (1,2,3) \rightarrow (2,3) \rightarrow (12)$

• bushy tree for 35 13 23 12

$\langle R_1, R_2, R_3, R_4, R_5 \rangle$
$\langle R_3 \bowtie R_5, R_1, R_2, R_4 \rangle$
$\langle (R_3 \bowtie R_5) \bowtie R_2, R_1, R_4 \rangle$
$\langle$ " $, R_1 \bowtie R_4 \rangle$
$\langle ($ " $) \bowtie ($ " $) \rangle$



24. $(((R_1 \bowtie R_2) \bowtie R_3) \bowtie R_4)$ $\underline{vs.}$ $((R_1 \bowtie R_2) \bowtie (R_3 \bowtie R_4))$

cardinality statistics $(\pi_{s_i n_i})$

	R_1	R_2	R_3	R_4
R_1	100	100	8	1
R_2		25	200	25
R_3			40	500
R_4				100

cost costs (Cout)

	R_1	R_2	R_3	R_4
R_1	0	100	108	109
R_2		0	200	225
R_3			0	500
R_4				0

split points

	R_1	R_2	R_3	R_4
R_1		1	2	3
R_2			2	3
R_3				3
R_4				

$Cout((R_1 \bowtie R_2) \bowtie R_3) = 100 + 0.2 \times 0.01 \times 100 \times 40 = 108$

$Cout(R_1 \bowtie (R_2 \bowtie R_3)) = 200 + \ldots > 108$

$Cout(((R_1 \bowtie R_2) \bowtie R_3) \bowtie R_4) = 100 + 8 + 0.01 \times 0.125 \times 100 \times 8 = 109$

Resulting tree: $(((R_1 \bowtie R_2) \bowtie R_3) \bowtie R_4)$ Cost $= \underline{109}$

26.

- Exhaustive transformative approach generates all possible trees

$$n_{trees} = C(n-1)\,n!$$
$$= \frac{1}{(n-1)+1}\binom{2(n-1)}{n-1} \cdot n! = \frac{1}{11}\binom{20}{10} \cdot n!$$

- DPsub : $O(3^n)$ on cliques

  Assignments for R_{left} and R_{right} : $2^n - 1$

  When both are empty : 1

  subtract case where both are {}

$$n_{trees} = 3^n - 2 \times (2^n - 1) - 1 = 3^n - 2^{n+1} + 1$$

27.

- $\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{n!}{(n-(n-k))!\,(n-k)!} = \binom{n}{n-k}$

$$\bullet \binom{n-1}{k} + \binom{n-1}{k-1} = \frac{(n-1)!}{k!\,(n-1-k)!} + \frac{(n-1)!}{(k-1)!\,(n-\cancel{1}-k+\cancel{1})!}$$

$$= \frac{(n-1)!}{k(k-1)!\,(n-1-k)!} + \frac{(n-1)!}{(k-1)!\,(n-k)(n-k-1)!}$$

$$= \frac{(n-1)!}{(k-1)!\,(n-1-k)!} \left(\frac{1}{k} + \frac{1}{n-k} \right)$$

$$= \quad n \quad \left(\frac{n-k+k}{k(n-k)} \right)$$

$$= \quad n \quad \left(\frac{n}{k(n-k)} \right) = \frac{n!}{k!\,(n-k)!} = \binom{n}{k}$$

28. $Yao = \overline{Y}_n^{N,m}(k)$ $\quad m = 3$ pages $\quad n =$ tuples $= 2$ $\quad N = m \times n = 6$

- 2 distinct tuples $\Rightarrow$ $\underline{k=2}$

$$\overline{Y}_2^{6,3}(k) \overset{k=2}{=} \overline{Y}_2^{6,3}(2) = 3 \times \overline{Y}_2^{6}(2)$$

$$= 3 \cdot (1-p) = 3 \cdot \left(1 - \frac{\binom{6-2}{2}}{\binom{6}{2}} \right)$$

$$= 3 \cdot \left(1 - \frac{4}{6} \cdot \frac{3}{5} \right) = 1.8$$

30. • Probability of having j pages skipped before first 1 is :

$$B_b^{B}(j) = \frac{\binom{B-j-1}{b-1}}{\binom{B}{b}} \quad \text{for } j \in \{0, 30, 60, 90\}$$

- Estimated number of pages skipped :

$$\overline{B}_b^{B} = \overline{B}_{10}^{100} = \frac{B-b}{b+1} = \frac{90}{11} = 8.18$$

- Pages between the beginning of the relation and the last page to be read ≡ after the last page

$$B - \bar{B}_b^B = 100 - 8.18 = 91.82$$

- Pages between first and last relation to be read

$$B - 2\bar{B}_b^B = 100 - 2 \times 8.18 = 83.64$$

31. $R.a. >= 55$

$$n_{elem} = \frac{60-55}{60-40} \cdot 4 + \frac{80-60}{80-60} \cdot 2 = \frac{1}{4} \times 4 + 2 = 3$$

32. $R.a = S.b$

$[0,20)$	$[0,10)$	$\frac{1}{2} \cdot 2 = 1$
$[0,20)$	$[10,20)$	$\frac{1}{2} \cdot 4 = 2$
$[20,40)$	$[20,40)$	$1 \cdot 3 = 3$
$[40,60)$	$[40,50)$	$2 \cdot 6 = 12$
$[40,60)$	$[50,100)$	$2 \cdot \frac{4}{5} = 1.6$
$[60,80)$	$[50,100)$	$2 \cdot \frac{2}{5} \cdot 4 = 3.2$
$[80,100)$	$[50,100)$	$0 \cdot \frac{2}{5} \cdot 4 = 0$

$$\Sigma = 22.8$$