DMD-MOS Image Plane Fit

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1 Introduction

Every point on the plane of the DMD can be identified by a coordinate (x, y). For some particular wavelength λ, a point (x, y) can be mapped onto a point on the image plane (CCD detector) with a coordinate (x_d, y_d). We define the coordinate system on the DMD plane such that the centre of the DMD is at the origin, $(x, y) = (0, 0)$. Similarly, we define the coordinate system on the image plane such that the centre of the CCD detector is at the origin, $(x_d, y_d) = (0, 0)$. Our goal is to find a mapping from (x, y) to (x_d, y_d) for any value of λ. Note that this mapping is non-injective. The centre wavelength λ_c of this system is 0.55 µm.

2 Simple Fit $(y = 0)$

We will first deal with the simple case that $y = 0$. A visualization of the DMD is shown in Figure 1.

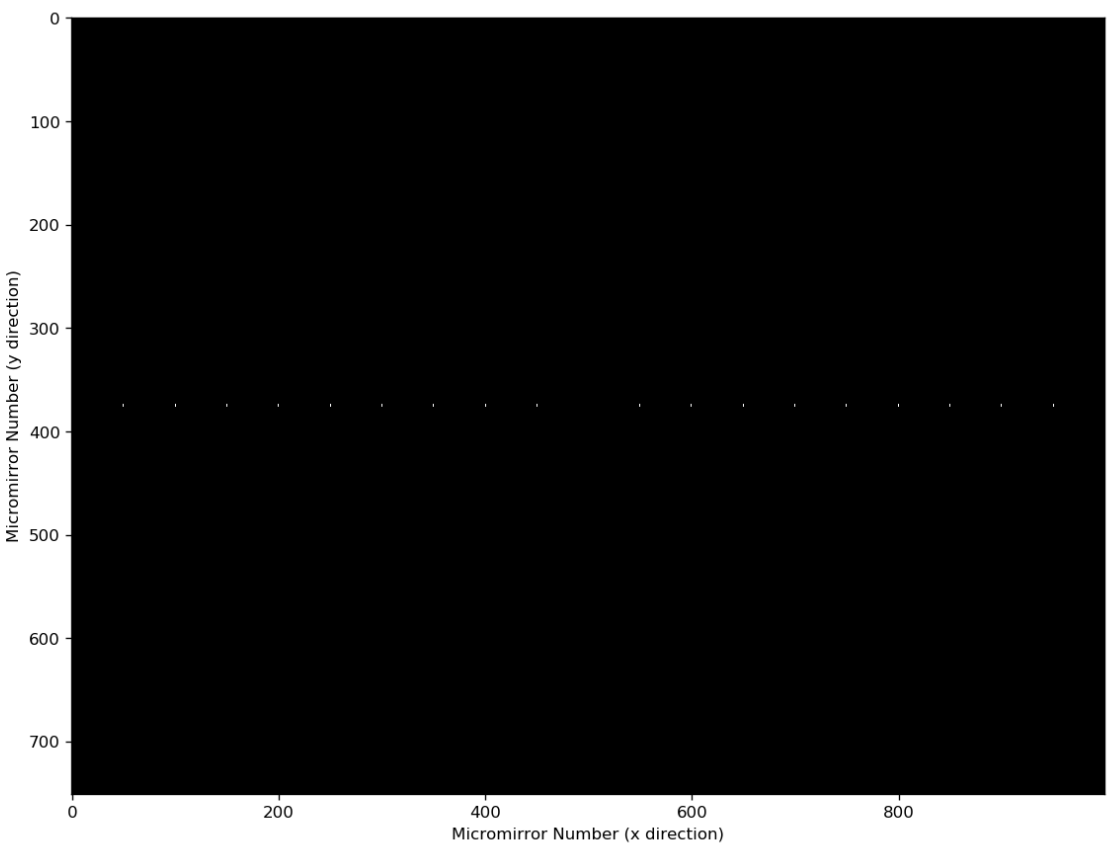


Figure 1: Visualization of the DMD. Squares in white represent a micromirror that is in the ON configuration.

The Geometric Image Analysis tool (GIA) in Zemax OpticStudio was used to generate simulated performance data of the DMD-MOS system. 1 million rays were launched into the system, with the DMD configuration in Figure 1. The result on the CCD is shown in Figure 2. 13 wavelengths were used, ranging from 0.4 µm to 0.7 µm.

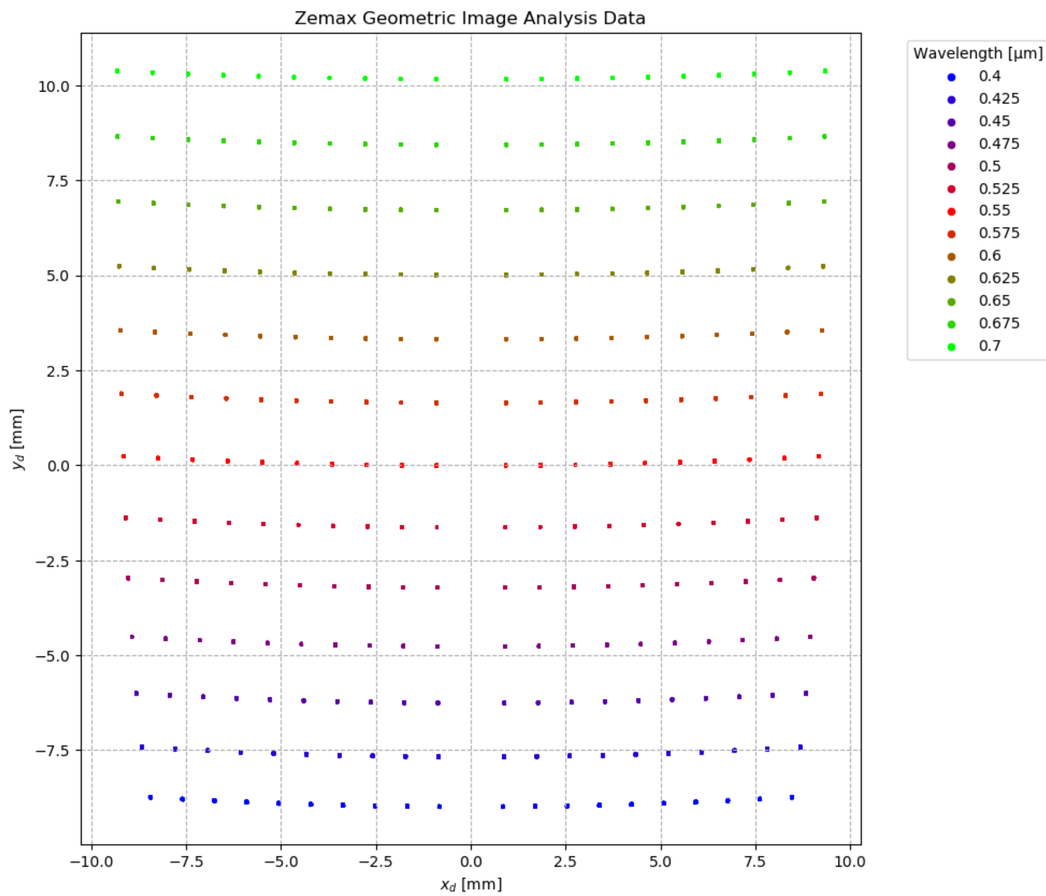


Figure 2: Zemax Geometric Image Analysis data for $y = 0$

Let M_S be the magnification of the spectrograph. Suppose an ON micromirror on the DMD has centre coordinate (x, y). Without distortion, this point will be mapped to some point on the CCD with $x_d = M_S x$, and a y_d that depends on M_S and λ. However, hyperspectral imaging systems have smile distortion and keystone distortion, which would need to be corrected.

2.1 Smile Distortion

Smile distortion is a shift in the spectra at some wavelength, which happens because the dispersion varies with field position. Figure 3 shows a close-up plot of the Zemax GIA data at λ_c. The rays launched from the DMD plane that land on the image plane are identified with blue points. Rays launched from the same ON micromirror land close together in clusters. Using k-means clustering, the landed rays (blue points) can be clustered together, and a centroid (average point) can be identified for each cluster, shown by the black points. The predicted x_d of the centroid of the rays coming from each ON micromirror (with position x) are shown with grey vertical lines. The predicted x_d is $M_S x$.

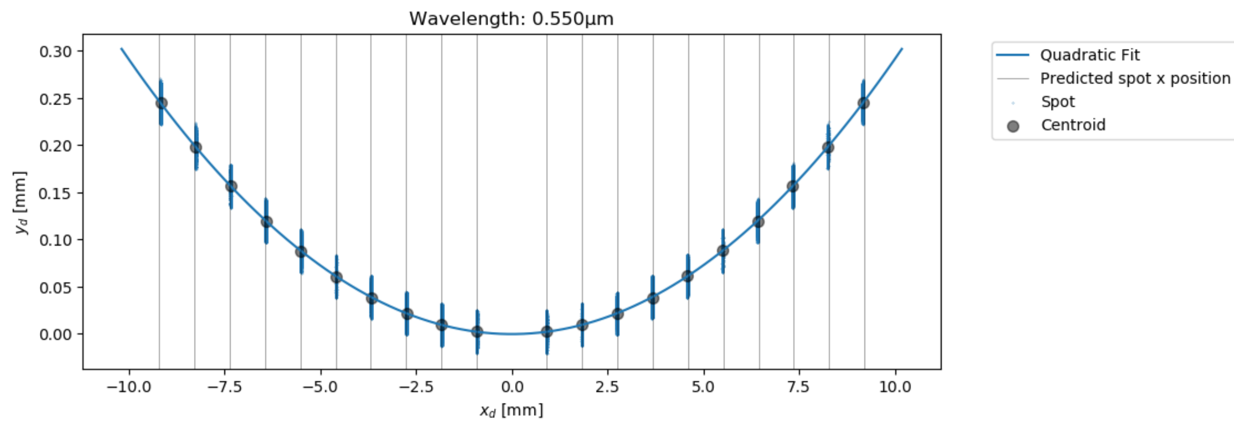


Figure 3: Close-up plot of the Zemax GIA data for λ_c and $y = 0$

Let us find y_d as a function of x_d, that is, $y_d = f(x_d)$. As seen in Figure 3, from the positions of the centroids, $f(x_d)$ appears to be a quadratic function. In addition, due to symmetry in the system, $f(x_d)$ must be an even function, that is, $f(x_d) = f(-x_d)$. We fit a quadratic function to the centroids, with the form:

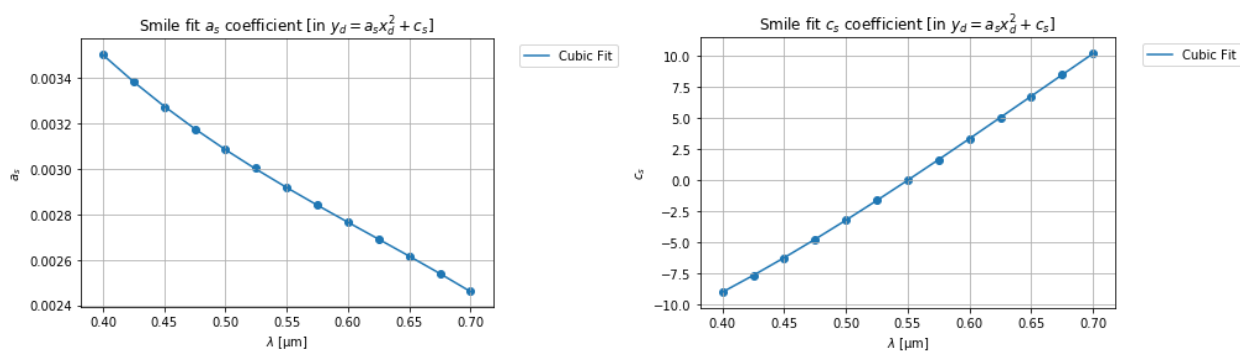
$$y_d = f(x_d) = a_s x_d^2 + c_s \tag{1}$$

where a_s and c_s are coefficients that are functions of λ. By fitting quadratic functions to each set of points corresponding to some λ, as in Figure 3, we can obtain a_s as a function of λ and c_s as a function of λ. These are shown in Figure 4, obtained with cubic fits:

$$a_s = \sum_{i=0}^{3} a_{si} \lambda^i \tag{2}$$

$$c_s = \sum_{i=0}^{3} c_{si} \lambda^i \tag{3}$$

where each a_{si} and c_{si} is some constant.



(a) a_s as a function of λ, with a cubic fit

(b) c_s as a function of λ, with a cubic fit

Figure 4: Smile fit coefficients as functions of λ, for $y = 0$

2.2 Keystone Distortion

Keystone distortion is a shift in the spectra along the spatial direction, which happens because band-to-band magnification varies with wavelength. There is a deviation between the predicted x_d, which is $M_S x$, and the actual x_d. This deviation is a function of x. Let the deviation be $g(x) \equiv x_d - (M_S x)$. $g(x)$ must satisfy $g(0) = 0$ because there is no keystone distortion at $x = 0$, and $g(x) = -g(-x)$ because of symmetry in the system. Given some x and the corresponding x_d from the Zemax GIA data, we can fit a cubic function to $g(x)$, with the form:

$$g(x) = a_k (M_S x)^3 + c_k (M_S x)$$

where a_k and c_k are coefficients that are functions of λ. Using the definition of $g(x)$, we can instead write:

$$x_d = a_k (M_S x)^3 + (c_k + 1)(M_S x) \tag{4}$$

The cubic fit for $g(x)$ is shown in Figure 5 for λ_c.

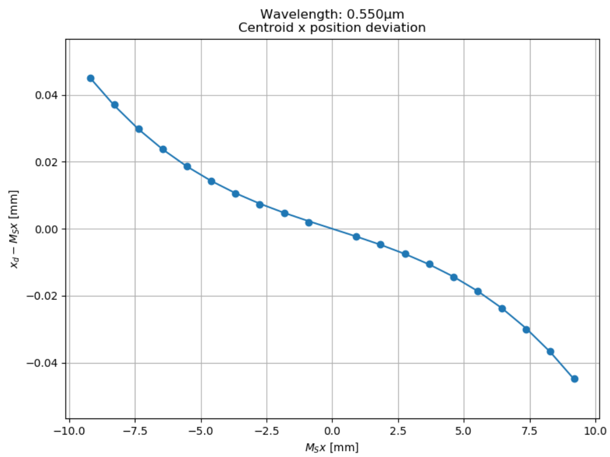


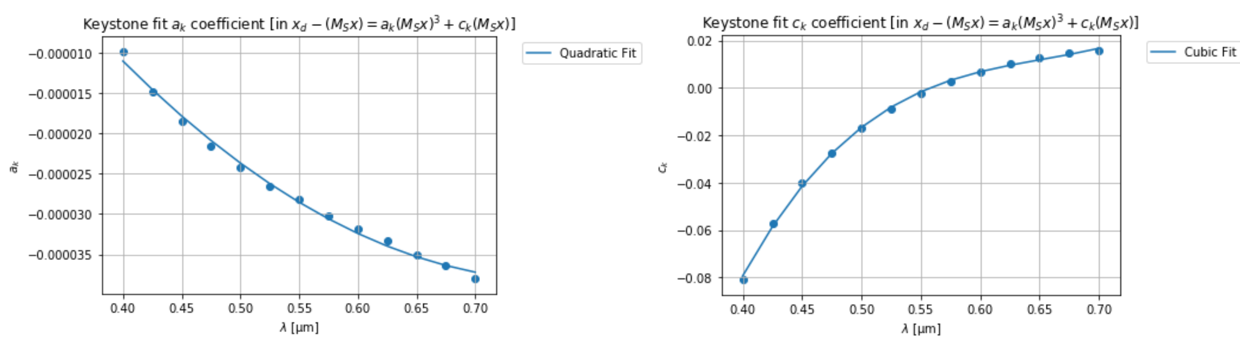
Figure 5: Deviation between the predicted and actual centroid positions for λ_c and $y = 0$

a_k and c_k are functions of λ. By finding the $g(x)$ for each λ, we can obtain a_k as a function of λ and c_k as a function of λ. We fit a quadratic function to a_k and a cubic function to c_k, shown in Figure 6. We have:

$$a_k = \sum_{i=0}^{2} a_{ki} \lambda^i \tag{5}$$

$$c_k = \sum_{i=0}^{3} c_{ki} \lambda^i \tag{6}$$

where each a_{ki} and c_{ki} is some constant.



(a) a_k as a function of λ, with a quadratic fit

(b) c_k as a function of λ, with a cubic fit

Figure 6: Keystone fit coefficients as functions of λ, for $y = 0$

2.3 Equations for x_d and y_d

By equations (4), (5), and (6), we have:

$$x_d = \left(\sum_{i=0}^{2} a_{ki}\lambda^i\right)(M_S x)^3 + \left(\left[\sum_{i=0}^{3} c_{ki}\lambda^i\right] + 1\right)(M_S x) \tag{7}$$

By equations (1), (2), (3), and (7), we have:

$$y_d = \left(\sum_{i=0}^{3} a_{si}\lambda^i\right)\left[\left(\sum_{i=0}^{2} a_{ki}\lambda^i\right)(M_S x)^3 + \left(\left[\sum_{i=0}^{3} c_{ki}\lambda^i\right] + 1\right)(M_S x)\right]^2 + \sum_{i=0}^{3} c_{si}\lambda^i \tag{8}$$

The values of the constants are shown in Table 1. Note that x is in units of mm and λ is units of µm for the fits.

Distortion Type	Constant	Value
Smile	a_{s0}	0.00782671
	a_{s1}	-0.01890454
	a_{s2}	0.02581286
	a_{s3}	-0.01394186
	c_{s0}	-14.86248407
	c_{s1}	-35.79126845
	c_{s2}	158.05186215
	c_{s3}	-79.78971198

Table 1: Values of the fit constants for the smile distortion fit coefficients, for $y = 0$

Distortion Type	Constant	Value
Keystone	a_{k0}	0.00007896
	a_{k1}	-0.00030339
	a_{k2}	0.00019631
	c_{k0}	-1.23198145
	c_{k1}	5.53880362
	c_{k2}	-8.34571333
	c_{k3}	4.25929110

Table 2: Values of the fit constants for the keystone distortion fit coefficients, for $y = 0$