GRAVITATIONAL LENSING

24 - WEAK LENSING BY GALAXY CLUSTERS

Massimo Meneghetti
AA 2017-2018

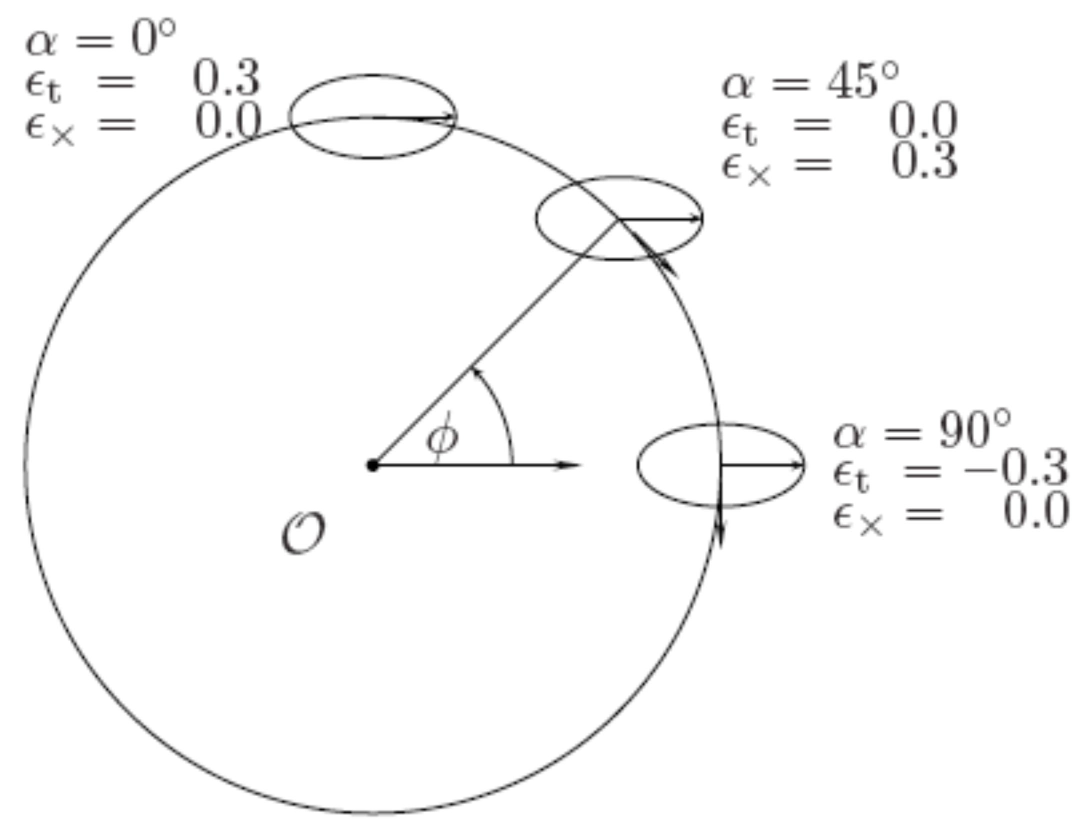
TANGENTIAL AND CROSS COMPONENT OF THE SHEAR

Given a direction φ we can define a tangential and a cross component of the ellipticity/shear relative to this direction.

$$\gamma_{\rm t} = -\mathcal{R}{\rm e}\left[\gamma\,{\rm e}^{-2{\rm i}\phi}\right] \quad , \quad \gamma_\times = -\mathcal{I}{\rm m}\left[\gamma\,{\rm e}^{-2{\rm i}\phi}\right]$$

Note that, under this convention, “tangential” means both tangentially and radially oriented ellipticities

With this we want to emphasize that lensing, being caused by a scalar potential is curl-free



The signs are chosen such that the tangential component is positive for tangentially distorted images, and it is negative for radially distorted images.

FIT OF THE TANGENTIAL SHEAR PROFILE

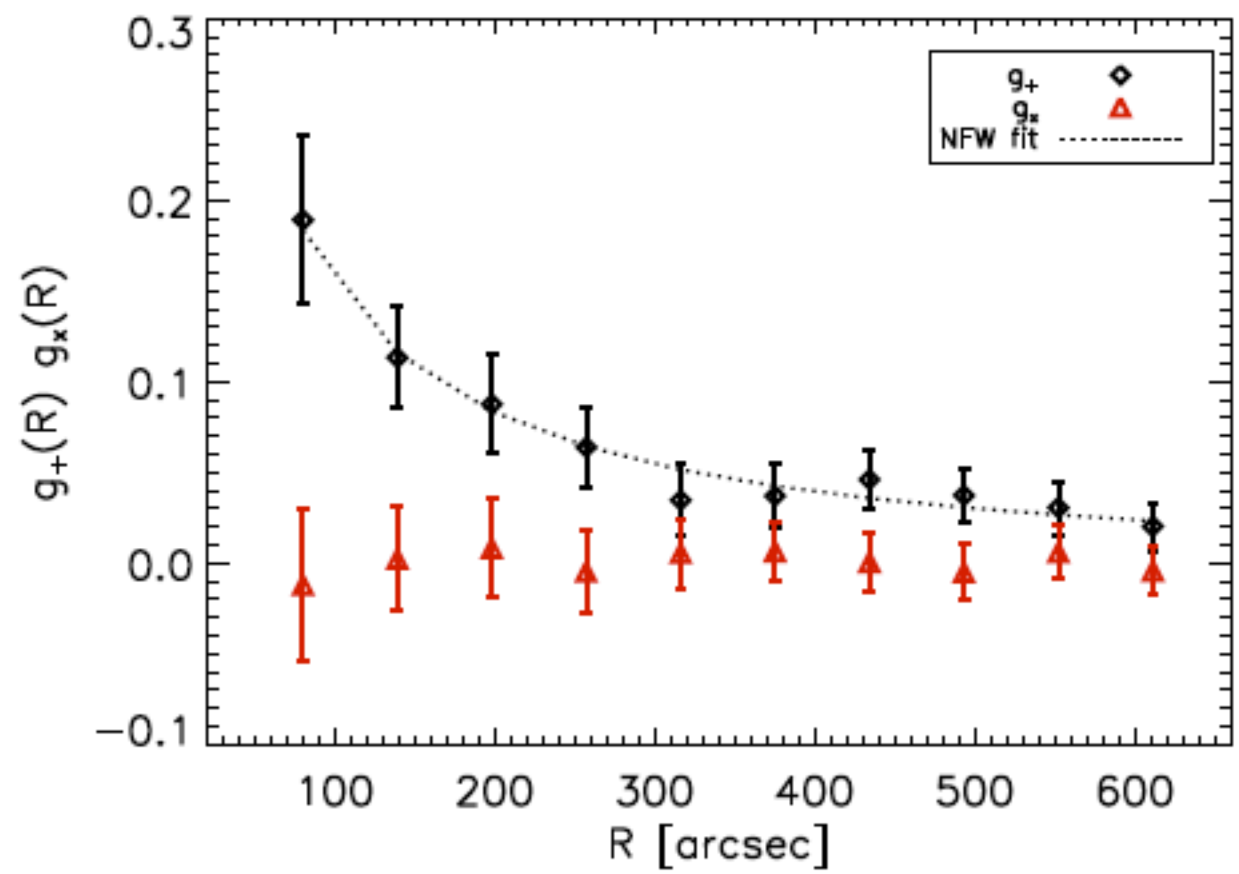
Having identified the galaxies in the background of the cluster, one can bin them in circular annuli and measure the average tangential and cross shear profiles.

The tangential shear profile, which contains the signal, can be fitted with some parametric model.
For example:

NFW $$\kappa(x) = \frac{\Sigma(\xi_0 x)}{\Sigma_{cr}} = 2\kappa_s \frac{f(x)}{x^2-1}$$

$$f(x) = \begin{cases} 1 - \frac{2}{\sqrt{x^2-1}} \arctan\sqrt{\frac{x-1}{x+1}} & (x > 1) \\ 1 - \frac{2}{\sqrt{1-x^2}} \operatorname{arctanh}\sqrt{\frac{1-x}{1+x}} & (x < 1) \\ 0 & (x = 1) \end{cases}$$

$$\gamma(x) = \overline{\kappa}(x) - \kappa(x)$$



$$l_\gamma = \sum_{i=1}^{N_\gamma} \left[\frac{|\epsilon_i - g(\theta_i)|^2}{\sigma^2[g(\theta_i)]} + 2\ln\sigma[g(\theta_i)] \right]$$

The profile of the cross component provides a check for systematics.

THE KAISER & SQUIRES INVERSION ALGORITHM

Shear and convergence in Fourier space

$$
\begin{aligned}
\kappa &= \frac{1}{2}(\psi_{11} + \psi_{22}) &\Rightarrow\quad \hat{\kappa} &= -\frac{1}{2}(k_1^2 + k_2^2)\hat{\psi} \\
\gamma_1 &= \frac{1}{2}(\psi_{11} - \psi_{22}) &\Rightarrow\quad \hat{\gamma}_1 &= -\frac{1}{2}(k_1^2 - k_2^2)\hat{\psi} \\
\gamma_2 &= \psi_{12} &\Rightarrow\quad \hat{\gamma}_2 &= -k_1 k_2 \hat{\psi}\ ,
\end{aligned}
$$

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$$\begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{pmatrix} = k^{-2} \begin{pmatrix} k_1^2 - k_2^2 \\ 2k_1k_2 \end{pmatrix} \hat{\kappa}$$

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using:

$$\left[k^{-2} \begin{pmatrix} k_1^2 - k_2^2 \\ 2k_1 k_2 \end{pmatrix} \right] [k^{-2} (\; k_1^2 - k_2^2 \quad 2k_1 k_2 \;)] = 1$$

$$\hat{\kappa} = k^{-2} (\; k_1^2 - k_2^2 \quad 2k_1 k_2 \;) \begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{pmatrix} = k^{-2}[(k_1^2 - k_2^2)\hat{\gamma}_1 + 2k_1 k_2 \hat{\gamma}_2]$$

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using:

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$$(f \hat{*} g) = \hat{f}\hat{g}$$

$$\hat{\kappa} = k^{-2} (\; k_1^2 - k_2^2 \quad 2k_1k_2 \;) \begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{pmatrix} = k^{-2}[(k_1^2 - k_2^2)\hat{\gamma}_1 + 2k_1k_2\hat{\gamma}_2]$$

THE KAISER & SQUIRES INVERSION ALGORITHM

Shear and convergence in Fourier space

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$$\gamma_2 = \psi_{12} \quad \Rightarrow \quad \hat{\gamma}_2 = -k_1 k_2 \hat{\psi} \; ,$$

$$\boxed{\kappa(\vec{\theta}) = \frac{1}{\pi} \int \mathrm{d}^2\theta' [D_1(\vec{\theta} - \vec{\theta}')\gamma_1 + D_2(\vec{\theta} - \vec{\theta}')\gamma_2]}$$

$$\begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{pmatrix} = k^{-2} \begin{pmatrix} k_1^2 - k_2^2 \\ 2k_1 k_2 \end{pmatrix} \hat{\kappa}$$

using:

$$\left[k^{-2} \begin{pmatrix} k_1^2 - k_2^2 \\ 2k_1 k_2 \end{pmatrix} \right] [k^{-2} (\; k_1^2 - k_2^2 \quad 2k_1 k_2 \;)] = 1$$

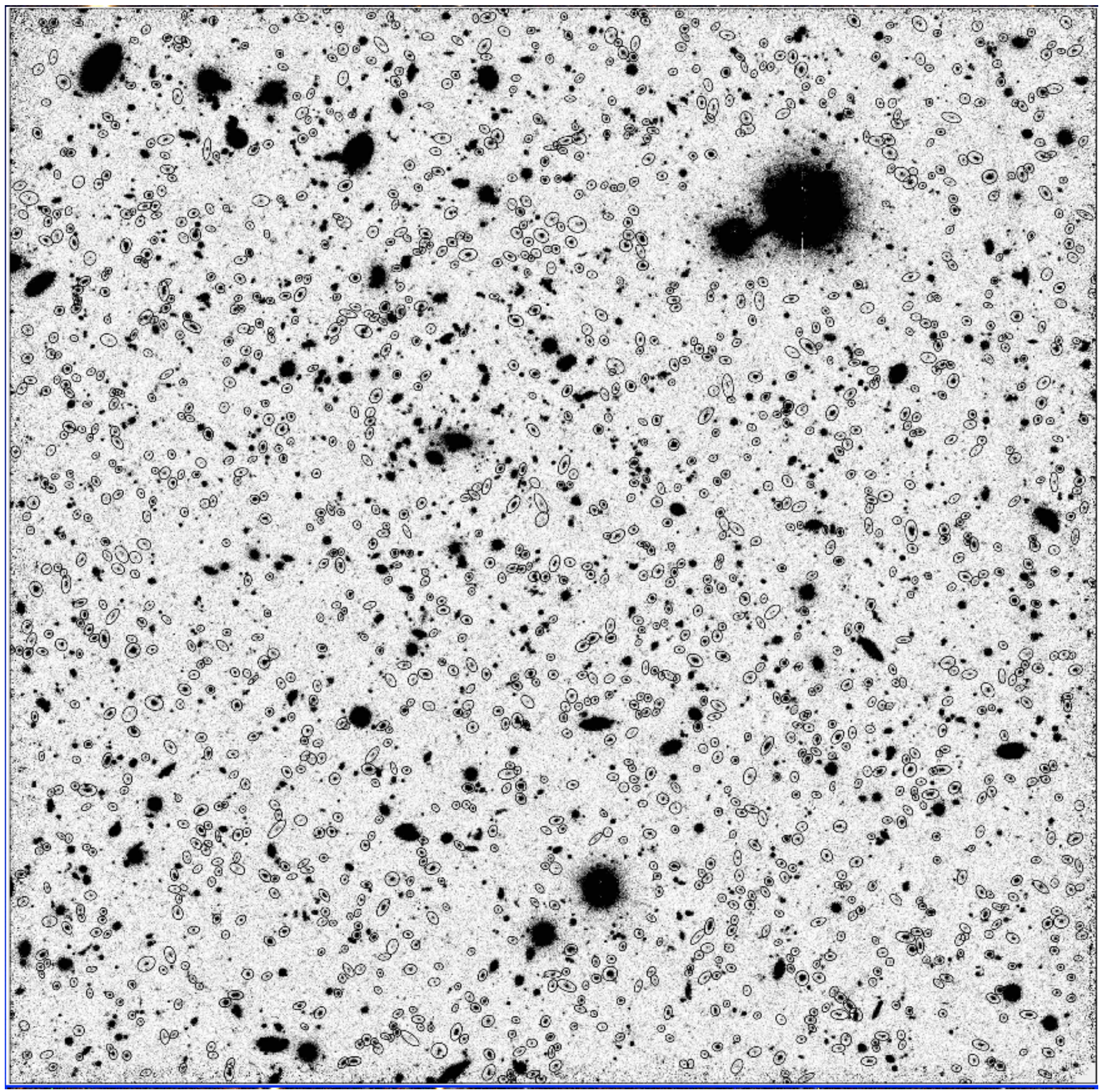
$$\widehat{(f * g)} = \hat{f}\hat{g}$$

$$\hat{\kappa} = k^{-2} (\; k_1^2 - k_2^2 \quad 2k_1 k_2 \;) \begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{pmatrix} = k^{-2}[(k_1^2 - k_2^2)\hat{\gamma}_1 + 2k_1 k_2 \hat{\gamma}_2]$$

THE KAISER & SQUIRES INVERSION ALGORITHM

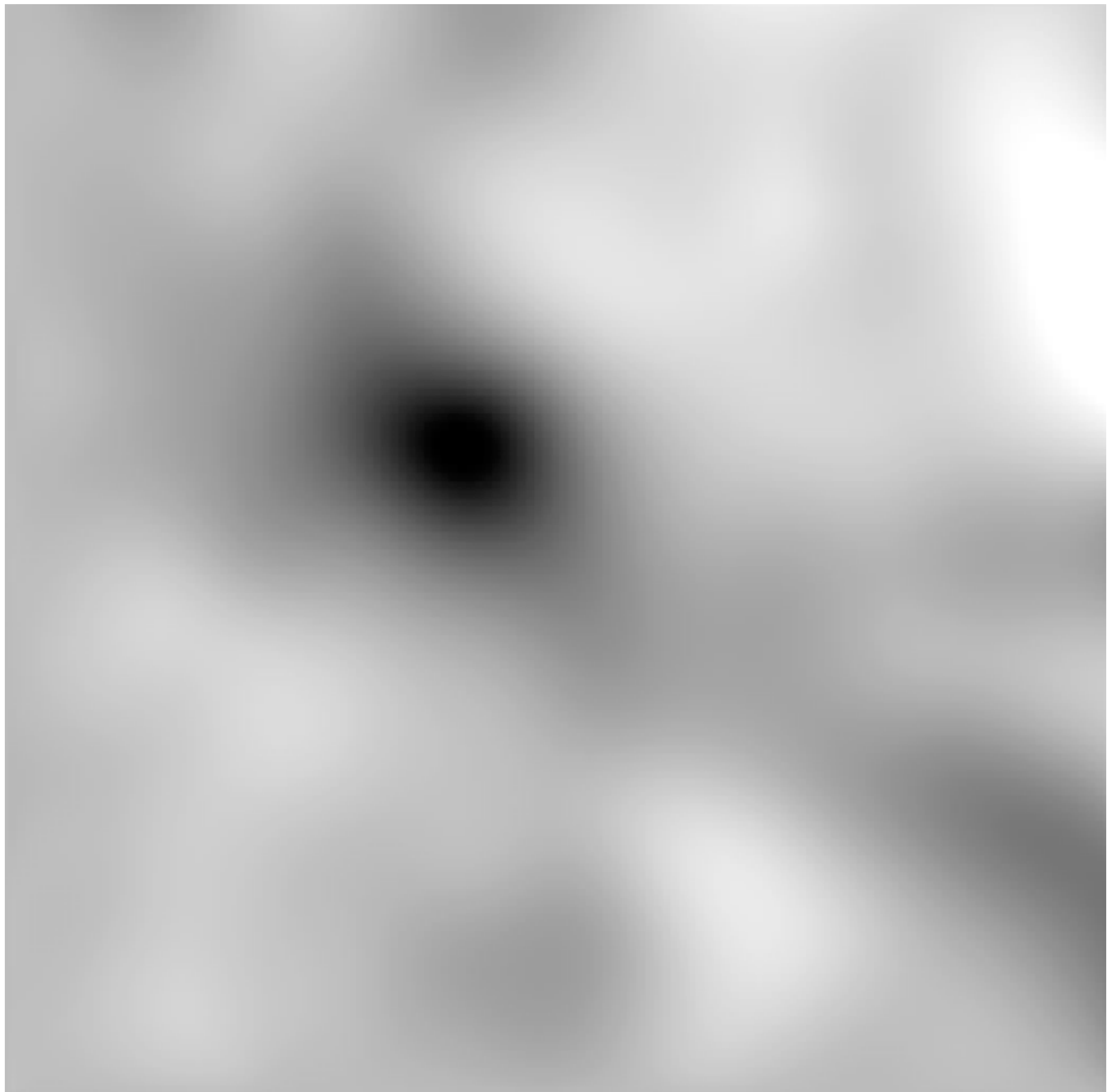
CL1232-1250
(Clowe et al.)

THE KAISER & SQUIRES INVERSION ALGORITHM



CL1232-1250
(Clowe et al.)

THE KAISER & SQUIRES INVERSION ALGORITHM



CL1232-1250
(Clowe et al.)

THE KAISER & SQUIRES INVERSION ALGORITHM

- infinite fields would be required: wide field + boundary conditions.
- ellipticity measures the reduced shear, not the shear:

$$\kappa(\vec{\theta}) = \frac{1}{\pi} \int \mathrm{d}^2\theta' [D_1(\vec{\theta} - \vec{\theta}')g_1(1-\kappa) + D_2(\vec{\theta} - \vec{\theta}')g_2(1-\kappa)]$$

This equation can be solved iteratively starting from $\kappa=0$

A FUNDAMENTAL LIMIT OF MASS MODELING: THE MASS-SHEET DEGENERACY

$$\psi(\vec{x}) \rightarrow \psi'(\vec{x}) = \frac{1-\lambda}{2}x^2 + \lambda\psi$$

$$\vec{\alpha}'(\vec{x}) = \vec{\nabla}\psi'(\vec{x}) = (1-\lambda)\vec{x} + \lambda\vec{\alpha}(\vec{x})$$

$$\vec{y}' = \vec{x} - \vec{\alpha}'(\vec{x}) = \lambda[\vec{x} - \vec{\alpha}(\vec{x})] = \lambda\vec{y}$$

Thus, under the transformation of the potential above, the image positions are unchanged, provided that we apply an isotropic scaling to the source plane.

A FUNDAMENTAL LIMIT OF MASS MODELING: THE MASS-SHEET DEGENERACY

$$\vec{\alpha}'(\vec{x}) = \vec{\nabla}\psi'(\vec{x}) = (1-\lambda)\vec{x} + \lambda\vec{\alpha}(\vec{x})$$

If we derive further, we obtain:

$$\begin{aligned} \kappa &\rightarrow \kappa' = (1-\lambda) + \lambda\kappa \\ \gamma &\rightarrow \gamma' = \lambda\gamma\ , \end{aligned}$$

This is called the "mass-sheet degeneracy" (Falco et al. 1985). Note that:

$$\begin{aligned} 1-\kappa-\gamma = \lambda_t &\rightarrow \lambda_t' = \lambda\lambda_t \\ 1-\kappa+\gamma = \lambda_r &\rightarrow \lambda_r' = \lambda\lambda_r\ . \end{aligned}$$

Thus, the transformation does not change the location of the critical lines

MASS SHEET DEGENERACY FOR GALAXY SHAPES

The mass sheet transformation on shear and convergence is:

$$\kappa \to \kappa' = (1-\lambda) + \lambda\kappa$$
$$\gamma \to \gamma' = \lambda\gamma$$

The ellipticity is then: $$\epsilon' = g' = \frac{\lambda\gamma}{1-(1-\lambda)-\lambda\kappa} = \frac{\lambda\gamma}{\lambda(1-\kappa)} = g = \epsilon$$

Thus, weak lensing is also variant under mass-sheet transformations!

AFFECTED QUANTITIES: TIME DELAYS AND MAGNIFICATIONS

$$
\begin{aligned}
\Delta t' &\propto \frac{1}{2}(\vec{x}-\vec{y}')^2 - \psi'(\vec{x}) \\
&= \frac{\vec{x}-\lambda\vec{y}}{2} - \lambda\psi(\vec{x}) - \frac{1-\lambda}{2}x^2 \\
&= \lambda\left[\frac{1}{2}(\vec{x}-\vec{y})^2 - \psi(\vec{x})\right] + \frac{\lambda(\lambda-1)}{2}y^2 \\
&= \lambda\Delta t + \mathrm{const}
\end{aligned}
$$

$$
\mu' = (\lambda_t'\lambda_r')^{-1} = (\det A')^{-1} = (\lambda^2 \det A)^{-1} = \frac{\mu}{\lambda^2}
$$

Time delays and magnifications are changed by mass-sheet transformations.

Thus: to determine slope of mass profiles, absolute masses (away from the ER), Hubble constant, MSD must first be broken!

BREAKING THE DEGENERACY

- Using sources at different redshifts
- Complementary measurements of the mass profiles
  - Example: using stellar kinematics, in the case of an elliptical galaxy
- Adopting a shape for the mass profile
- Assuming that the convergence goes to zero at large distances from the center of the lens
- Measuring the magnification statistically, or via galaxy number counts

USING GALAXY NUMBER COUNTS

$$n(>S) = n_0(S/\mu)/\mu$$

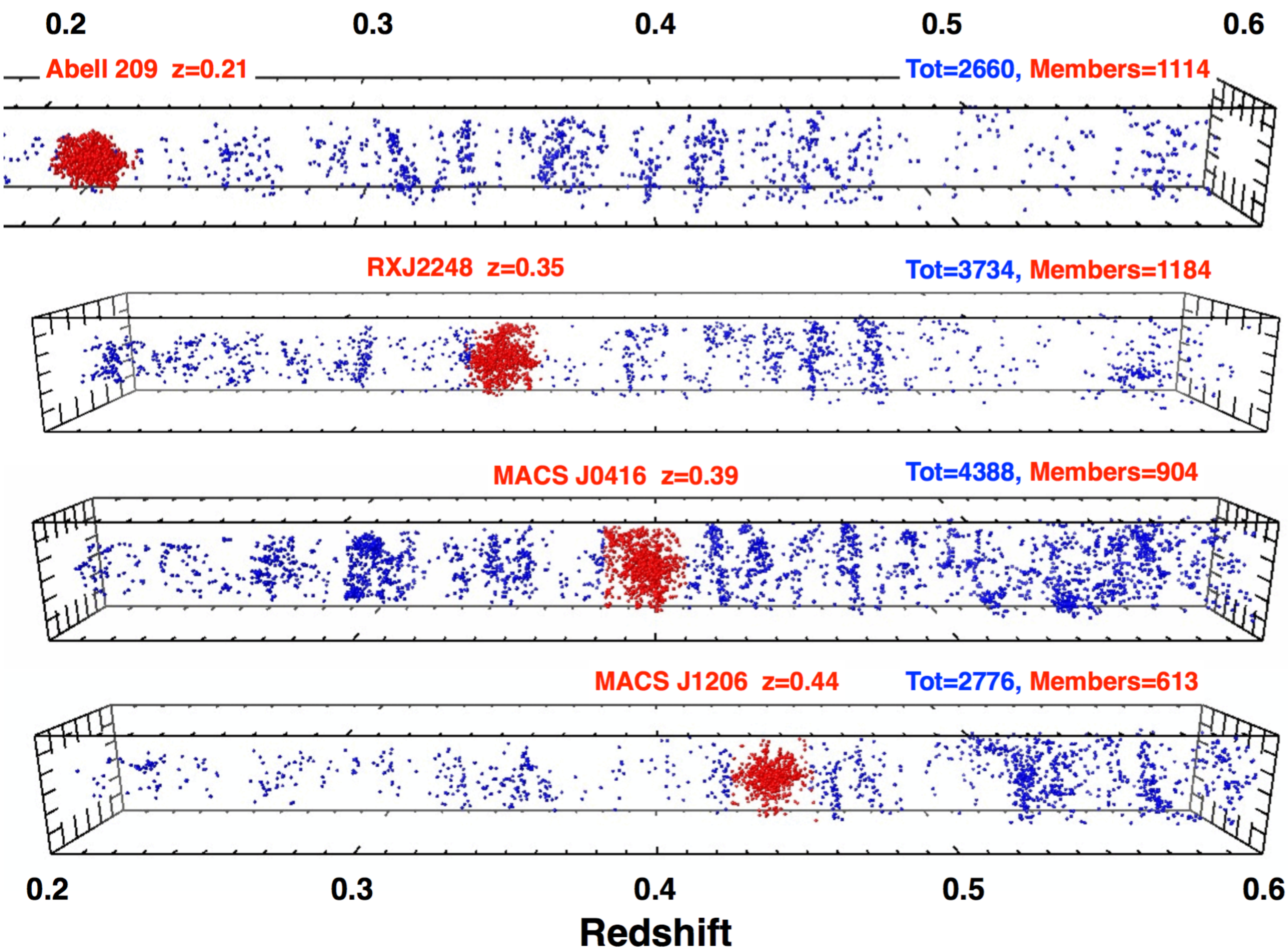
$$n_0(>S) \propto S^{-\alpha}$$

$$n(>S) \propto \frac{S^{-\alpha}}{\mu^{1-\alpha}}$$

$$n(>S)/n_0(>S) = \mu^{\alpha-1}$$

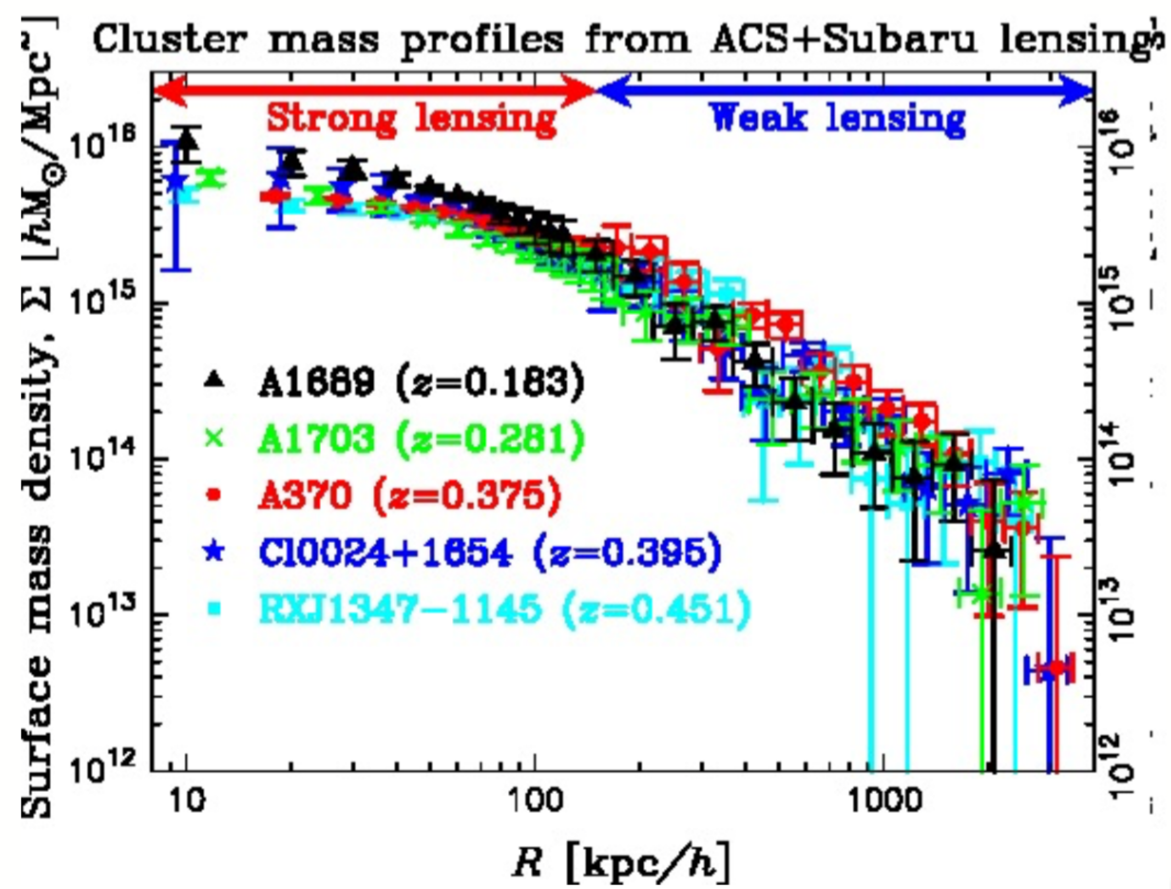
Knowing the unlensed number density of galaxies, and the slope of the number counts, one can estimate the magnification and break the degeneracy

YET ANOTHER LIMIT: PERTURBATIONS ALONG THE LINE OF SIGHT



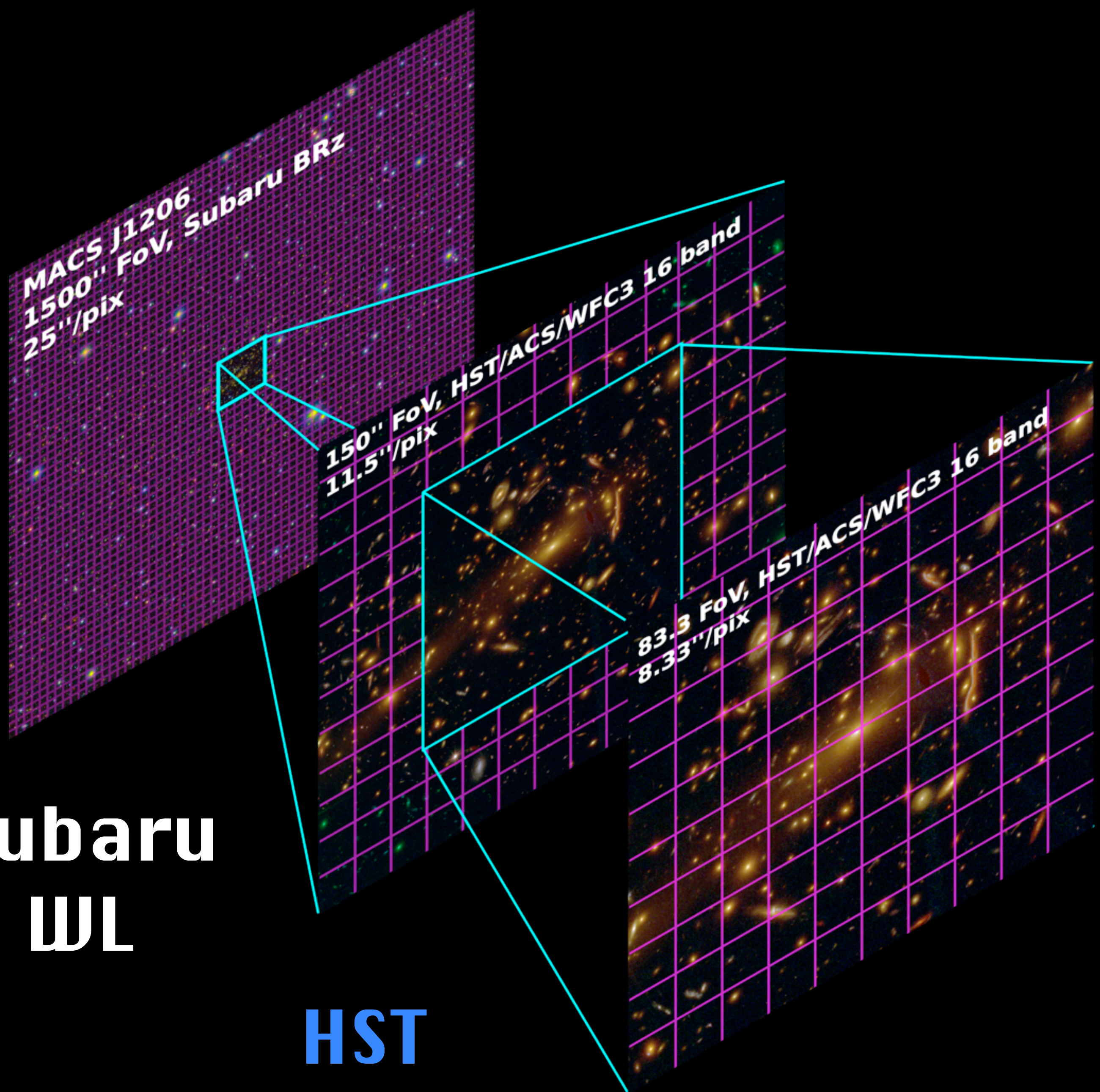
MASS PROFILES

- SL in clusters probes scales ~10-50”. Assuming z~0.5, this corresponds to scales ~60-300 kpc
- typical scale radii for cluster-sized halos are >200 kpc
- thus, SL alone cannot constrain well the scale radius, neither the virial radius, i.e. the concentration
- weak lensing probes the mass distribution outside the SL region
- the combination of SL and WL is a powerful method to measure the total mass profile



Umetsu et al. 2011

CLASH reconstructions



Subaru WL

HST WL

HST SL

SaWLens

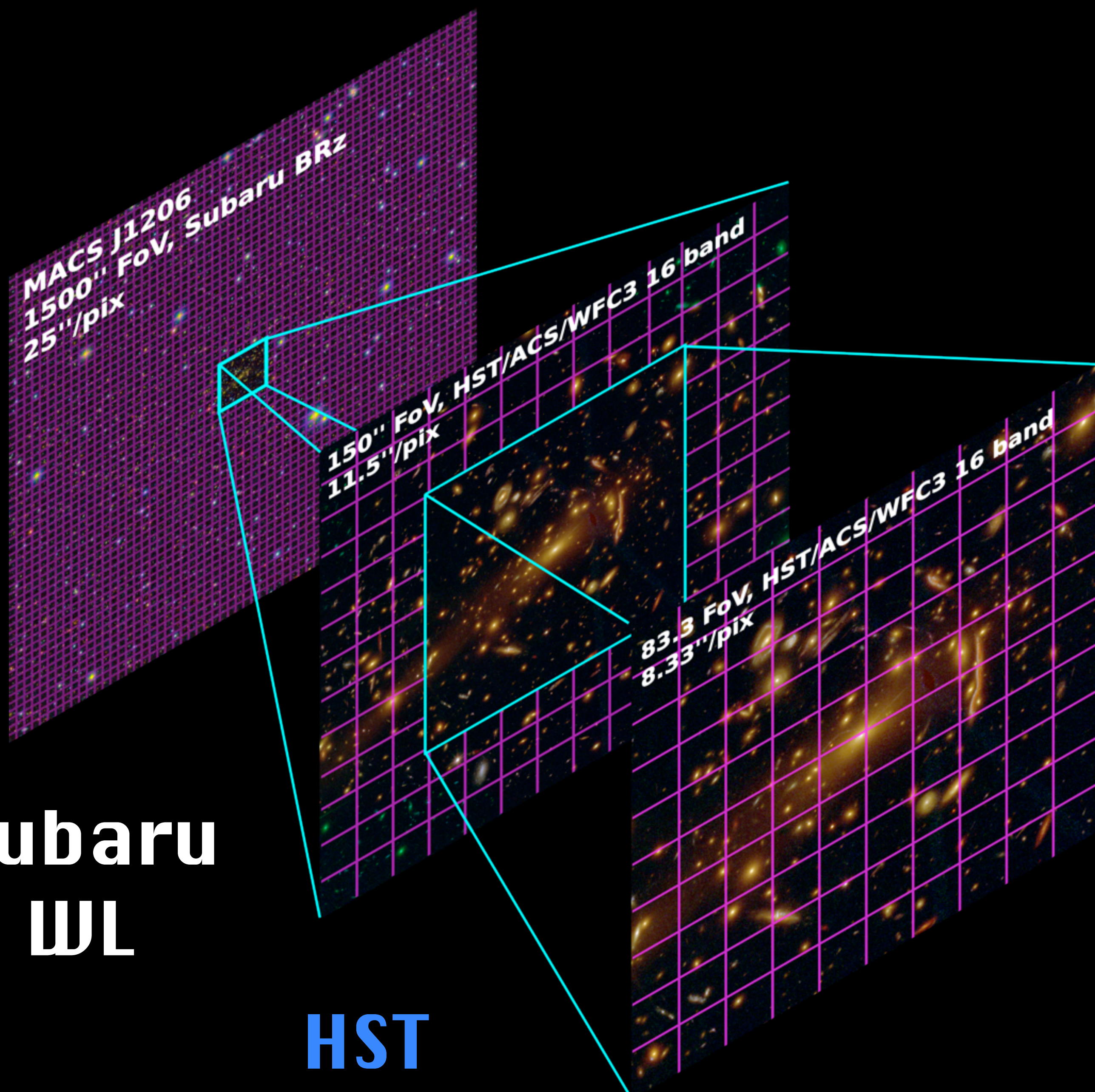
implements WL, SL and (Flexion)

operates on adaptively refined grids (AMR)

non-parametric method, this means that we make no assumptions on the lens' mass profile

JM et al. 2009

CLASH reconstructions



Subaru
WL

HST
WL

HST
SL

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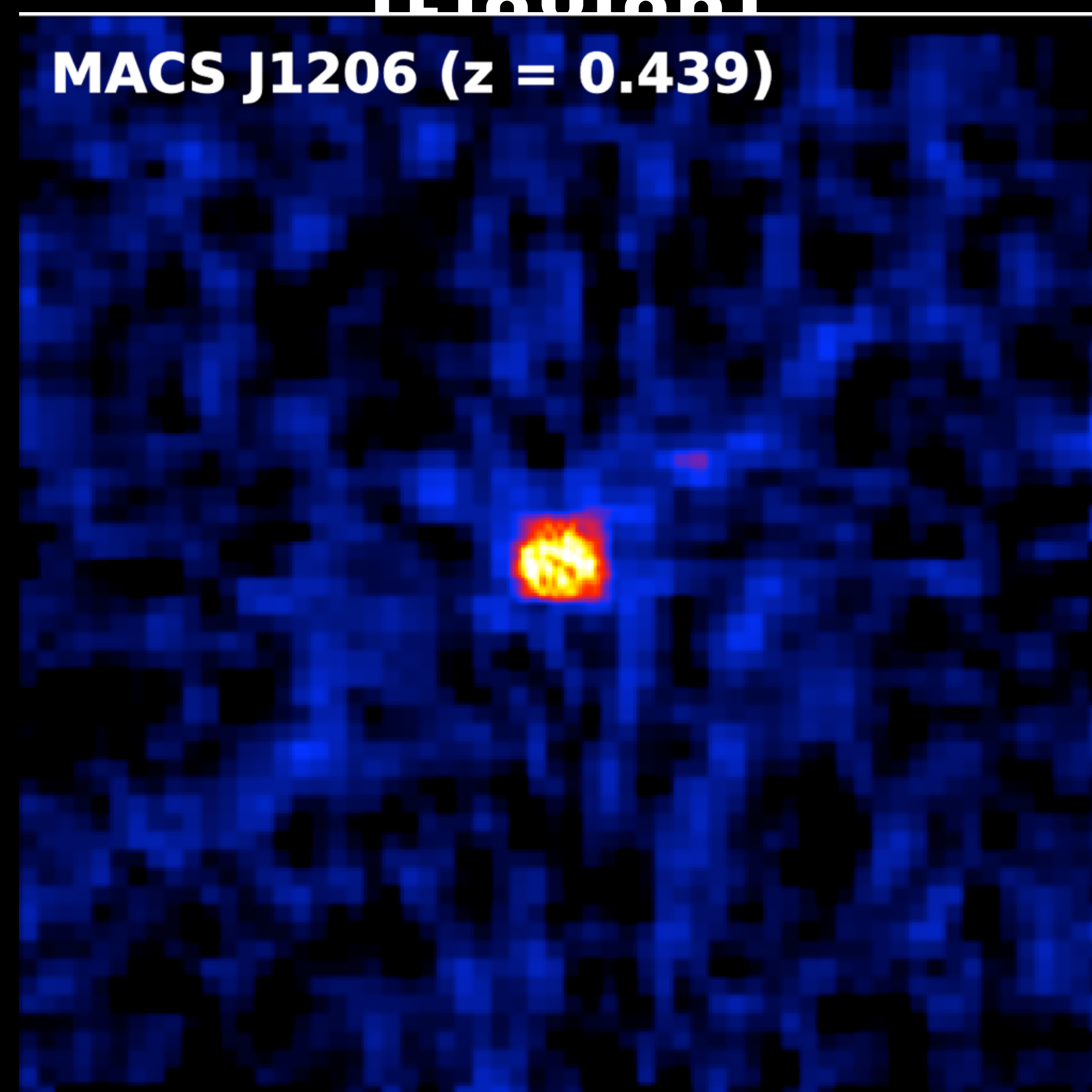
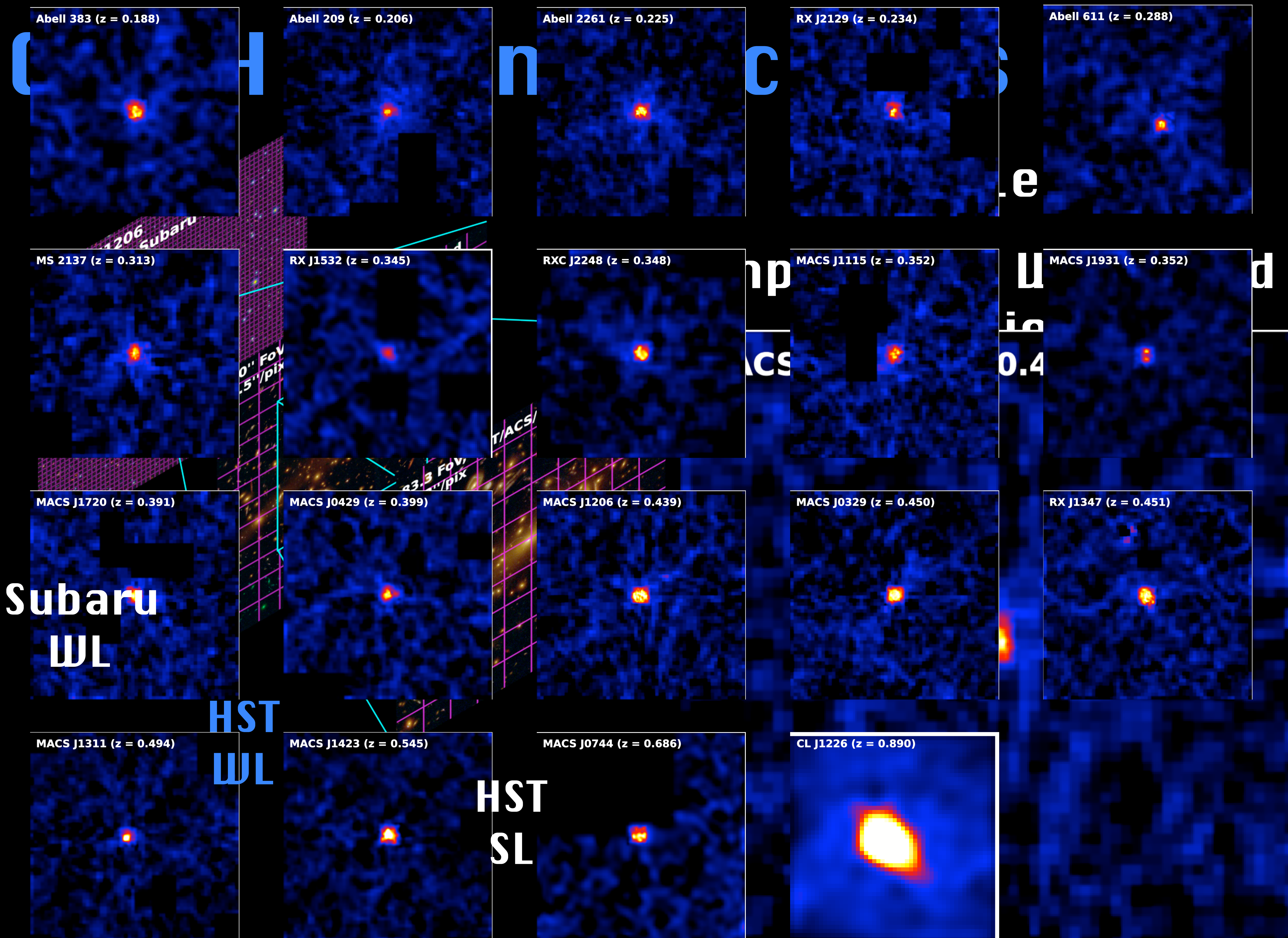
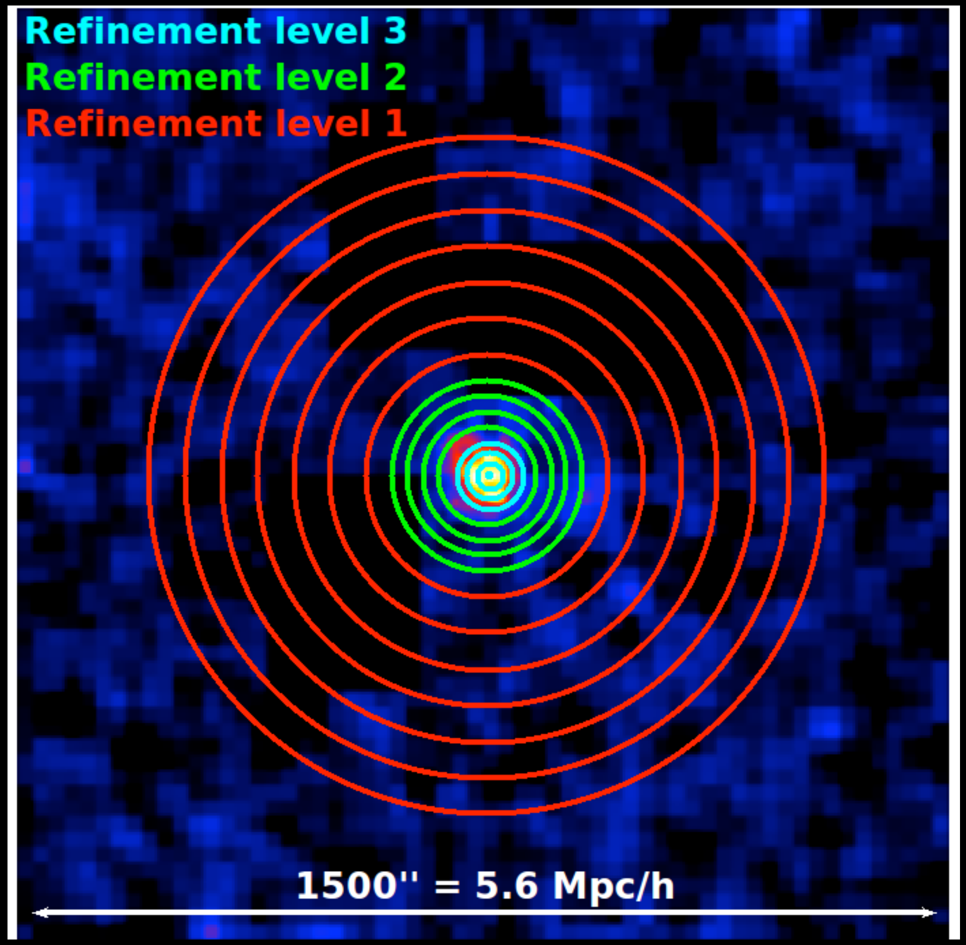


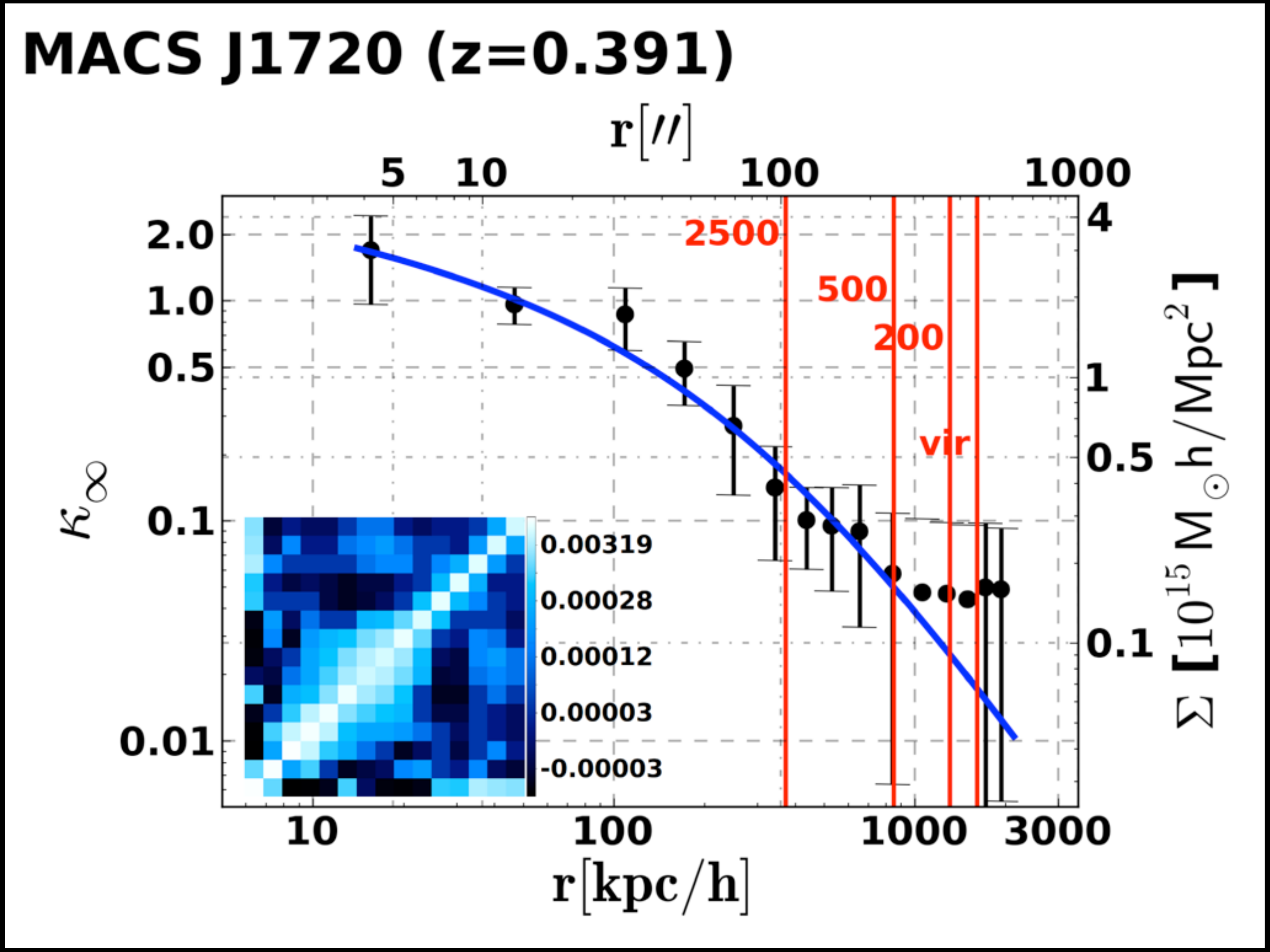


Abell 383 (z = 0.188)
Abell 209 (z = 0.206)
Abell 2261 (z = 0.225)
RX J2129 (z = 0.234)
Abell 611 (z = 0.288)
MS 2137 (z = 0.313)
RX J1532 (z = 0.345)
RXC J2248 (z = 0.348)
MACS J1115 (z = 0.352)
MACS J1931 (z = 0.352)
MACS J1720 (z = 0.391)
MACS J0429 (z = 0.399)
MACS J1206 (z = 0.439)
MACS J0329 (z = 0.450)
RX J1347 (z = 0.451)
MACS J1311 (z = 0.494)
MACS J1423 (z = 0.545)
MACS J0744 (z = 0.686)
CL J1226 (z = 0.890)
Subaru
WL
HST
WL
HST
SL

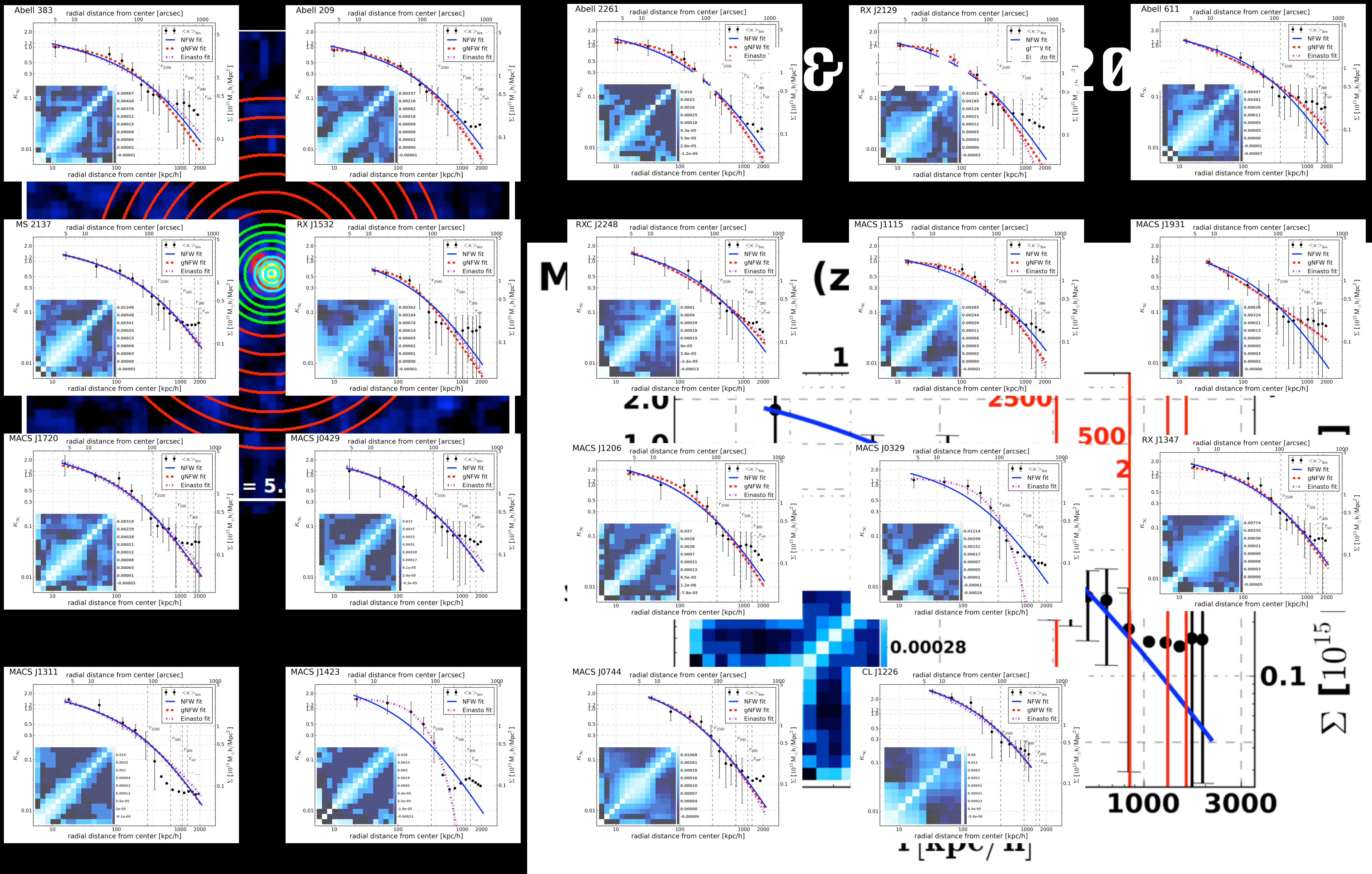
CLASH profiles



JM & CLASH 2014



CLASH profiles

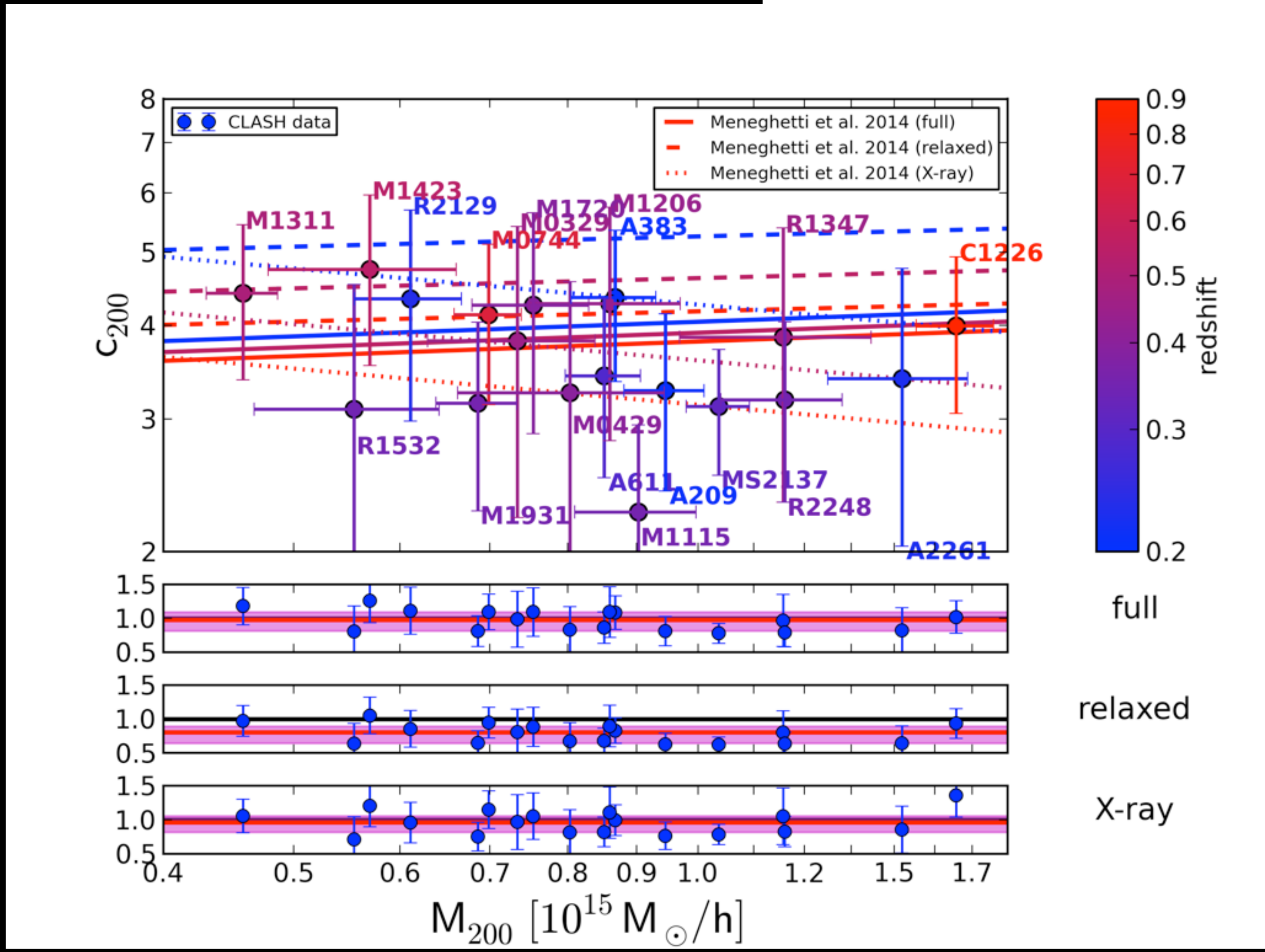


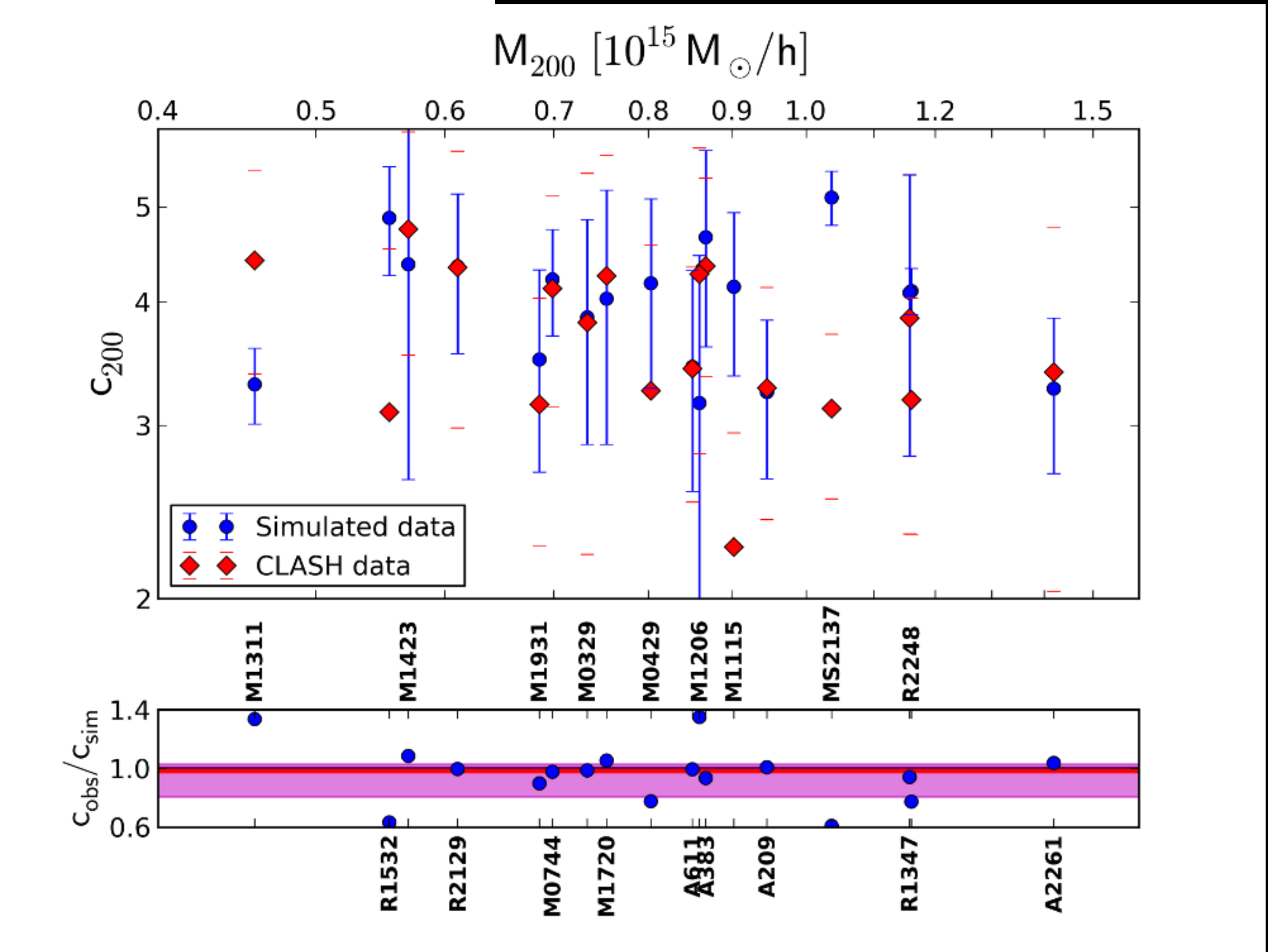
Tailored comparisons

Table 9
Goodness-of-fit: Meneghetti et al. 2014

Sample	$\langle c_{obs}/c_{sim} \rangle$	Q_2	Q_1	Q_3	χ^2	p-value
3D full	1.05 ± 0.16	1.09	0.90	1.20	5.7	0.97
3D relaxed	0.85 ± 0.15	0.88	0.71	0.97	18.7	0.18
2D full	0.97 ± 0.15	0.98	0.82	1.09	7.3	0.92
2D relaxed	0.79 ± 0.14	0.81	0.64	0.89	33.7	0.00
2D rel.+SL	0.82 ± 0.18	0.81	0.69	0.94	28.6	0.01
2D X-ray	0.96 ± 0.18	0.96	0.82	1.06	9.5	0.80

Note. — The column explanations are identical to Tab. 8.



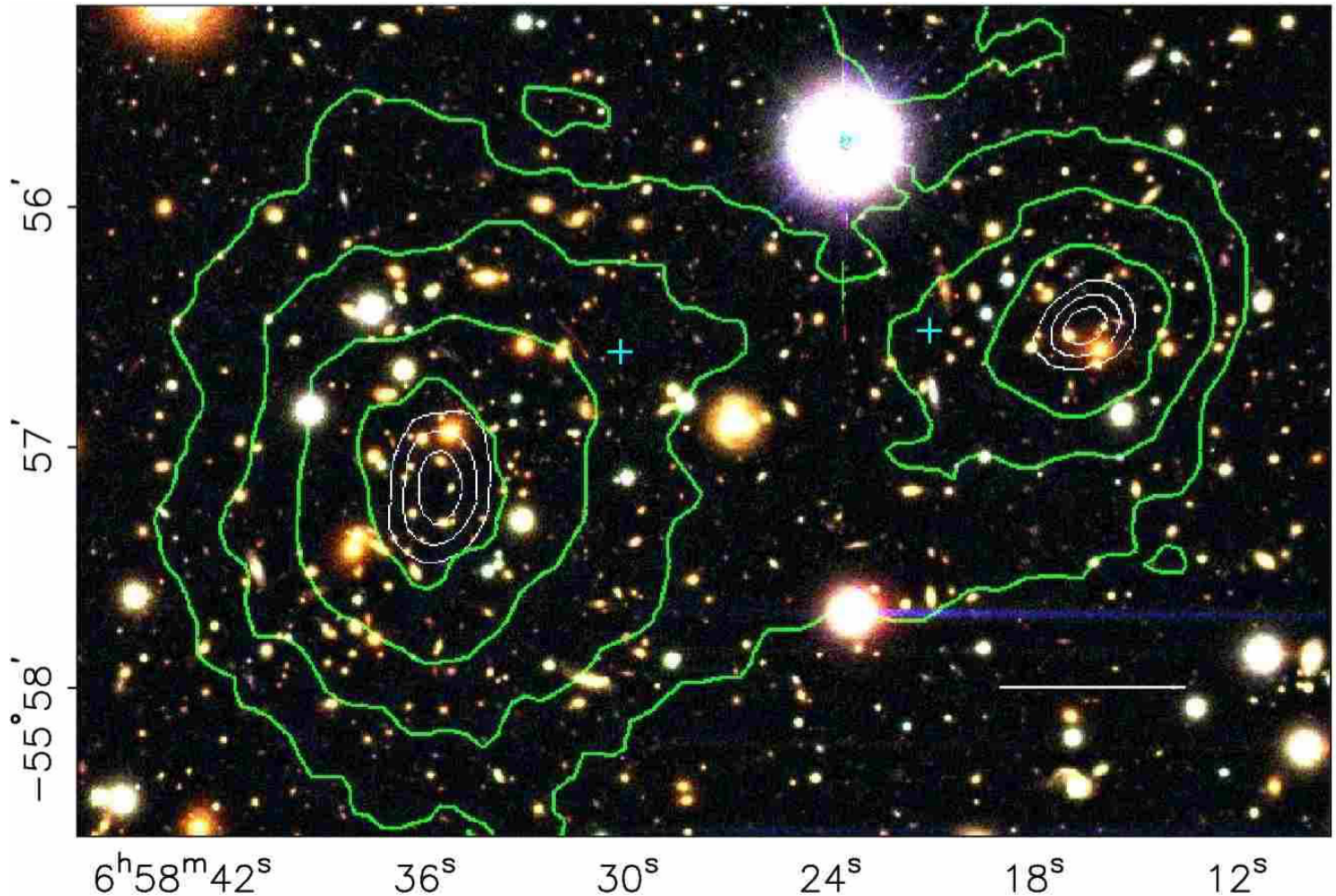


JM & CLASH 2014

THE NATURE OF DM FROM WEAK LENSING

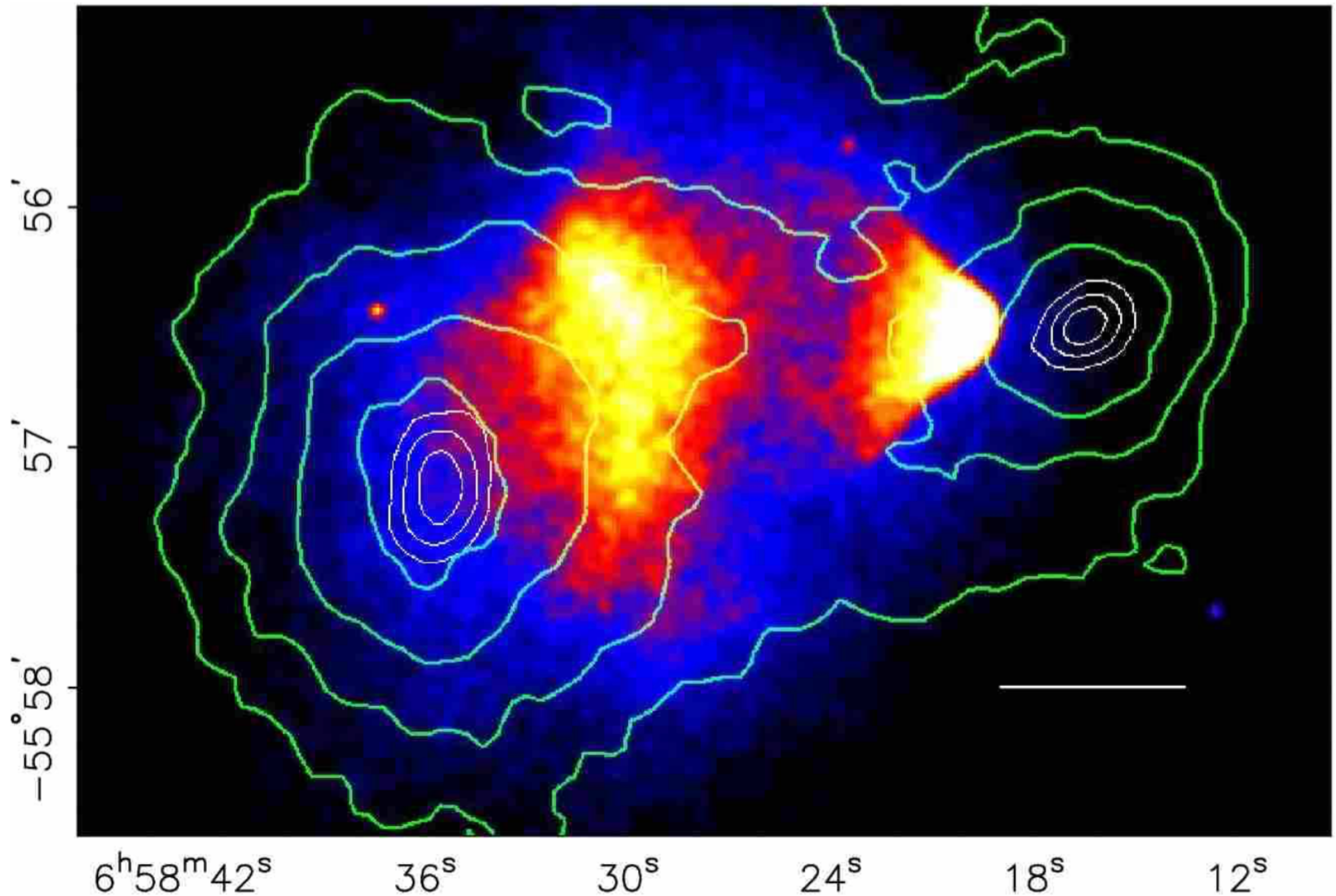
Clusters are dominated by collisionless matter:

The Bullet Cluster is a pair of colliding galaxy clusters (Clowe et al. 2006)



THE NATURE OF DM FROM WEAK LENSING

X-ray emission from the bullet cluster

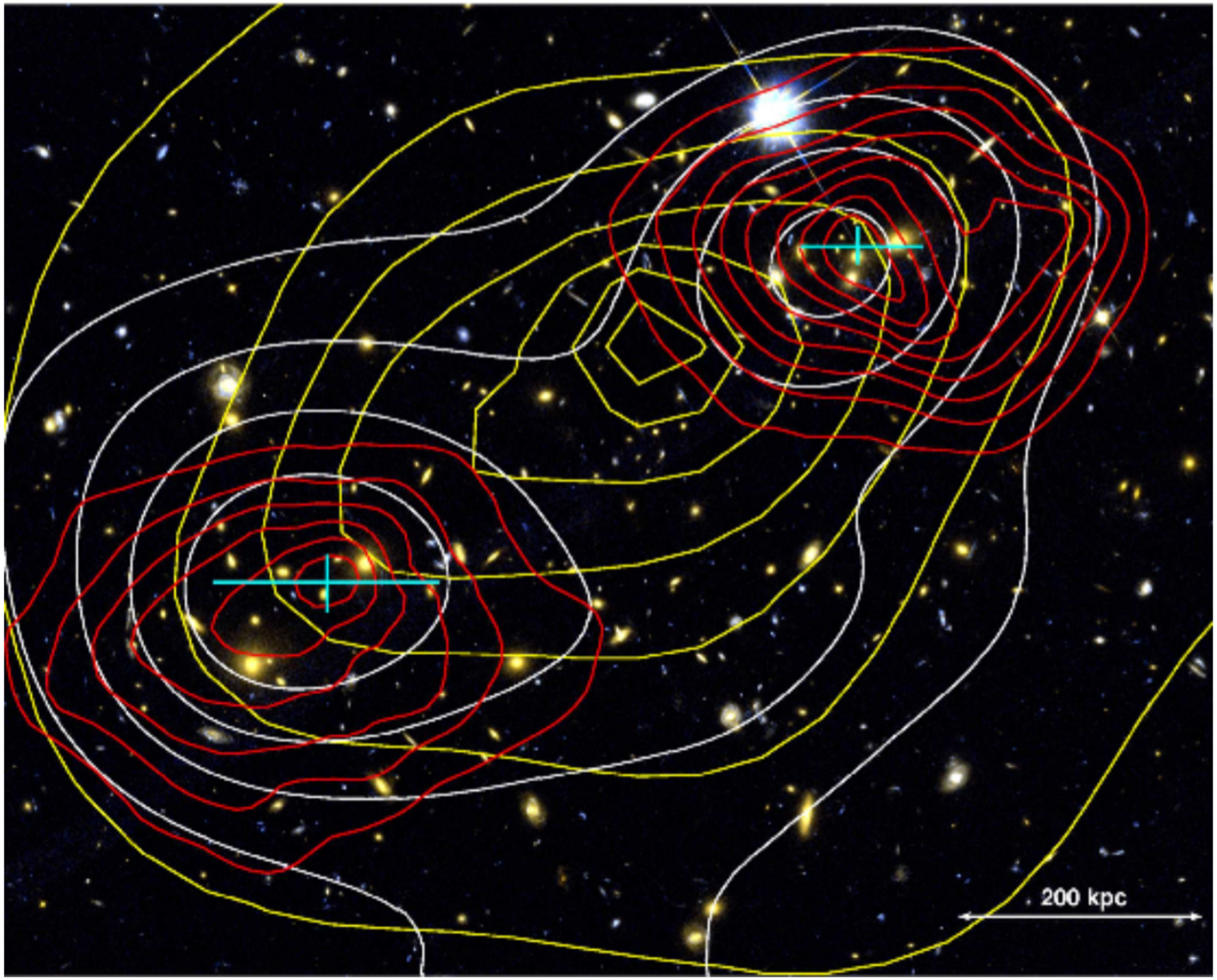


THE NATURE OF DM FROM WEAK LENSING

- Lensing shows that most of the mass is located near the galaxies,
- and not centered on the gas, which is displaced by the collision.
- $\Rightarrow$ Most of the mass in this cluster pair must behave collisionless, like galaxies.
- Most of the mass is dark matter – the bullet cluster can not be explained by changing the law of gravity without invoking collisionless dark matter.
- The bullet cluster is not the only case where this clear distinction can be made...

THE NATURE OF DM FROM WEAK LENSING

The cluster MACS J0025.4−1222 ($z = 0.59$)



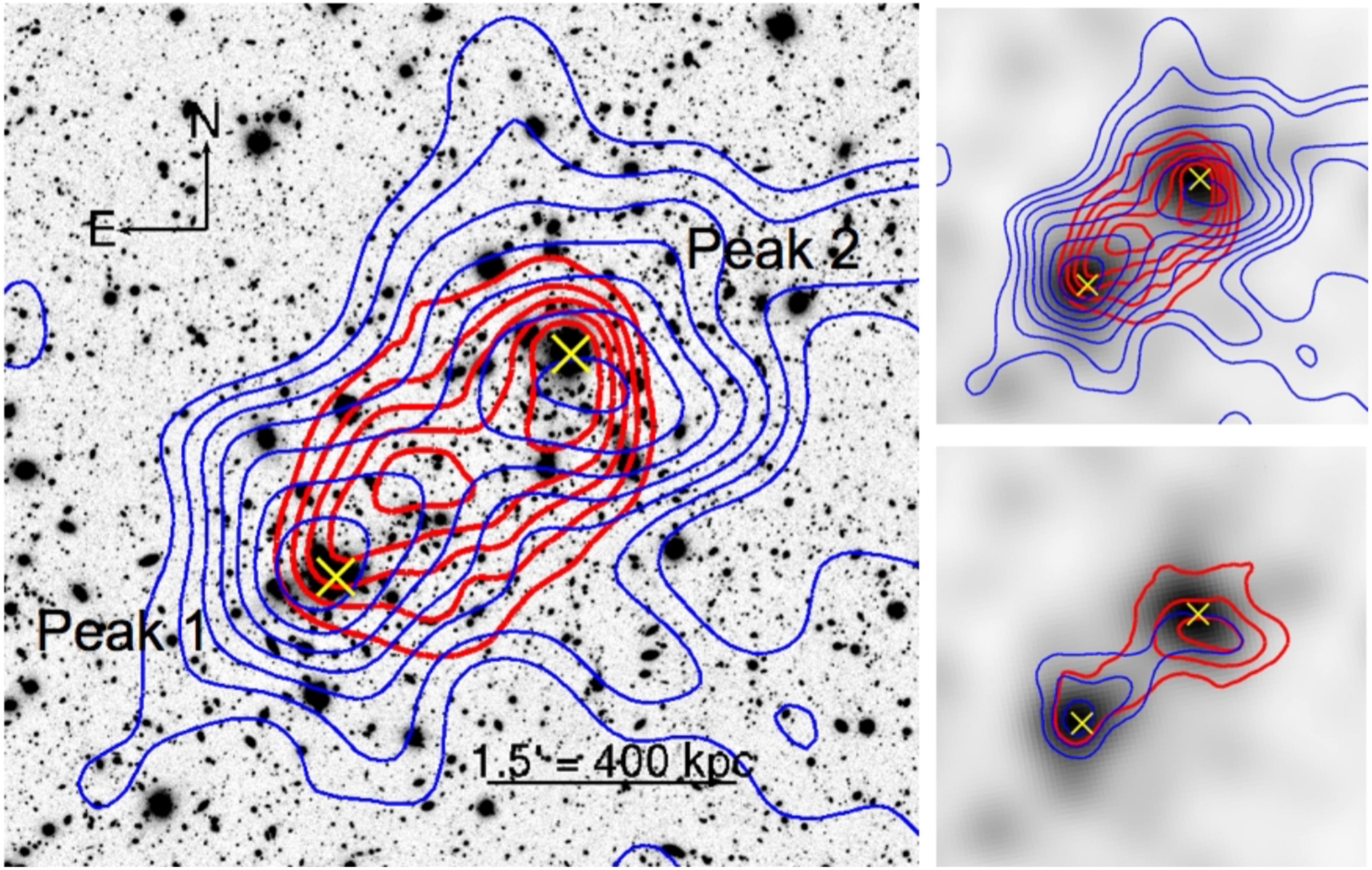
(Bradac et al. 2008)

red:
surface mass density;

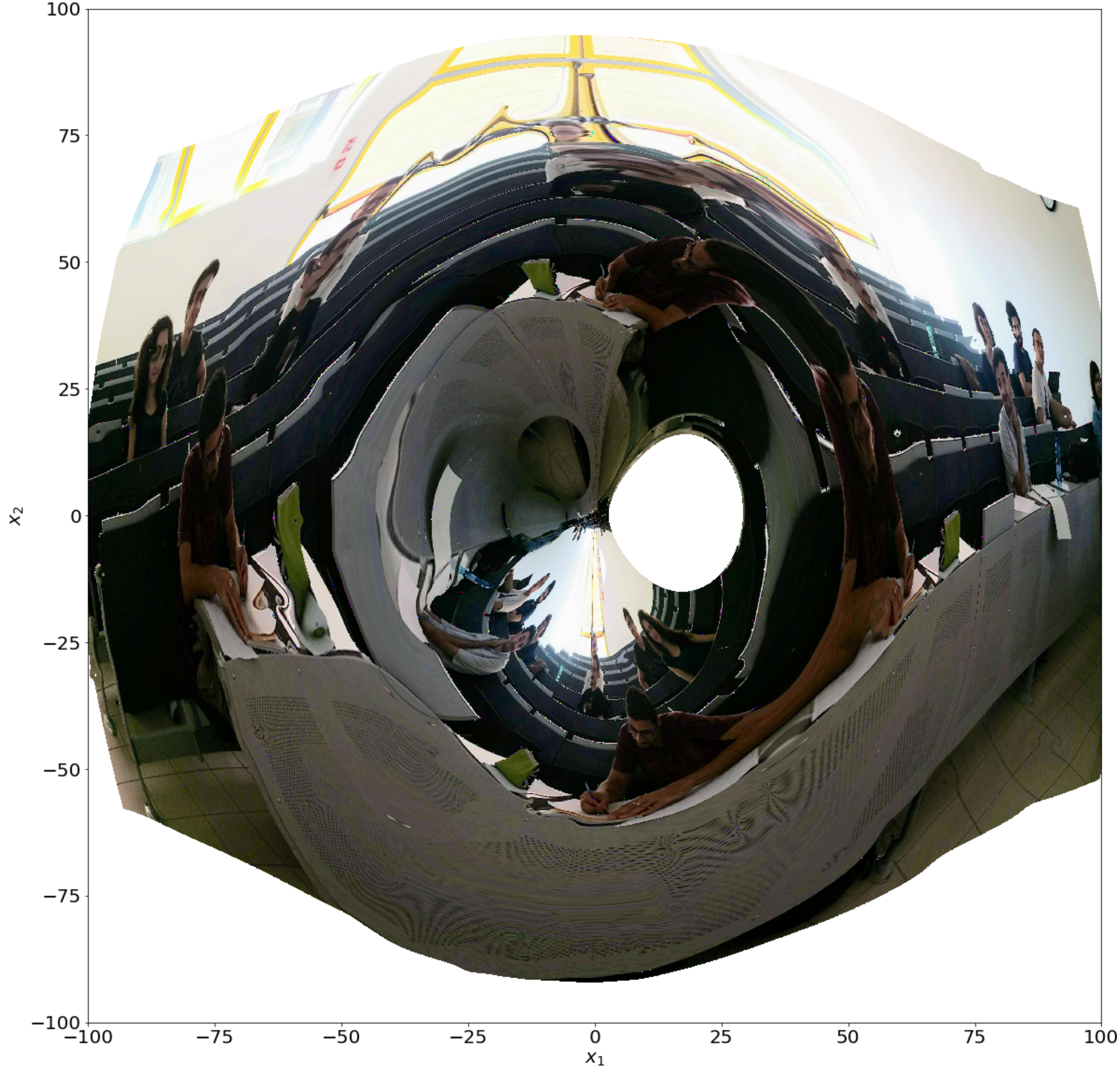
yellow:
X-ray emission;

white:
smoothed optical light.

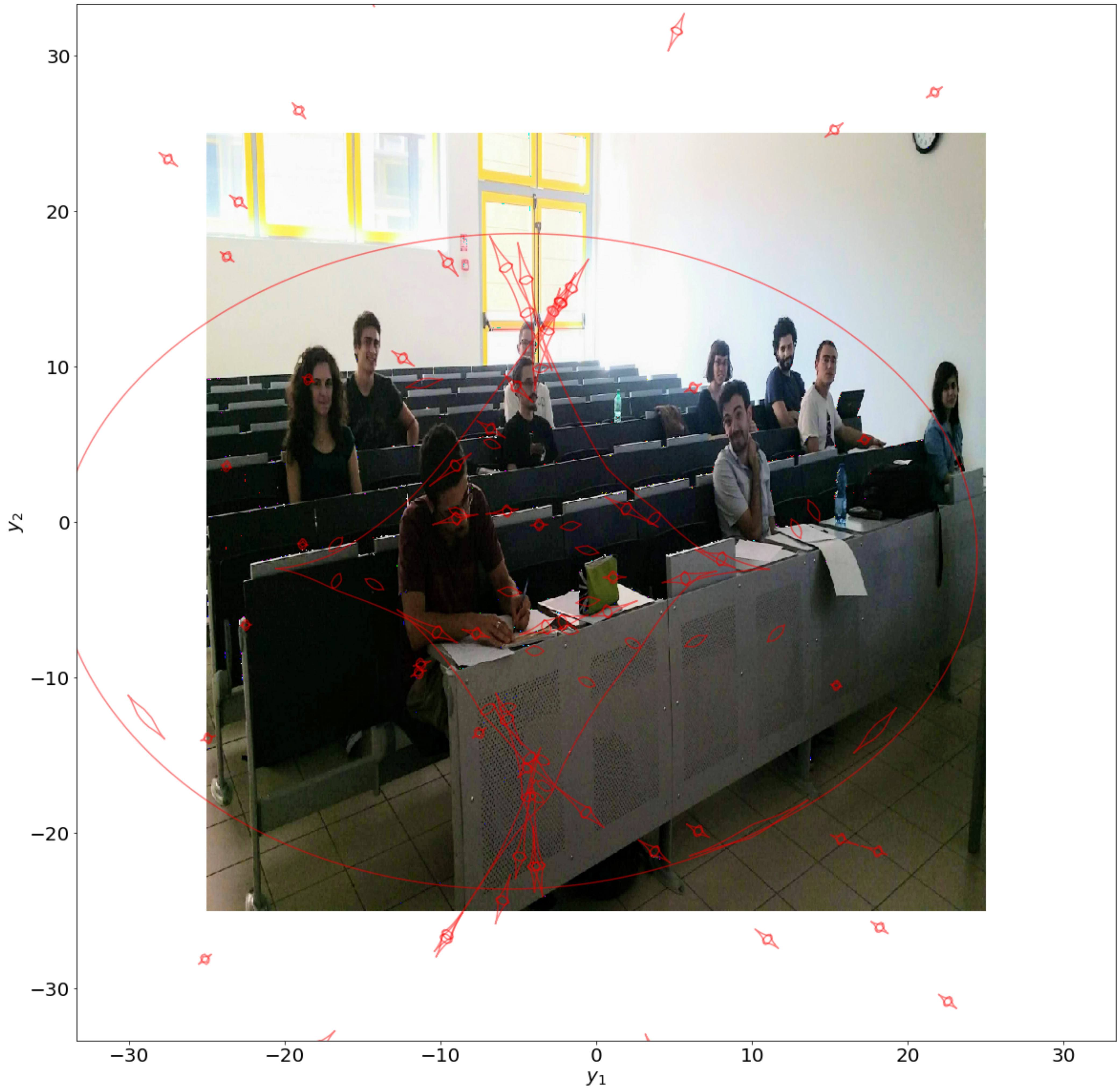
THE NATURE OF DM FROM WEAK LENSING

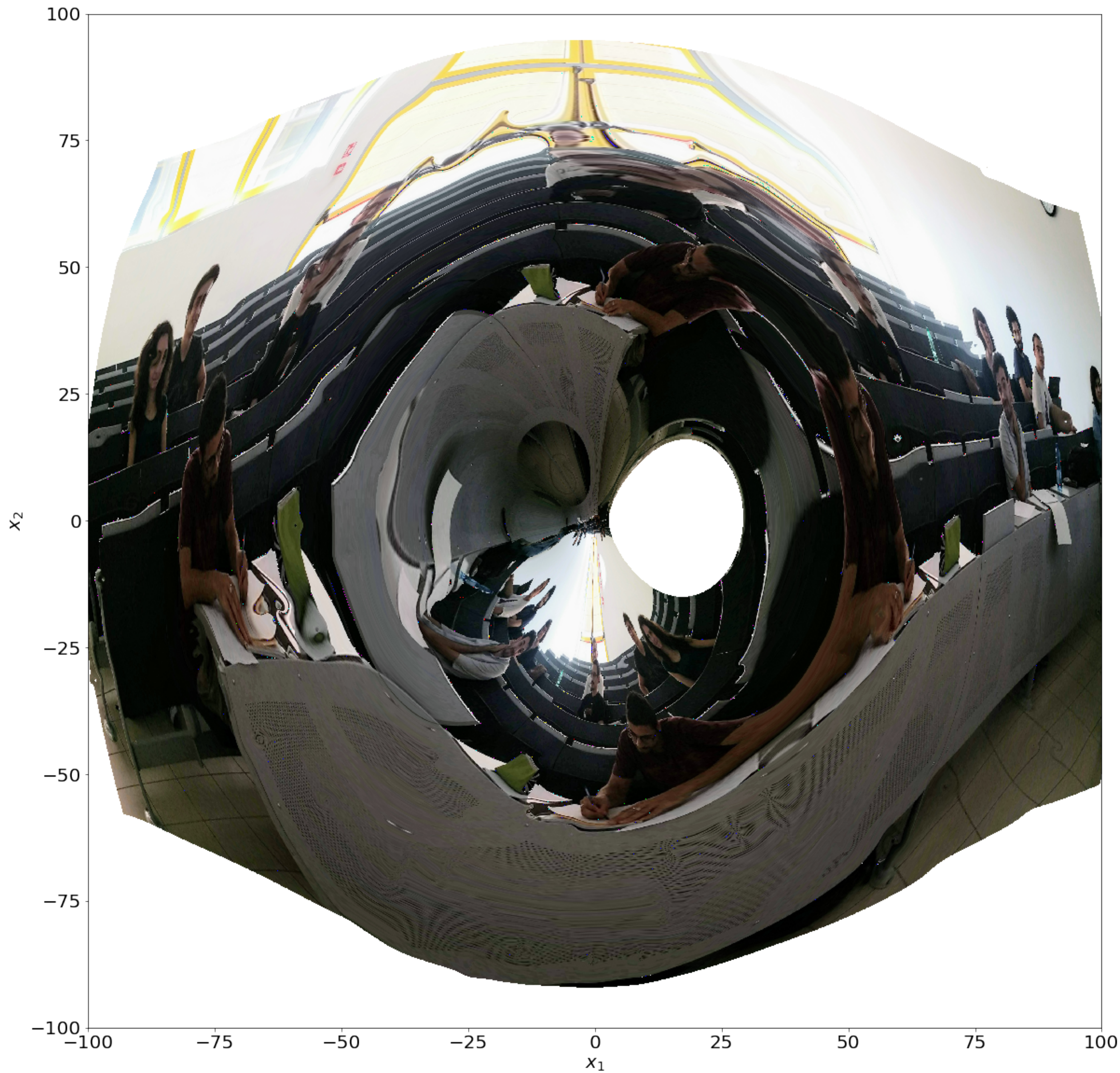


A1758N (Ragozzine & Clowe 2011)
Blue: mass reconstruction; red: X-ray emission

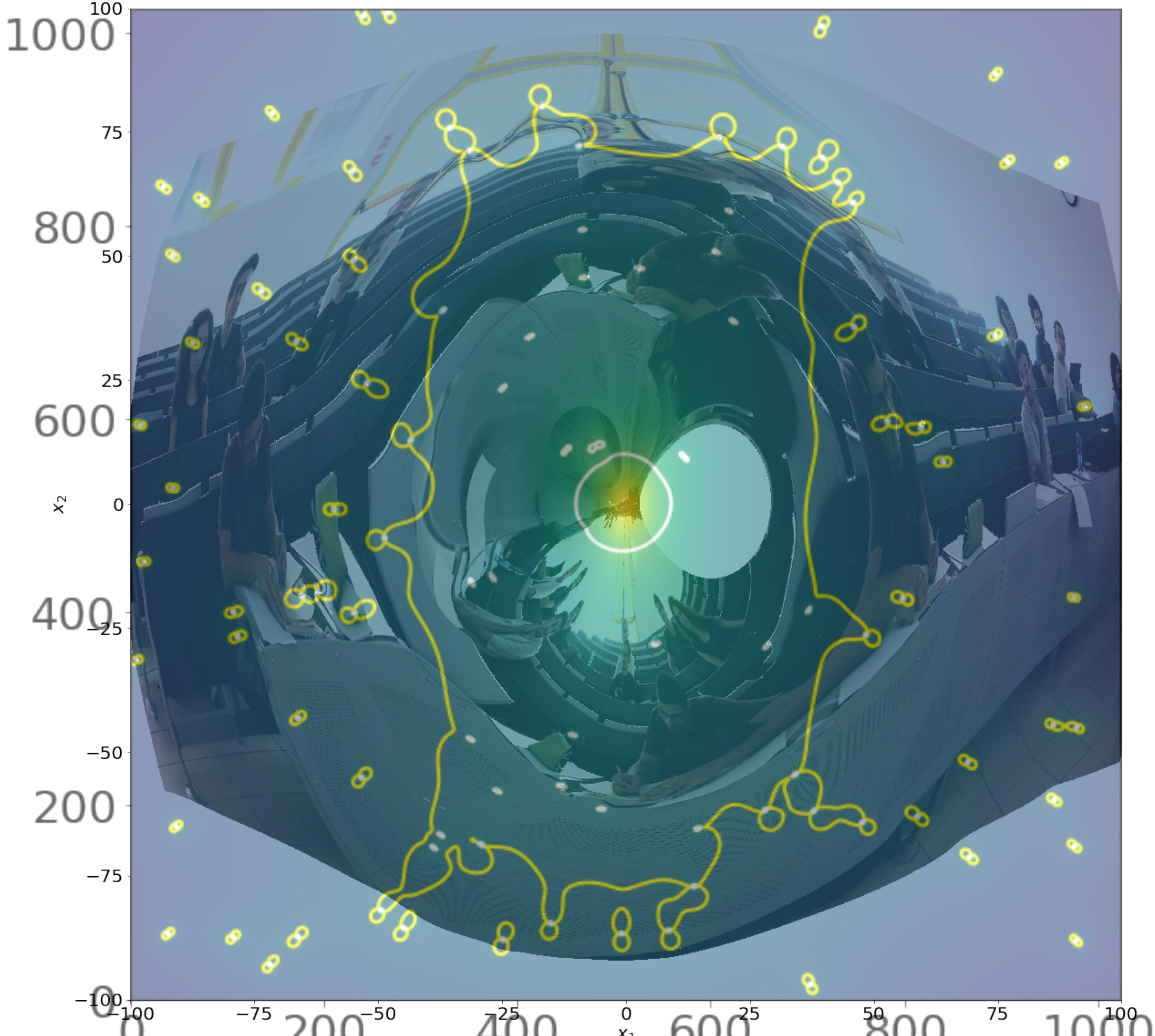


100
75
50
25
0
−25
−50
−75
−100
x_2
−100
−75
−50
−25
0
25
50
75
100
x_1


30
20
10
0
−10
−20
−30
y_2
−30
−20
−10
0
10
20
30
y_1



100
75
50
25
0
−25
−50
−75
−100
x_2
−100
−75
−50
−25
0
25
50
75
100
x_1



x_1
x_2
−100 −75 −50 −25 0 25 50 75 100
100 75 50 25 0 −25 −50 −75 −100
0 200 400 600 800 1000
1000 800 600 400 200 0