GRAVITATIONAL LENSING

6 – TIME DELAYS (2), IMAGE PARITY

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TOTAL TIME DELAY

$$
\begin{aligned}
t_{tot} &= t_{geom} + t_{grav} \\
&= \frac{1}{2c}\frac{D_S D_L}{D_{LS}}(\theta - \beta)^2 - \frac{1}{c}\frac{D_S D_L}{D_{LS}}\hat{\Psi}(\theta) \\
&= \frac{1}{c}\frac{D_S D_L}{D_{LS}}\left[\frac{1}{2}(\theta - \beta)^2 - \hat{\Psi}(\theta)\right]
\end{aligned}
$$

Accounting for the expansion of the universe and for the fact that this is a surface:

$$
t_{tot}(\vec{\theta}) = \frac{1+z_L}{c}\frac{D_S D_L}{D_{LS}}\left[\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \hat{\Psi}(\vec{\theta})\right]
$$

TOTAL TIME DELAY

$$t_{tot}(\vec{\theta}) = \frac{1+z_L}{c}\frac{D_S D_L}{D_{LS}}\left[\frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \hat{\Psi}(\vec{\theta})\right]$$

$$\tau(\vec{\theta}) = \frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \hat{\Psi}(\vec{\theta})$$

Fermat potential

$$D_{\Delta t} = (1+z_L)\frac{D_S D_L}{D_{LS}}$$

Time delay distance

TIME DELAY SURFACE

$$t(\vec{\theta}) = t_{geom} + t_{grav} \propto \left(\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \hat{\Psi} \right)$$

$$\vec{\nabla} t(\vec{\theta}) \propto \left(\vec{\theta} - \vec{\beta} - \vec{\nabla}\hat{\Psi} \right)$$

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Lens equation!

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Images form at the stationary points of t!

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$$T_{ij} = \frac{\partial^2 t(\vec{\theta})}{\partial\theta_i \partial\theta_j} \propto (\delta_{ij} - \Psi_{ij})$$

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Lens equation!

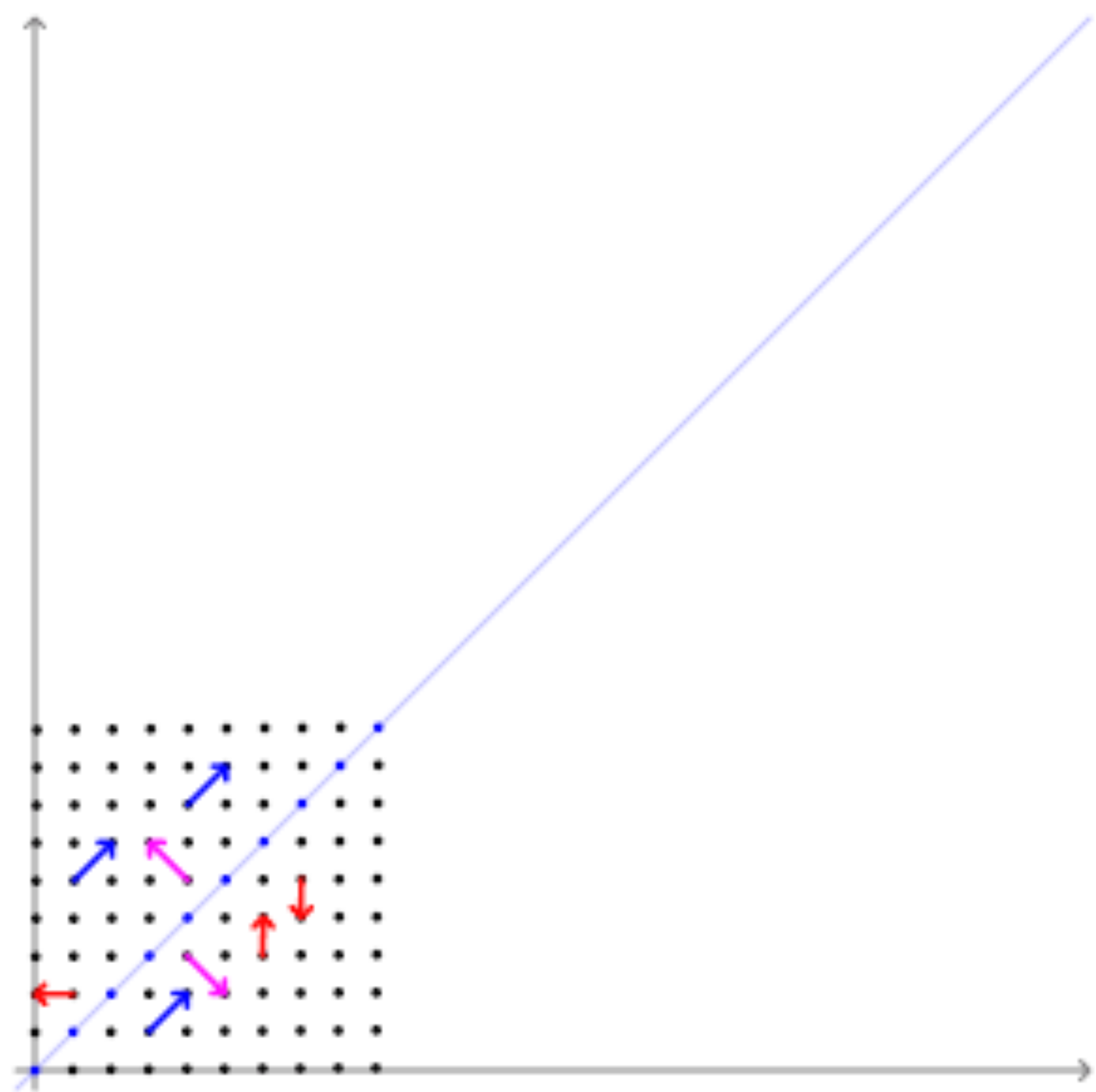
Images form at the stationary points of t!

$$T_{ij} = \frac{\partial^2 t(\vec{\theta})}{\partial\theta_i \partial\theta_j} \propto (\delta_{ij} - \Psi_{ij})$$ ***This is the Jacobian!***

TYPES OF IMAGES

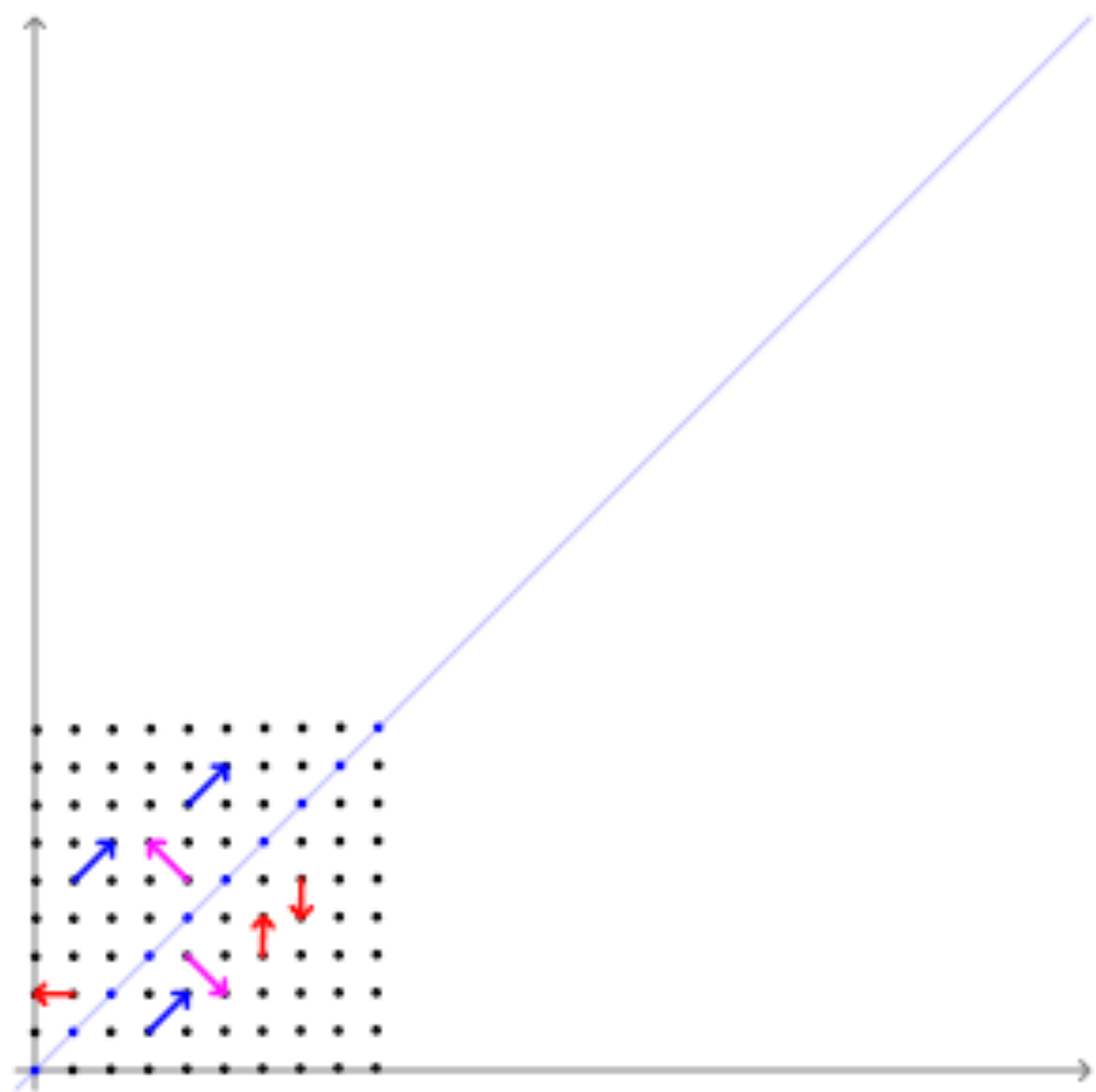
- minima (eigenvalues of A are both positive, hence $\det A>0$ and $\mathrm{Tr}\, A>0$; positive magnification)
- saddle (eigenvalues have opposite signs, thus $\det A<0$; negative magnification)
- maxima (eigenvalues are both negative, hence $\det A>0$ and $\mathrm{Tr}\, A<0$; positive magnification)

POSITIVE AND NEGATIVE MAGNIFICATION



Lens mapping described by the lensing Jacobian: maps squares into parallelograms

POSITIVE AND NEGATIVE MAGNIFICATION



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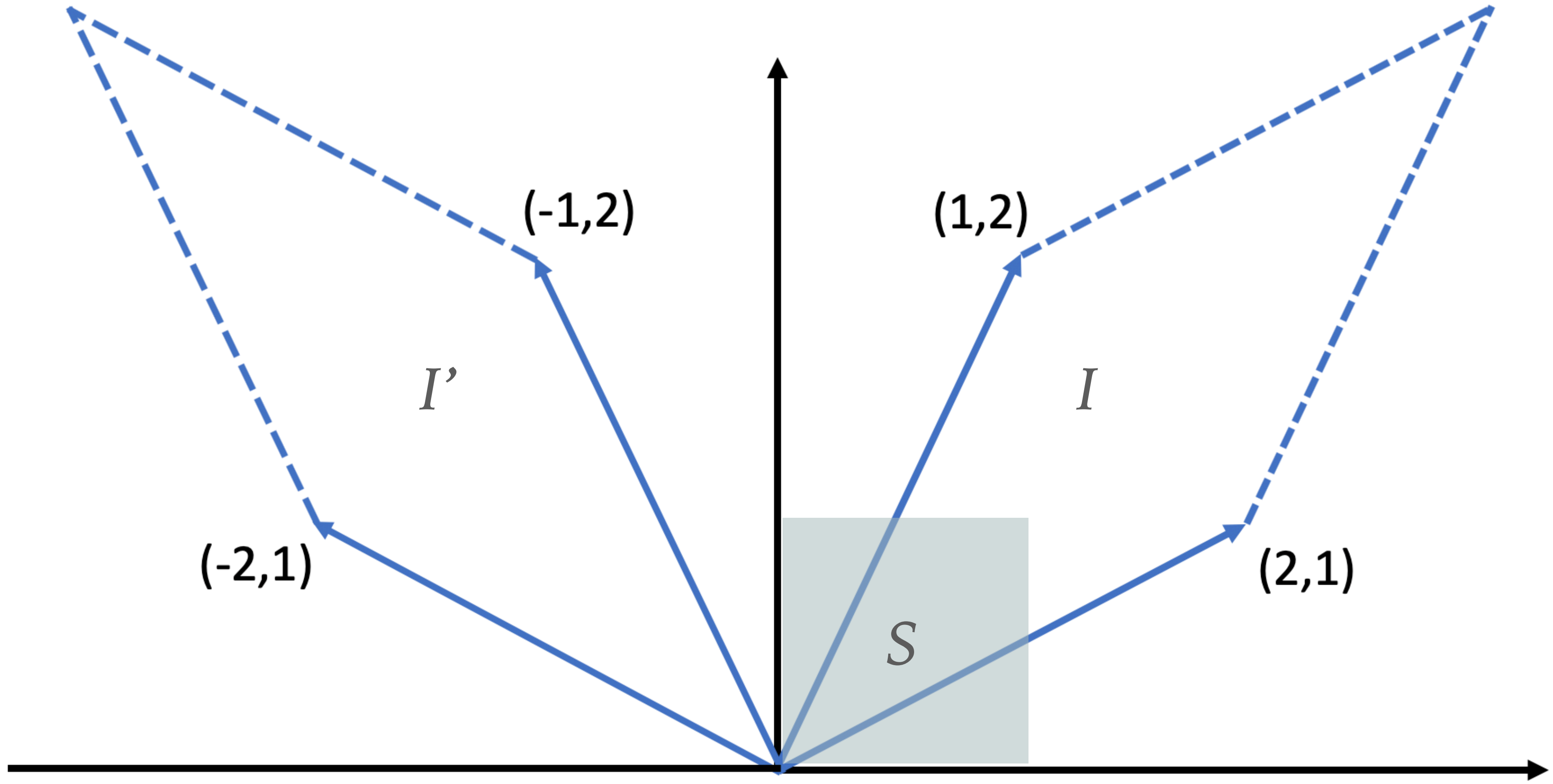


$$I' = \det\begin{pmatrix} -2 & 1 \\ -1 & 2 \end{pmatrix} = -3 \qquad I = \det\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = 3$$

POSITIVE AND NEGATIVE MAGNIFICATION

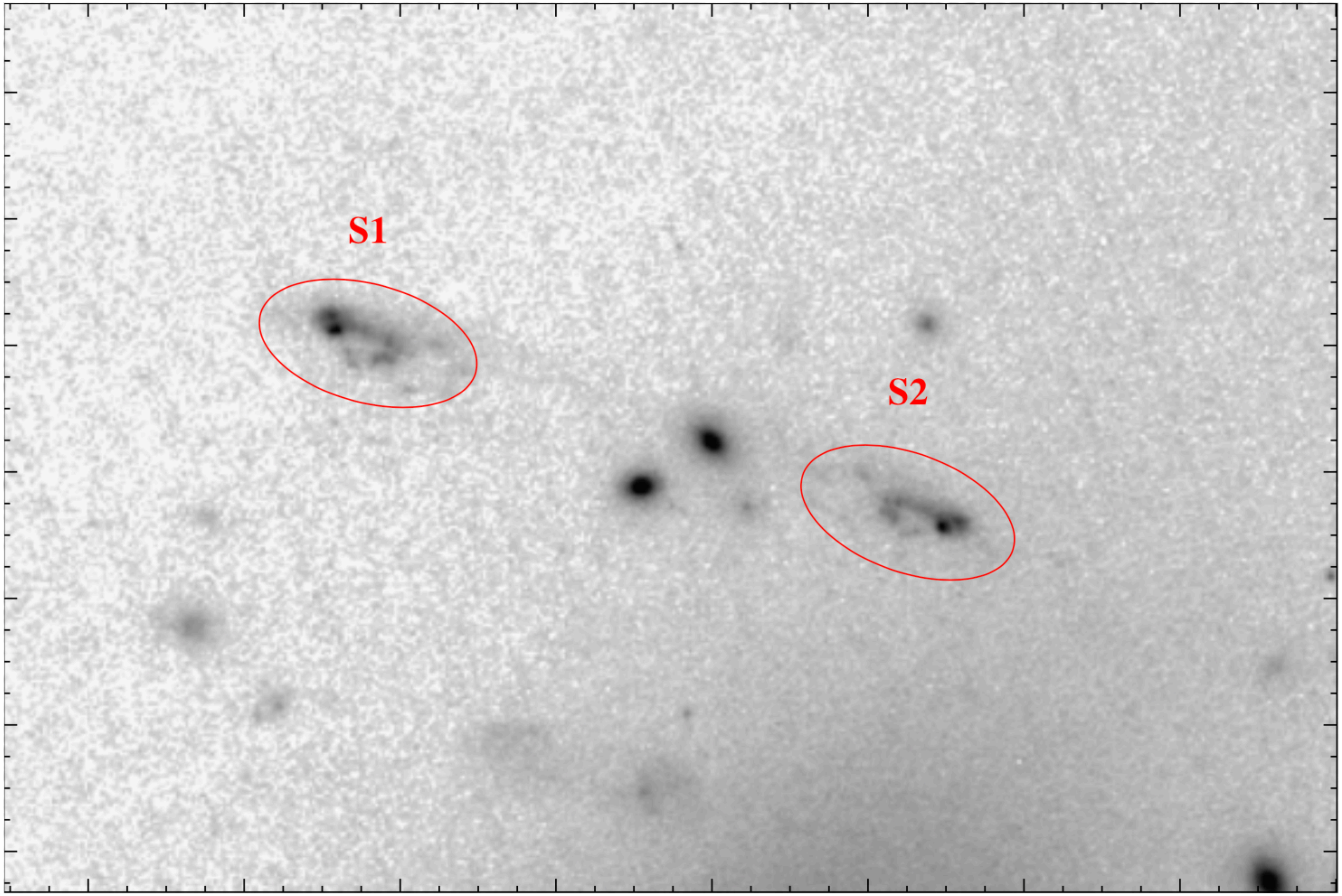


Fig. 10 The lensed pair S1–S2 in AC114. This galaxy at $z = 1.867$ displays the surprising morphology of a hook, with an obvious change in parity (Smail et al. 1995; Campusano et al. 2001)

Kneib & Natarajan 2012

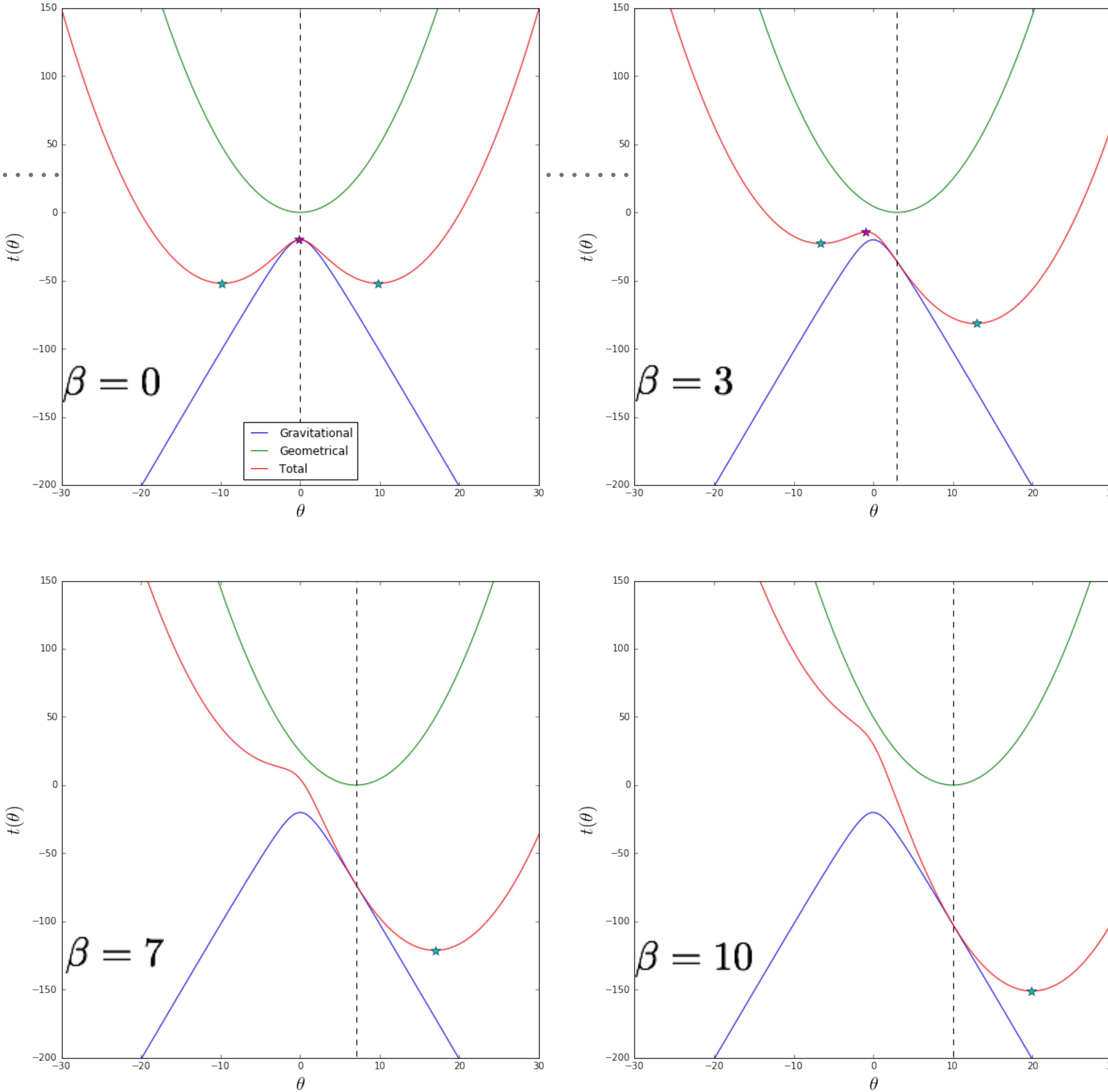
EXAMPLE OF TIME DELAY SURFACE

Toy potential:

$$\hat{\Psi}(\theta) = K\sqrt{\theta^2 + \theta_c^2}$$

Assuming axial-symmetry, we can discuss the time-delay function instead of the time delay surface.

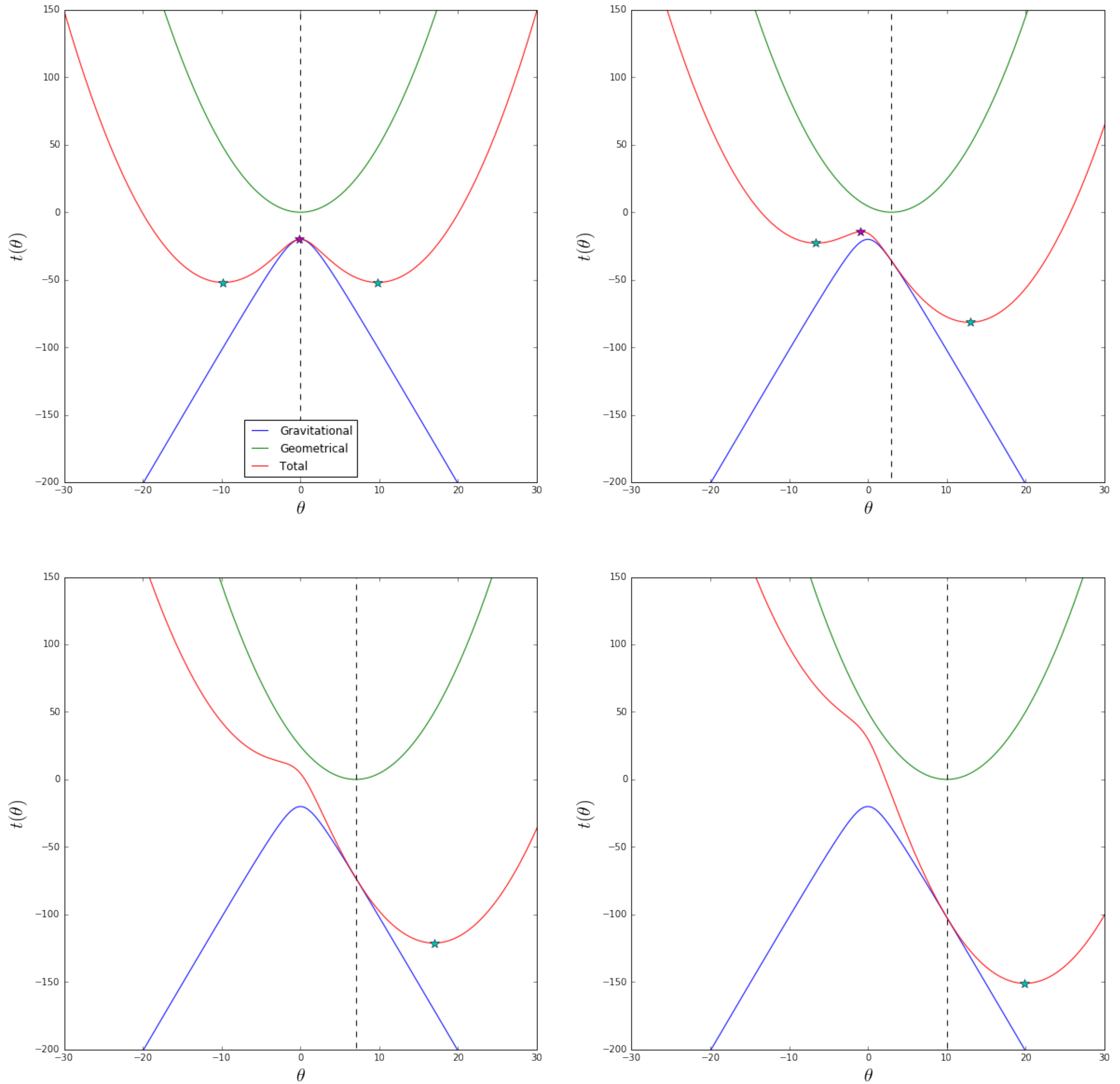
$$t(\theta) \propto \frac{1}{2}(\theta - \beta)^2 - K\sqrt{\theta^2 + \theta_c^2}$$



$\beta = 0$
$\beta = 3$
$\beta = 7$
$\beta = 10$
Gravitational
Geometrical
Total
$t(\theta)$
θ

SOME INTERESTING PROPERTIES

- image multiplicity depends on the relative position of lens and source
- couples of images disappear after approaching each other
- the time-delay function is flat when this happens!
- det A=0 means infinite magnification: the images disappear on the critical lines!
- this happens every time a source crosses a caustic!

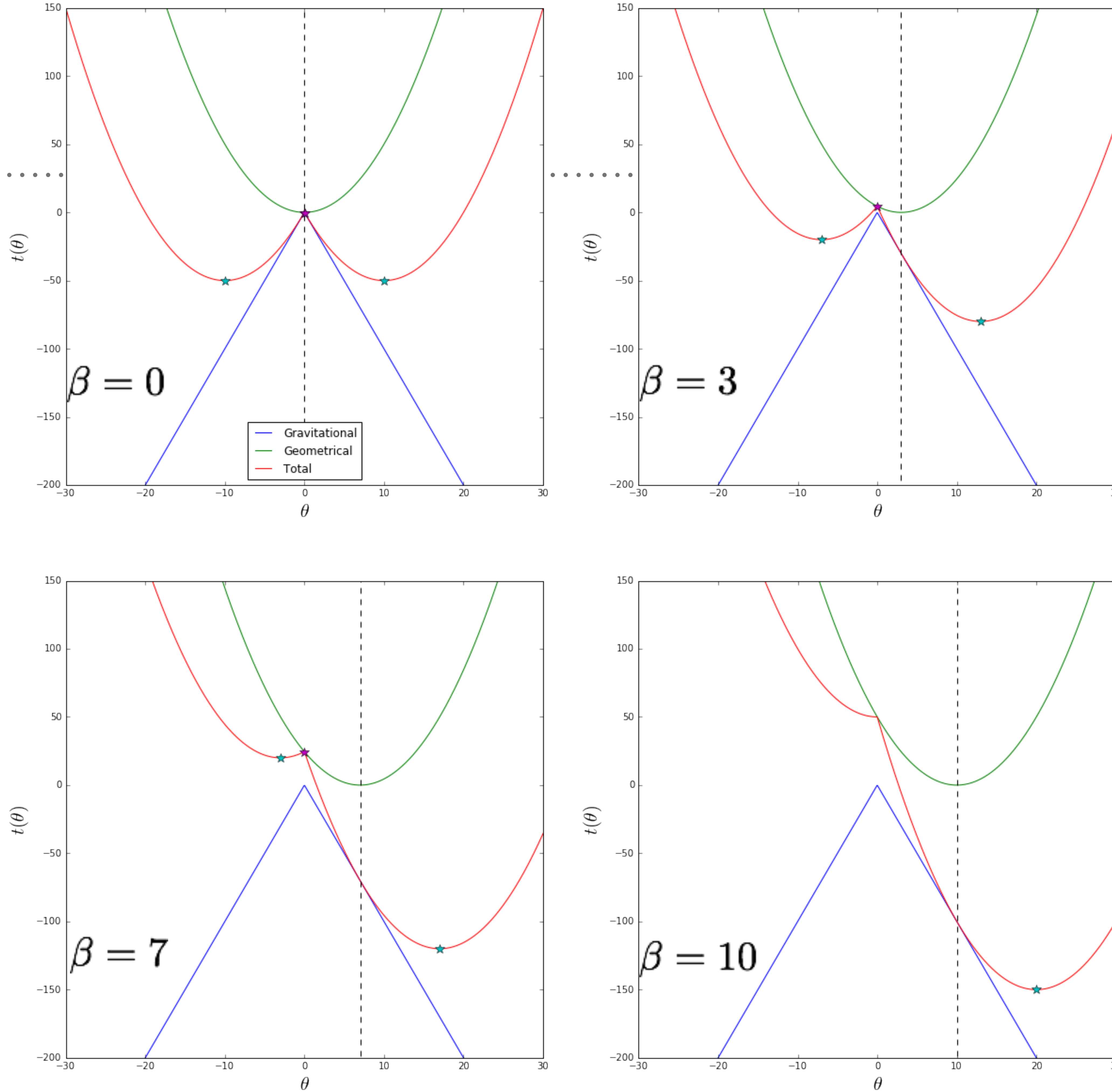


EXAMPLE OF TIME DELAY SURFACE

Let's change potential:

$$\hat{\Psi}(\theta) = K|\theta|$$

The lens model is the same as before, but the core has been removed



$\beta = 0$
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$\beta = 7$
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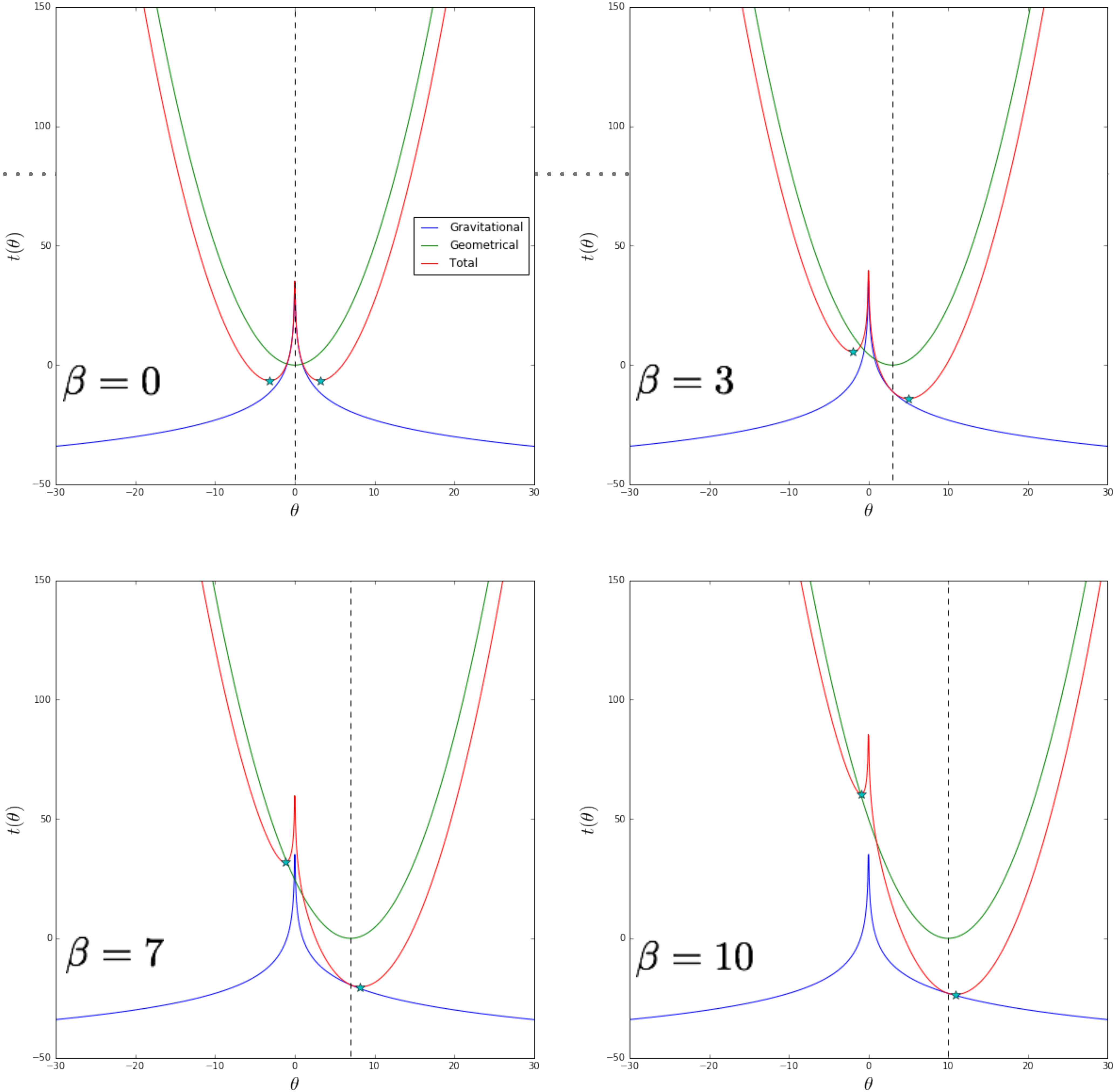
Gravitational
Geometrical
Total
$t(\theta)$
θ

EXAMPLE OF TIME DELAY SURFACE

Yet another potential

$$\hat{\Psi}(\theta) = K \ln|\theta|$$

This is the lensing potential of the point mass…



Gravitational
Geometrical
Total
$\beta = 0$
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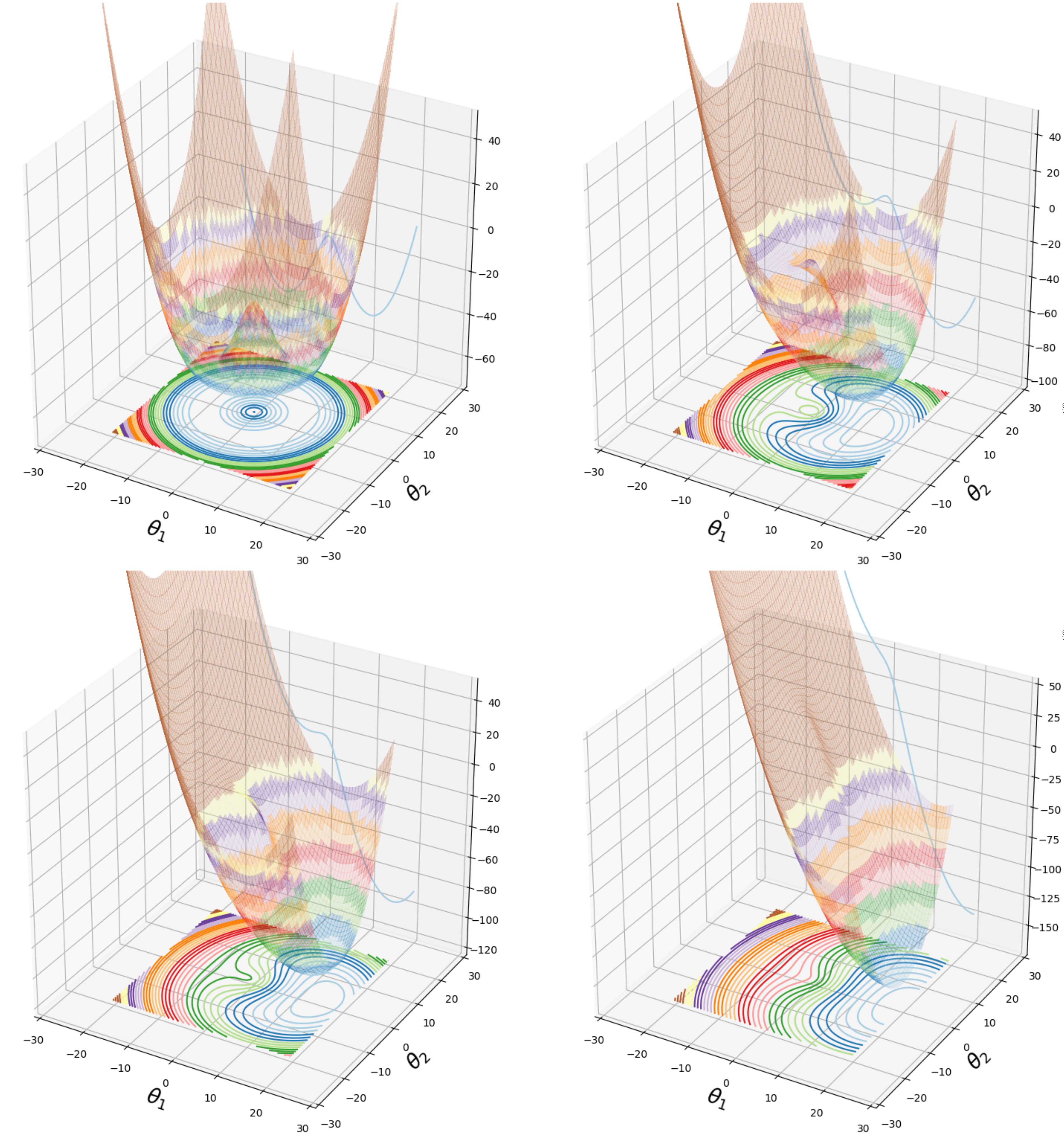
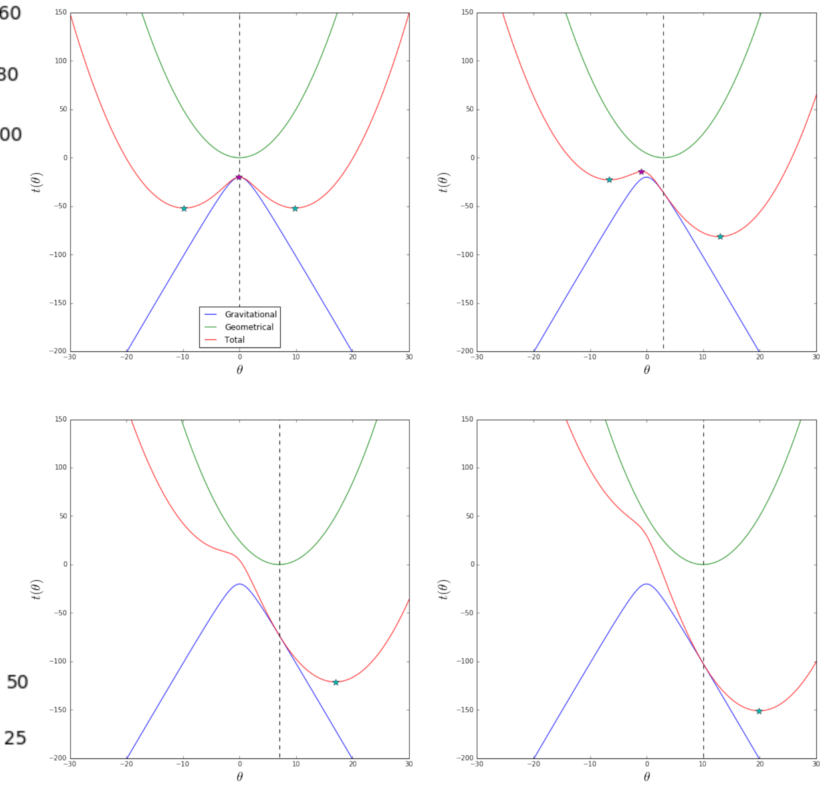
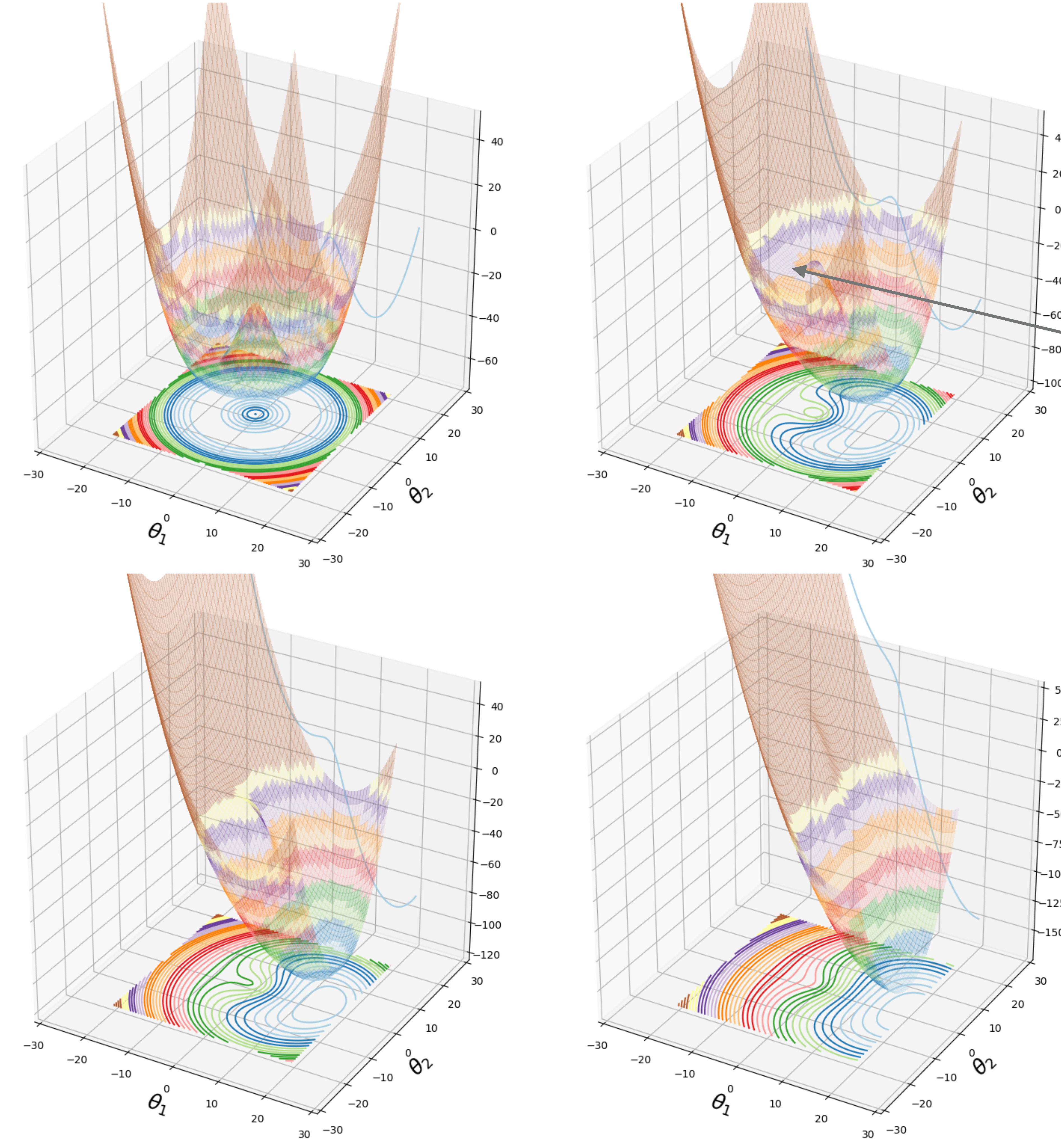
$\beta = 7$
$\beta = 10$
$t(\theta)$
θ



…but $t(\theta)$ is a surface!





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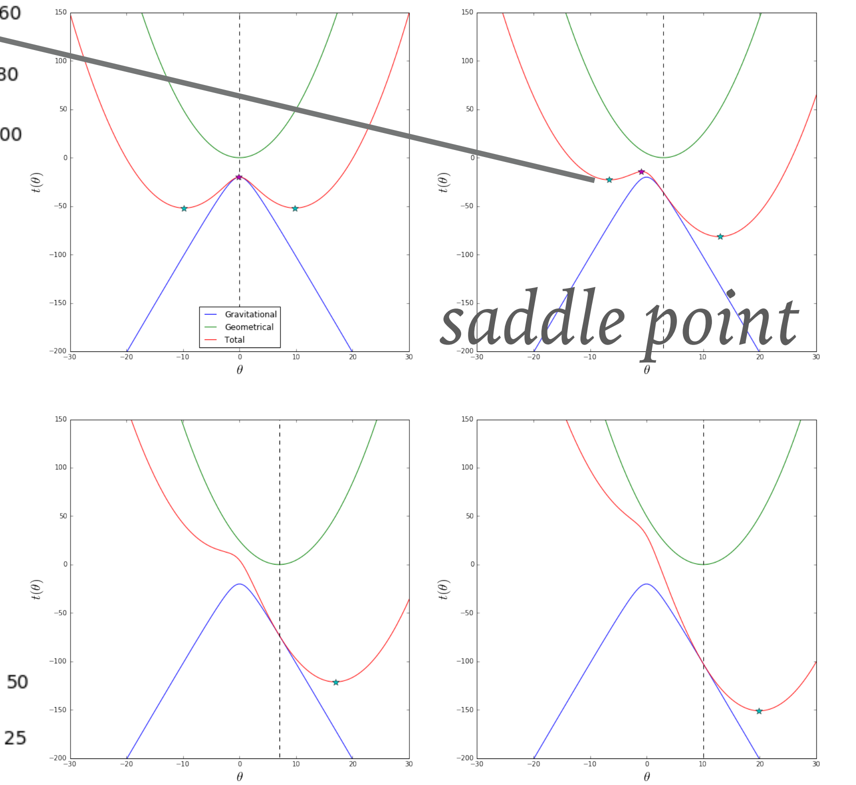


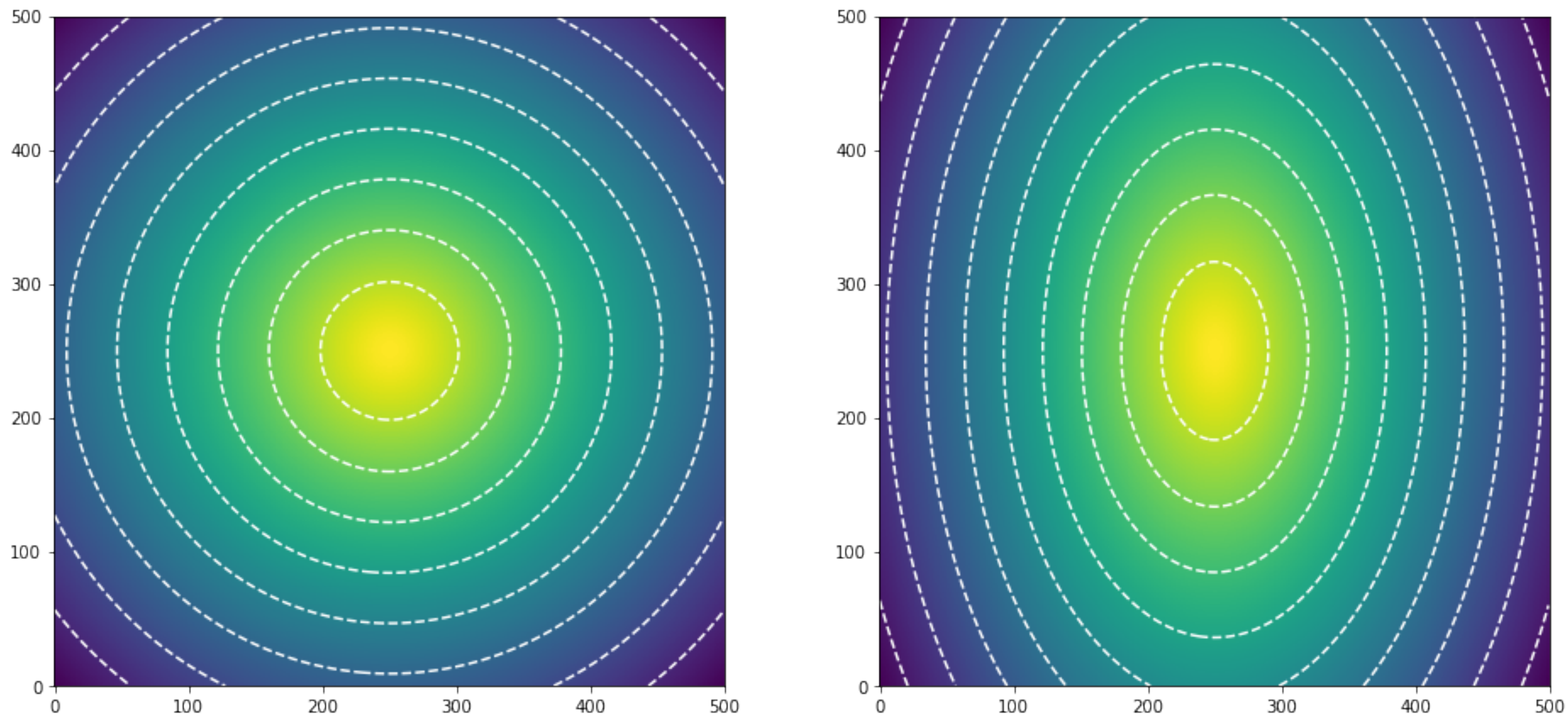
saddle point

INTRODUCING ELLIPTICITY

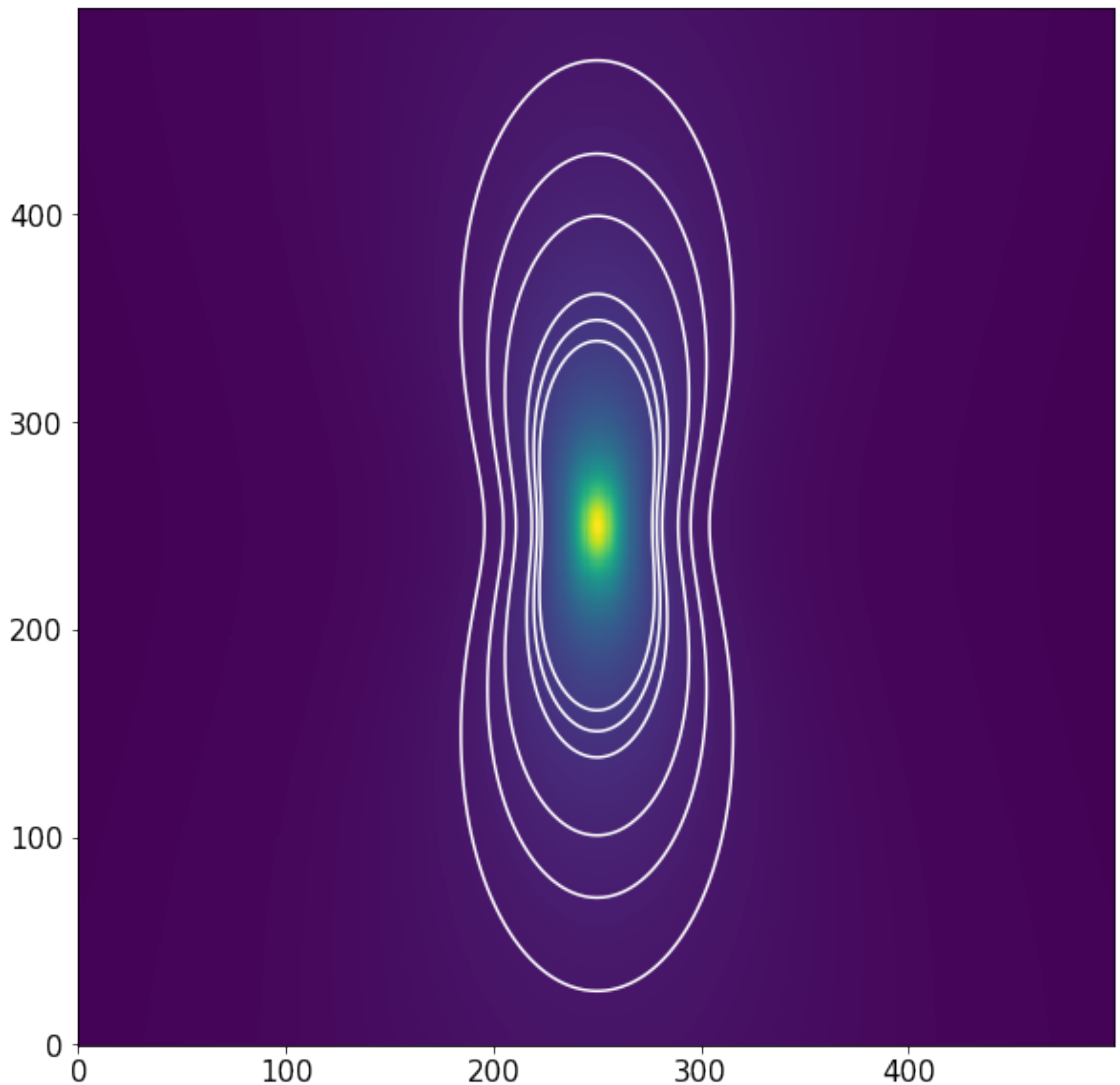
One easy way to make a lens elliptical is by modifying the potential as follows:

$$|\theta| \rightarrow \sqrt{\frac{\theta_1^2}{1-\epsilon} + \theta_2^2(1-\epsilon)}$$

This makes the potential constant over ellipses rather than on circles.

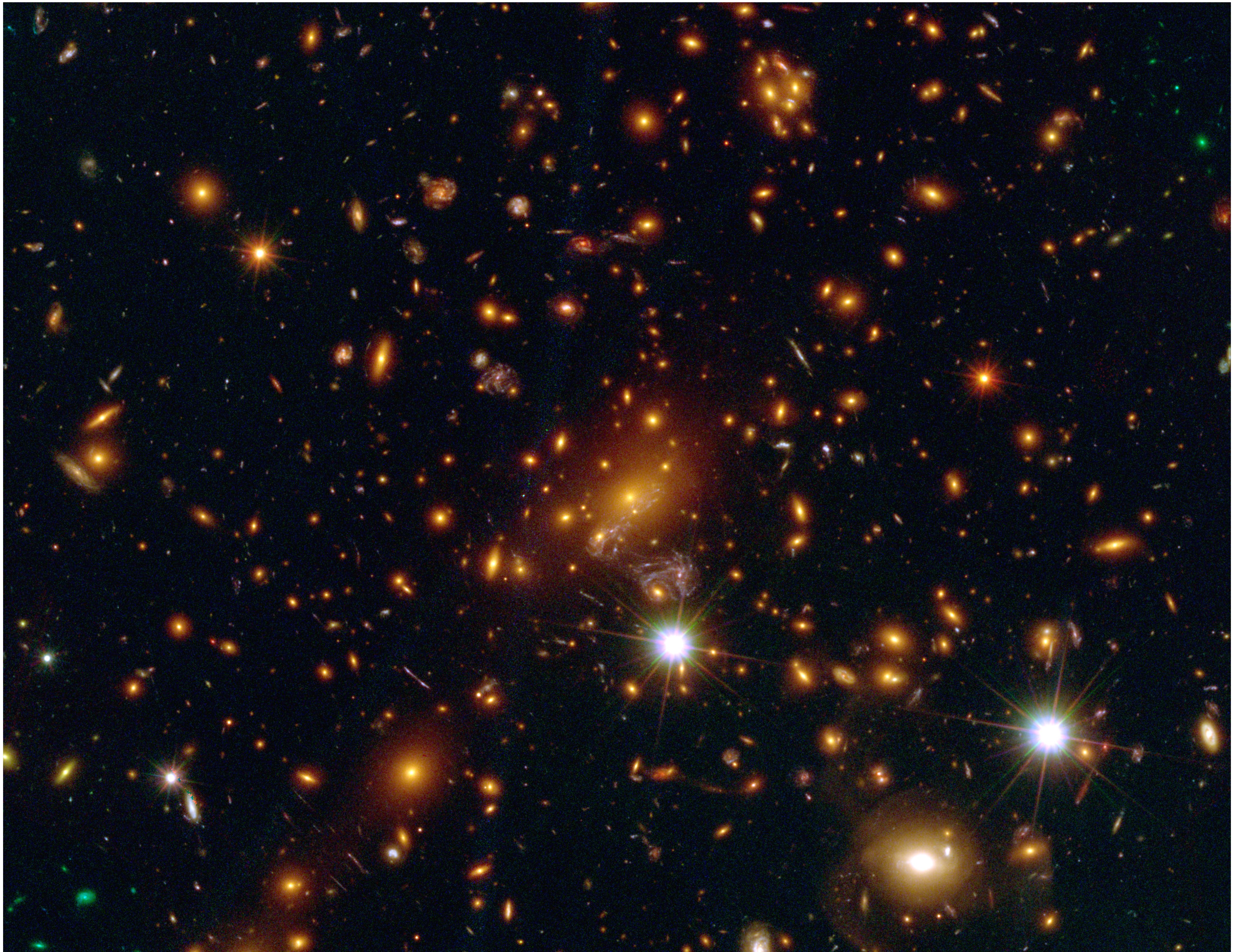


CAUTION: PSEUDO ELLIPTICAL LENSES



To Jupyter Notebook 6_2020

SN REFSDAL IN MACS 1149



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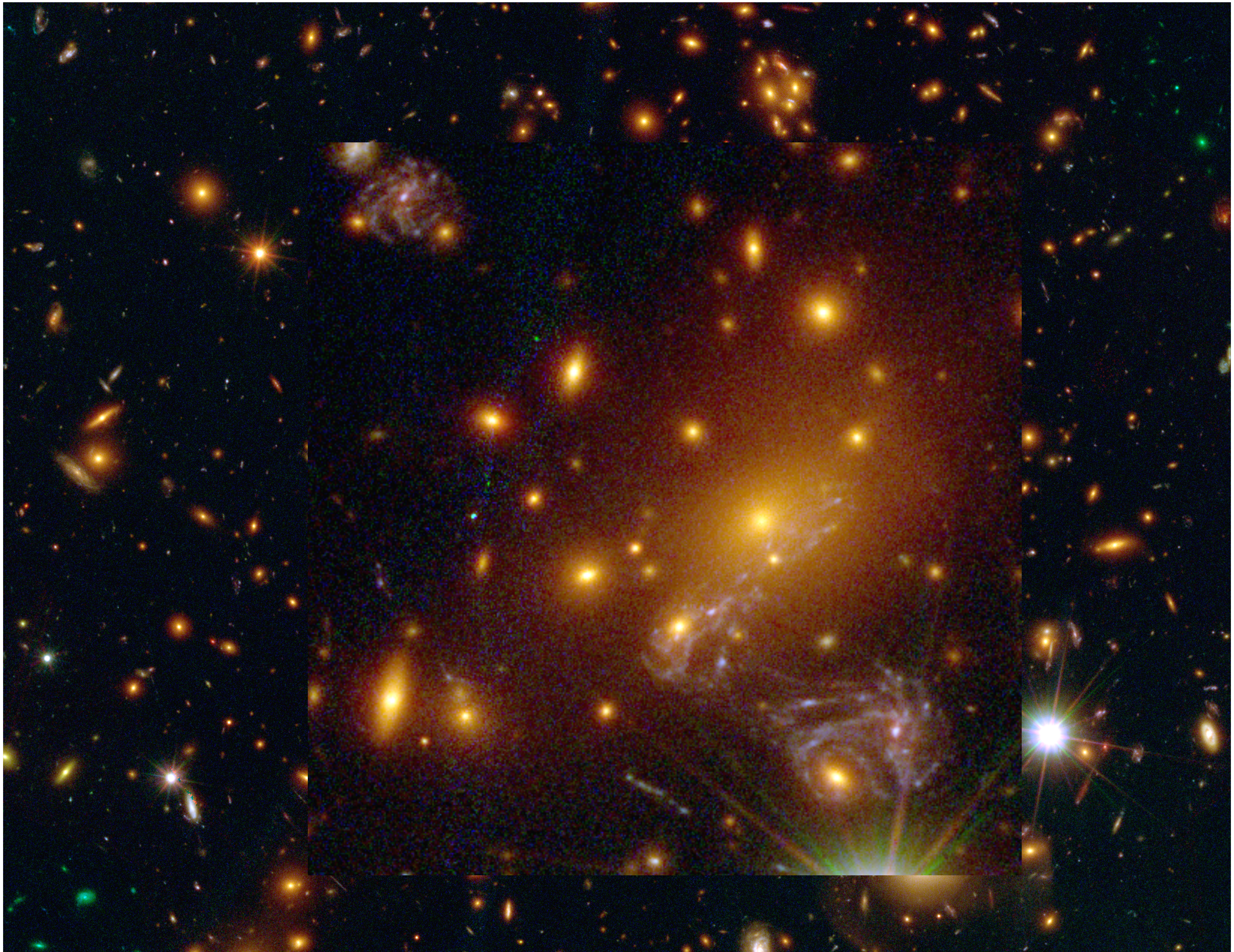

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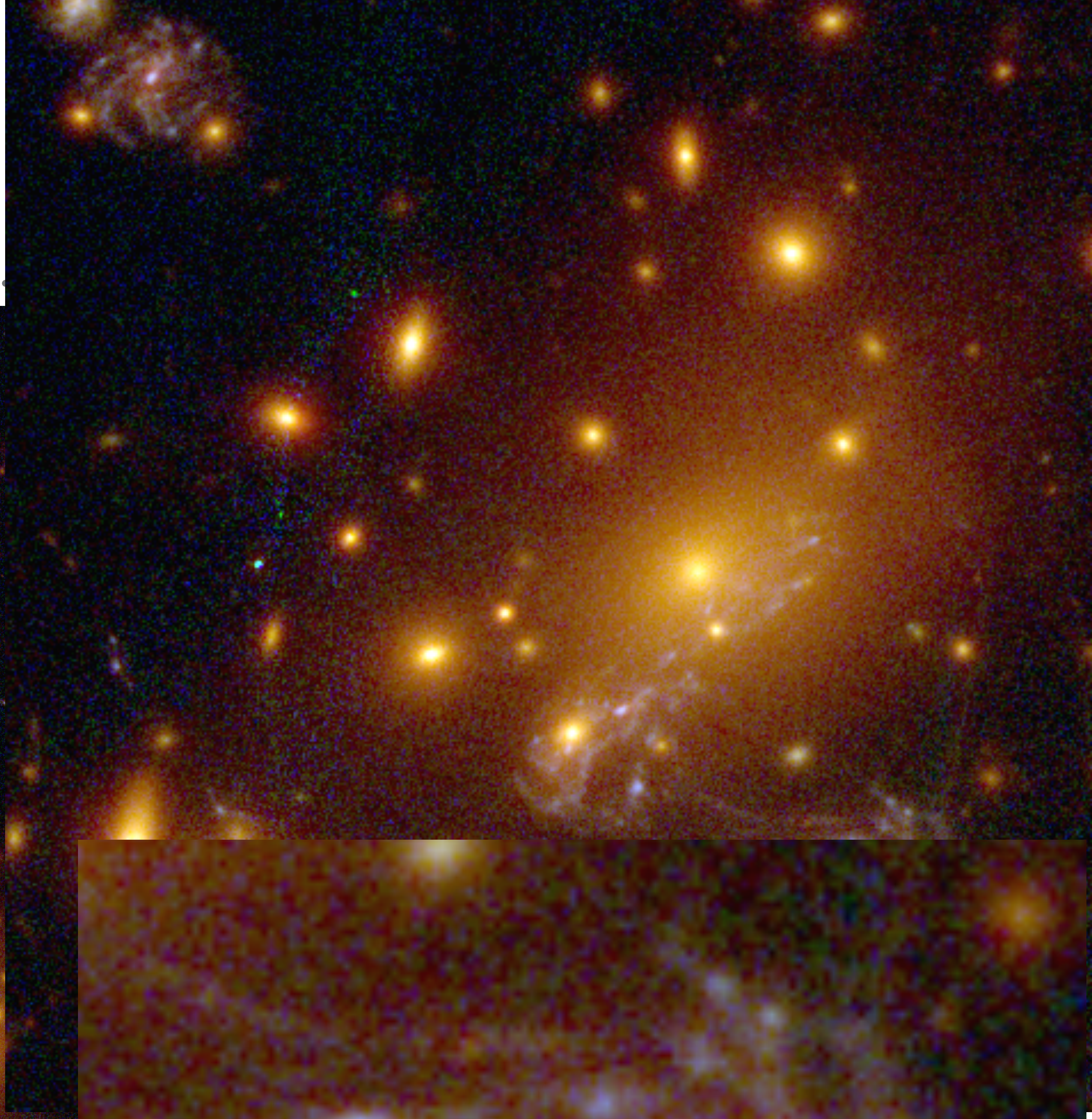

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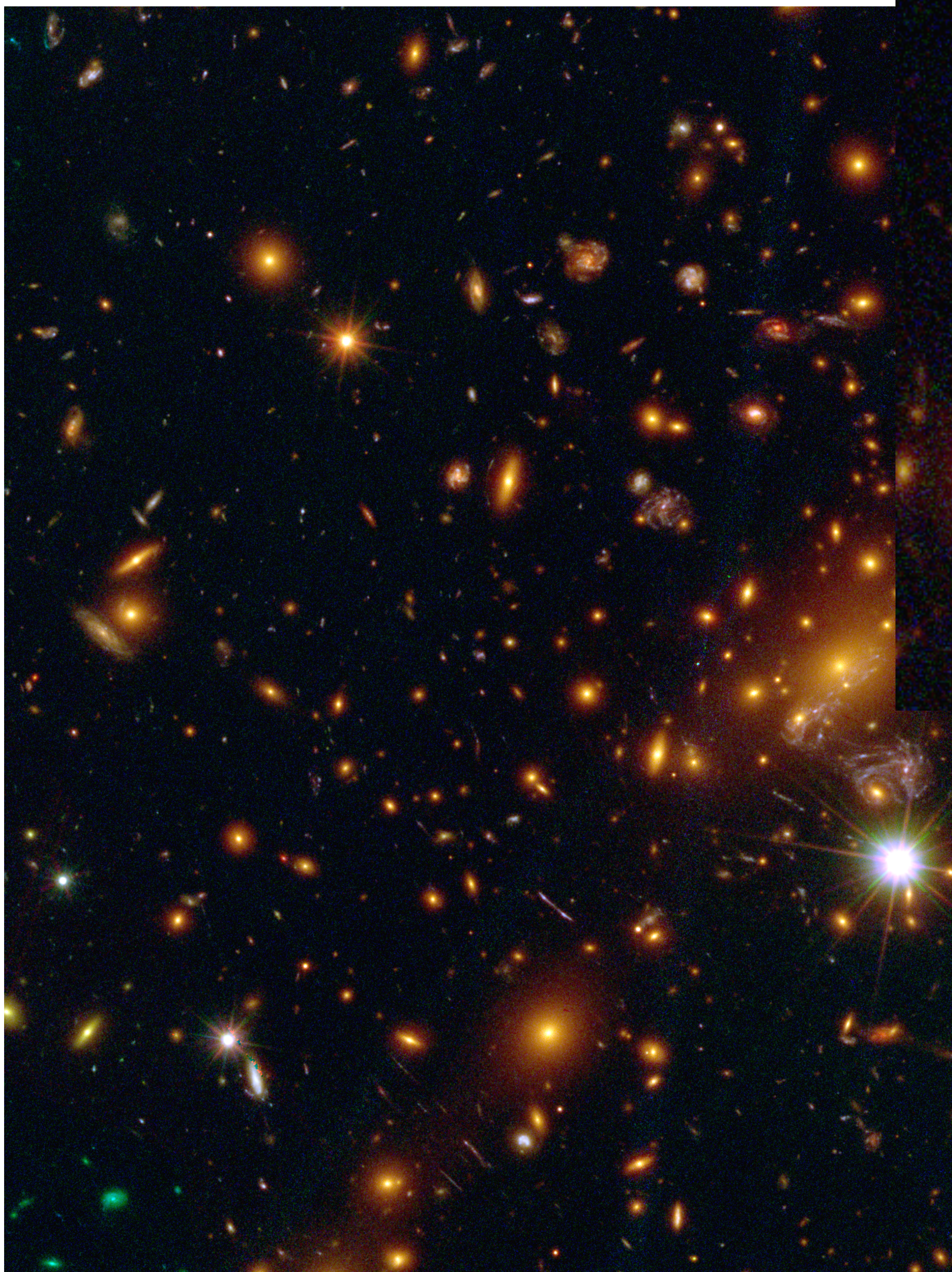

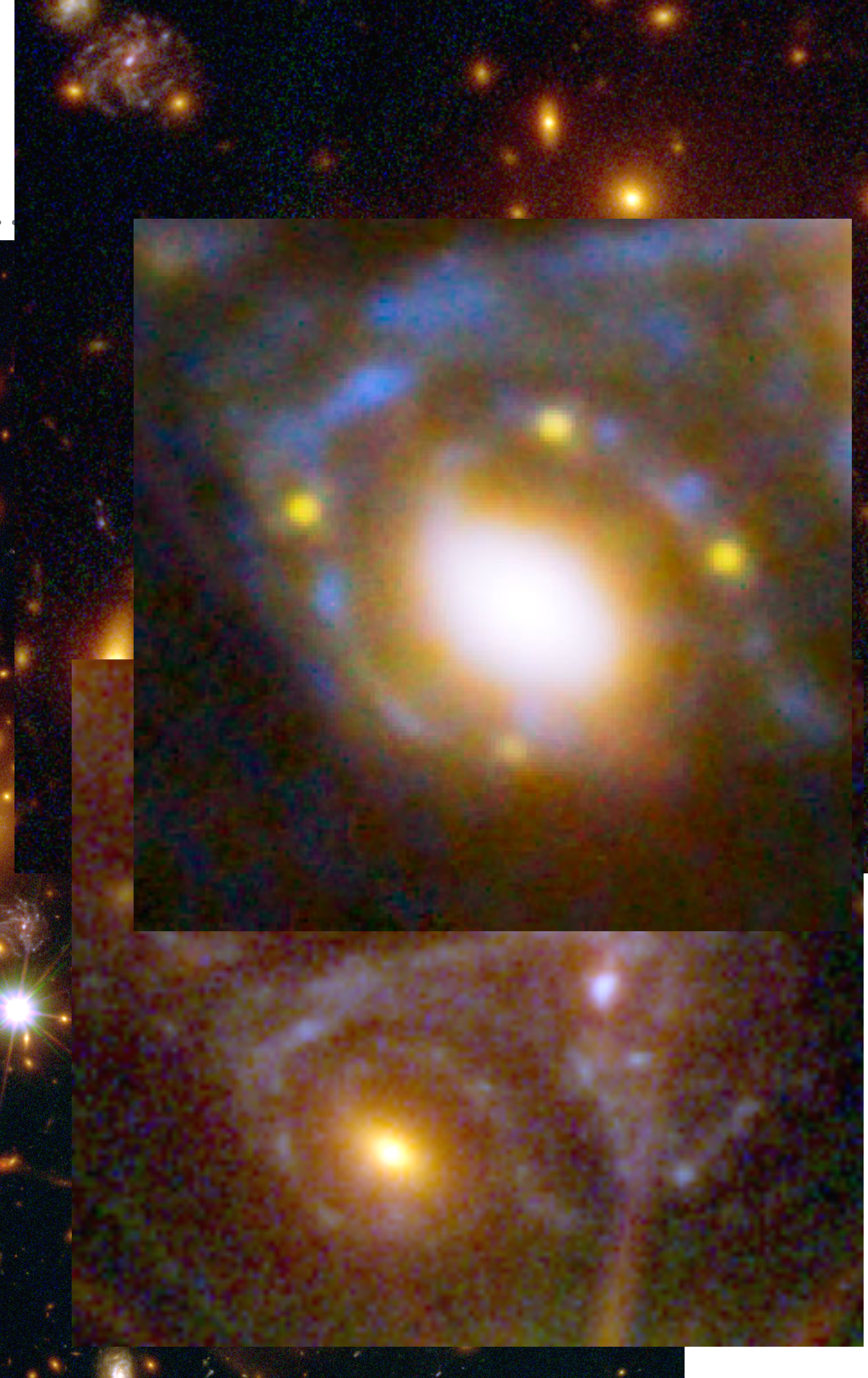

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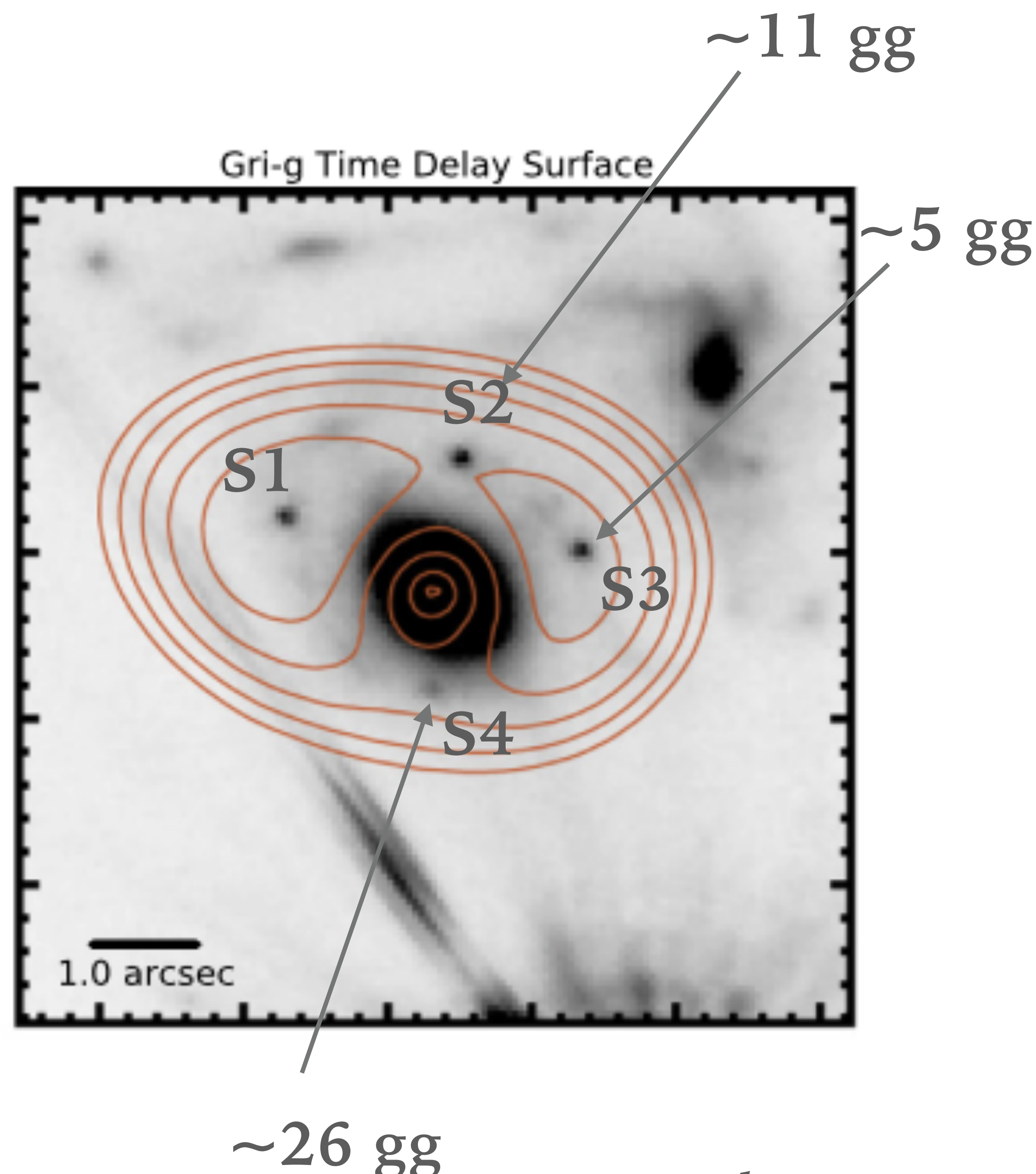
Nov. 2014 (Kelly et al.)



Treu et al. 2016

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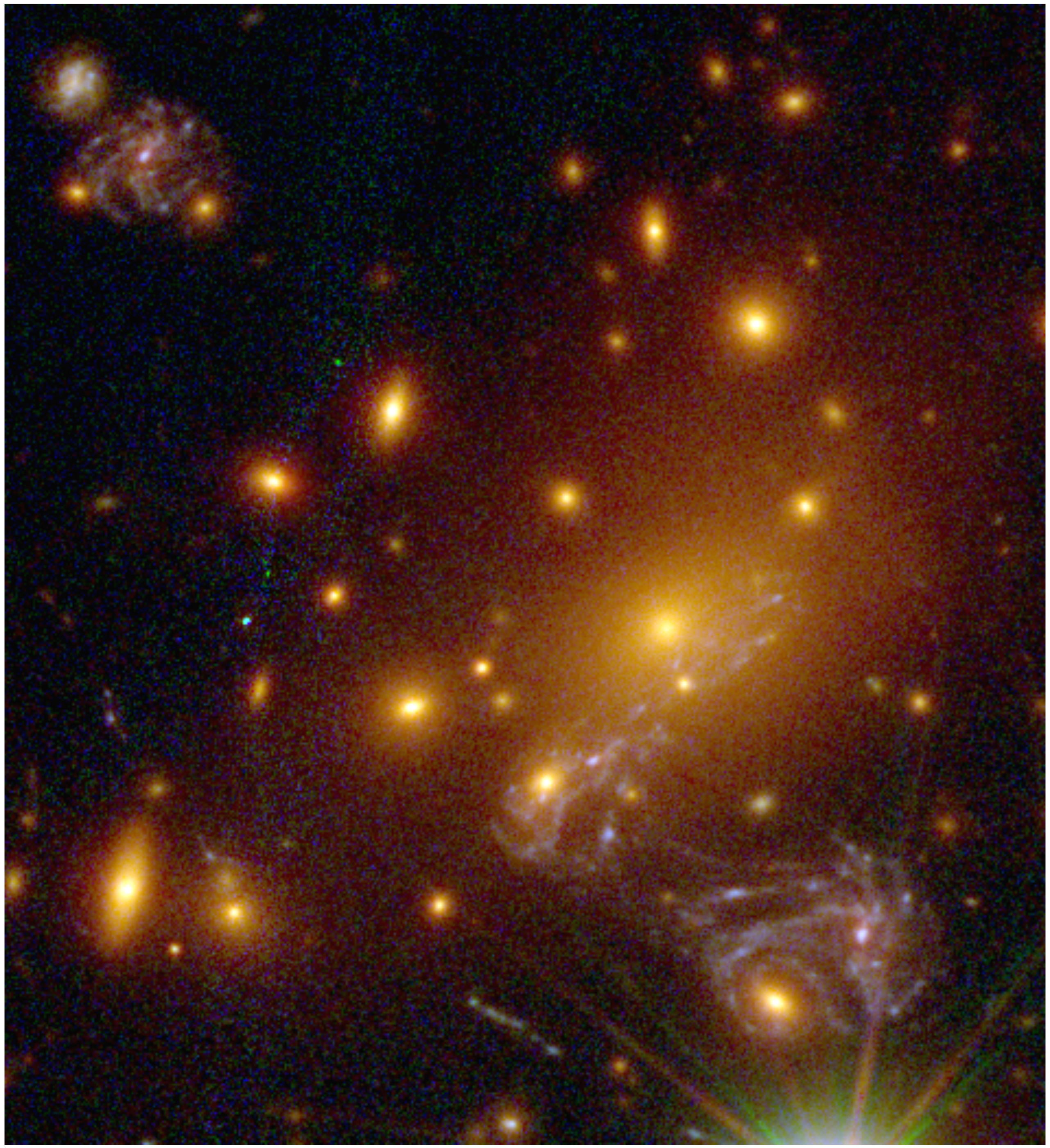

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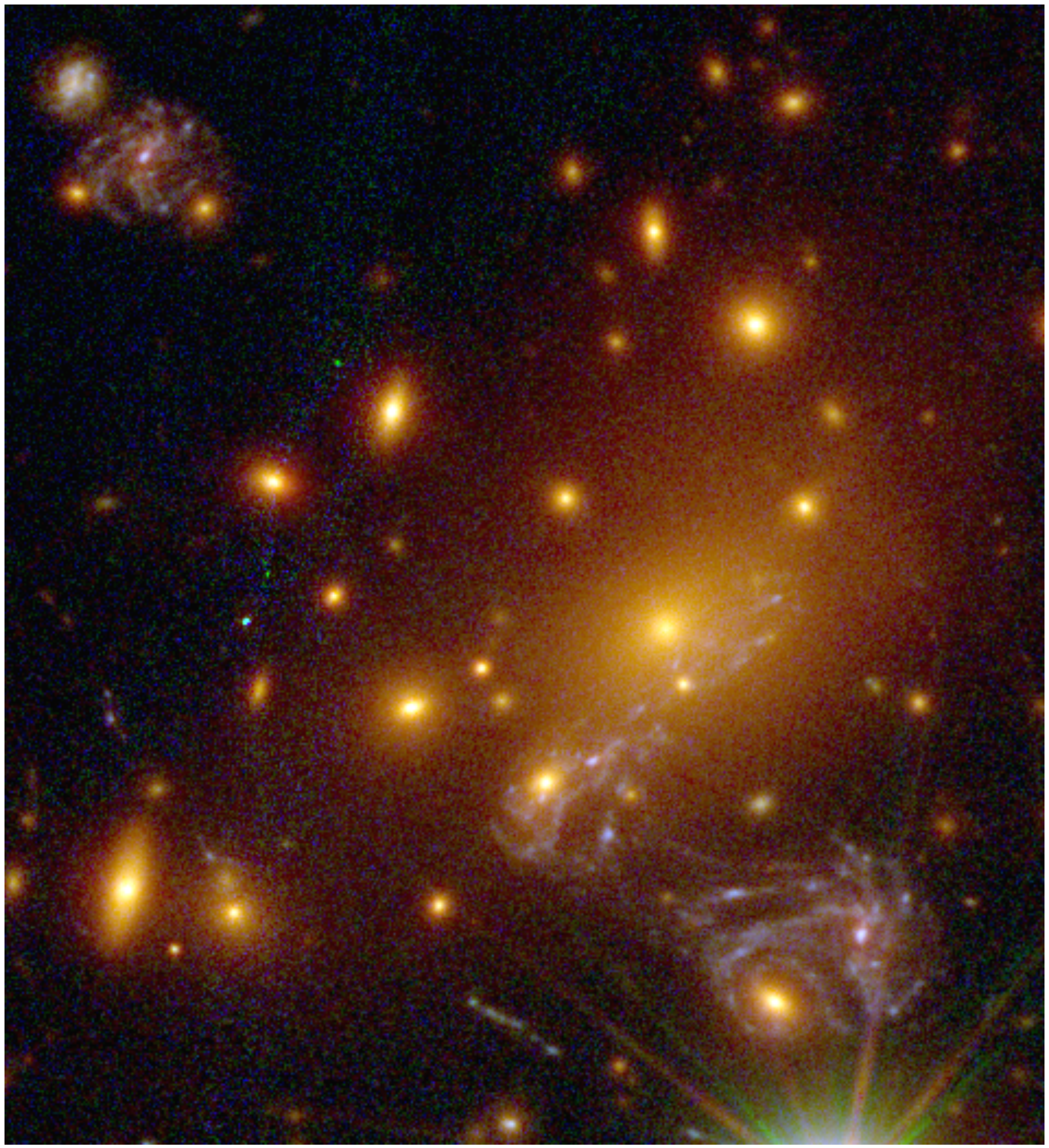

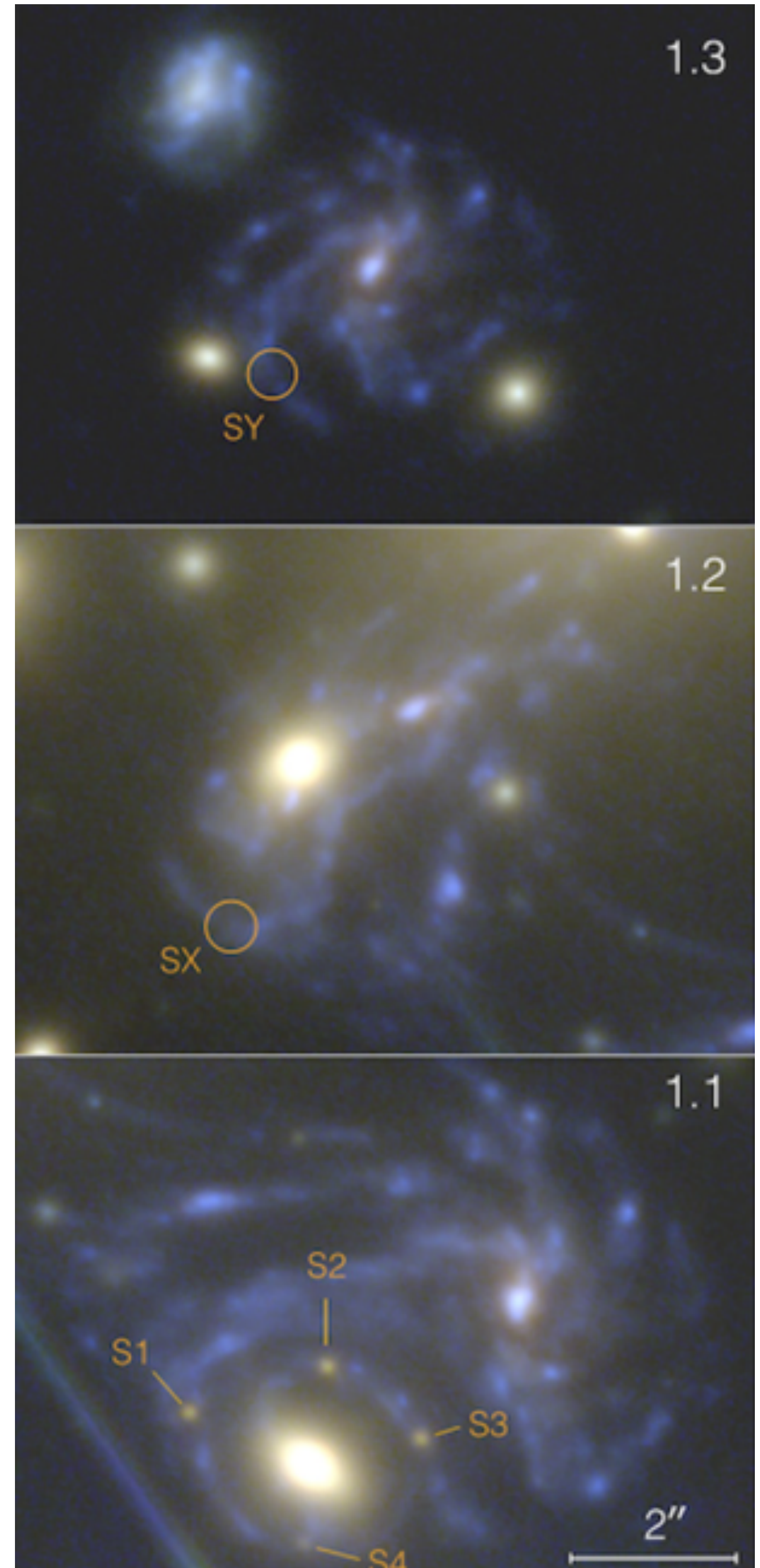

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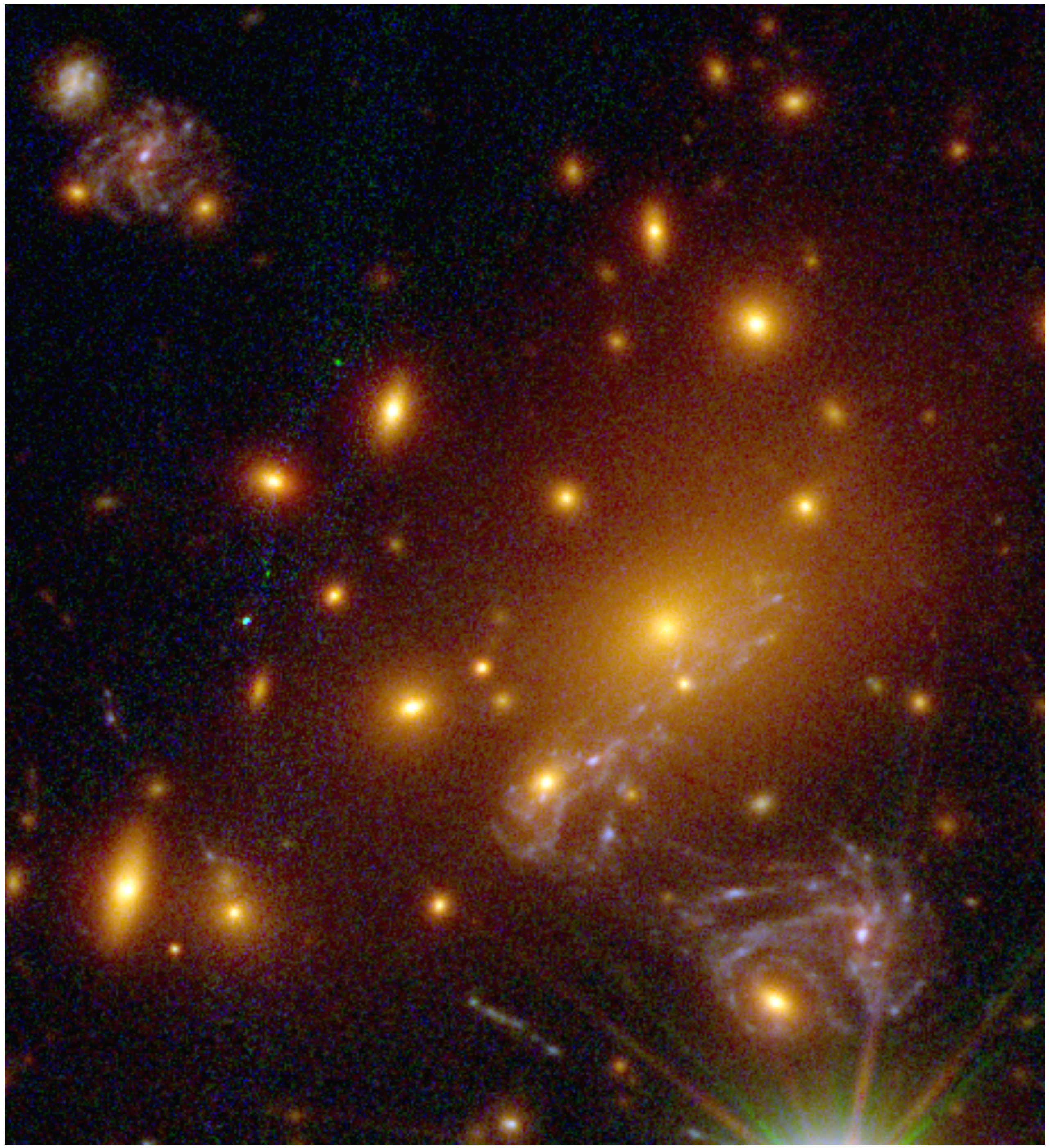

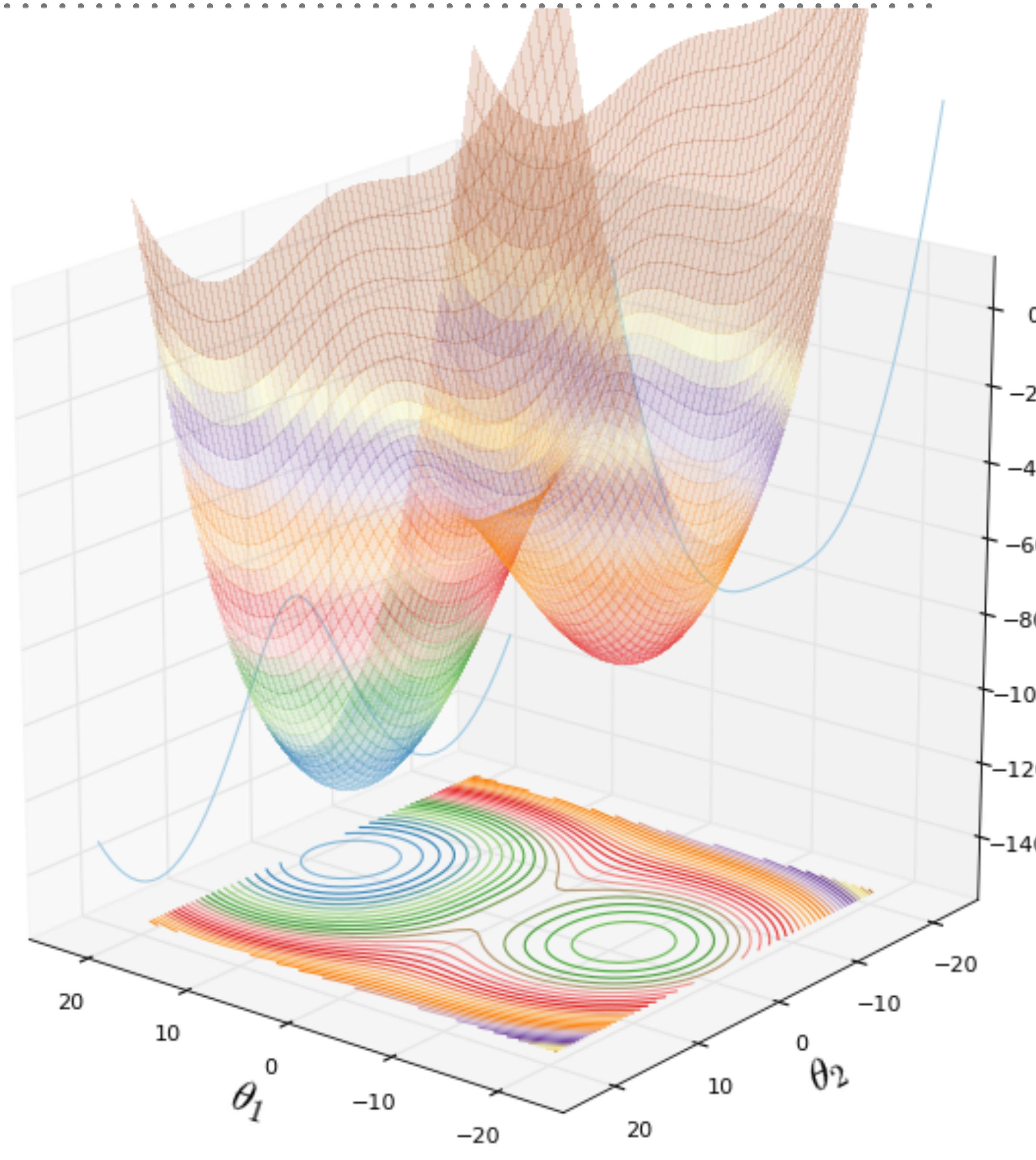

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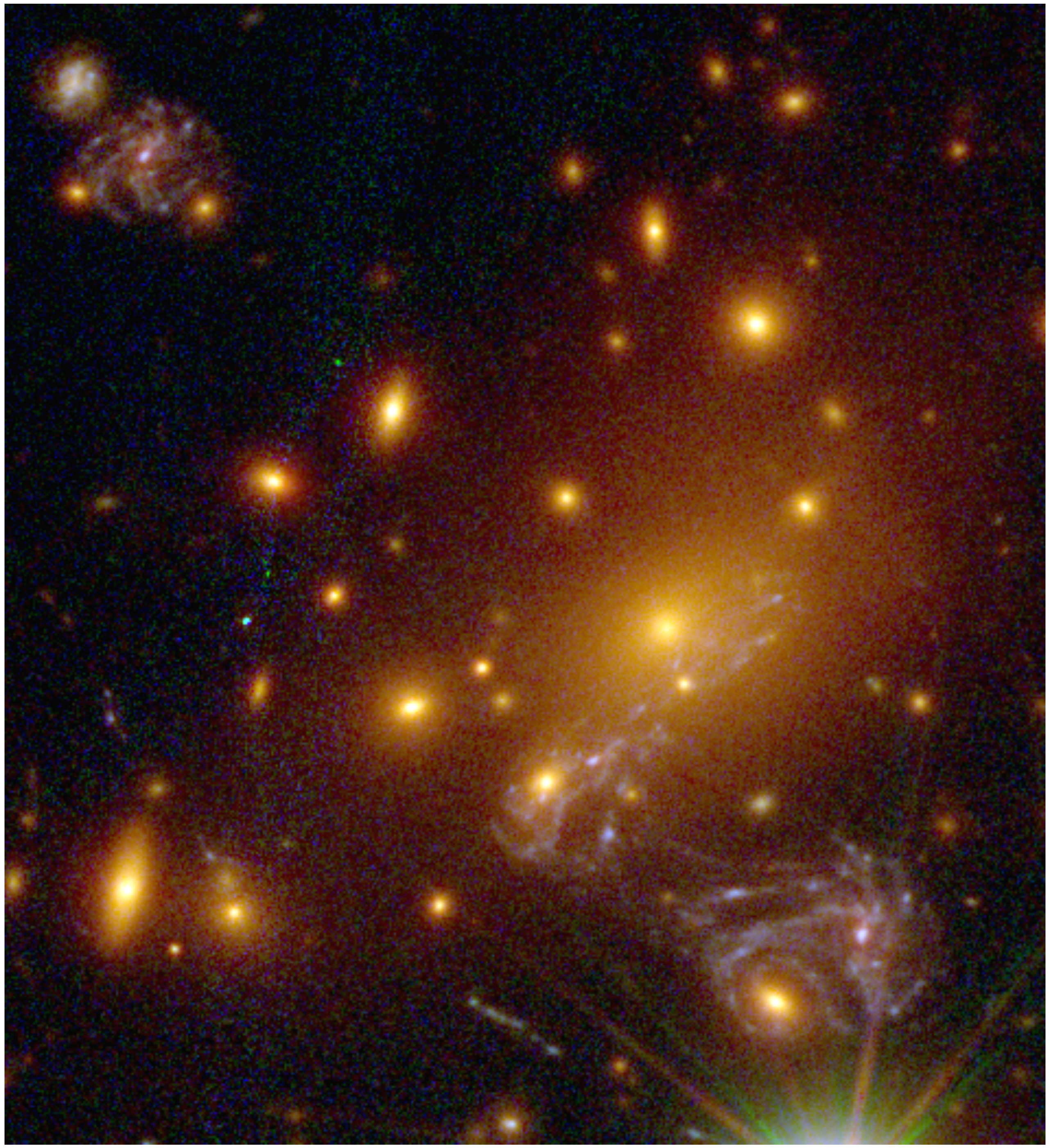

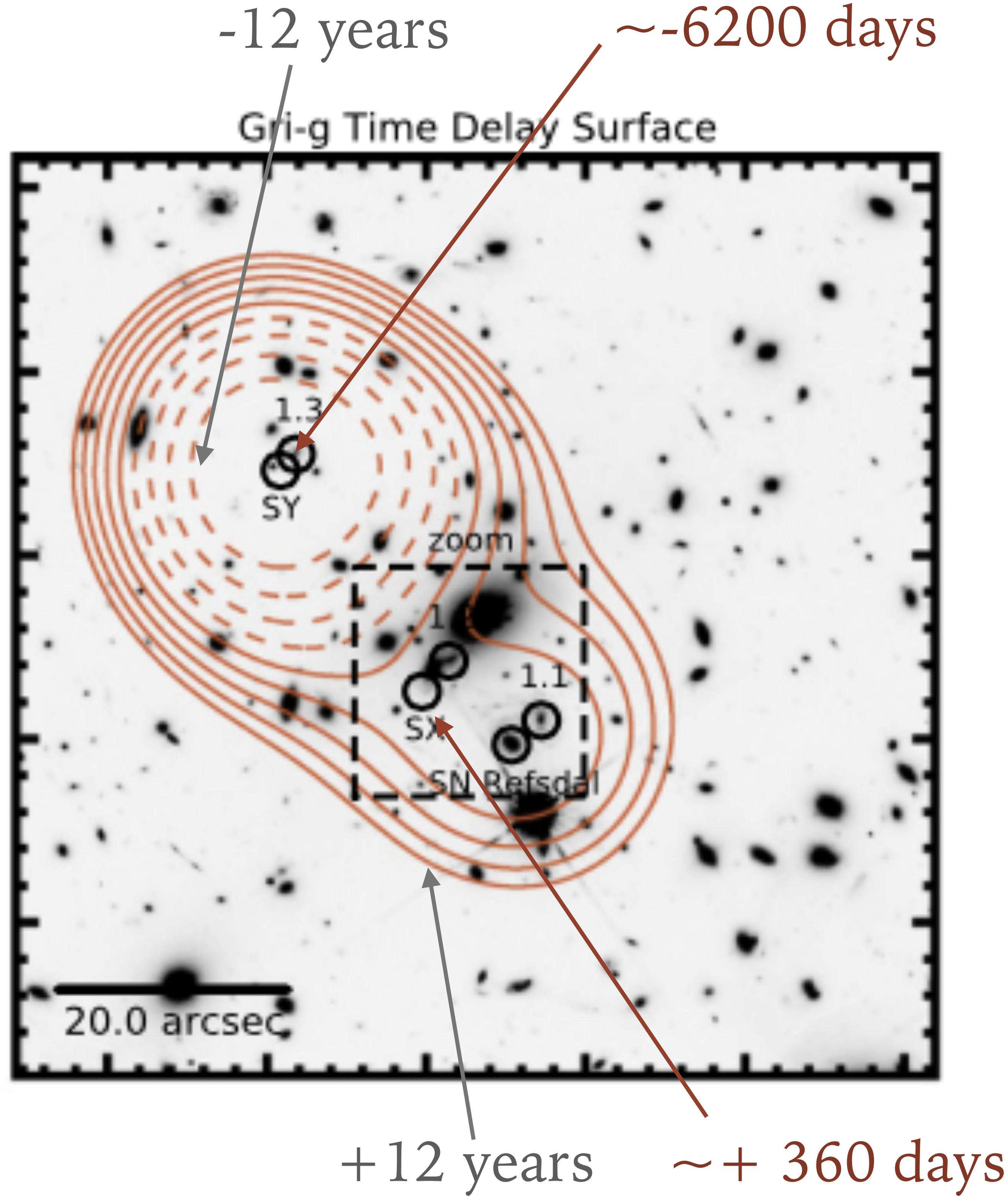

SN REFSDAL IN MACS 1149

16/12/2016…

Time delay

(SX- S1)
345+-10gg