$$\frac{d^2x(t)}{dt^2} + \omega_0^2 x(t) = 0 \Rightarrow \frac{d^2x(t)}{dt^2} = \underbrace{\left(-\omega_0^2 x(t)\right)}_{f(x)}$$

$$\rightarrow x(t-\Delta t) = x(t) - \Delta t\, x'(t) + \frac{\Delta t^2}{2} x''(t)$$

$$\Rightarrow \frac{x''(t)}{2} = \frac{x(t-\Delta t) - x(t) + \Delta t\, x'(t)}{\Delta t^2}$$

$$\rightarrow x(t+\Delta t) = x(t) + \Delta t\, x'(t) + \frac{\Delta t^2}{2} x''(t)$$

$$\Rightarrow \frac{x''(t)}{2} = \frac{x(t+\Delta t) - x(t) - \Delta t\, x'(t)}{\Delta t^2}$$

Donc $x''(t) = \dfrac{x(t-\Delta t) - 2x(t) + x(t+\Delta t)}{\Delta t^2}$

$$\frac{d^2x}{dt^2} + \omega_0^2 x(t) = 0 \Rightarrow \frac{d^2x}{dt^2} = -\omega_0^2 x \qquad \omega_0 = \sqrt{\frac{k}{m}}$$

On pose $v = \dfrac{dx}{dt} \Rightarrow$ (S) : $\begin{cases} \dfrac{dx}{dt} = v \\ \dfrac{dv}{dt} = -\omega_0^2 x \end{cases}$

$$\Rightarrow \underset{\begin{pmatrix} x' \\ v' \end{pmatrix}}{X'(t)} = f(\underset{\begin{pmatrix} x \\ v \end{pmatrix}}{X}, t) \quad \text{avec} \quad f(X,t) = \begin{pmatrix} v(t) \\ -\omega_0^2 x(t) \end{pmatrix}$$

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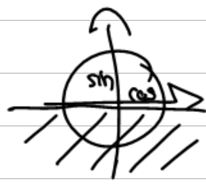
sol° exacte : $x(t) = x_0 \cos(\underset{\downarrow\, 5}{\omega_0} t + \phi_0)$

$x(0) = 0 \Rightarrow x_0 \cos(\phi_0) = 0$

et $v(0) = 1 \Rightarrow \dfrac{dx}{dt}(0) = 1 \Rightarrow -x_0 \omega_0 \sin(\phi_0) = 1$



$\Rightarrow$ soit $x_0 = 0$ Imp soit $\cos(\phi_0) = 0 \Rightarrow \phi_0 = \dfrac{\pi}{2}$

$\Rightarrow -x_0 \omega_0 = 1 \Rightarrow x_0 = \dfrac{-1}{5}$