GRAVITATIONAL LENSING

11 - MICROLENSING PARALLAX AND ASTROMETRIC MICROLENSING

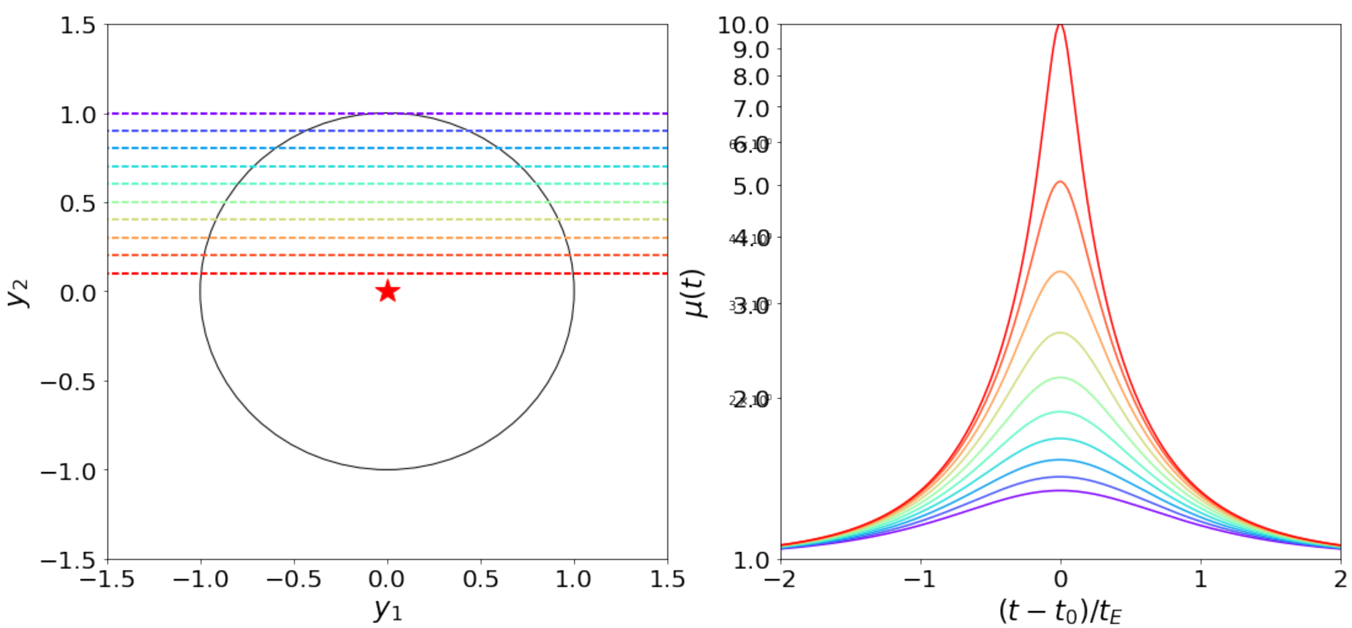
Massimo Meneghetti

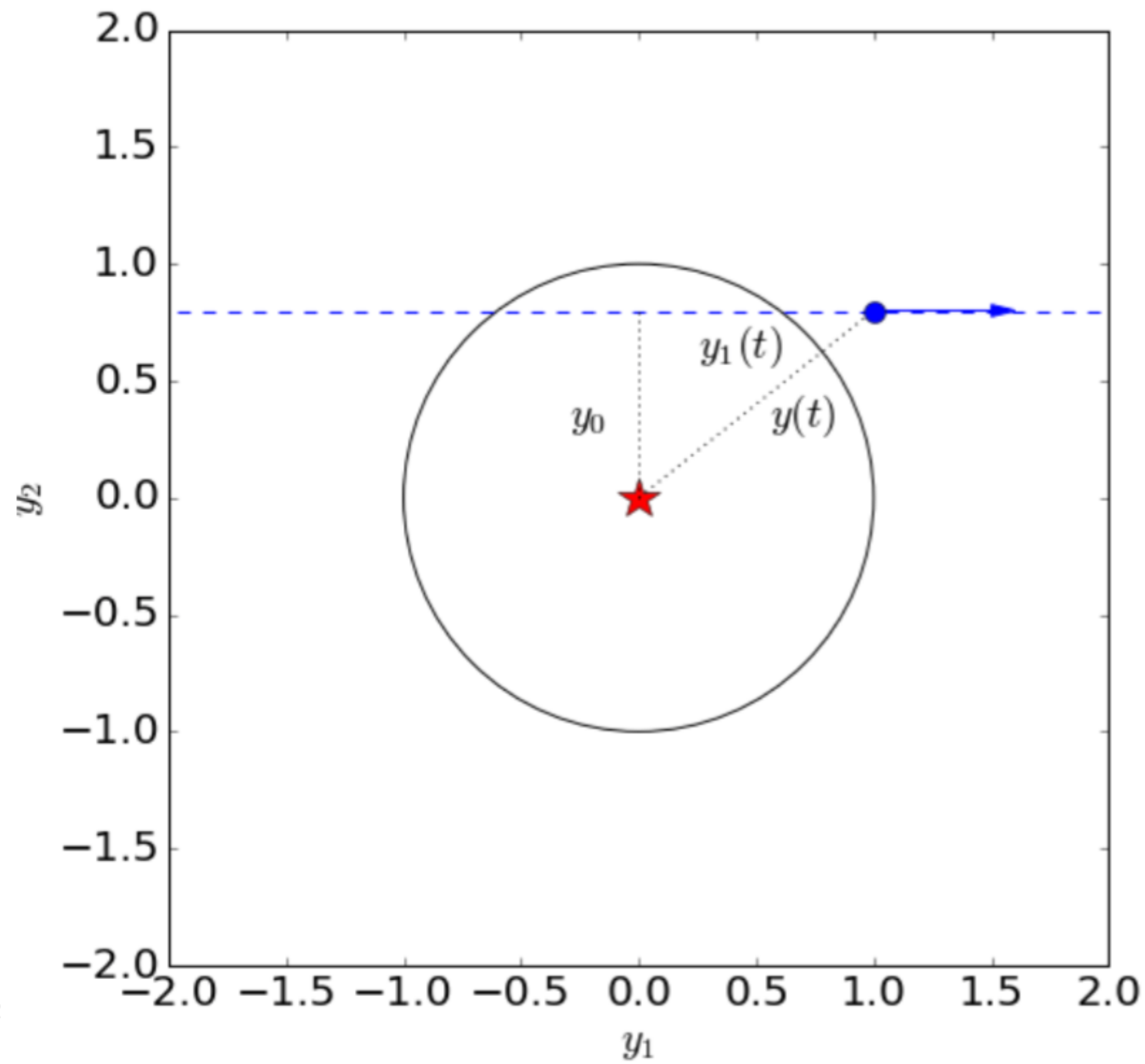
AA 2017-2018

CHANGES IN PERSPECTIVE

$$y(t) = \sqrt{y_0^2 + y_1^2(t)} = \sqrt{y_0^2 + \frac{(t-t_0)^2}{t_E^2}}$$

$$\mu(t) = \frac{y^2(t)+2}{y(t)\sqrt{y^2(t)+4}}$$





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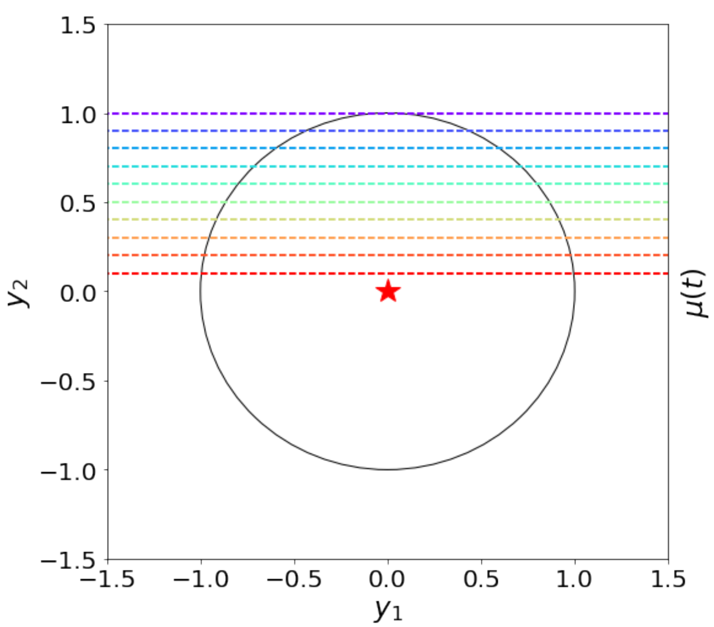

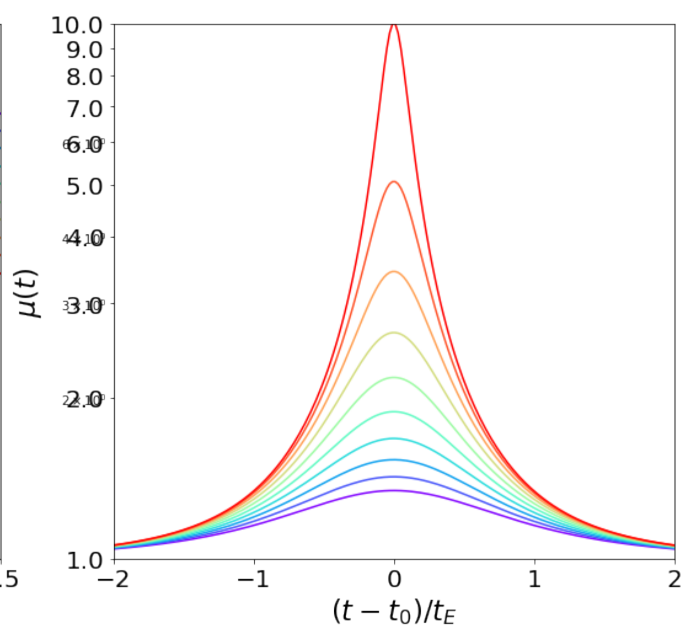

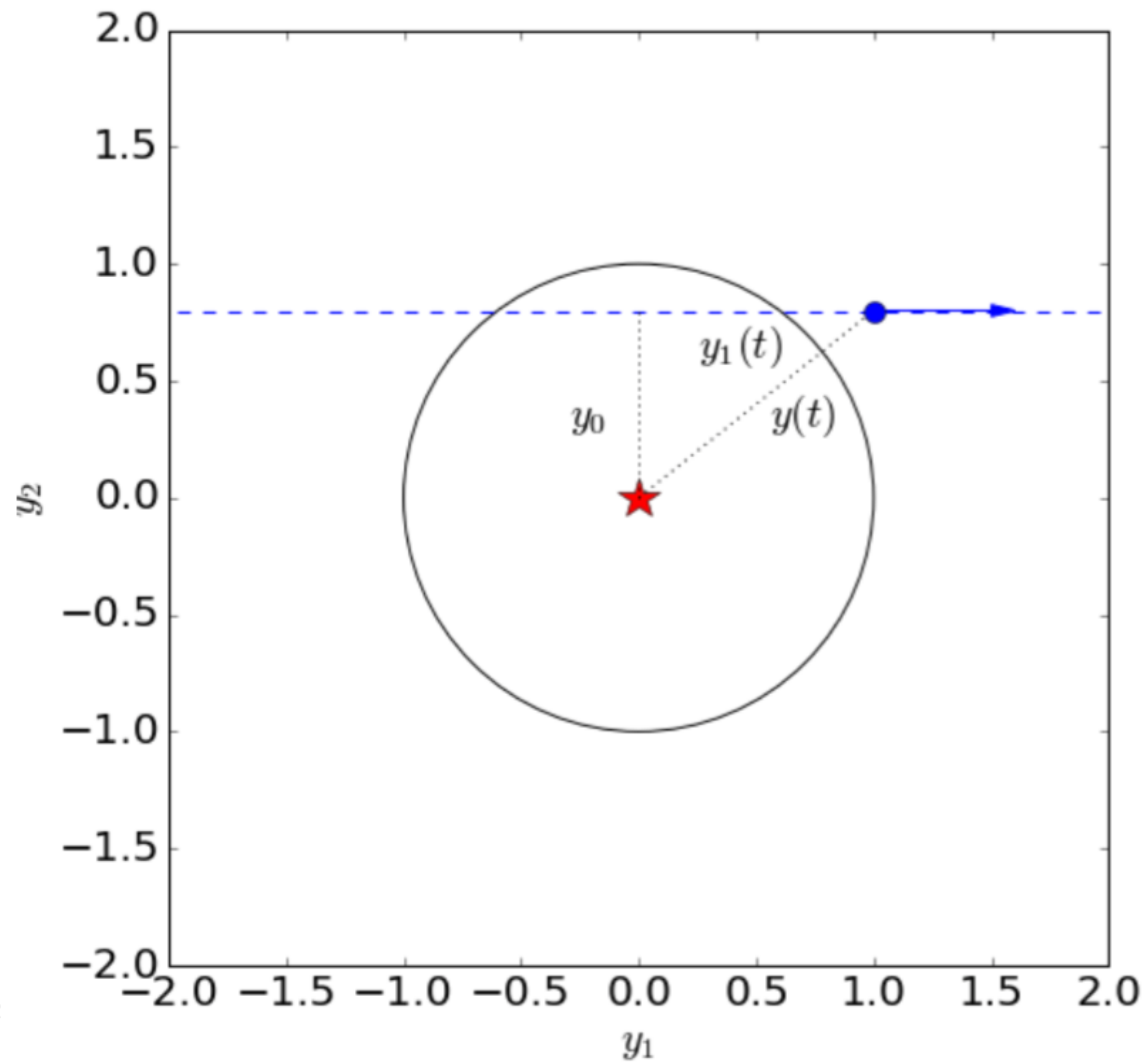

Q: What happens if by means of perspective effects the lens position changes relative to the source?

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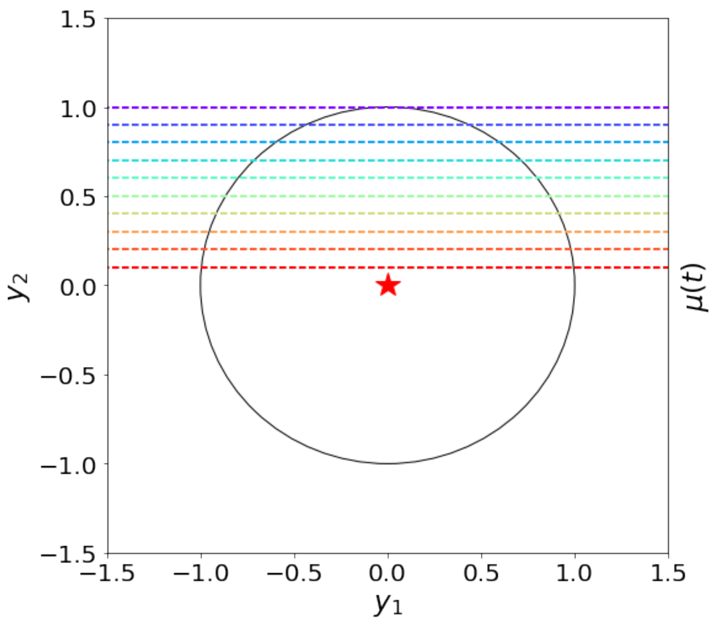

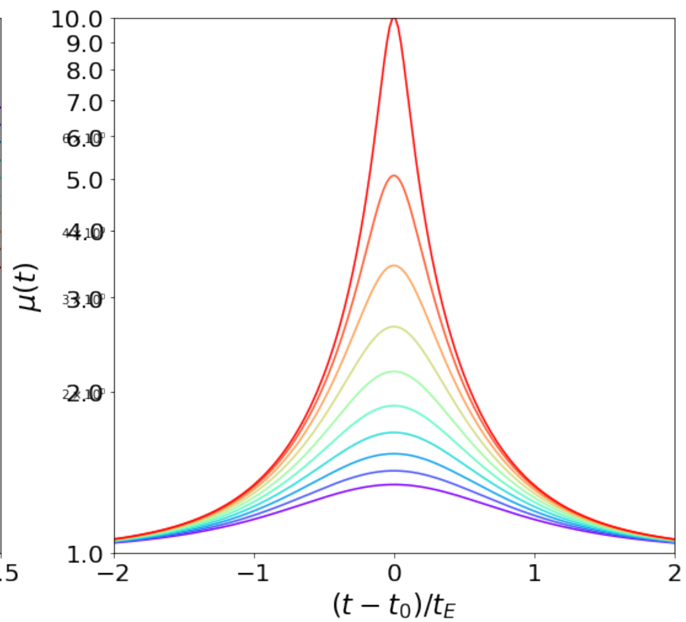

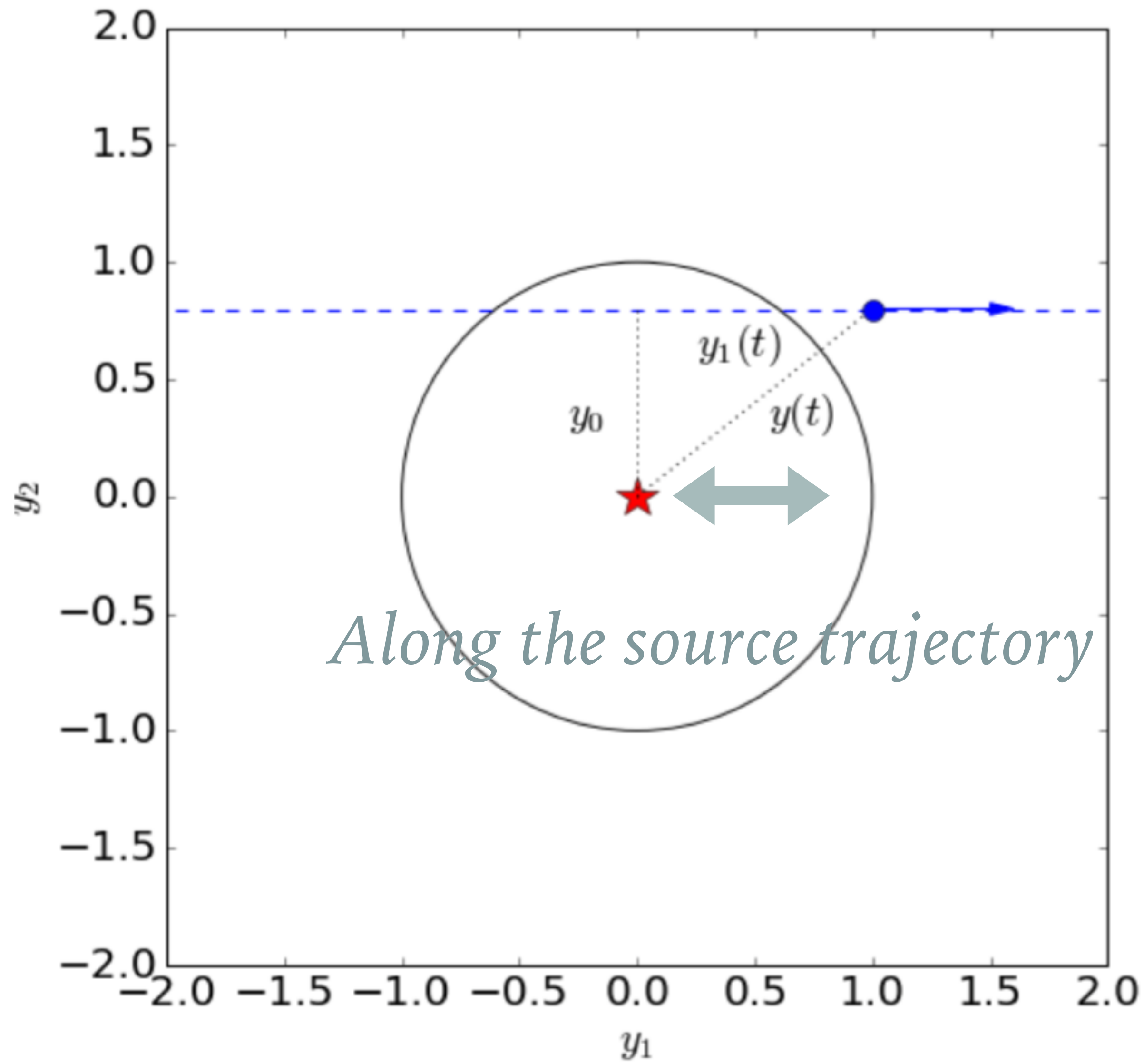

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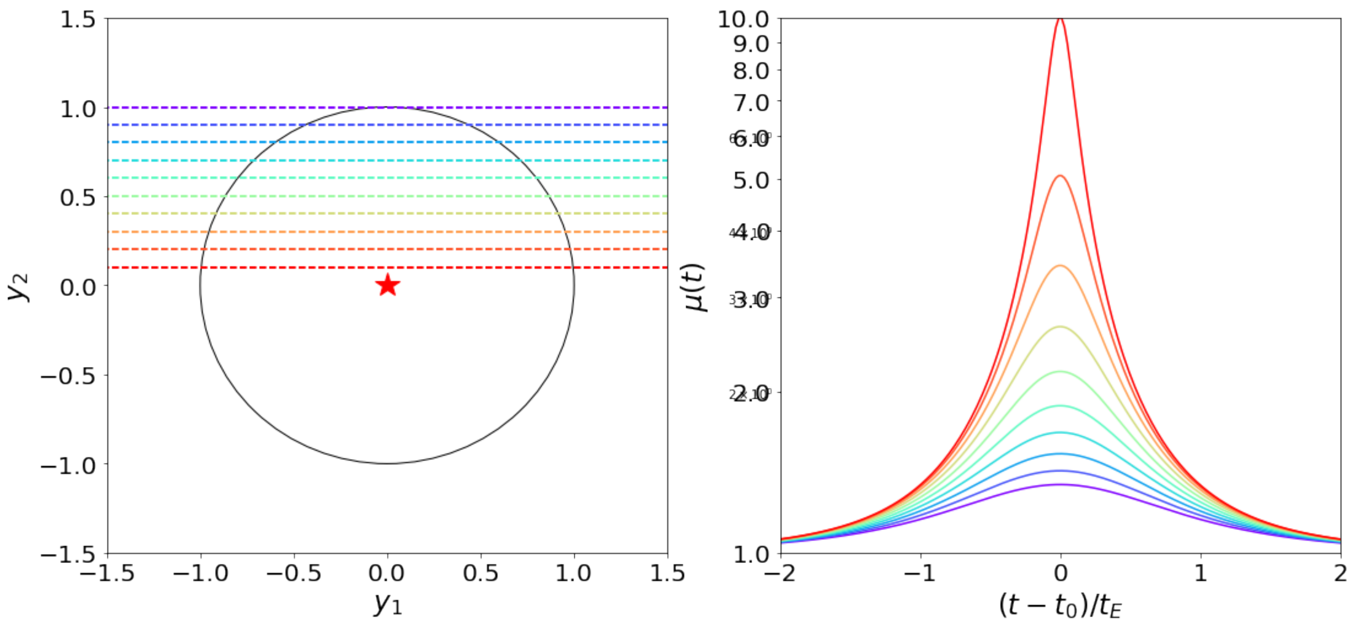

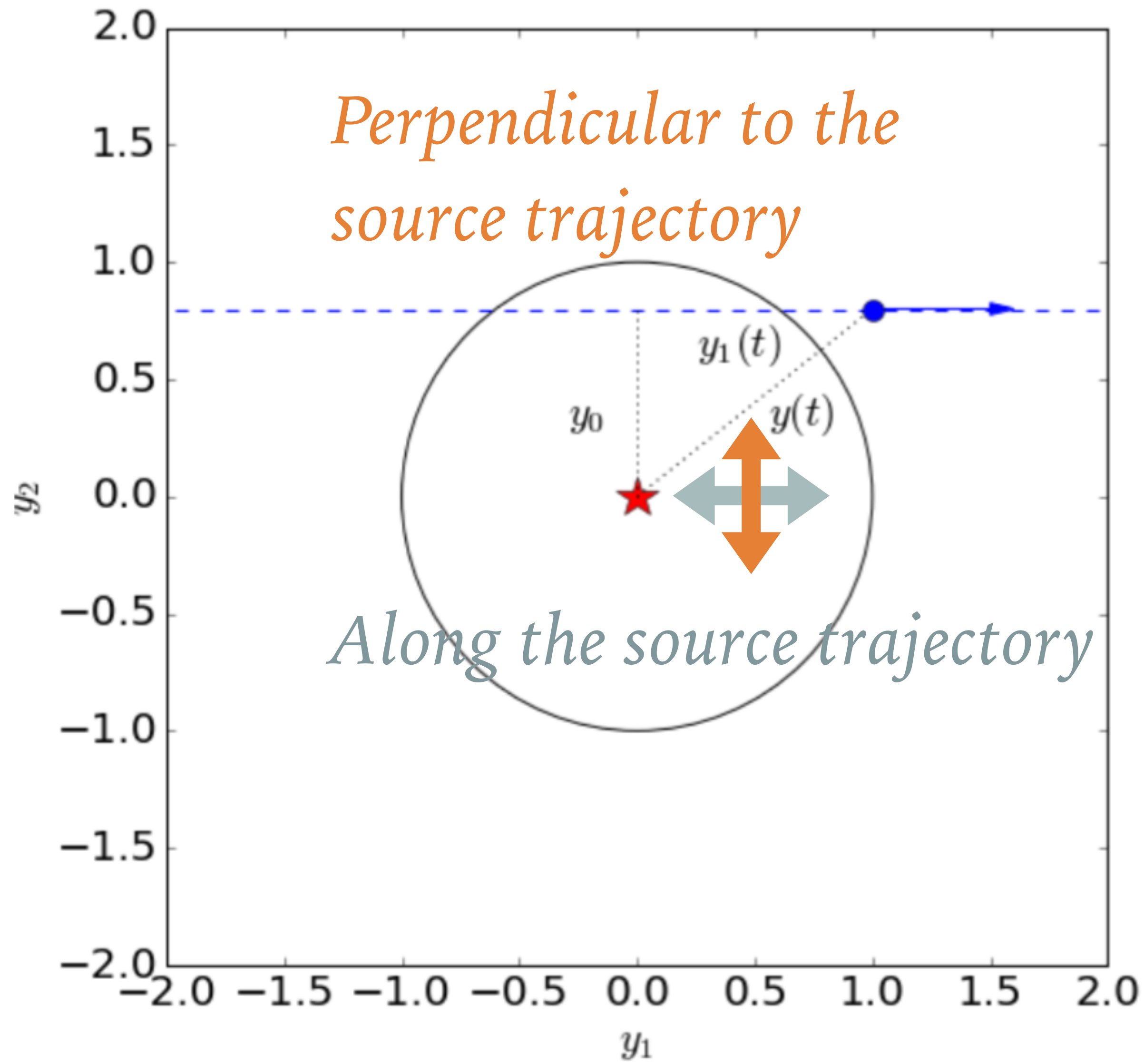

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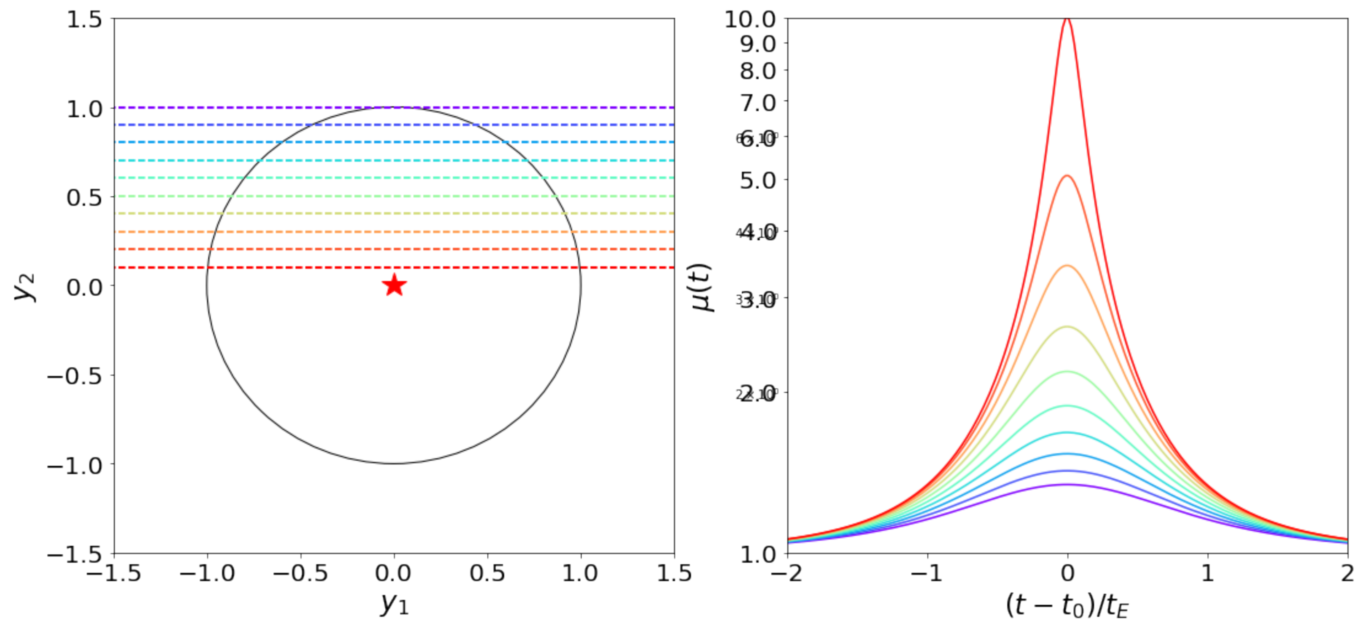

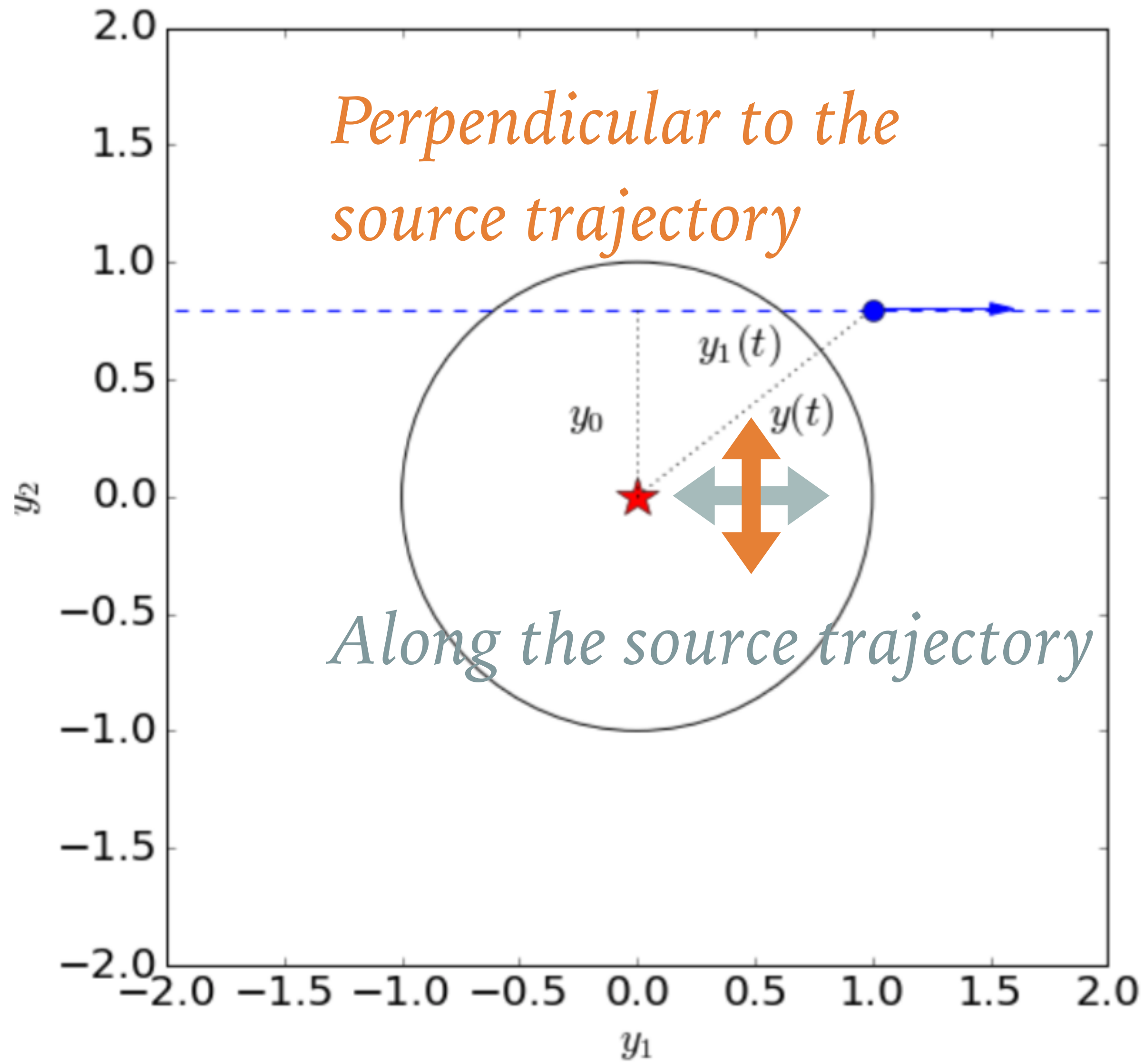

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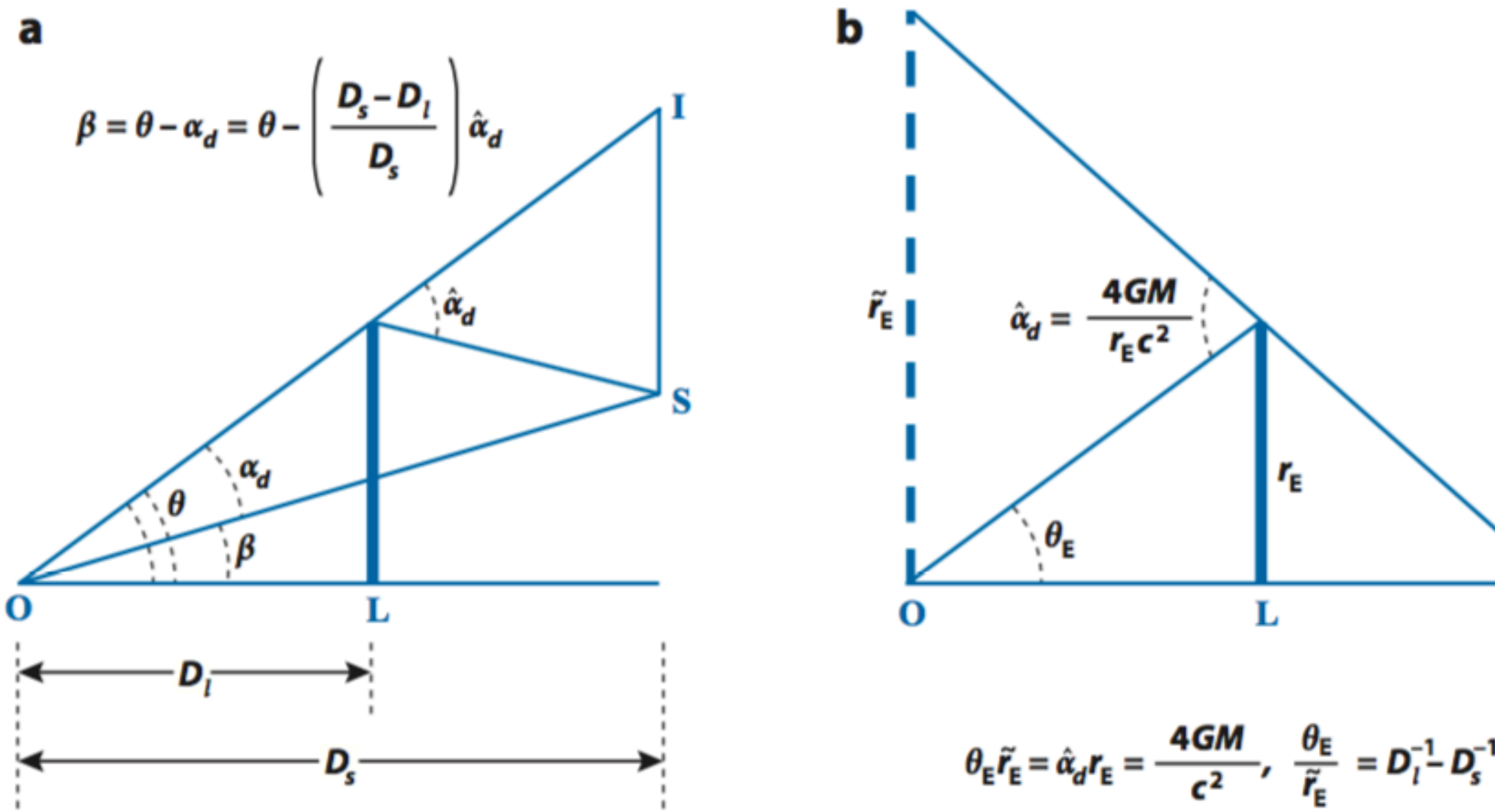
A: We will see a parallax effect on the light curve

MICROLENSING PARALLAX

Gaudi (2012)



$$\pi_{rel} = \frac{1}{D_L} - \frac{1}{D_S}$$

$$\boxed{\frac{1}{\tilde{r}_E} = \frac{\pi_{rel}}{\theta_E} \equiv \pi_E}$$

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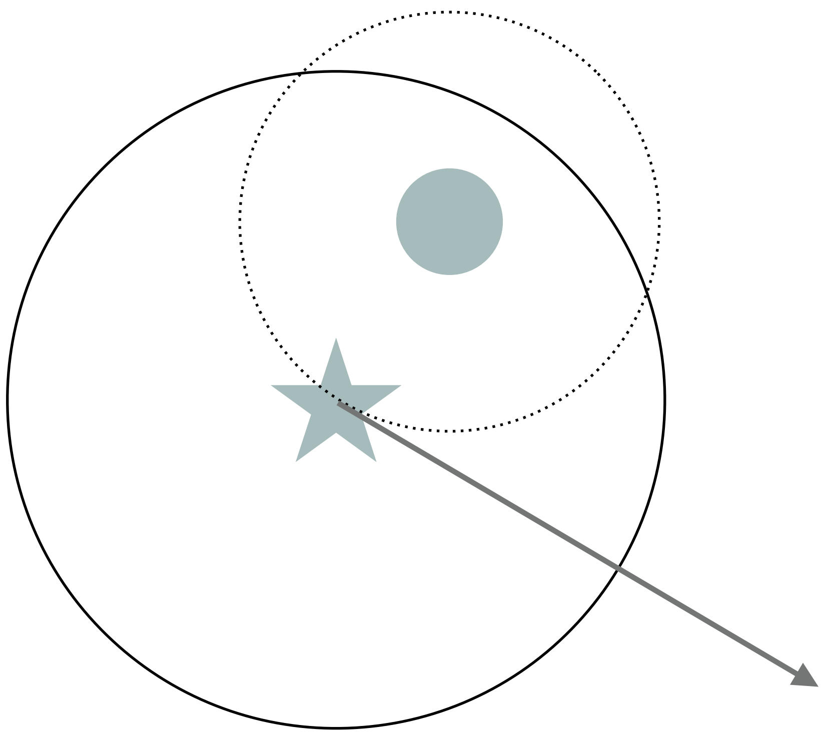
If we can measure the microlensing parallax, then we measure the mass as a function of the distance. If we can measure θ_E*, then we break the lensing degeneracy and measure the lens mass (without even seeing the lens light!)*

MICROLENSING PARALLAX

- This may happen for different reasons (several kind of parallax effects).
- Due to the motion of the observer:
  - annual/orbital parallax: the earth moves around the sun
- Due to the separation between observers
  - satellite parallax: when we can look at the same microlensing event from two positions simultaneously from a space telescope and from the earth
  - terrestrial parallax: when we can look at the same microlensing event from two positions on the earth

IMPORTANT: MICROLENSING PARALLAX IS A VECTOR!



Magnification does not depend on the direction of proper motion

$$\mu(t) = \frac{y^2(t) + 2}{y(t)\sqrt{y^2(t) + 4}}$$

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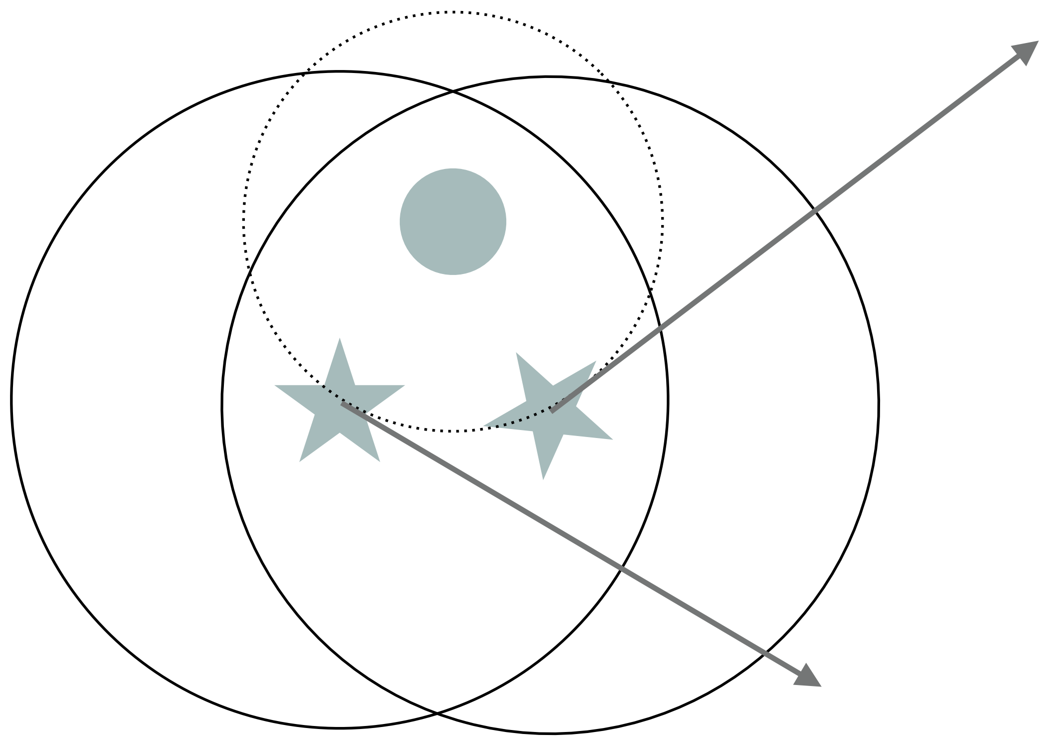

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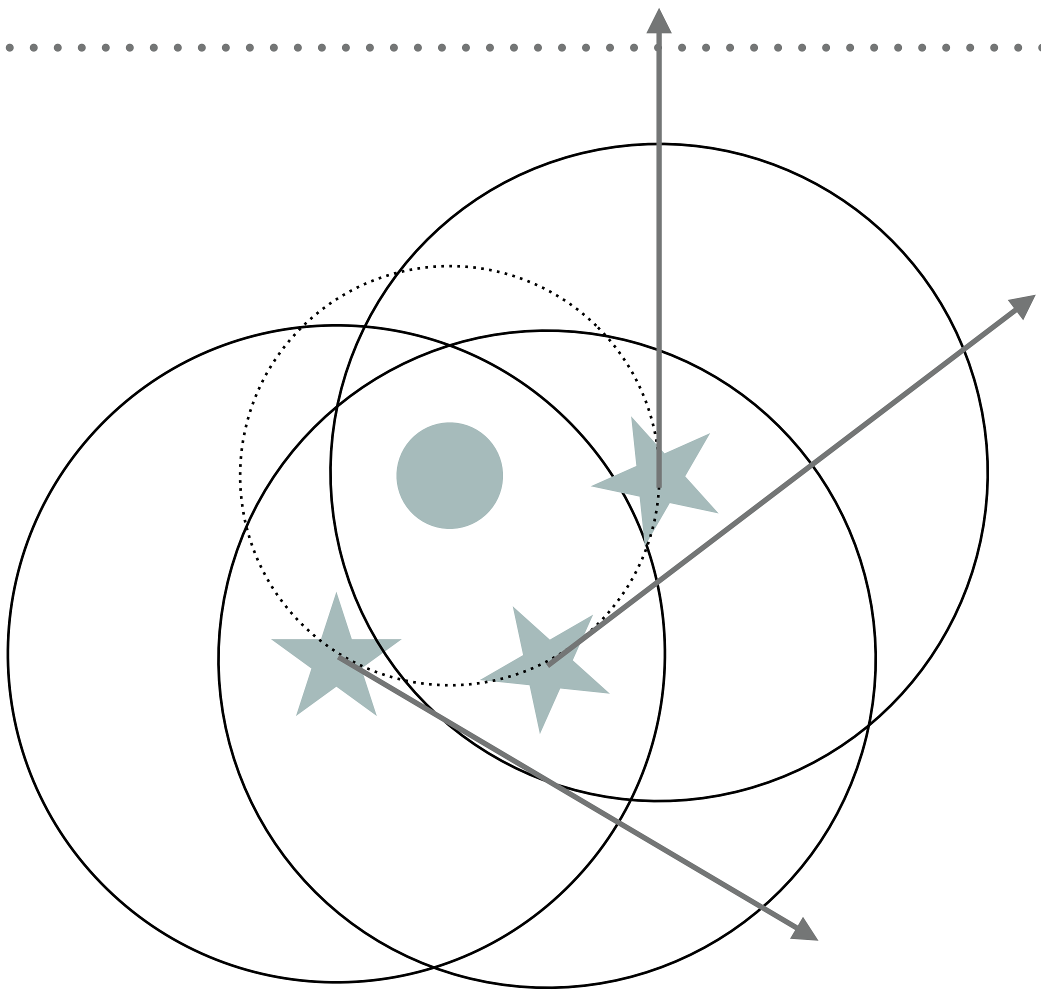

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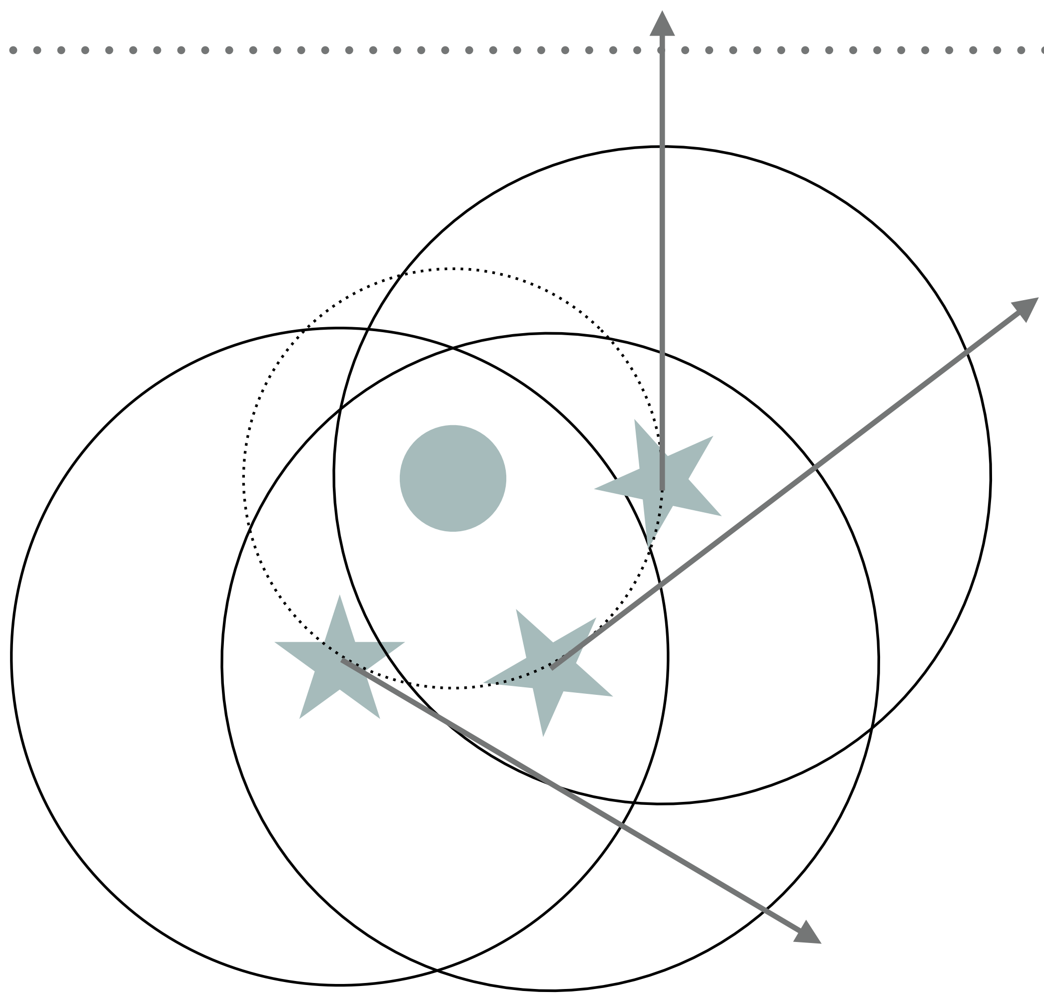

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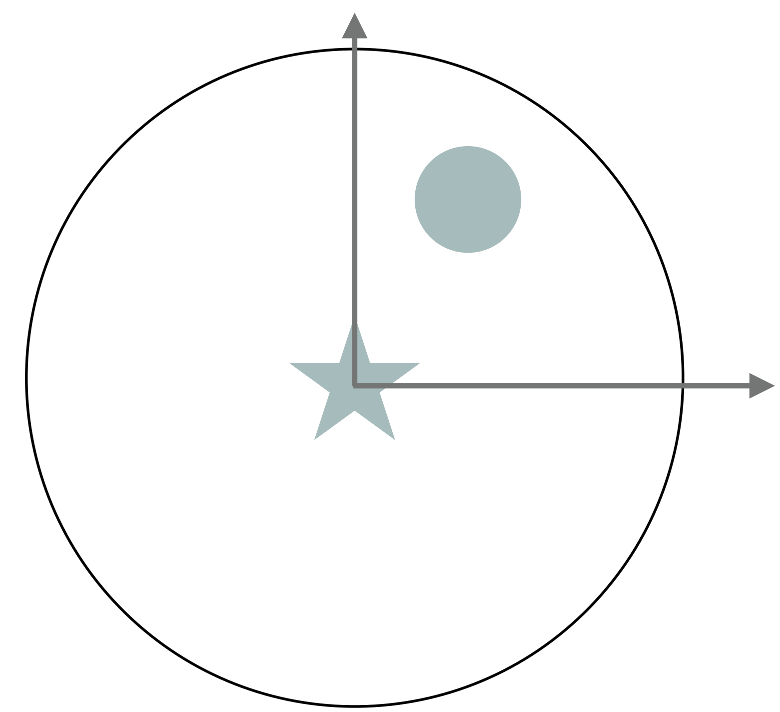
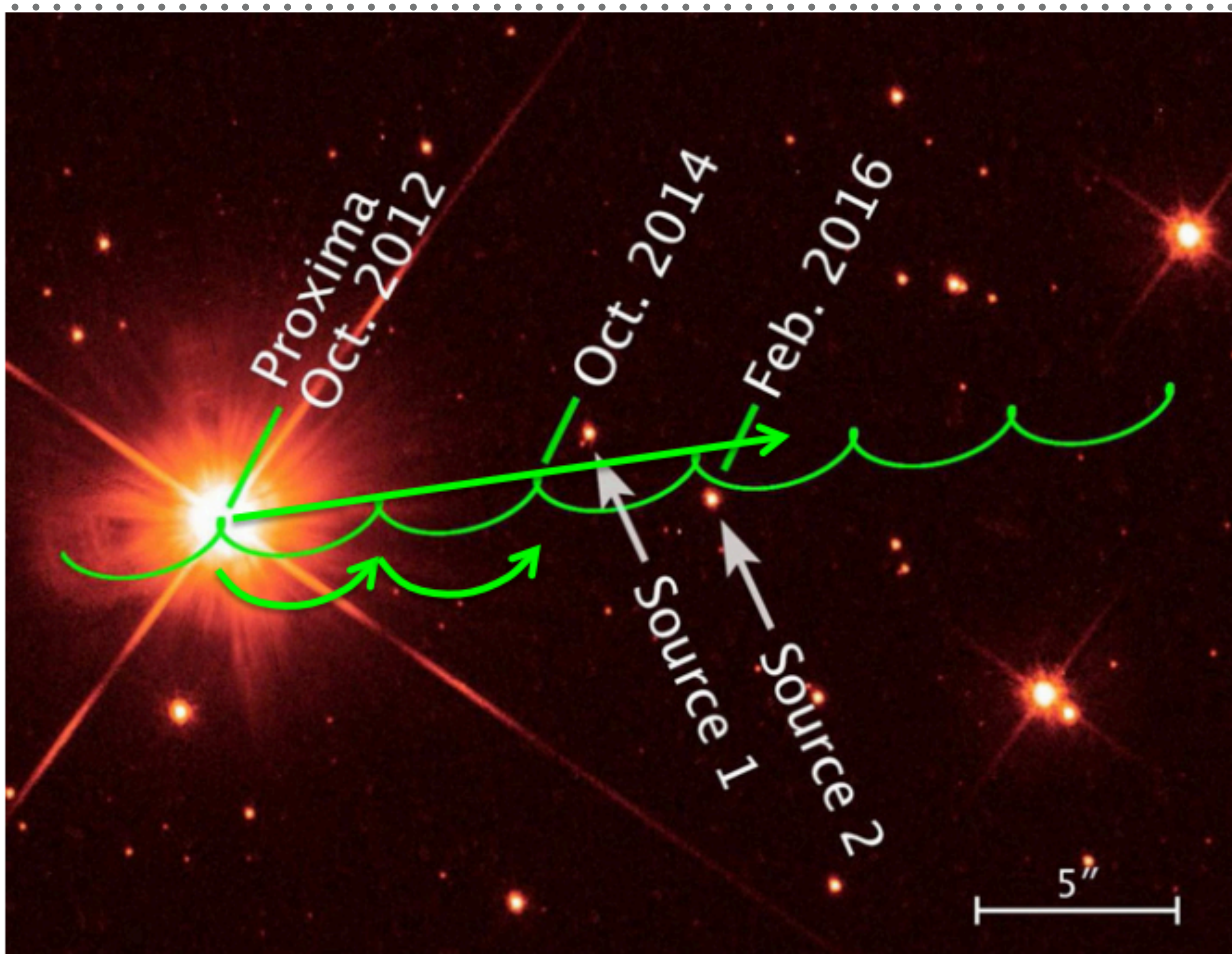
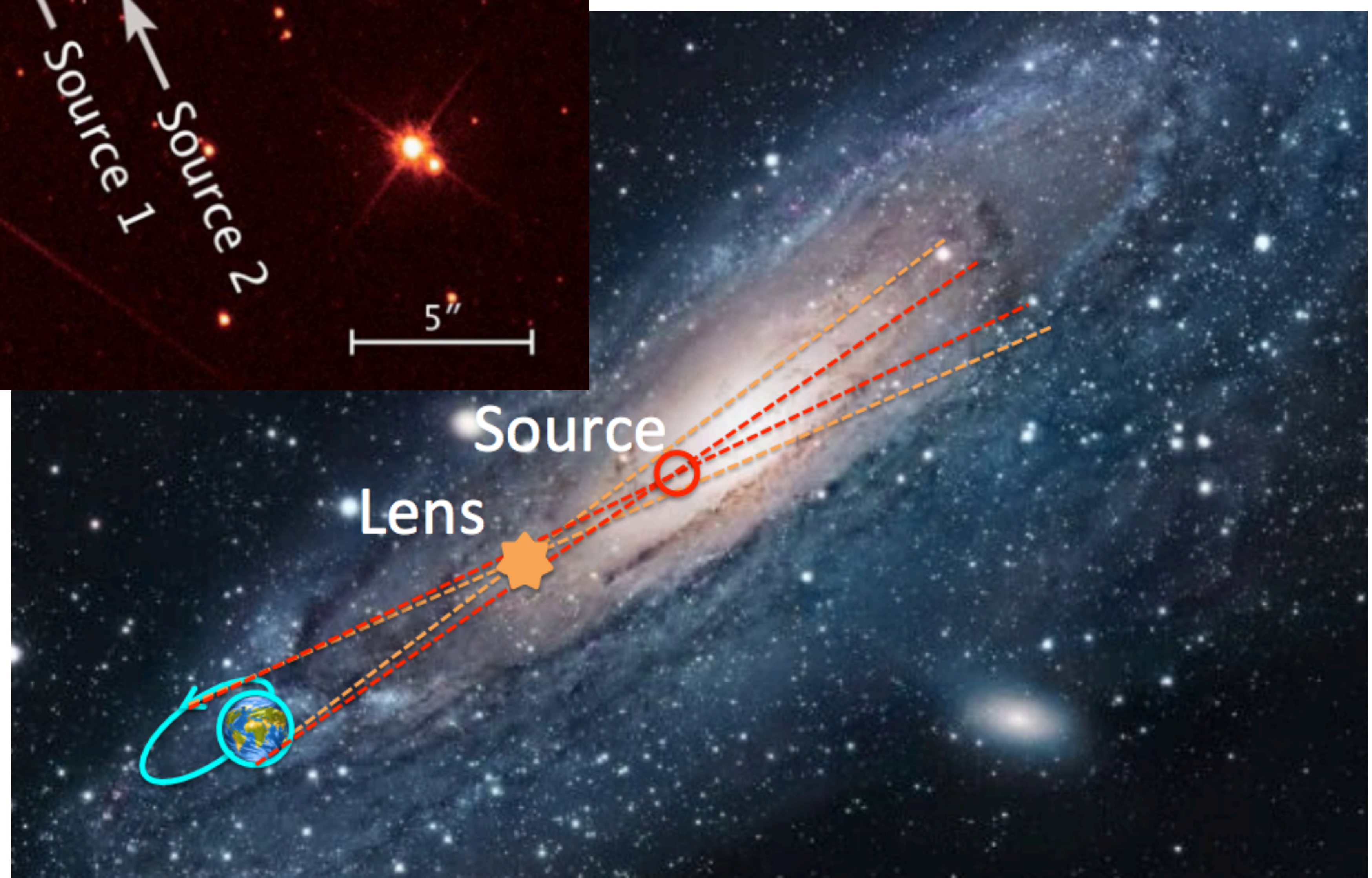


…but microlensing parallax does!

Depending on the lens displacement relative to the source (parallel or perpendicular to the proper motion), we will see different effects

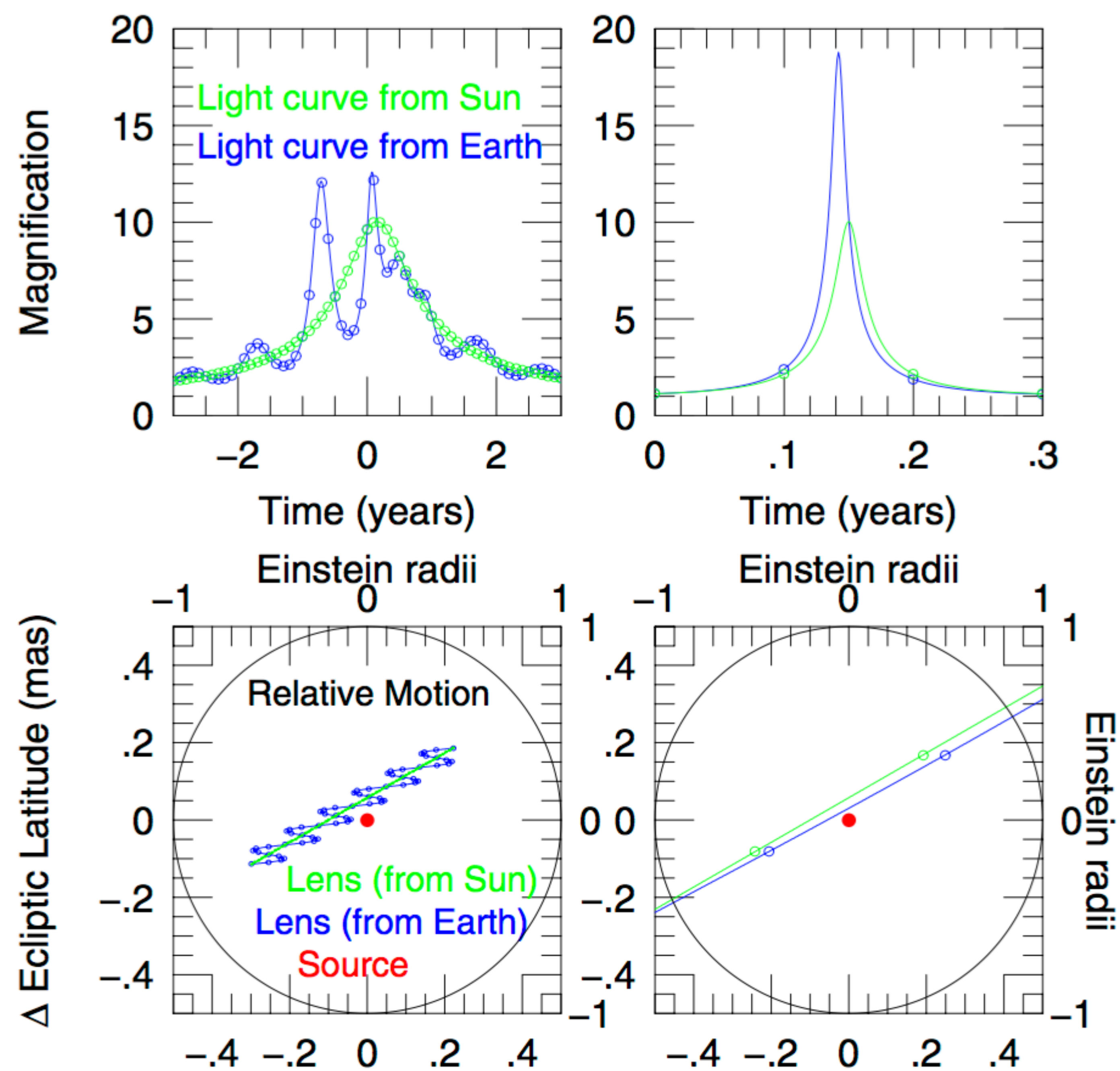
ORBITAL PARALLAX





ORBITAL PARALLAX

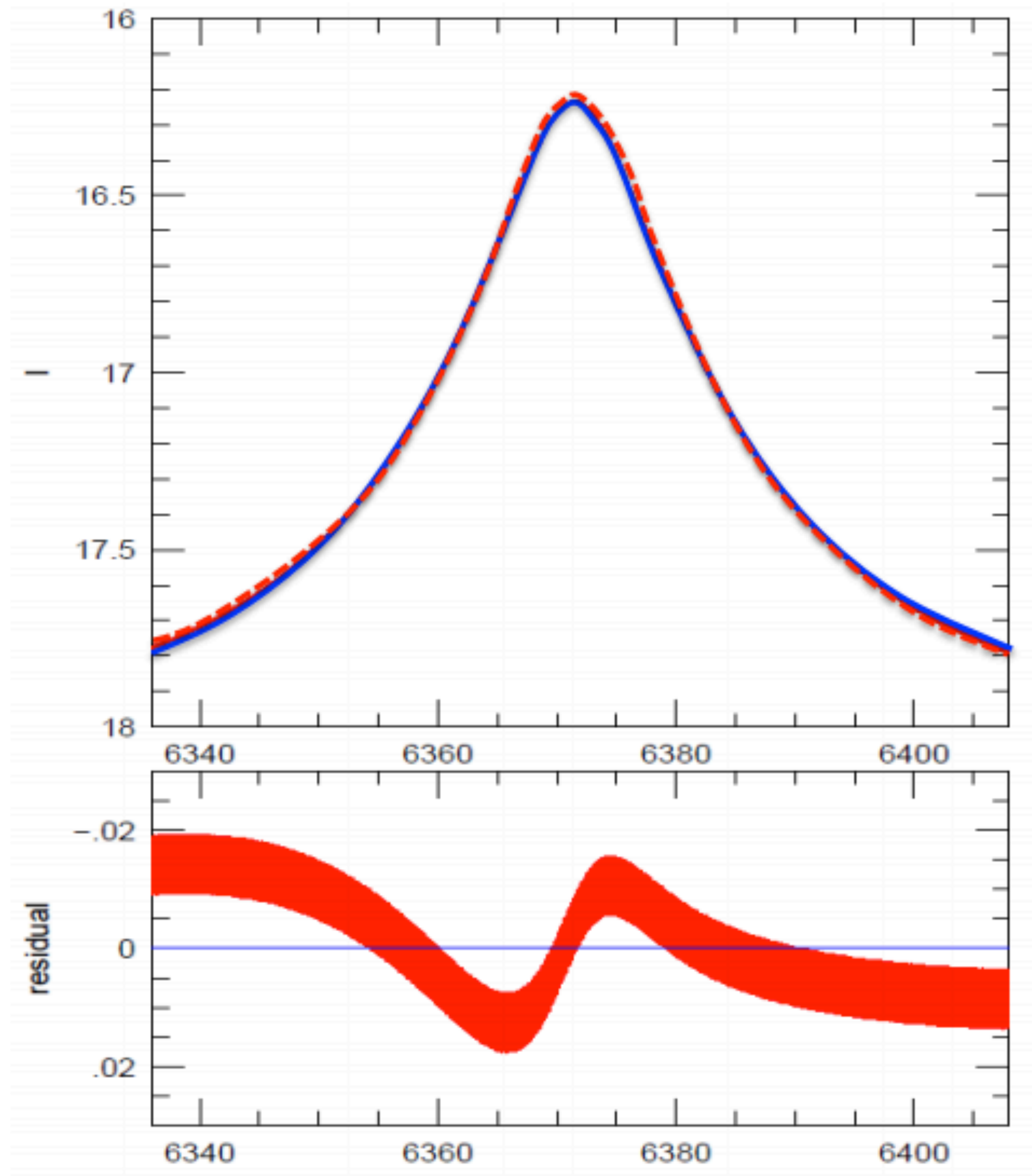
- on the left: what we would see if the μ_{hel}=0.1 mas/year
- on the right: the typical μ_{hel}=5 mas/year



Gould & Horne, 2013

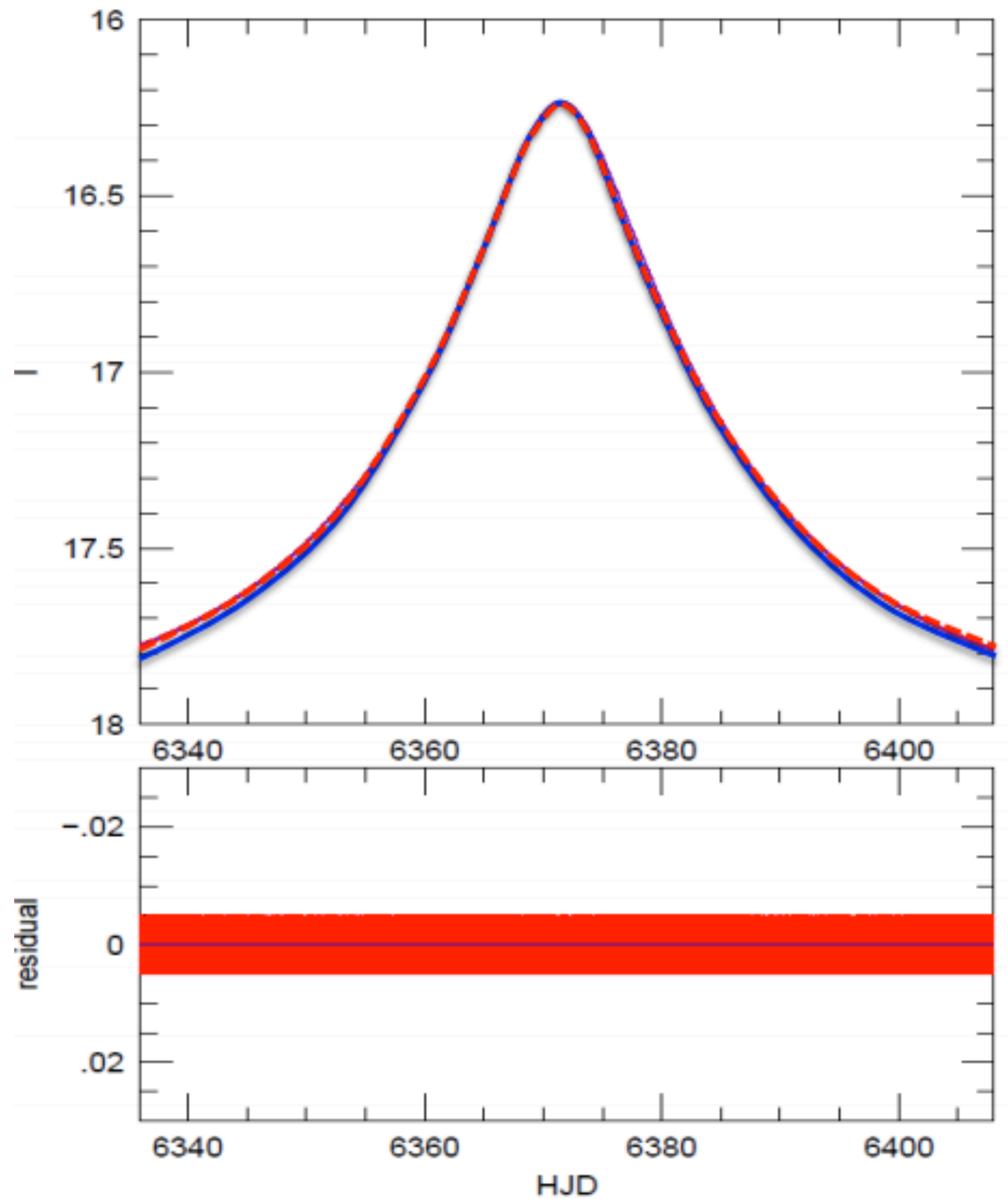
COMPONENT PARALLEL TO THE LENS TRAJECTORY

Asymmetric distortion of the light curve due to acceleration of the lens



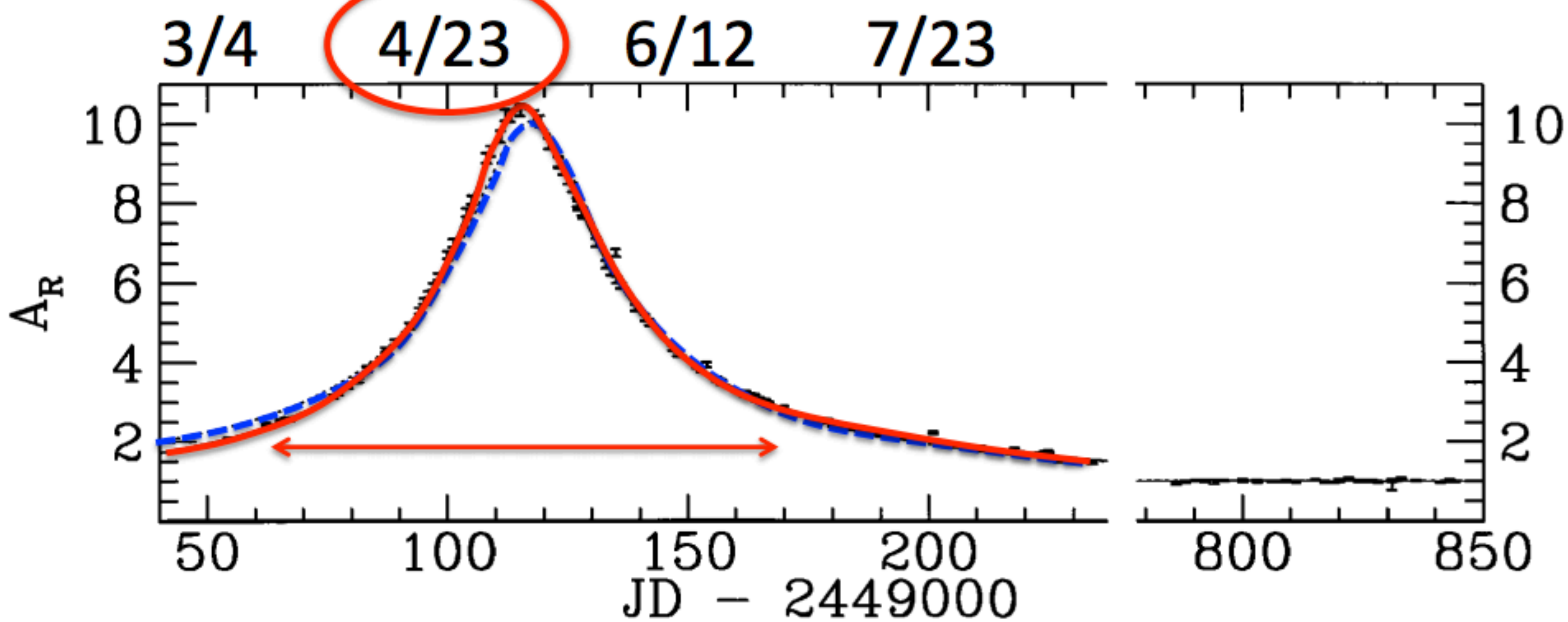
COMPONENT PERPENDICULAR TO THE LENS TRAJECTORY

Symmetric distortion of the light curve due to motion perpendicular to lens trajectory

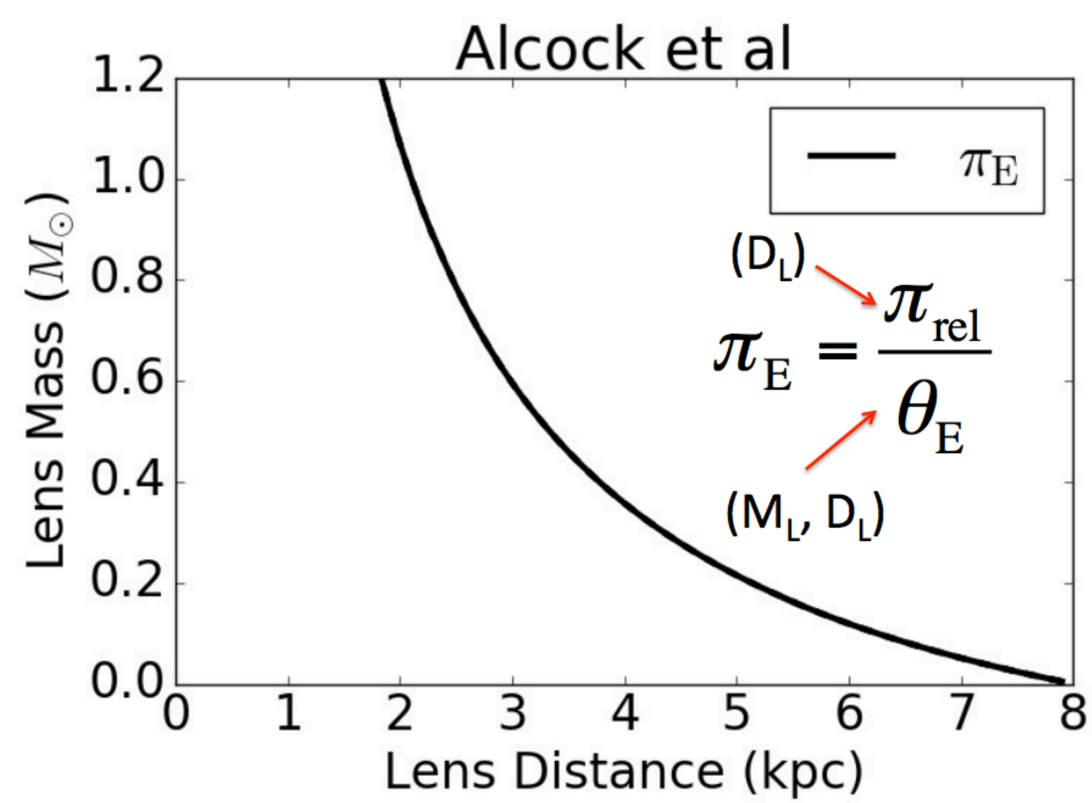


FIRST DETECTION OF MICROLENS PARALLAX

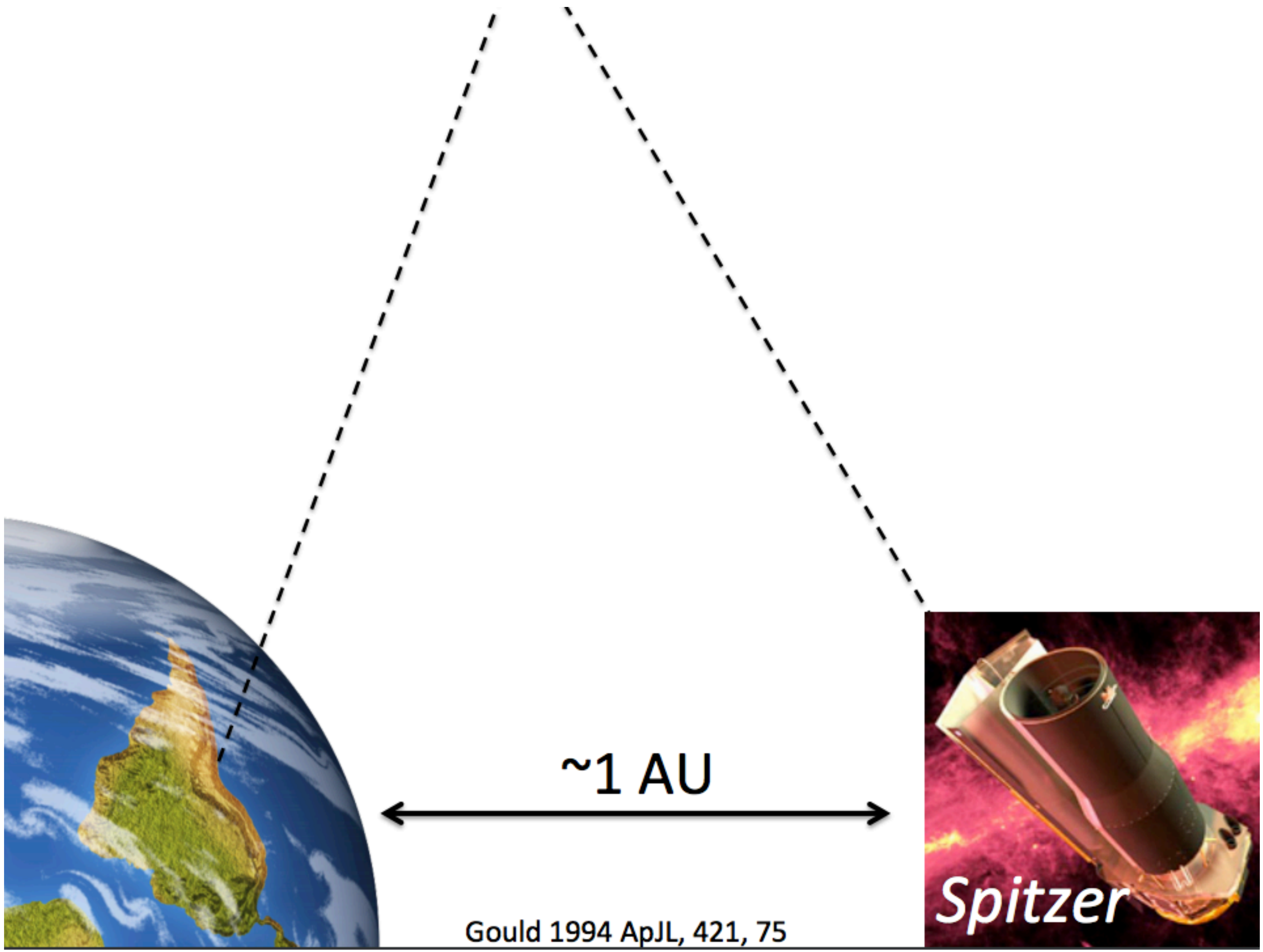
Alcock et al. 1995



Even without an estimate of θ_E, measuring the parallax still allows to measure the mass vs distance



SATELLITE PARALLAX



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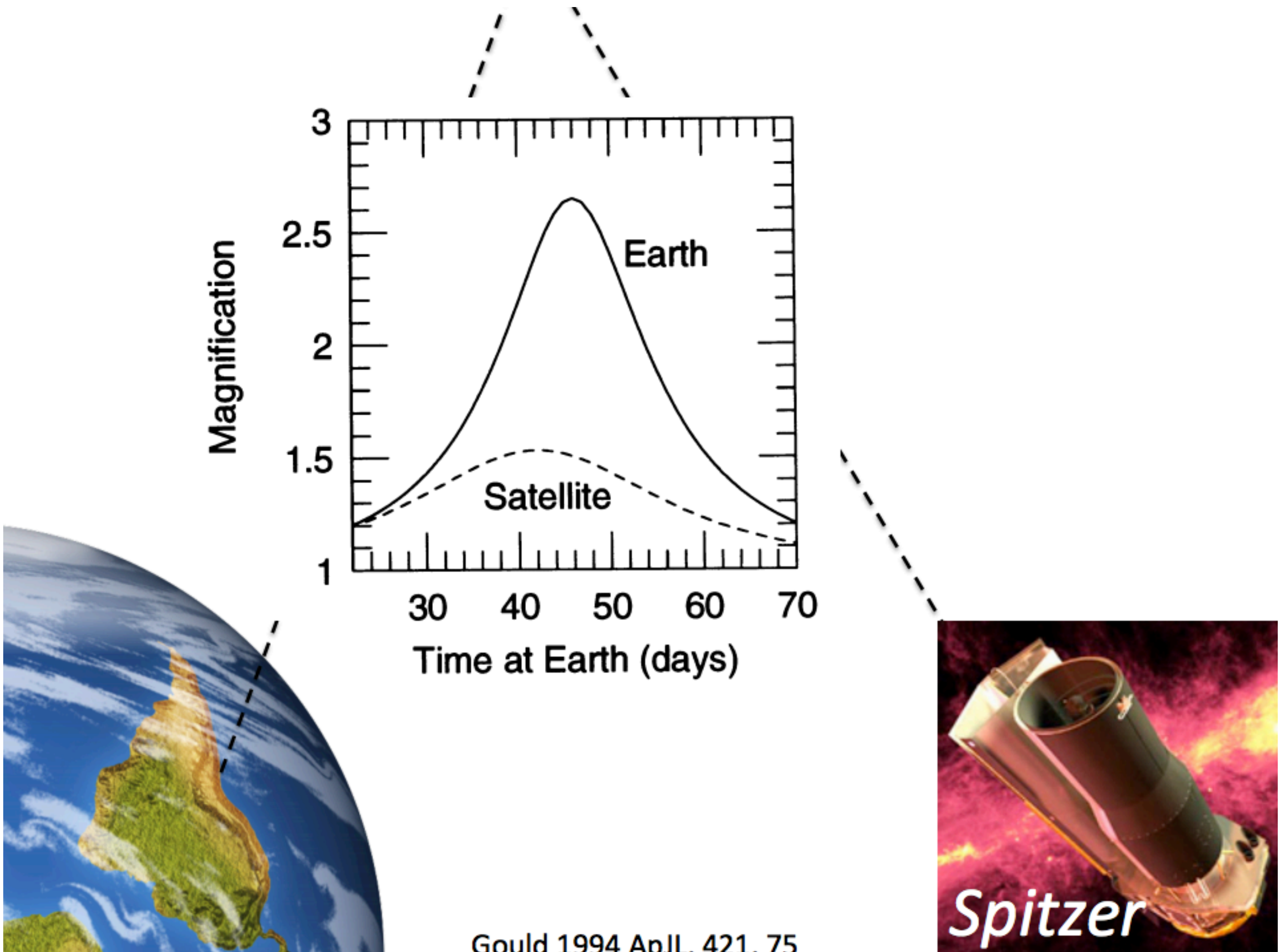

Gould 1994 ApJL, 421, 75

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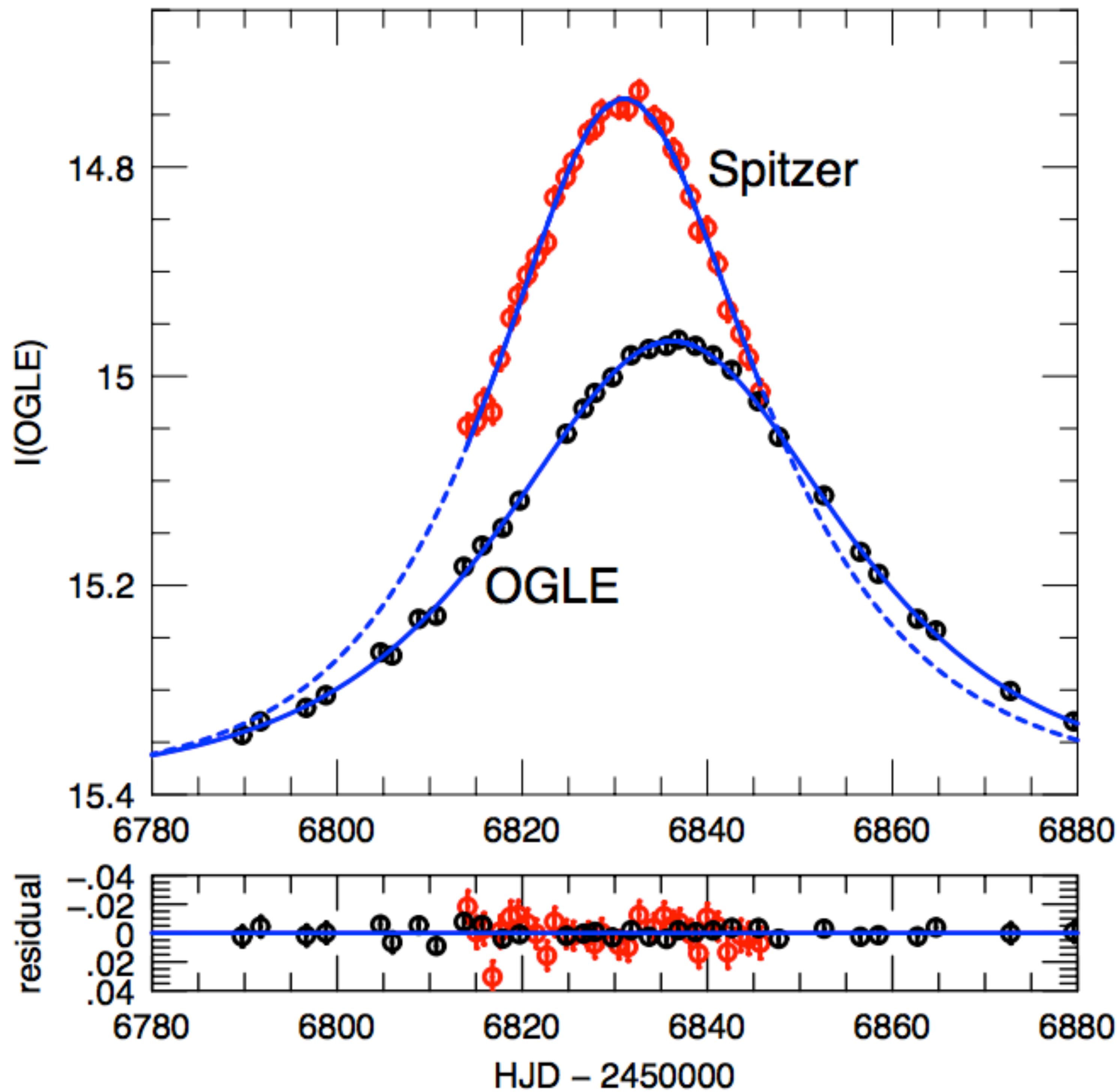

Yee et al. 2015, ApJ, 802, 76

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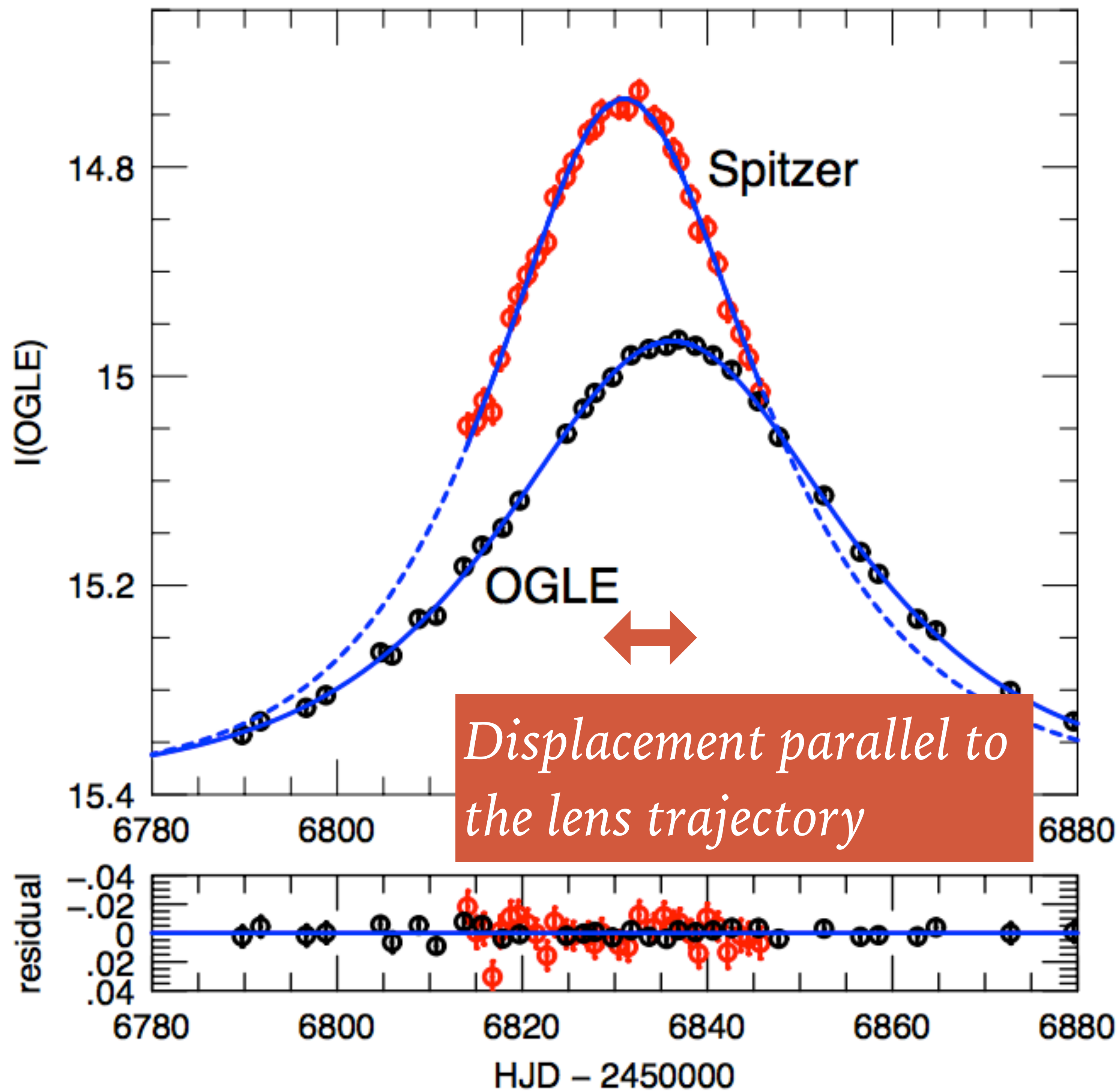

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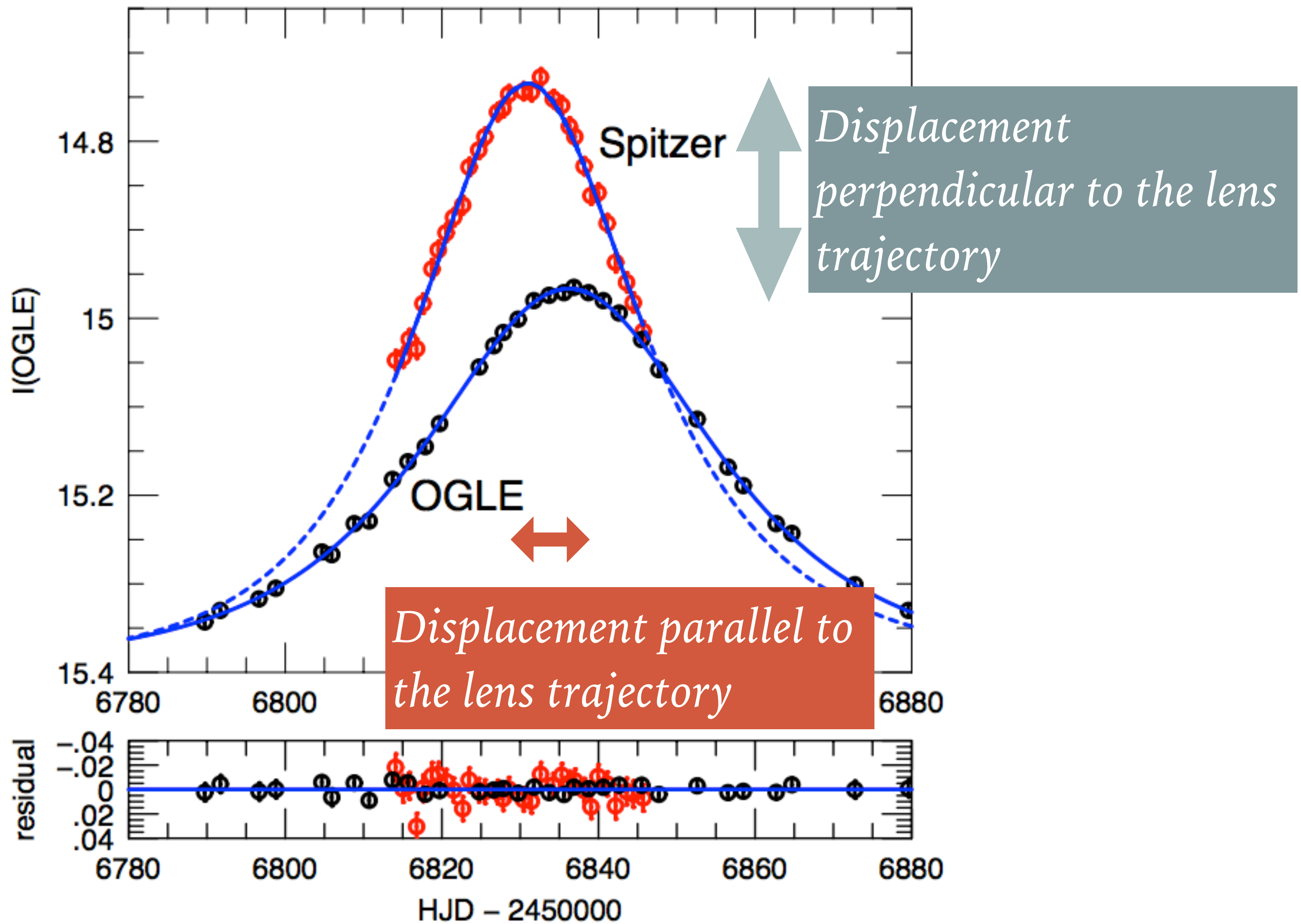
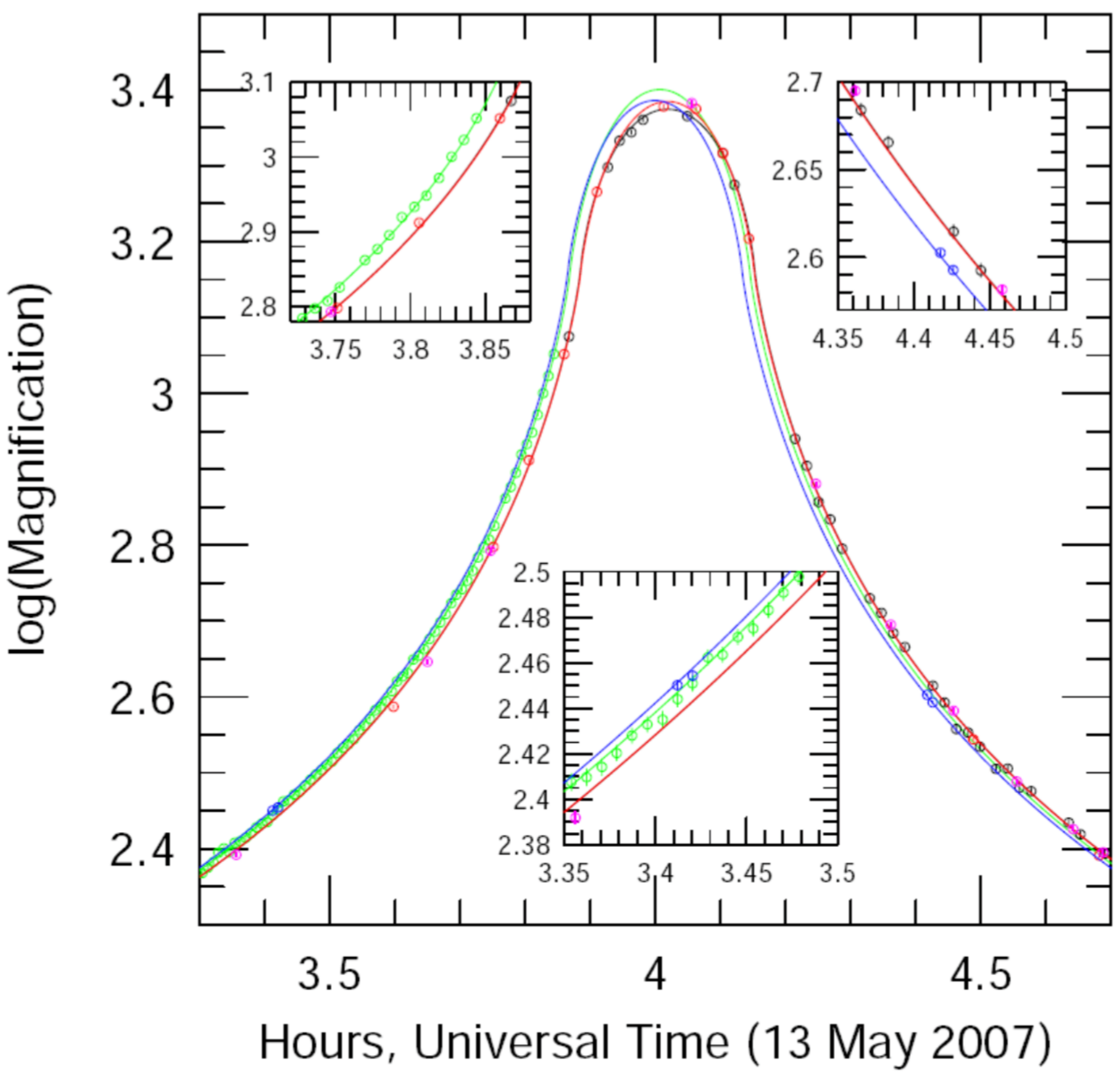


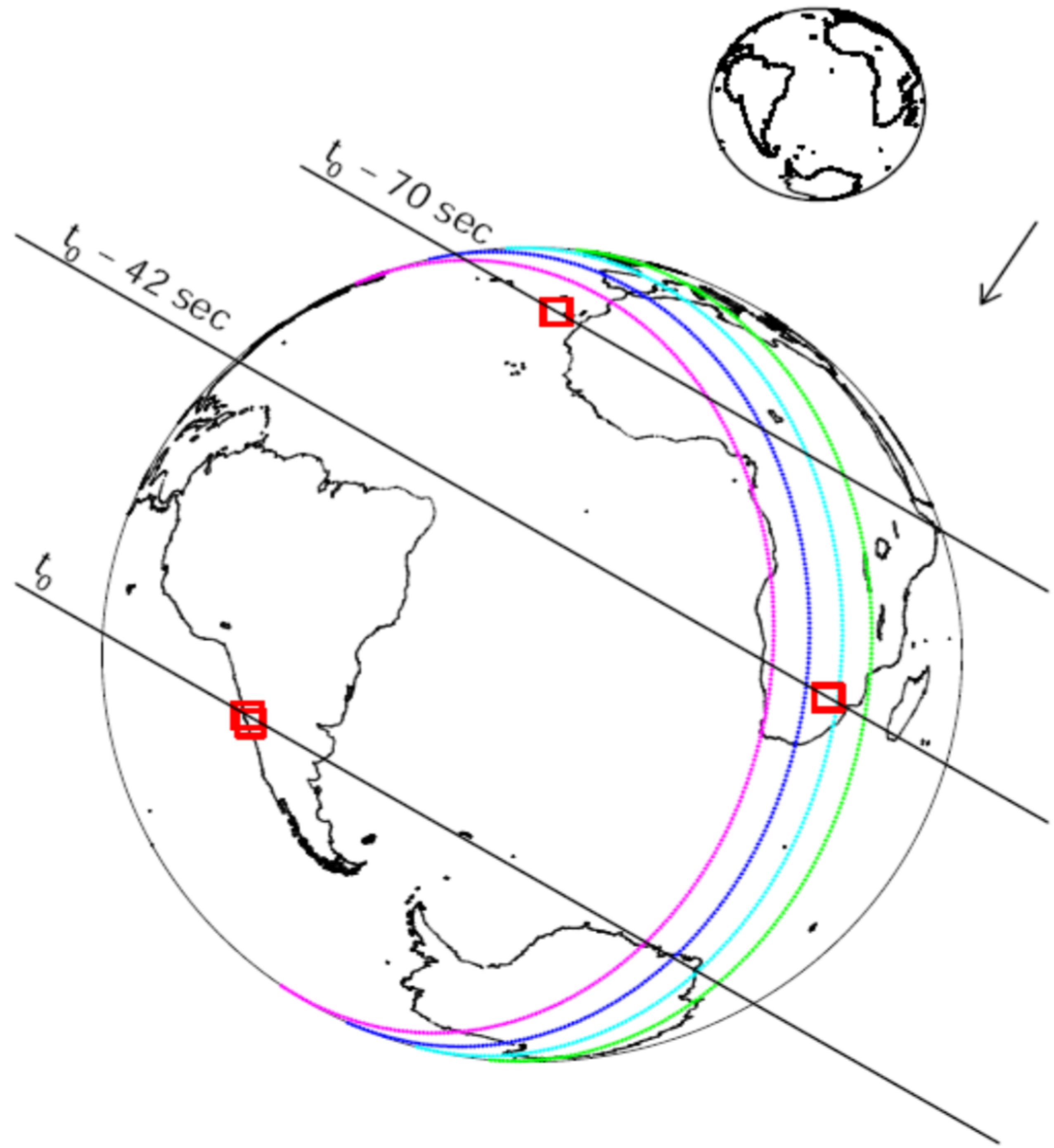
Yee et al. 2015, ApJ, 802, 76

MICROLENS PARALLAX (TERRESTRIAL)

OGLE-2007-BLG-224

Canaries South Africa Chile





Gould & Horne, 2013

WHAT IS ASTROMETRIC MICROLENSING?

- during a microlensing event, the two images of the source cannot be resolved ($\theta_E \sim 1$mas)
- their positions and the magnifications change as a function of time
- in particular, the image forming outside the Einstein ring dominates, in terms of flux for most of the time
- what an observer will see is one source at the light centroid, which will move as a function of time depending on where the two images form and on how much flux they emit

THE EQUATIONS

$$
\begin{aligned}
x_{\pm,\parallel} &= \frac{1}{2}(1 \pm Q) y_{\parallel} \\
x_{\pm,\perp} &= \frac{1}{2}(1 \pm Q) y_{\perp}
\end{aligned}
$$

$$
\begin{aligned}
\mu_{\pm}(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \\
&= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right) \\
&= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right)
\end{aligned}
$$

$$
Q = \frac{\sqrt{y^2+4}}{y}
$$

$$
\vec{x}_c = \frac{\vec{x}_+ |\mu_+| + \vec{x}_- |\mu_-|}{|\mu_+| + |\mu_-|}
$$

$$
\delta \vec{x}_c = \vec{x}_c - \vec{y}
$$

LIGHT CENTROID SHIFT AMPLITUDE

$$
\begin{aligned}
\delta x_c &= \frac{\frac{1}{4}\left[(y+\sqrt{y^2+4})\left(1+\frac{y^2+2}{y\sqrt{y^2+4}}\right)-(y-\sqrt{y^2+4})\left(1-\frac{y^2+2}{y\sqrt{y^2+4}}\right)\right]}{\frac{y^2+2}{y\sqrt{y^2+4}}}-y \\
&= \frac{\frac{1}{4}\left(y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}+\frac{y^2+2}{y}-y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}-\frac{y^2+2}{y}\right)}{\frac{y^2+2}{y\sqrt{y^2+4}}}-y \\
&= \frac{y}{y^2+2}\,.
\end{aligned}
$$

Given the sign, the shift points in the same direction of y.

Note that $y \gg \sqrt{2}$, $\delta x_c \approx \frac{1}{y}$

Thus, the shift decreases relatively slow with y… remember the scaling of μ?

LIGHT CENTROID SHIFT AMPLITUDE

$$
\begin{aligned}
\delta x_c &= \frac{\frac{1}{4}\left[(y+\sqrt{y^2+4})\left(1+\frac{y^2+2}{y\sqrt{y^2+4}}\right)-(y-\sqrt{y^2+4})\left(1-\frac{y^2+2}{y\sqrt{y^2+4}}\right)\right]}{\frac{y^2+2}{y\sqrt{y^2+4}}}-y \\
&= \frac{\frac{1}{4}\left(y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}+\frac{y^2+2}{y}-y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}-\frac{y^2+2}{y}\right)}{\frac{y^2+2}{y\sqrt{y^2+4}}}-y \\
&= \frac{y}{y^2+2}.
\end{aligned}
$$

In addition

$$\frac{d(\delta x_c)}{dy} = \frac{2-y^2}{(y^2+2)^2}$$

the shift is maximum at $y=\sqrt{2}$, $\delta x_c = \delta x_{c,max} = (2\sqrt{2})^{-1}$

This corresponds to $\sim 0.354\theta_E$ *which is above the accuracy of GAIA*