

$$\frac{\partial^2 \alpha(t)}{\partial t^2} + \omega_0^2 \alpha(t) = 0 \Rightarrow \frac{\partial^2 \alpha(t)}{\partial t^2} = \underbrace{(-\omega_0^2 \alpha(t))}_{f(\alpha)}$$

$$\begin{aligned} \rightarrow \alpha(t - \Delta t) &= \alpha(t) - \Delta t \alpha'(t) + \frac{\Delta t^2}{2} \alpha''(t) \\ \Rightarrow \frac{\alpha''(t)}{2} &= \frac{\alpha(t - \Delta t) - \alpha(t) + \Delta t \alpha'(t)}{\Delta t^2} \end{aligned}$$

$$\begin{aligned} \rightarrow \alpha(t + \Delta t) &= \alpha(t) + \Delta t \alpha'(t) + \frac{\Delta t^2}{2} \alpha''(t) \\ \Rightarrow \frac{\alpha''(t)}{2} &= \frac{\alpha(t + \Delta t) - \alpha(t) - \Delta t \alpha'(t)}{\Delta t^2} \end{aligned}$$

$$\text{Donc } \alpha''(t) = \frac{\alpha(t - \Delta t) - 2\alpha(t) + \alpha(t + \Delta t)}{\Delta t^2}$$

$$\frac{\partial^2 \alpha}{\partial t^2} + \omega_0^2 \alpha(t) = 0 \Rightarrow \frac{\partial^2 \alpha}{\partial t^2} = -\omega_0^2 \alpha \quad \omega_0 = \sqrt{\frac{k}{m}}$$

$$\text{On pose } v = \frac{\partial \alpha}{\partial t} \Rightarrow \textcircled{a} : \begin{cases} \frac{\partial \alpha}{\partial t} = v \\ \frac{\partial v}{\partial t} = -\omega_0^2 \alpha \end{cases}$$

$$\Rightarrow \begin{pmatrix} \alpha' \\ v \end{pmatrix}'(t) = \underset{\begin{pmatrix} v \\ -\omega_0^2 \alpha \end{pmatrix}}{f(X, t)} \quad \text{avec } f(X, t) = \begin{pmatrix} v(t) \\ -\omega_0^2 \alpha(t) \end{pmatrix}$$

$$\text{sol}^0 \text{ exacte: } \alpha(t) = \alpha_0 \cos(\underbrace{\omega_0 t}_s + \phi_0)$$

$$\begin{aligned} \text{et } \alpha(0) = 1 &\Rightarrow \alpha_0 \cos(\phi_0) = 1 \\ v(0) = 1 &\Rightarrow \frac{\partial \alpha}{\partial t}(0) = 1 \Rightarrow -\alpha_0 \omega_0 \sin(\phi_0) = 1 \end{aligned}$$

$$\begin{aligned} \Rightarrow \text{soit } \alpha_0 &= 0 & \text{soit } \cos(\phi_0) &= 0 \Rightarrow \phi_0 = \frac{\pi}{2} \\ \text{Imp} & & \Rightarrow -\alpha_0 \omega_0 &= 1 \Rightarrow \alpha_0 = -\frac{1}{\omega_0} \end{aligned}$$

