MIT 14.385: Nonlinear Econometric Analysis, Fall 2022
Homework 2 for part 2.

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In this problem, you are asked to implement some simulations and estimators in R. Your code should run from start to end in one execution, producing all the output. Output and discussion of findings should be integrated in a report generated in R-Markdown (or Quarto). Figures and tables should be clearly labeled and interpretable. The findings should be discussed in the context of the theoretical results that we derived in class.

You are asked to simulate data for a Bernoulli bandit problem, where

$$D_t \in \{1, \ldots, k\}, \qquad Y_t = Y^{D_t}, \qquad Y_t^d \sim Ber(\theta^d).$$

and treatment is assigned using Thompson sampling with a uniform prior, $(\theta^1, \ldots, \theta^k) \sim U([0,1]^k)$.

1. Set up a function which accepts a sample size T and a k-vector $(\theta^1, \ldots, \theta^k)$ as its arguments, and returns a history $(D_t, Y_t)_{t=1}^T$ generated based on the Bernoulli bandit model and Thompson sampling.

2. Write a second function which takes the same arguments, plus a number of replications R, and evaluates the first function R times (using parallel computing; for instance the *future* package).

   This function should return 4 vectors of length T: The averages of Y_t, θ^{D_t}, $\mathbf{1}(D_t = \max \theta^d)$, and $\max \theta^d - \theta^{D_t}$, for each time periord t.

3. Pick a fixed vector of parameters $(\theta^1, \ldots, \theta^k)$ and a time horizon T and use the second function to plot cumulative average regret as a function of t, using a large number of replications R (such as $R = 10.000$). Repeat this for several different choices of $(\theta^1, \ldots, \theta^k)$.

   How does the result relate to the theoretical regret rate bound discussed in class, and to Agrawal and Goyal (2012)?

4. Now let $k = 2$, fix $\theta^1 = .5$ and $T = 200$. Plot cumulative average regret for T as a function of θ^2, for $\theta^2 \in [0, 1]$. Do the same for the share of observations assigned to the optimal treatment.

   How does the result relate to the local-to-zero asymptotics discussed in class, and to Figure 3 in Wager and Xu (2021)?