Social foundations for statistics and machine learning

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Today's argument

- Decision theory provides the foundation for both statistics and machine learning:
  - Experimental design / estimation / inference / policy choice,
    and algorithmic decision making / machine learning
    are modeled as **decision problems**
  - in the presence of an unknown **state** of the world.
  - The state of the world impacts the distribution of observed **data**,
  - as well as the **loss** function used to ultimately evaluate the decision.
- This single-agent framework provides important insights.
- But it cannot address important scientific and societal challenges:
  1. Replication crisis, publication bias, p-hacking, pre-registration,
     reforms of statistics teaching and the publication system.
  2. The social impact of artificial intelligence,
     questions of discrimination and inequality, value alignment.

Today's argument, continued

- Both scientific knowledge production and the deployment of technology are inherently social:
    1. Different agents have different (conflicting) objectives ("loss functions").
    2. The objectives of statistical inference or machine learning algorithms are socially determined – **who's objectives matter**?
    3. Replication crisis and reform proposals, as well as conflicts over the impact of AI can only be understood if we take this into account.
- These points have of course been recognized by the humanities:
    - Philosophy, sociology, and history of science, science and technology studies.
    - These fields however do not develop **formal** and **prescriptive** recommendations for quantitative empirical researchers, or AI engineers.

Economics to the rescue

- Economics is well positioned to fill this gap:
  - We share the languages of **constrained optimization** and **probability theory** with statistics and machine learning.
  - In contrast to these fields, we are used to thinking about **multiple agents** with unequal endowments, conflicting interests and private information.
- Today, I will discuss two projects that are part of this general agenda:

Kasy, M. and Spiess, J. (2021). The value of pre-analysis plans: Statistical decisions subject to implementability.

Kasy, M. and Abebe, R. (2021). Fairness, equality, and power in algorithmic decision making.

AI as decision theory

The textbook "Artificial intelligence: a modern approach" (Russell and Norvig, 2016) defines the goal of AI as the derivation of

> *" general principles of rational agents and on components for constructing them."*

> *"An agent is anything that can be viewed as perceiving its environment through sensors and acting upon that environment through actuators."*

> *"For each possible percept sequence, a rational agent should select an action that is expected to* ***maximize its performance measure, given the evidence*** *provided by the percept sequence and whatever built-in knowledge the agent has."*

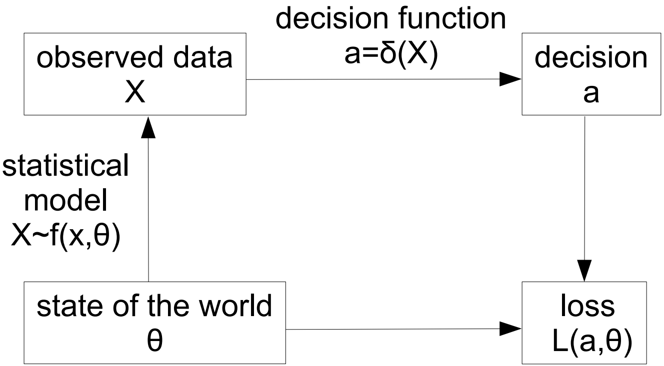
Statistics as decision theory

Similarly, the Bayesian statistics textbook by Robert (2007) states:

> "*Considering that the overall purpose of most inferential studies is to provide the statistician (or a client) with a decision, it seems reasonable to ask for an* ***evaluation*** *criterion* ***of decision procedures*** *that assesses the* ***consequences of each decision*** *and depends on the parameters of the model, i.e., the true* ***state of the world*** *(or of Nature).*"

> "*[...] implies a reinforced axiomatization of the statistical inferential framework, called Decision Theory. This augmented theoretical structure is necessary for Statistics to reach a coherence otherwise unattainable.*"

Decision theory – General setup



Examples of decision problems

- **Estimation:**
  - Find an a which is close to some function μ of θ.
  - Typical loss function: $L(a, \theta) = (a - \mu(\theta))^2$.
- **Testing:**
  - Decide whether $H_0 : \theta \in \Theta_0$ is true.
  - Typical loss function: $L(a, \theta) = \mathbf{1}(a = 1,\ \theta \in \Theta_0) + c \cdot \mathbf{1}(a = 0,\ \theta \notin \Theta_0)$.
- **Targeted treatment assignment:**
  - Assign treatment W as a function of features X, $W = \delta(X)$.
  - Typical utility function: $E[\delta(X) \cdot (M - c)]$,
    for treatment effects M, treatment cost c.

Notions of risk

- **Risk function:**
  Expected loss, averaging over sampling distribution, given the state of the world:

$$R(\delta, \theta) = E_\theta[L(\delta(X), \theta)].$$

- **Bayes risk and worst case risk:**
  Average of the risk function (over π), maximum of the risk function (over Θ):

$$R(\delta, \pi) = \int R(\delta, \theta)\pi(\theta)d\theta, \qquad \overline{R}(\delta, \Theta) = \sup_{\theta \in \Theta} R(\delta, \theta).$$

- **Regret:**
  Difference between risk function and the oracle-optimal loss.

$$Reg(\delta, \theta) = R(\delta, \theta) - \inf_a L(a, \theta).$$

P-hacking and pre-analysis plans

- Trial registration and pre-analysis plans (PAPs) have become a standard requirement for experimental research.
  - For clinical studies in medicine starting in the 1990s.
  - For experimental research in economics more recently.
- Standard justification: Guarantee validity of inference.
  - P-hacking, specification searching, and selective publication distort inference.
  - Tying researchers' hands prevents selective reporting.
  - "PAPs are to frequentist inference what RCTs are to causality."
- Counter-arguments:
  - Pre-specification is costly.
  - Interesting findings are unexpected and flexibility is necessary.

No commitment (pre-registration) in decision theory

- Two alternatives, in a generic decision problem:
  1. We can commit to (**pre-register**) a rule $\delta(\cdot)$ before observing X.
  2. We can pick $\delta(X)$ after observing X.
- By the **law of iterated expectations**

$$
\begin{aligned}
R(\delta, \pi) &= E[L(\delta(X), \theta)] \\
&= E[E[L(\delta(X), \theta)|X]] \\
&= \sum_x E[L(\delta(x), \theta)|X = x] \cdot P(X = x).
\end{aligned}
$$

- Therefore:
  - Picking the optimal $\delta(\cdot)$ (to minimize the sum) is the same
  - as picking the optimal $\delta(x)$ for every value of x (each term of the sum).
  - The decision-problem is **dynamically consistent**.

A mechanism design perspective

Claim: Concerns about p-hacking, publication bias, pre-registration are at their core about divergent interests between multiple actors.

Q: How to incorporate this social dimension into prescriptive methodology?

A: Model statistical inference as a mechanism design problem!

- Take the perspective of a reader of empirical research who wants to implement a statistical decision rule (mapping from full data to a decision).
- Not all rules are implementable when researchers have divergent interests and private information about the data, and they can selectively report to readers.
- Agenda: Characterize optimal decision rules subject to implementability.

Setup

- Two agents: Decision-maker and analyst.
- The analyst observes a vector

  $$X = (X_1, \ldots, X_{\bar{n}}),$$

  where

  $$X_i \overset{\text{iid}}{\sim} \text{Ber}(\theta).$$

- Analyst: Reports a subvector X_I to the decision-maker, where

  $$I \subset \{1, \ldots, \bar{n}\}.$$

- Decision-maker: Makes a decision

  $$a \in \{0, 1\},$$

  based on this report.

Prior and objectives

- Common prior:
$$\theta \sim \text{Beta}(\alpha, \beta).$$

- Analyst's objective:
$$u^{\text{an}} = a - c \cdot |I|.$$
$|I|$ is the size of the reported set,
c is the cost of communicating an additional component.

- Decision-maker's objective:
$$u^{\text{d-m}} = a \cdot (\theta - \underline{\theta}).$$
$\underline{\theta}$ is a commonly known parameter.
Minimum value of θ beyond which the decision-maker would like to choose $a = 1$.

Timeline

1. The decision-maker commits to a decision rule
$$a = a(J, I, X_I).$$
2. The analyst reports a PAP
$$J \subseteq \{1, \dots, \bar{n}\}.$$
3. The analyst next observes X, chooses $I \subseteq \{1, \dots, \bar{n}\}$, and reports
$$(I, X_I).$$
4. The decision rule is applied and utilities are realized.

Implementability

- Let x denote values that the random vector X may take.
- Reduced form mapping (statistical decision rule)

$$x \mapsto \bar{a}(x).$$

- $\bar{a}(x)$ is implementable
  if there exist mappings $I(x)$ and $a(I, x_I)$
  such that for all x

$$\bar{a}(x) = a(I(x), x_{I(x)}),$$

  and

$$I(x) \in \underset{I}{\operatorname{argmax}}\ a(I, x_I) - c \cdot |I|.$$

Notation

- Successes among all components: $s(X) = \sum_{i=1}^{\bar{n}} X_i$.
  Successes among the subset I: $s(X_I) = \sum_{i \in I} X_i$.

- Maximal number of components the analyst is willing to submit:

$$\bar{n}^{PC} = \max\{n : \ 1 - cn \geq 0\} = \lfloor 1/c \rfloor .$$

- First-best cutoff for the decision-maker:

$$\underline{s}^*(n) = \min\left\{\underline{s} : \ E[\theta \big| s(X_{1,\dots,n}) = \underline{s}] \geq \underline{\theta}\right\} .$$

- Minimal cutoff for the decision-maker:

$$\underline{s}^{min}(n) = \min\left\{\underline{s} : \ E[\theta \big| s(X_{1,\dots,n}) \geq \underline{s}] \geq \underline{\theta}\right\} .$$

Symmetric decision rules

- Denote $t(X_I) = |I| - s(X_I)$.
- Consider now, for general $\bar{n}$, symmetric rules of the form

$$a(s(X_I), t(X_I)),$$

Proposition (Optimal symmetric decision rule)

The optimal reduced-form decision rule that is symmetrically implementable takes the form

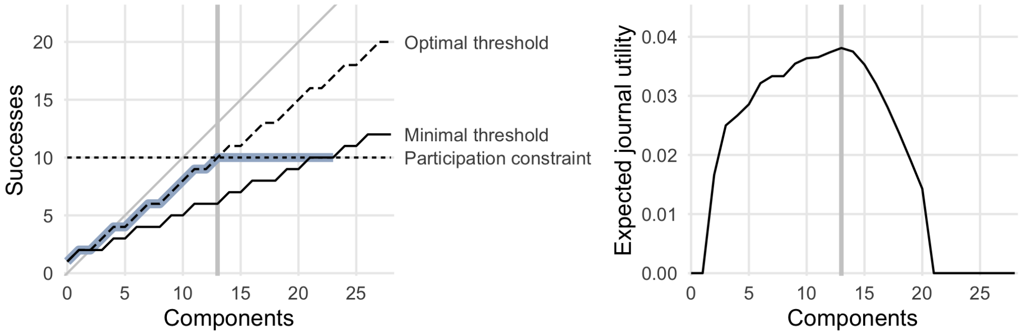
$$\bar{a} = \mathbf{1}(s(X) \geq \min(\underline{s}^*, \bar{n}^{PC})),$$

if $\bar{n}^{PC} \geq \underline{s}^{min}$, *and can be implemented by*

$$a = \mathbf{1}(s(X_I) \geq \min(\underline{s}^*, \bar{n}^{PC})).$$

Otherwise the optimal decision rule is given by $a \equiv 0$.

Symmetric cutoff without PAP, uniform prior



If the number of components $\bar{n}$ is to the right of the maximum $\bar{n}^*$:

- PAPs increase decision-maker welfare
- by forcing the analyst to ignore all components $i > \bar{n}^*$.

Algorithmic fairness and economic inequality

- Standard definitions of algorithmic **fairness**:
  - Absence of "bias" – conflating the statistical and the social notion of bias.
  - Similar to "taste based discrimination" in economics, defined as a deviation from profit maximization.
  - Fairness as a decision problem, **aligning treatment and latent merit**.
- This contrasts starkly with social choice theory in economics, and with the theory of justice in political philosophy:
  - Social welfare is typically defined based on individuals' welfare.
  - Key points of contention: How to measure individual welfare, how to trade off welfare across individuals $\Rightarrow$ distributional conflict.
  - Policies (and algorithms!) are evaluated based on their **consequences for social welfare**.
- These perspectives have very different implications, as I will elaborate.

Fairness in algorithmic decision making – Setup

- Binary treatment W, treatment return M (heterogeneous), treatment cost c.
  Decision maker's objective

$$\mu = E[W \cdot (M - c)].$$

- All expectations denote averages across individuals (not uncertainty).

- M is unobserved, but predictable based on features X.
  For $m(x) = E[M|X = x]$, the optimal policy is

$$w^*(x) = \mathbf{1}(m(X) > c).$$

Examples

- Bail setting for defendants based on predicted recidivism.
- Screening of job candidates based on predicted performance.
- Consumer credit based on predicted repayment.
- Screening of tenants for housing based on predicted payment risk.
- Admission to schools based on standardized tests.

Definitions of fairness

- Most definitions depend on **three ingredients**.
  1. Treatment W (job, credit, incarceration, school admission).
  2. A notion of merit M (marginal product, credit default, recidivism, test performance).
  3. Protected categories A (ethnicity, gender).
- I will focus on the following **definition of fairness**:

$$\pi = E[M|W=1, A=1] - E[M|W=1, A=0] = 0$$

"Average merit, among the treated, does not vary across the groups a."

This is called "predictive parity" in machine learning,
the "hit rate test" for "taste based discrimination" in economics.

- "Fairness in machine learning" literature: **Constrained optimization**.

$$w^*(\cdot) = \underset{w(\cdot)}{\operatorname{argmax}}\, E[w(X) \cdot (m(X) - c)] \qquad \text{subject to} \qquad \pi = 0.$$

Fairness and $\mathscr{D}$'s objective

Observation

Suppose that W, M are binary ("classification"), and that

1. $m(X) = M$ *(perfect predictability), and*
2. $w^*(x) = \mathbf{1}(m(X) > c)$ *(unconstrained maximization of $\mathscr{D}$'s objective μ).*

Then $w^*(x)$ satisfies predictive parity, i.e., $\pi = 0$.

In words:

- If $\mathscr{D}$ is a firm that is maximizing profits and observes everything then their decisions are fair by assumption.
  - No matter how unequal the resulting outcomes within and across groups.
- Only deviations from profit-maximization are "unfair."

Three normative limitations of "fairness" as predictive parity

Notions of fairness of this form have several key limitations:

1. They legitimize and perpetuate **inequalities justified by "merit."**
   Where does inequality in M come from?
2. They are **narrowly bracketed**.
   Inequality in W in the algorithm,
   instead of some outcomes Y in a wider population.
3. Fairness-based perspectives **focus on categories** (protected groups)
   and ignore within-group inequality.

Social welfare as an alternative framework

- The framework of fairness / bias / discrimination contrasts with perspectives focused on *consequences for social welfare*.

- Common presumption for most theories of justice:

  Normative statements about society
  are based on statements about individual welfare

- Formally:
  - Individuals $i = 1, \ldots, n$
  - Individual i's welfare Y_i
  - Social welfare as function of individuals' welfare

$$SWF = F(Y_1, \ldots, Y_n).$$

- Key points of contention:
  1. Who is included among the individuals i? Who's lives matter?
  2. How to measure individual welfare Y_i?
  3. How to trade off welfare across individuals i?

The impact on inequality or welfare as an alternative to fairness

- Outcomes are determined by the **potential outcome equation**

$$Y = W \cdot Y^1 + (1 - W) \cdot Y^0.$$

- The **realized outcome** distribution is given by

$$p_{Y,X}(y,x) = \left[p_{Y^0|X}(y,x) + w(x) \cdot \left(p_{Y^1|X}(y,x) - p_{Y^0|X}(y,x)\right)\right] \cdot p_X(x).$$

- What is the impact of $w(\cdot)$ on a **statistic** ν?

$$\nu = \nu(p_{Y,X}).$$

  Examples: Variance, quantiles, between group inequality, **social welfare**.

- Cf. Distributional decompositions in labor economics!

When fairness and equality are in conflict

- Fairness is about **treating** people of the same "**merit**" independently of their **group** membership.
- Equality is about the (counterfactual / causal) **consequences** of an algorithm for the distribution of **welfare** of different **people**.

Examples when they are in conflict:

1. Increased surveillance / **better prediction** algorithms:
   Lead to treatments more aligned with "merit"
   Good for fairness, bad for equality.
2. Affirmative action / **compensatory interventions** for pre-existing inequalities:
   Bad for fairness, good for equality.

Conclusion

- These examples show the limitations of decision theory
  for understanding important current debates.

- Many others (in econometrics, economic theory, computer science, and elsewhere) are exploring related ideas!

The road ahead:

1. Reconceptualize statistics as knowledge production and communication in a social context, involving diverse actors with divergent interests.

   Provide formal, prescriptive guidance for researchers based on this perspective.

2. Develop an understanding of algorithmic decision making / machine learning / AI based on the awareness that different people have different objectives, and control rights over data and algorithms make a difference.

Thank you!