RHUL Psychology Statistical modelling notebook

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Contents

Chapter 1

About

This online book is created and maintained by Matteo Lisi and is meant to be a shared resource for staff and students at the Department of Psychology of Royal Holloway, University of London. It will contain a miscellanous set of tutorial, examples, case studies, workshops materials and any other useful material related to data analysis and modelling. These will be added and revised over time, based on the most common questions and requests that I receive.

This is a work in progress and may contain imprecisions and typos. If you spot any please let me know at matteo.lisi [at] rhul.ac.uk. The materials that will be included builds upon and draw from existing literature on statistics and modelling. I will endeavor to properly cite existing books and papers; but if any author feels that I have not given them fair acknowledgement, please let me know and I will make amend.

Chapter 2

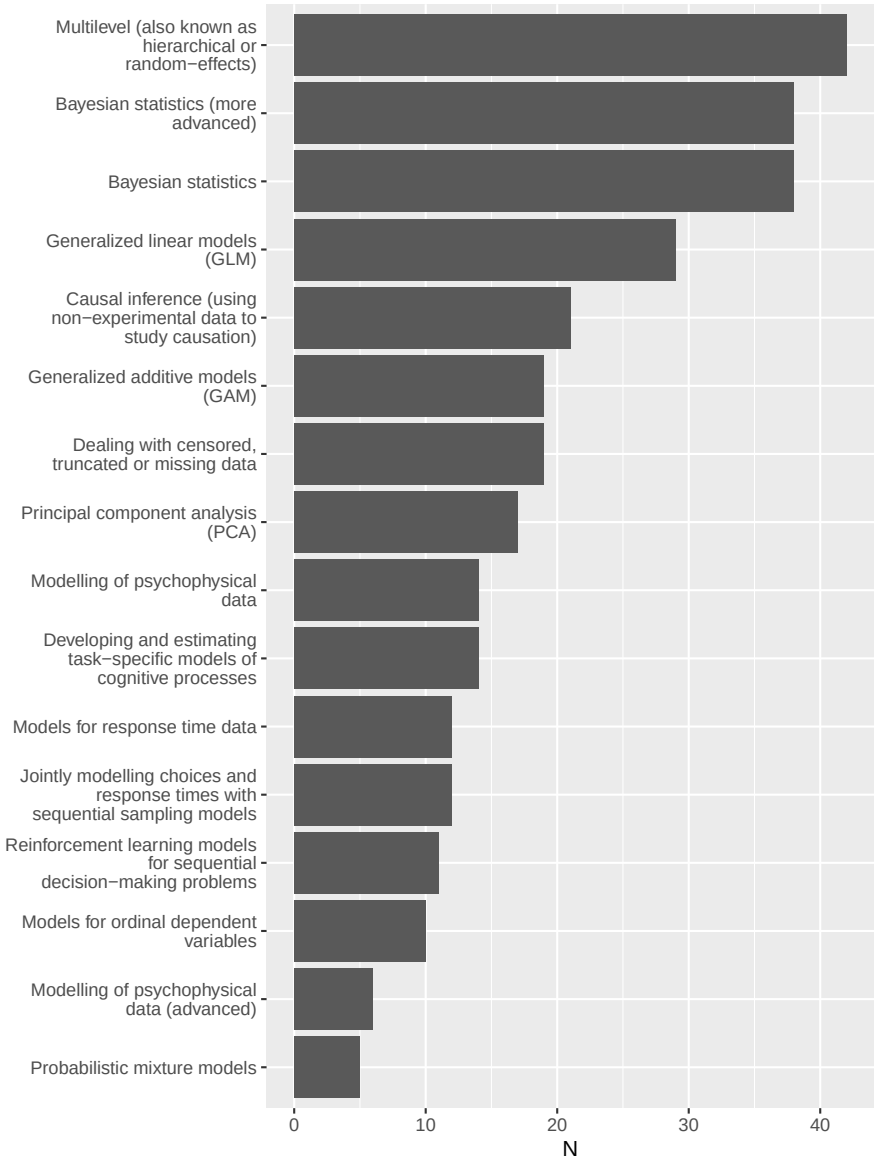
Departmental survey about statistical methods

I used an anonymous survey to ask colleagues some questions about which topics may be more interesting or useful in their research.

2.1 March 2022

2.1.1 Question 1

In the first question people indicated topics of interests. The winner are multi-level models, followed closely by Bayesian statistics.



There were some additional suggestions.

```
#> [1] "power analyses using Shiny apps"
#> [2] "agent-based models"
#> [3] "this may be covered in the above, but approaches to analysing experience sampli
#> [4] "Methods for longitudinal analyses"
#> [5] "Network modelling"
#> [6] "Neural networks, Markov processes"
```

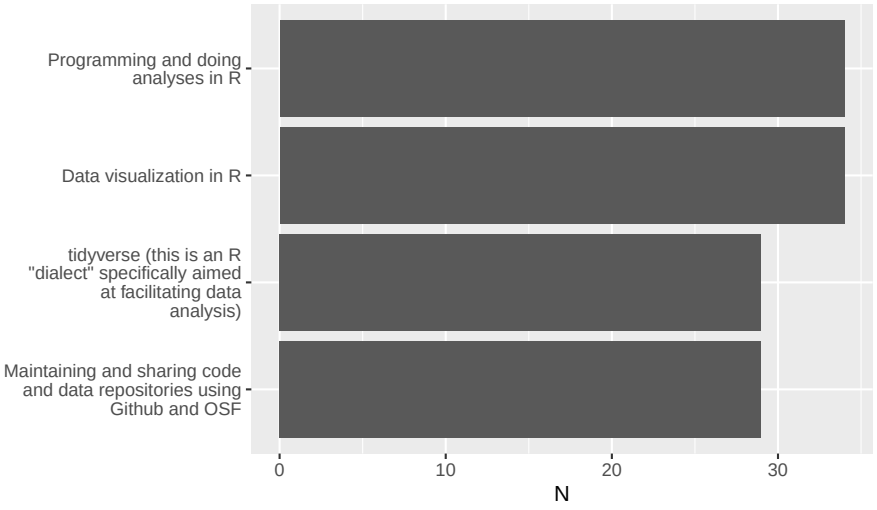
```
#> [7] "Random forests and related"
#> [8] "causal modelling using regression models - path models etc"
#> [9] "prediction modelling"
```

A few other topics were mentioned in the comment section:

- Shiny apps
- Network modelling
- Longitudinal analyses
- Random forests
- Neural network

2.1.2 Question 2

Here people indicated their interest for topics related to data analysis.

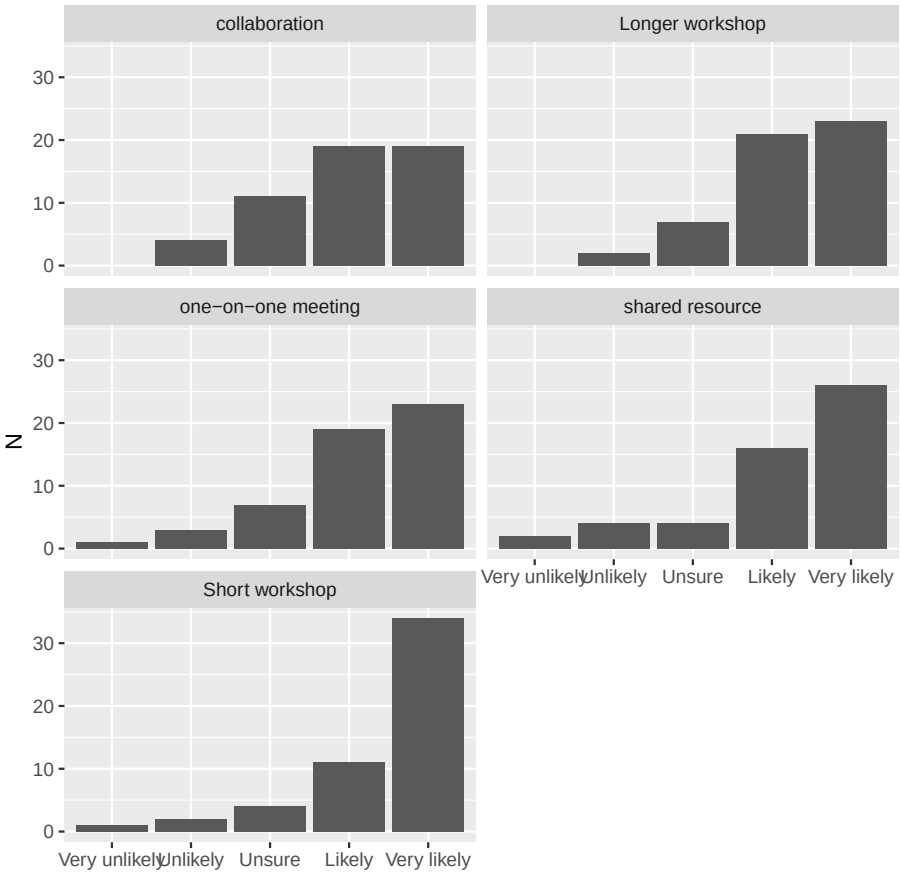


Other things mentioned in the comments were:

- SPM
- Docker
- Python

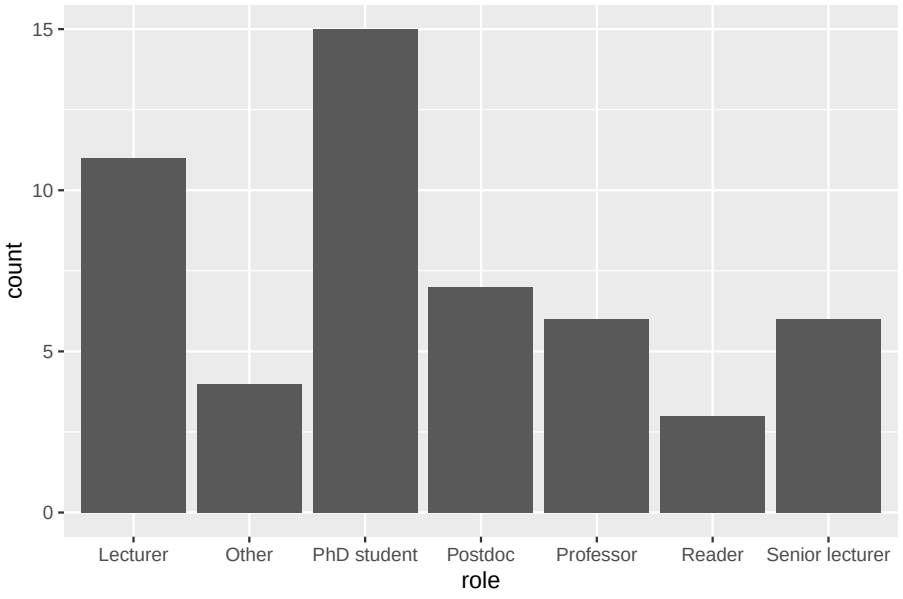
2.1.3 Question 4

This question was about likelihood of using different formats of support



2.1.4 Respondents' status

The final questions asked about the status / career level.


15
10
5
0
count
Lecturer
Other
PhD student
Postdoc
Professor
Reader
Senior lecturer
role

Chapter 3

Introduction to R

Most of the practical statistical tutorials and recipes in this book use the software R, so this section provides some introduction to R for the uninitiated.

3.1 Installing R

The base R system can be downloaded at the following link, which provides installers for both Windows, Mac and Linux:

https://cran.rstudio.com/

In addition to the base R system, it is useful to have also R-studio, which is an IDE (Integrated Development Environment) for R, and provides both an editor, a graphical interface and much more. It can be downloaded from:

https://www.rstudio.com/products/rstudio/download/

3.2 First steps

R is a programming language and free software environment for statistical computing and graphics. It is an *interpreted language*, which means that to give

instructions to the computer you do not have to compile it first in machine language, everything is done 'on the fly' through a command line interpreter, e.g. if you type `2+2` in the command line R, the computer will reply with the answer (try this on your computer):

```
2+2
#> [1] 4
```

Typically the normal workflow involve writing and saving a series of instructions in a script file, which can be executed (either step by step or all at once). This makes statistical analyses transparent and reproducible since everyone in possess of the same data and script should be able to obtain the same results.

In an R script you can use the `#` sign to add comments, so that you and others can understand what the R code is about. Comments are not run as R code, so they will not influence your result. See the next example:

```
# calculate 3 + 4
3 + 4
#> [1] 7
```

3.2.1 Arithmetic with R

In its most basic form, R can be used as a simple calculator. Consider the following arithmetic operators:

- Addition: `+`
- Subtraction: `-`
- Multiplication: `*`
- Division: `/`
- Exponentiation: `^`
- Modulo: `%%`

The last two might need some explaining:

The `^` operator raises the number to its left to the power of the number to its right: for example `3^2` is 9.

The modulo returns the remainder of the division of the number to the left by the number on its right, for example 5 modulo 3 (or `5 %% 3`) is 2.

Try on your computer:

- Type `2^5` in the editor to calculate 2 to the power of 5.
- Type 28 %% 6 to calculate 28 modulo 6.

3.2.2 Variable assignment

A basic concept in programming (statistical or not) is called a *variable*.

A variable allows you to store a value (e.g. 2) or an object (e.g. a function description) in R. You can then later use this variable's name to easily access the value or the object that is stored within this variable.

You can assign a value `2` to a variable `my_var` with the command

```
my_var <- 2
```

Note that you would have obtained the same result using:

```
2 -> my_var
```

that is, the *assignment operator* works in both directions `<-` and `->`.

The variable can then be used in any computation, for example:

```
my_var + 2
#> [1] 4
```

Try on your computer:

- Assigns the value 42 to the variable x in the editor, then print out its value.
- Suppose you have a fruit basket with five apples. As a data analyst, you want to store the number of apples in a variable with the name `my_apples`.
- You decide to add six oranges to your fruit basket. Create the variable `my_oranges` and assign the value 6 to it. Next, you want to calculate how many pieces of fruit you have in total. Since you have given meaningful names to these values, you can now code this in a clear way by typing `my_apples + my_oranges`
- Assign the result of adding `my_apples` and `my_oranges` to a new variable `my_fruit`

3.2.3 Basic data types in R

Despite common knowledge we just added apples and oranges. The `my_apples` and `my_oranges` variables both contained a number (a numerical value) in the previous exercise. The `+` operator works with numeric variables in R. If you really tried to add "apples" and "oranges", and assigned a *text value* to the variable my_oranges, you would be trying to assign the addition of a numeric and a character variable to the variable `my_fruit`. This is not possible, and R will give you an error message.

```
# Assign a value to the variable my_apples
my_apples <- 5

# Assign a text value
my_oranges <- "six"

#
```

```
my_fruit <- my_apples + my_oranges
#> Error in my_apples + my_oranges: non-numeric argument to binary operator
```

In fact R works with numerous data types, and some of these are not numerical (so they can't be added, subtracted, etc.). Some of the most basic types to get started are:

- Decimal values like 4.5 are called numerics.
- Natural numbers like 4 are called integers. Integers are also numerics.
- Boolean values (`TRUE` or `FALSE`, abbreviated `T` and `F`) are called logical[1].
- Text (or string) values are called characters.

Try on your computer:

- Assign `my_numeric` variable to `42`.
- Assign `my_character` variable to "universe". (Note that the quotation marks indicate that "universe" is a character.)
- Assign `my_logical` variable to `FALSE`. (Note that R is case sensitive!)

3.3 Using R for statistical analyses

3.3.1 Example 1: *How old is the universe?*

In this example[2] we will see how to import data into R and perform a simple linear regression analysis.

According to the standard big-bang model, the universe expands uniformly and locally, according to Hubble's law

$$y = \beta x$$

where y and x are the relative velocity and distance of a galaxy, respectively; and β is "Hubble's constant". Note that this is a simple linear equation, in which β indicate how much y changes for each unitary increase in x.

[1] Note that you can add or multiply logical Boolean values: internally `FALSE` are treated as zeroes, and `TRUE` as ones.

[2] Taken from Simon Wood's book on GAM(Wood, 2017).

According to this model β^{-1} gives the approximate age of the universe, but β is unknown and must somehow be estimated from observations of y and x, made for a variety of galaxies at different distances from us. Luckily we have available data from the Hubble Space Telescope. Velocities are assessed by measuring the Doppler effect red shift in the spectrum of light that we receive from the Galaxies. Distance is estimated more indirectly, by using the discovery that in certain class of stars (Cepheids), which display fluctuations in diameter and temperature over a stable period, there is a systematic relationship between the period and their luminosity.

We can load the data in R using the following code

```
d <- read.table("https://raw.githubusercontent.com/mattelisi/RHUL-stats/main/data/hubble.txt", he
```

`read.table` is a generic function to import dataset in text files (e.g. .csv files) into R. We use the argument `header=T` to specify that the first line of the dataset gives the names of the columns. To see the help of this function, and what other arguments and features are available type `?read.table` in the R command line.

We can use the command `str()` to examine what we have imported

```
str(d)
#> 'data.frame':    24 obs. of  3 variables:
#>  $ Galaxy  : chr  "NGC0300" "NGC0925" "NGC1326A" "NGC1365" ...
#>  $ velocity: int  133 664 1794 1594 1473 278 714 882 80 772 ...
#>  $ distance: num  2 9.16 16.14 17.95 21.88 ...
```

This tells us that our data frame has 3 variables:

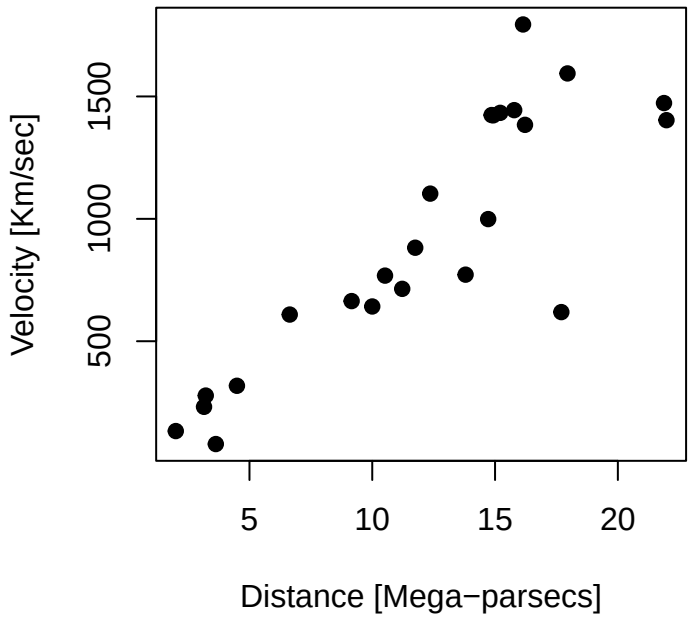
- `Galaxy`, the 'names' of the galaxies in the dataset
- `velocity`, their relative velocity in Km/sec
- `distance`, their distance expressed in Mega-parsecs[3]

We can plot[4] them using the following code:

```
plot(d$distance, # indicate which variable on X axis
     d$velocity, # indicate which variable on Y axis
     xlab="Distance [Mega-parsecs]",
     ylab="Velocity [Km/sec]",
     pch=19) # set the type of point
```

[3] 1Mega-parsec $= 3.09 \times 10^{19}$Km

[4] See `?plot` for more info about how to customize plots in R.



It is clear, from the figure, that the observed data do not follow Hubble's law exactly, but given the how these measurements were obtained (there is uncertainty about the true values of the distance and velocities) it would be surprising if they did. Given the apparent variability, what can be inferred from these data? In particular:

1. what value of β is most consistent with the data?
2. what range of β values is consistent with the data?

In order to make inferences we make some assumptions about the nature of the measurement noise. Specifically, we assume that measurements errors are well-characterized by a Gaussian distribution. This result in the following model:

$$y = \beta x + \epsilon$$
$$\epsilon \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$$

which is essentially a linear regression but without the intercept.

We can fit the model with the function `lm` in R.

```
hub.m <- lm(velocity ~ 0 + distance, d)
summary(hub.m)
```

```
#>
#> Call:
#> lm(formula = velocity ~ 0 + distance, data = d)
#>
#> Residuals:
#>    Min     1Q Median     3Q    Max
#> -736.5 -132.5  -19.0  172.2  558.0
#>
#> Coefficients:
#>          Estimate Std. Error t value Pr(>|t|)
#> distance   76.581      3.965   19.32 1.03e-15 ***
#> ---
#> Signif. codes:
#> 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#>
#> Residual standard error: 258.9 on 23 degrees of freedom
#> Multiple R-squared:  0.9419, Adjusted R-squared:  0.9394
#> F-statistic: 373.1 on 1 and 23 DF,  p-value: 1.032e-15
```

So, based on this data, our estimate of the Hubble constant is 76.58 with a standard error of 3.96.

So, how old?

The Hubble constant estimate have units of $\frac{\text{Km/sec}}{\text{Mega-parsecs}}$. A Mega-parsecs is 3.09×10^{19}Km, so we divide our estimate of $\hat{\beta}$ by this amount. The reciprocal of $\hat{\beta}$ then gives the approximate age of the universe (in seconds). In R we can calculate it (in years) as follow

```
hubble.const <- coef(hub.m)/(3.09 * 10^(19))
age <- 1/hubble.const
age/(60^2 * 24 * 365)
#>    distance
#> 12794692825
```

giving an estimate of about 13 billion years.

3.4 Other learning resources

There is plenty of resources on the web to learn R. I will recommend a couple that I think are particularly well-done and useful:

- Software Carpentry tutorials on R for Reproducible Scientific Analysis
- The free book Learning Statistics with R by Danielle Navarro

Chapter 4

Meta-analyses

For running meta-analyses, we recommend the `metafor` package for R (see link 1, link 2).

A comprehensive, hands-on guide on how to use this package is provided in the book by Harrer and colleagues (Harrer et al., 2021), freely available at this link.

An alternative to the `metafor` package is to Bayesian multilevel modelling (also discussed in the book by Harrer and colleagues). A more technical discussion of Bayesian multilevel modelling for meta-analyes is provided in this paper by Williams, Rast and Bürkner (Williams et al., 2018).

Chapter 5

Missing data

5.1 Types of missing data

Following the work of Rubin(RUBIN, 1976), missing data are typically grouped in 3 categories:

- Missing completely at random (**MCAR**). This assumes that the probability of being missing is the same for all cases; this implies that the mechanisms that causes missingness is not related in any way to the data. For example, say, there's a known unpredictable error on the server side that prevented recording some responses for some respondents to a survey. As the missingness is entirely independent on the respondents' characteristics, this would be MCAR. When the data are MCAR we can ignore a lot of the complexities and just do a *complete-case* analysis (that is, simply exclude incomplete observations from the dataset). A part from possible loss of information, doing a complete case analysis should not introduce bias in the results. In practice, however, it is difficult to establish whether the data are truly MCAR. Ideally, to argue that data are MCAR, one should have a good idea of the mechanisms that caused missingness (more on this below). Formally, data is MCAR if

  $$\Pr(R = 0|Y_{\text{obs}}, Y_{\text{mis}}, \psi) = \Pr(R = 0|\psi)$$

  where R is an indicator variable that is set to 0 for missing data and 1 otherwise; $Y_{\text{obs}}, Y_{\text{mis}}$ indicate observed and missing data, respectively; and ψ is simply a parameter that determine the overall (fixed) probability of being missing.

- Missing at random (**MAR**). A less strong assumption about missingness is that it is systematically related to the observed but not the unobserved data. For example, data are MAR if in a study male respondents are less likely to complete a survey on depression severity than female respondents

- that is, the probability of reaching the end of the survey is related to their sex (fully observed) but not the severity of their symptoms. Formally, data is MAR if

$$\Pr(R = 0|Y_{\text{obs}}, Y_{\text{mis}}, \psi) = \Pr(R = 0|Y_{\text{obs}}, \psi)$$

When data are missing at random (MAR) the results of complete case analyses may be biased and a common approach to deal with this is to use imputation. Stef van Buuren has a freely available online book on this topic(van Buuren, 2018). Among other things, it illustrates how to do multiple imputation in R with examples.

- Missing not at random (**MNAR**). This means that the probability of being missing varies for reasons that are unknown to us, and may depends on the missing values themselves. Formally this means that $\Pr(R = 0|Y_{\text{obs}}, Y_{\text{mis}}, \psi)$ does not simplify in any way. This case is the most hard to handle: a complete case analyses may or may not be biased, but there is no way of knowing it and we may have to find more information about what caused missingness.

5.2 Deciding whether the data are MCAR

As MCAR is the only scenario in which it is safe to do a complete case analysis, it would seem useful to have way to test this assumption. Some approaches have been proposed to test whether the data are MCAR, but they are not widely used and it's not clear how useful they are in practice. For example one could run a logistic regression with "missingness" as dependent variable (e.g. an indicator variable set to 1 if data is missing and 0 otherwise), and all other variables as predictors - if the data are MCAR then none of the predictors should predict missingness. A popular alternative, implemented in several software packages is Little's test(Little, 1988).

Technically, these approaches can help determine whether the missingness depends on some observed variables (that is, if they are MAR), but strictly speaking cannot exclude missingness due to unobserved variables (MNAR scenario). Nevertheless, if one has good reasons to believe that the data are MCAR, and want to add some statistical test that corroborate this assumption, these could be reasonable tests to do. However, it remains important to also discuss openly possible reasons and mechanisms of missingness, and explain why we deem it a priori plausible that the data are MCAR. In fact, *statistical tests alone cannot tell whether data are missing completely at random.* The terms MCAR, MAR and MNAR refers to the *causal* mechanisms that is responsible for missing data and, strictly speaking, causal claims cannot be decided uniquely on the basis of a simple statistical test. If the data "pass" the test it would provide some additional support to the assumption that they are MCAR, but in and of itself the test alone does not fully satisfy the assumptions of MCAR. To see why note that MCAR (as formally defined above) assumes also that there should be no relationship between the missingness on a particular variable and the values of

that same variable: but since this is a question about what is missing from the data, it cannot be tested with any quantitative analysis of the available data. Finally, it should be added that as these are null-hypothesis significance test, a failure to reject the null hypothesis does not, in and of itself, provide evidence for the null hypothesis (that the data are MCAR). It may be also that we don't have enough power to reliably detect the pattern in the missingness.

5.3 Causal analysis and Bayesian imputation

The best and most principled approach to deal with missingness (at least in my opinion) is to think hard about the causal mechanisms that may determine missingness, and use our assumption about the causal mechanisms to perform a full Bayesian imputation (that is , treating the missing data as parameter and estimating them).

I plan to create and include here a worked example of how to do this; in the meantime interested readers are referred to Chapter 15 (in particular section 15.2) of the excellent book by Richard McElreath *Statistical Rethinking*(McElreath, 2020) which present a very accessible worked example of how to do this in R.

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