



UNIVERSITY OF  
**OXFORD**

# Interventions for mitigating Algorithmic Inequality in Social Networks

*Ana-Andreea Stoica, Max Planck Institute for Intelligent Systems*

*May, 2023*

**MAX PLANCK INSTITUTE**  
FOR INTELLIGENT SYSTEMS



# Algorithms may amplify patterns of discrimination

KHARI JOHNSON BUSINESS MAY 16, 2022 10:25 AM

## Feds Warn Employers Against Discriminatory Hiring Algorithms

As AI invades the interview process, the DOJ and EEOC have provided guidance to protect people with disabilities from bias.

TECH POLICY

## Facebook's ad algorithms are still excluding women from seeing jobs

Its ad-delivery system is excluding women from opportunities without regard to their qualifications. That would be illegal under US employment law.

By Karen Hao

April 9, 2021

BUSINESS

## HUD is reviewing Twitter's and Google's ad practices as part of housing discrimination probe

By Tracy Jan and Elizabeth Dwoskin  
March 28, 2019 at 6:59 p.m. EDT

## The Death and Life of an Admissions Algorithm

U of Texas at Austin has stopped using a machine-learning system to evaluate applicants for its Ph.D. in computer science. Critics say the system exacerbates existing inequality in the field.

By Lilah Burke // December 14, 2020

# How do we use networks to design algorithms?

1. Using networks to diagnose *when* and *how* an algorithm may amplify bias
2. Using networks to test algorithms: randomized controlled trials
3. Build interventions to mitigate algorithmic bias
  - a. In designing fair information diffusion campaigns
  - b. In designing fair committees in opinion aggregation settings

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# Overview of published and ongoing projects

## 1. Diagnosing *when* and *how* an algorithm is amplifying bias

- A.-A. Stoica, C. Riederer, and A. Chaintreau. “*Algorithmic glass ceiling in social networks: the effects of social recommendations on network diversity*”. The Web Conference. 2018.
- A.-A. Stoica and A. Chaintreau. “*Bias in spectral embeddings: the case of recommendation algorithms on social networks*”. Manuscript in preparation. 2022.

## 2. Building interventions for mitigating such bias

- A.-A. Stoica, J.X. Han, and A. Chaintreau. “*Seeding network influence and the benefit of diversity*”. The Web Conference. 2020.
- A.-A. Stoica, A. Chakraborty, P. Dey, and K.P. Gummadi. “*Minimizing margin of victory for political and educational districting*”. AAMAS. 2020.
- A.-A. Stoica and C. Papadimitriou. “*Strategic clustering*”. In submission. 2022.

# Information diffusion

(Social influence maximization problem)

- Given a network  $G$ , with diffusion model as independent cascade with probability  $p$ , pick the best  $k$  early-adopters ('seeds') that maximize outreach:<sup>1</sup>

$$S^* = \operatorname{argmax}_{S \subseteq V(G)} \mathbb{E}(|\phi_G(S, p)|),$$

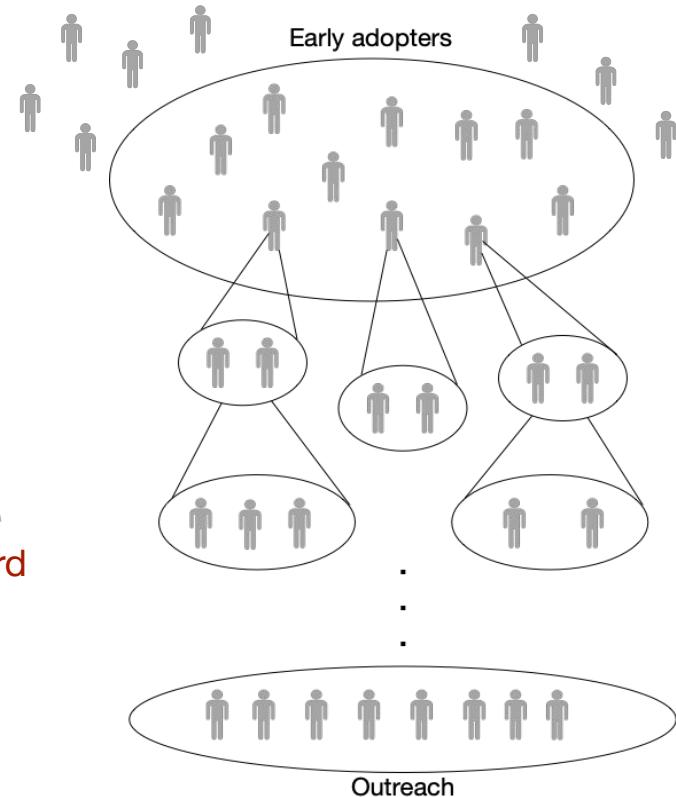
s.t.  $|S| \leq k$

NP-hard

- Algorithms that choose based on:

- Centrality: degree, distance centrality, ...
- Iteratively: greedy

↑  
Agnostic to communities



<sup>1</sup> Kempe, David, Jon Kleinberg, and Éva Tardos. "Maximizing the spread of influence through a social network." In Proceedings of the ninth ACM SIGKDD Conference, pp. 137-146. 2003.

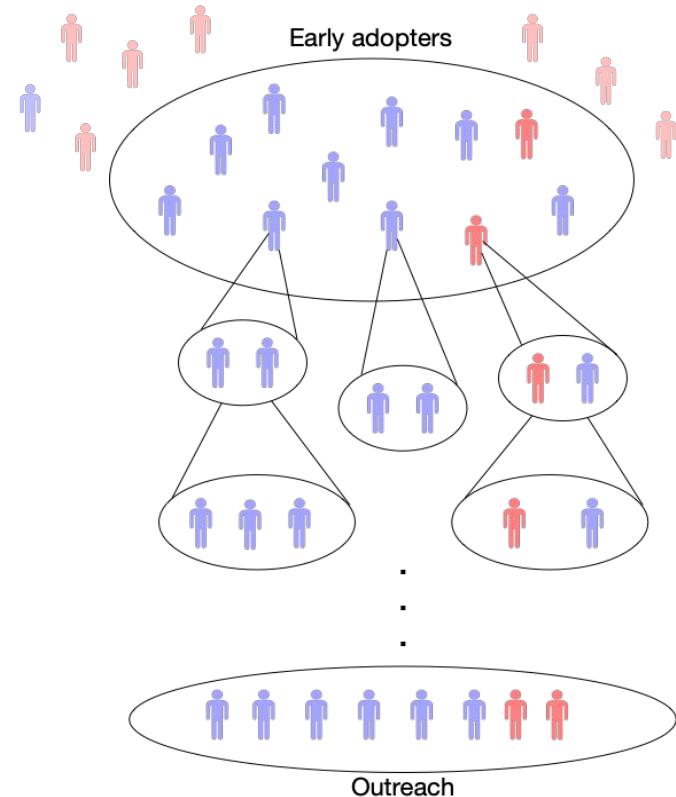
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  - Iteratively: greedy



⇒ Bias in centrality measures and social structure gets reproduced<sup>2</sup>

<sup>2</sup> Fish, Benjamin, et al. "Gaps in information access in social networks". *The World Wide Web Conference*. ACM, 2019.

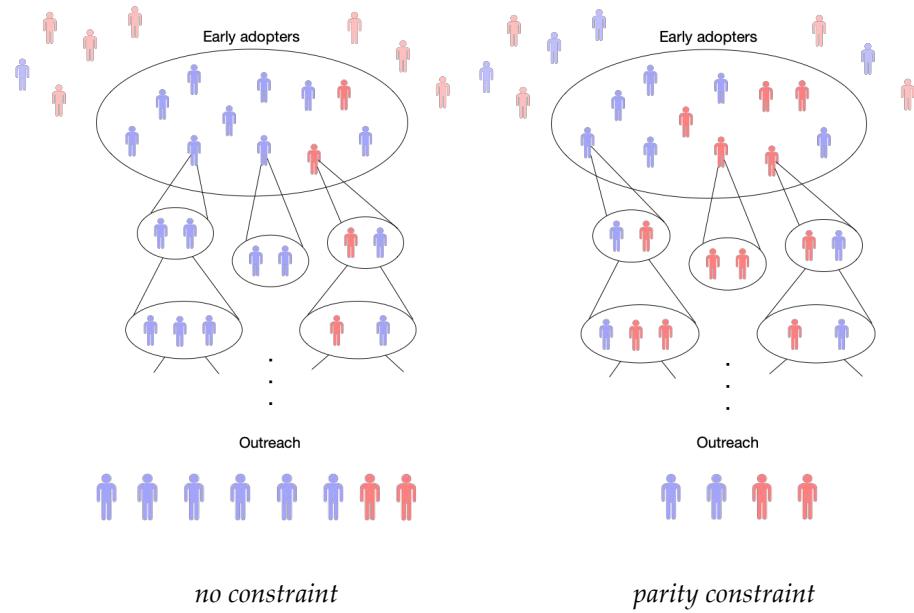
# Information diffusion

- Parity constraint in an optimization function:

$$S^* = \operatorname{argmax}_{S \subseteq V(G)} \mathbb{E}(|\phi_G(S, p)|),$$

s.t.  $|S| \leq k$  and  $\frac{\mathbb{E}(|\phi_G(S, p) \cap R|)}{\mathbb{E}(|\phi_G(S, p) \cap B|)} \simeq \frac{|R|}{|B|}$

→ Fairness-efficiency trade-off



## Our approach:

- Partially known networks  $\Rightarrow$  centrality measures (# of connections etc)
- Model of network growth & tap into inactive communities
- Theoretical conditions for when **equity increases efficiency (outreach)**

# Information diffusion

Just a Few Seeds More:  
Value of Network Information for Diffusion\*

Mohammad Akbarpour<sup>†</sup>  
Suraj Malladi<sup>‡</sup>  
Amin Saberi<sup>§</sup>



Random seeding with extra  $x$  nodes is comparable to optimal seeding (for small  $x$ )

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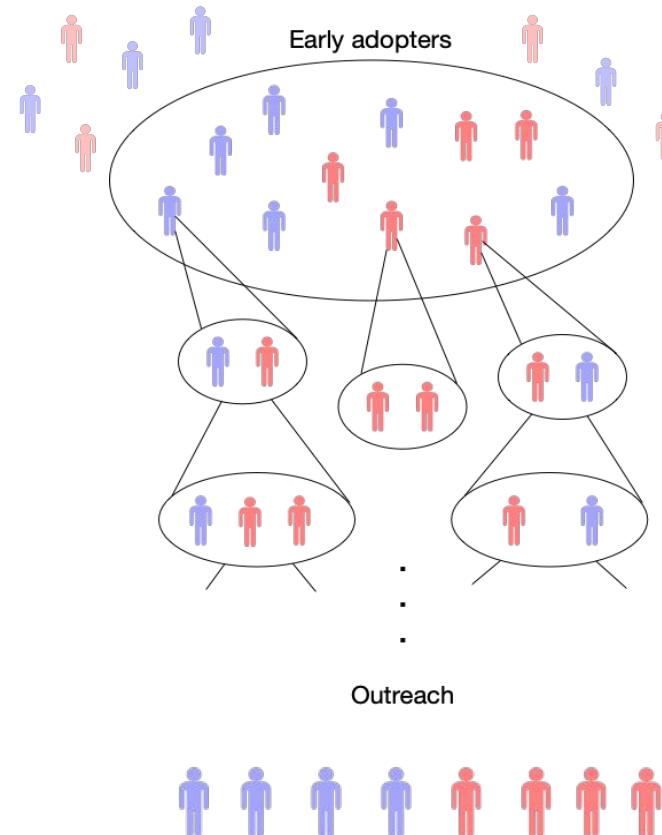
# Information diffusion

- Our vision: bias as a sign of inefficiency
  - Diversity: tap into inactivated communities in the *early adopters* set

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s.t.  $|S| \leq k$  and  $\frac{\mathbb{E}(|S \cap R|)}{\mathbb{E}(|S \cap B|)} \simeq \frac{|R|}{|B|}$

- Seeding can be done with awareness of labels:  
**statistical parity** in your campaign (even if choosing less connected people)
  - Parity seeding (strict)
  - Diversity seeding (relaxed)



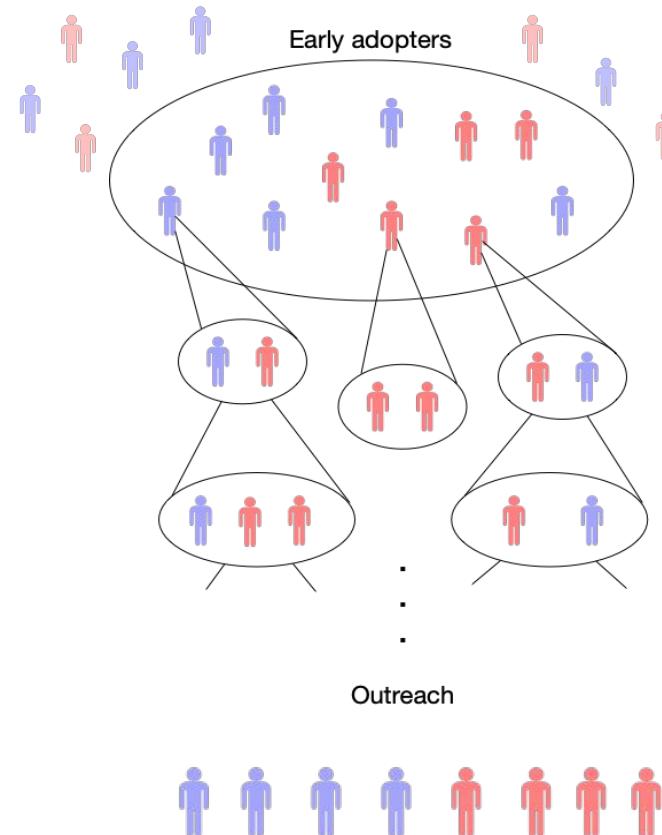
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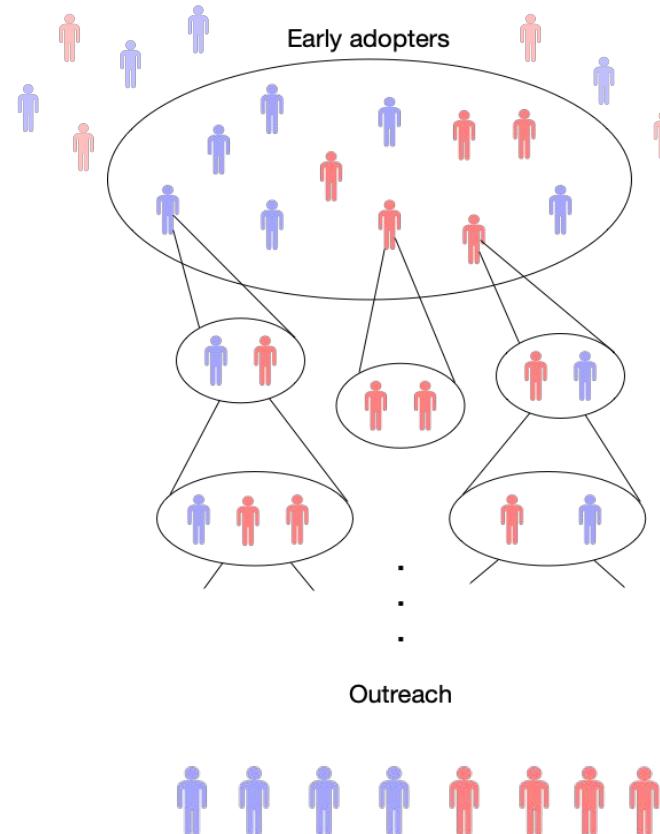
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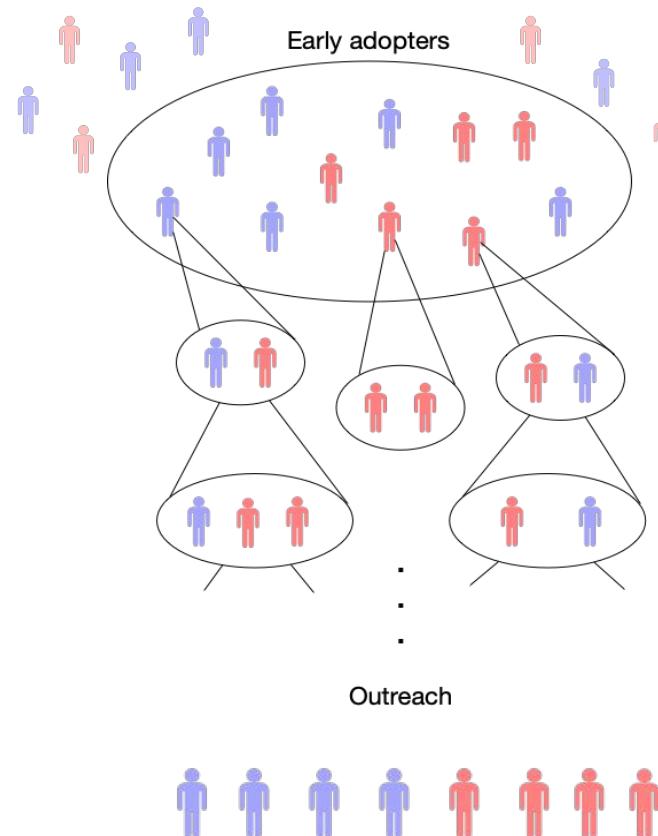
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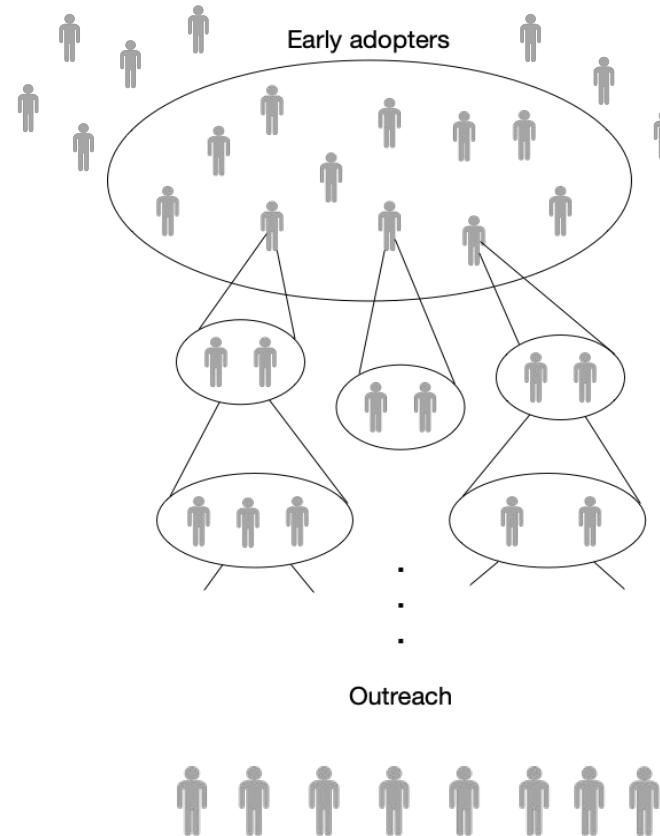
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$$\frac{\mathbb{E}(|S \cap R|)}{\mathbb{E}(|S \cap B|)} \pm \epsilon = \frac{|R|}{|B|}$$

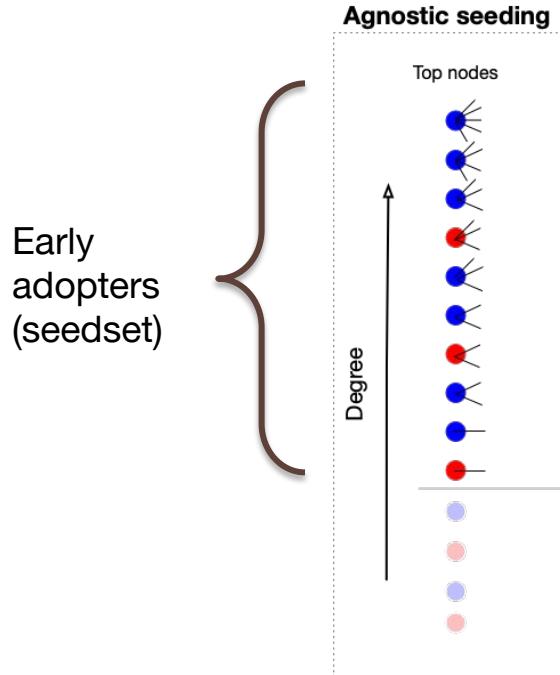


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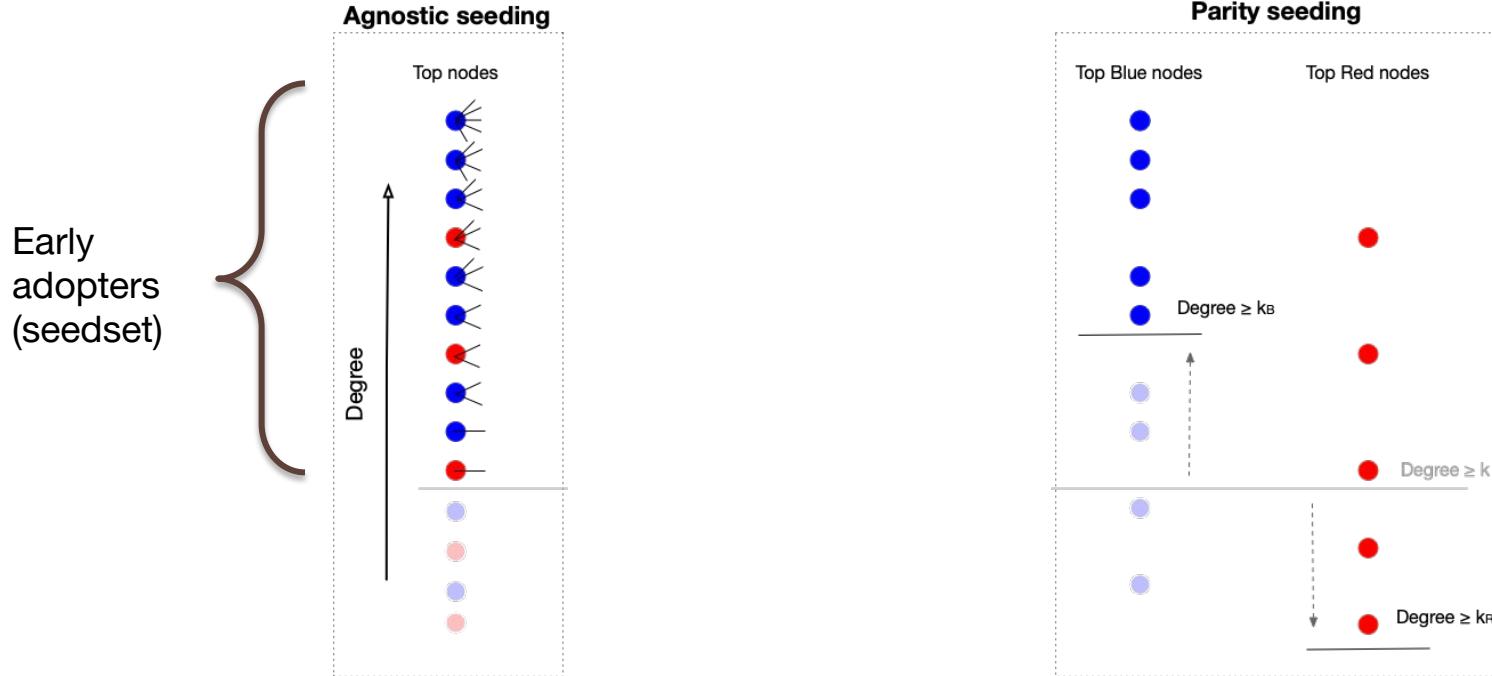
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**statistical parity** in your campaign (even if choosing less connected people)
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- Baseline: Seeding can be done **agnostically**: ignore labels, already takes into account network structure



# Color-agnostic v. Diversity Seeding



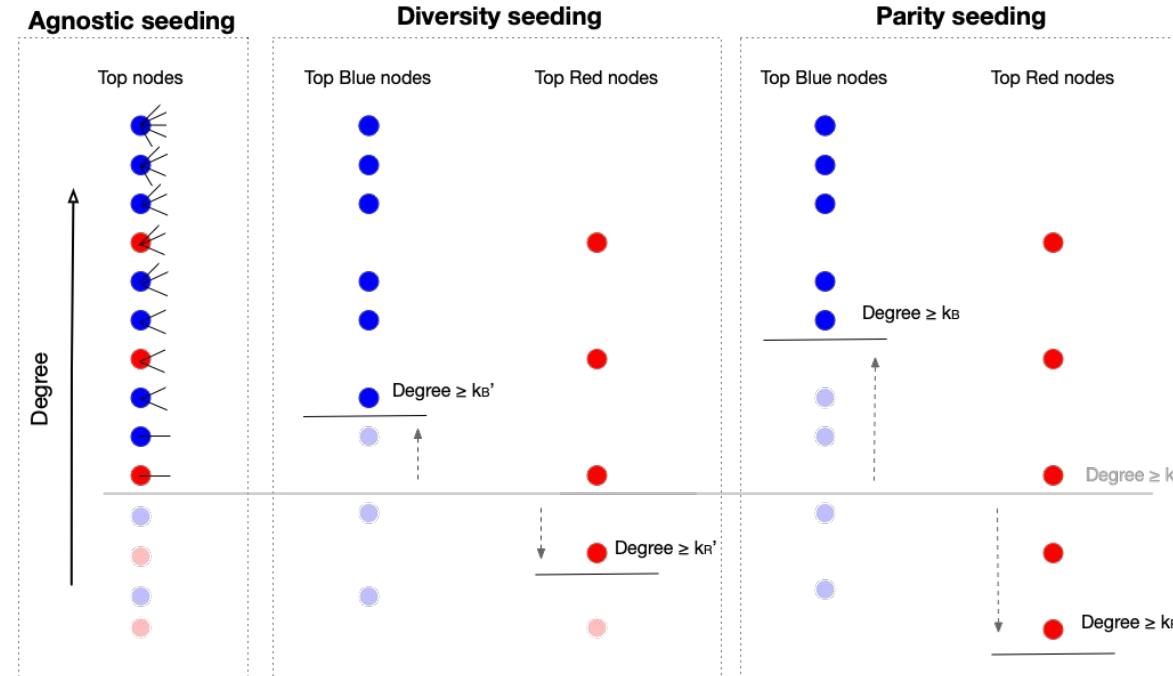
# Color-agnostic v. Diversity Seeding



Keeping the same budget!

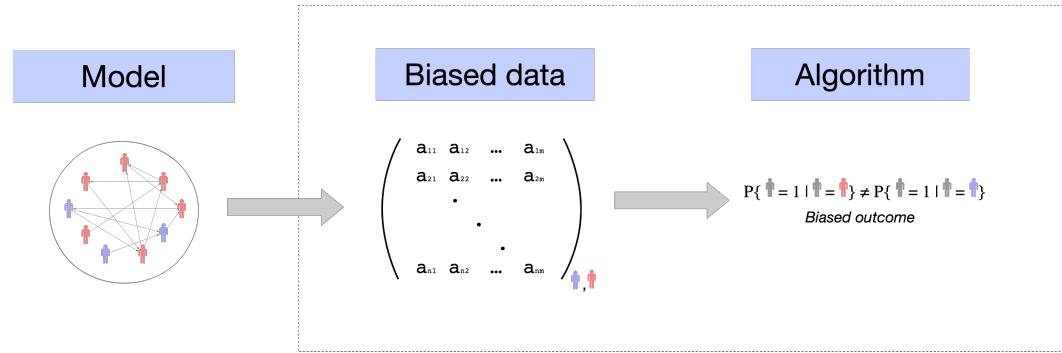
# Color-agnostic v. Diversity Seeding

Early adopters  
(seedset)



Keeping the same budget!

# Networks modeling for building more diverse and efficient heuristics



Models of network evolution:

- Explain where inequality or bias originates and how it propagates in an algorithm
- Useful to prove guarantees about interventions to mitigate bias

# Biased preferential attachment model (BPAM)

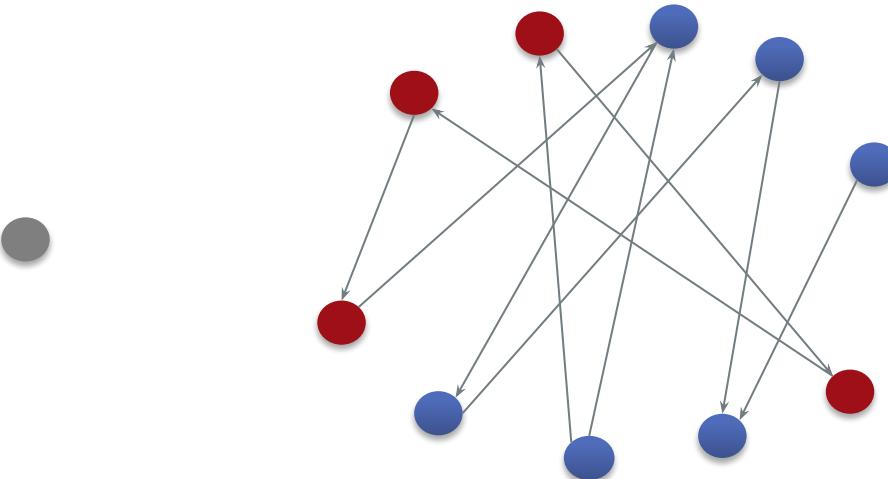
**Minority-majority:** red label and blue label

- Fraction of red nodes =  $r < \frac{1}{2}$

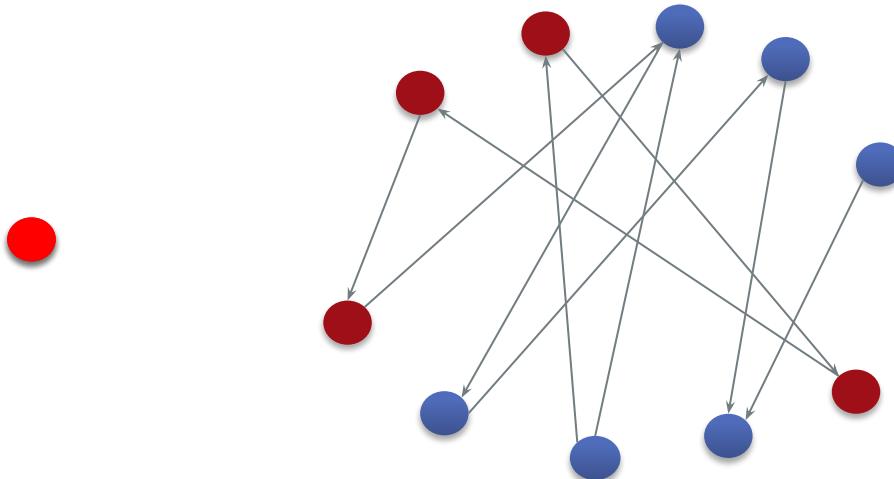
**Preferential attachment** (rich-get-richer): nodes connect w.p. proportional to degree

**Homophily:** if different labels, connection is accepted w.p.  $\rho$

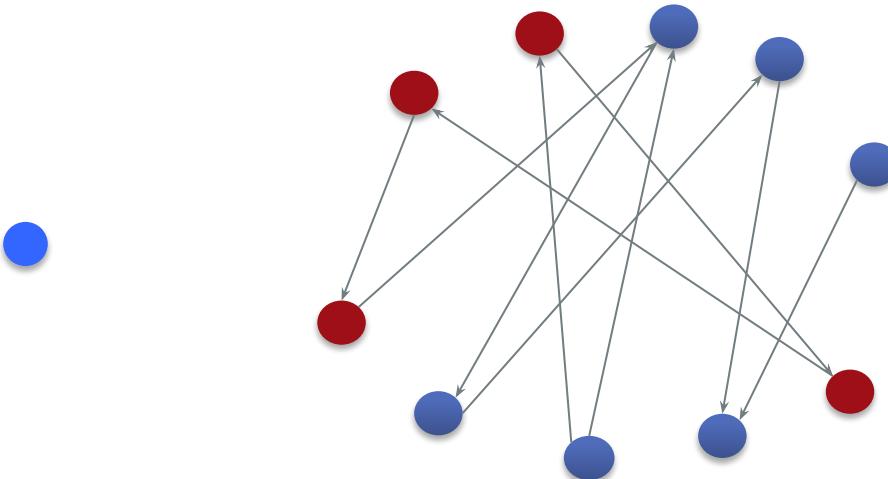
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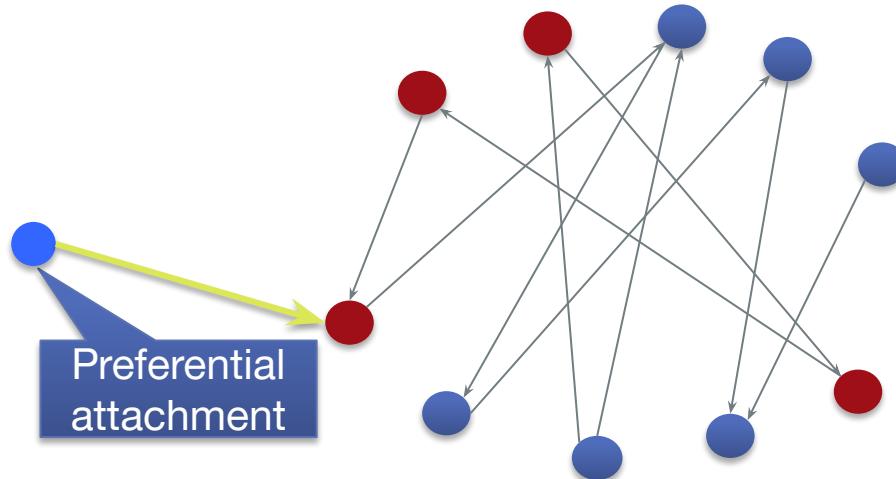
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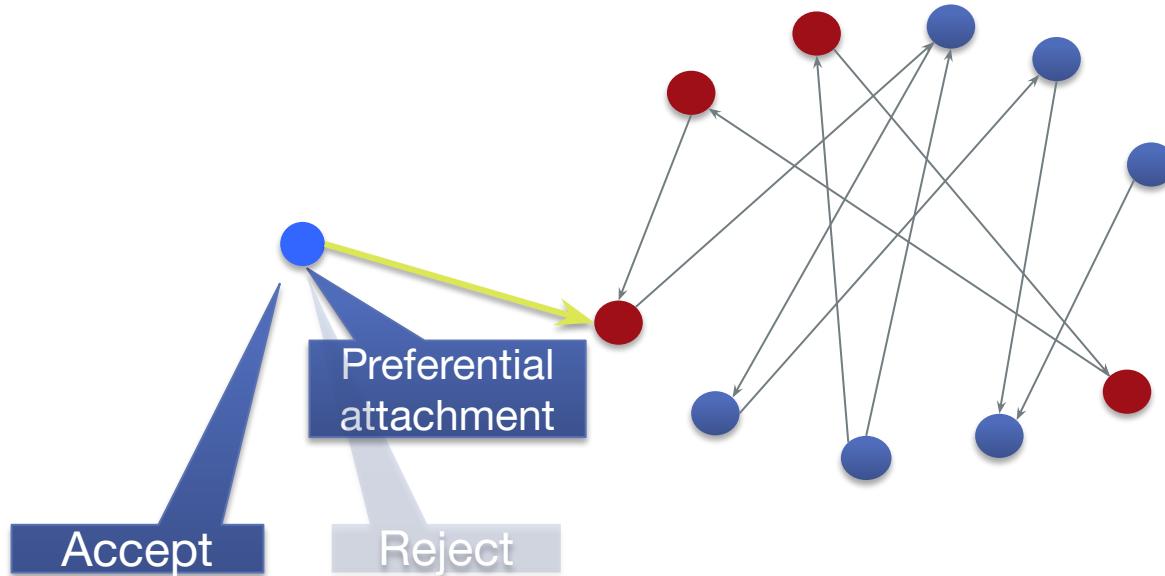
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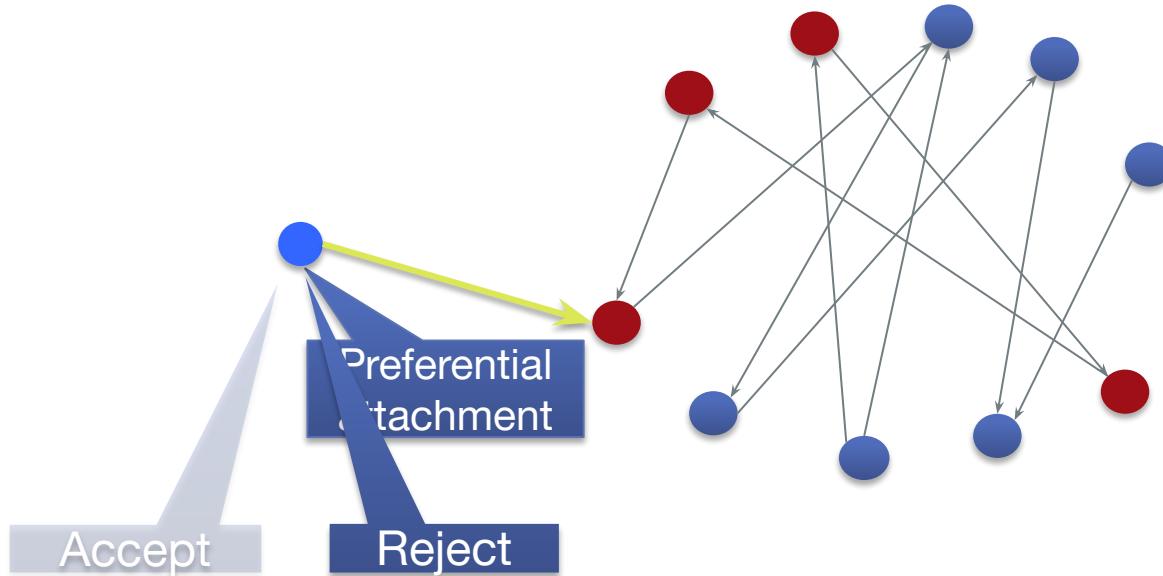
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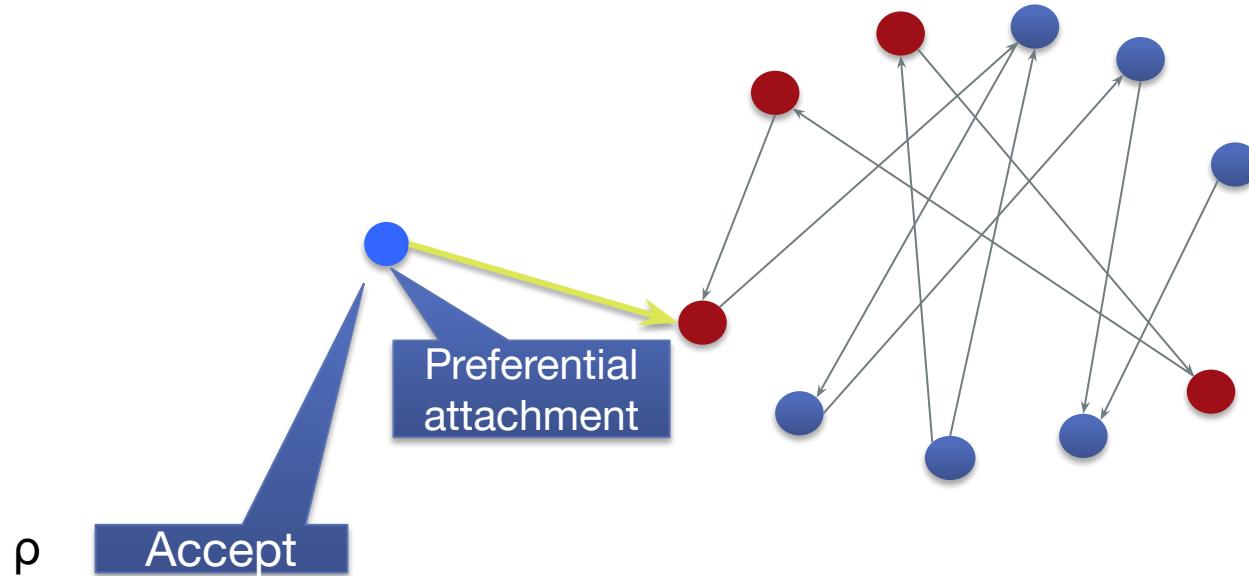
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⇒ known to exhibit inequality in the degree distribution of the two communities<sup>4</sup>

$$top_k(\mathbf{R}) \sim k^{-\beta(R)}$$

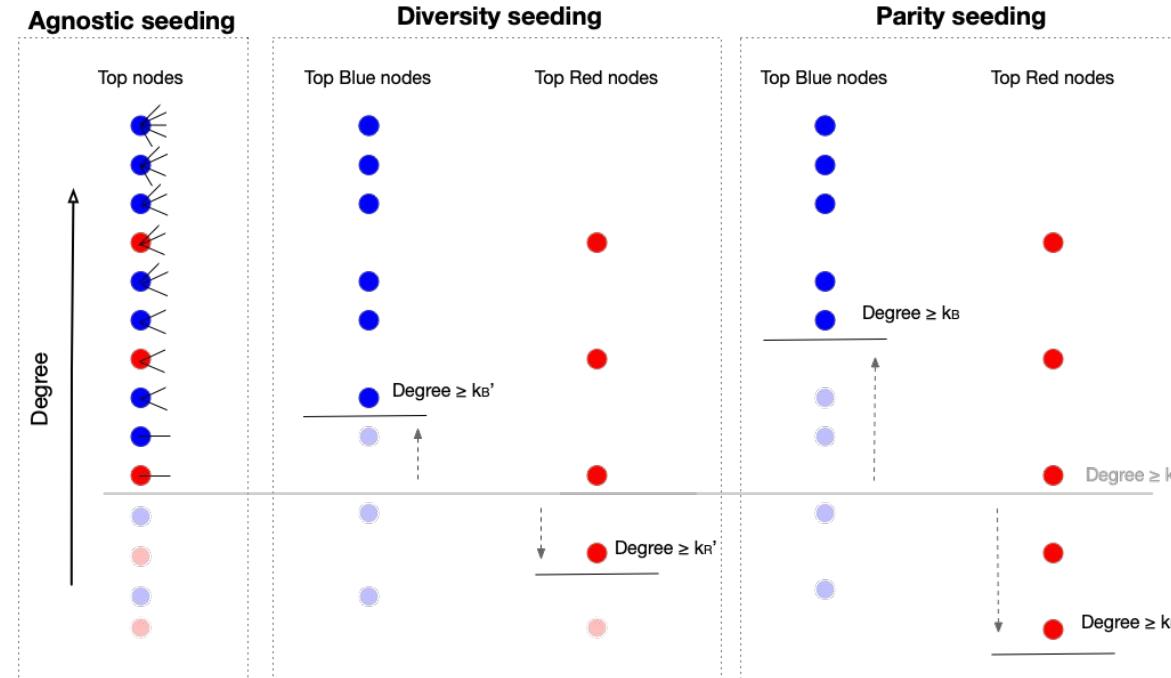
Thm [Avin et al]:  $\beta(\mathbf{R}) > 3 > \beta(\mathbf{B})$

$$top_k(\mathbf{B}) \sim k^{-\beta(B)}$$

<sup>4</sup>Avin, Chen et al. "Homophily and the glass ceiling effect in social networks." ITCS. 2015

# Color-agnostic v. Diversity Seeding

Early adopters  
(seedset)



Keeping the same budget!

# Theoretical analysis of diversity interventions

**Theorem:** for the graph sequences  $G(n)$  generated from the BPAM:

1. Diversity seeding and parity seeding leads to fairer outreach for the same budget

$$abs \left( \frac{\mathbb{E}(|\phi(S_{\text{diversity,parity}}) \cap \textcolor{red}{R}|)}{\mathbb{E}(|\phi(S_{\text{diversity,parity}}) \cap \textcolor{blue}{B}|)} - \frac{|\textcolor{red}{R}|}{|\textcolor{blue}{B}|} \right) \leq abs \left( \frac{\mathbb{E}(|\phi(S_{\text{agnostic}}) \cap \textcolor{red}{R}|)}{\mathbb{E}(|\phi(S_{\text{agnostic}}) \cap \textcolor{blue}{B}|)} - \frac{|\textcolor{red}{R}|}{|\textcolor{blue}{B}|} \right)$$

2.  $\exists k^*$  (closed form) such that when  $k > k^*$ , diversity seeding and parity seeding can outperform agnostic seeding in outreach

$$\mathbb{E}(\phi(S_{\text{diversity}})) > \mathbb{E}(\phi(S_{\text{parity}})) > \mathbb{E}(\phi(S_{\text{agnostic}})),$$

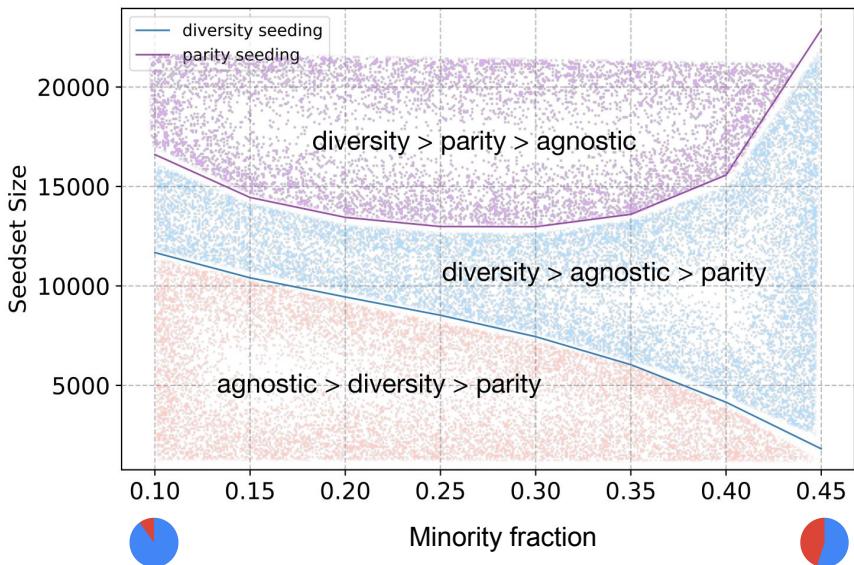
$$\text{given } |S_{\text{diversity}}| = |S_{\text{parity}}| = |S_{\text{agnostic}}| = k$$

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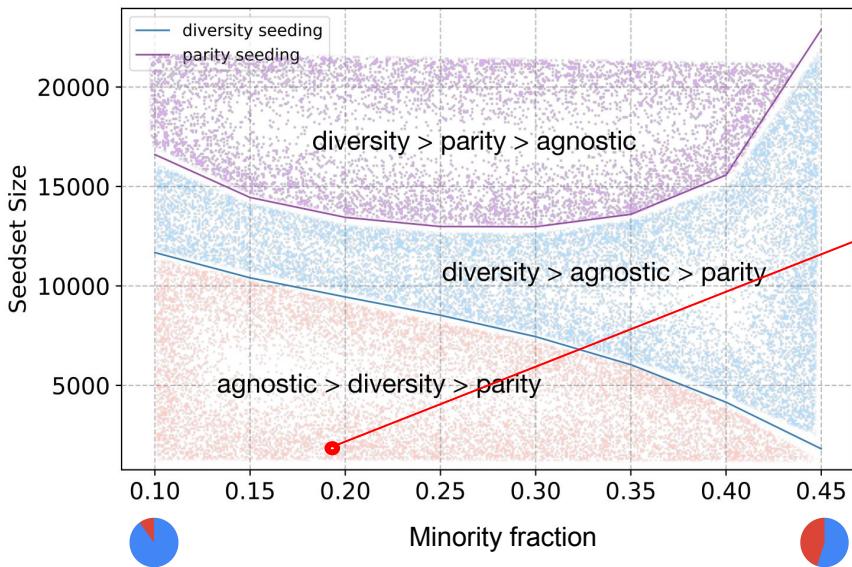
# Theoretical analysis of diversity interventions



Network of ~53,000 nodes, 2 communities, homophily  $\rho = 0.135$

- Compute regions where each heuristic performs better than the agnostic one
- As communities become more equal, need fewer seeds for diversity heuristic to be more efficient
- Not the same thing happens with the parity heuristic!

# Theoretical analysis of diversity interventions

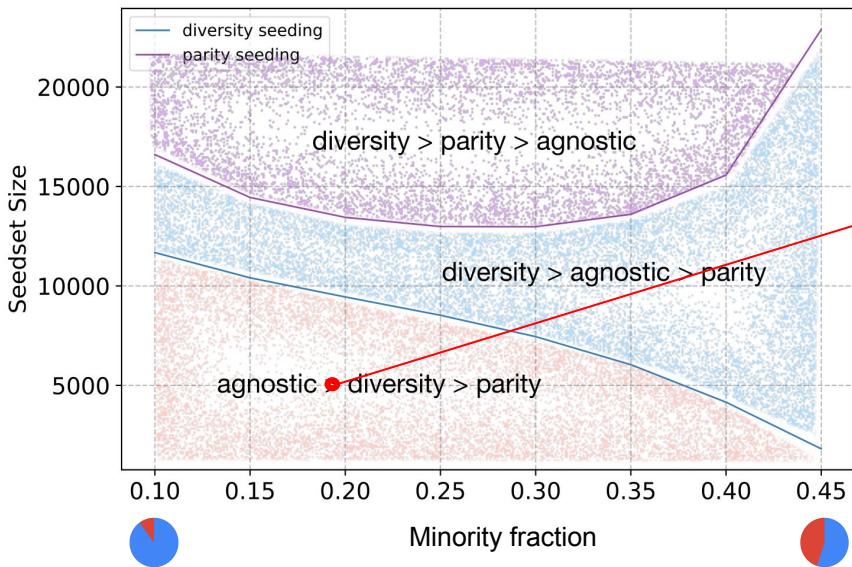


Network of ~53,000 nodes, 2 communities, homophily  $\rho = 0.135$

# DBLP citation dataset: men and women

$p = 0.01$	1,000 seeds		
	Agnostic seeding	Parity seeding	Diversity seeding
Total outreach	<b>1,149.15</b>	<span style="color:red">\downarrow</span> 1,147.874	<span style="color:red">\downarrow</span> 1,149.1
F outreach	191.95	<span style="color:green">\uparrow</span> 210.456	<span style="color:green">\uparrow</span> 196.6
M outreach	<b>957.2</b>	<span style="color:red">\downarrow</span> 937.418	<span style="color:red">\downarrow</span> 952.5
F % in outreach	0.167	<span style="color:green">\uparrow</span> 0.183	<span style="color:green">\uparrow</span> 0.171

# Theoretical analysis of diversity interventions

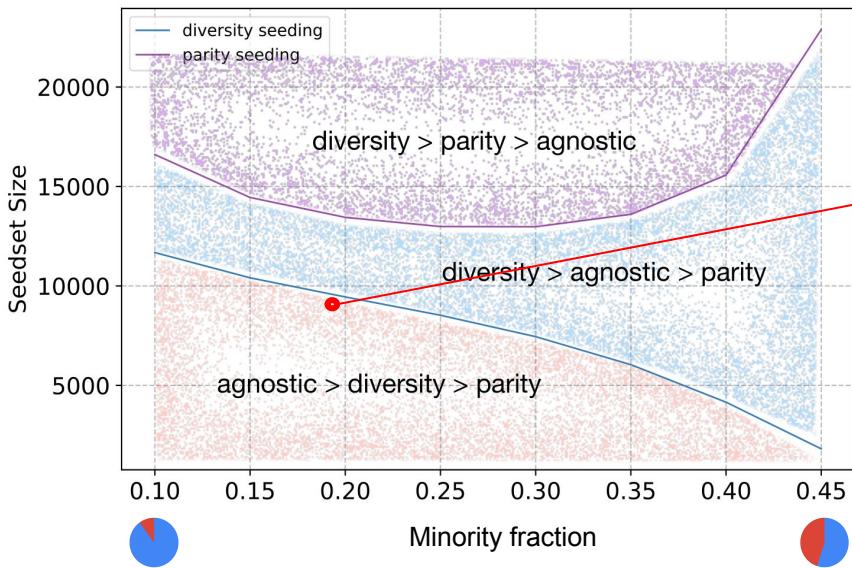


Network of ~53,000 nodes, 2 communities, homophily  $\rho = 0.135$

# DBLP citation dataset: men and women

$p = 0.01$	5,000 seeds		
	Agnostic seeding	Parity seeding	Diversity seeding
Total outreach	5,410.748	↓5,408.762	↑5411.191
F outreach	862.191	↑1,004.232	↑892.11
M outreach	4,548.557	↓4,404.53	↓4,519.081
F % in outreach	0.15934	↑0.18567	↑0.165

# Theoretical analysis of diversity interventions



# DBLP citation dataset: men and women

$p = 0.01$	9,100 seeds		
	Agnostic seeding	Parity seeding	Diversity seeding
Total outreach	9,554.934	↑9,555.559	↑9,556.349
F outreach	1,581.842	↑1,776.037	↑1,679.423
M outreach	7,973.092	↓7,779.522	↓7,876.926
F % in outreach	0.16555	↑0.186	↑0.176

Network of ~53,000 nodes, 2 communities, homophily  $\rho = 0.135$

# Future directions

- Other models beyond independent cascade?
  - Linear threshold model<sup>5</sup>
- Theoretical analysis for different centrality metrics?
- Network formation & causality questions
  - Am I friends with people because we influenced each other or the other way around?<sup>6</sup>

<sup>5</sup> Ali, J., Babaei, M., Chakraborty, A., Mirzasoleiman, B., Gummadi, K.P. and Singla, A., 2022, May. On the fairness of time-critical influence maximization in social networks. *ICDE*. 2022.

<sup>6</sup> Cristali I, Veitch V. Using Embeddings for Causal Estimation of Peer Influence in Social Networks. arXiv preprint arXiv:2205.08033. 2022.

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# Opinion dynamics models

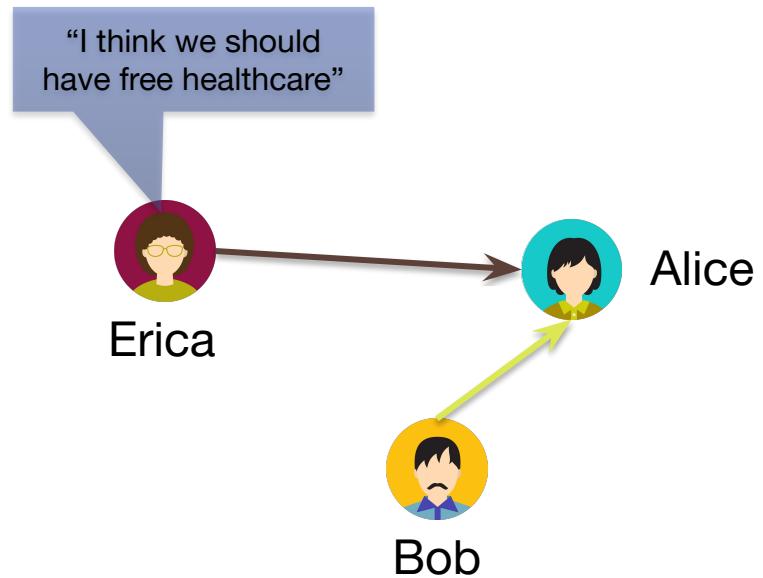
- Political purposes: understanding voting patterns and changes
- Policy purposes:
  - Education policy
  - Healthcare policy
  - Collective action (union formation)
  - Local decisions, e.g. transportation

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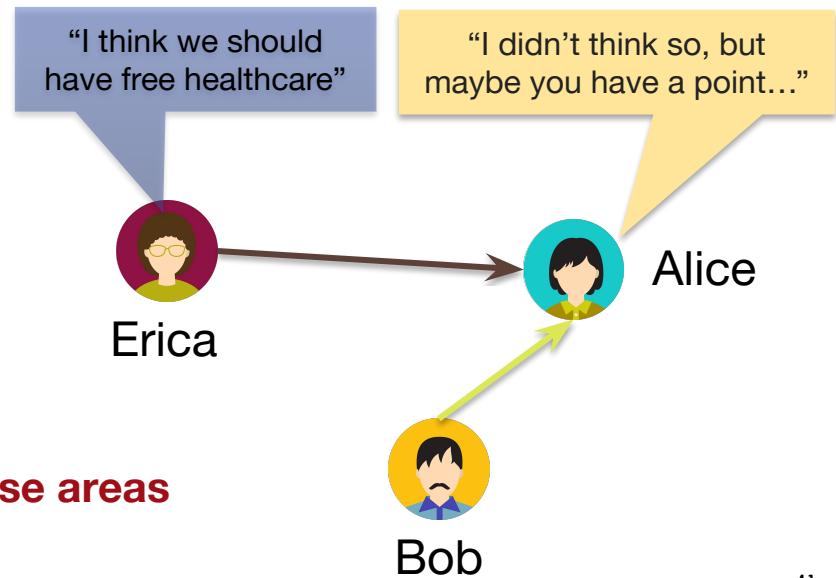
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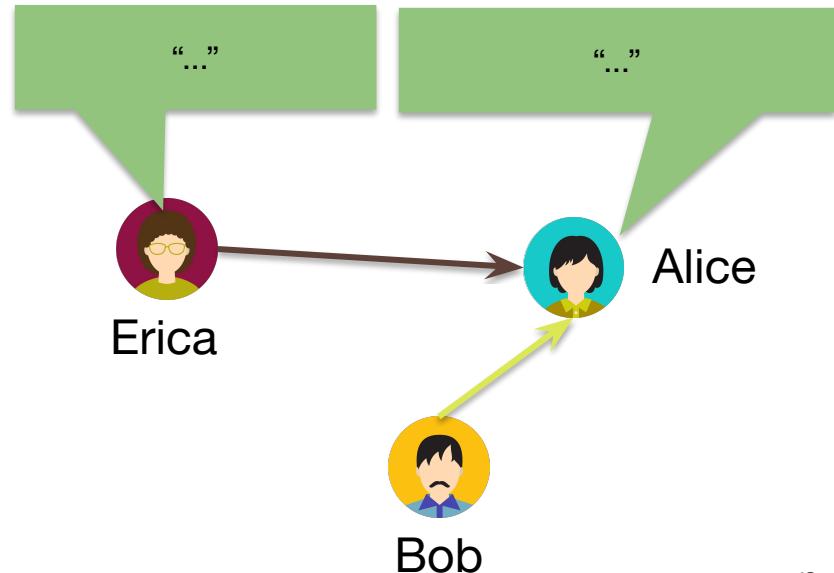
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⇒ final consensus governs decisions in these areas



# Opinion dynamics models

- [Golub & Jackson]<sup>7</sup> describe network conditions to get consensus



<sup>7</sup>Golub, B. and Jackson, M.O., 2010. Naive learning in social networks and the wisdom of crowds. *American Economic Journal: Microeconomics*, 2(1), pp.112-149.

# Opinion dynamics models

- [Golub & Jackson]<sup>7</sup> describe network conditions to get consensus
- DeGroot model<sup>8</sup> of opinion aggregation:

For a population of n agents with initial opinions  $\{x_1(0), x_2(0), \dots, x_n(0)\}$

and a network with adjacent matrix A, opinions update at every timestep t:

$$x_i(t+1) = \sum_{j=1}^n A_{ij} x_j(t)$$

Consensus is reached as  $t \rightarrow \infty : \mathbf{x}(\infty) = \mathbf{e} \cdot \mathbf{x}(0)$

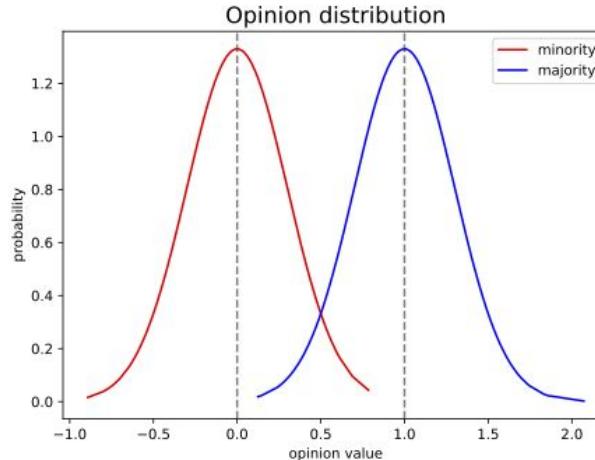
Eigencentrality matters!

<sup>7</sup>Golub, B. and Jackson, M.O., 2010. Naive learning in social networks and the wisdom of crowds. *American Economic Journal: Microeconomics*, 2(1), pp.112-149.

<sup>8</sup>DeGroot, Morris H. (1974). 'Reaching a Consensus', *Journal of the American Statistical Association* 69(345): 118–121.

# Opinion dynamics models

- [\[Golub & Jackson\]<sup>7</sup>](#) describe network conditions to get consensus
- If different groups have different opinions, how does consensus look like?



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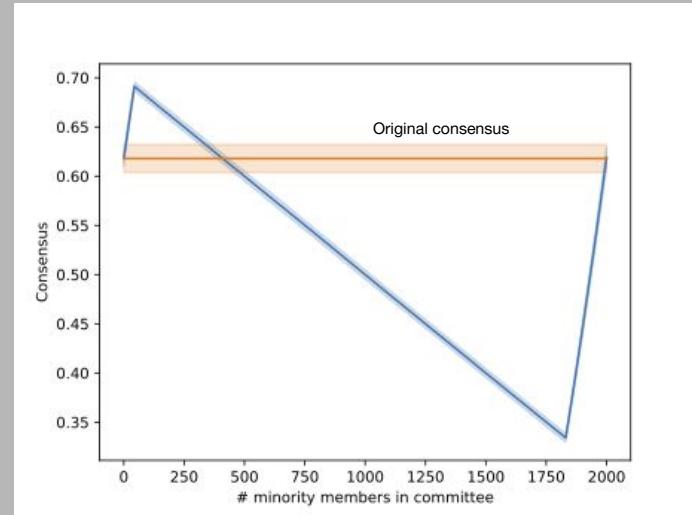
# How does a committee affect consensus?

Modeling choices for committees:

- Choose a proportion  $p$  of the population in the committee
- Assume that consensus **first** occurs in the committee, and then in the general population
  - 2-step process:
    - for a committee  $C \subset [n], (x_i(0))_{i \in C} \xrightarrow{t \rightarrow \infty} (x_i(\infty))_{i \in C}$  (assume that committee forms a click)
    - initial opinions of the population:  $\{(x_i(\infty))_{i \in C}, (x_i(0))_{i \notin C}\}$
- Fairness: how many of each group do we choose?
  - Proportional to their numbers in the population
  - Which individuals do we choose? The most central ones

# How does a committee affect consensus?

If we choose a committee with proportions equal to the general population (21% minority), we actually skew the consensus more towards the majority!

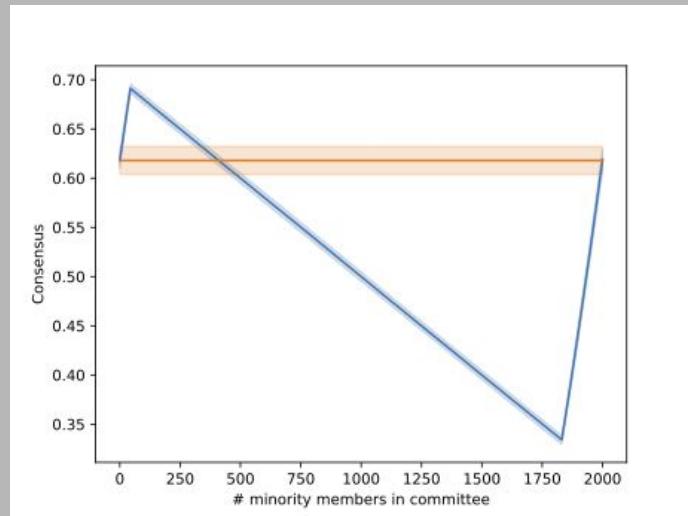


DBLP data, 53,000 nodes, 21% women

# What interventions can we enact?

1. Choose more minority members in the committee

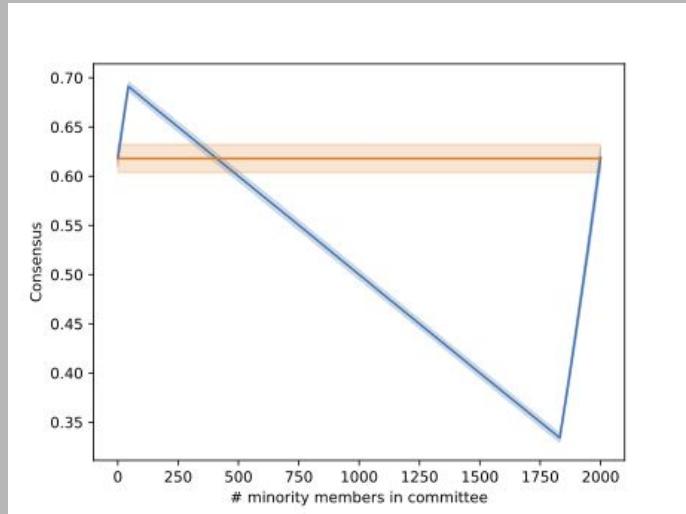
⇒ proportional representation can hurt



DBLP data, 53,000 nodes, 21% women

# What interventions can we enact?

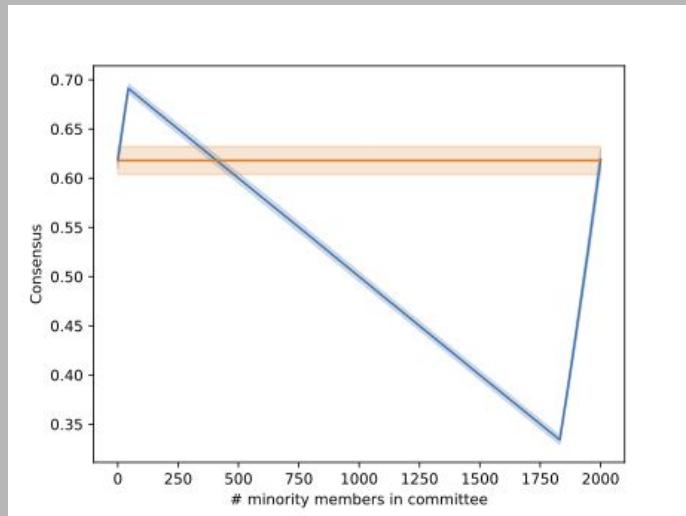
1. Choose more minority members in the committee
2. Choose less central minority members in the committee



DBLP data, 53,000 nodes, 21% women

# What interventions can we enact?

1. Choose more minority members in the committee
2. Choose less central minority members in the committee
3. Change the way committee aggregates

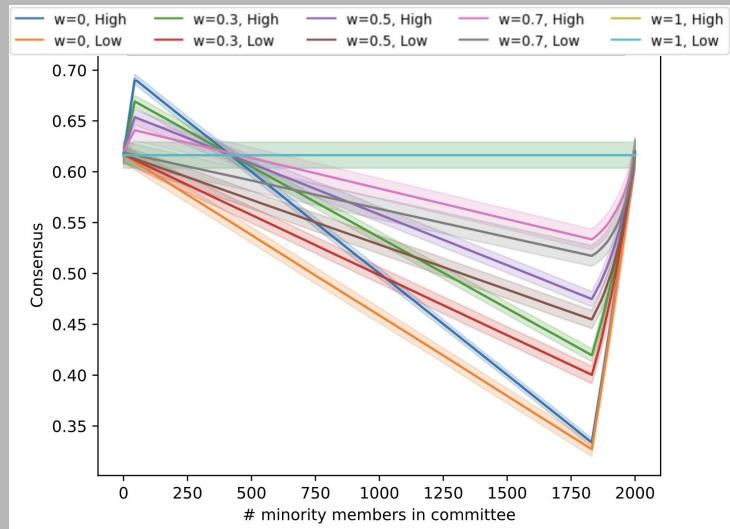


DBLP data, 53,000 nodes, 21% women

# What interventions can we enact?

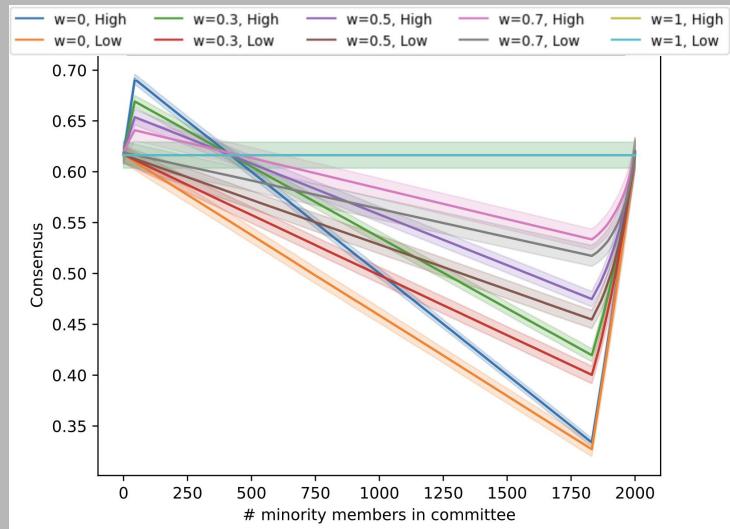
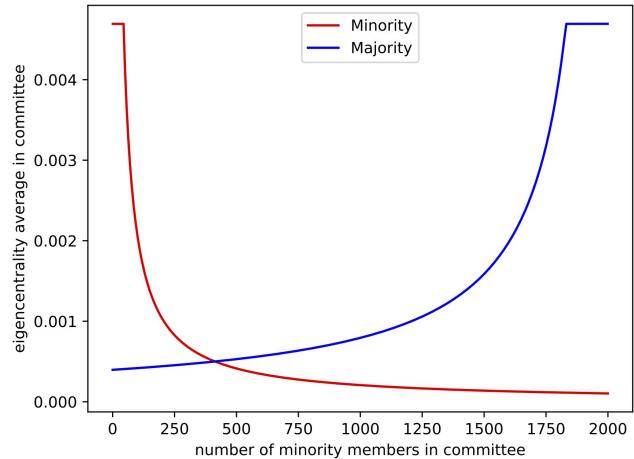
1. Choose more minority members in the committee
2. Choose less central minority members in the committee (Low vs. High)
3. Change the way committee aggregates
  - Committee forms a clique
  - Committee aggregates proportional to their network eigencentrality

$$\mathbf{e} = w \cdot \mathbf{e}_{\text{original}} + (1 - w) \cdot \mathbf{e}_{\text{clique}}$$



DBLP data, 53,000 nodes, 21% women

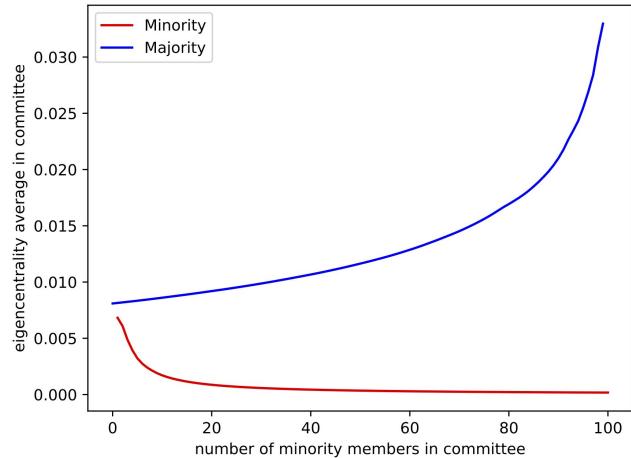
# What interventions can we enact?



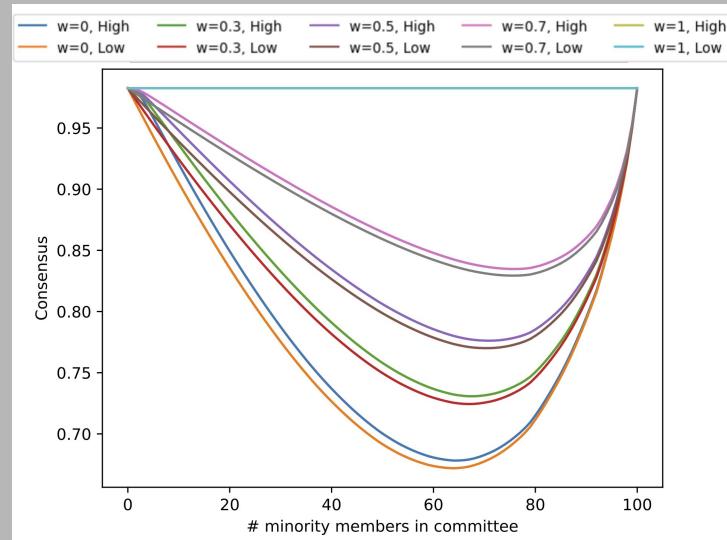
DBLP data, 53,000 nodes, 21% women

⇒ Theoretical explanation for when consensus is skewed towards one of the communities

# What interventions can we enact?

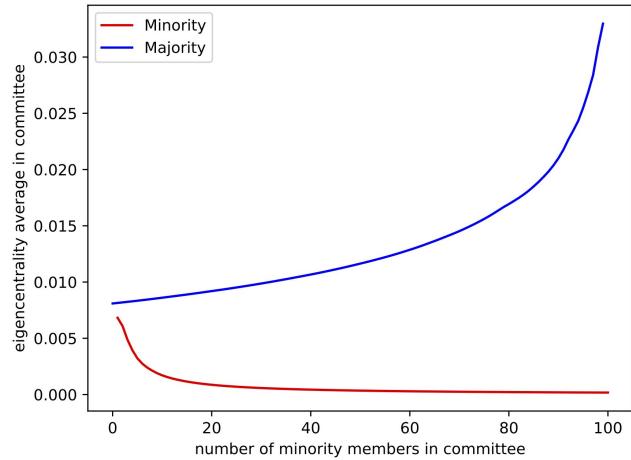


⇒ if the minority eigencentrality is very low, committee impact is the same

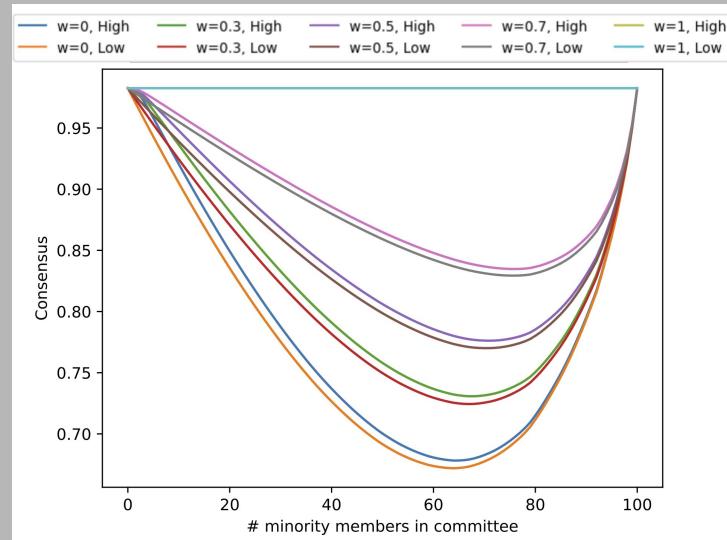


APS citation data, 1,281 nodes, 33% minority (two different fields of physics)

# What interventions can we enact?

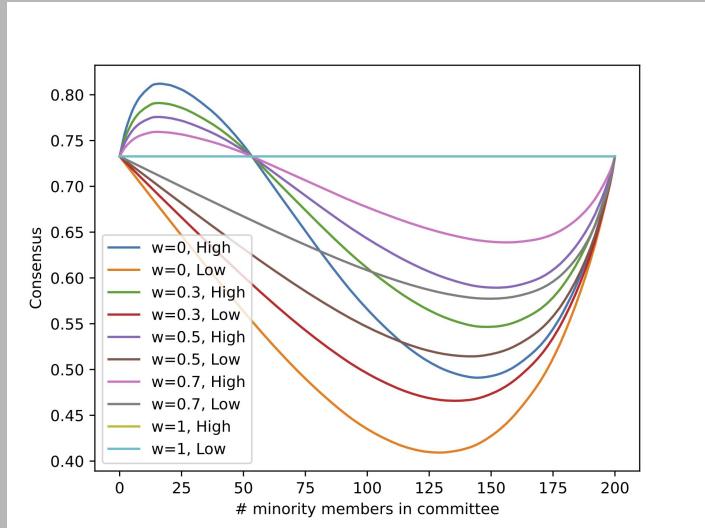
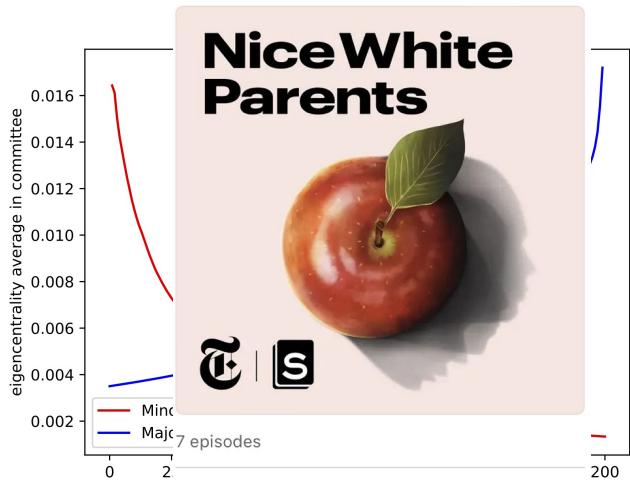


⇒ if the minority eigencentrality is very low, committee impact is the same



APS citation data, 1,281 nodes, 33% minority (two different fields of physics)

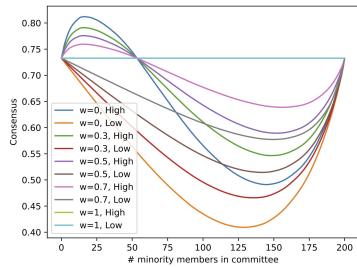
# What interventions can we enact?



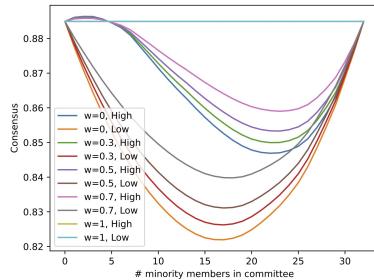
Add Health data: schools with different demographics: hispanic minority of 22% (out of ~1,100 students)

<https://addhealth.cpc.unc.edu/>

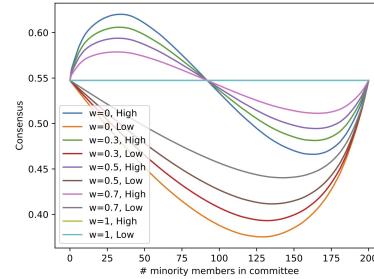
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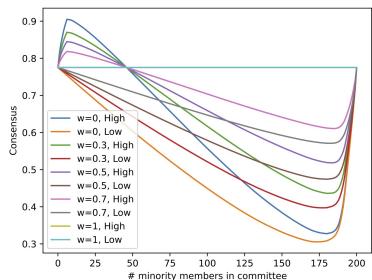
Hispanic minority of 22%



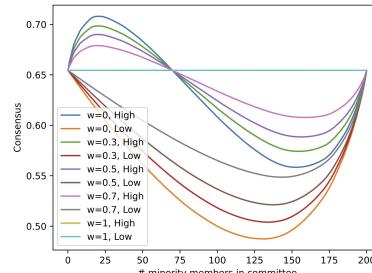
Black minority of 11%



Hispanic minority of 43%



Hispanic minority of 20%



Black minority of 34%

# Future directions

- Normative questions regarding interventions & policy implications
- Experimental analysis of opinion aggregation among different communities
  - Deliberation studies

# Future directions

- Normative questions regarding interventions & policy implications
- Experimental analysis of opinion aggregation among different communities
- Long-term implications of committees & incentives

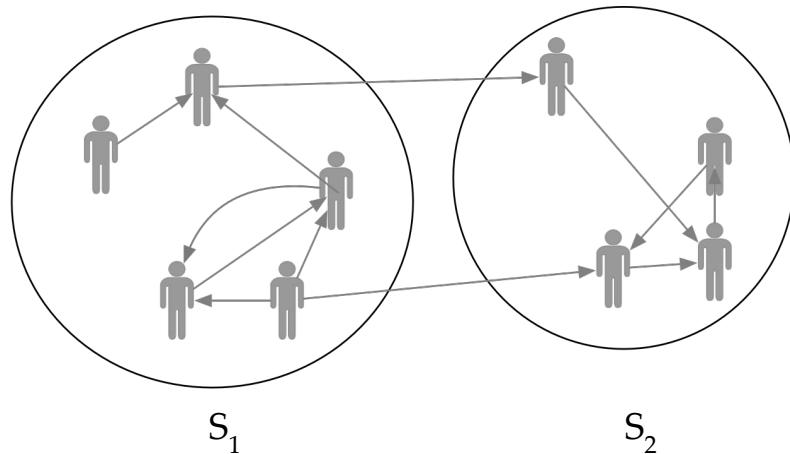
Thank you!

# Additional slides

# Utility and quality in clustering

## Cluster quality:

- Optimization problem that minimizes inter-group connections:
- $$L_{norm}(S_1, S_2) = \sum_{i \in S, j \in T} a_{ij} \left( \frac{1}{vol(S_1)} + \frac{1}{vol(S_2)} \right)$$
- Proxy: conductance
  - Algorithms: **spectral clustering**<sup>12</sup>

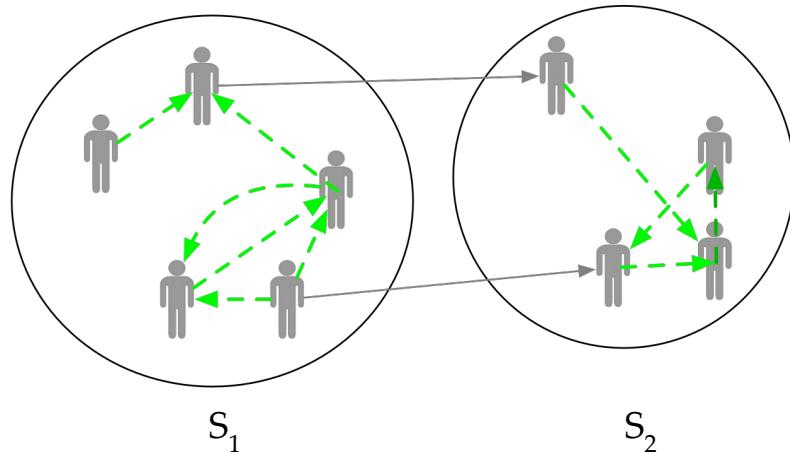


<sup>12</sup> Shi, Jianbo, and Jitendra Malik. "Normalized cuts and image segmentation." IEEE Transactions on pattern analysis and machine intelligence 22, no. 8: 888-905. 2000.

# Utility and quality in clustering

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- Optimization problem that minimizes inter-group connections:
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- Proxy: conductance
  - Algorithms: **spectral clustering**<sup>12</sup>



$$f(u) := \sum_{v \in N} \mathbb{P}((u, v) \in E'), \forall u \in N'$$

$E'$  is the set of edges within clusters

<sup>12</sup> Shi, Jianbo, and Jitendra Malik. "Normalized cuts and image segmentation." IEEE Transactions on pattern analysis and machine intelligence 22, no. 8: 888-905. 2000.

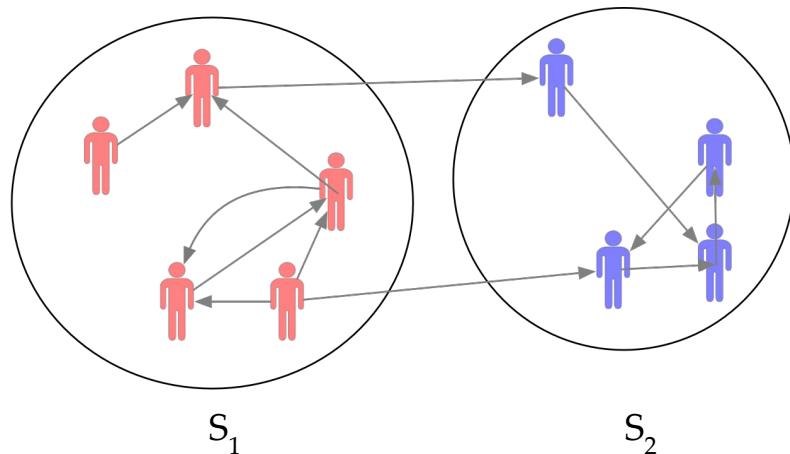
# Utility and quality in clustering

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What about proportional representation?

<sup>12</sup> Shi, Jianbo, and Jitendra Malik. "Normalized cuts and image segmentation." IEEE Transactions on pattern analysis and machine intelligence 22, no. 8: 888-905. 2000.

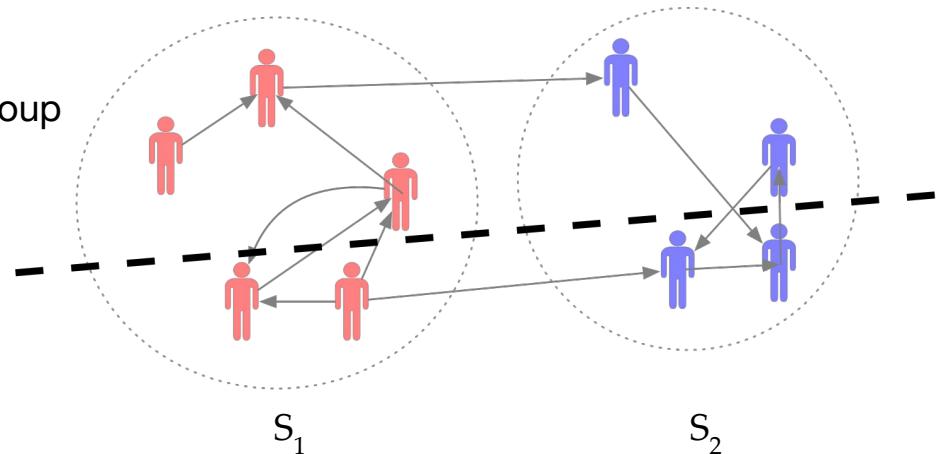
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- Proxy: conductance
- Algorithms: **spectral clustering**<sup>12</sup>



Proportionality as a constraint in SC<sup>13</sup>

$f(\text{red}) \rightarrow$

<sup>12</sup> Shi, Jianbo, and Jitendra Malik. "Normalized cuts and image segmentation." IEEE Transactions on pattern analysis and machine intelligence 22, no. 8: 888-905. 2000.

<sup>13</sup> Kleindessner, Matthäus, Samira Samadi, Pranjal Awasthi, and Jamie Morgenstern. "Guarantees for spectral clustering with fairness constraints." In ICML pp. 3458-3467. PMLR. 2019.

# Clustering with utility

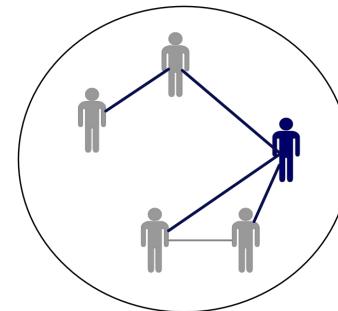
## Summary of results:

- Proposed a **new utility metric** that captures immediate connections and cluster cohesiveness (normalized  $f$ )

**Closeness utility:**  $cls(i, S) = \frac{deg_S(i)}{\sum_{j \in S} dist_G(i, j)}$



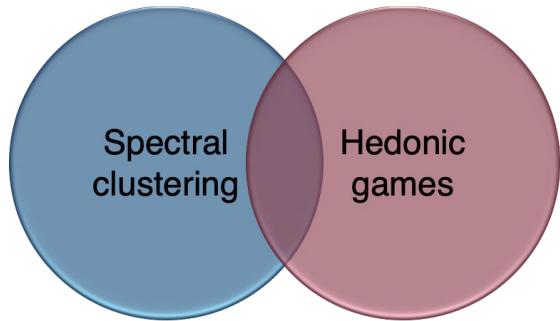
Equilibrium



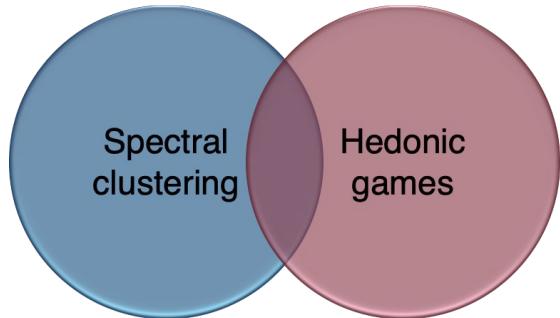
$$cls(i, S) = \frac{3}{5}$$

- Theoretical results:
  - characterizing the **structure** of a Pareto optimal partition
  - Polytime algs:** A welfare optimal partition can be found in polynomial time
  - Bounds on Price of Pareto Optimality:** at most 4/3, and there are graphs for which the PPO is arbitrarily close to 4/3
- Empirical methods for comparing with optimization-based clustering:
  - Clustering games

# Clustering with utility



# Clustering with utility



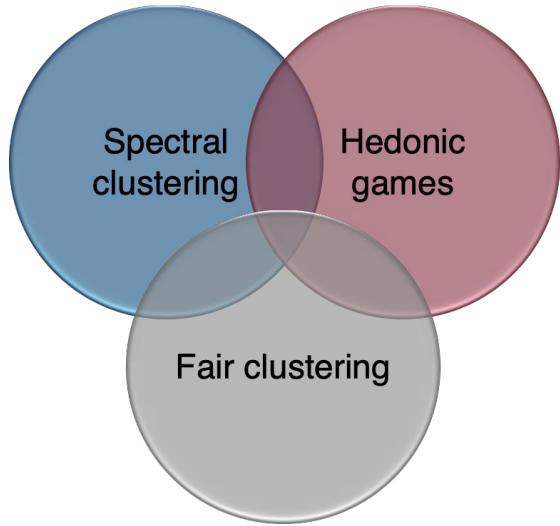
Closeness utility:

⇒ better conductance at equilibrium than other competing metrics

## Clustering Games:

- Fixed number of clusters  $k$
- Initial partition
- Allow nodes to choose clusters until Nash equilibrium

# Clustering with utility



## Clustering Games:

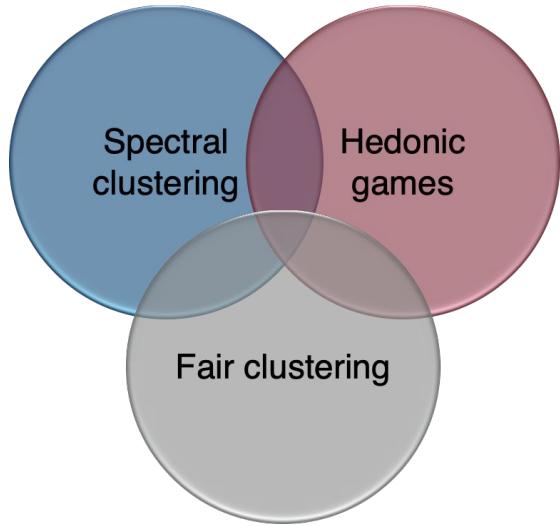
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Closeness utility:

- ⇒ better conductance at equilibrium than other competing metrics
- ⇒ improves diversity in real data

$$\text{balance}(C) = \min \left( \frac{\#R \text{ in } C}{\#B \text{ in } C}, \frac{\#B \text{ in } C}{\#R \text{ in } C} \right)$$

# Clustering with utility



## Clustering Games:

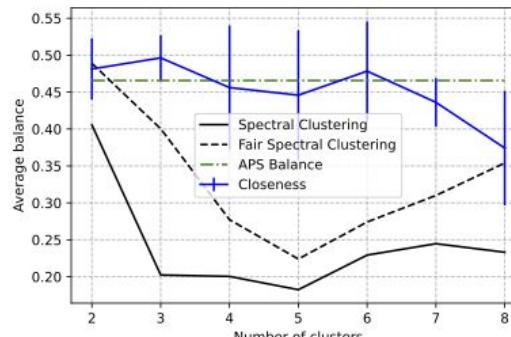
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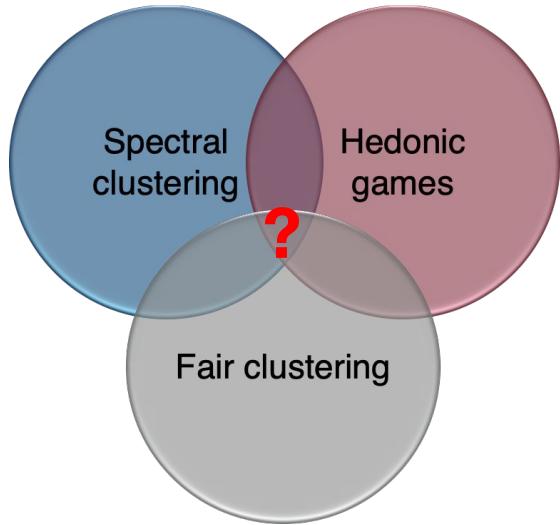
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APS citation data

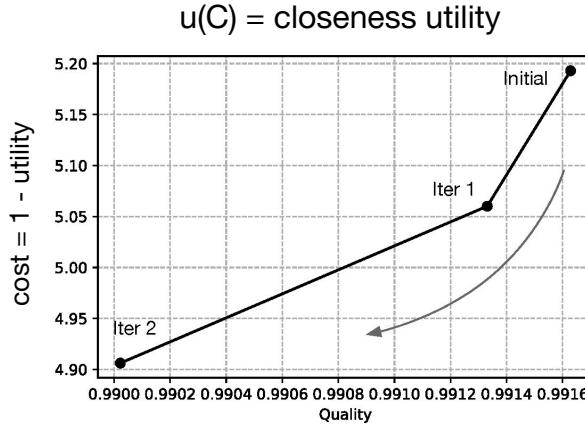


# Clustering with utility



Clustering Games:

- Fixed number of clusters  $k$
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- Allow nodes to choose clusters until Nash equilibrium



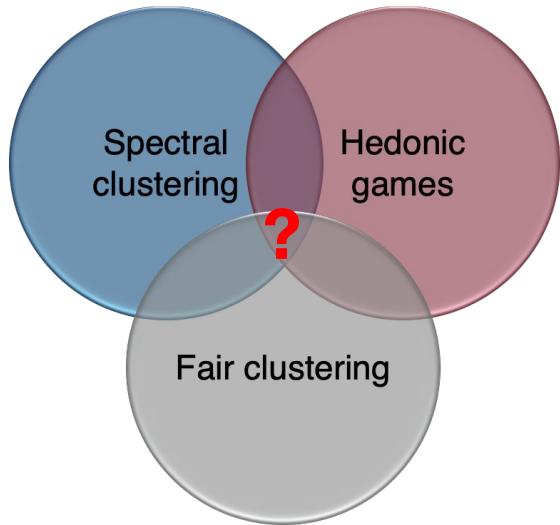
**Polynomial time algorithm for tracing  
the quality-utility trade-off**

<sup>14</sup> Stoica and Papadimitriou. "Strategic clustering." In submission. 2022.

<sup>15</sup> Golitschek, M.v. "Optimal cycles in doubly weighted graphs and approximation of bivariate functions by univariate ones". Numerische Mathematik 39 (1), 65–84. 1982.

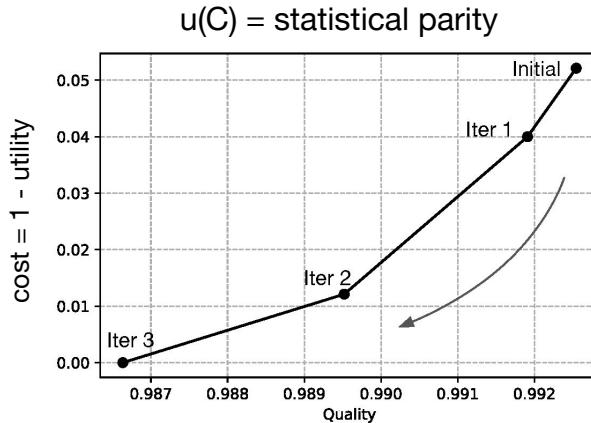
<sup>16</sup> Lawler, E.L. "Optimal cycles in doubly weighted directed linear graphs". In Proceedings of the International Symposium of Theory of Graphs. 209–232. 1966.

# Clustering with utility



Clustering Games:

- Fixed number of clusters  $k$
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## Recommendation

**Algorithmic glassceiling in social networks: the effect of recommendation on network diversity** [*with C. Riederer and A. Chaintreau, WWW'18*]

## Information Campaigns

**Seeding network influence and the benefits of diversity** [*with J.X. Han and A. Chaintreau, WWW'20*]

## Grouping

**Minimizing Margin of Victory for Fair Political and Educational Districting** [*with A. Chakraborty, P. Dey, and K. P. Gummadi, AAMAS'20*]

## Clustering

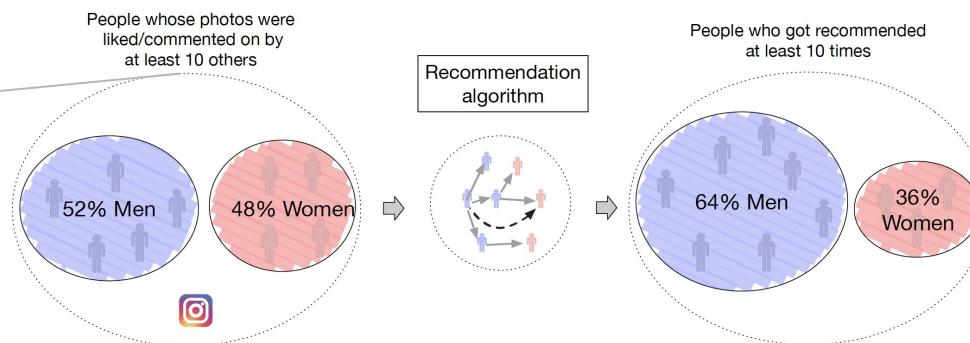
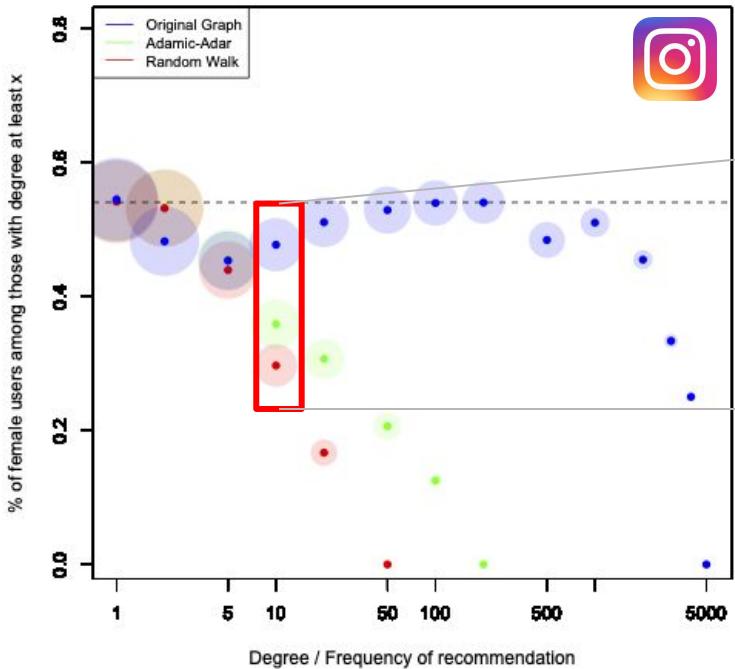
**Strategic clustering** [*with C. Papadimitriou, in submission*]

## Spectral embeddings

**Bias reflection in spectral properties** [*with A. Chaintreau, ongoing*]

# Distributional inequality in social recommendations

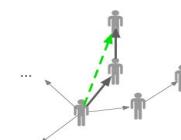
Instagram activity graph of likes and comments



Adamic Adar index:

$$A(x, y) = \sum_{u \in N(x) \cap N(y)} \frac{1}{\log |N(u)|}$$

Random walk:



<sup>1</sup> Stoica, Ana-Andreea, Christopher Riederer, and Augustin Chaintreau. "Algorithmic Glass Ceiling in Social Networks: The effects of social recommendations on network diversity." In Proceedings of the 2018 World Wide Web Conference, pp. 923-932. 2018.

# Model for biased networks

Biased preferential attachment model:

- Minority-majority: blue ( $B$ ) label and red ( $R$ ) label (% of red nodes  $< \frac{1}{2}$ )
- Rich-get-richer: nodes connect w.p. proportional to degree
- Homophily: if different labels, connection is accepted with a certain probability  
⇒ known to exhibit inequality in the degree distribution of the two communities<sup>3</sup>

$$top_k(R) \sim k^{-\beta(R)}$$

$$top_k(B) \sim k^{-\beta(B)}$$

$$\beta(R) > 3 > \beta(B)$$

Necessary and sufficient conditions: **groups, homophily, preferential attachment**

<sup>3</sup>Avin, Chen, et al. "Homophily and the glass ceiling effect in social networks." ITCS. 2015.

# Degree distribution

Organic growth:

$$\text{top}_k(\mathbf{R}) \sim k^{-\beta(R)}$$

$$\text{top}_k(\mathbf{B}) \sim k^{-\beta(B)}$$

Recommendation model:

$$\text{top}'_k(\mathbf{R}) \sim k^{-\beta_{rec}(R)}$$

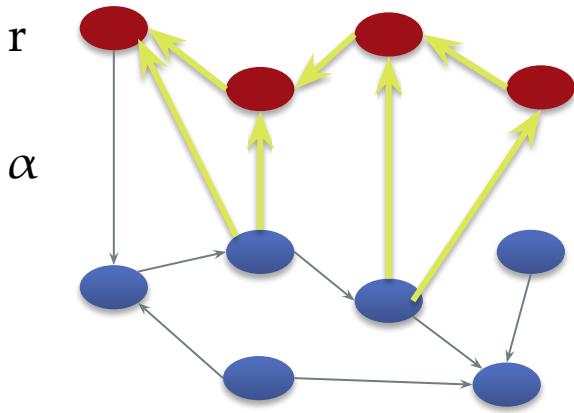
$$\text{top}'_k(\mathbf{B}) \sim k^{-\beta_{rec}(B)}$$

**Theorem:** For  $0 < r < \frac{1}{2}$  and  $0 < \rho < 1$ , for the graph sequences  $G(n)$  for the organic model and  $G'(n)$  for the recommendation model, the red and blue populations exhibit a power law degree distribution with coefficients:

$$\begin{array}{c} \beta_{rec}(\mathbf{R}) > \beta(\mathbf{R}) > 3 > \beta(\mathbf{B}) > \\ \beta_{rec}(\mathbf{B}) \end{array}$$

gap

# Proof sketch



'Wealth' of red nodes:

- Fraction of edges towards R

$$\alpha_t = \sum_{v \in R} \text{in deg}(v) / t$$

Define a function F as the rate of growth of  $\alpha_t$

- F has a fixed point  $a \Rightarrow \alpha_t \rightarrow \alpha < r$

Organic growth

$\alpha$

>

Recommendation model

$\alpha'$

# Proof sketch

Evolution equation:

- When does a node of degree  $k$  get a new link

Randomly

Preferential attachment

$T_t^R$  = rate at which R nodes receive edges through **randomness**

$k \cdot C_t^R$  = rate at which R nodes receives edges through **preferential attachment**

$$top_k(\mathbf{R}) \sim k^{-\beta(R)}$$

$$\beta(R) = 1 + \frac{1}{C^R}$$

$$top_k(\mathbf{B}) \sim k^{-\beta(B)}$$

$$\beta(B) = 1 + \frac{1}{C^B}$$

# Proof sketch

Goal: compute evolution equation and close it down to an solution...

$$\beta_{rec}(R) > \rho(B) > \beta_{rec}(B)$$

Big mess!



Key idea: at equilibrium, the rate at which red edges appear must equal the current fraction of red edges, as it does not evolve anymore

**Invariant equation modeling asymptotic dynamics of degree distribution**

# Invariant equation

Organic growth:

$$\alpha \cdot C^R + r \cdot T^R = \alpha$$

Recommendation model:

$$\alpha' \cdot C'^R + r \cdot T'^R = \alpha'$$

$$\alpha > \alpha' \Rightarrow C^R > C'^R \Rightarrow \beta'(R) > \beta(R)$$



$$\beta'(R) > \beta(R) > 3 > \beta(B) > \beta'(B)$$

# Degree distribution

Organic growth:

$$top_k(R) \sim k^{-\beta(R)}$$

$$top_k(B) \sim k^{-\beta(B)}$$

Recommendation model:

$$top_k'(R) \sim k^{-\beta_{rec}(R)}$$

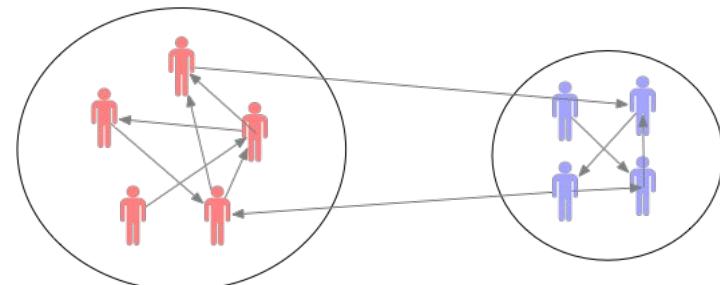
$$top_k'(B) \sim k^{-\beta_{rec}(B)}$$

Majority has degree advantage + homophily:

$$\beta_{rec}(R) > \beta(R) > 3 > \beta(B) > \beta_{rec}(B)$$

Minority has degree advantage + homophily:

$$\beta_{rec}(B) > \beta(B) > 3 > \beta(R) > \beta_{rec}(R)$$



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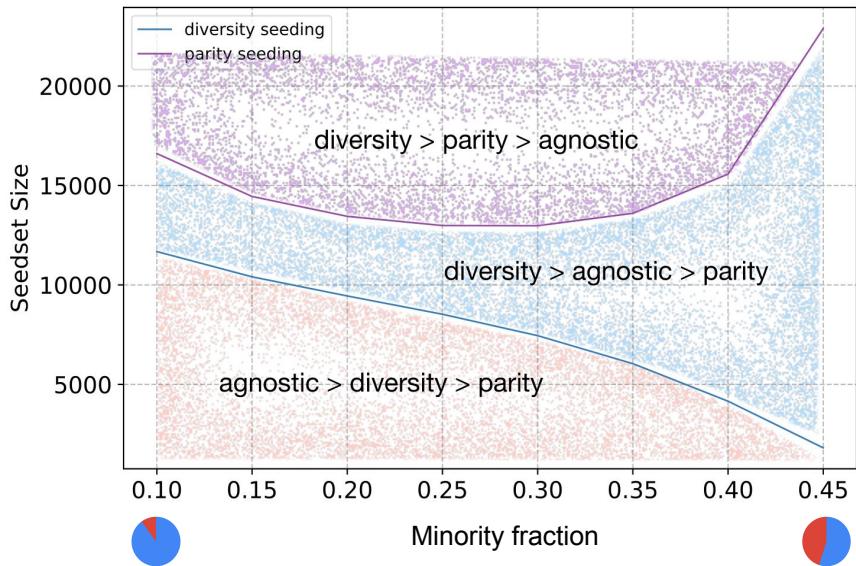
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# Theoretical analysis of diversity interventions



Network of ~53,000 nodes, 2 communities, homophily  $\rho = 0.135$

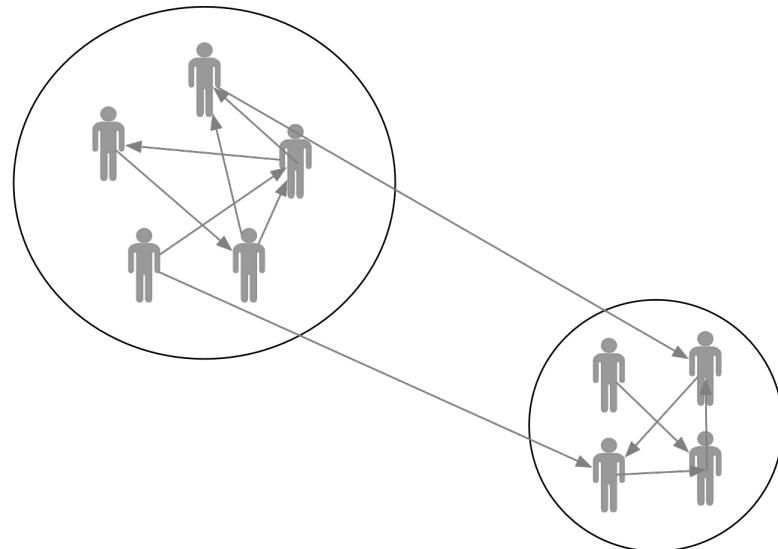
- Compute regions where each heuristic performs better than the agnostic one
- As communities become more equal, need fewer seeds for diversity heuristic to be more efficient
- Not the same thing happens with the parity heuristic!

# Social Influence Maximization

Summary of results:

- **Our vision:** a biased system is an inefficient system

Awareness of community affiliation can identify when centrality is inefficient (gets overlapping spheres of influence)

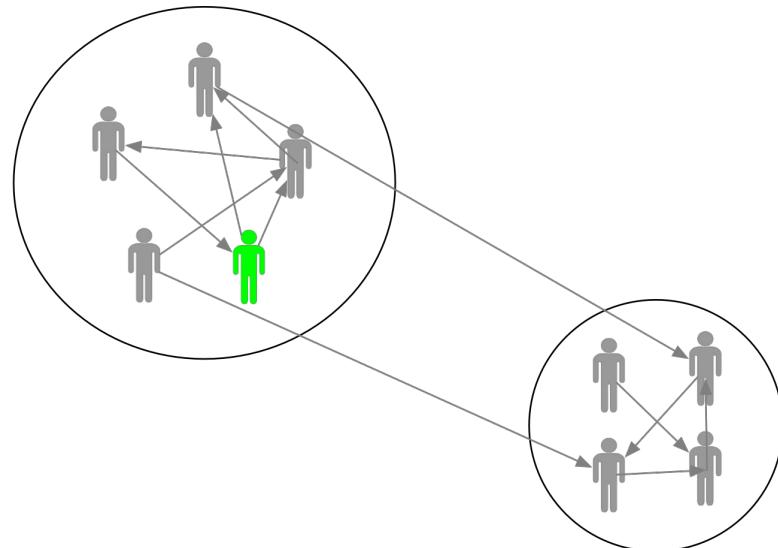


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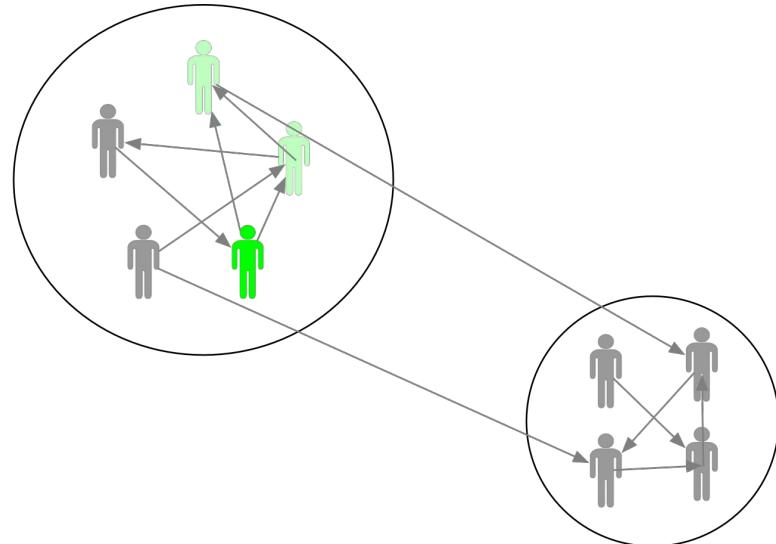


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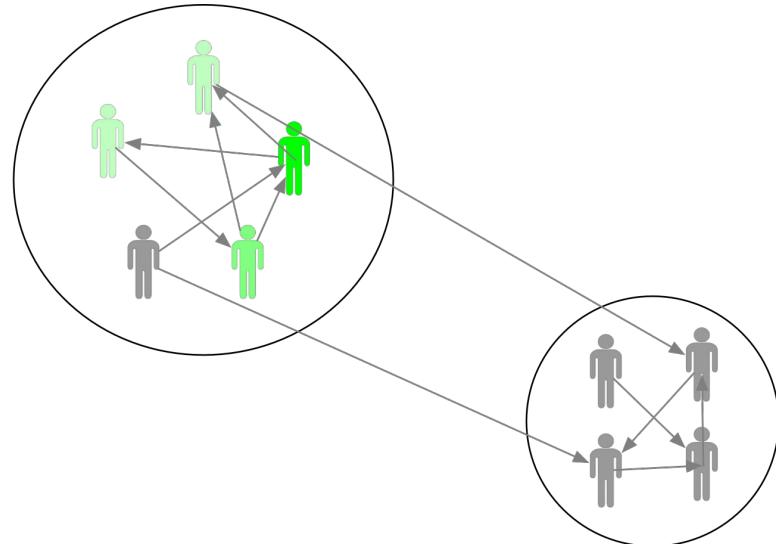


# Social Influence Maximization

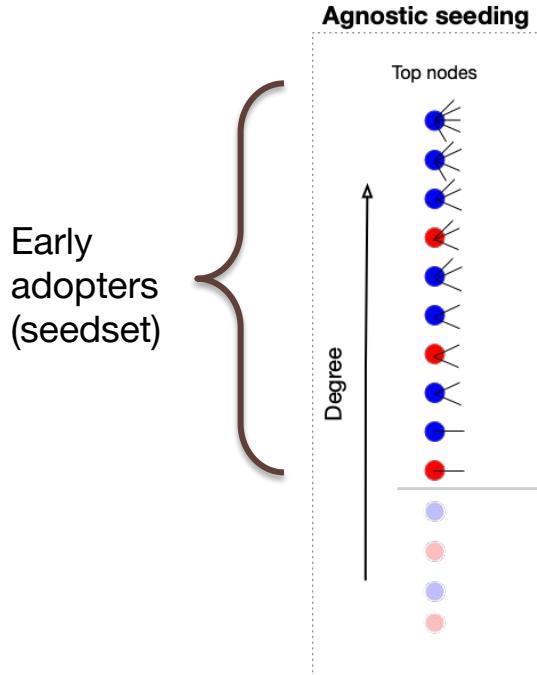
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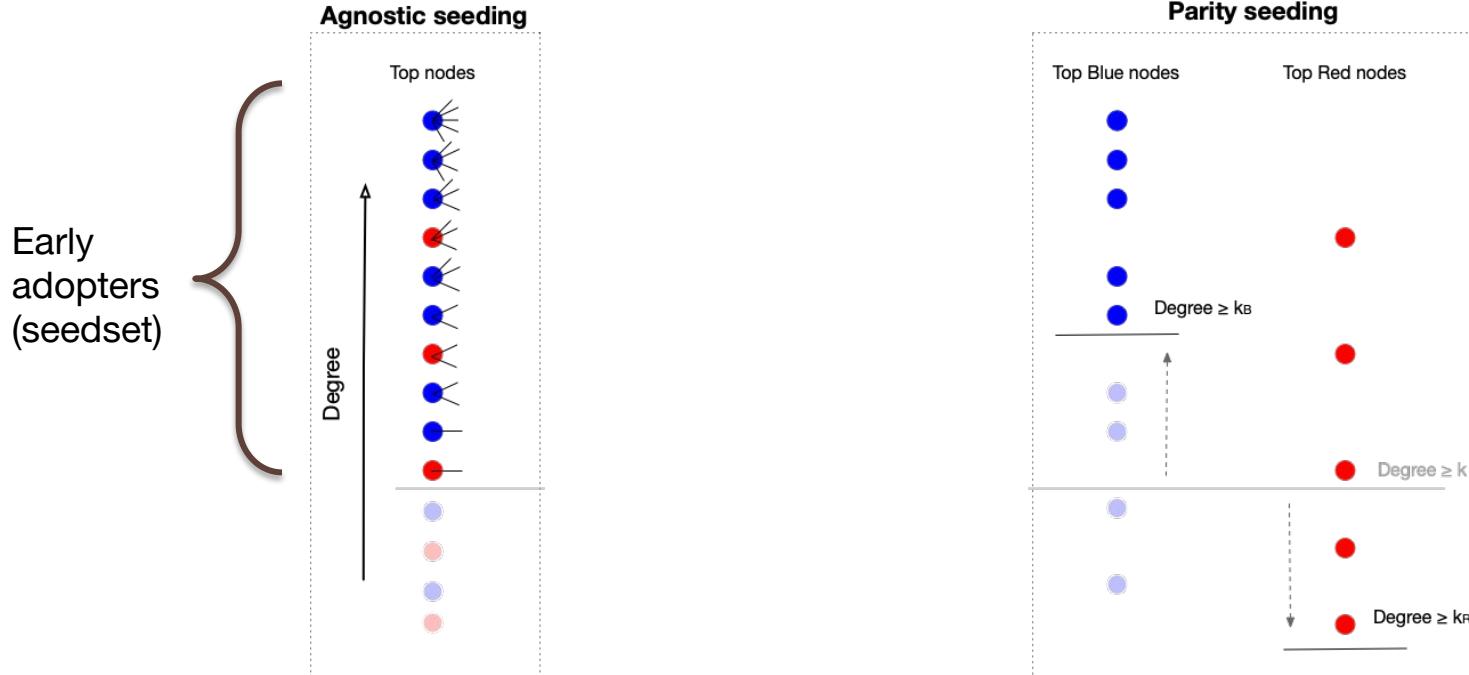
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# Color-agnostic v. Diversity Seeding



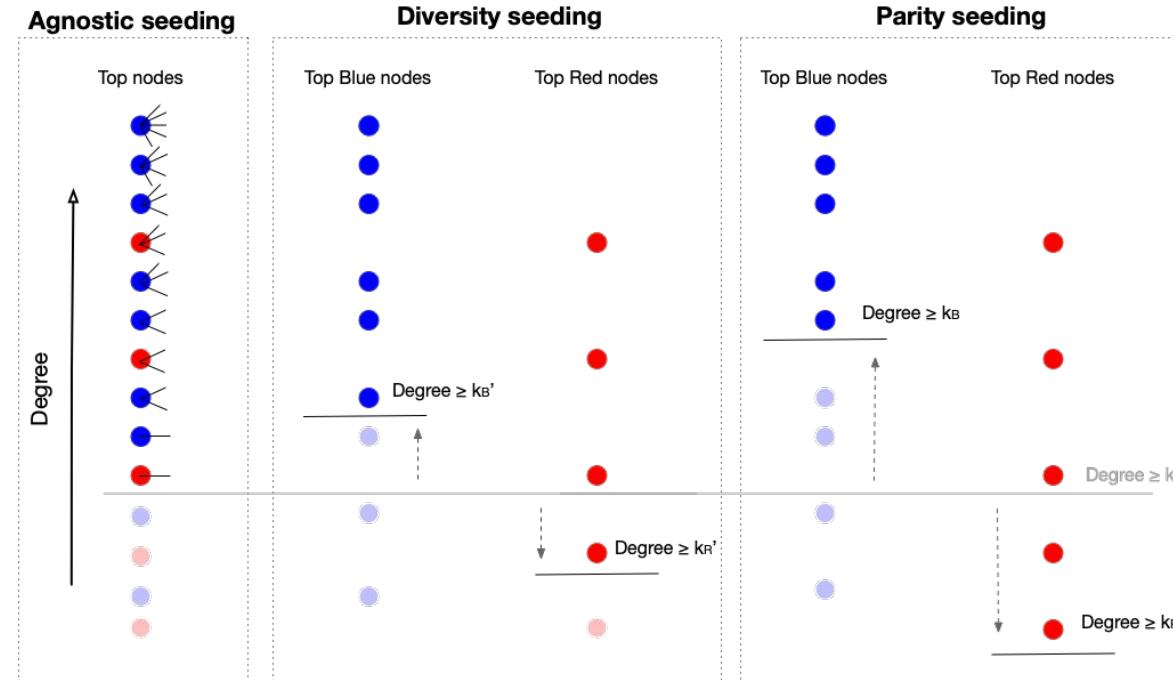
# Color-agnostic v. Diversity Seeding



Keeping the same budget!

# Color-agnostic v. Diversity Seeding

Early adopters  
(seedset)



Keeping the same budget!

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**Strategic clustering** [with C. Papadimitriou, in submission]

## Spectral embeddings

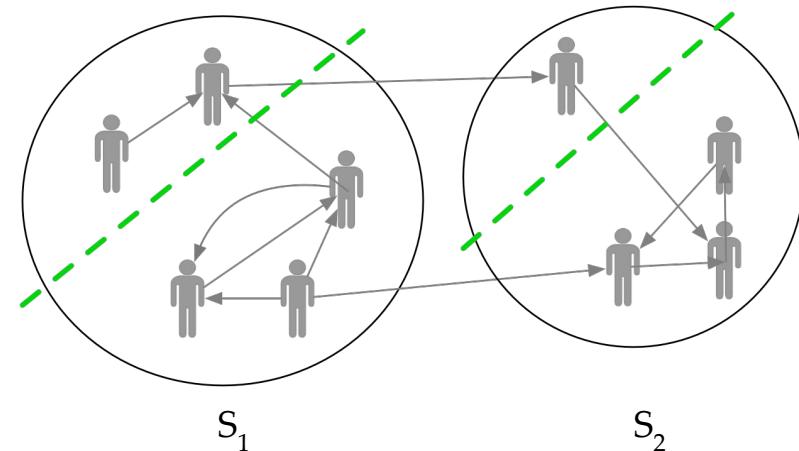
**Bias reflection in spectral properties** [with A. Chaintreau, ongoing]

# Utility and quality in clustering

Cluster utility:

- Hedonic games
  - Individual utility inducing preferences over clusters as an order  $\succ_i$
  - Natural cluster formation through equilibrium conditions
  - Nash equilibrium:

$$\forall i, S_{\Pi}(i) \succ_i S_k \cup \{i\} \forall S_k \in \Pi \cup \{\emptyset\}$$

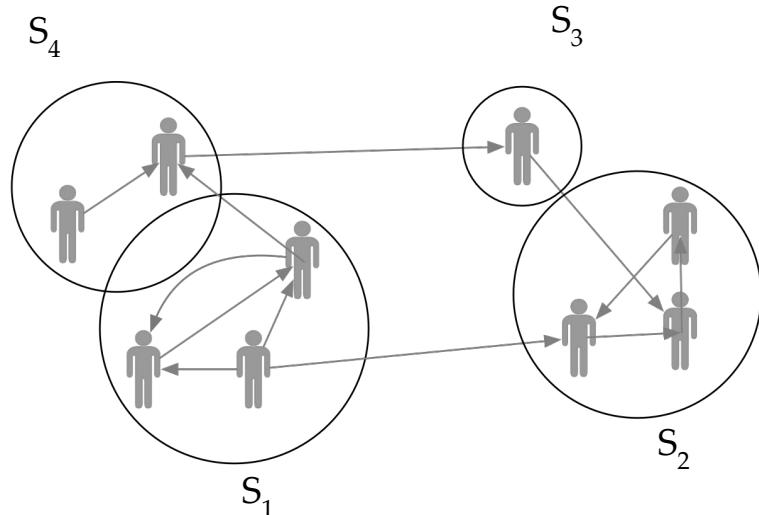


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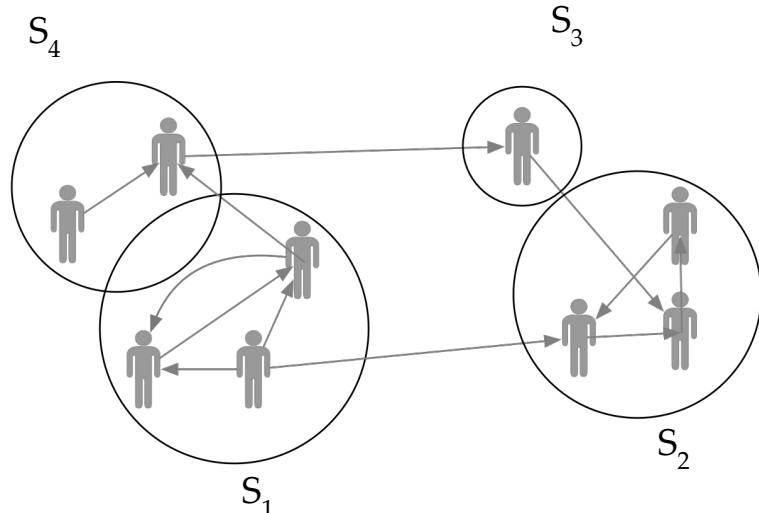


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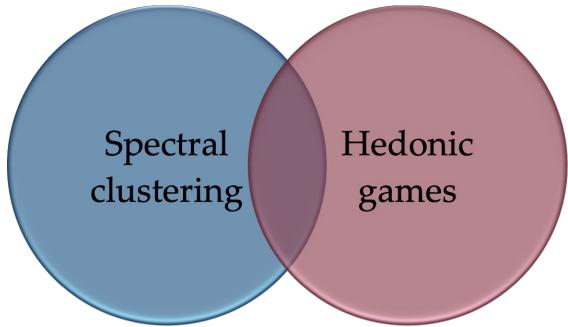
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Are groups stable and follow the incentives of their members?

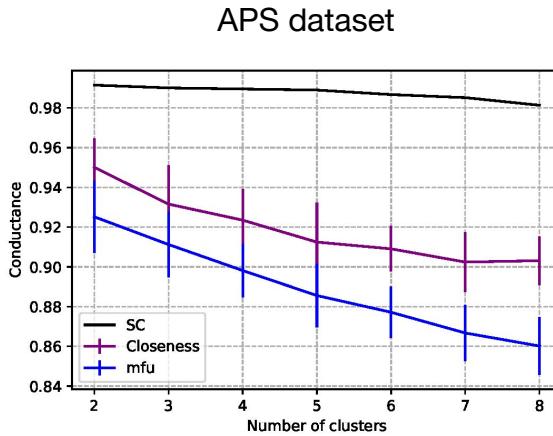
# Clustering games



Clustering Games:

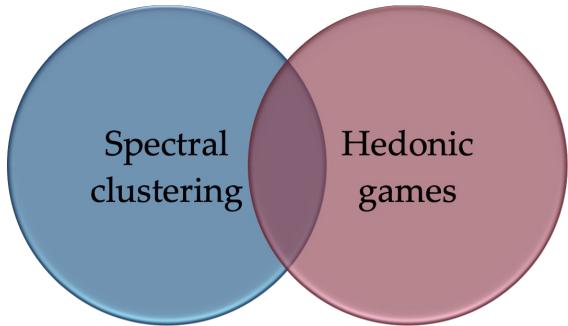
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- Allow nodes to choose clusters until Nash equilibrium

Games on real data:



⇒ closeness utility leads to better conductance

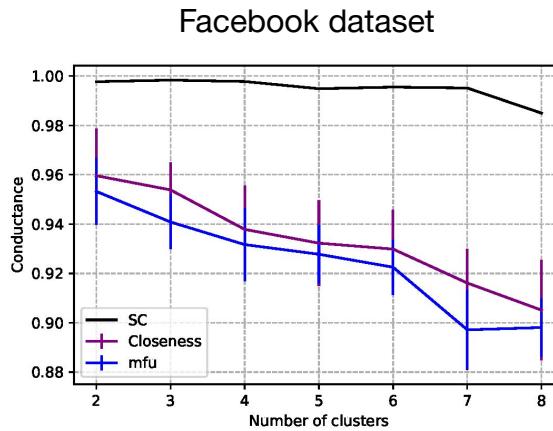
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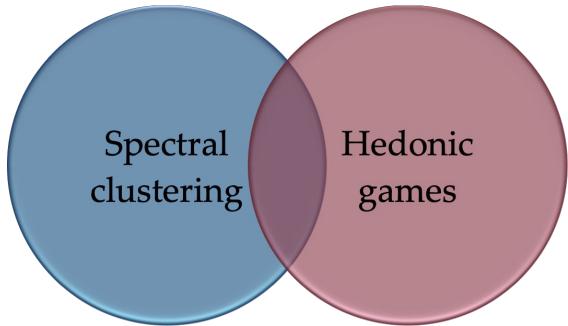
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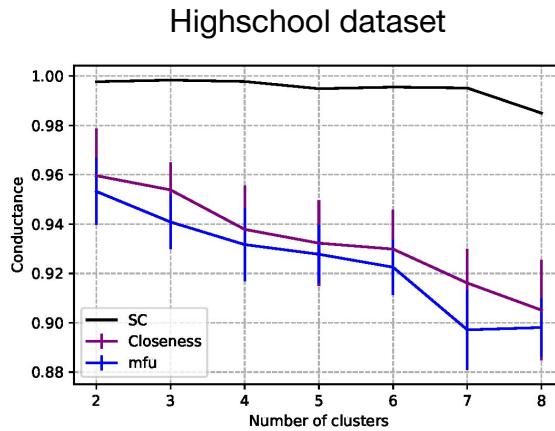
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# Tracing the quality-utility trade-off

**Poly-time algorithm** for tracing the quality-welfare

tradeoff boundary:

- For a clustering  $C$ , quality metric  $q(C)$ , utility metric  $f(C)$
- Goal: starting with the optimal  $C$  in quality, find the next  $C'$  which results in the largest ratio of increment in utility over the decrement in quality:

$$C' = \operatorname{argmax}_{C' \in \operatorname{changes}(C)} \frac{f(C') - f(C)}{q(C) - q(C')}$$

<sup>16</sup> Golitschek, M.v. "Optimal cycles in doubly weighted graphs and approximation of bivariate functions by univariate ones". *Numerische Mathematik* 39 (1), 65–84. 1982.

<sup>17</sup> Lawler, E.L. "Optimal cycles in doubly weighted directed linear graphs". In *Proceedings of the International Symposium of Theory of Graphs*. 209–232. 1966.

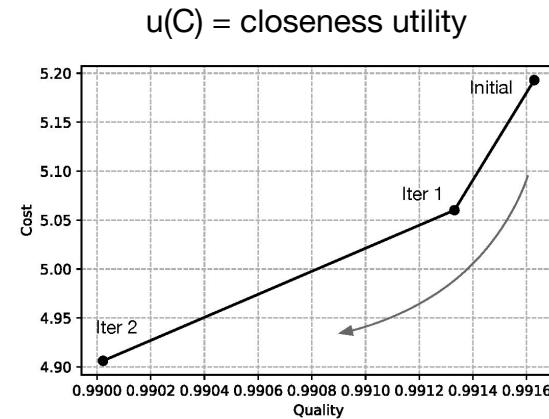
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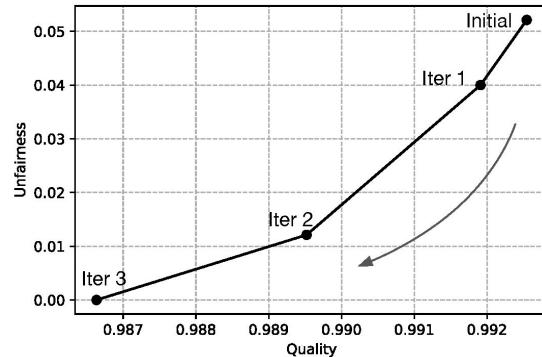
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$u(C) = \text{statistical parity}$



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# Overview of Thesis Research Projects

## Recommendation

**Algorithmic glassceiling in social networks: the effect of recommendation on network diversity** [with C. Riederer and A. Chaintreau, WWW'18]

## Information Campaigns

**Seeding network influence and the benefits of diversity** [with J.X. Han and A. Chaintreau, WWW'20]

## Grouping

**Minimizing Margin of Victory for Fair Political and Educational Districting** [with A. Chakraborty, P. Dey, and K. P. Gummadi, AAMAS'20]

## Clustering

**Strategic clustering** [with C. Papadimitriou, in submission]

## Spectral embeddings

**Bias reflection in spectral properties** [with A. Chaintreau, ongoing]

# Bias amplification in spectral embeddings

Summary of results:

- Theoretical analysis for when average PageRank (PR) for two communities reflects network ‘hierarchy’  $\Rightarrow$  generalizable method for a variety of classes and network models
- Quantifying bias amplification / reduction: depends on the ranking algorithm used

