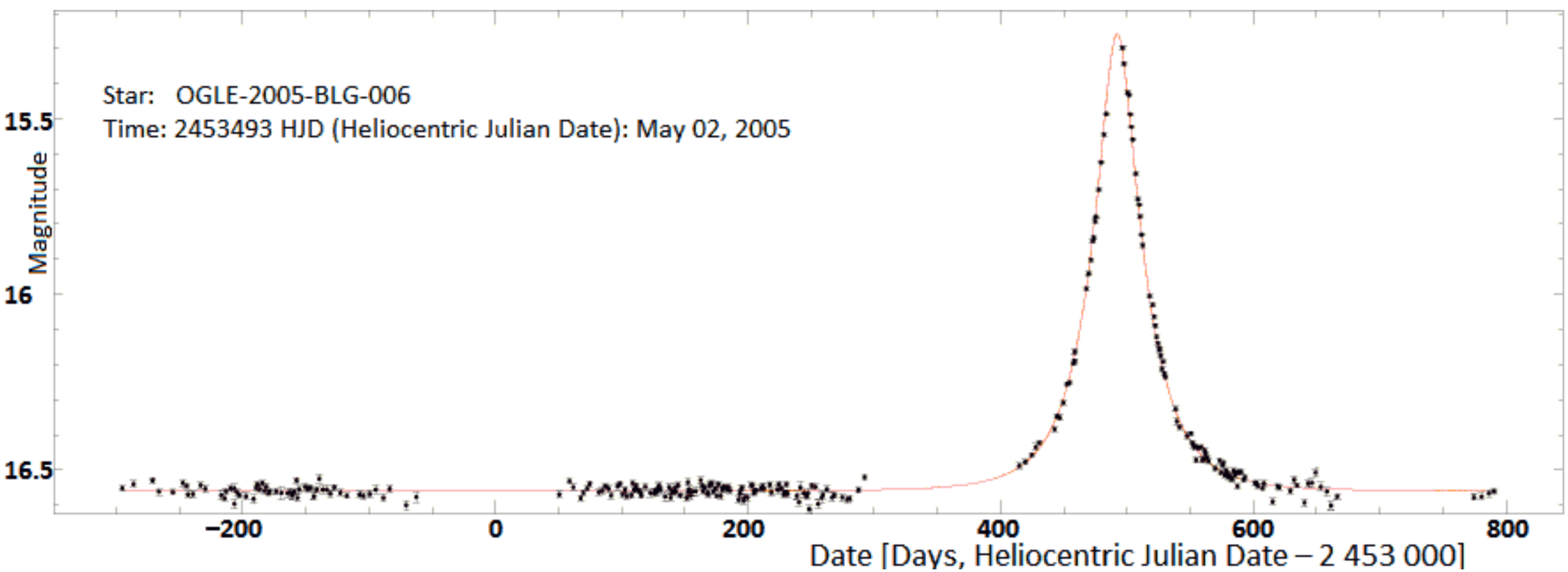
GRAVITATIONAL LENSING

11 – MICROLENSING LIGHT CURVES: BREAKING THE MICROLENSING DEGENERACIES

Massimo Meneghetti

AA 2018-2019

EXAMPLE OF STANDARD LIGHT CURVE



MICROLENSING DEGENERACY AND ITS BREAKERS

- As seen, from the standard light curve we can measure the Einstein crossing time, which is a degenerate combination of the lens mass, distance and velocity
- Thus it is impossible to characterise the lens and the source of a single event through light curve measurement alone…
- … unless special circumstances are present!

FINITE SOURCE EFFECTS

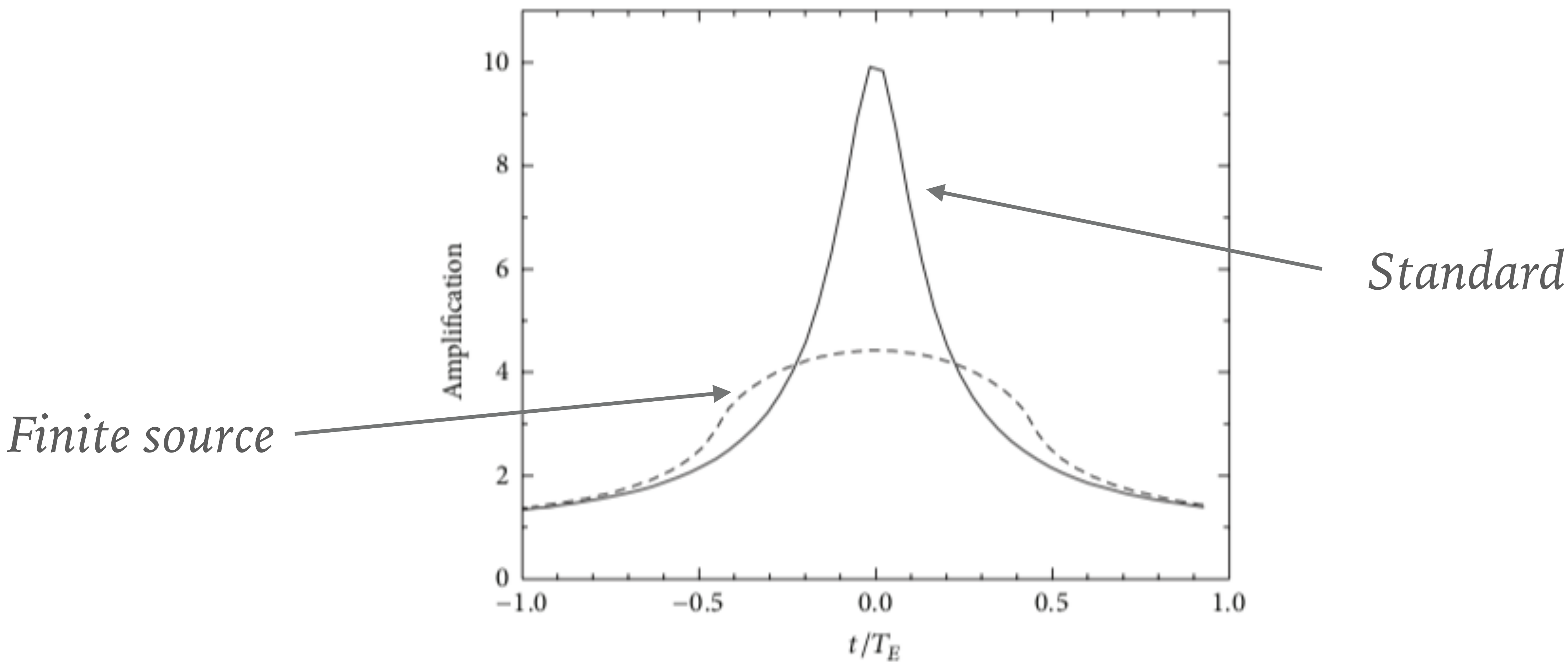
- So far, we have assumed that source are point-like
- In practice, while the lens transits the surface of an extended source during the course of microlensing, this approximation is no longer valid
- The correct calculation of the source magnification in this case requires to integrate the magnification over the surface of the source:

$$\mu_{FS}(y \,|\, \rho_s) = \frac{\int_0^{2\pi} \int_0^{\rho_s} d\varphi dr r \mu \left(\sqrt{(y + r\cos\varphi)^2 + (r\sin\varphi)^2} \right)}{\int_0^{2\pi} \int_0^{\rho_s} d\varphi dr r}$$

$$\rho_s = \frac{\beta_*}{\theta_E}$$

FINITE SOURCE EFFECTS

Hamolli et al (2015)



We can fit the light curve with an additional parameter (ρ_S), and use some empirical relation to measure the source size from the source color and magnitude. For example, Kervella et al. (2004) find:

$$\log(2\beta_*) = 0.0755(V-K) + 0.517 - 0.2K$$

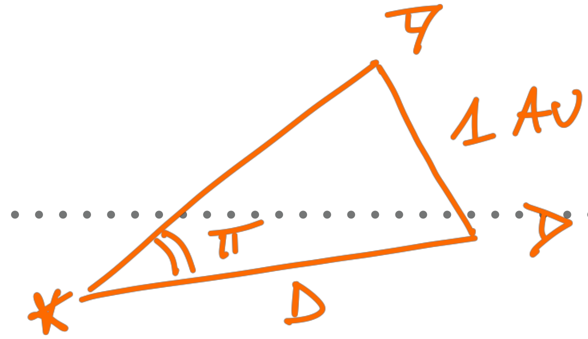
Then, we can infer the Einstein radius.

ASTROMETRIC MEASUREMENT OF THE RELATIVE PROPER MOTION

- Another method to measure the Einstein radius is by measuring the relative proper motion of source and lens.
- This requires to see the lens!
- If we have high resolution imaging with HST or ground based AO, we can measure the position of lens and source at some time after the maximum of the light curve Δt
- If we measure a shift $\Delta\theta$ then $\mu_{rel} = \Delta\theta/\Delta t$ and

$$\theta_E = t_E \times \mu_{rel}$$

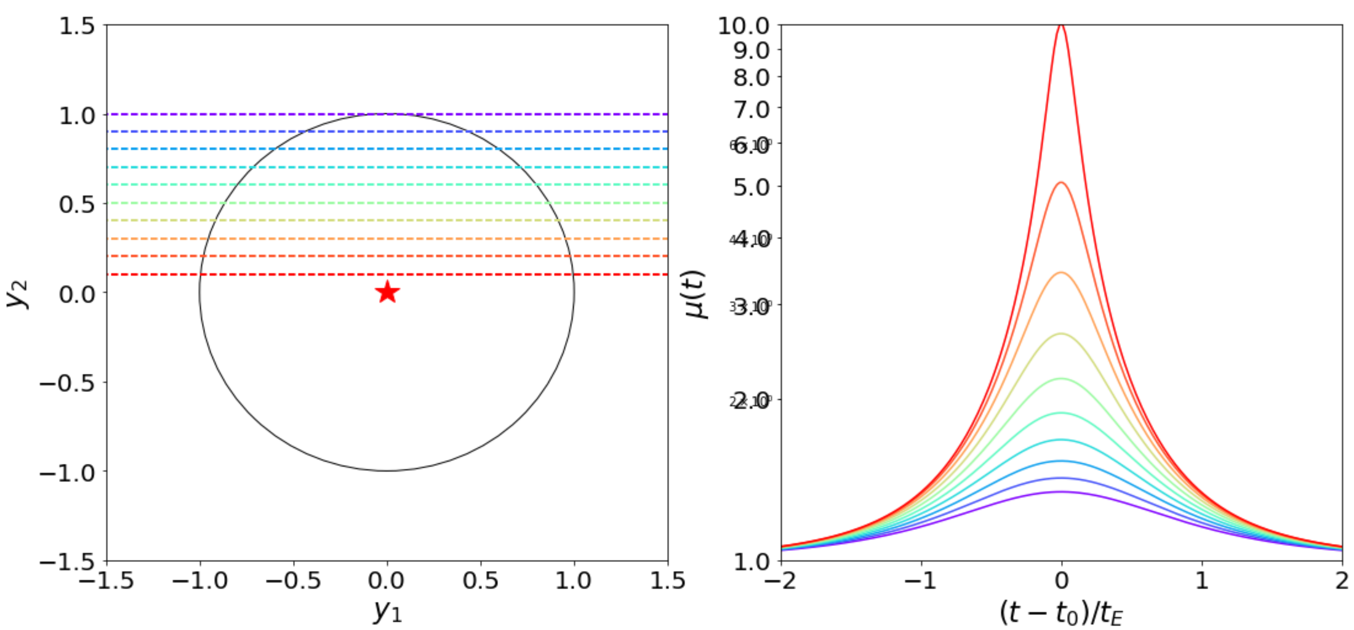
CHANGES IN PERSPECTIVE…

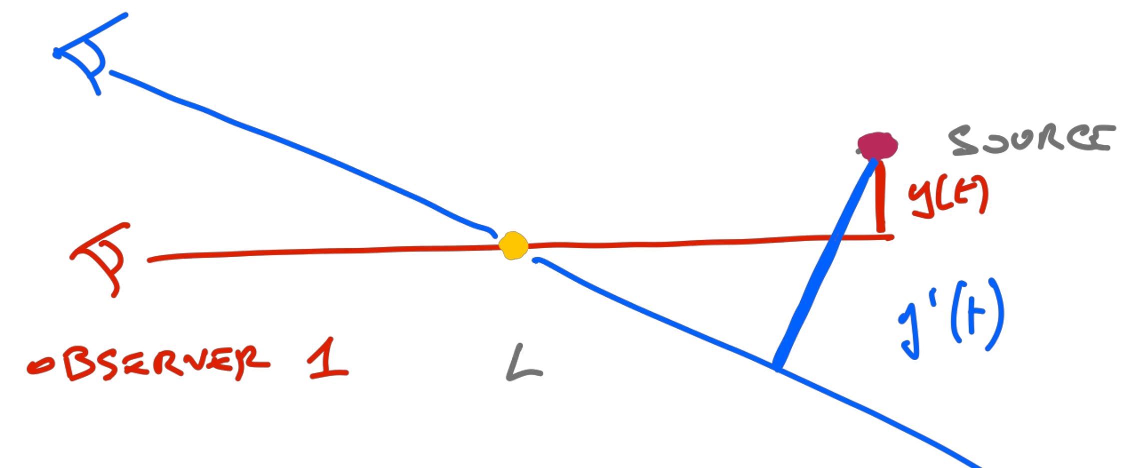


$$y(t) = \sqrt{y_0^2 + y_1^2(t)} = \sqrt{y_0^2 + \frac{(t-t_0)^2}{t_E^2}}$$

$\pi = \frac{1\,AU}{D}$ PARALLAX

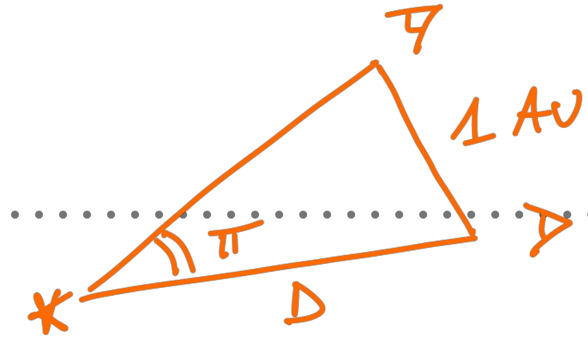
$$\mu(t) = \frac{y^2(t)+2}{y(t)\sqrt{y^2(t)+4}}$$





$$\mu(y'(t)) \neq \mu(y(t))$$

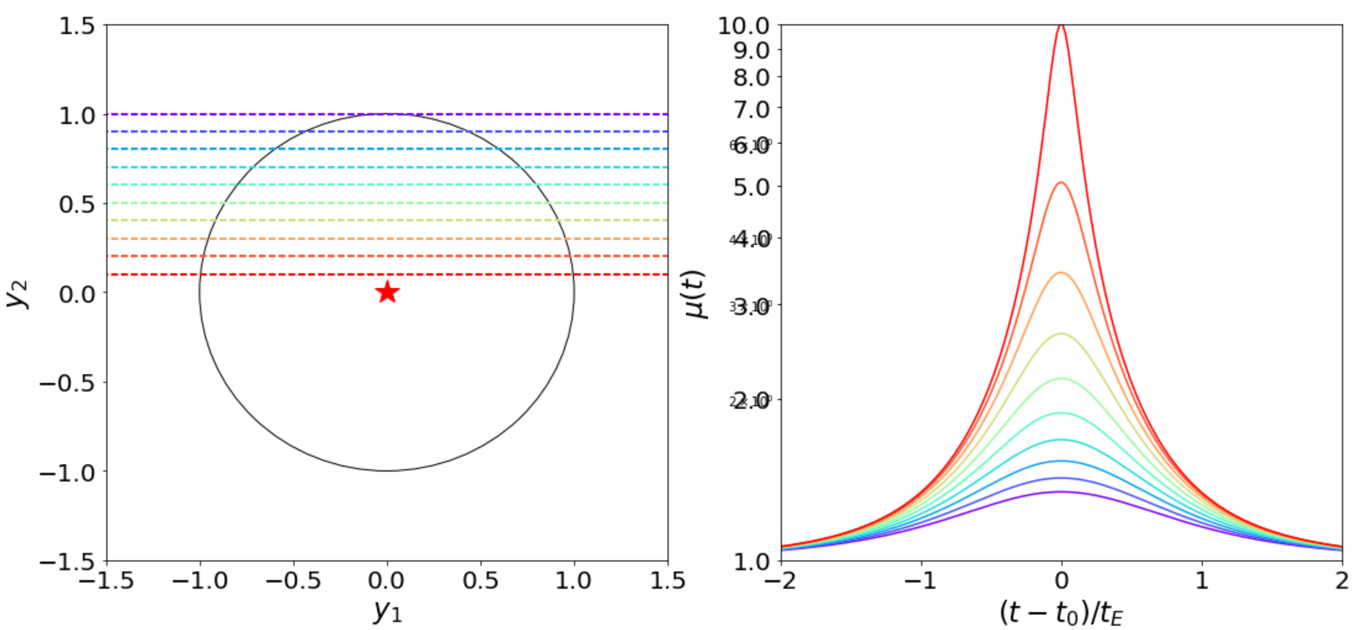
CHANGES IN PERSPECTIVE…

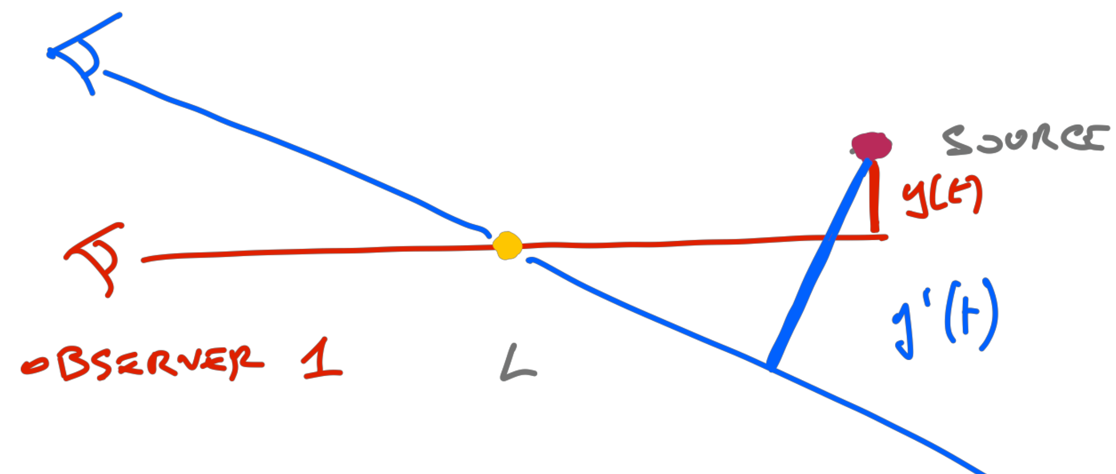


$\pi = \frac{1\,AU}{D}$ PARALLAX

$$y(t) = \sqrt{y_0^2 + y_1^2(t)} = \sqrt{y_0^2 + \frac{(t-t_0)^2}{t_E^2}}$$

$$\mu(t) = \frac{y^2(t)+2}{y(t)\sqrt{y^2(t)+4}}$$

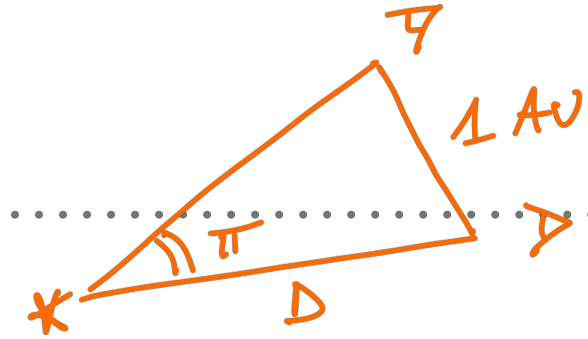




$\mu(y'(t)) \neq \mu(y(t))$

Two observers looking at the same Microlensing event will see two different light curves (under some circumstances).

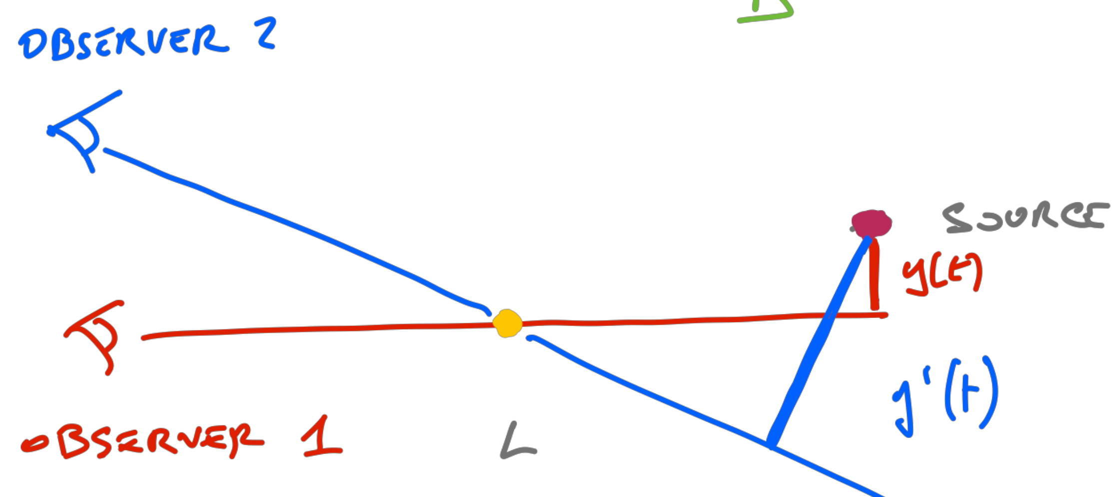
CHANGES IN PERSPECTIVE...



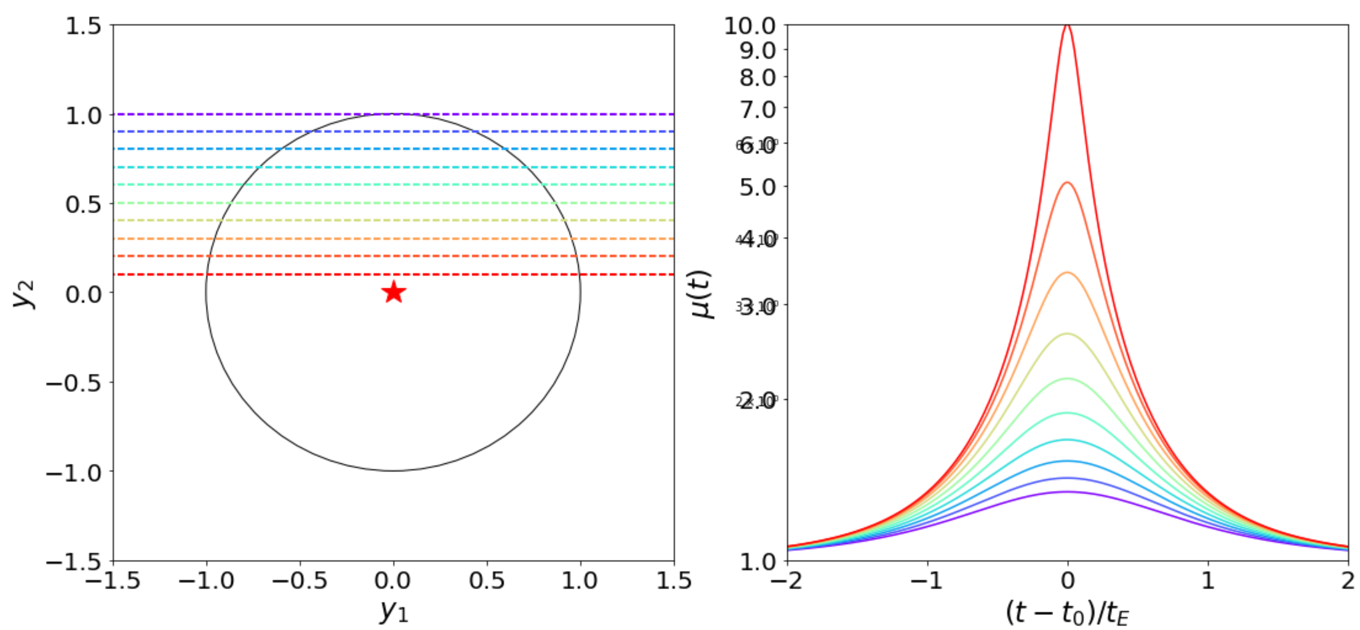
$$\pi = \frac{1\,AU}{D} \quad \text{PARALLAX}$$

$$y(t) = \sqrt{y_0^2 + y_1^2(t)} = \sqrt{y_0^2 + \frac{(t-t_0)^2}{t_E^2}}$$

$$\mu(t) = \frac{y^2(t)+2}{y(t)\sqrt{y^2(t)+4}}$$



$$\mu(y'(t)) \neq \mu(y(t))$$



Two observers looking at the same Microlensing event will see two different light curves (under some circumstances).

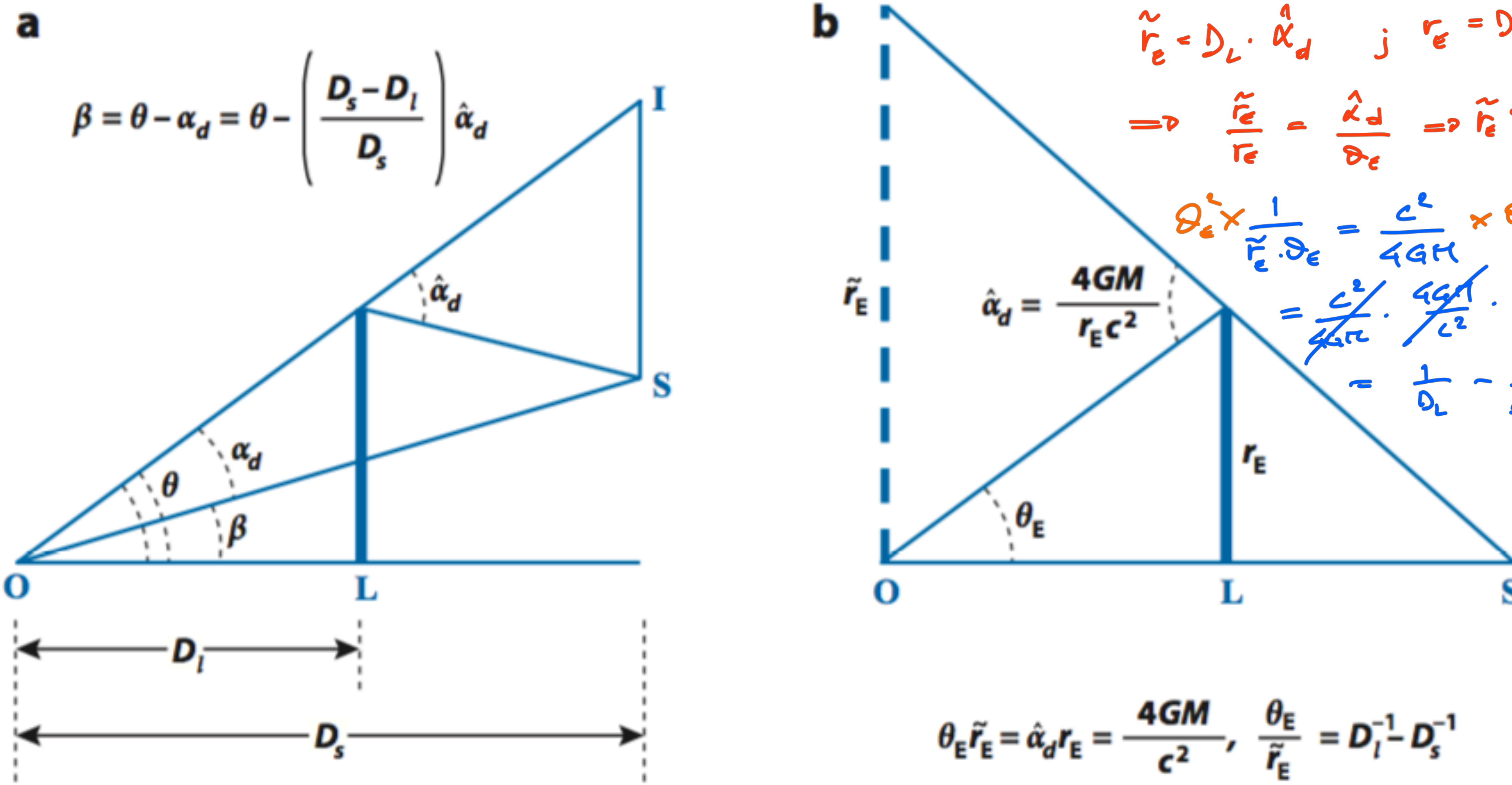
This is a parallax effect! But what is the relevant baseline???

MICROLENSING PARALLAX

$$\theta_E = \sqrt{\frac{4GM}{c^2}\,\frac{D_{LS}}{D_L \cdot D_S}}$$

Gaudi (2012)

for galactic microlensing: $\underline{\underline{D_{LS} = D_S - D_L}}$



$\tilde{r}_E = D_L \cdot \hat{\alpha}_d$; $r_E = D_L \cdot \theta_E$

$\Rightarrow \frac{\tilde{r}_E}{r_E} = \frac{\hat{\alpha}_d}{\theta_E} \Rightarrow \tilde{r}_E \theta_E = r_E \hat{\alpha}_d$

$$\theta_E^2 \times \frac{1}{\tilde{r}_E \cdot \theta_E} = \frac{c^2}{4GM} \times \theta_E^2$$

$$= \cancel{\frac{c^2}{4GM}} \cdot \cancel{\frac{4GM}{c^2}} \cdot \frac{D_S - D_L}{D_L D_S}$$

$$= \frac{1}{D_L} - \frac{1}{D_S}$$

$$\pi_{rel} = \frac{1}{D_L} - \frac{1}{D_S}$$

$$\boxed{\frac{1}{\tilde{r}_E} = \frac{\pi_{rel}}{\theta_E} \equiv \pi_E}$$

MICROLENSING PARALLAX

$$\frac{1}{\tilde{r}_E} = \frac{\pi_{rel}}{\theta_E} \equiv \pi_E$$

MICROLENSING PARALLAX

$$\frac{1}{\tilde{r}_E} = \frac{\pi_{rel}}{\theta_E} \equiv \pi_E$$

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_L D_S}} = \sqrt{\frac{4GM}{c^2} \pi_{rel}}$$

MICROLENSING PARALLAX

$$\boxed{\frac{1}{\tilde{r}_E} = \frac{\pi_{rel}}{\theta_E} \equiv \pi_E}$$

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S}} = \sqrt{\frac{4GM}{c^2}\pi_{rel}}$$

$$\theta_E = \frac{4G}{c^2}M\frac{\pi_{rel}}{\theta_E} = kM\pi_E \qquad k = 8.15\text{mas } M_\odot^{-1}$$

MICROLENSING PARALLAX

$$\boxed{\frac{1}{\tilde{r}_E} = \frac{\pi_{rel}}{\theta_E} \equiv \pi_E}$$

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S}} = \sqrt{\frac{4GM}{c^2}\pi_{rel}}$$

$$\theta_E = \frac{4G}{c^2} M \frac{\pi_{rel}}{\theta_E} = kM\pi_E \qquad k = 8.15\text{mas } M_\odot^{-1}$$

$$M = \frac{\theta_E}{k\pi_E}$$

MICROLENSING PARALLAX

$$\frac{1}{\tilde{r}_E} = \frac{\pi_{rel}}{\theta_E} \equiv \pi_E$$

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S}} = \sqrt{\frac{4GM}{c^2}\pi_{rel}}$$

$$\theta_E = \frac{4G}{c^2} M \frac{\pi_{rel}}{\theta_E} = kM\pi_E \qquad k = 8.15\mathrm{mas}\ M_\odot^{-1}$$

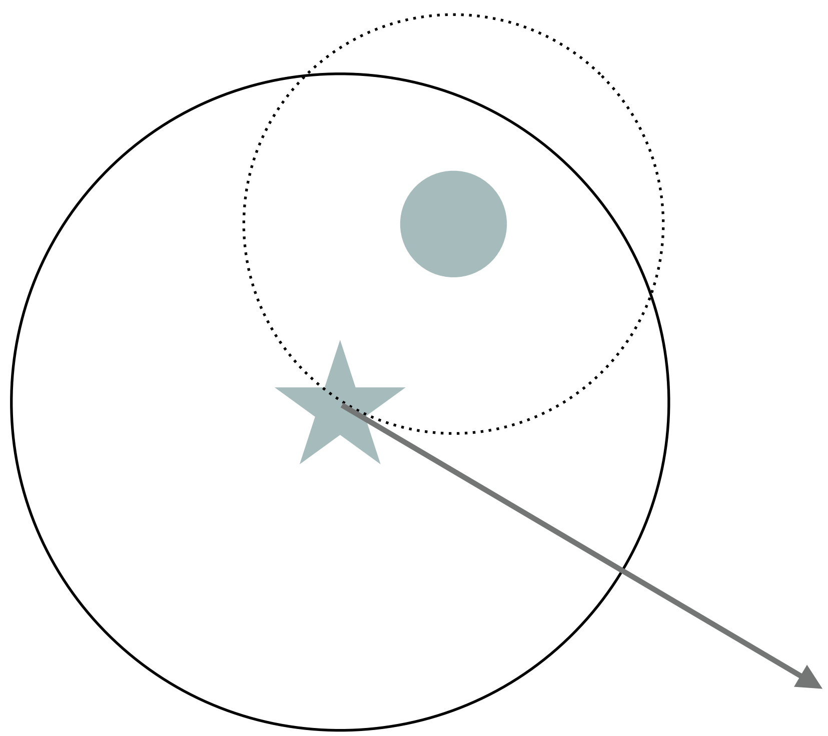
$$M = \frac{\theta_E}{k\pi_E}$$

If we can measure the microlensing parallax, then we measure the mass as a function of the distance. If we can measure θ_E, then we break the lensing degeneracy and measure the lens mass (without even seeing the lens light!)

MICROLENSING PARALLAX

- This may happen for different reasons (several kind of parallax effects).
- Due to the motion of the observer:
  - annual/orbital parallax: the earth moves around the sun
- Due to the separation between observers
  - satellite parallax: when we can look at the same microlensing event from two positions simultaneously from a space telescope and from the earth
  - terrestrial parallax: when we can look at the same microlensing event from two positions on the earth

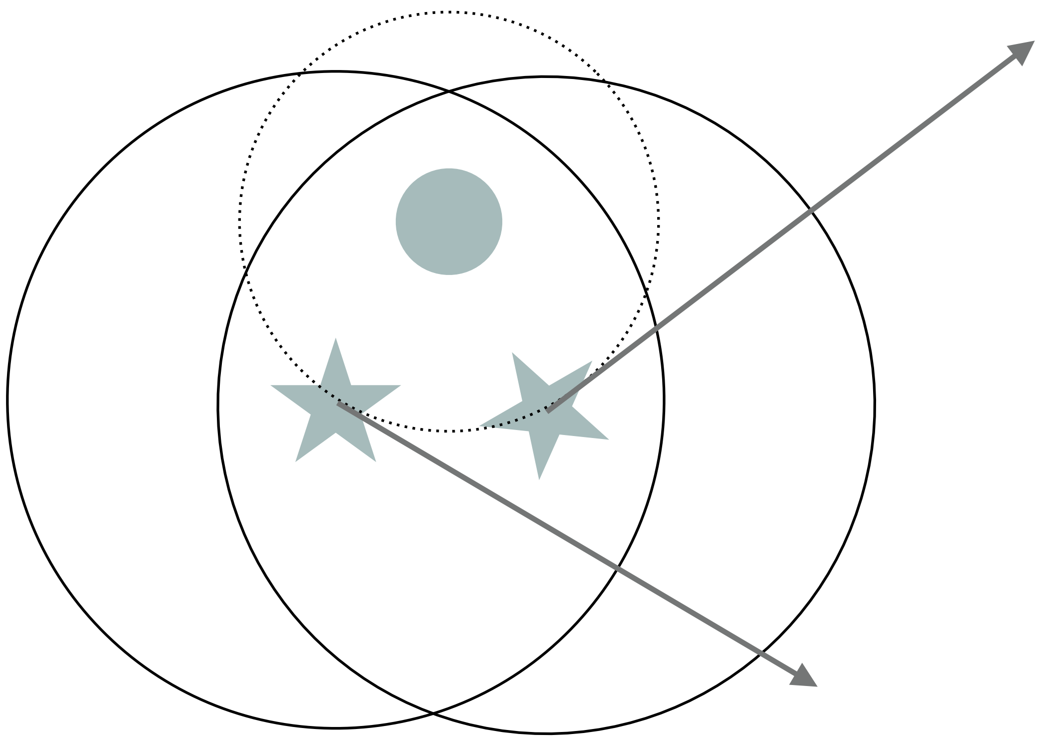
IMPORTANT: MICROLENSING PARALLAX IS A VECTOR!



Magnification does not depend on the direction of proper motion

$$\mu(t) = \frac{y^2(t) + 2}{y(t)\sqrt{y^2(t) + 4}}$$

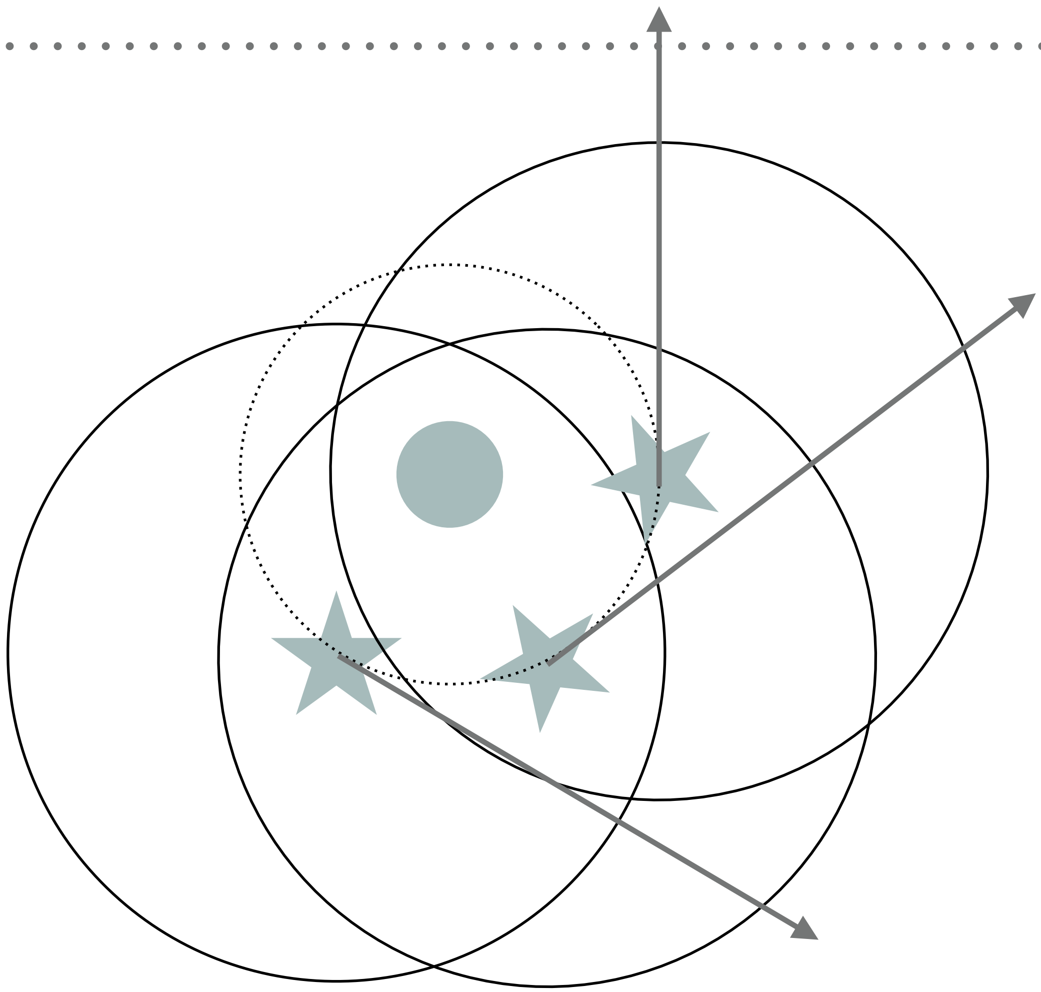
IMPORTANT: MICROLENSING PARALLAX IS A VECTOR!



Magnification does not depend on the direction of proper motion

$$\mu(t) = \frac{y^2(t) + 2}{y(t)\sqrt{y^2(t) + 4}}$$

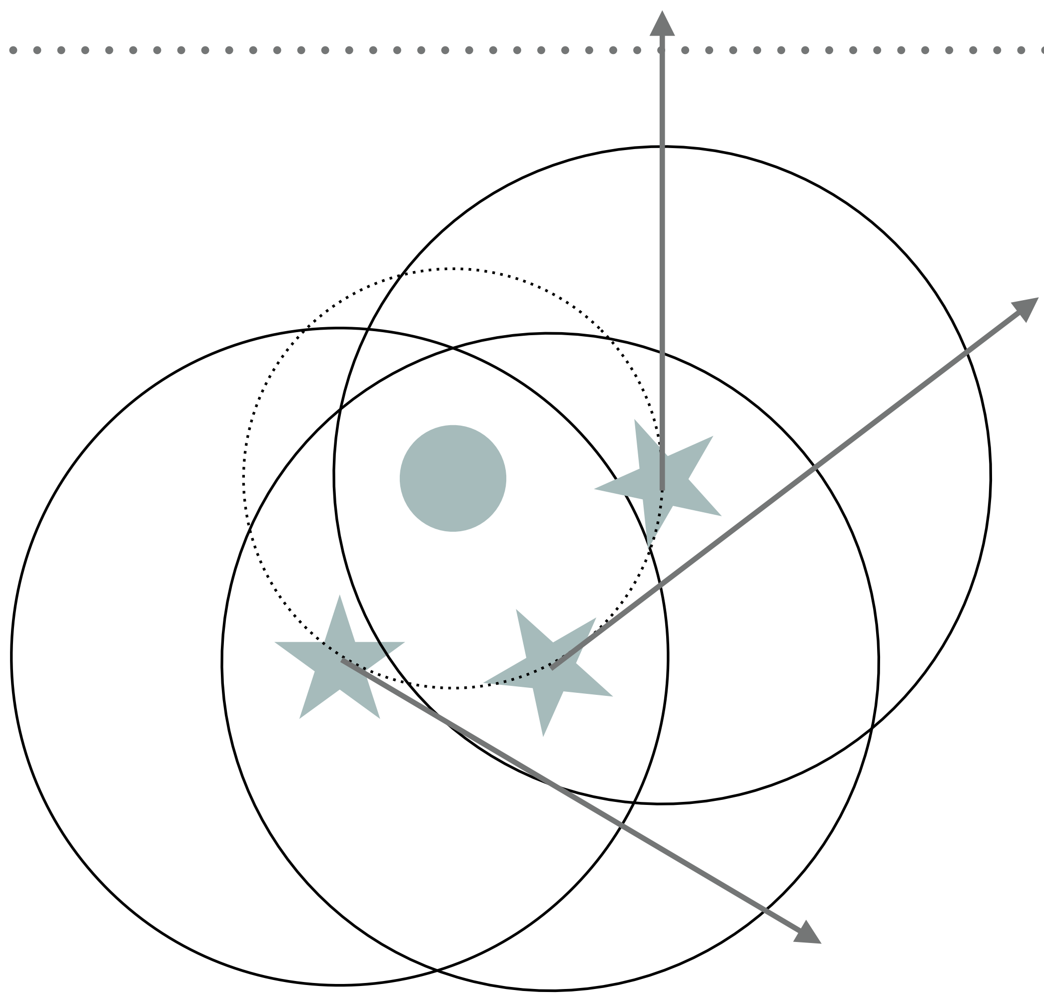
IMPORTANT: MICROLENSING PARALLAX IS A VECTOR!

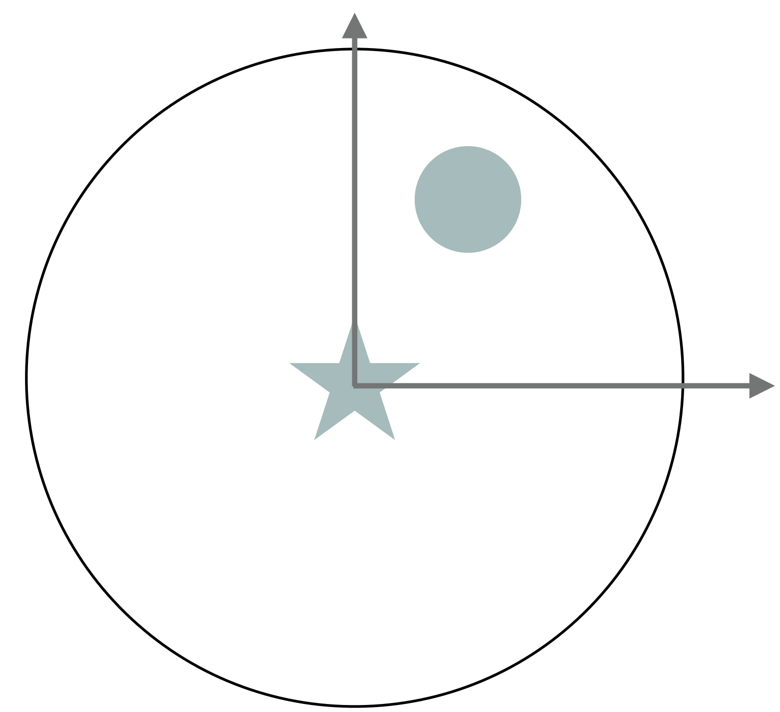


Magnification does not depend on the direction of proper motion

$$\mu(t) = \frac{y^2(t) + 2}{y(t)\sqrt{y^2(t) + 4}}$$

IMPORTANT: MICROLENSING PARALLAX IS A VECTOR!





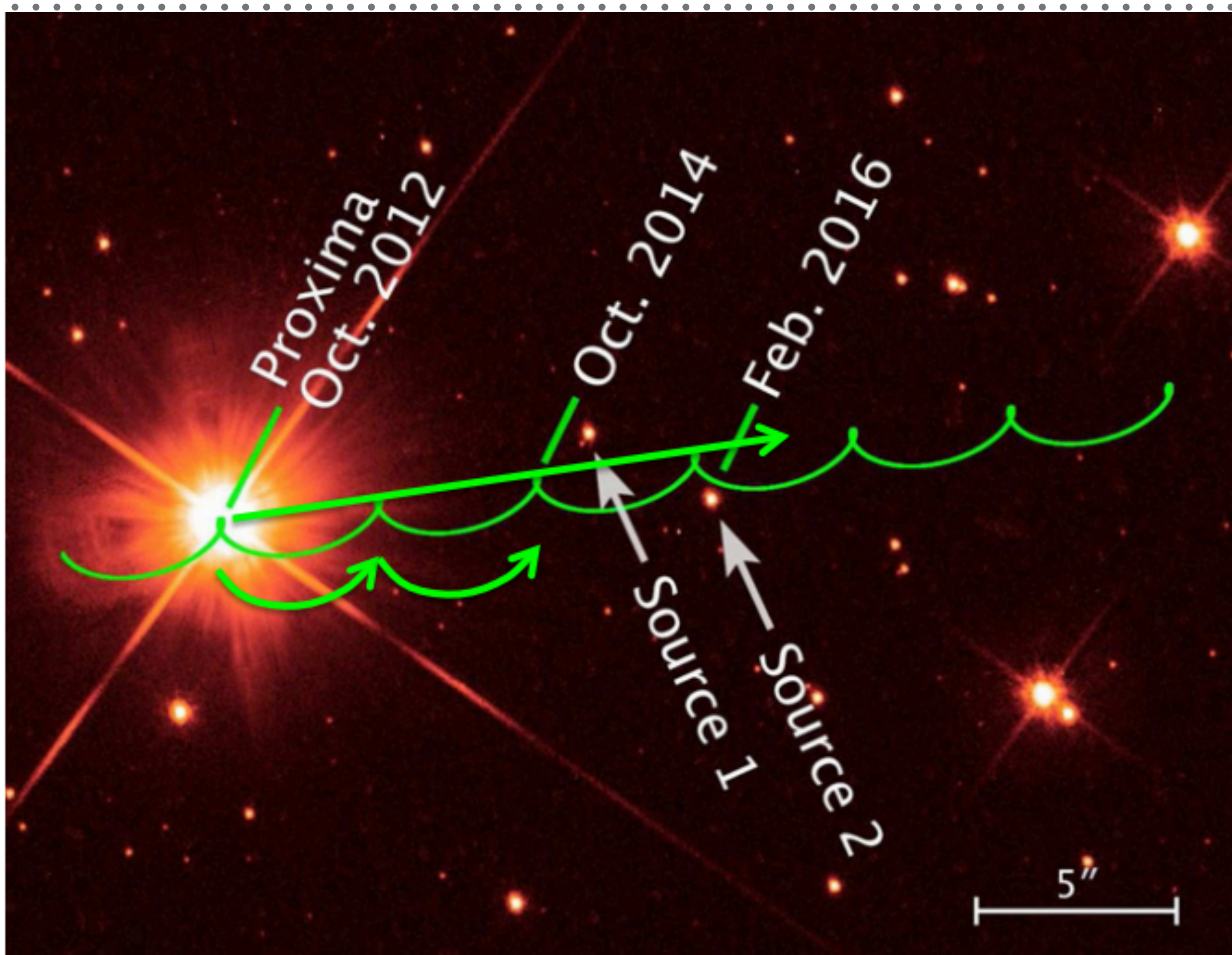
Magnification does not depend on the direction of proper motion

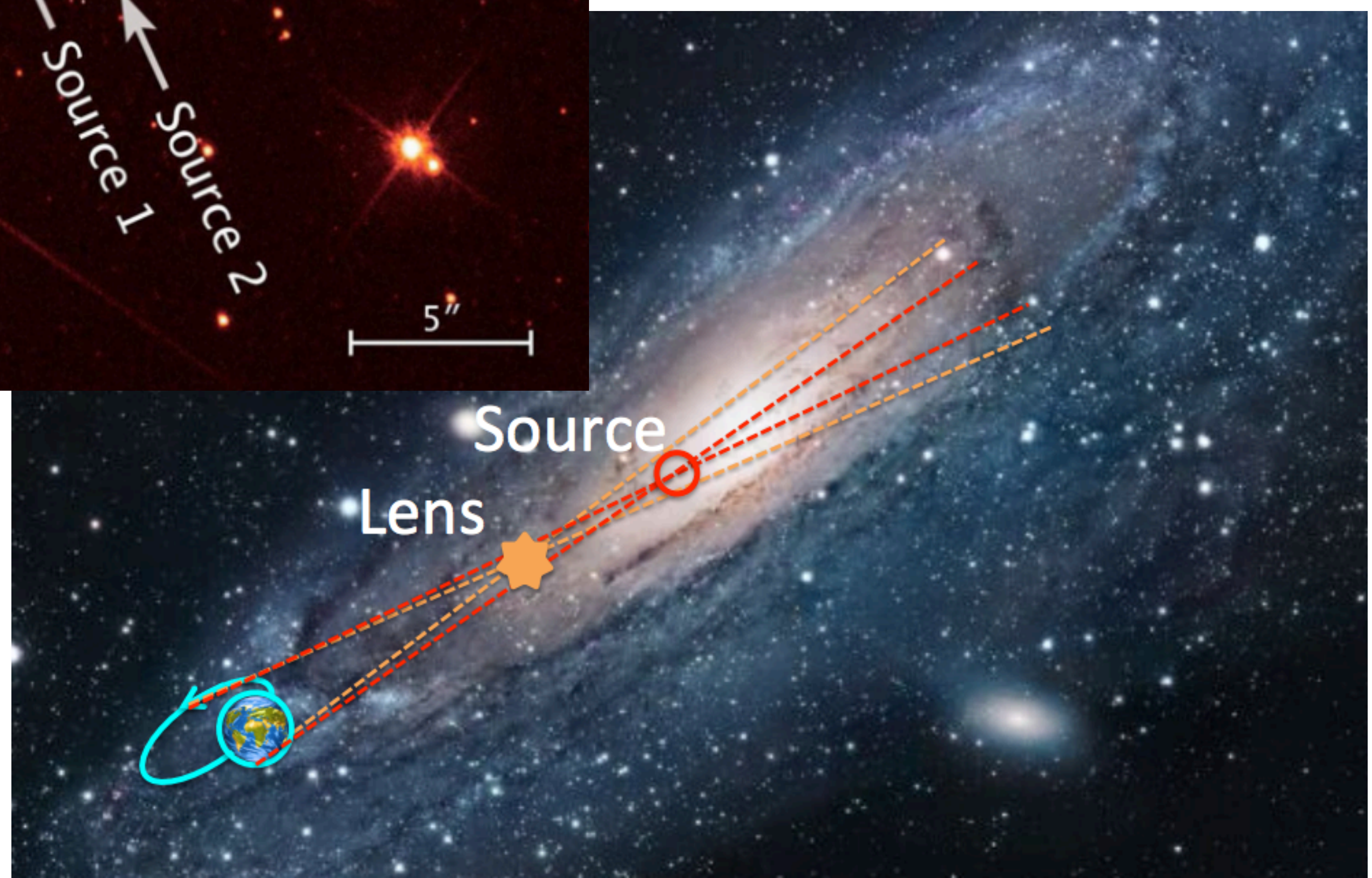
$$\mu(t) = \frac{y^2(t) + 2}{y(t)\sqrt{y^2(t) + 4}}$$

…but microlensing parallax does!

Depending on the lens displacement relative to the source (parallel or perpendicular to the proper motion), we will see different effects

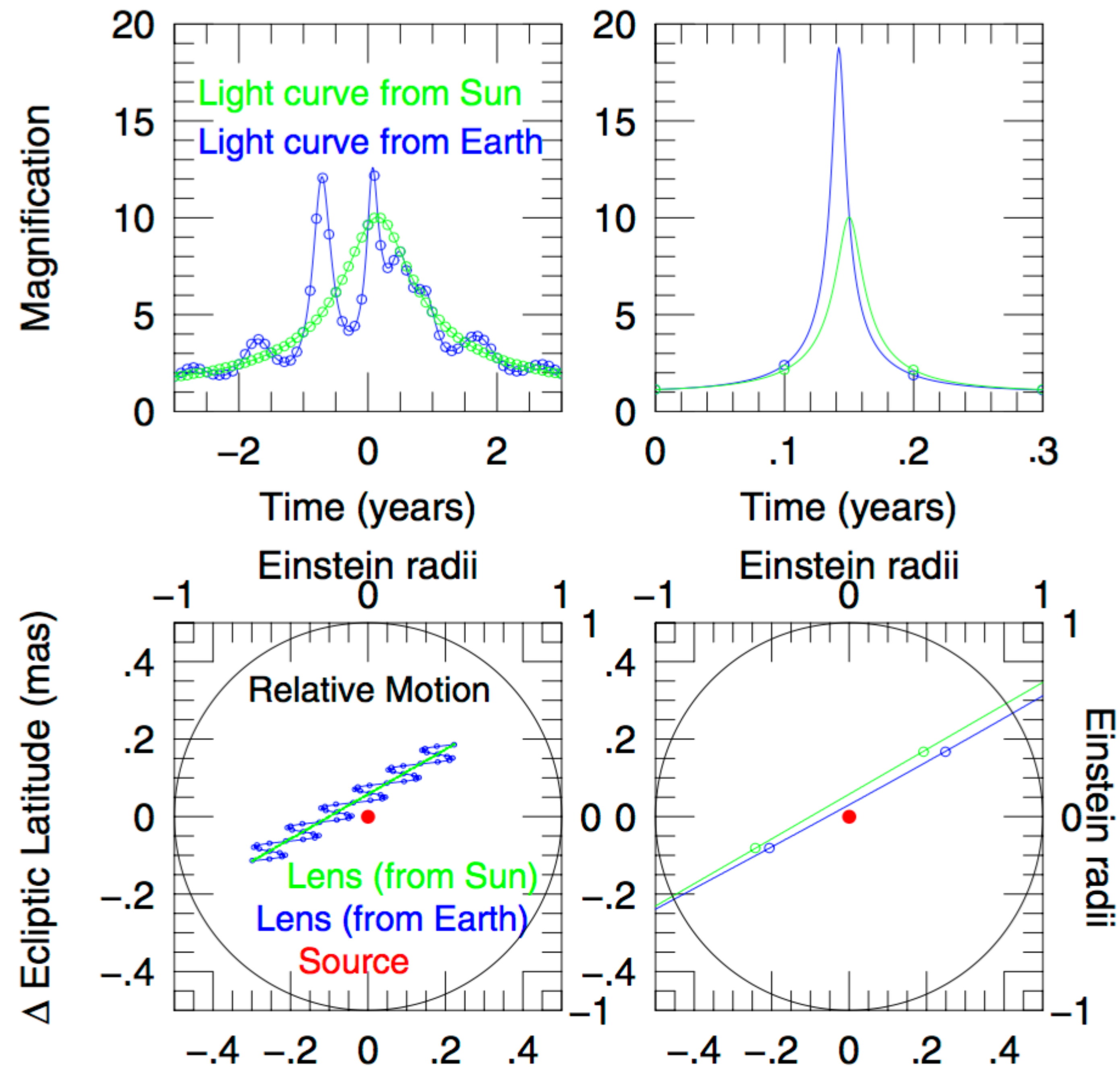
ORBITAL PARALLAX





ORBITAL PARALLAX

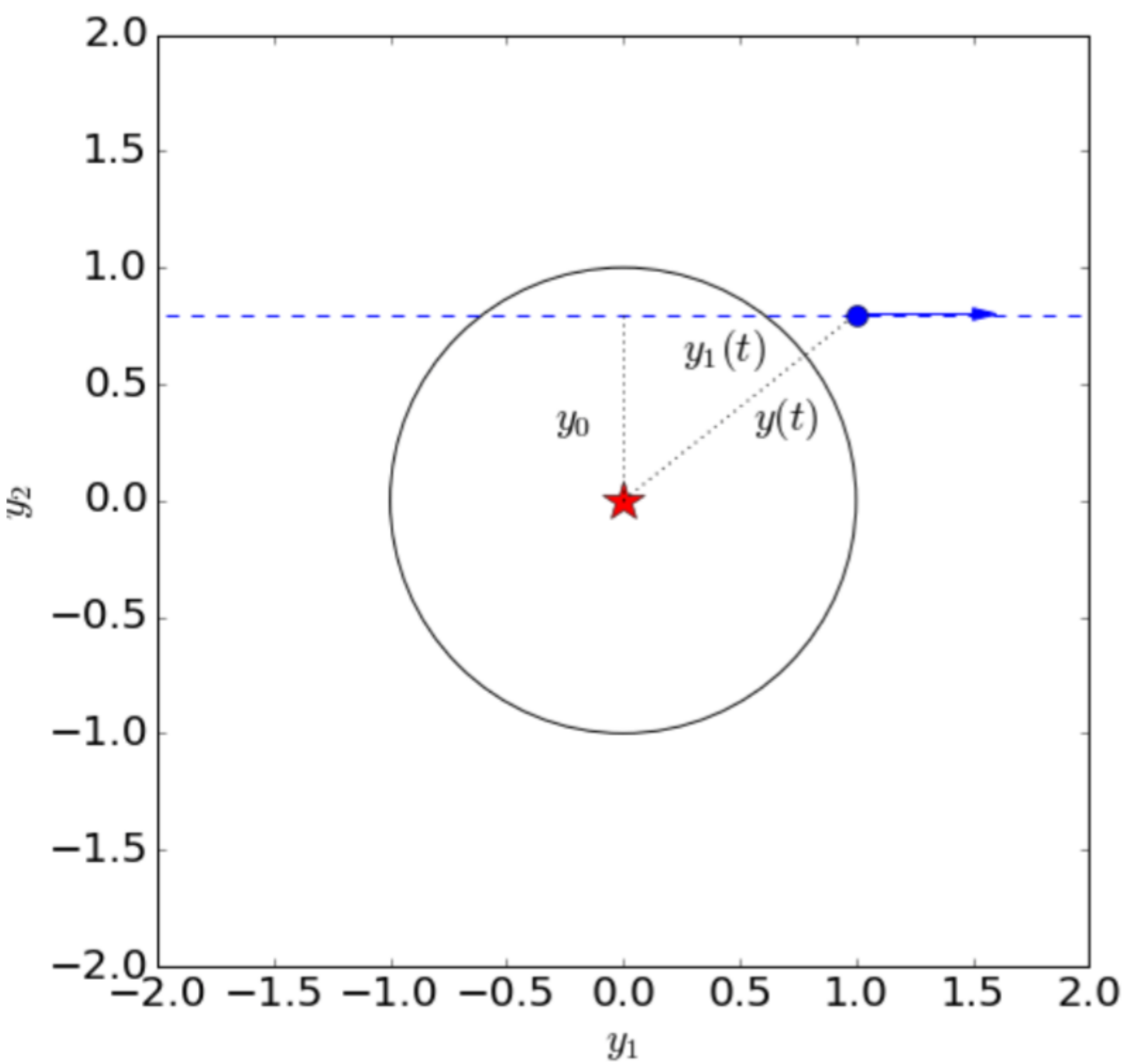
- on the left: what we would see if the μ_{hel}=0.1 mas/year
- on the right: the typical μ_{hel}=5 mas/year

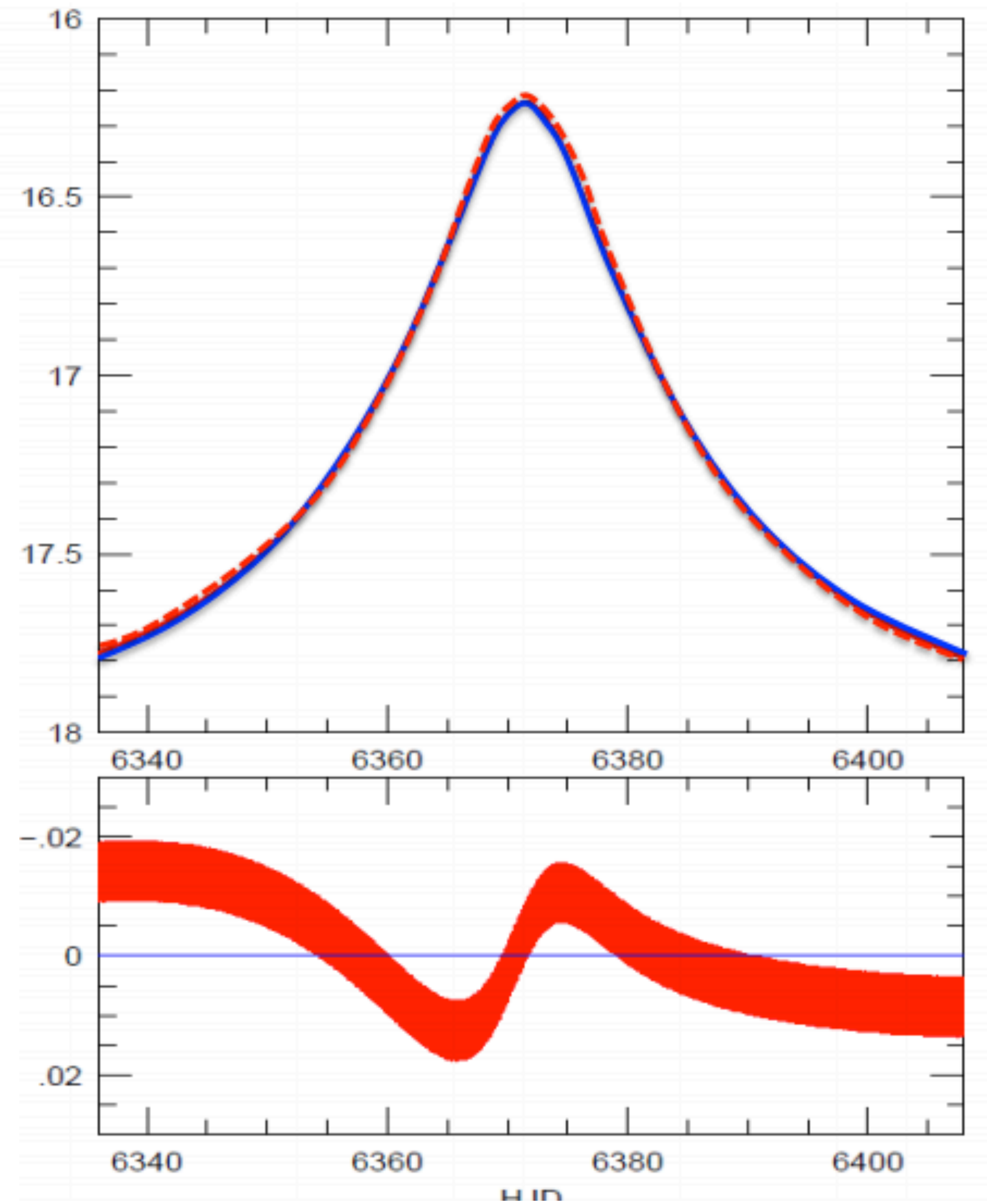


Gould & Horne, 2013

COMPONENT PARALLEL TO THE LENS TRAJECTORY

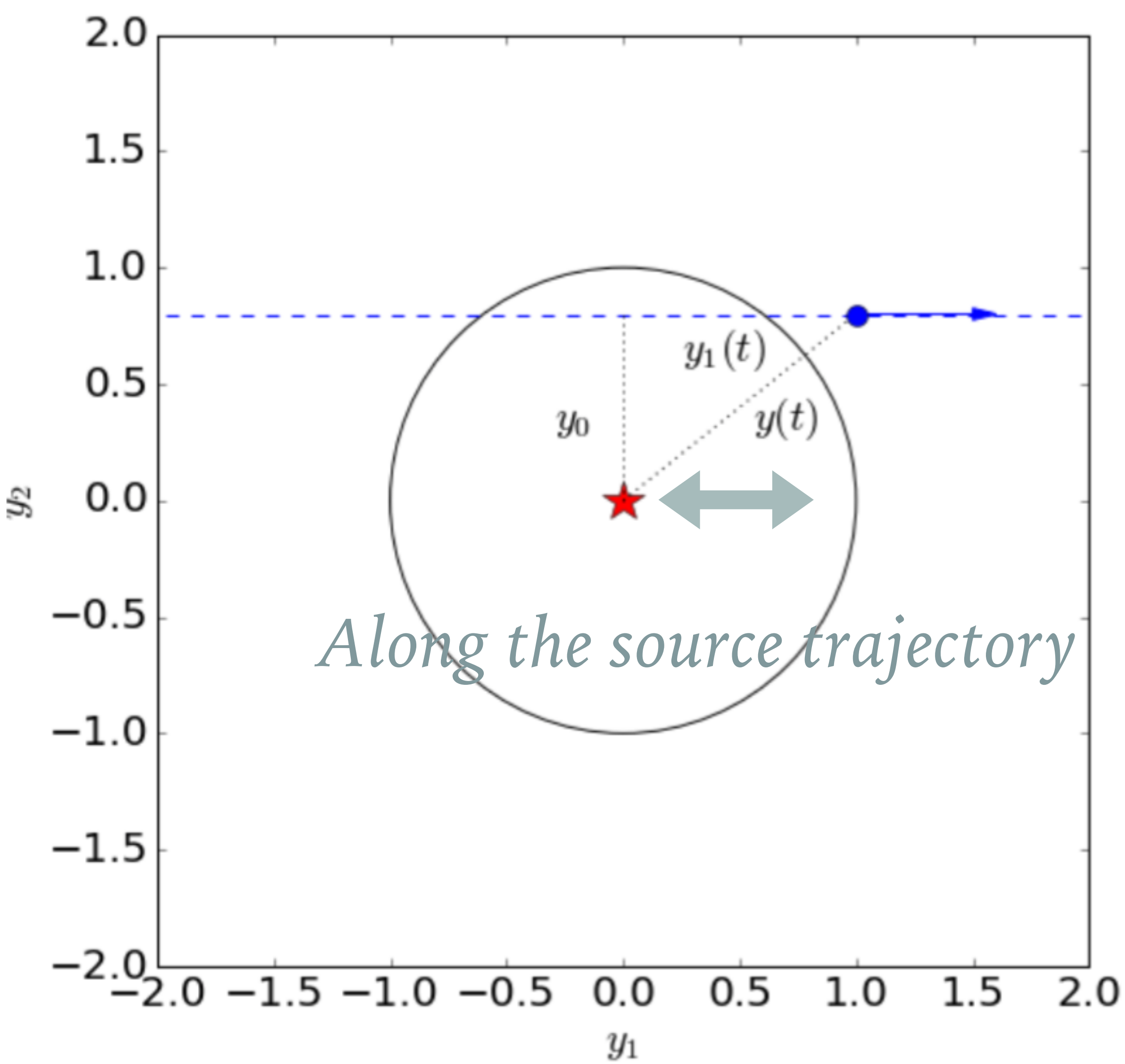
Asymmetric distortion of the light curve due to acceleration of the lens

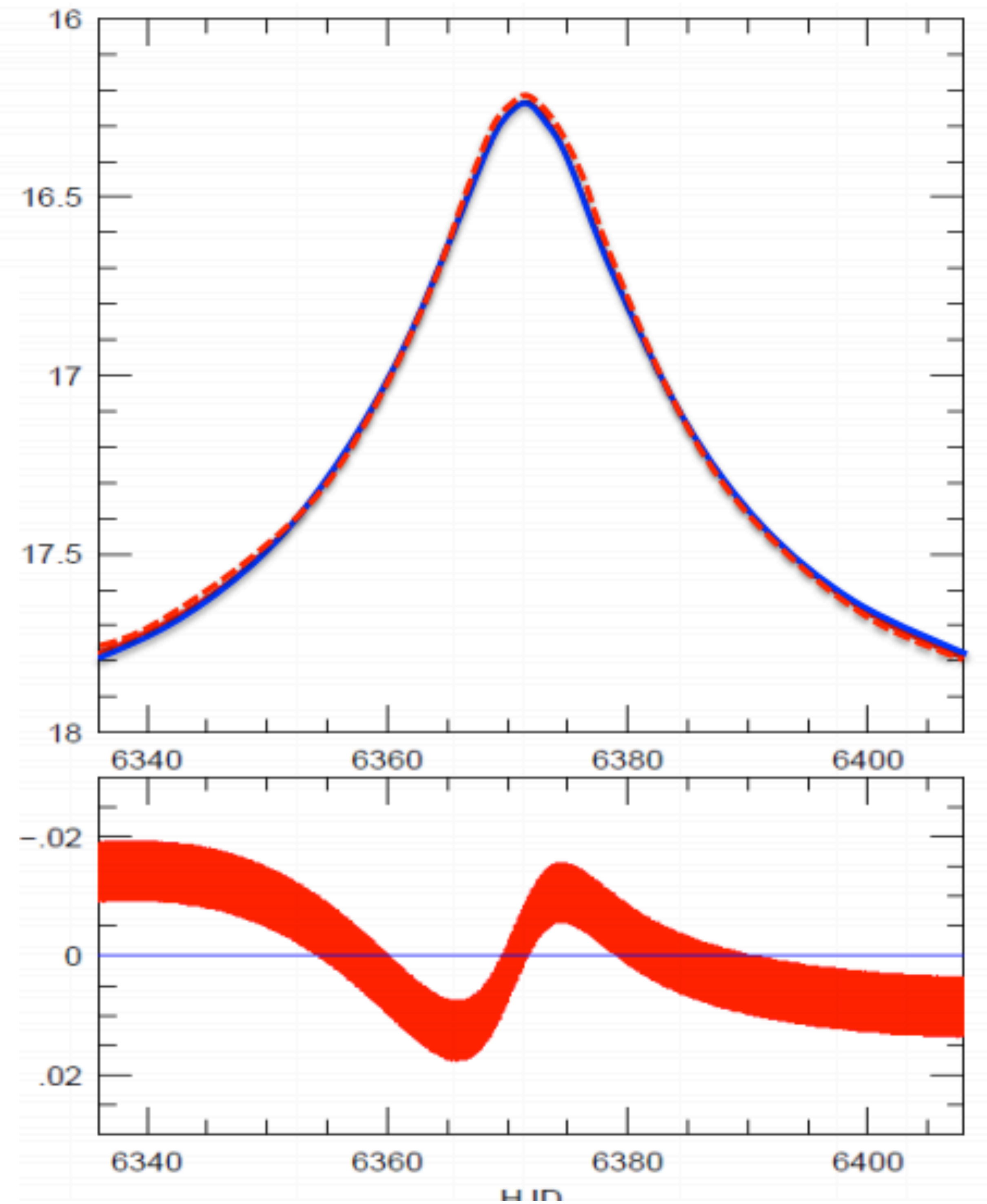




COMPONENT PARALLEL TO THE LENS TRAJECTORY

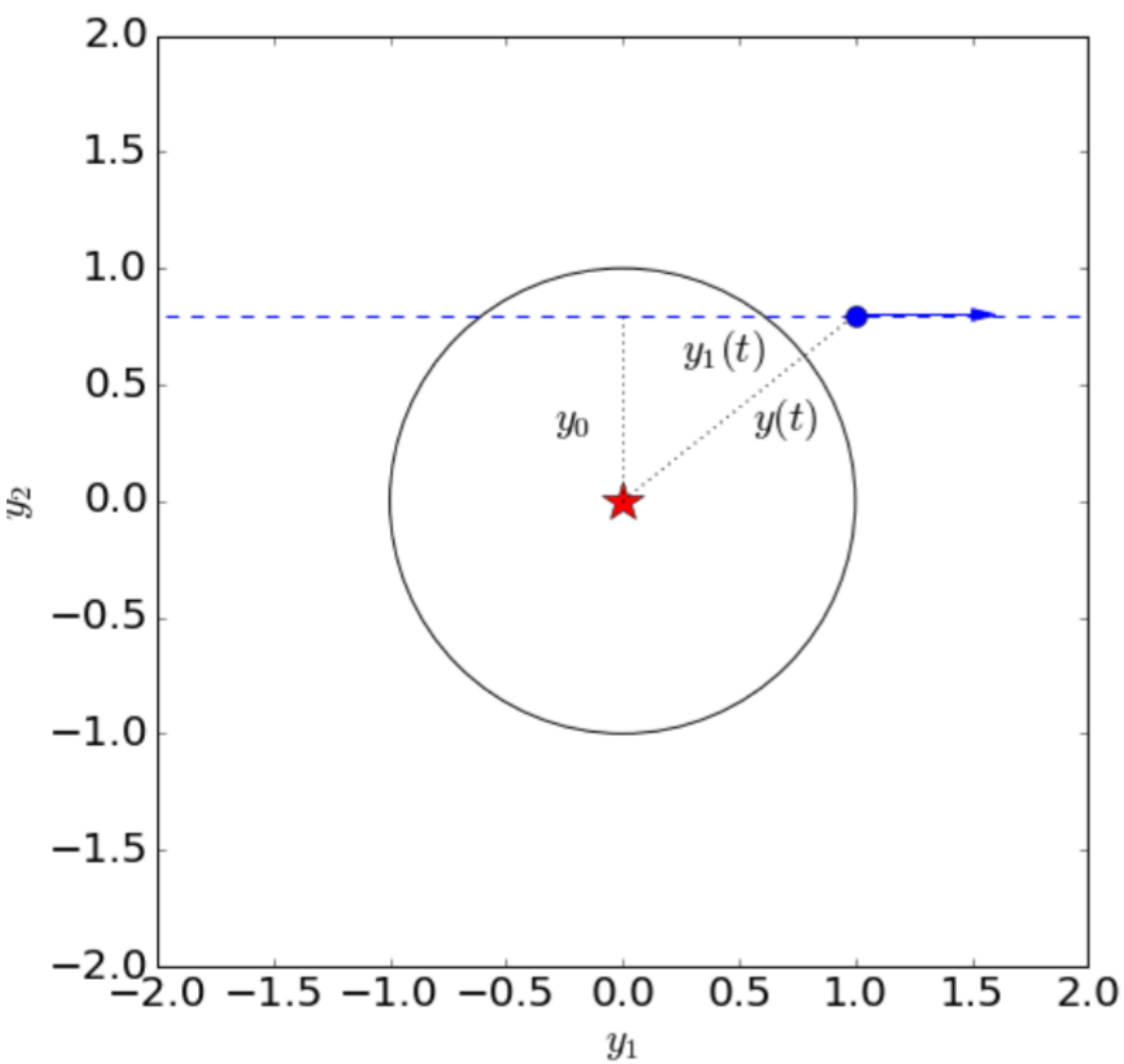
Asymmetric distortion of the light curve due to acceleration of the lens

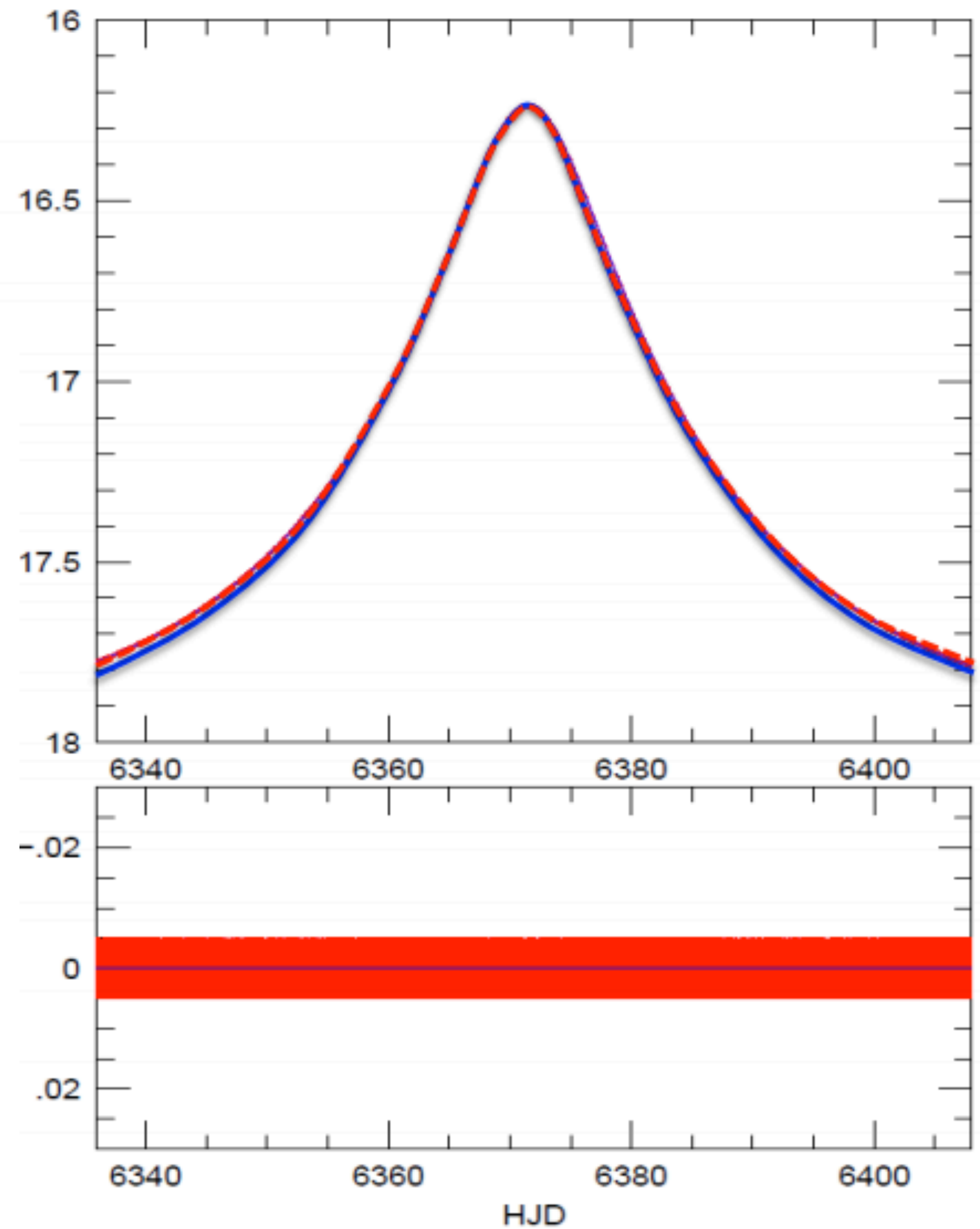




COMPONENT PERPENDICULAR TO THE LENS TRAJECTORY

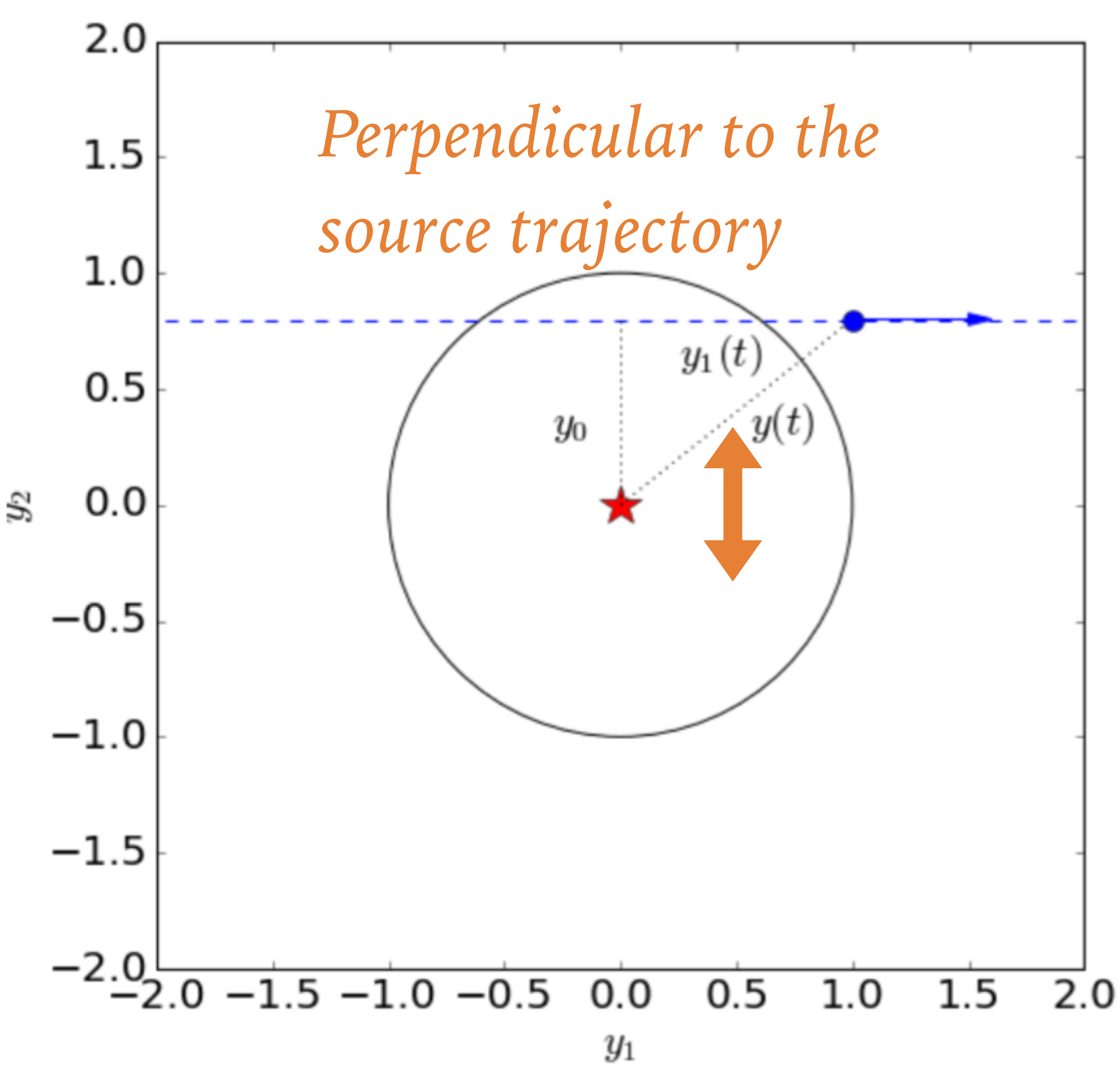
Symmetric distortion of the light curve due to motion perpendicular to lens trajectory

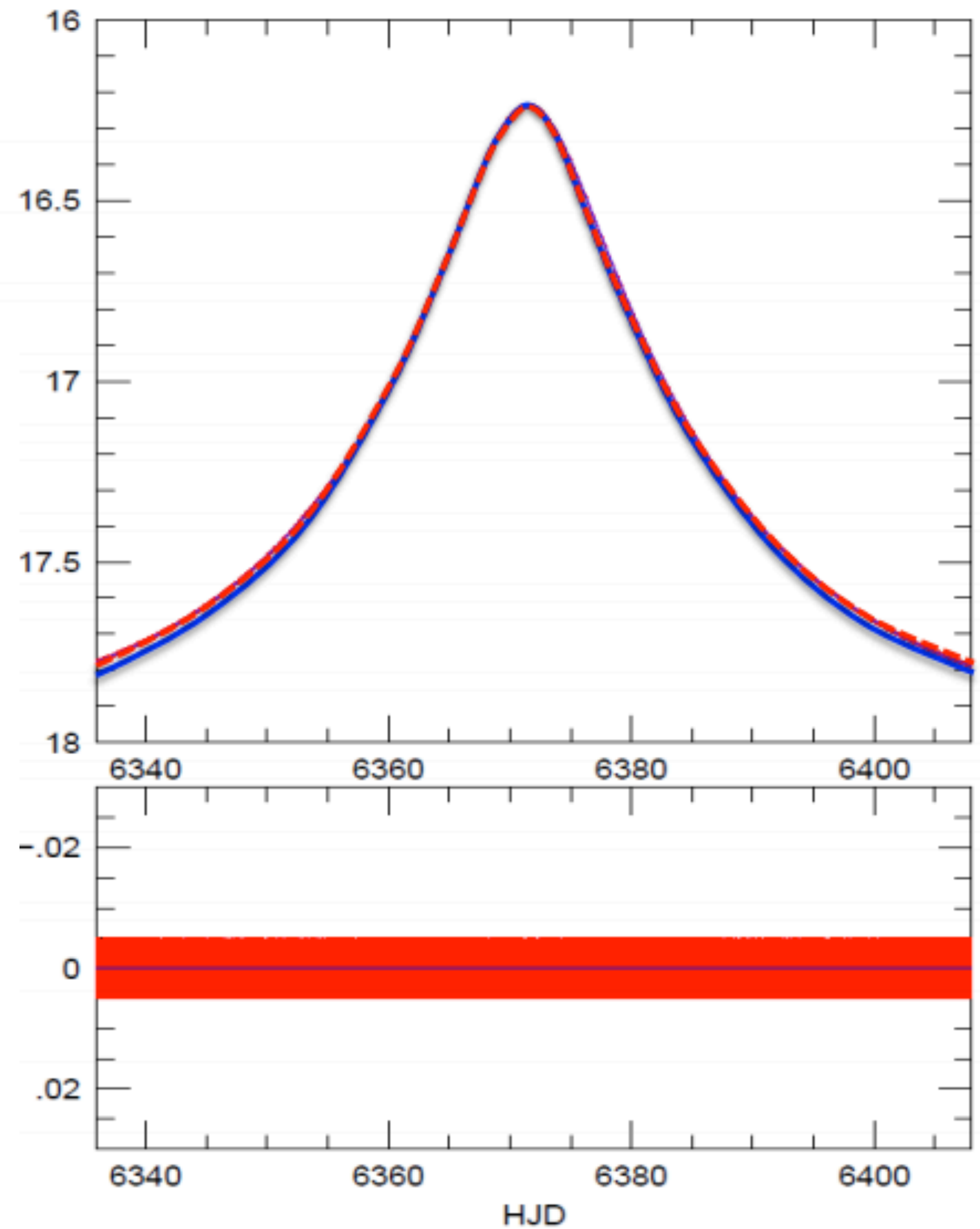




COMPONENT PERPENDICULAR TO THE LENS TRAJECTORY

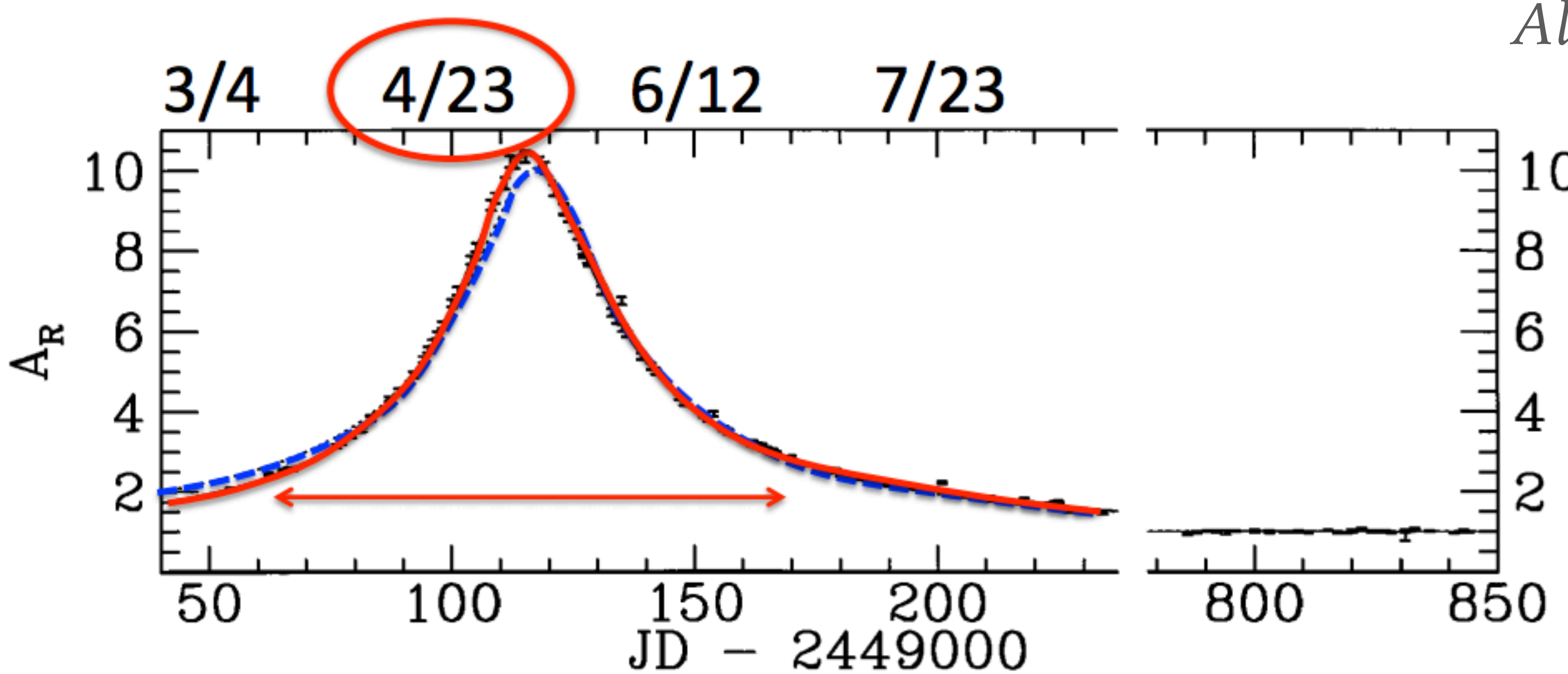
Symmetric distortion of the light curve due to motion perpendicular to lens trajectory



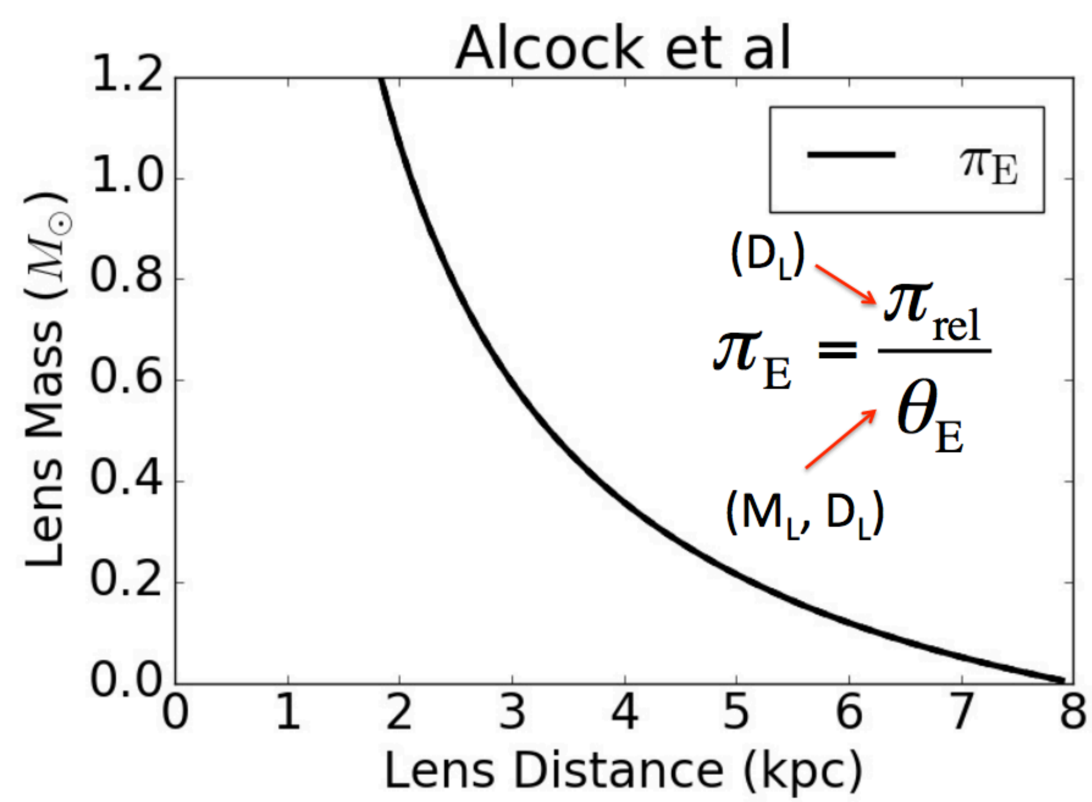


FIRST DETECTION OF MICROLENS PARALLAX

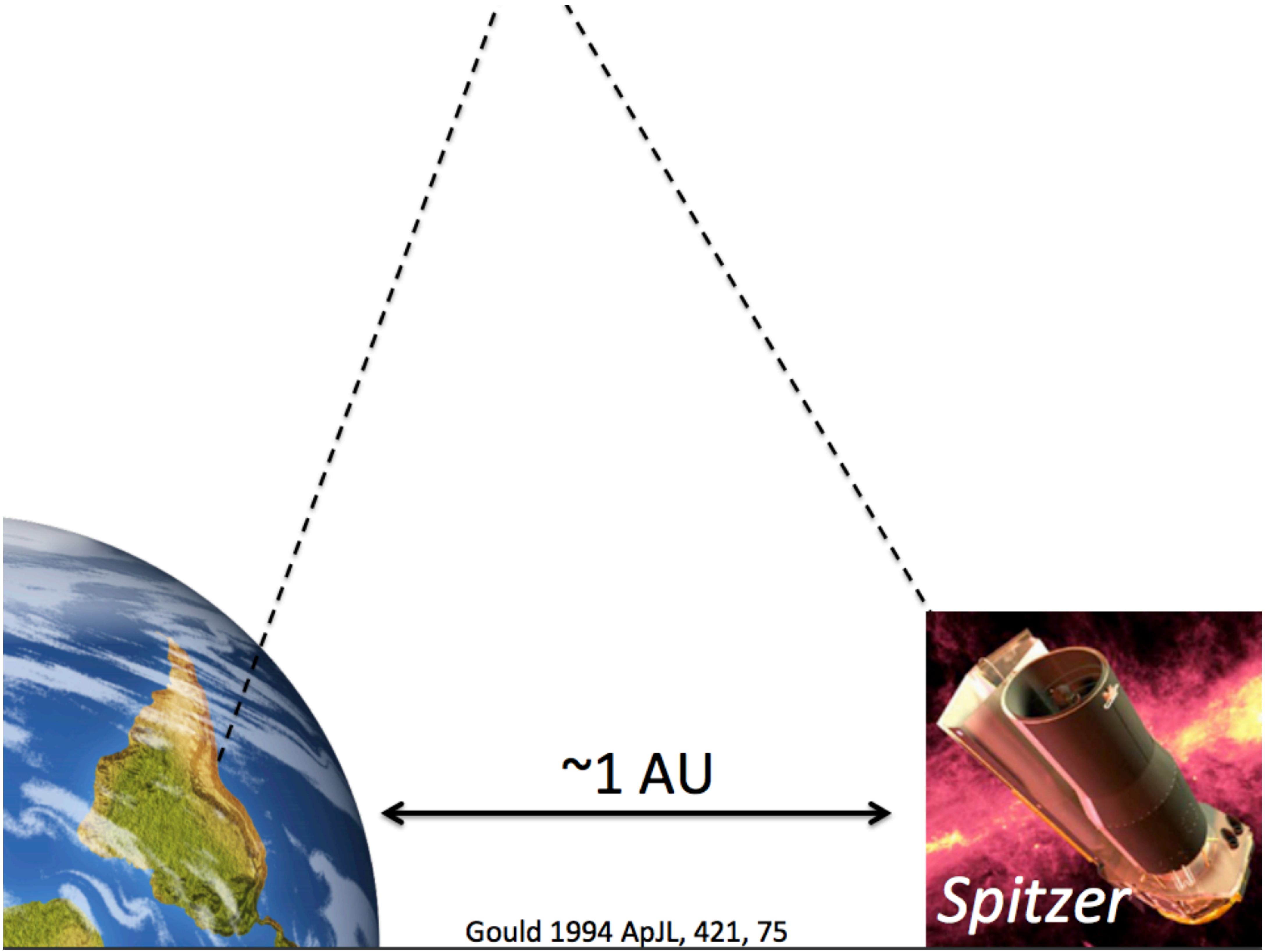
Alcock et al. 1995



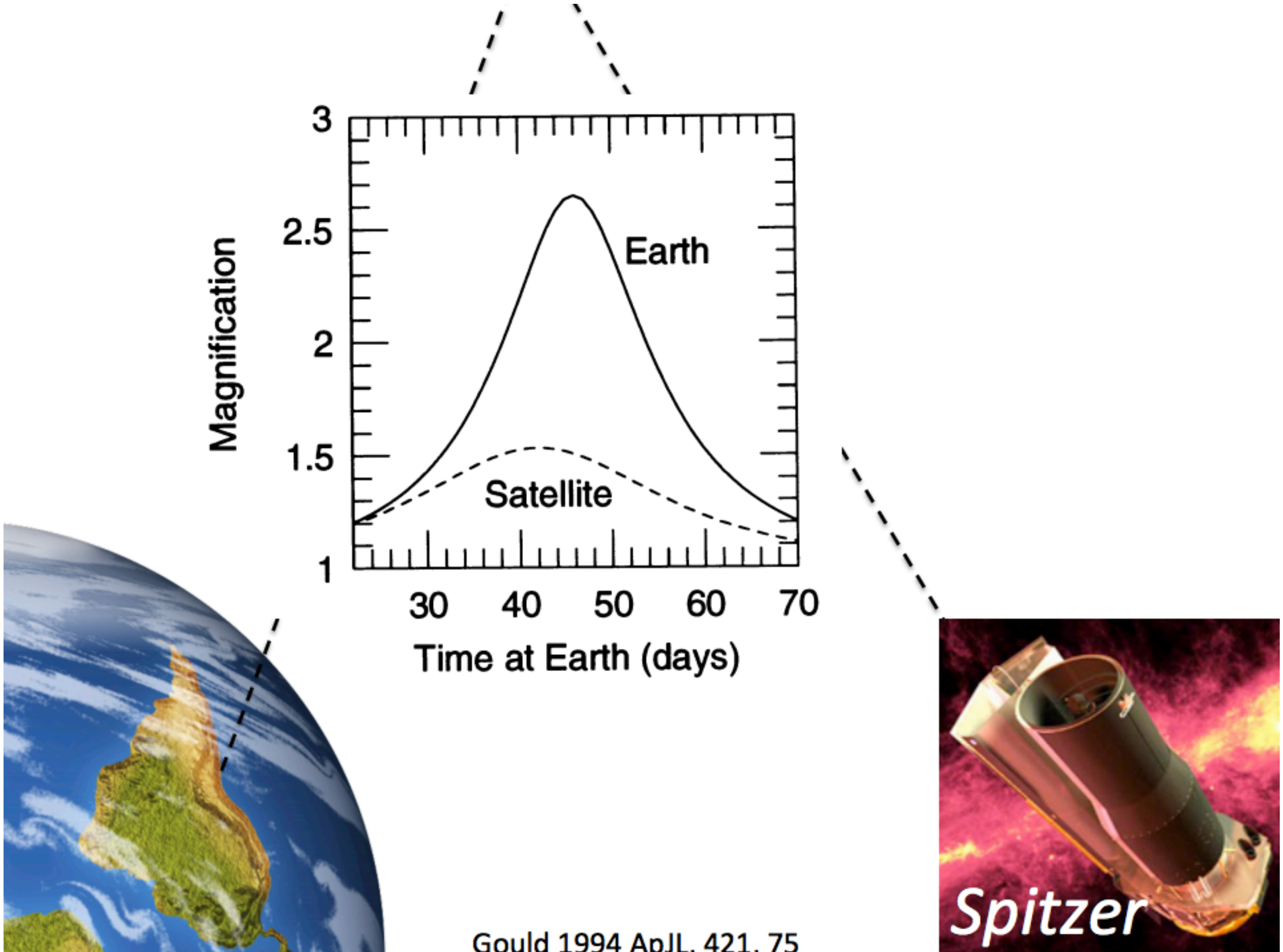
Even without an estimate of θ_E, measuring the parallax still allows to measure the mass vs distance



SATELLITE PARALLAX

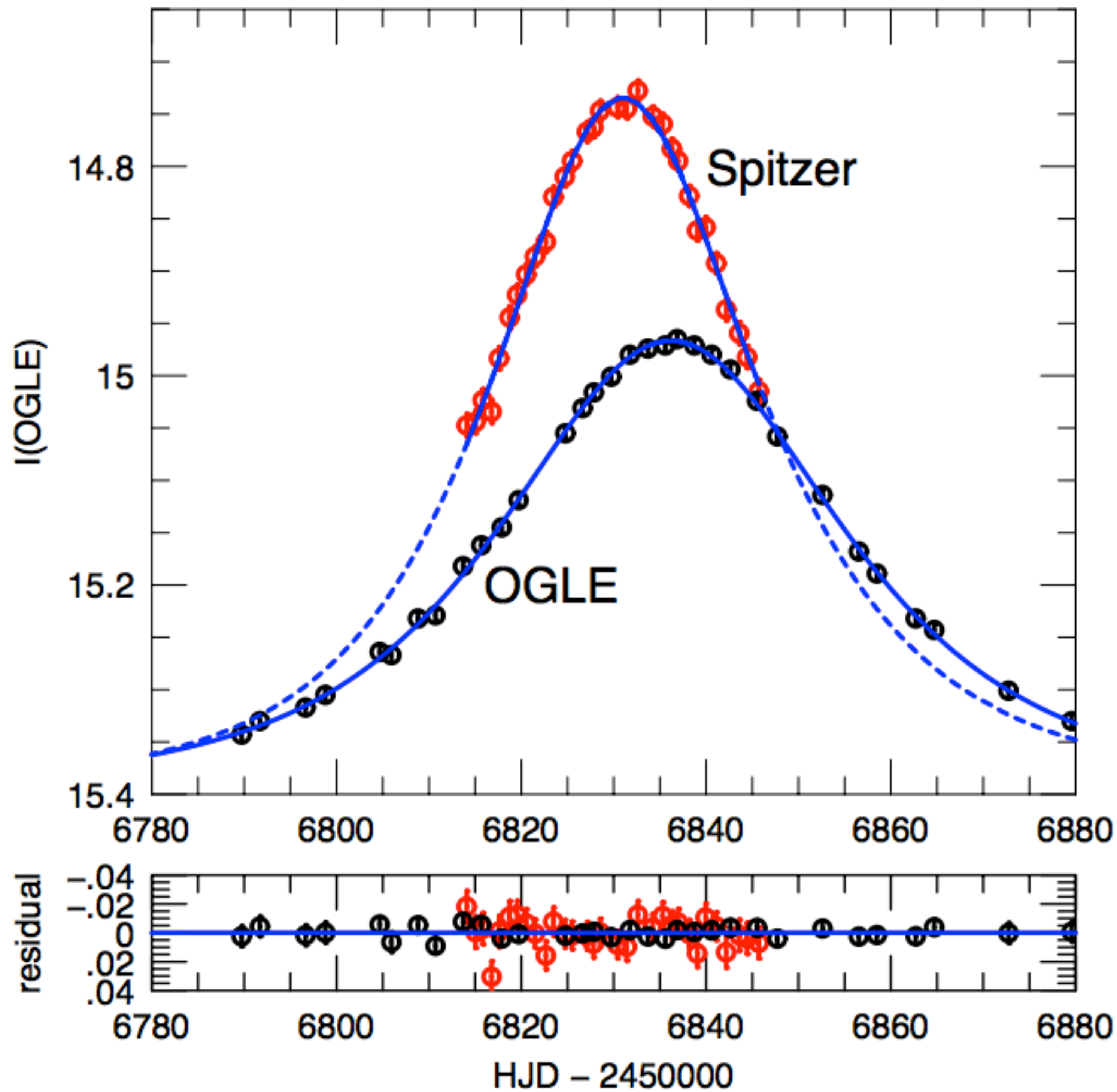


SATELLITE PARALLAX



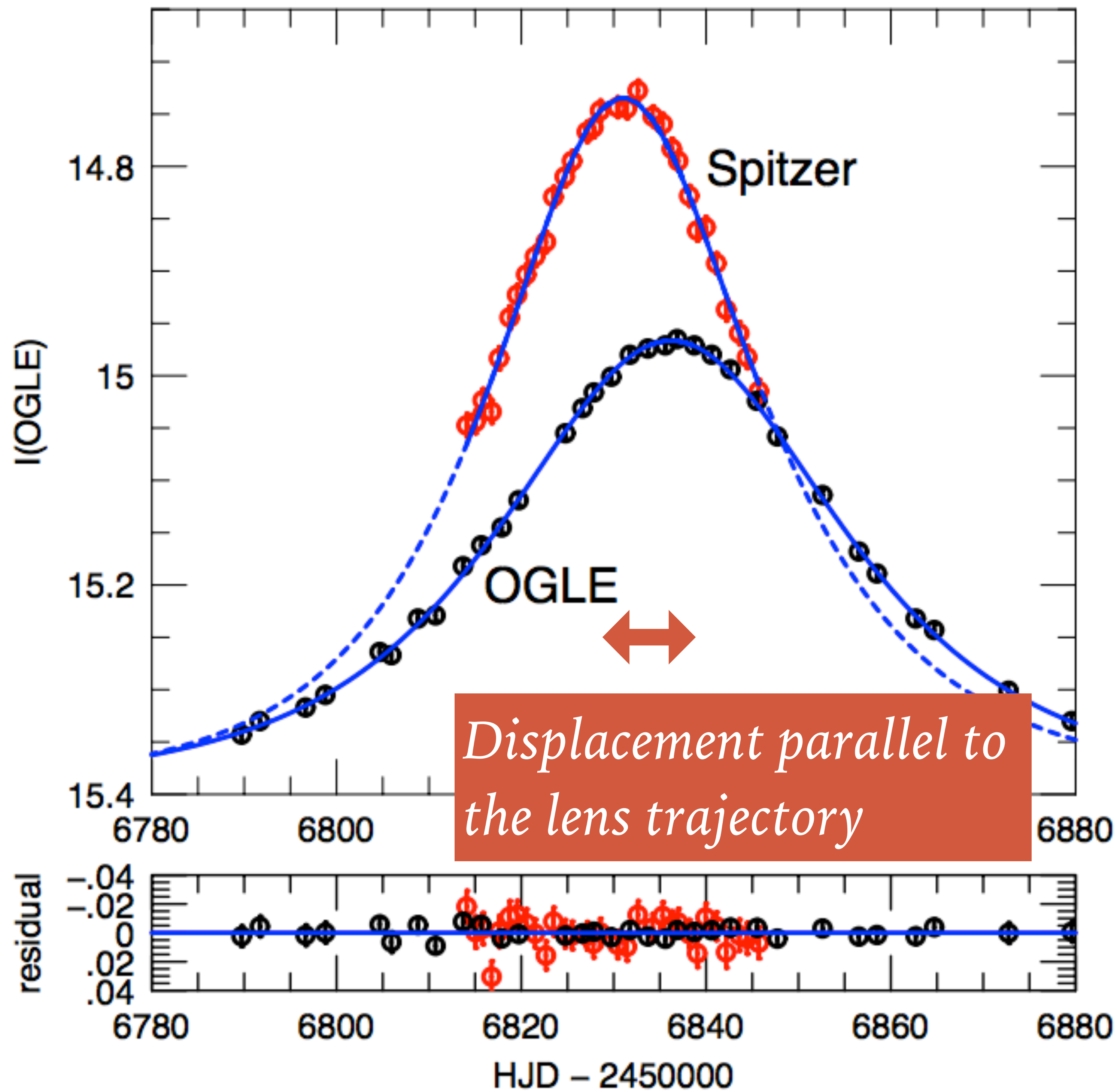
Gould 1994 ApJL, 421, 75

SATELLITE PARALLAX



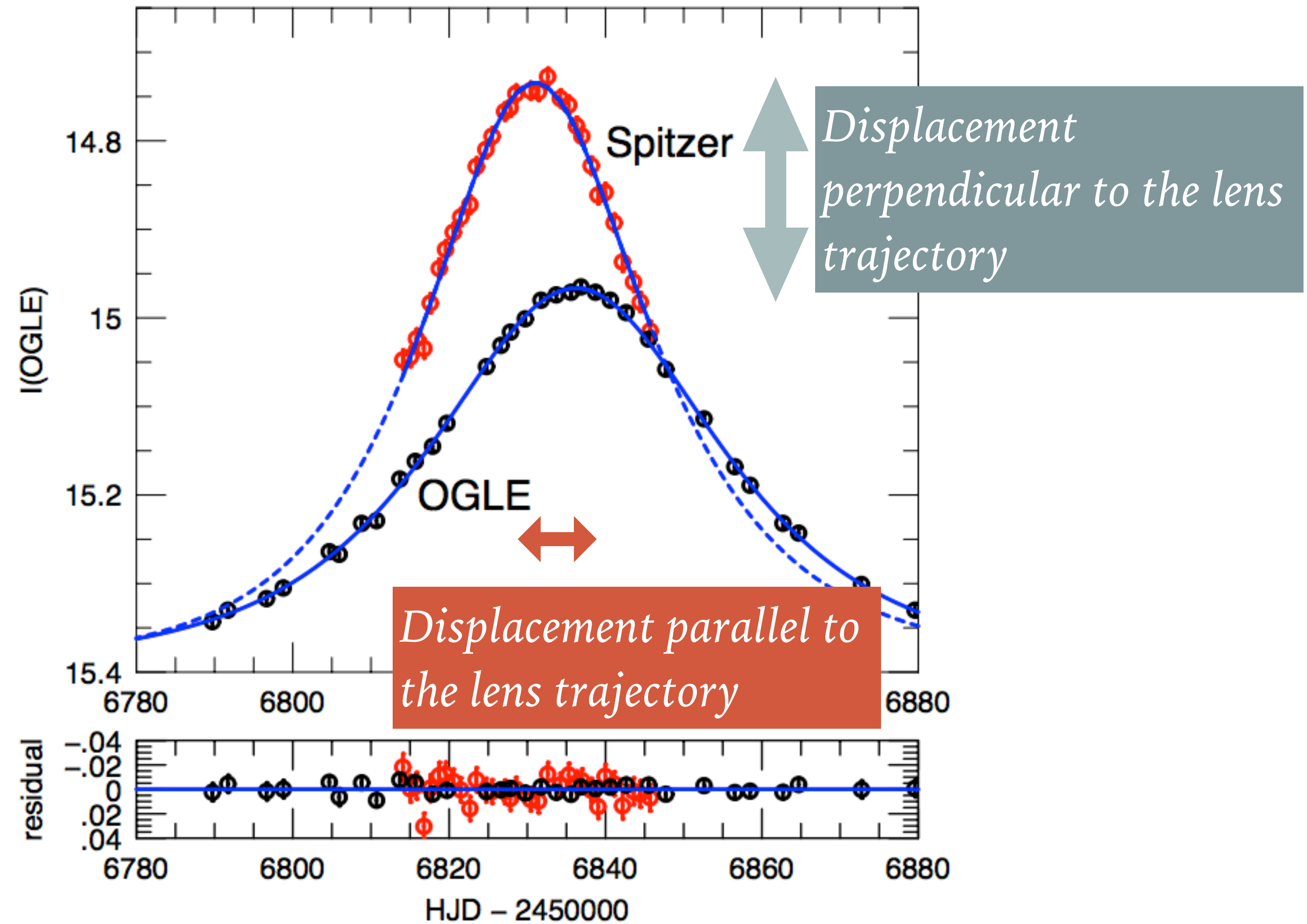
Yee et al. 2015, ApJ, 802, 76

SATELLITE PARALLAX



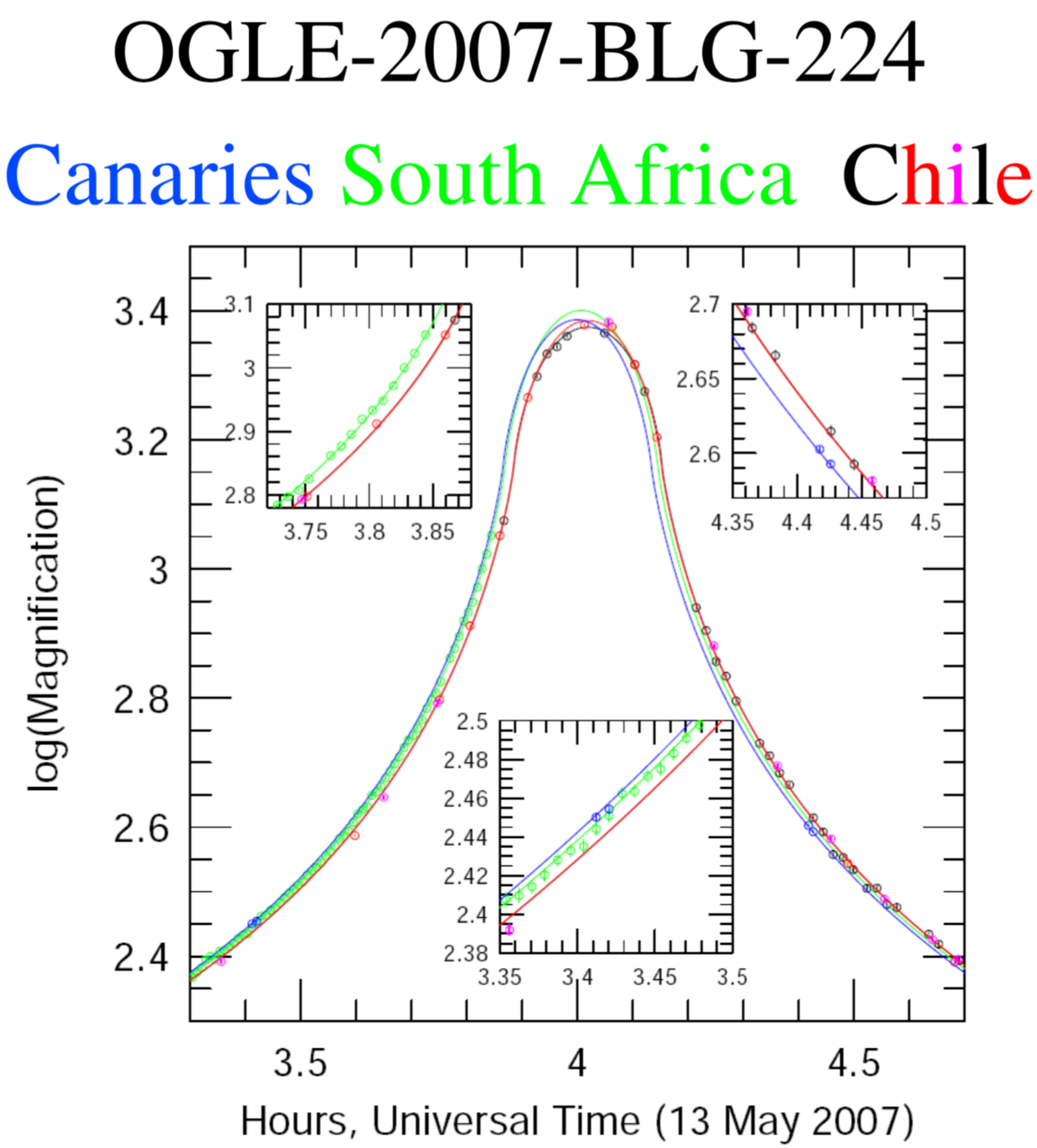
Yee et al. 2015, ApJ, 802, 76

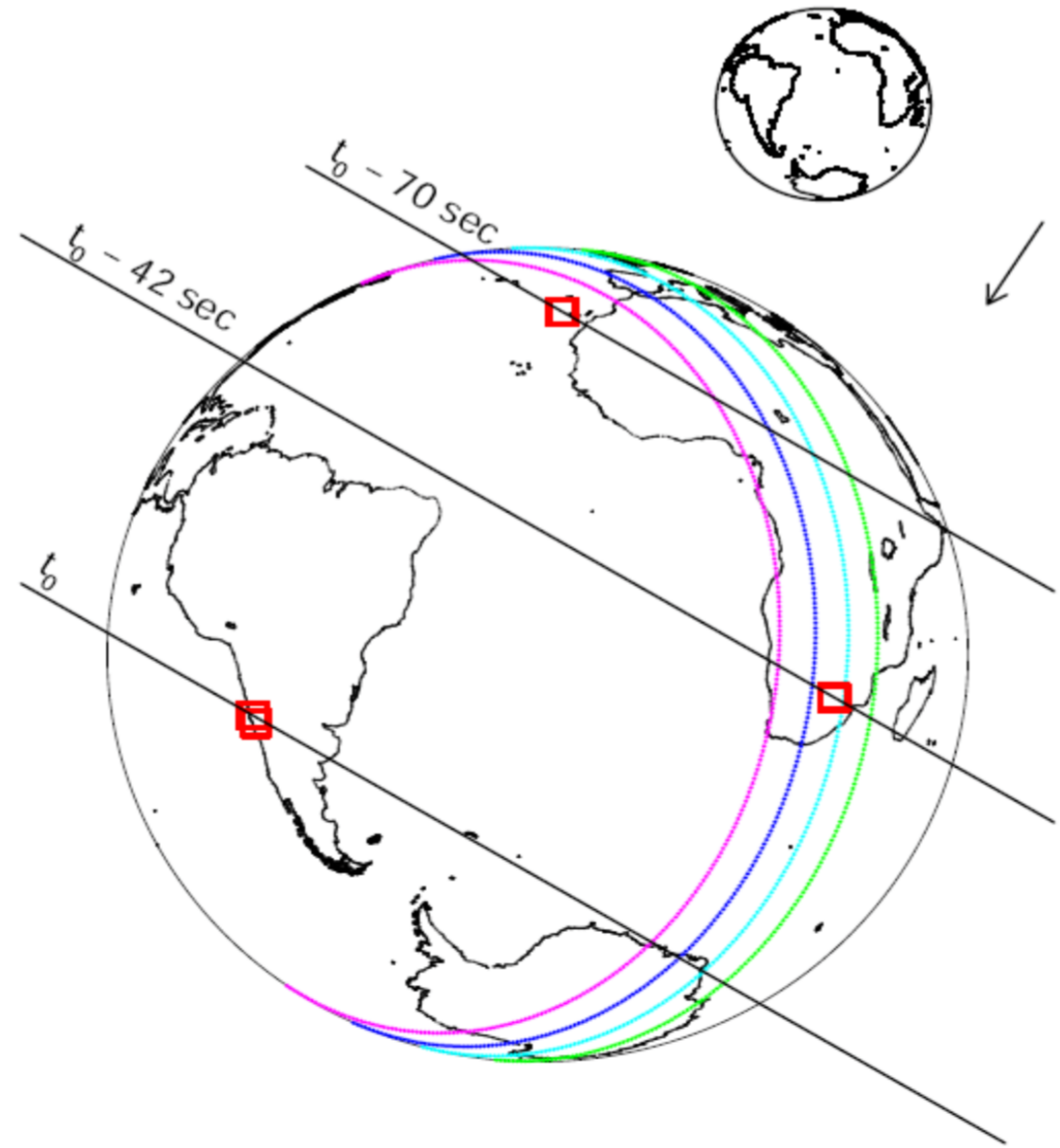
SATELLITE PARALLAX



Yee et al. 2015, ApJ, 802, 76

MICROLENS PARALLAX (TERRESTRIAL)



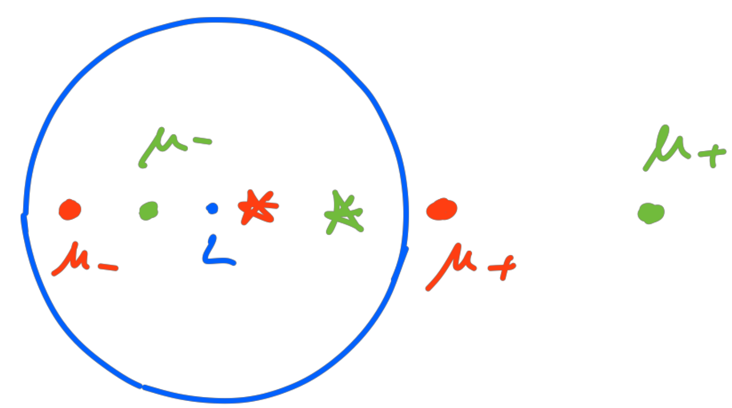


Gould & Horne, 2013

WHAT IS ASTROMETRIC MICROLENSING?

- during a microlensing event, the two images of the source cannot be resolved ($\theta_E \sim 1$mas)
- their positions and the magnifications change as a function of time
- in particular, the image forming outside the Einstein ring dominates, in terms of flux for most of the time
- what an observer will see is one source at the light centroid, which will move as a function of time depending on where the two images form and on how much flux they emit

$$
\begin{aligned}
\mu_{\pm}(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \\
&= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right) \\
&= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right)
\end{aligned}
$$



THE EQUATIONS

$$x_\pm = \frac{1}{2}\left[y \pm \sqrt{y^2+4}\right]$$

$$\begin{aligned} x_{\pm,\parallel} &= \frac{1}{2}(1 \pm Q)y_\parallel \\ x_{\pm,\perp} &= \frac{1}{2}(1 \pm Q)y_\perp \end{aligned}$$

$$\begin{aligned} \mu_\pm(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \\ &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right) \\ &= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right) \end{aligned}$$

$$Q = \frac{\sqrt{y^2+4}}{y}$$

$$\vec{x}_c = \frac{\vec{x}_+|\mu_+| + \vec{x}_-|\mu_-|}{|\mu_+| + |\mu_-|}$$

$$\delta\vec{x}_c = \vec{x}_c - \vec{y}$$

LIGHT CENTROID SHIFT AMPLITUDE

$$
\begin{aligned}
\delta x_c &= \frac{\frac{1}{4}\left[(y+\sqrt{y^2+4})\left(1+\frac{y^2+2}{y\sqrt{y^2+4}}\right)-(y-\sqrt{y^2+4})\left(1-\frac{y^2+2}{y\sqrt{y^2+4}}\right)\right]}{\frac{y^2+2}{y\sqrt{y^2+4}}}-y \\
&= \frac{\frac{1}{4}\left(y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}+\frac{y^2+2}{y}-y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}-\frac{y^2+2}{y}\right)}{\frac{y^2+2}{y\sqrt{y^2+4}}}-y \\
&= \frac{y}{y^2+2}\,.
\end{aligned}
$$

Given the sign, the shift points in the same direction of y.

Note that $y \gg \sqrt{2}$, $\delta x_c \approx \frac{1}{y}$

Thus, the shift decreases relatively slow with y… remember the scaling of μ?

LIGHT CENTROID SHIFT AMPLITUDE

$$
\begin{aligned}
\delta x_c &= \frac{\frac{1}{4}\left[(y+\sqrt{y^2+4})\left(1+\frac{y^2+2}{y\sqrt{y^2+4}}\right)-(y-\sqrt{y^2+4})\left(1-\frac{y^2+2}{y\sqrt{y^2+4}}\right)\right]}{\frac{y^2+2}{y\sqrt{y^2+4}}}-y \\
&= \frac{\frac{1}{4}\left(y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}+\frac{y^2+2}{y}-y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}-\frac{y^2+2}{y}\right)}{\frac{y^2+2}{y\sqrt{y^2+4}}}-y \\
&= \frac{y}{y^2+2}\,.
\end{aligned}
$$

In addition

$$
\frac{d(\delta x_c)}{dy}=\frac{2-y^2}{(y^2+2)^2}
$$

the shift is maximum at $y=\sqrt{2}$, $\delta x_c = \delta x_{c,max} = (2\sqrt{2})^{-1}$

This corresponds to $\sim 0.354\theta_E$ *which is above the accuracy of GAIA*