



UNIVERSITY OF
OXFORD

Diagnosing Algorithmic Inequality in Social Networks

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MAX PLANCK INSTITUTE
FOR INTELLIGENT SYSTEMS



Networks and inequality: empirical studies

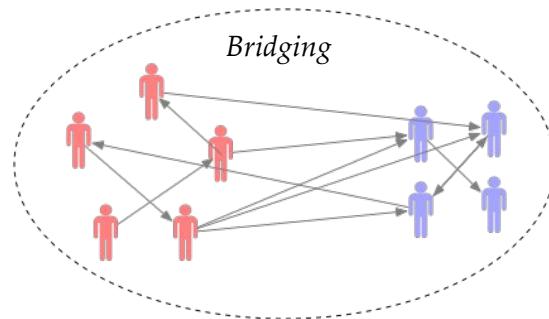
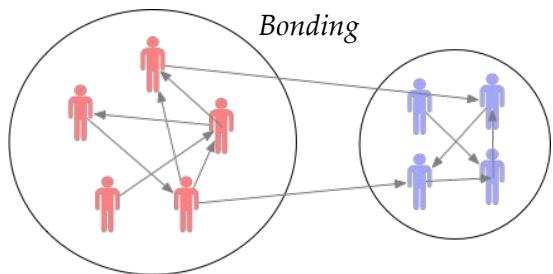
Empirical studies set the grounds for models



Structures & patterns that we see in real networks:

- Bigger cities \Rightarrow higher average degree & communication activity volume [[Schlapher et al, 2014](#)]
- $P[a\text{-}c \text{ edge} \mid a\text{-}b \text{ edge}, b\text{-}c \text{ edge}]$ independent of city size [[Schlapher et al, 2014](#)]
- Decreased communication to and from a certain area \leftrightarrow poverty [[Smith-Clarke et al, 2014](#)]

Social network utility: social capital [[Putnam, 2000](#)]



Bonding and bridging communities in empirical studies

- [Gündoğdu et al, 2019] finds that **poverty** correlates to ‘bridging’ communities and **wealth** to ‘bonding’ communities
- Network of 378 mobile cell towers in Côte d'Ivoire
 - edges weighted by amount of communication of users in the cell towers
 - aggregate by area (commune)
- ‘Bonding’ (closed) or ‘bridging’ (open) measures:

Table 6. Mean values of the degree centrality, betweenness centrality, effective size, efficiency, and local clustering coefficient in the communication network. These measures were calculated for each of the ten communes of Abidjan.

Commune	Degree	Open structure		Closed structure	
		Between. centrality	Eff. size	Efficiency	Local clust. coeff.
Abobo	1200.200	1128.866	624.555	0.521	0.946
Adjame	1195.565	1123.036	609.005	0.509	0.946
Attecoube	1202.538	1147.459	619.986	0.516	0.945
Cocody	1141.789	1054.691	560.872	0.476	0.948
Koumassi	1188.118	1125.058	654.206	0.551	0.947
Marcory	1025.103	841.657	573.949	0.552	0.958
Plateau	1001.588	789.809	427.779	0.394	0.961
Port-Bouet	1067.750	896.124	607.666	0.568	0.956
Treichville	1113.850	950.254	612.798	0.550	0.954
Yopougon	1175.697	1143.794	599.460	0.502	0.946

Extra slide on network measures

Open / bridging structures:

- Betweenness centrality: $g(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$ for σ_{st} = # shortest paths
- Effective size:
 - Define a transition matrix as T with $t_{a_i, a_l} = \frac{w_{a_i, a_l} + w_{a_l, a_i}}{\sum_{a_m} (w_{a_i, a_m} + w_{a_m, a_i})}$
 - Define a marginal strength matrix M with $m_{a_j, a_l} = \frac{w_{a_j, a_l} + w_{a_l, a_j}}{\max_{a_m} (w_{a_j, a_m} + w_{a_m, a_j})}$
 - The effective size of a node a_i is $s_{a_i} = \sum_{a_j \in N_{a_i}} \left[1 - \sum_{a_l} t_{a_i, a_l} m_{a_j, a_l} \right]$
- Efficiency is effective size normalized by degree

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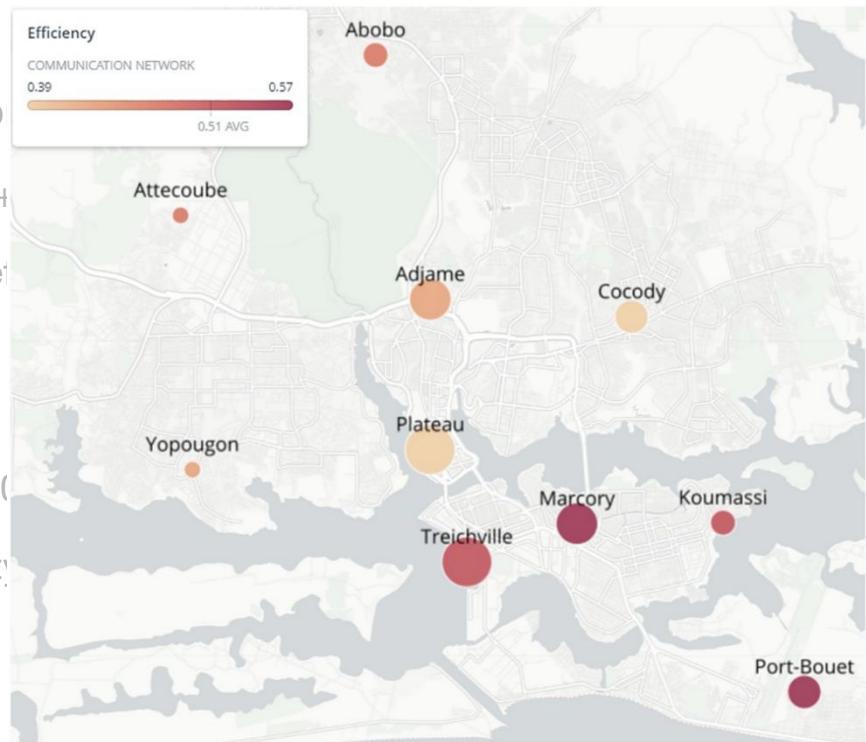
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Bonding and bridging communities in empirical studies

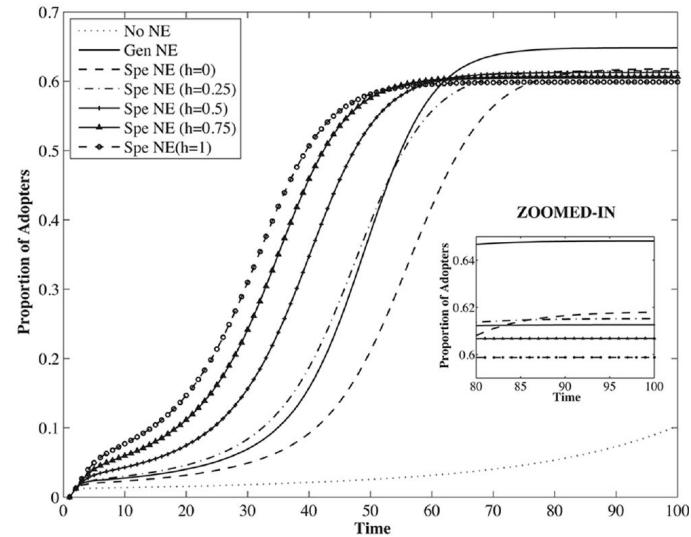
- [Gündoğdu et al, 2019] finds that **poverty** correlates to ‘bridging’ communities and **wealth** to ‘bonding’ communities:
- Revenue (and capital) budget per person
 - + correlated with clustering coefficient +
 - - correlated with betweenness -0.63^* , etc (social structures)
- % land covered by slums:
 - - correlated with clustering coefficient -0.57^*
 - + correlated with betweenness centrality 0.51^*



Homophily and inequality

[DiMaggio & Garip, 2011] shows that homophily (bonding) leads to increasing inter-group **inequality** r.e. Internet adoption in the US:

- 2,257 African-American and white respondents to the 2002 General Social Survey (GSS), which included items on network size, race, education, and income
 - Create networks with individual features, vary homophily
- Simulate diffusion through threshold model + a fixed initial price of Internet



⇒ Homophily decreases adoption with time

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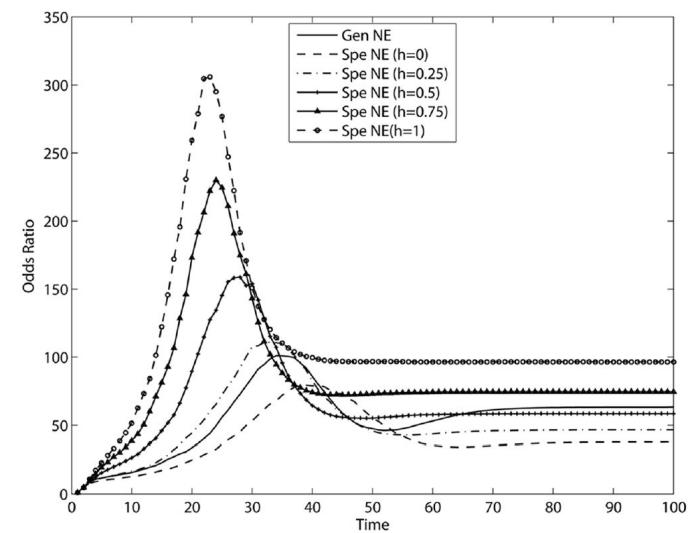


FIG. 4.—Odds ratios of diffusion rates for highest- as compared to lowest-income classes in six conditions of externalities and homophily.

⇒ **Homophily increases inter-group inequality in adoption**

Homophily and inequality

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- 2,257 African-American and white respondents from 2002 General Social Survey (GSS), which includes network size, race, education, and income
 - Create networks with individual features, v. homophily
- Simulate diffusion through threshold model + a 1% price of Internet

TABLE 2
LINEAR REGRESSION OF ADOPTION LEVELS ON EXPERIMENTAL CONDITIONS

	RACE			INCOME		EDUCATION	
	ALL	Whites	Blacks	High	Low	BA	Less than High School
No network externalities	-.516**	-.536**	-.399**	-.685**	-.238**	-.611**	-.351**
General network externalities030**	.028**	.043**	.032**	.017**	.023**	.030**
Homophily = .25 ...	-.003**	-.001	-.012**	.009**	-.014**	.005**	-.011**
Homophily = .5	-.005**	-.002**	-.024**	.017**	-.028**	.010**	-.024**
Homophily = .75 ...	-.011**	-.006**	-.040**	.024**	-.046**	.012**	-.043**
Homophily = 1	-.019**	-.012**	-.061**	.029**	-.067**	.015**	-.068**
Intercept618**	.647**	.454**	.925**	.249**	.788**	.392**
R ²99	.99	.97	.99	.96	.99	.96

NOTE.—All independent variables are binary. Both dependent and independent variables are measured on the final period of simulations ($t = 100$). Reference: homophily = 0; $N = 7,000$.

* $P < .05$.

** $P < .01$.

⇒ Internet adoption increases among the most prosperous in the presence of homophily

Homophily and inequality

[[Okafor, 2022](#)] shows that tighter-knit minorities can actually improve their chance at better career opportunities through job referrals

Empirical studies on the Internet [Barabasi-Albert,1999]

Power law degree distribution in online networks: $P(k) \sim k^\gamma$

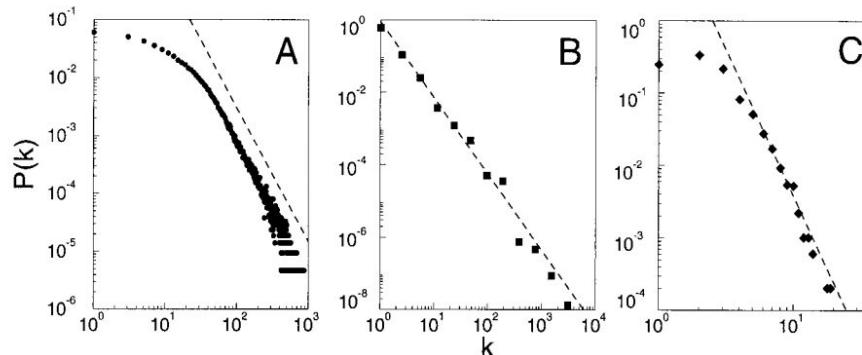


Fig. 1. The distribution function of connectivities for various large networks. (A) Actor collaboration graph with $N = 212,250$ vertices and average connectivity $\langle k \rangle = 28.78$. (B) WWW, $N = 325,729$, $\langle k \rangle = 5.46$ (6). (C) Power grid data, $N = 4941$, $\langle k \rangle = 2.67$. The dashed lines have slopes (A) $\gamma_{\text{actor}} = 2.3$, (B) $\gamma_{\text{www}} = 2.1$ and (C) $\gamma_{\text{power}} = 4$.

Social capital



Resources, opportunities, ...

Social influence & opportunities

The Diffusion of Microfinance

Abhijit Banerjee,* Arun G. Chandrasekhar,* Esther Duflo,* Matthew O. Jackson*

Introduction: How do the network positions of the first individuals in a society to receive information about a new product affect its eventual diffusion? To answer this question, we develop a model of information diffusion through a social network that discriminates between information passing (individuals must be aware of the product before they can adopt it, and they can learn from their friends) and endorsement (the decisions of informed individuals to adopt the product might be influenced by their friends' decisions). We apply it to the diffusion of microfinance loans, in a setting where the set of potentially first-informed individuals is known. We then propose two new measures of how "central" individuals are in their social network with regard to spreading information; the centrality of the first-informed individuals in a village helps significantly in predicting eventual adoption.

 **Access to information is access to opportunity**

How do we use networks to design algorithms?

1. Using networks to diagnose *when* and *how* an algorithm may amplify bias
 - a. Unify unsupervised graph problems
 - b. Define theoretical formulation for capturing distributional inequality
 - c. Leverage network models for re-creating the root cause of bias
2. Using networks to test algorithms: randomized controlled trials

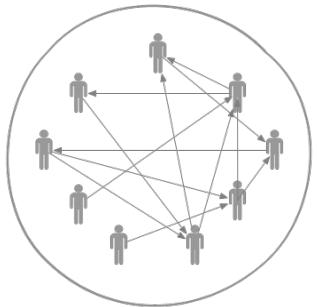
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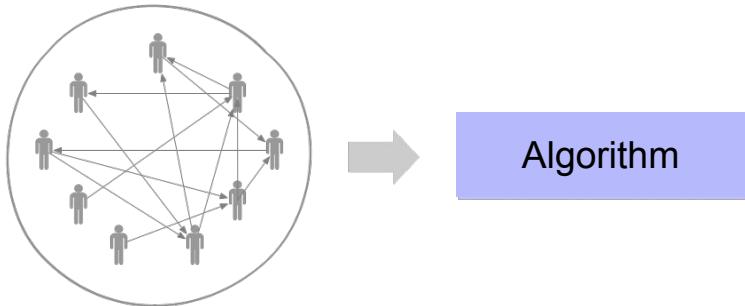
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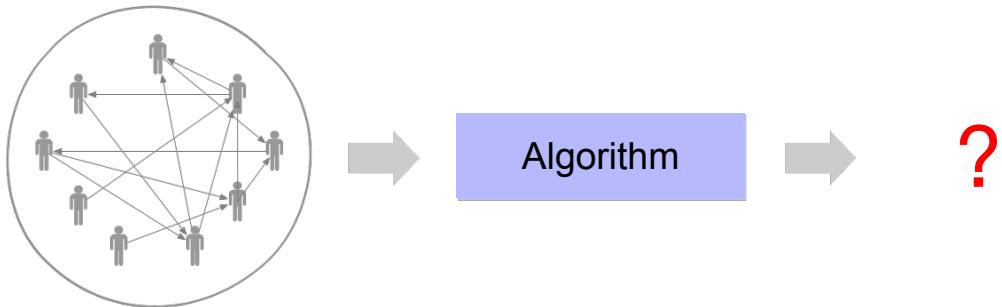
Diagnosing algorithmic bias



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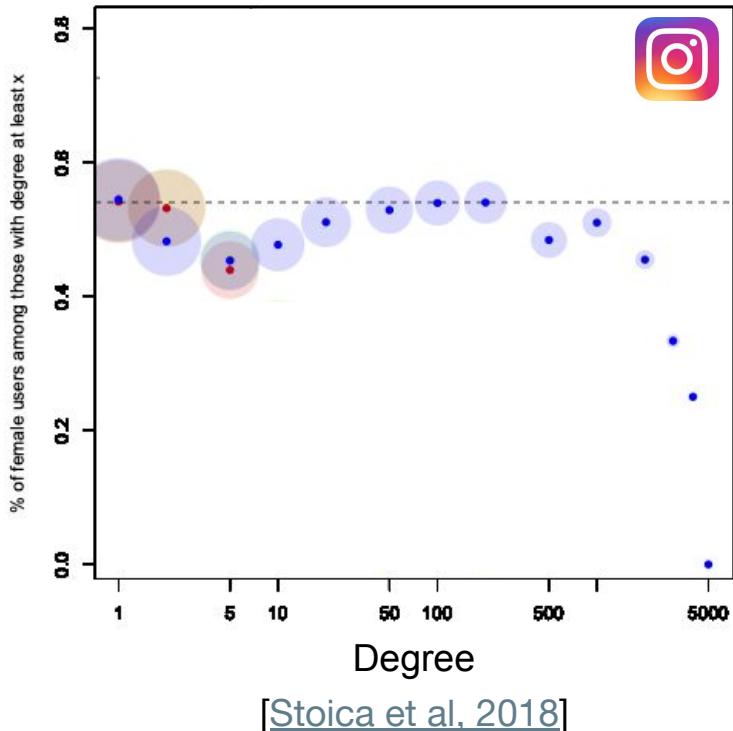


Diagnosing algorithmic bias



Distributional inequality in social capital

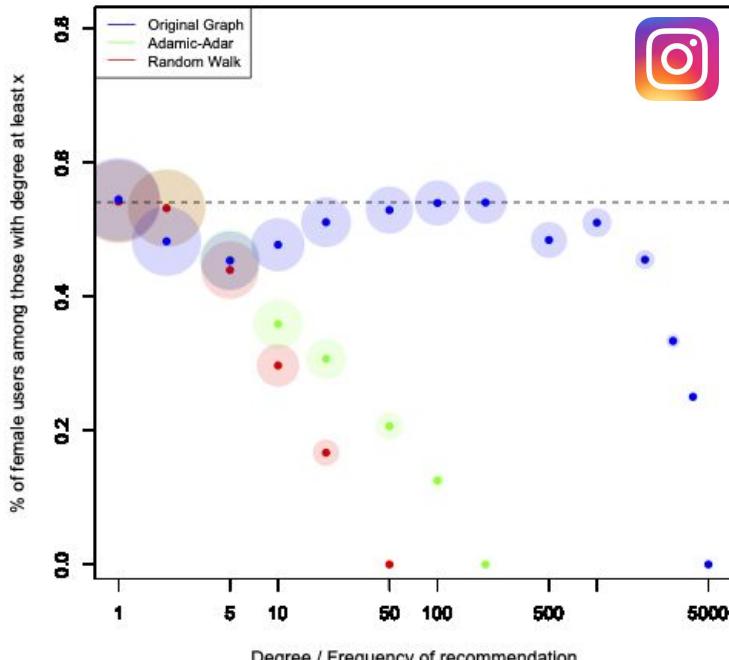
Instagram activity graph of likes and comments



- Groups: men (46%) and women (54%)
- Only **organic** connections
- Representation of women is *increasingly worse* for popular accounts

Distributional inequality in social recommendations

Instagram activity graph of likes and comments



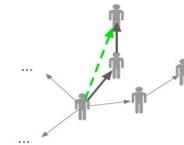
[Stoica et al, 2018]

- Common recommendation algorithms amplify degree inequality between men and women!
- Utility is equivalent to the number of connections after recommendation: $\deg_{RG}(u)$

Adamic Adar index:

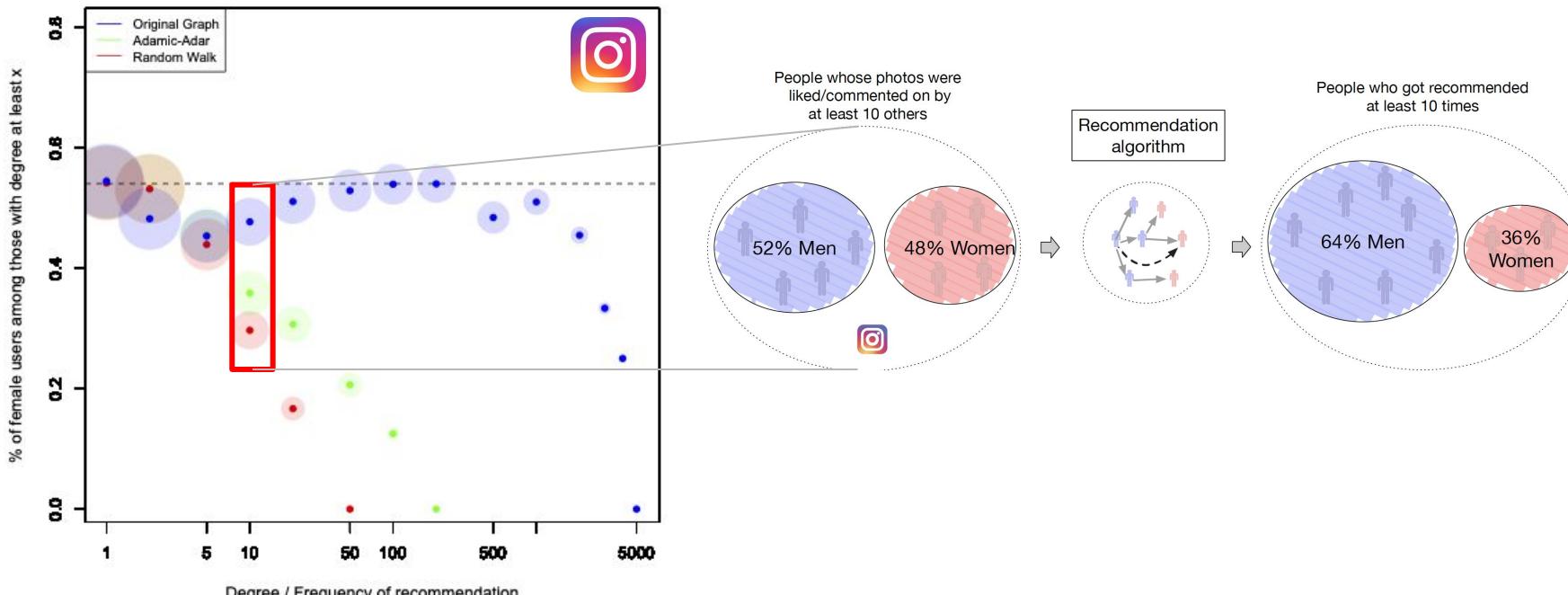
$$A(x, y) = \sum_{u \in N(x) \cap N(y)} \frac{1}{\log |N(u)|}$$

Random walk:



Distributional inequality in social recommendations

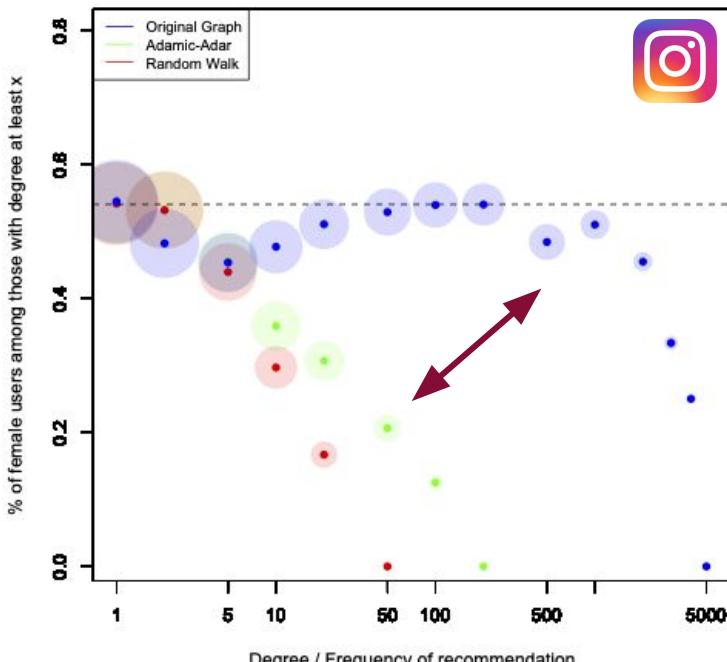
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Distributional inequality in social recommendations

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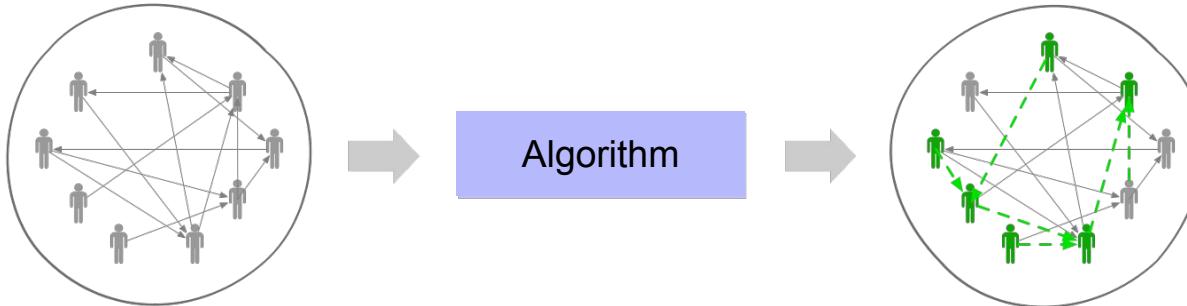


[Stoica et al, 2018]

- Common recommendation algorithms amplify degree inequality between men and women!
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→ **Algorithmic amplification of bias**

Diagnosing algorithmic bias



Benefit of connections activated by an algorithm:

Recommendation

→ Receive new connections through recommendations

Information diffusion

→ Be exposed to an information campaign

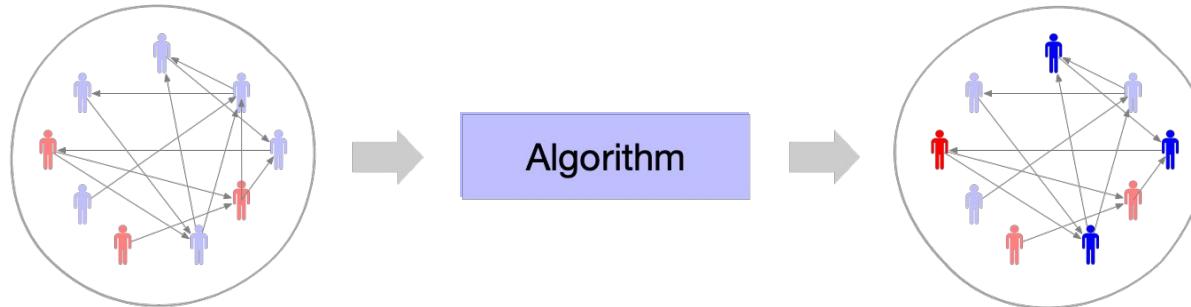
Clustering

→ Be targeted for assistance, help, new products or services, ...

Ranking

→ Receive exposure by showing up in search results

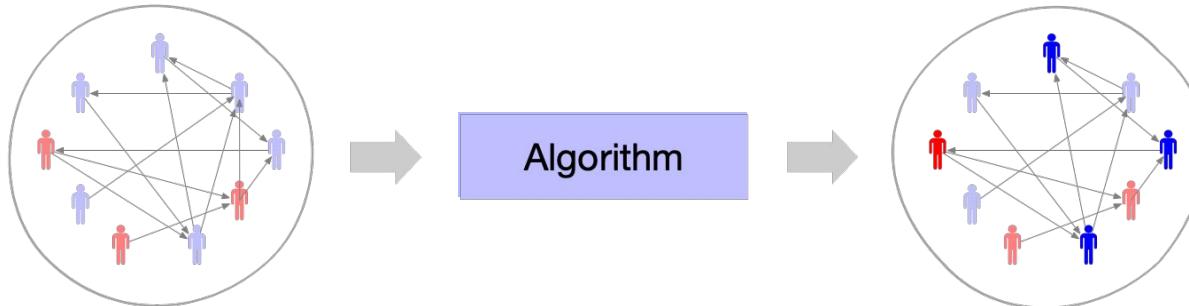
Diagnosing algorithmic bias: is it always a problem?



Benefit of connections activated by an algorithm:

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Inequality in information diffusion

Empirical: Internet adoption increases among the most prosperous in the presence of homophily

[DiMaggio & Garip, 2011]

CS (algorithmic): Defined as the social influence maximization problem

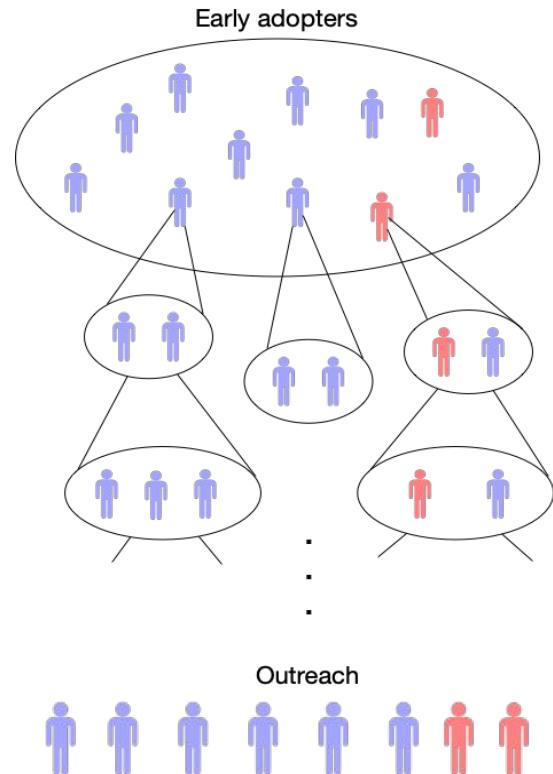
- Algorithms: greedy, centrality based (degree, distance centrality, etc)



[Fish et al., 2019]



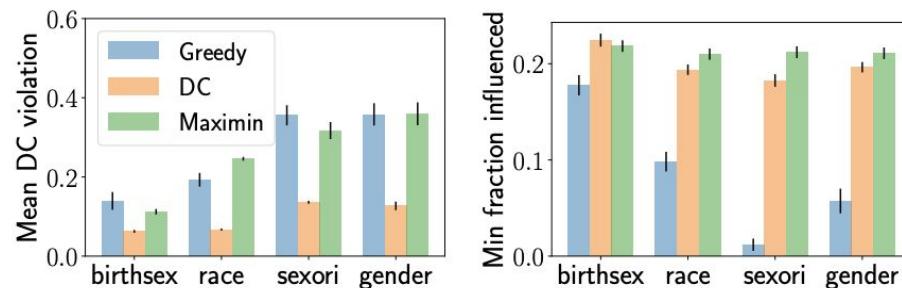
[Tsang et al., 2019]
 [Ali et al., 2019]
 [Stoica et al., 2020]



Inequality in information diffusion

Group: ➔ [\[Tsang et al, 2019\]](#)

- Influence function: $I_G(S)$ in graph G with seedset S , $I_{G,R}(S)$ for a group R
- Utility function: $\mu^{maximin}(S) = \min_{C_i} \frac{I_{G,C_i}(S)}{|C_i|}$ (Maximin)
- How many seeds should each group get? Proportional to their size: $k_i = \frac{k * |C_i|}{|V(G)|}$
- How much *influence* should each group get?
 - Here: at least as much as if they left the network $I_{G[C_i]}(k_i)$ (Group Rationality, DC)



Inequality in information diffusion

Individual: → [Fish et al, 2019]

- Set-up: graph G , initial seedset S (e.g. degree heuristic), choose additional k seeds that maximize reach through independent cascade
- Utility functions:
 - [Kempe et al, 2003]: $\mu_{reach}(S) = \frac{1}{|V(G)|} * \sum_{v \in V(G)} p_v$
 - Here: $\mu_{-\infty}(S, G) = \min_{v \in V(G)} p_v \Rightarrow \text{maximin access}$ (NP-complete)
- Approximation algorithms (compare with TIM+):
 - Myopic: greedy version of choosing vertices in increasing order of p_v (estimating & updating p_v in rounds through Monte-Carlo simulations)
 - Naive Myopic: picks k nodes with smallest p_v

Inequality in information diffusion

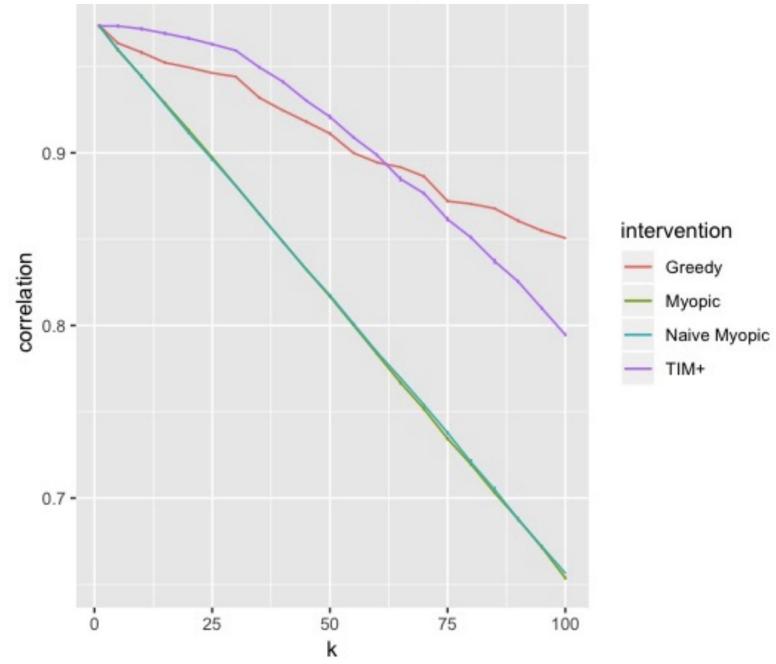
Social influence maximization [Fish et al, 2019]

- **Performance:** Myopic actually gets more nodes in the outreach (measured as $\mu_{\text{reach}}(S)$)
- **Inequality:** correlation between p_v and degree for greedy and TIM+ \Rightarrow reflects the network structure
 \Rightarrow they lower that correlation

Community differences?

Model?

Where does inequality come from?



Inequality in information diffusion

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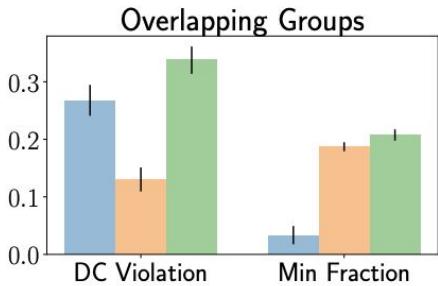
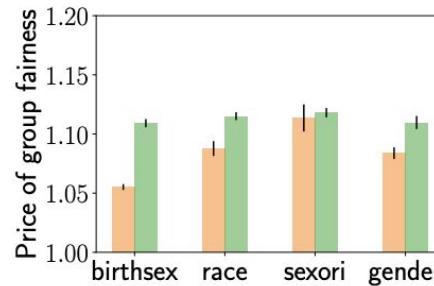
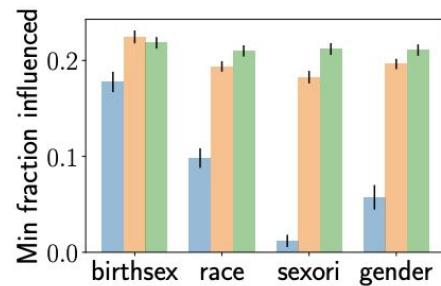
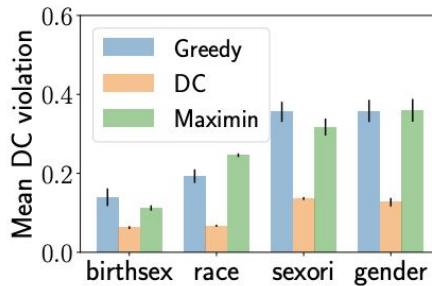
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$$\mu^{rational} = I_G(S) \text{ if } I_{G,C_i}(S) \geq I_{G[C_i]} \forall i \text{ and } 0 \text{ o/w}$$

Inequality in information diffusion

Group: → [\[Tsang et al, 2019\]](#)

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Inequality in information diffusion

Community differences?

Model?

Where does inequality come from?

$$\text{[Ali et al. 2019]: } \max_{i,j} \left| \frac{I_{G,C_i}^t(S)}{|C_i|} - \frac{I_{G,C_j}^t(S)}{|C_j|} \right|$$

- Diversity and time constraint in greedy \Rightarrow tradeoff; + seedsets and + times reduce disparity

Inequality in information diffusion

Community differences?

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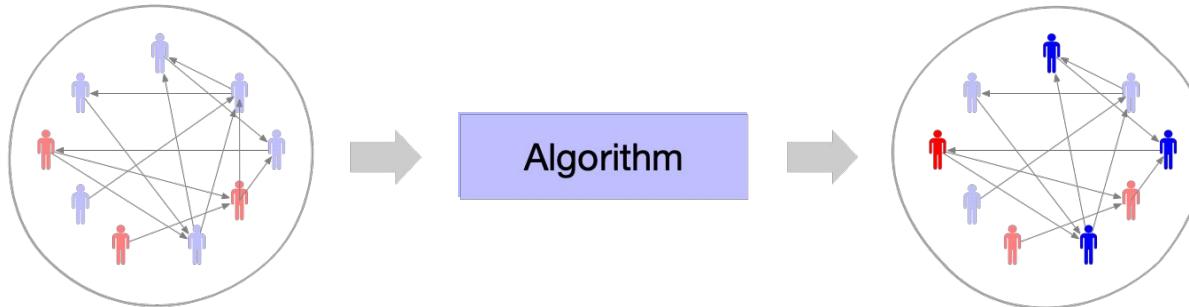
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[Stoica et al. 2020]: avg community difference + model of biased networks with homophily

- Diversity constraint in degree alg \Rightarrow no tradeoff when seedset increases

Diagnosing algorithmic bias



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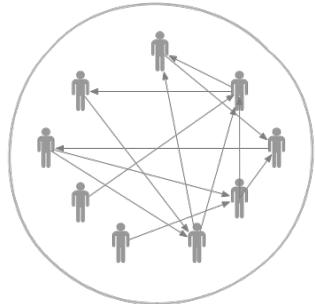
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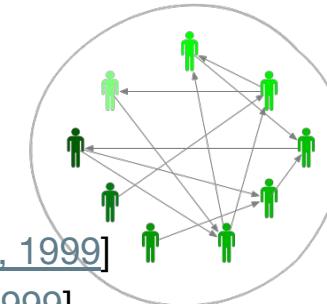
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Bias in ranking algorithms

Original graph $G = (N, E)$



Activated graph $G' = (N', E')$



Ranking

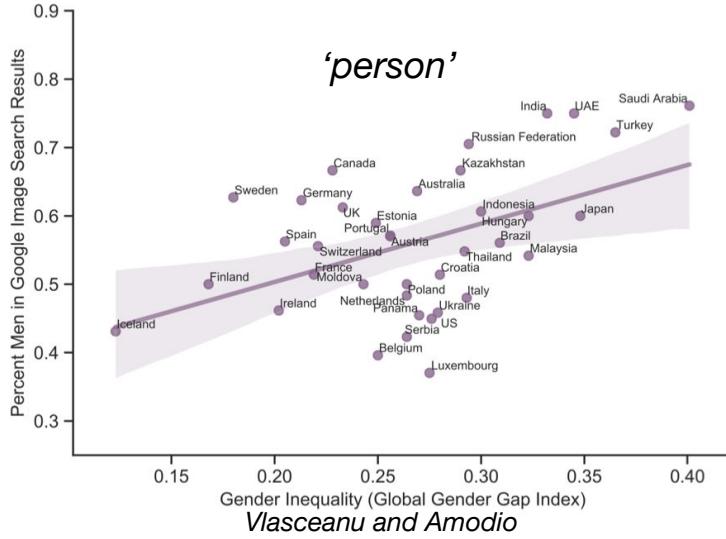
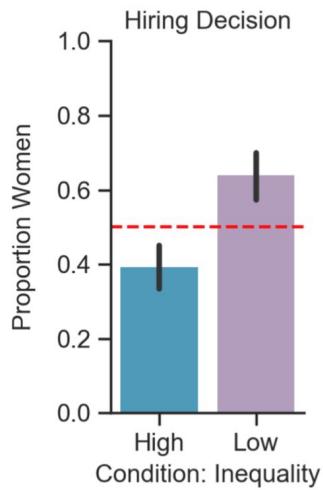
- PageRank [\[Page & Brin, 1999\]](#)
- HITS [\[Kleinberg, 1999\]](#)

Application to ranking algorithms:

- Content search: Google, Bing, ...
- Credibility / popularity metric

Minorities get ‘pushed down’

Bias in ranking algorithms

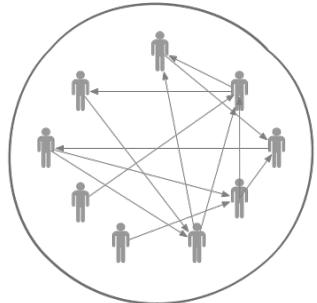


Minorities get ‘pushed down’

[[Espin-Noboa et al, 2022](#)]
[\[Vlasceanu & Amodio, 2022\]](#)

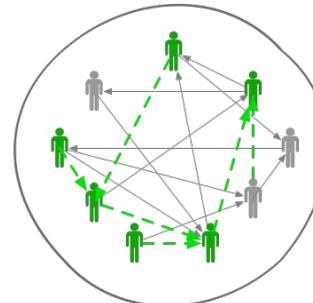
Diagnosing algorithmic bias: a unified formulation

Original graph $G = (N, E)$



Algorithm

Activated graph $G' = (N', E')$



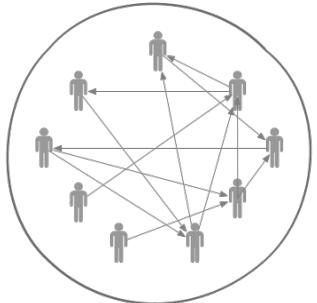
An algorithms outputs a subset of the nodes and a set of edges: $\mathcal{A} : G \rightarrow G', G' = (N', E')$

Evaluate the output through a gain function $f : G' \rightarrow \mathbb{R}$ that models one's social capital under \mathcal{A}

$$f(u) := \sum_{v \in N} \mathbb{P}((u, v) \in E'), \forall u \in N'$$

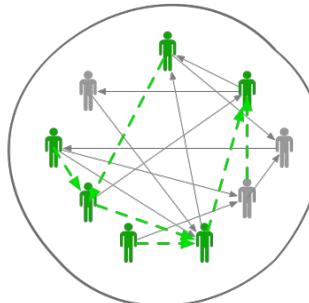
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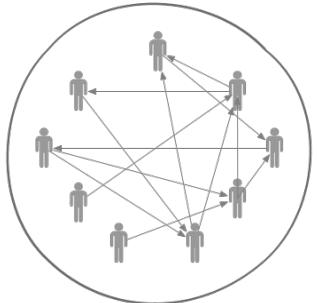
Recommendation

E' is the set of newly created edges
 $f(\cdot)$ is the number of new connections

$$f(u) := \sum_{v \in N} \mathbb{P}((u, v) \in E'), \forall u \in N'$$

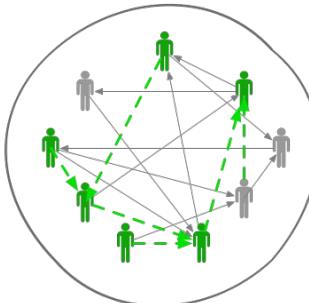
Diagnosing algorithmic bias: a unified formulation

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Activated graph $G' = (N', E')$



Recommendation

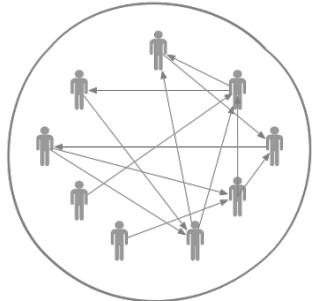
Information diffusion

$$f(u) := \sum_{v \in N} \mathbb{P}((u, v) \in E'), \forall u \in N'$$

E' is the set of edges that actually transmit information
 $f(\cdot)$ is the probability of getting the information

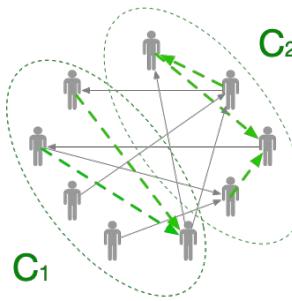
Diagnosing algorithmic bias: a unified formulation

Original graph $G = (N, E)$



Algorithm

Activated graph $G' = (N', E')$



Recommendation

Information diffusion

Clustering

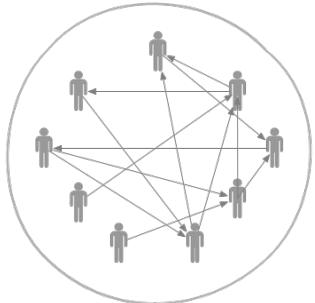
$$f(u) := \sum_{v \in N} \mathbb{P}((u, v) \in E'), \forall u \in N'$$

E' is the set of edges within clusters

$f(\cdot)$ is the in-cluster degree

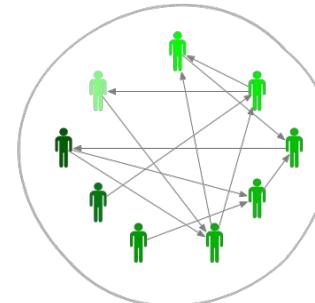
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Recommendation

Information diffusion

Clustering

Ranking

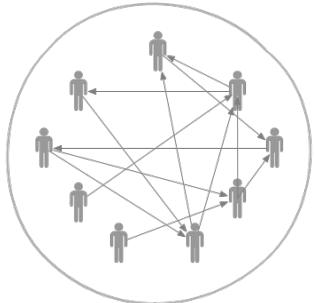
$$E' = \emptyset$$

$f(\cdot)$ is the ranking score

$$f(u) := \sum_{v \in N} \mathbb{P}((u, v) \in E'), \forall u \in N'$$

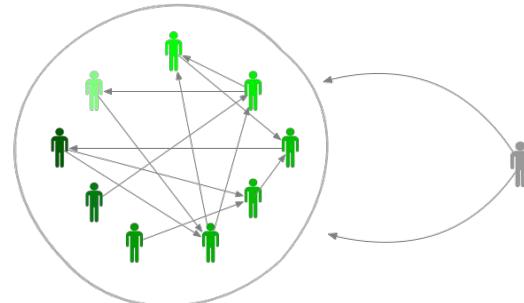
Diagnosing algorithmic bias: a unified formulation

Original graph $G = (N, E)$



Algorithm

Activated graph $G' = (N', E')$



Recommendation

Information diffusion

Clustering

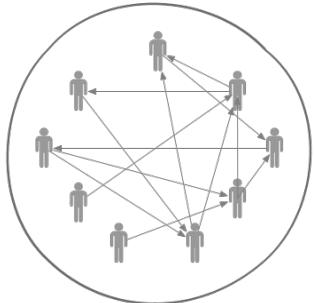
Ranking

$$f(u) := \sum_{v \in N} \mathbb{P}((u, v) \in E'), \forall u \in N'$$

E' is the set of edges to be created with the new node
 $f(\cdot)$ is the ranking score

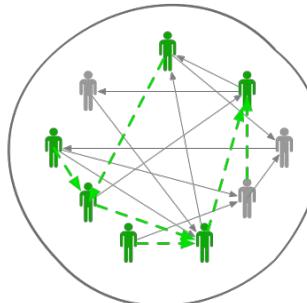
Diagnosing algorithmic bias: a unified formulation

Original graph $G = (N, E)$



Algorithm

Activated graph $G' = (N', E')$



Recommendation

$$E' \cap E = \emptyset$$

Information diffusion

$$E' \subseteq E$$

Clustering

$$E' \subseteq E$$

Ranking

$$E' \cap E = \emptyset$$

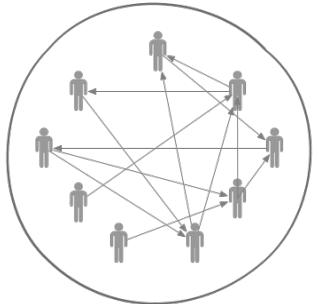
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How do we use networks to design algorithms?

1. Diagnose *when* and *how* an algorithm may amplify bias
 - a. Unify unsupervised graph problems
 - b. Define theoretical formulation for capturing distributional inequality
 - c. Leverage network models for re-creating the root cause of bias
2. Using networks to test algorithms: randomized controlled trials

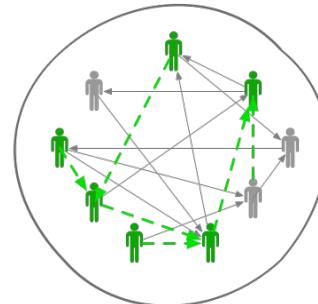
Diagnosing algorithmic bias

Original graph $G = (N, E)$



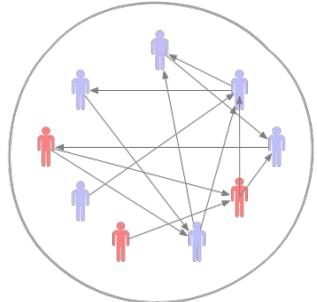
Algorithm

Activated graph $G' = (N', E')$



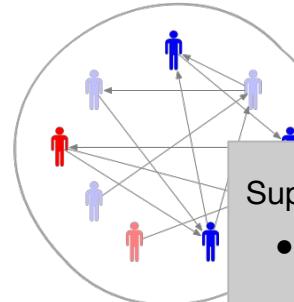
Diagnosing algorithmic bias: impact on different groups

Original graph $G = (N, E)$



Unsupervised learning

Activated graph $G' = (N', E')$



Supervised learning:

- Decision-making: select people who receive a positive outcome
- Known ground truth

- Independence (average comparison):

$$E[f(\textcolor{red}{\text{person}})] = E[f(\textcolor{blue}{\text{person}})] \Leftrightarrow P\{\textcolor{gray}{\text{person}} = 1 | \textcolor{gray}{\text{person}} = \textcolor{red}{\text{person}}\} = P\{\textcolor{gray}{\text{person}} = 1 | \textcolor{gray}{\text{person}} = \textcolor{blue}{\text{person}}\}$$

- Analyze distributional inequality in f :

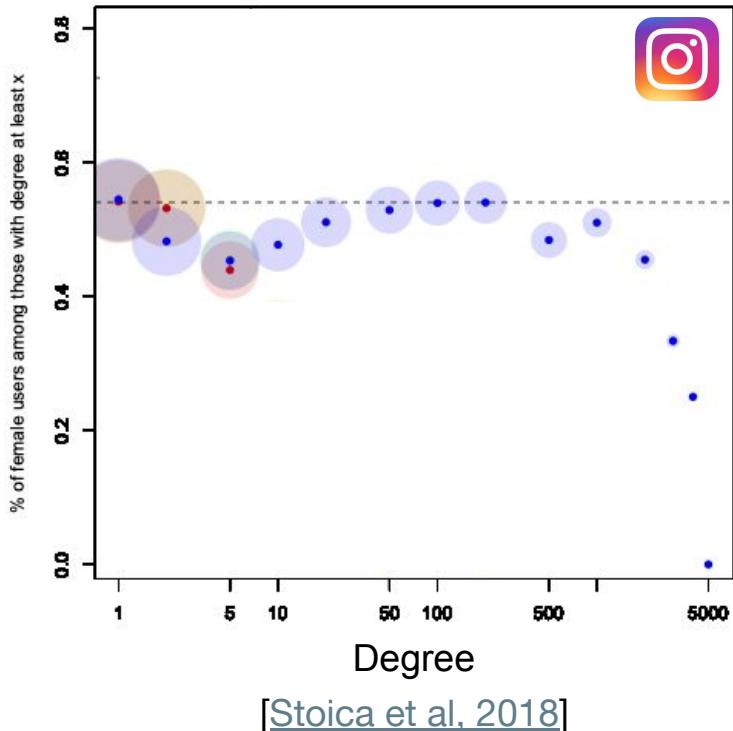
behavior of

$$\frac{P[f(\textcolor{gray}{\text{person}}) > r | \textcolor{gray}{\text{person}} = \textcolor{red}{\text{person}}]}{P[f(\textcolor{gray}{\text{person}}) > r | \textcolor{gray}{\text{person}} = \textcolor{blue}{\text{person}}]}$$

Group fairness:

Distributional inequality in social capital

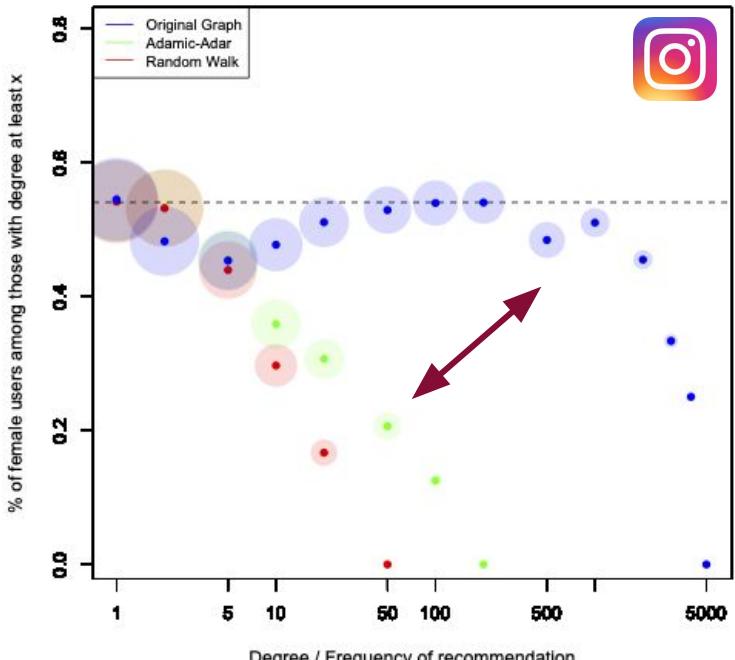
Instagram activity graph of likes and comments



- Groups: men (46%) and women (54%)
- Only **organic** connections
- $f(u) = \deg_{OG}(u)$
- Representation of each group on average does not tell the entire story:
 $E[f(\text{women})] = 2.25$
 $E[f(\text{men})] = 2.52$
- Representation of women is *increasingly* worse for popular accounts

Distributional inequality in social recommendations

Instagram activity graph of likes and comments



[Stoica et al, 2018]

- Common recommendation algorithms amplify degree inequality between men and women!
- Utility is equivalent to the number of connections after recommendation:

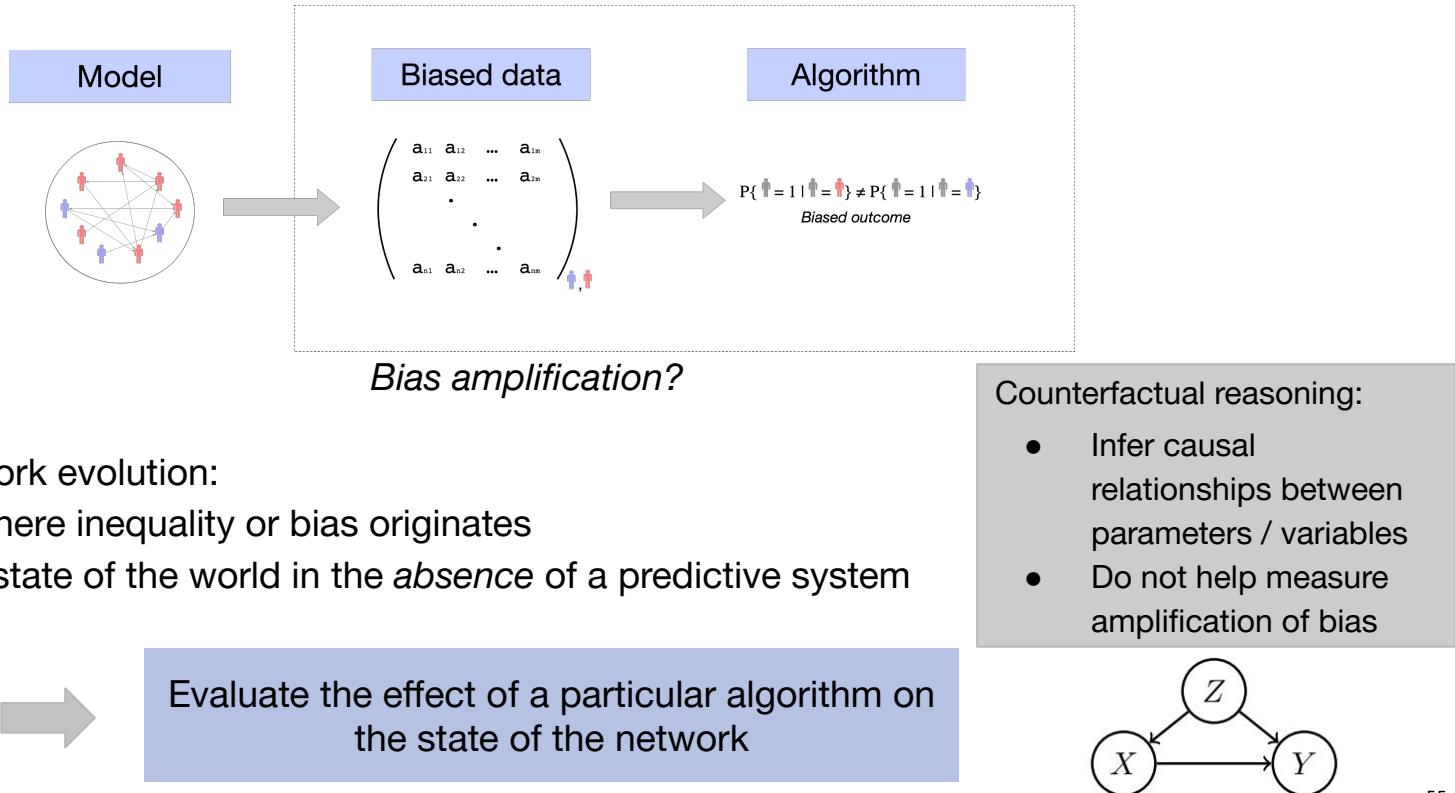
$$f(u) = \deg_{RG}(u)$$

→ **Algorithmic amplification of bias**

How do we use networks to design algorithms?

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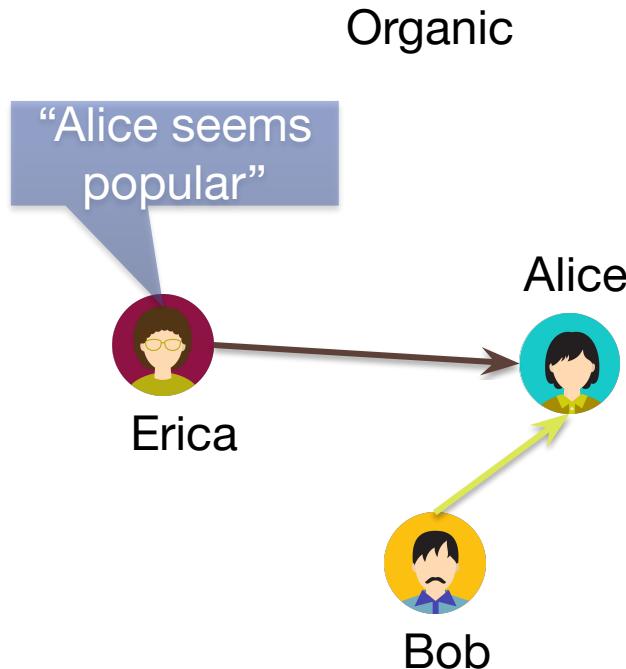
Networks modeling for finding the root cause of bias



Preferential attachment with homophily [Avin et al, 2015]

Model ingredients:

- **Minority-majority:** **B** label and **R** label
 - Fraction of **R** nodes = $r < \frac{1}{2}$
- **Preferential attachment** (rich-get-richer): nodes connect w.p. proportional to degree



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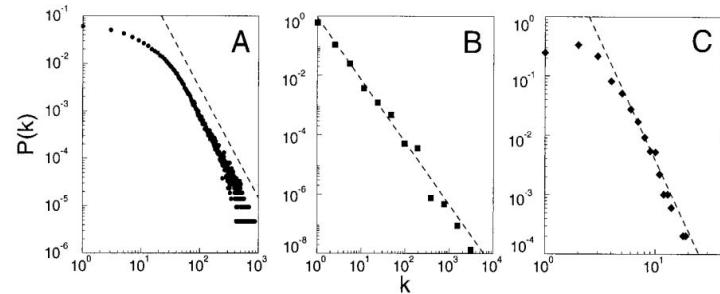


Fig. 1. The distribution function of connectivities for various large networks. (A) Actor collaboration graph with $N = 212,250$ vertices and average connectivity $\langle k \rangle = 28.78$. (B) WWW, $N = 325,729$, $\langle k \rangle = 5.46$ (6). (C) Power grid data, $N = 4941$, $\langle k \rangle = 2.67$. The dashed lines have slopes (A) $\gamma_{\text{actor}} = 2.3$, (B) $\gamma_{\text{www}} = 2.1$ and (C) $\gamma_{\text{power}} = 4$.

[Barabasi-Albert,1999]

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- **Homophily:** if different labels, connection is accepted w.p. ρ

Preferential attachment with homophily [Avin et al, 2015]

Degree distribution follows a power law at equilibrium:

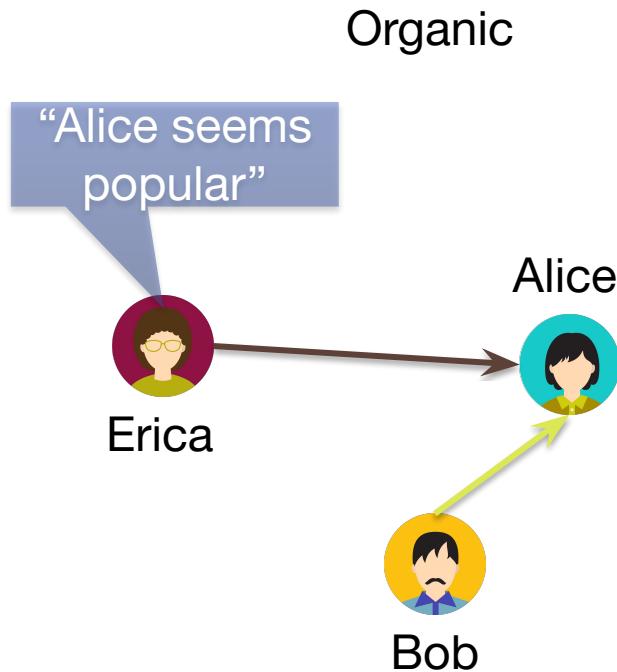
$$\text{top}_k(\mathbf{R}) \sim k^{-\beta(R)}$$

$$\text{top}_k(\mathbf{B}) \sim k^{-\beta(B)}$$

Theorem:

$$\beta(\mathbf{R}) > 3 > \beta(\mathbf{B})$$

gap



Preferential attachment with homophily [Avin et al, 2015]

Data: DBLP dataset of mentors-mentees

- ~400k people, male (79%) and female (21%)
- Female mentors avg. deg: 4.60
- Male mentors avg. deg: 5.25

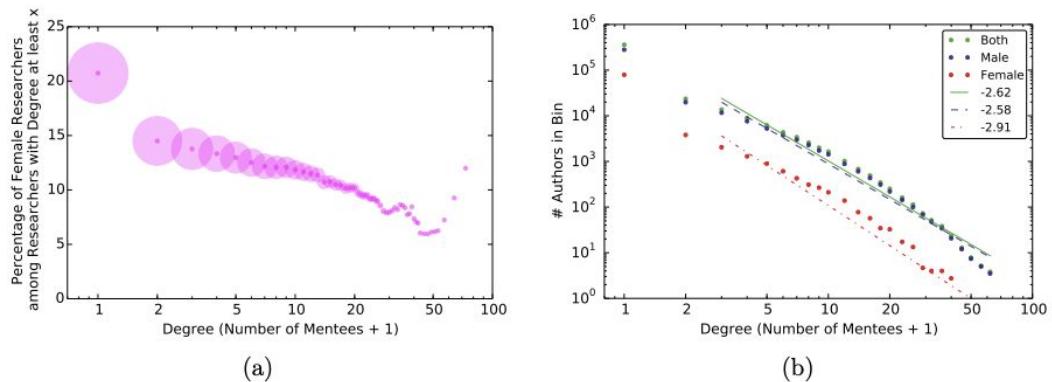


Figure 6: Glass ceiling effect in mentor graph: (a) percentage of females in the mentor population of degree at least k . Female start with 21% in the population and drop to below 15% when considering degree at least 2 (faculty members). It continues to decrease (ignoring small samples at the end, see text). Vertex size and darker color represent larger sample space. (b) The power-law-like degree distribution for both females and males. The exponent β for females is higher than for males, demonstrating the glass ceiling effect.

Preferential attachment with homophily [Avin et al, 2015]

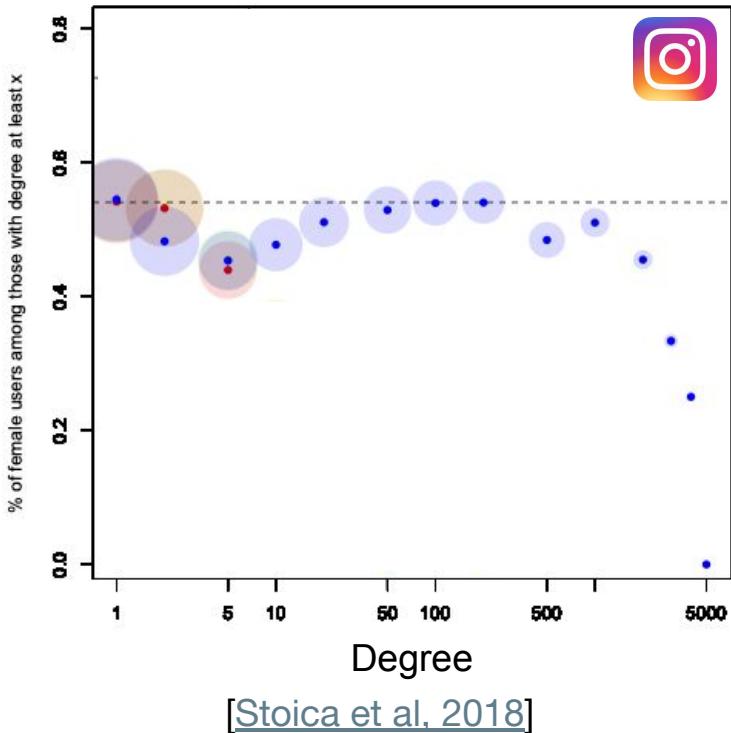
Measures of inequality between R and B :

- Power inequality: $\lim_{n \rightarrow \infty} \frac{\frac{1}{n(R)} \sum_{v \in R} \delta(v)}{\frac{1}{n(B)} \sum_{v \in B} \delta(v)} \leq c$ for some constant c
- Tail glass ceiling effect: there exists an increasing function $k(n)$ such that:

$$\lim_{n \rightarrow \infty} top_{k(n)}(B) = \infty \quad \text{and} \quad \lim_{n \rightarrow \infty} \frac{top_{k(n)}(R)}{top_{k(n)}(B)} = 0$$
- Strong glass ceiling effect: $\lim_{n \rightarrow \infty} \frac{\frac{1}{n(R)} \sum_{v \in R} \delta(v)^2}{\frac{1}{n(B)} \sum_{v \in B} \delta(v)^2} = 0$

Distributional inequality in social capital

Instagram activity graph of likes and comments



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Preferential attachment with homophily [[Avin et al, 2015](#)]

Main results:

- Minority-majority
- Preferential attachment
- Homophily
- Power inequality
- Tail glass ceiling effect
- Strong glass ceiling effect

Preferential attachment with homophily [[Avin et al, 2015](#)]

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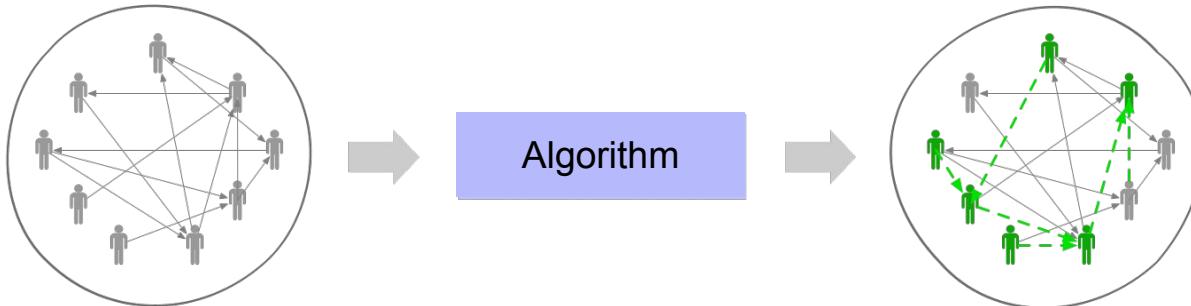
Main results:

- Minority-majority
- Preferential attachment
- Homophily



- Power inequality?
- Tail glass ceiling effect?
- Strong glass ceiling effect

Diagnosing algorithmic bias



Benefit of connections activated by an algorithm:

Recommendation

→ Receive new connections through recommendations

Information diffusion

→ Be exposed to an information campaign

Clustering

→ Be targeted for assistance, help, new products or services, ...

Ranking

→ Receive exposure by showing up in search results

Bias amplification in recommendation algorithms

Summary of results:

- Experimental results show a **bias amplification**
- Build a **theoretical explanation** for when bias amplifies in recommendation based on an evolving network model
- **Main ingredients** for bias creation and amplification:
 - Disparity in group sizes: **minority (R)**, **majority (B)**
 - Preferential attachment (rich-get-richer effect)
 - Homophily (nodes in the same community connect)
 - **Recommendations based on random walk of length 2**

Model evolution with recommendations

At timestep t, a new edge is formed:

Organic growth:

[Avin et al, 2015]

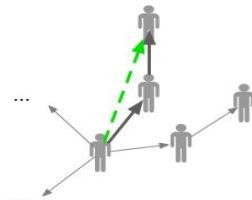
New node connects:

- randomly
- preferential attachment + homophily

Biased Preferential Attachment Model (BPAM)

Recommendation model:

- organic growth
- **existing node connects through a random walk of length 2**



Degree distribution

Organic growth:

$$\text{top}_k(R) \sim k^{-\beta(R)}$$

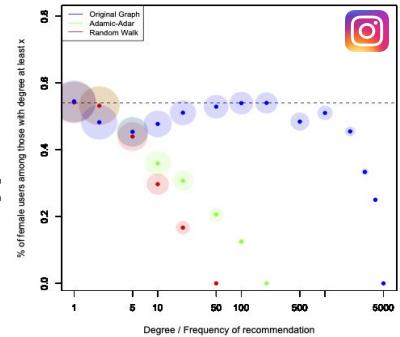
$$\text{top}_k(B) \sim k^{-\beta(B)}$$

$$\frac{\text{P}[\text{f}(\text{gray person}) > r \mid \text{gray person} = \text{red person}]}{\text{P}[\text{f}(\text{gray person}) > r \mid \text{gray person} = \text{blue person}]}$$

Recommendation model:

$$\text{top}_k'(R) \sim k^{-\beta_{rec}(R)}$$

$$\text{top}_k'(B) \sim k^{-\beta_{rec}(B)}$$



Theorem: For $0 < r < \frac{1}{2}$ and $0 < \rho < 1$, for the graph sequences $G(n)$ for the organic model and $G'(n)$ for the recommendation model, the red and blue populations exhibit a power law degree distribution with coefficients:

$$\beta_{rec}(R) > \beta(R) > 3 > \beta(B) > \beta_{rec}(B)$$

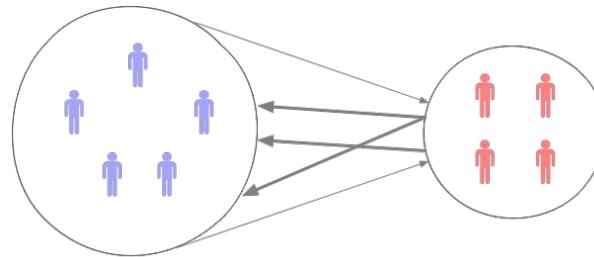
gap

Bias amplification for whom?

Symmetric homophily predicts majority advantage:

$$\beta_{rec}(R) > \beta(R) > 3 > \beta(B) > \beta_{rec}(B)$$

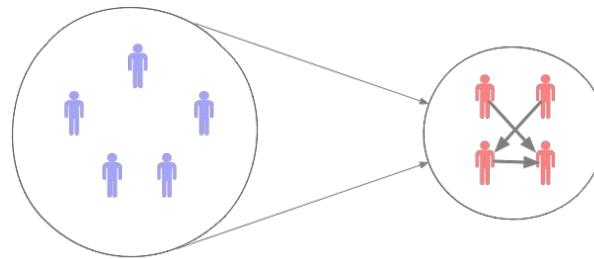

gap



Asymmetric homophily leads to a reversal of bias (amplification):

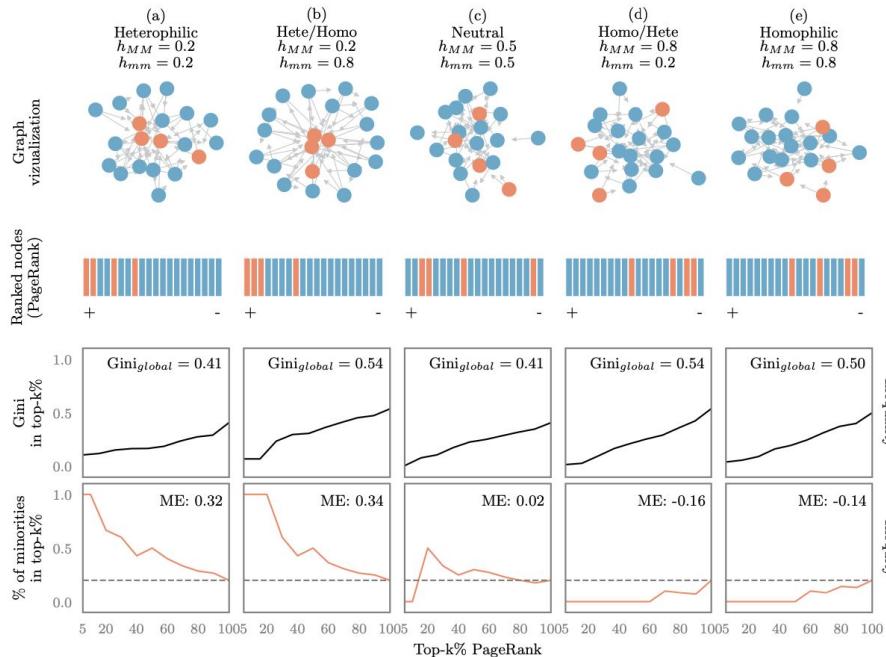
$$\beta_{rec}(B) > \beta(B) > 3 > \beta(R) > \beta_{rec}(R)$$


gap



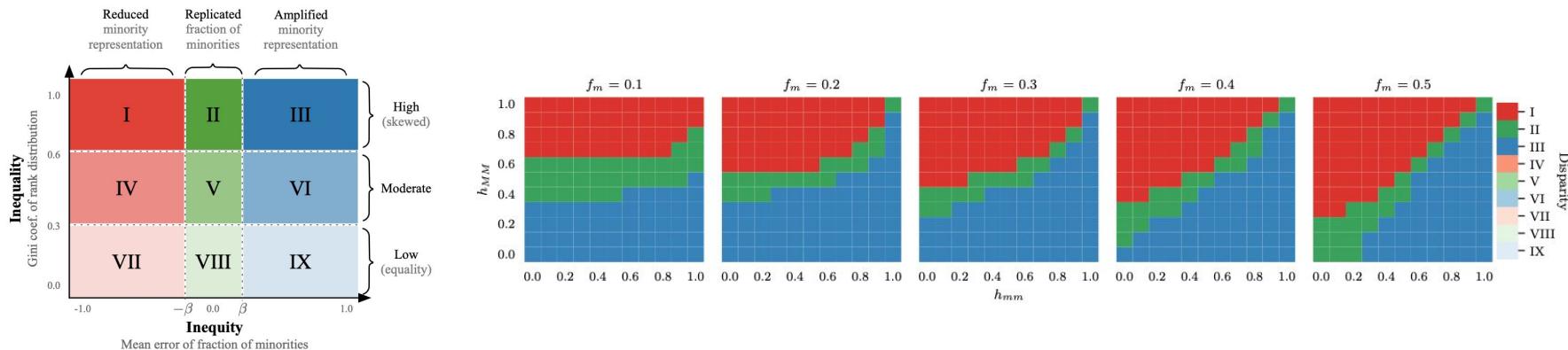
Bias amplification: recommendation and ranking

[Espin-Noboa et al, 2022] show the role of homophily/heterophily in the biased preferential attachment model in down-ranking minorities



Bias amplification: recommendation and ranking

[Espin-Noboa et al, 2022] show the role of homophily/heterophily in the biased preferential attachment model in down-ranking minorities: differentiated homophily



Preferential attachment as a unique equilibrium [Avin et al, 2018]⁵

Social capital: degree as utility function

Evolutionary game:

- Time t : new node v_t chooses a connection to u w.o. π_t over degree sequence D_t
- u accepts the connection w.p. α_t (and sends a neighbor w.p. $1 - \alpha_t$ who accepts)

Utility(u) = $E[\text{degree}(u)]$

Strategies: $\Pi = (\pi_t)$; wealth: $\alpha = (\alpha_t)$



Preferential attachment strategy is the **unique universal Nash equilibrium**

Knowledge of the network is essential

in diagnosing the impact of an algorithm

on different groups in a population

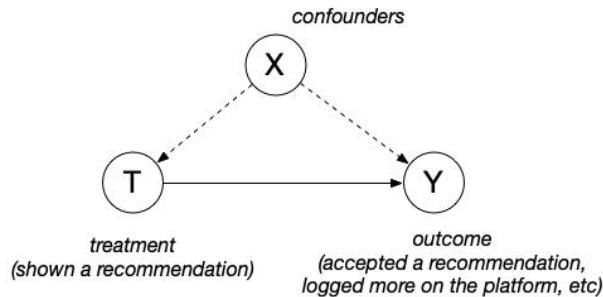
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Causality inference experiments on networks

Network experiments

- pharmaceutical companies researching the efficacy of a new medication
- policy makers understanding the impact of social welfare programs
- social media companies evaluating the impact of different recommendation algorithms on user engagement across their platforms



Potential outcomes model

Set-up: population of n individuals, a central planner that administers a treatment

- Treatment: binary variable T (let's assume a Bernoulli randomized design, $T \sim \text{Bin}(n,p)$)
- Confounders: known attributes (potentially) X
- Outcome: real-valued Y

What are we estimating?

$$TTE := \frac{1}{n} \sum_{i=1}^n (Y_i(1) - Y_i(0))$$

Classic (non-network) model:

- Stable Unit Treatment Value Assumption (SUTVA)

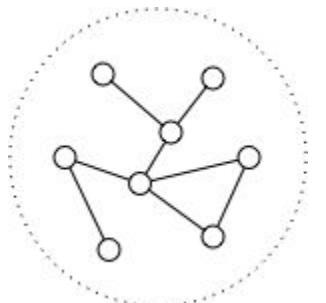
$$Y_i(T) = c_0 + c_i \cdot T_i \Rightarrow TTE = \frac{1}{n} \sum_{i=1}^n c_i$$

Network interference model:

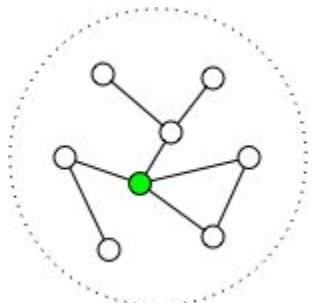
- No more SUTVA!

$$Y_i(T) = \sum_{S' \subseteq N_i} c_{i,S'} \prod_{j \in S'} T_j \Rightarrow TTE = \frac{1}{n} \sum_{i=1}^n \sum_{S' \subseteq N_i} c_{i,S'}$$

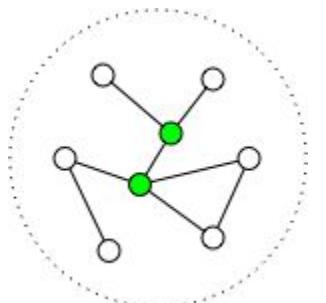
Network interference



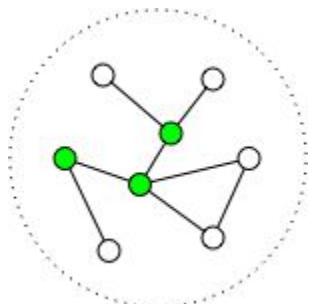
Network interference



Network interference

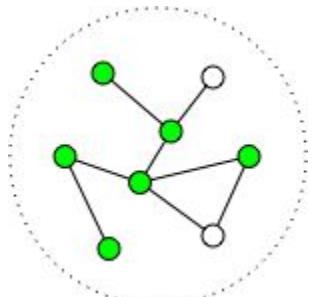


Network interference



Network interference

What is the issue?

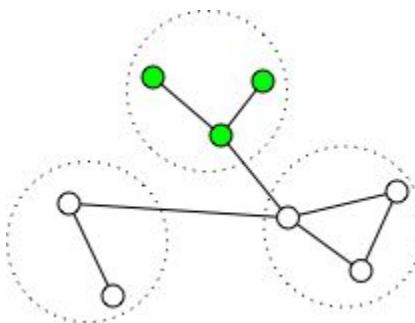


An estimator will have variance as large as the maximal degree: $O\left(\frac{Y_{max}^2 d^2}{np^d}\right)$ [Aronow et al, 2017]

Horvitz-Thompson estimator:

$$\frac{1}{n} \sum_{i=1}^n Y_i^{obs} \left(\frac{\mathbb{I}(T \text{ treats all of } N_i)}{\mathbb{P}(T \text{ treats all of } N_i)} - \frac{\mathbb{I}(T \text{ does not treat all of } N_i)}{\mathbb{P}(T \text{ does not treat all of } N_i)} \right)$$

Network interference: solutions



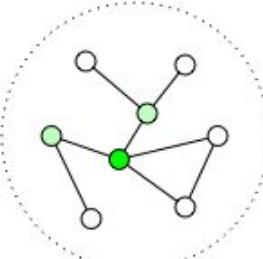
Randomized **clustered** design:

- Cluster the network
- Assume interference only within clusters
- Assign treatment at the level of the cluster

[\[Ugander et al, 2013\]](#)

[\[Eckles et al, 2016\]](#)

Network interference: bounds



		Assumptions on Network Structure		
Assumptions on Model Structure		C Disconnected Subcommunities	κ -restricted Growth	Fully General
	Linear			OLS, <i>Bernoulli RD</i> ; [40, 17, 6, 9, 29, 10]
	Generalized Linear			Regression/Machine Learning, <i>Bernoulli RD</i> ; [10]
	β -order Interactions	Directions for Future Work		Pseudoinverse estimator, <i>Bernoulli RD</i> ; $O\left(\frac{Y_{\max}^2 d^{2\beta+2}}{np^\beta}\right)$
	Arbitrary Neighborhood Interference	<i>Horvitz-Thompson</i> , Cluster RD; $O\left(\frac{Y_{\max}^2}{Cp}\right)$; [34, 31, 18, 39]	<i>Horvitz-Thompson</i> , Randomized Cluster RD; $O\left(\frac{Y_{\max}^2 \kappa^4 d^2}{np}\right)$; [17, 14, 41, 42]	<i>Horvitz-Thompson</i> , <i>Bernoulli RD</i> ; $O\left(\frac{Y_{\max}^2 d^2}{np^d}\right)$; [1]

[[Ugander et al. 2013](#)]
[\[Eckles et al. 2016\]](#)

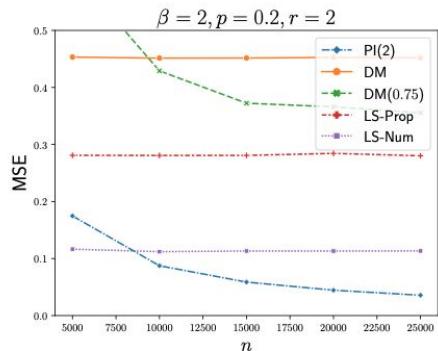
[[Cortez-Rodriguez et al. 2022](#)]

Network interference

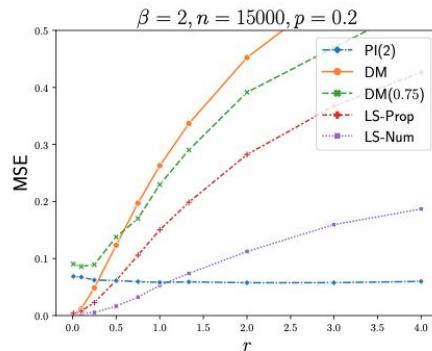
[Cortez-Rodriguez et al. 2022] proposes a new variant of the Horvitz-Thompson estimator:

$$\frac{1}{n} \sum_{i=1}^n Y_i^{obs} \sum_{S \subseteq N_i, |S| \leq \beta} g(S) \prod_{j \in S} \left(\frac{T_j}{\mathbb{P}(T_j = 1)} - \frac{1 - T_j}{\mathbb{P}(T_j = 0)} \right),$$

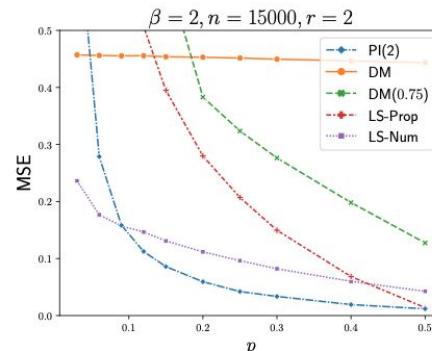
$$g(S) = \prod_{s \in S} (1 - \mathbb{P}(T_s = 1)) - \prod_{s \in S} (-\mathbb{P}(T_s = 1)), \forall S \subseteq [n]$$



(a) Varying population size



(b) Varying direct:indirect effects



(c) Varying treatment budget

DM = difference in means, LS = least squares

Conclusions and open directions

- Generalizing beyond parametric network models
 - What network properties cause bias to be projected onto different embeddings?

Conclusions and open directions

- Generalizing beyond parametric network models
- Bridging causality and fairness
 - How can infer the causal connection between algorithms and bias?

Conclusions and open directions

- Generalizing beyond parametric network models
- Bridging causality and fairness
- Feedback loops and long-term effects
 - Asymptotic analysis? Modeling feedback as strategic behavior?

Conclusions and open directions

- Generalizing beyond parametric network models
- Bridging causality and fairness
- Feedback loops and long-term effects
- Multi-objective optimization
 - How do we balance multiple objectives? How do we incorporate fairness beyond a constraint?

Conclusions and open directions

- Generalizing beyond parametric network models
- Bridging causality and fairness
- Feedback loops and long-term effects
- Multi-objective optimization
- Interdisciplinary studies
 - How can we bridge methods from social sciences, optimization, graph-theoretical modeling to understand patterns of connection / behavior and model the right objectives?

Conclusions and open directions

Thank you!

- Generalizing beyond parametric network models
- Bridging causality and fairness
- Feedback loops and long-term effects
- Multi-objective optimization
- Interdisciplinary studies

RESEARCH-ARTICLE

Bridging Machine Learning and Mechanism Design towards Algorithmic Fairness

Authors:  Jessie Finocchiaro,  Roland Maio,  Faidra Monachou,  Gourab K Patro,  Manish Raghavan,  Ana-Andreea Stoica,  Stratis Tsirfis [Authors Info & Claims](#)



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Additional slides

Biased preferential attachment model illustration

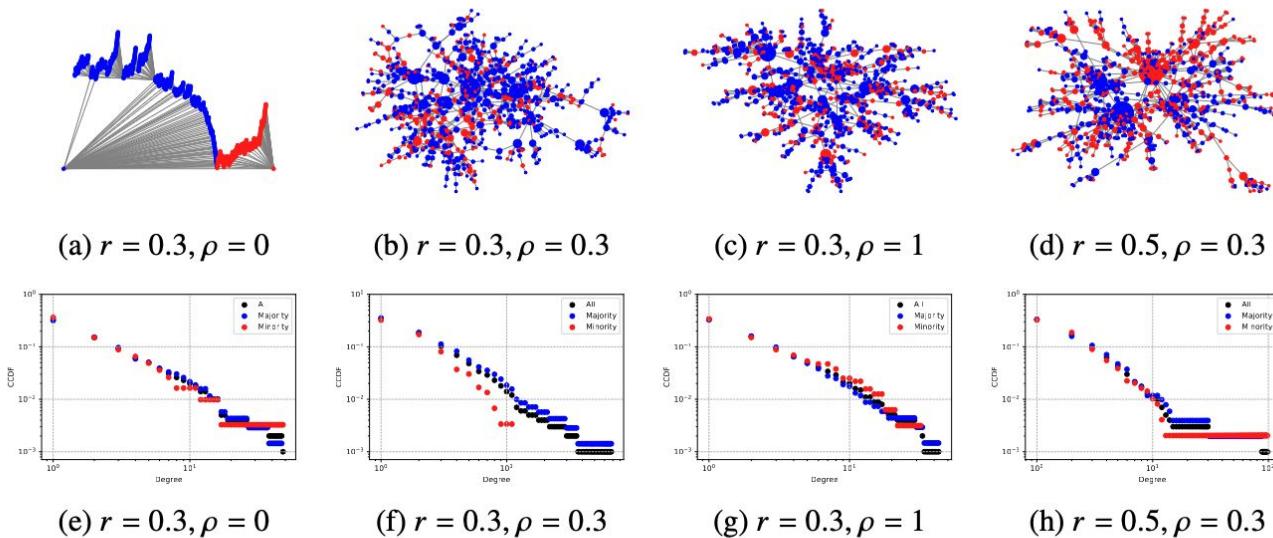


Figure 3.4: Networks generated from the Biased Preferential Attachment model (top row) and their respective cumulative complementary distribution functions, by community (bottom row), for different parameters.

Model for biased networks

Biased preferential attachment model:

- Minority-majority: blue (B) label and red (R) label (% of red nodes $< \frac{1}{2}$)
- Rich-get-richer: nodes connect w.p. proportional to degree
- Homophily: if different labels, connection is accepted with a certain probability
⇒ known to exhibit inequality in the degree distribution of the two communities³

$$top_k(R) \sim k^{-\beta(R)}$$

$$top_k(B) \sim k^{-\beta(B)}$$

$$\beta(R) > 3 > \beta(B)$$

Necessary and sufficient conditions: **groups, homophily, preferential attachment**

³Avin, Chen, et al. "Homophily and the glass ceiling effect in social networks." ITCS. 2015.

Degree distribution

Organic growth:

$$\text{top}_k(\mathbf{R}) \sim k^{-\beta(R)}$$

$$\text{top}_k(\mathbf{B}) \sim k^{-\beta(B)}$$

Recommendation model:

$$\text{top}'_k(\mathbf{R}) \sim k^{-\beta_{rec}(R)}$$

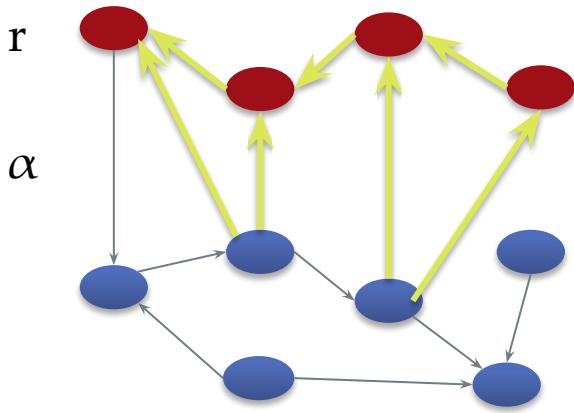
$$\text{top}'_k(\mathbf{B}) \sim k^{-\beta_{rec}(B)}$$

Theorem: For $0 < r < \frac{1}{2}$ and $0 < \rho < 1$, for the graph sequences $G(n)$ for the organic model and $G'(n)$ for the recommendation model, the red and blue populations exhibit a power law degree distribution with coefficients:

$$\begin{array}{c} \beta_{rec}(\mathbf{R}) > \beta(\mathbf{R}) > 3 > \beta(\mathbf{B}) > \\ \beta_{rec}(\mathbf{B}) \end{array}$$

gap

Proof sketch



'Wealth' of red nodes:

- Fraction of edges towards R

$$\alpha_t = \sum_{v \in R} \text{in deg}(v) / t$$

Define a function F as the rate of growth of α_t

- F has a fixed point $a \Rightarrow \alpha_t \rightarrow \alpha < r$

Organic growth

α

>

Recommendation model

α'

Proof sketch

Evolution equation:

- When does a node of degree k get a new link

Randomly

Preferential attachment

T_t^R = rate at which R nodes receive edges through **randomness**

$k \cdot C_t^R$ = rate at which R nodes receives edges through **preferential attachment**

$$top_k(\mathbf{R}) \sim k^{-\beta(R)}$$

$$\beta(R) = 1 + \frac{1}{C^R}$$

$$top_k(\mathbf{B}) \sim k^{-\beta(B)}$$

$$\beta(B) = 1 + \frac{1}{C^B}$$

Proof sketch

Goal: compute evolution equation and close it down to an solution...

$$\beta_{rec}(R) > \rho(B) > \beta_{rec}(B)$$

Big mess!



Key idea: at equilibrium, the rate at which red edges appear must equal the current fraction of red edges, as it does not evolve anymore

Invariant equation modeling asymptotic dynamics of degree distribution

Invariant equation

Organic growth:

$$\alpha \cdot C^R + r \cdot T^R = \alpha$$

Recommendation model:

$$\alpha' \cdot C'^R + r \cdot T'^R = \alpha'$$

$$\alpha > \alpha' \Rightarrow C^R > C'^R \Rightarrow \beta'(R) > \beta(R)$$



$$\beta'(R) > \beta(R) > 3 > \beta(B) > \beta'(B)$$

Degree distribution

Organic growth:

$$top_k(R) \sim k^{-\beta(R)}$$

$$top_k(B) \sim k^{-\beta(B)}$$

Recommendation model:

$$top_k'(R) \sim k^{-\beta_{rec}(R)}$$

$$top_k'(B) \sim k^{-\beta_{rec}(B)}$$

Majority has degree advantage + homophily:

$$\beta_{rec}(R) > \beta(R) > 3 > \beta(B) > \beta_{rec}(B)$$

Minority has degree advantage + homophily:

$$\beta_{rec}(B) > \beta(B) > 3 > \beta(R) > \beta_{rec}(R)$$

