GRAVITATIONAL LENSING

7 - TIME DELAYS

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GRAVITATIONAL TIME DELAY

- In a lensing phenomenon, light travels with an effective velocity $c' < c$. As seen, this implies an effective refractive index $n > 1$
- The effective refractive index is expressed in terms of the Newtonian potential
- If we compare the travel times of two photons, one traveling at velocity c and the other at velocity c', we notice that the second accumulates a time delay t_{grav}
- This time delay is called *gravitational* time delay, or *Shapiro* time delay (Shapiro, 1964)

$$n = 1 - \frac{2\Phi}{c^2}$$

$$\begin{aligned} t_{grav} &= \int \frac{dz}{c'} - \int \frac{dz}{c} \\ &= \frac{1}{c} \int (n-1) dz \\ &= -\frac{2}{c^3} \int \Phi dz \end{aligned}$$

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$$\hat{\Psi}(\vec{\theta}) = \frac{D_{LS}}{D_L D_S} \frac{2}{c^2} \int \Phi(\vec{\theta}, z) dz$$

$$t_{grav} = -\frac{1}{c} \frac{D_S D_L}{D_{LS}} \hat{\Psi}$$

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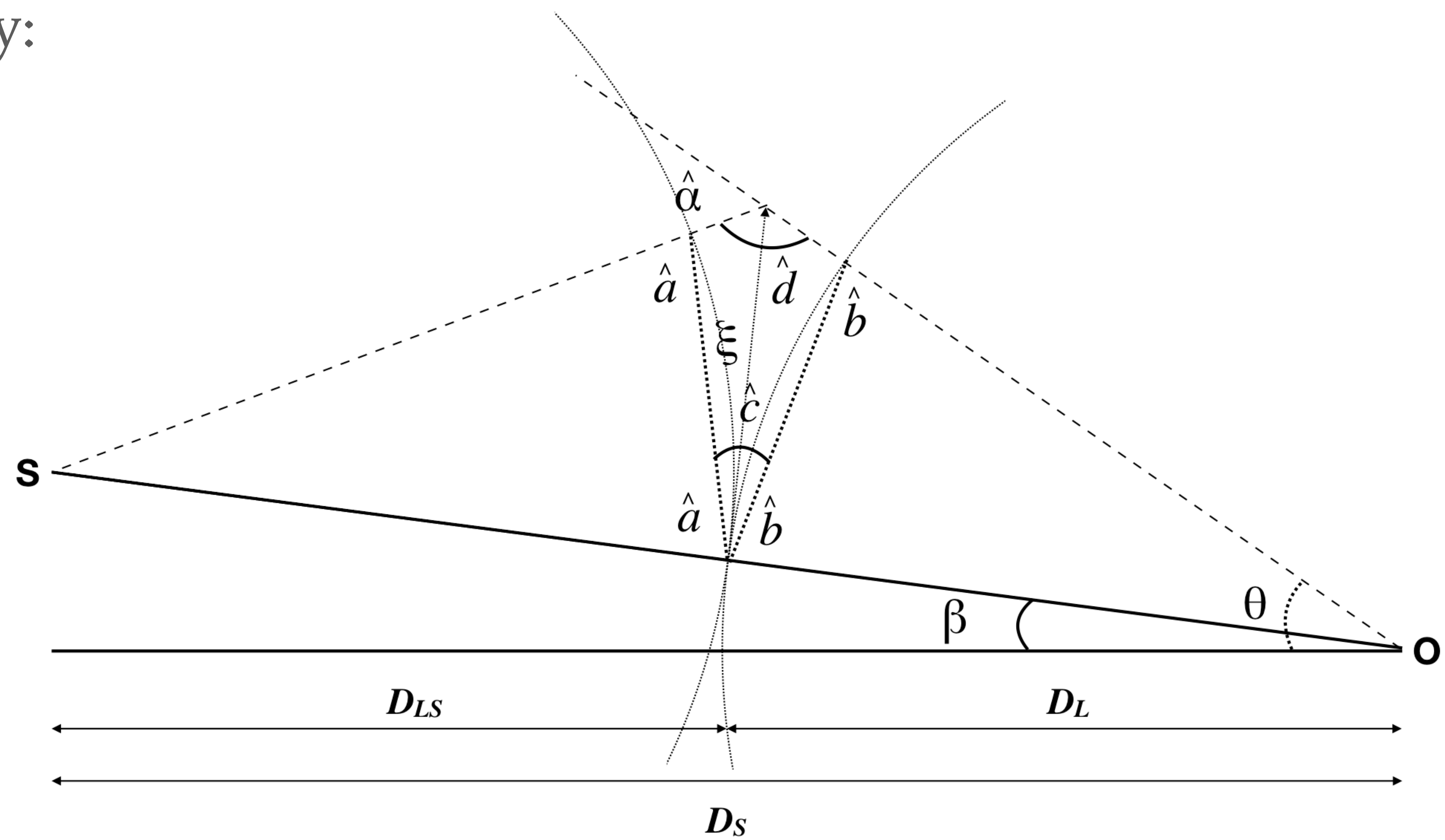
$$t_{grav} = -\frac{1}{c}\frac{D_S D_L}{D_{LS}}\hat{\Psi}$$

This time delay does not account yet for the different path of photons!

GEOMETRICAL TIME DELAY

- We need to combine the gravitational time delay to the so called *geometrical* time delay:

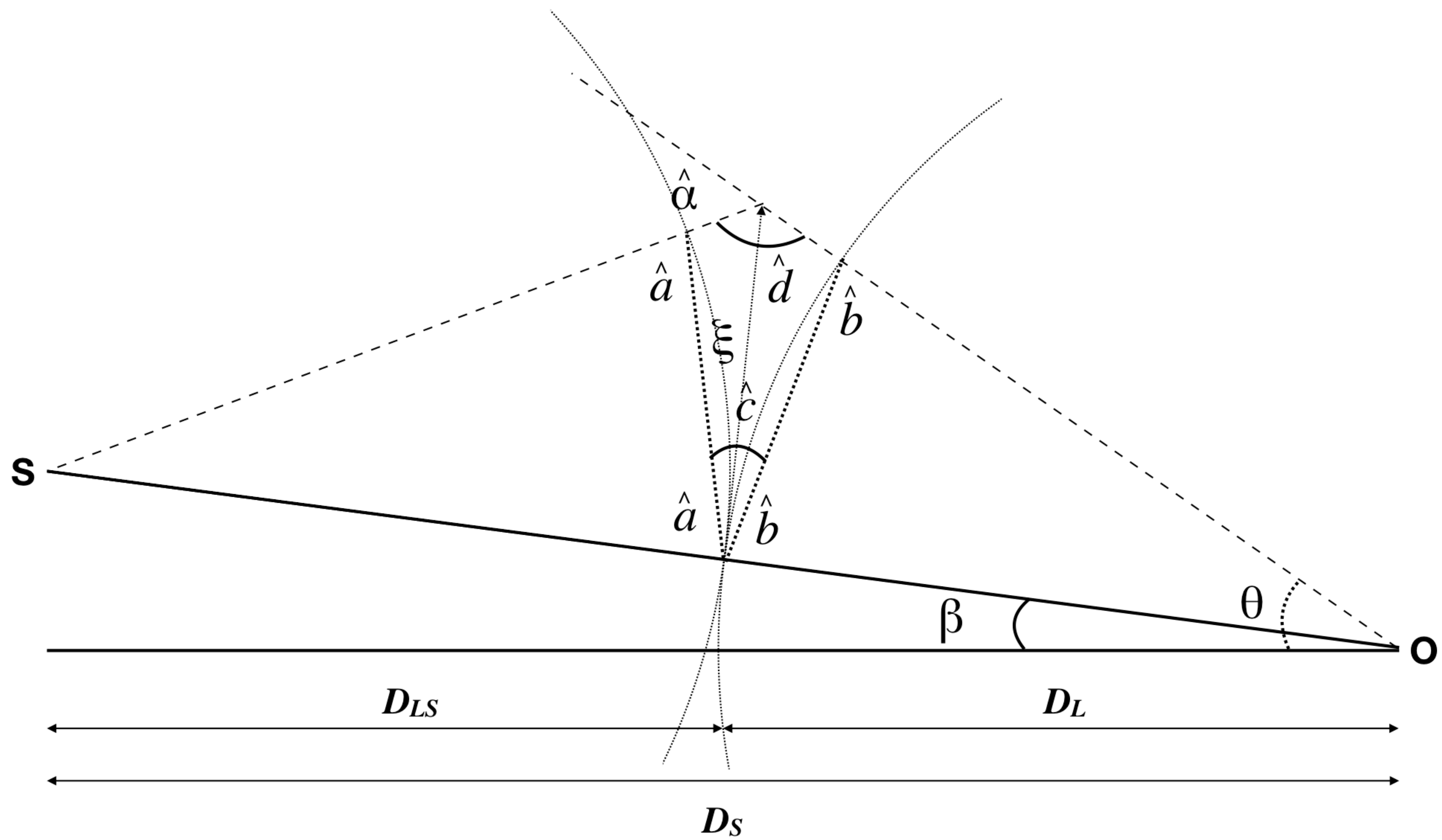


$$(\pi - \hat{a}) + (\pi - \hat{b}) + \hat{c} + \hat{d} = 2\pi \Rightarrow \hat{a} + \hat{b} = \hat{c} + \hat{d}$$

$$\hat{a} + \hat{b} + \hat{c} = \pi \Rightarrow 2\hat{c} = \pi - \hat{d}$$

$$\hat{\alpha} + \hat{d} = \pi \Rightarrow \hat{d} = \pi - \hat{\alpha}$$

GEOMETRICAL TIME DELAY



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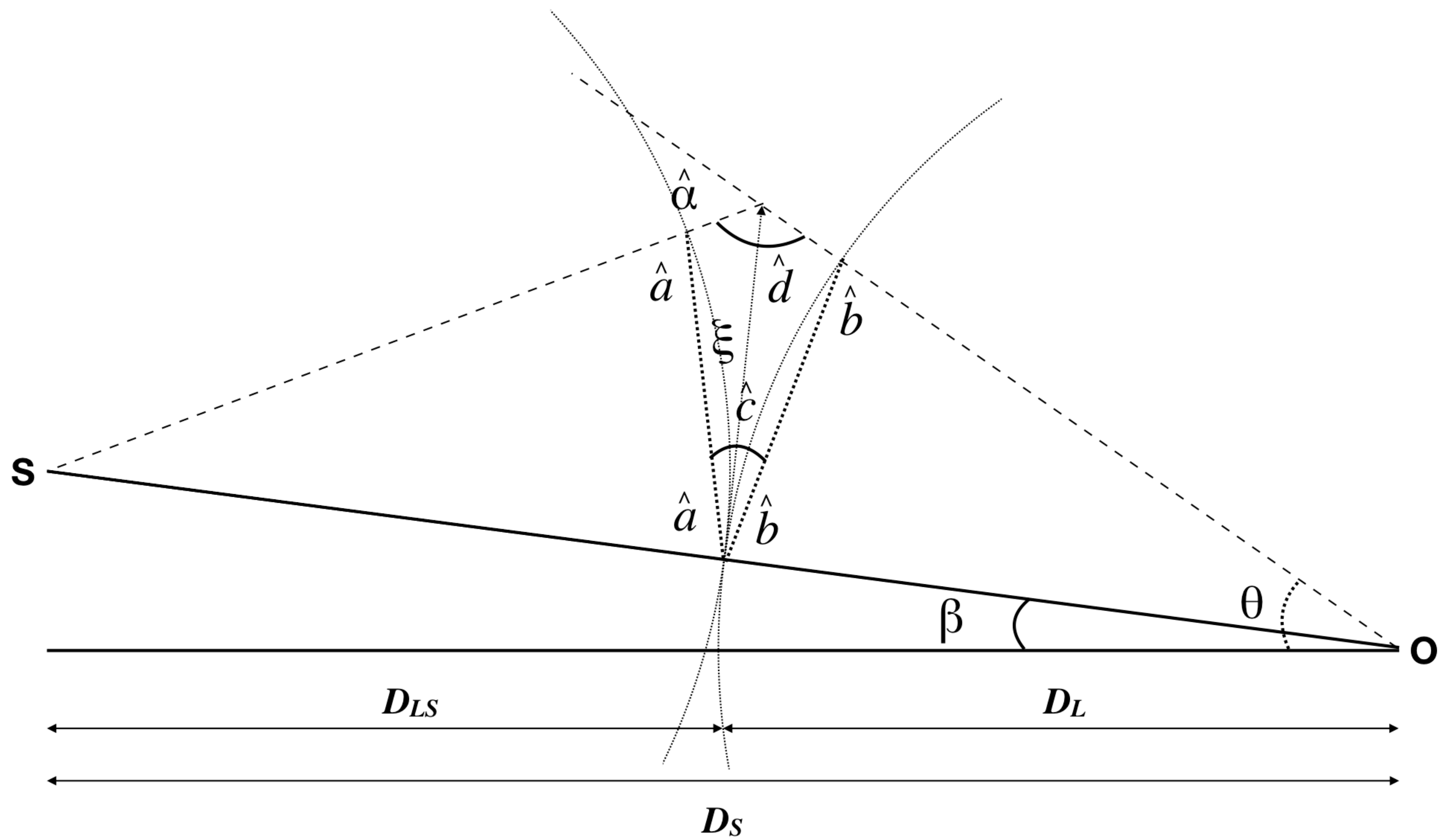
$$\begin{aligned} \Rightarrow \hat{c} &= \frac{\hat{\alpha}}{2} \\ &= \frac{1}{2}\frac{D_S}{D_{LS}}\alpha \\ &= \frac{1}{2}\frac{D_S}{D_{LS}}(\theta - \beta) \end{aligned}$$

GEOMETRICAL TIME DELAY



$$\Rightarrow \hat{c} = \frac{\hat{\alpha}}{2} = \frac{1}{2}\frac{D_S}{D_{LS}}\alpha = \frac{1}{2}\frac{D_S}{D_{LS}}(\theta - \beta)$$

$$\xi = D_L(\theta - \beta)$$

$$t_{geom} = \frac{1}{c}\xi\hat{c} = \frac{1}{2c}\frac{D_S D_L}{D_{LS}}(\theta - \beta)^2$$

TOTAL TIME DELAY

$$t_{geom} = \frac{1}{2c}\frac{D_S D_L}{D_{LS}}(\theta - \beta)^2$$

$$t_{grav} = -\frac{1}{c}\frac{D_S D_L}{D_{LS}}\hat{\Psi}$$

$$\begin{aligned} t_{tot} &= t_{geom} + t_{grav} \\ &= \frac{1}{2c}\frac{D_S D_L}{D_{LS}}(\theta - \beta)^2 - \frac{1}{c}\frac{D_S D_L}{D_{LS}}\hat{\Psi}(\theta) \\ &= \frac{1}{c}\frac{D_S D_L}{D_{LS}}\left[\frac{1}{2}(\theta - \beta)^2 - \hat{\Psi}(\theta)\right] \end{aligned}$$

TOTAL TIME DELAY

$$
\begin{aligned}
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&= \frac{1}{c}\frac{D_S D_L}{D_{LS}}\left[\frac{1}{2}(\theta-\beta)^2 - \hat{\Psi}(\theta)\right]
\end{aligned}
$$

Accounting for the expansion of the universe and for the fact that this is a surface:

$$
t_{tot}(\vec{\theta}) = \frac{1+z_L}{c}\frac{D_S D_L}{D_{LS}}\left[\frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \hat{\Psi}(\vec{\theta})\right]
$$

TOTAL TIME DELAY

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$$\tau(\vec{\theta}) = \frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \hat{\Psi}(\vec{\theta})$$

Fermat potential

$$D_{\Delta t} = (1+z_L)\frac{D_S D_L}{D_{LS}}$$

Time delay distance

TIME DELAY SURFACE

$$t(\vec{\theta}) = t_{geom} + t_{grav} \propto \left(\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \hat{\Psi} \right)$$

$$\vec{\nabla} t(\vec{\theta}) \propto \left(\vec{\theta} - \vec{\beta} - \vec{\nabla} \hat{\Psi} \right)$$

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Lens equation!

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Images form at the stationary points of t!

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Lens equation!

Images form at the stationary points of t!

$$T_{ij} = \frac{\partial^2 t(\vec{\theta})}{\partial\theta_i \partial\theta_j} \propto (\delta_{ij} - \Psi_{ij})$$

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Lens equation!

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$$T_{ij} = \frac{\partial^2 t(\vec{\theta})}{\partial\theta_i \partial\theta_j} \propto (\delta_{ij} - \Psi_{ij})$$ ***This is the Jacobian!***

TYPES OF IMAGES

- minima (eigenvalues of A are both positive, hence $\det A>0$ and $\mathrm{Tr}\,A>0$; positive magnification)
- saddle (eigenvalues have opposite signs, thus $\det A<0$; negative magnification)
- maxima (eigenvalues are both negative, hence $\det A>0$ and $\mathrm{Tr}\,A<0$; positive magnification)
- Let see some examples…

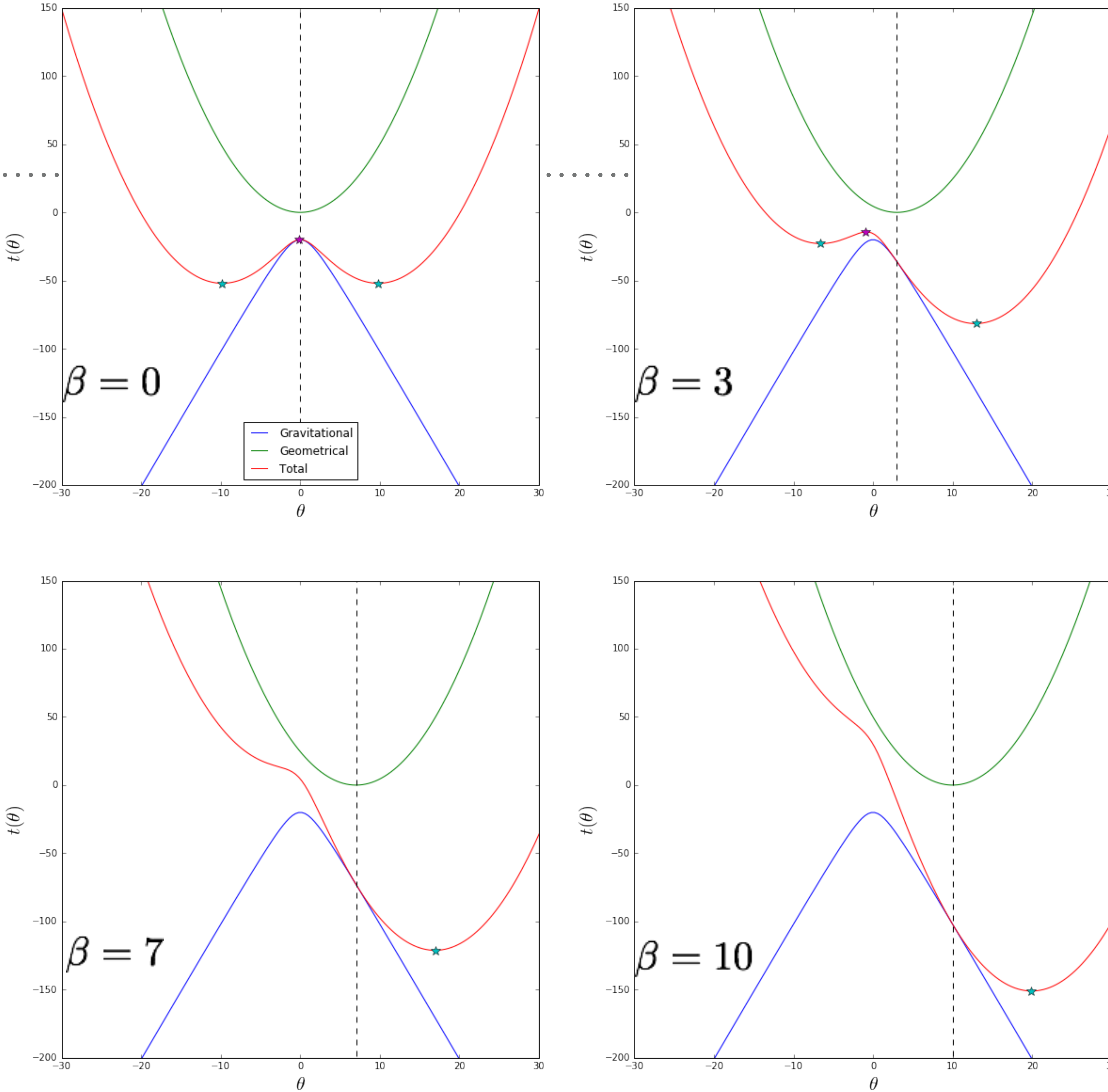
EXAMPLE OF TIME DELAY SURFACE

Toy potential:

$$\psi(\theta) \propto \frac{1}{\sqrt{\theta^2+\theta_c^2}}$$

Assuming axial-symmetry, we can discuss the time-delay function instead of the time delay surface.

$$t(\theta) \propto \left[\frac{1}{2}(\theta-\beta)^2 - \psi(\theta)\right]$$



Gravitational
Geometrical
Total
$\beta = 0$
$\beta = 3$
$\beta = 7$
$\beta = 10$
$t(\theta)$
θ

SOME INTERESTING PROPERTIES

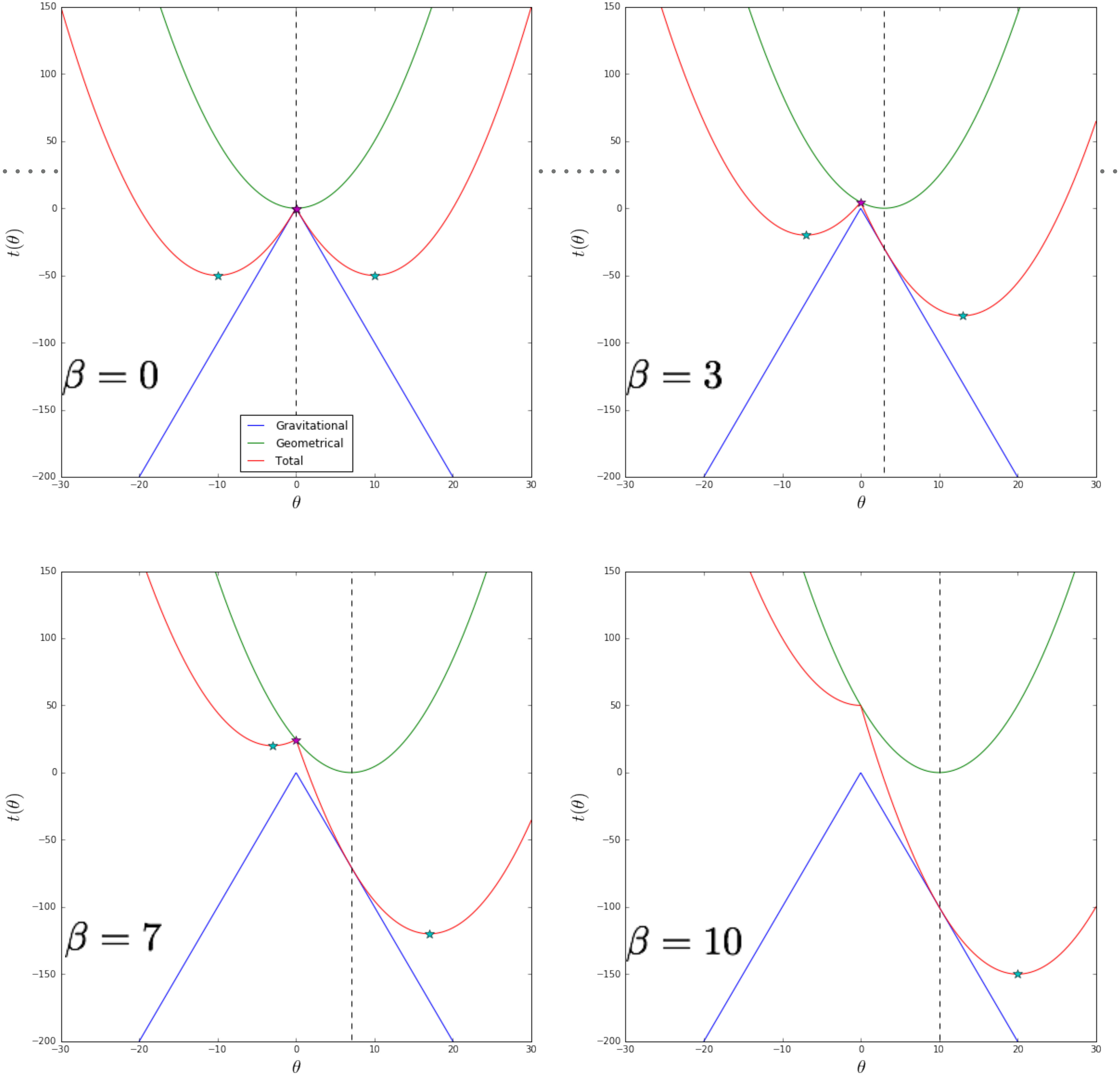
- image multiplicity depends on the relative position of lens and source
- couples of images disappear after approaching each other
- the time-delay function is flat when this happens!
- det A=0 means infinite magnification: the images disappear on the critical lines!
- this happens every time a source crosses a caustic!

EXAMPLE OF TIME DELAY SURFACE

Let's change potential:

$$\psi(\theta) \propto \frac{1}{|\theta|}$$

The lens model is the same as before, but the core has been removed



$\beta = 0$
$\beta = 3$
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Gravitational
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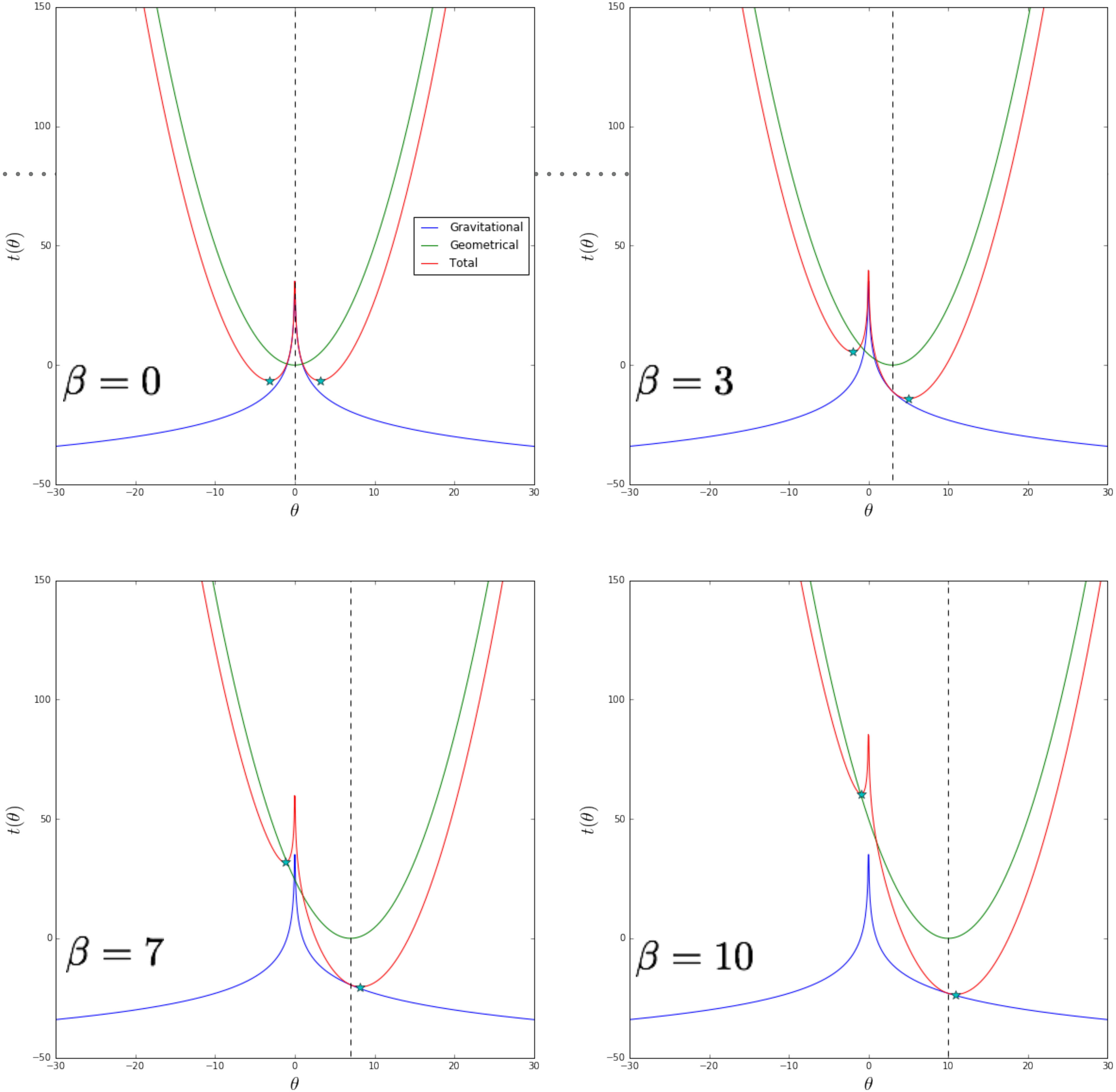
θ

EXAMPLE OF TIME DELAY SURFACE

Yet another potential

$$\psi(\theta) \propto \ln(|\theta|)$$

This is the lensing potential of the point mass…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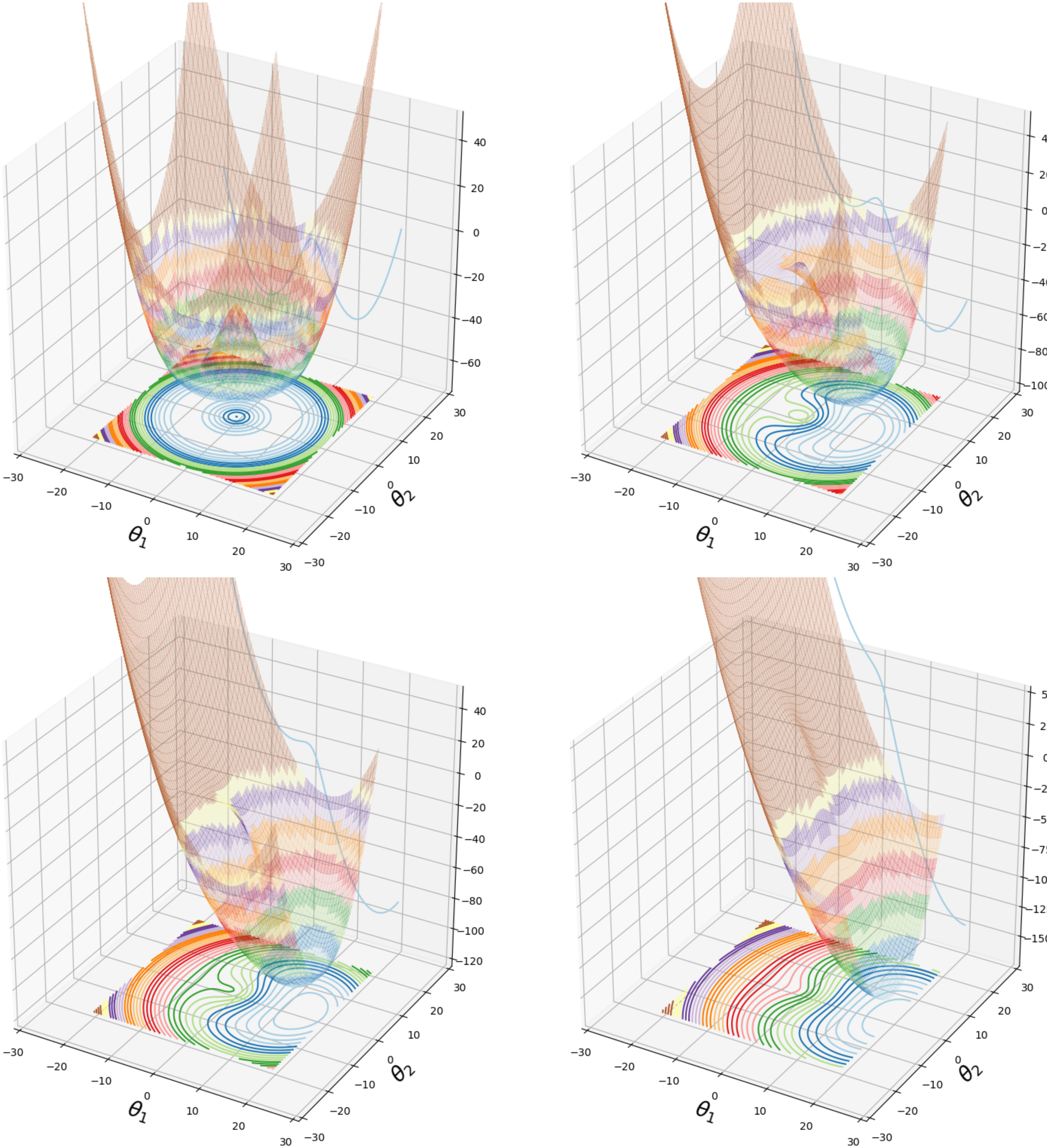
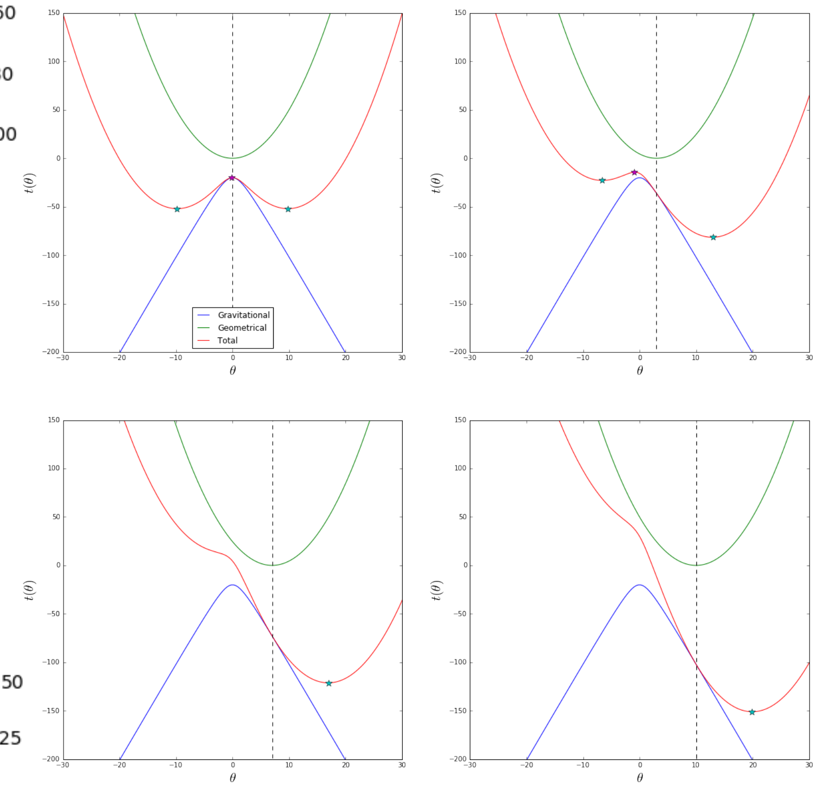
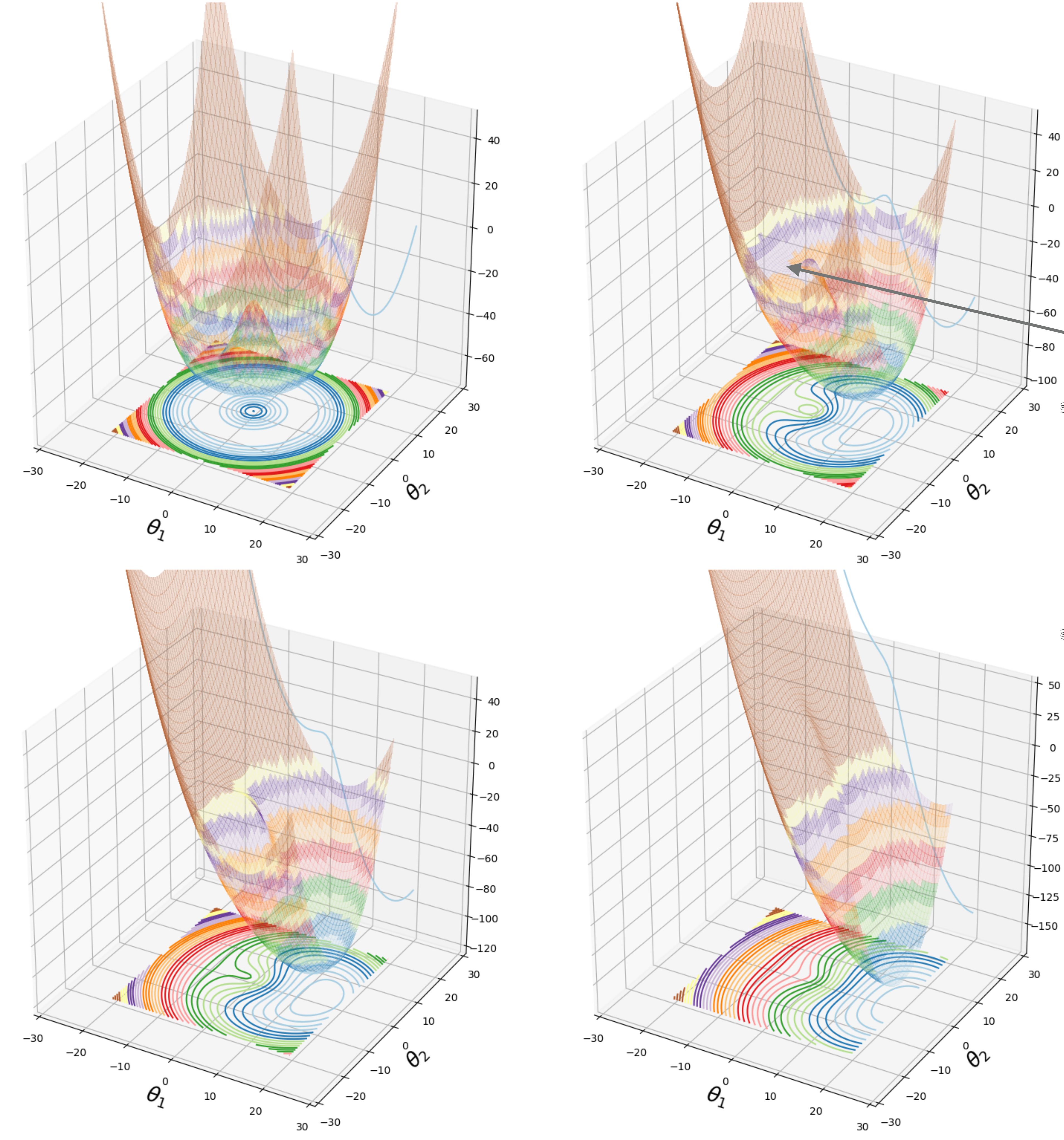
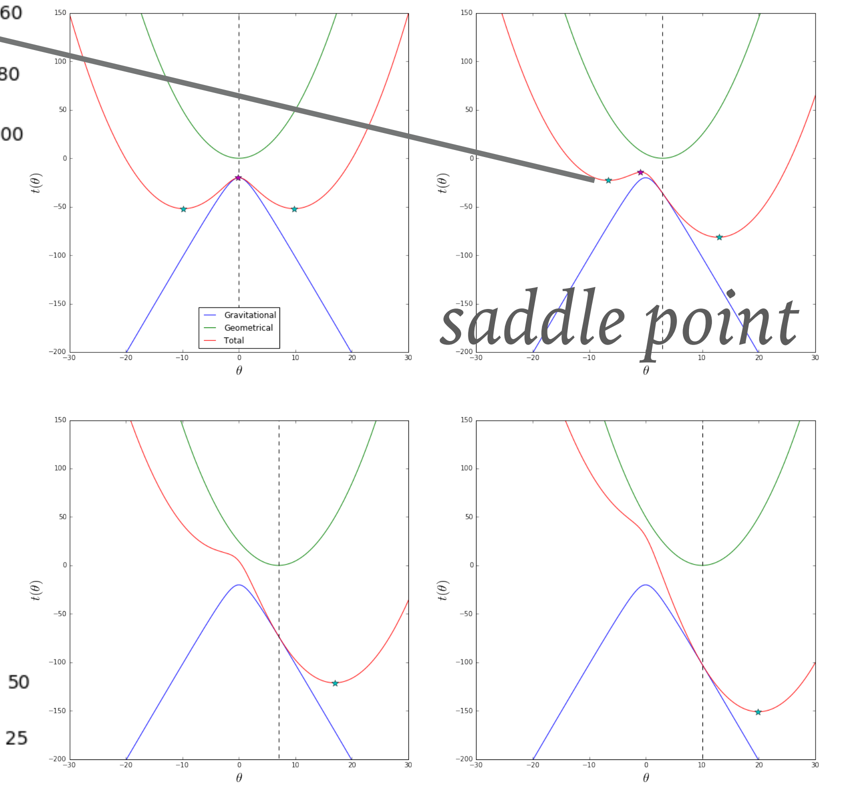


…*but t(θ) is a surface!*



…but $t(\theta)$ is a surface!





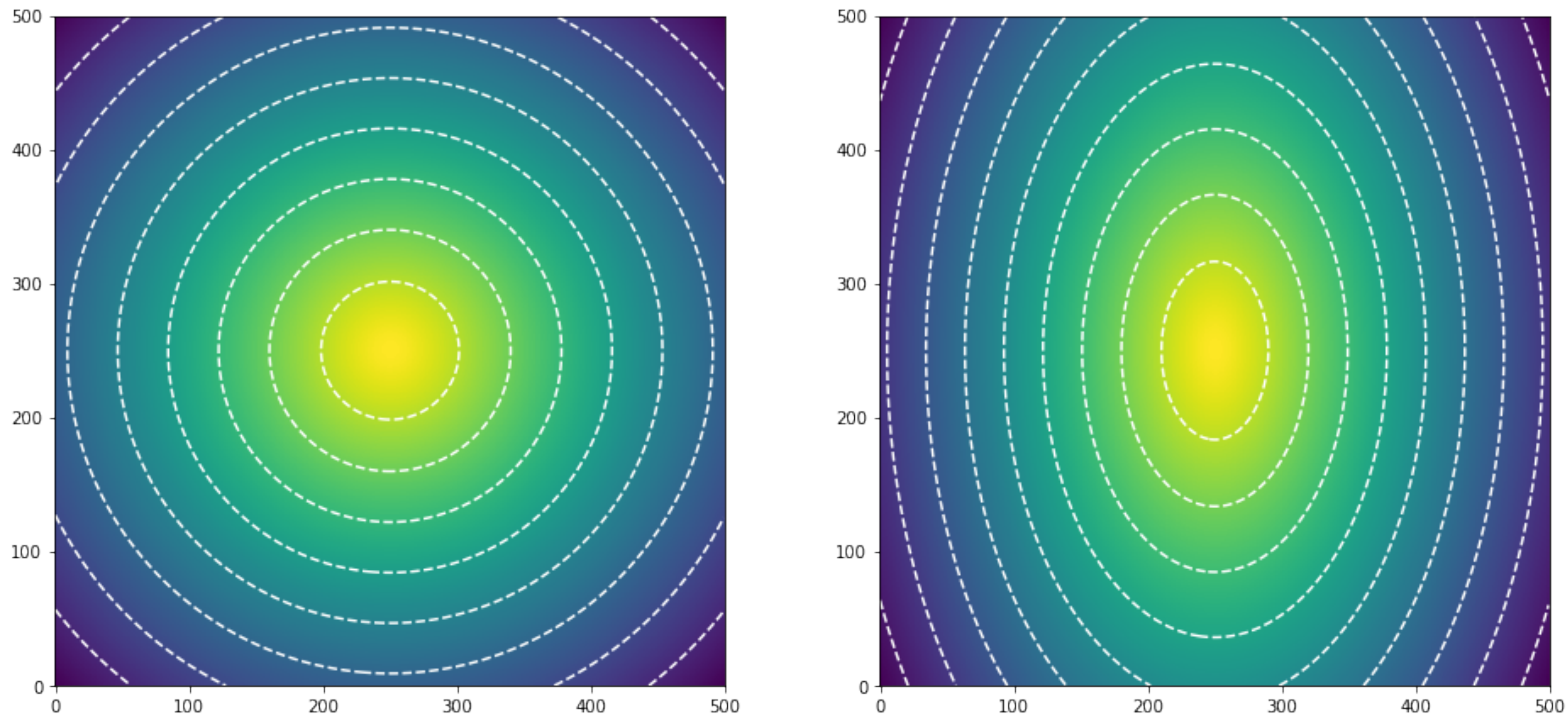
saddle point

INTRODUCING ELLIPTICITY

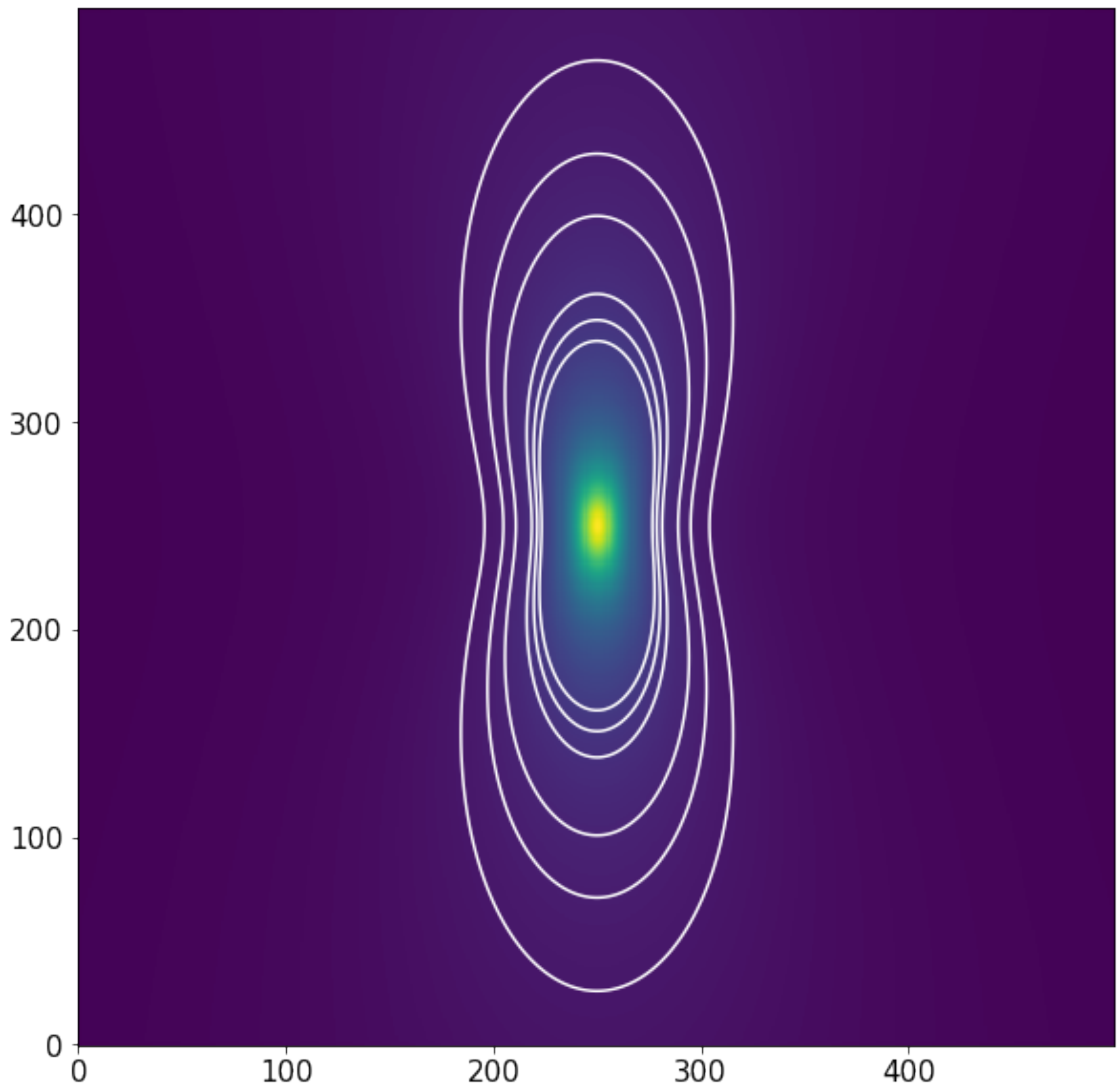
One easy way to make a lens elliptical is by modifying the potential as follows:

$$|\theta| \rightarrow \sqrt{\frac{\theta_1^2}{1-\epsilon} + \theta_2^2(1-\epsilon)}$$

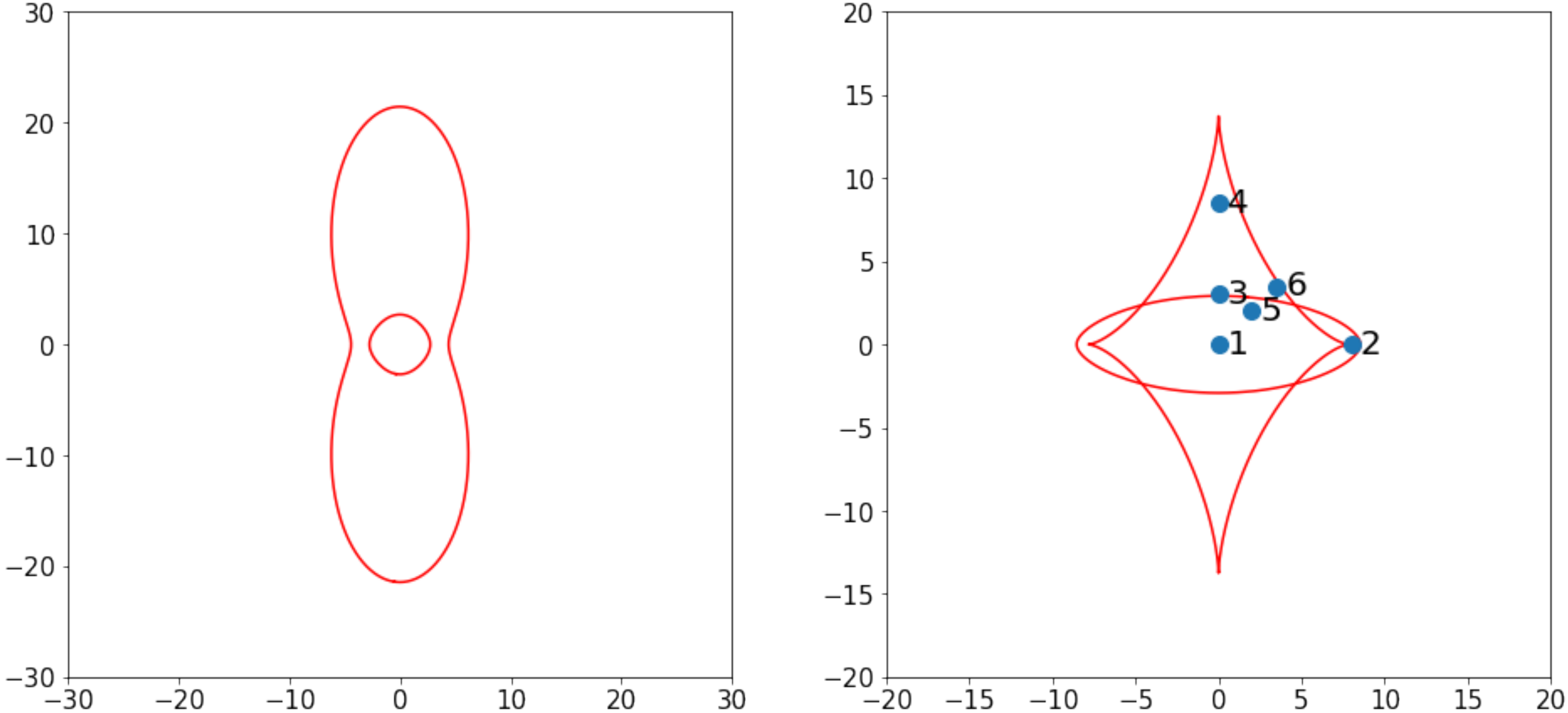
This makes the potential constant over ellipses rather than on circles.



CAUTION: PSEUDO ELLIPTICAL LENSES



CRITICAL LINES AND CAUSTICS



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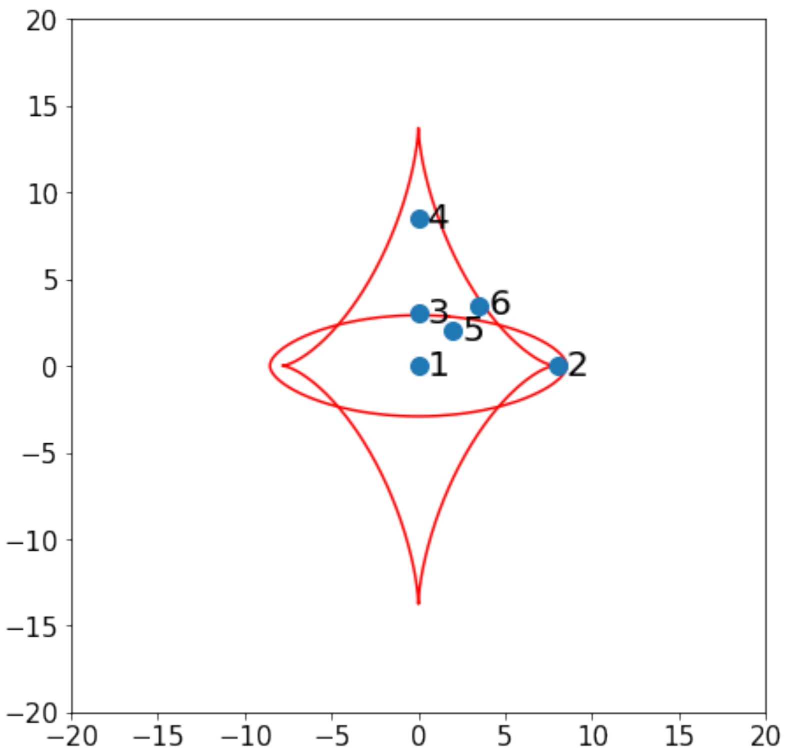

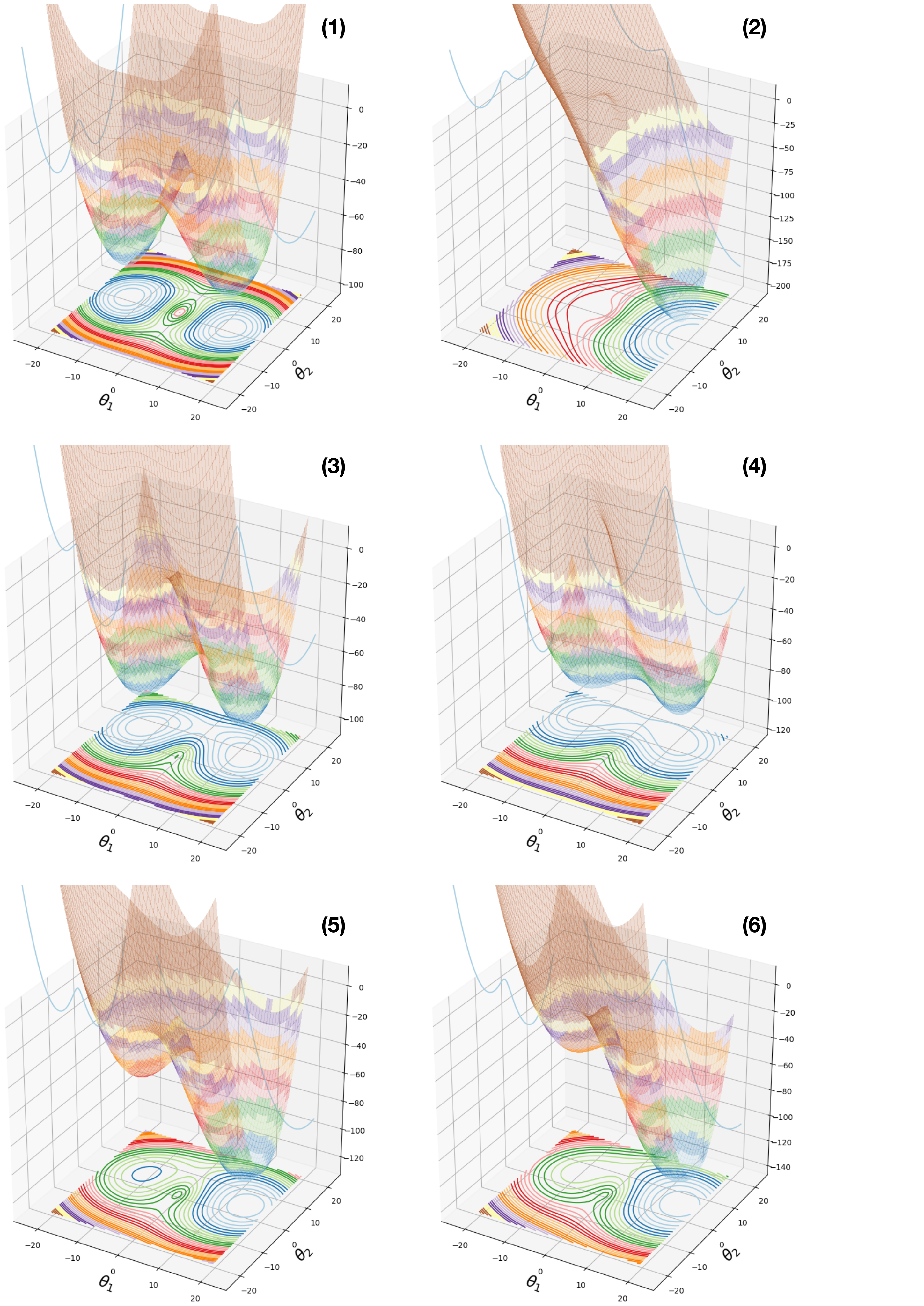

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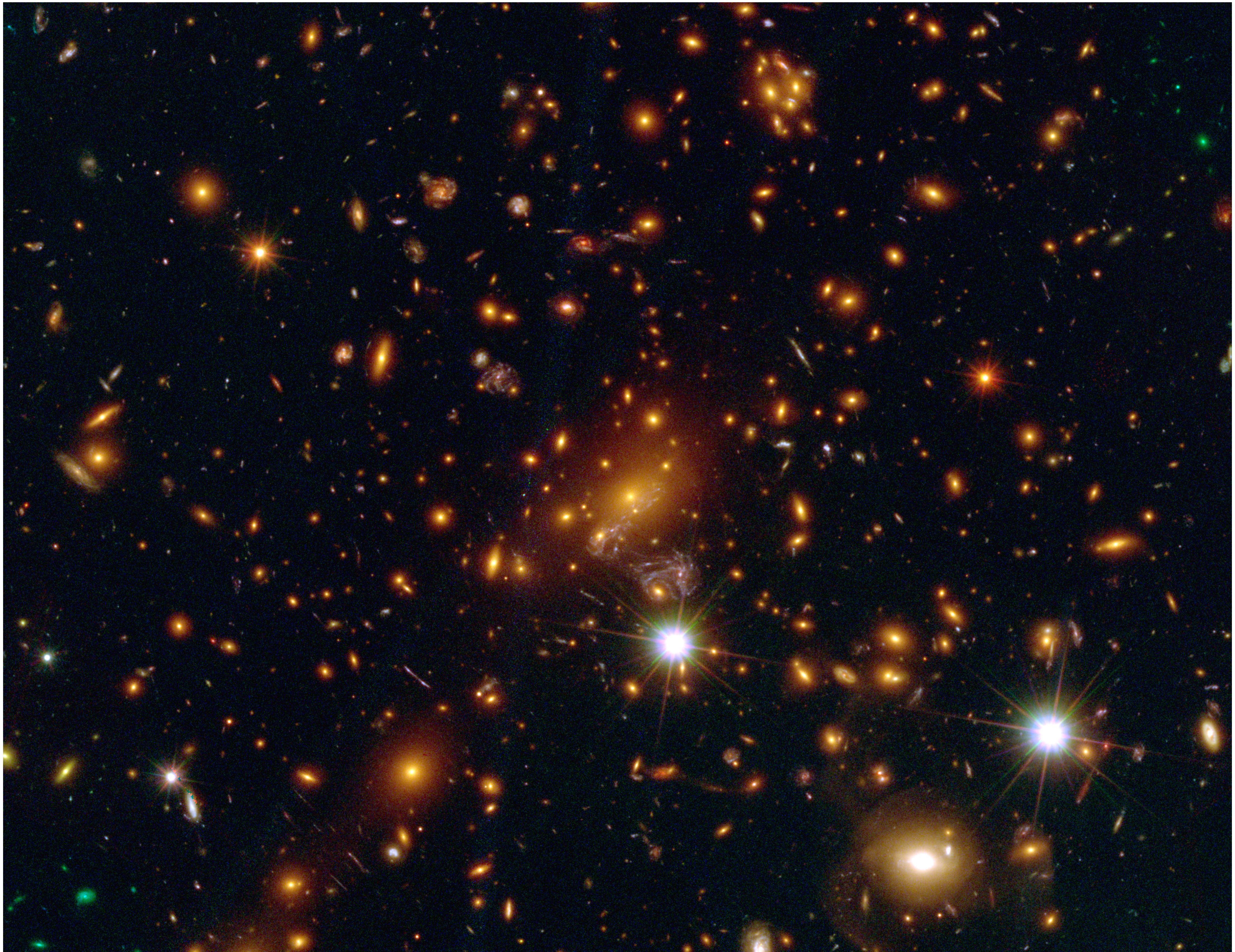

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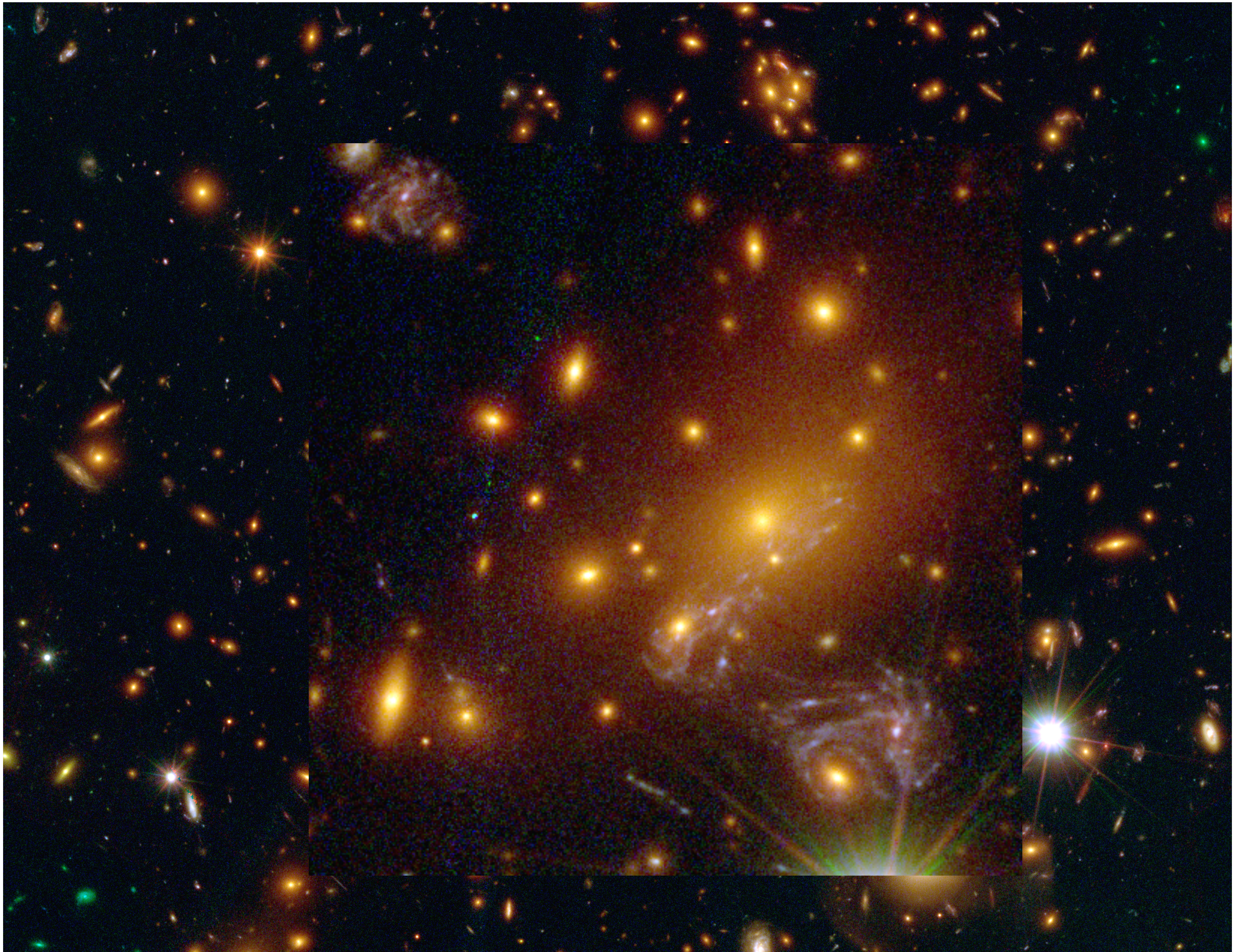

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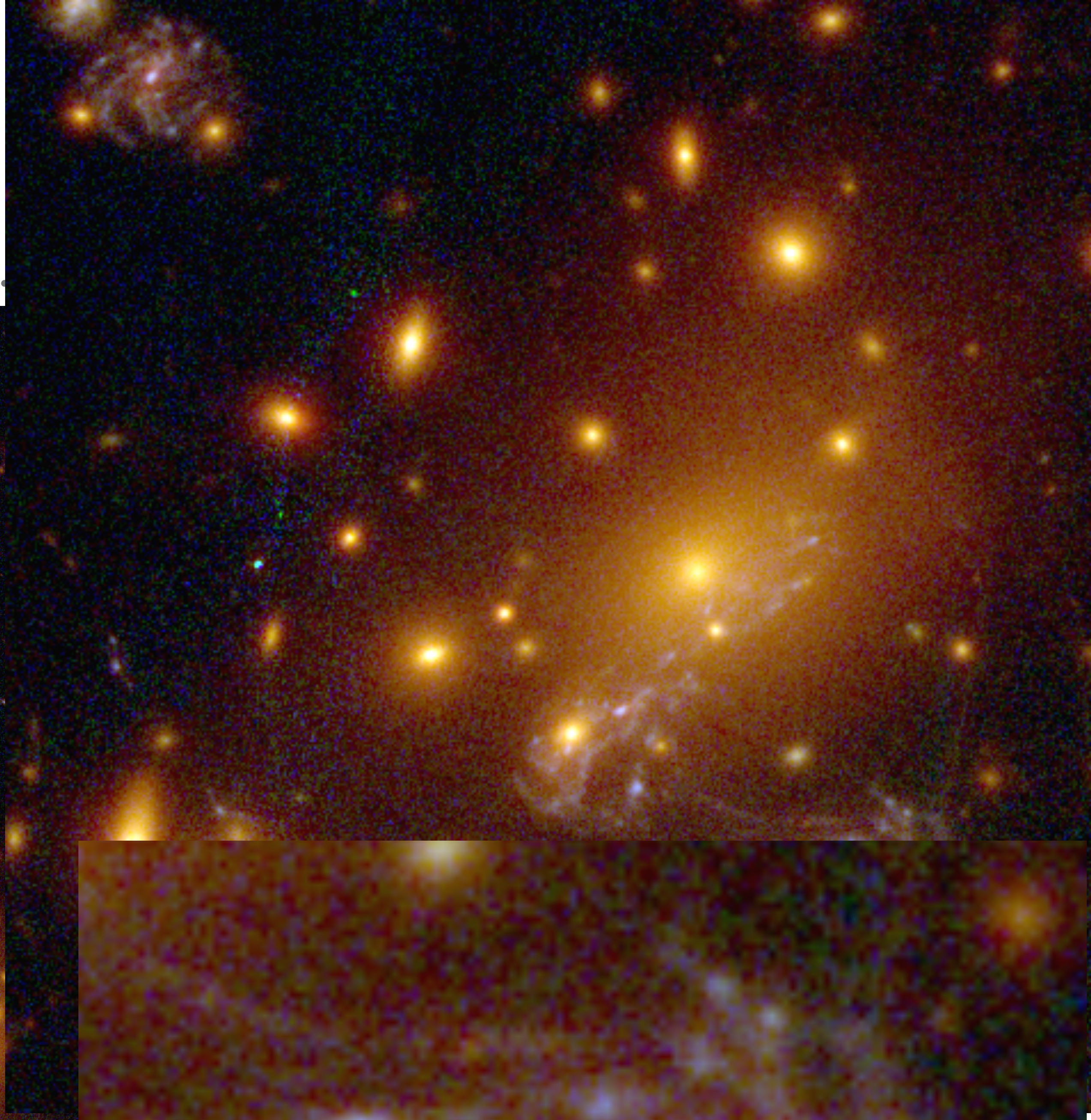

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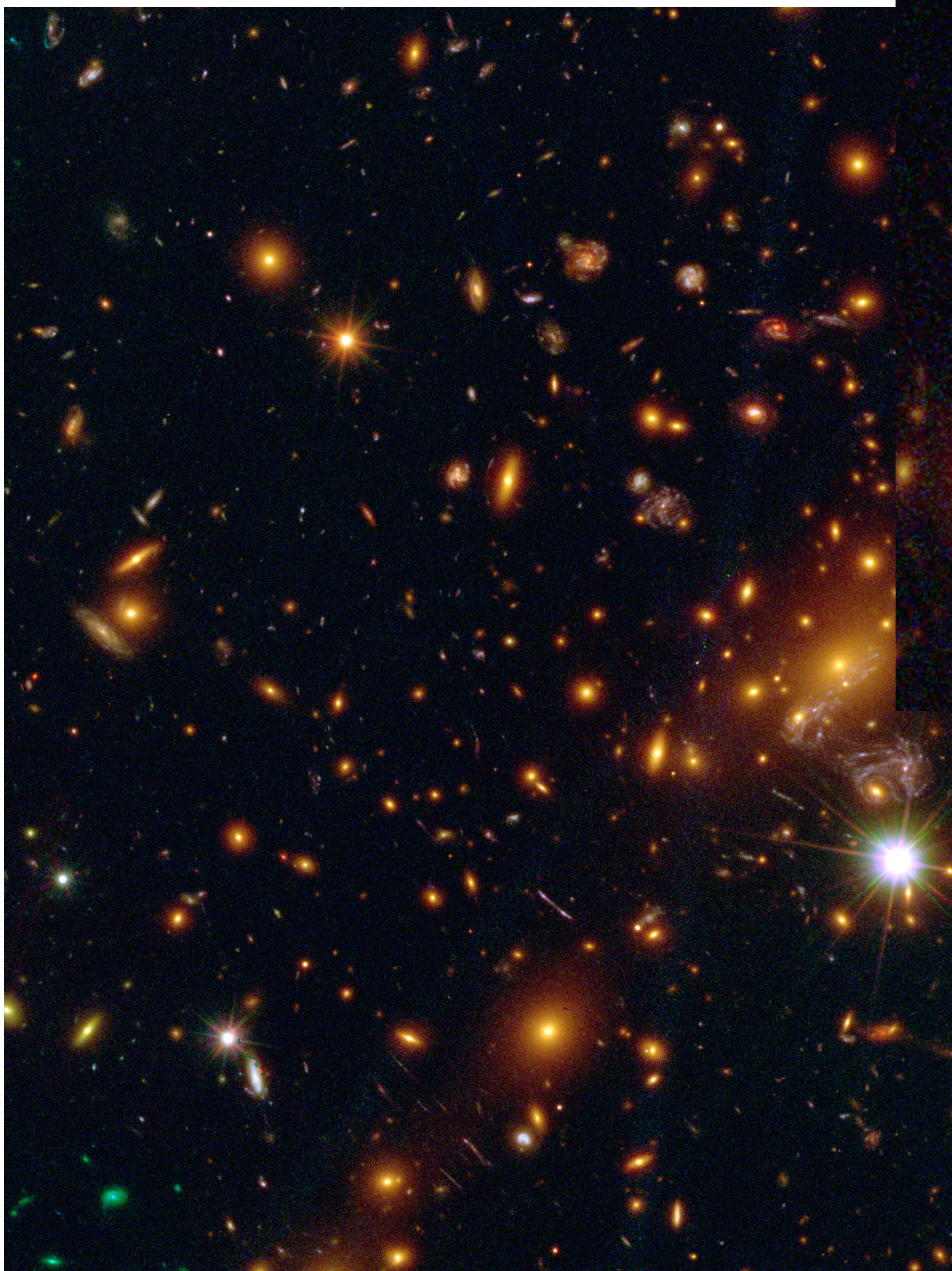

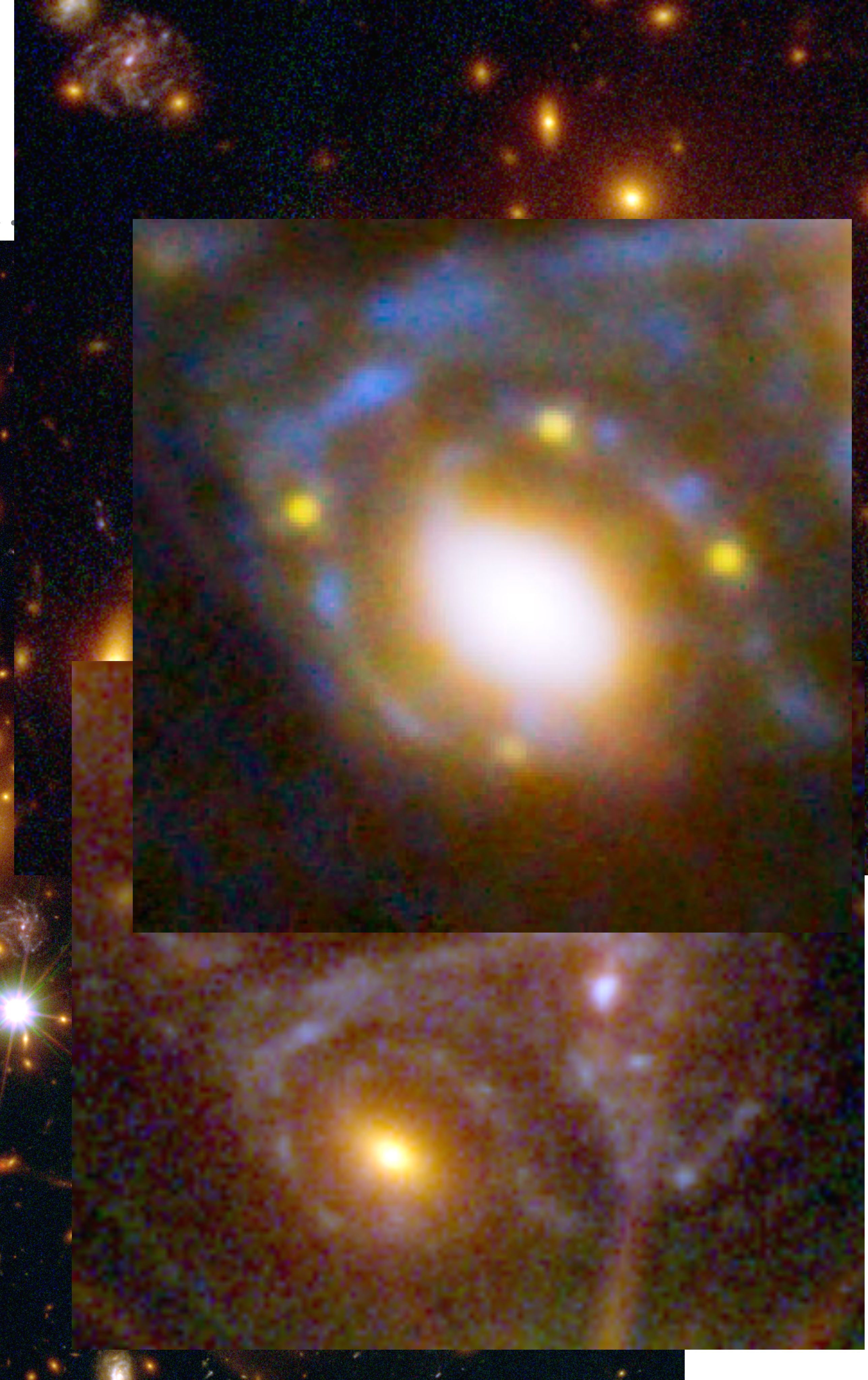

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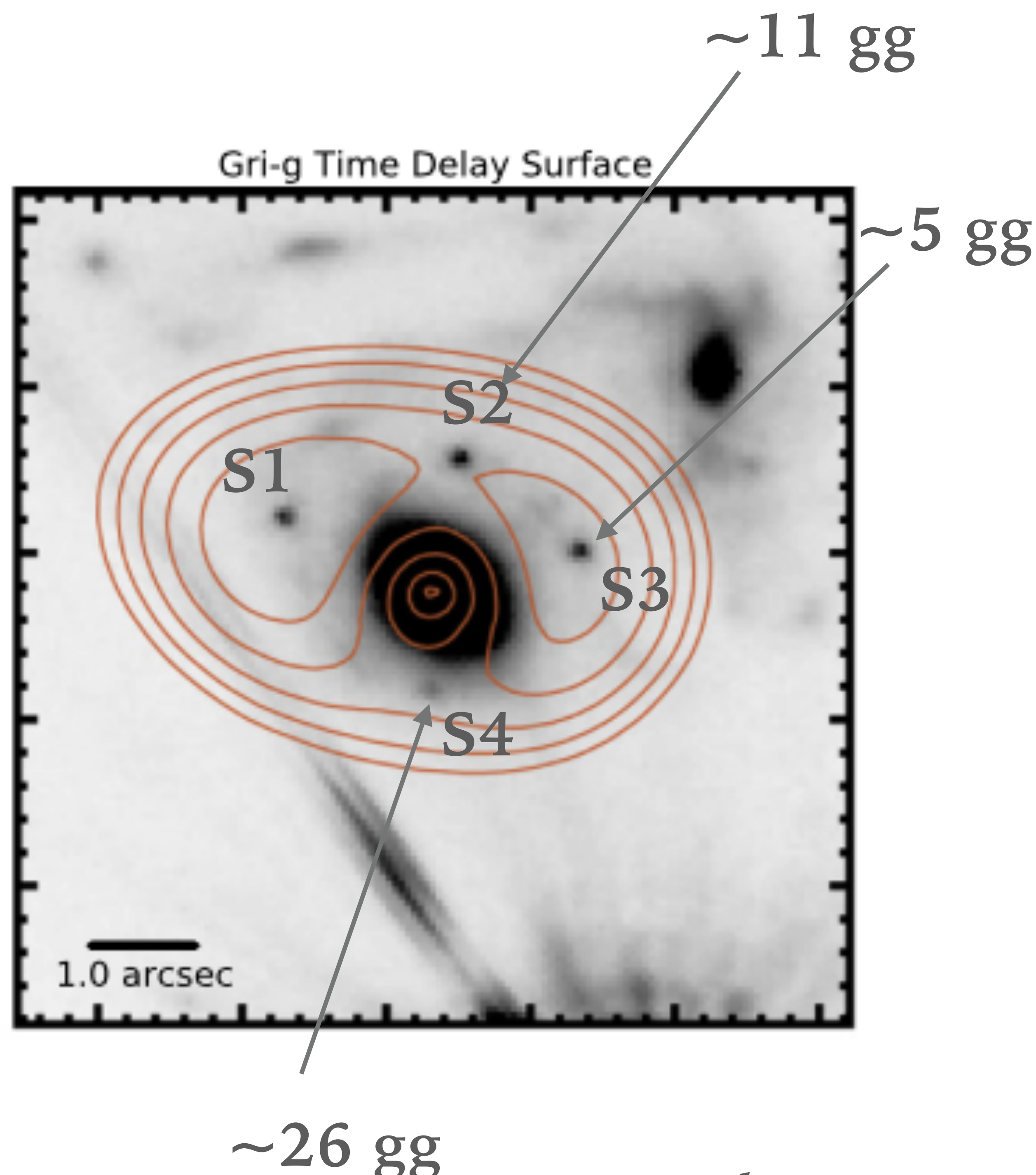
Nov. 2014 (Kelly et al.)



Treu et al. 2016

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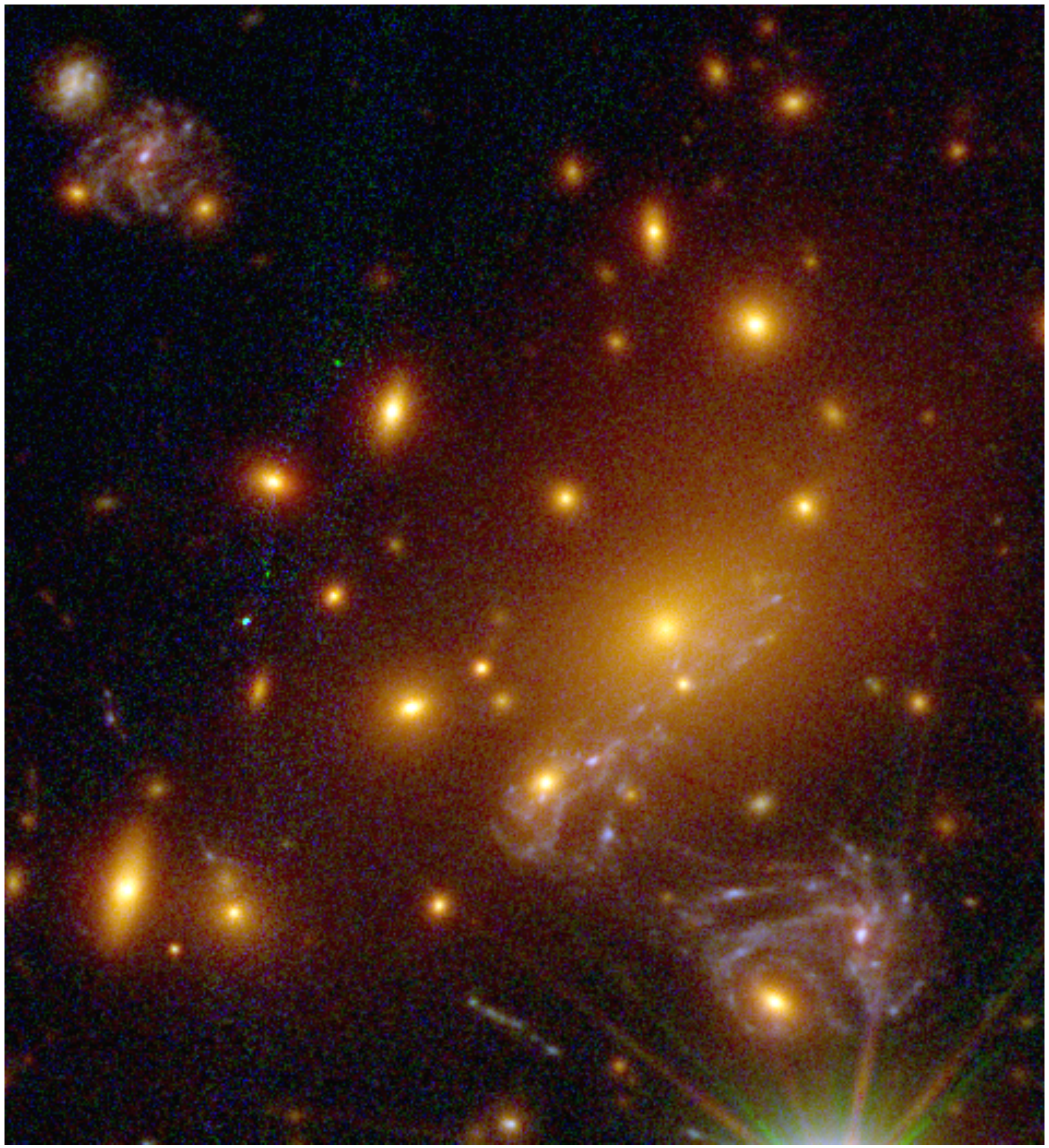

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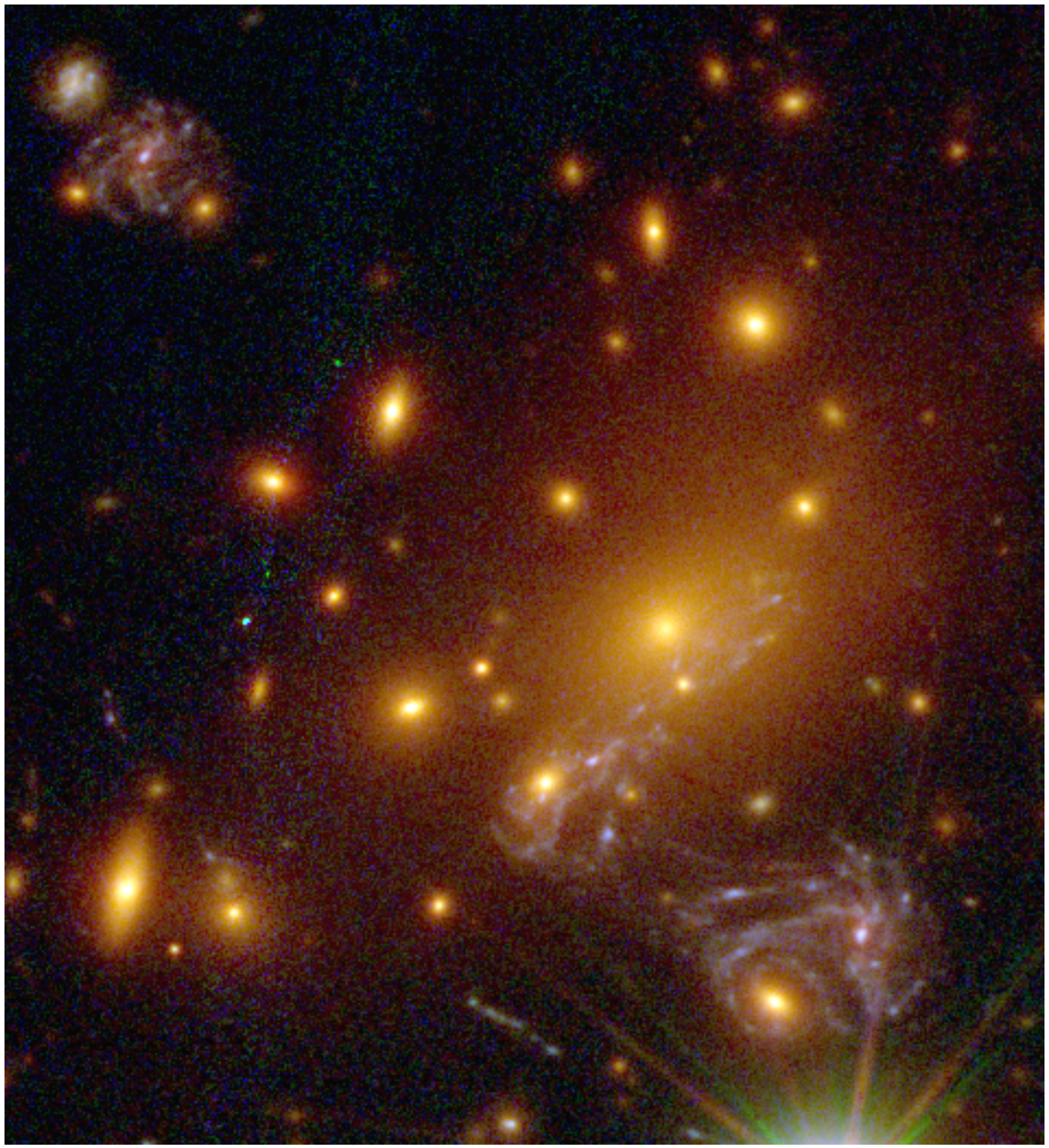

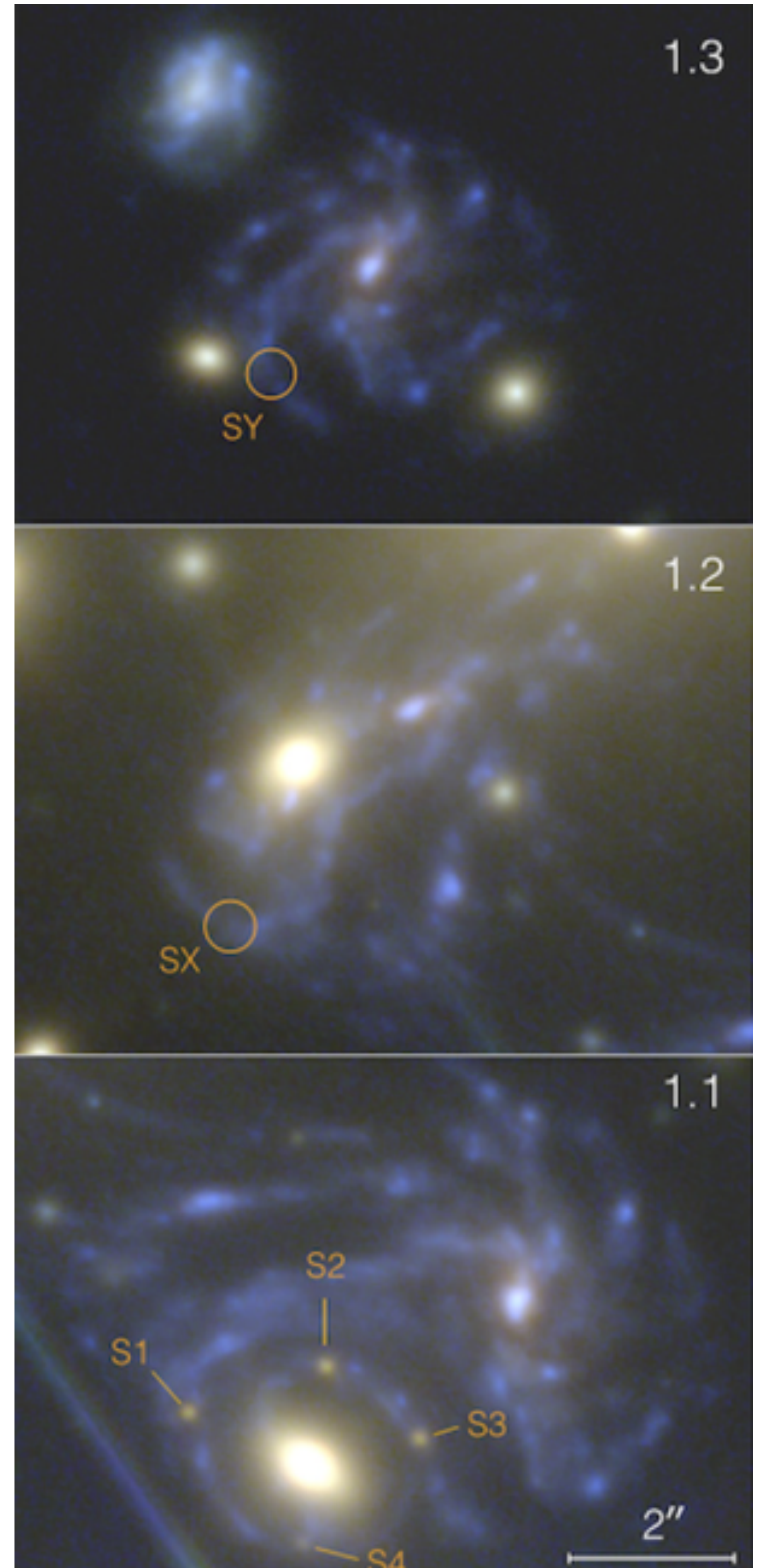

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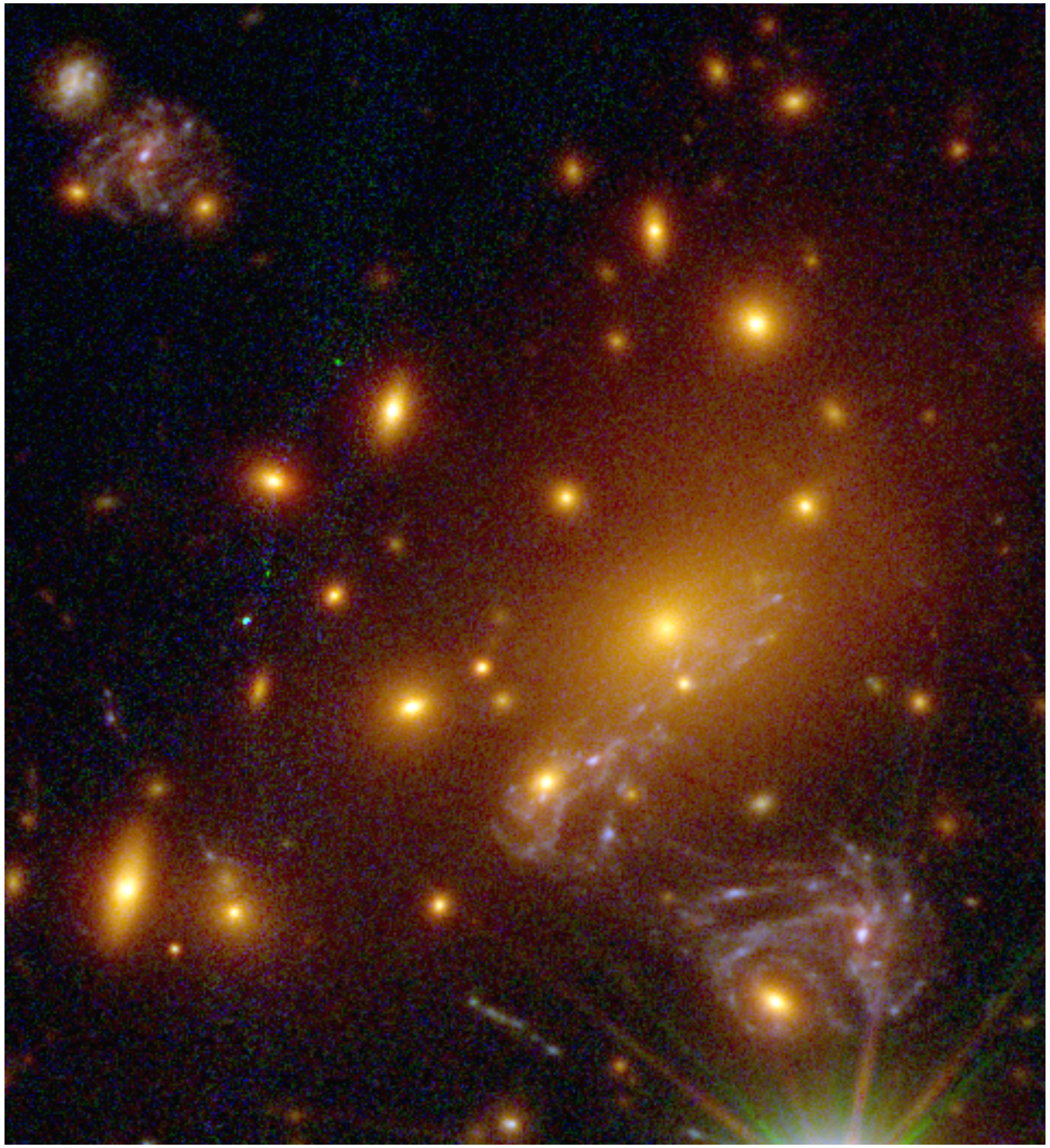

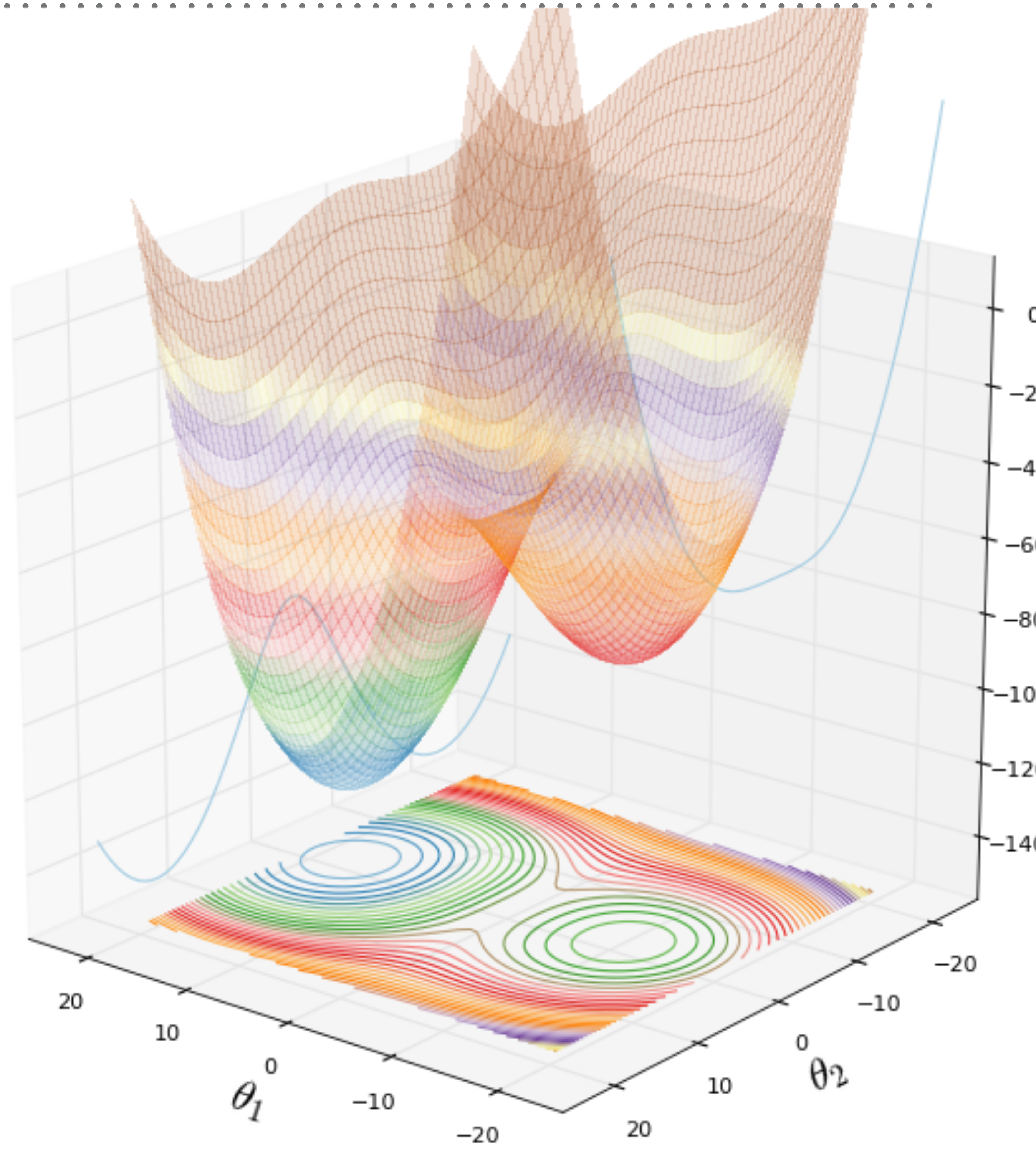

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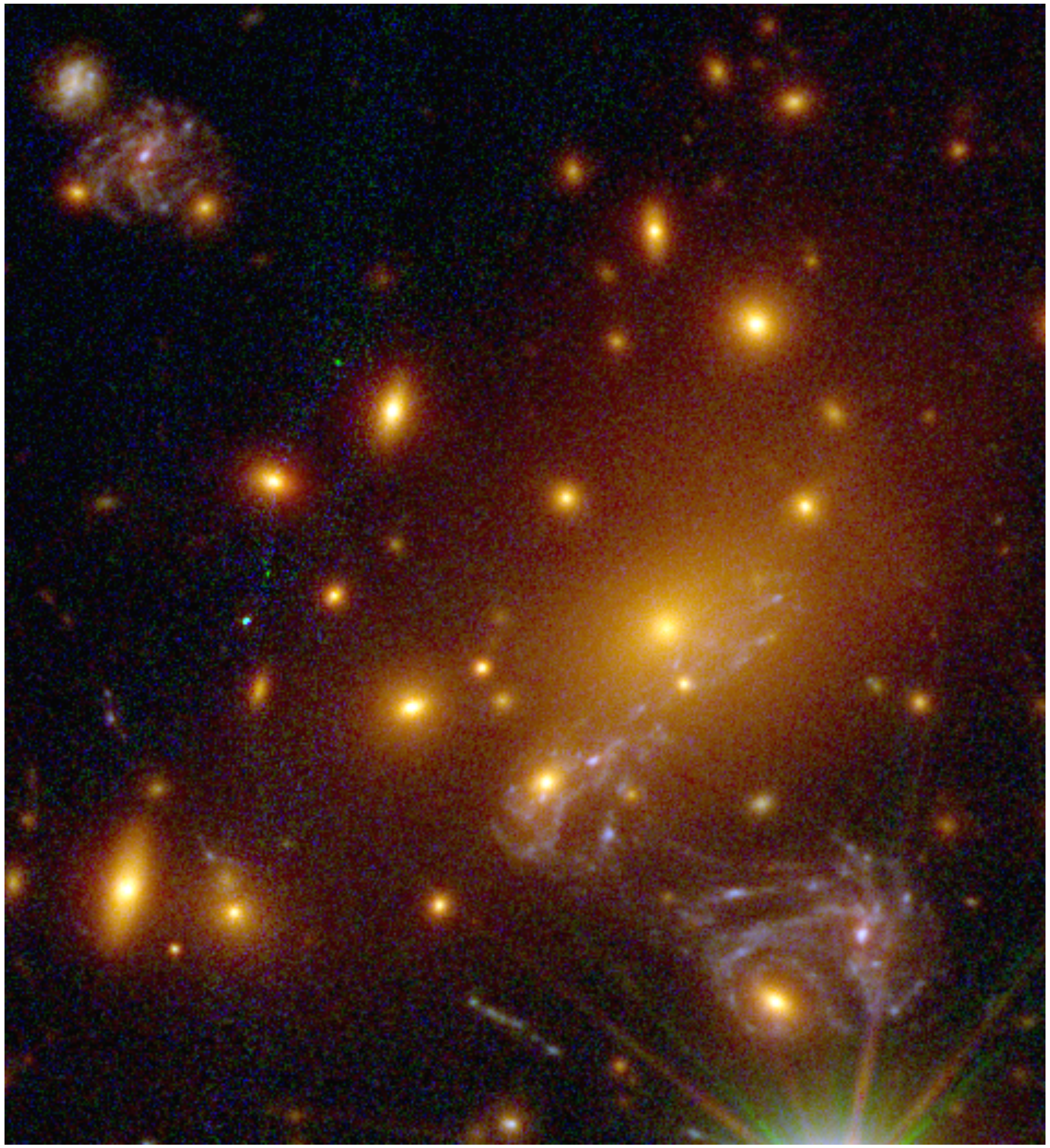

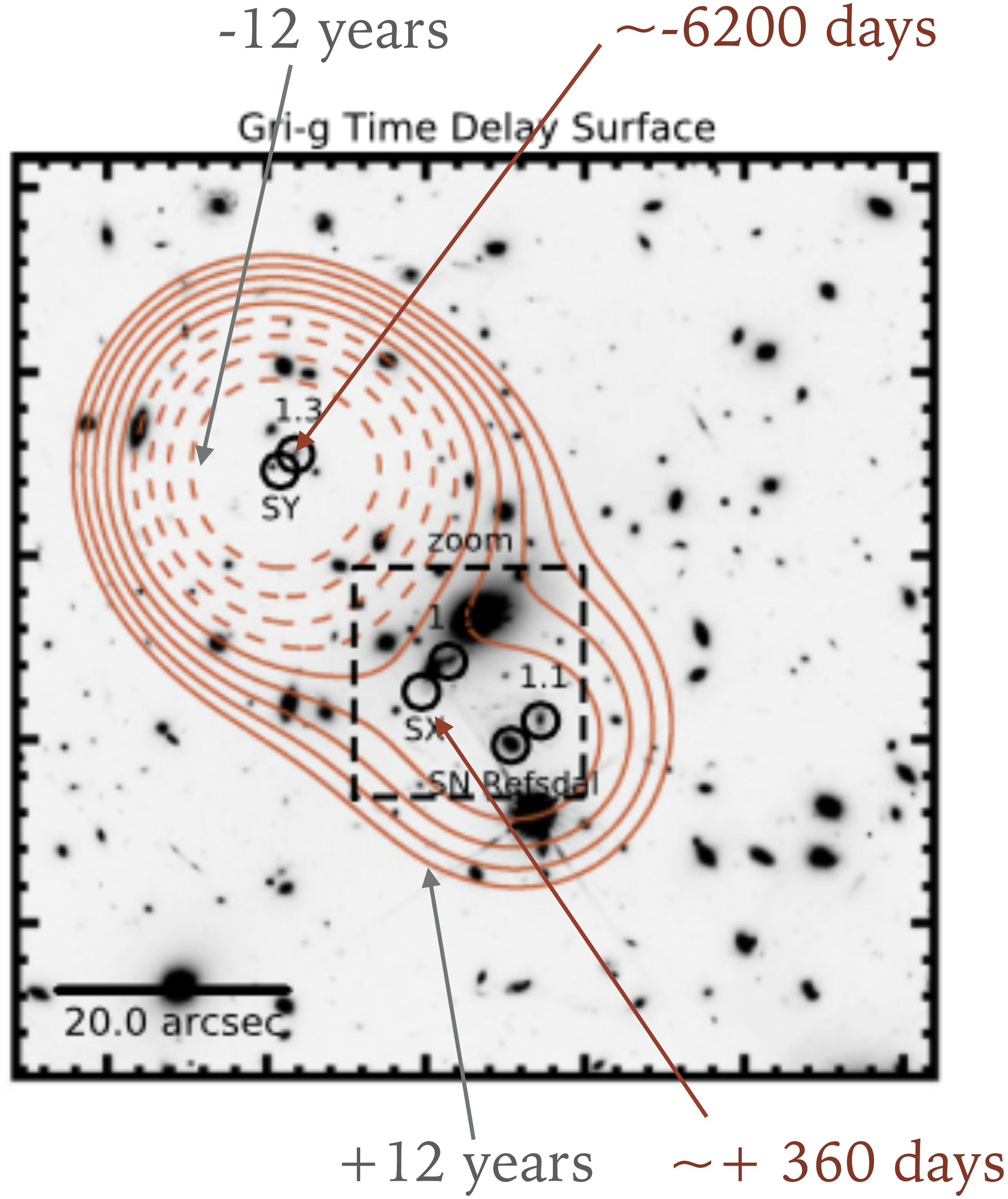

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16/12/2016…

Time delay

(SX- S1)
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