RUSSELL's Metatheoretic Study - Abstract

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We are working on the formalization of RUSSELL's type theory in the COQ proof assistant [3]. The type system of RUSSELL is based on the Calculus of Constructions with Σ-types (dependent sums), extended by an equivalence on types which subsumes β-conversion. The extension permits to identify types and subsets based on them in a manner similar to the *Predicate Subtyping* feature of PVS.

We are aiming at a complete proof of RUSSELL's metatheoretic properties (structural properties, Subject Reduction, maybe Strong Normalization), the refining steps which led us to the algorithmic system and the corresponding typing algorithm and also the correctness of an interpretation from RUSSELL to the Calculus of Inductive Constructions with metavariables.

We started the development using the formalization of the Calculus of Constructions by Bruno Barras [2]. We kept the standard de Bruijn encoding for variable bindings and defined our judgements using *dependent* inductive predicates. This alone causes some problems for the faithful formalization of the paper results. The proofs offer several other technical difficulties including:

- Elimination of transitivity in a system with an *untyped* type conversion relation.
- Subject Reduction, which is *not* directly provable for the declarative system. Here we adapted the technique developed by Robin Adams [1]. It includes a new term algebra, with associated reduction operations and a new type system for which we have to prove metatheoretic properties.
- Correction of the interpretation: the target system has metavariables, introducing a second, *unusual* kind of variable binding.

We will present these difficulties and our recipes for solving them in the COQ environment. Additionally, we will identify more general problems arising in COQ proofs and suggest possible solutions.

Disclaimer: this is a work in progress ; to date we have proved elimination of transitivity and subject reduction should follow soon.

References

[1] ADAMS, R. Pure Type Systems with Judgemental Equality. *Journal of Functional Programming 16* (2006), 219–246. http://www.cs.rhul.ac.uk/~robin/ptseq8.ps.gz.

[2] BARRAS, B. Coq en coq. Rapport de Recherche 3026, INRIA, Oct. 1996.

[3] SOZEAU, M. Russell Metatheoretic Study in Coq, experimental development, 2006. http://www.lri.fr/~sozeau/research/russell.en.html.