GRAVITATIONAL LENSING

5 - 2ND ORDER LENSING EFFECTS

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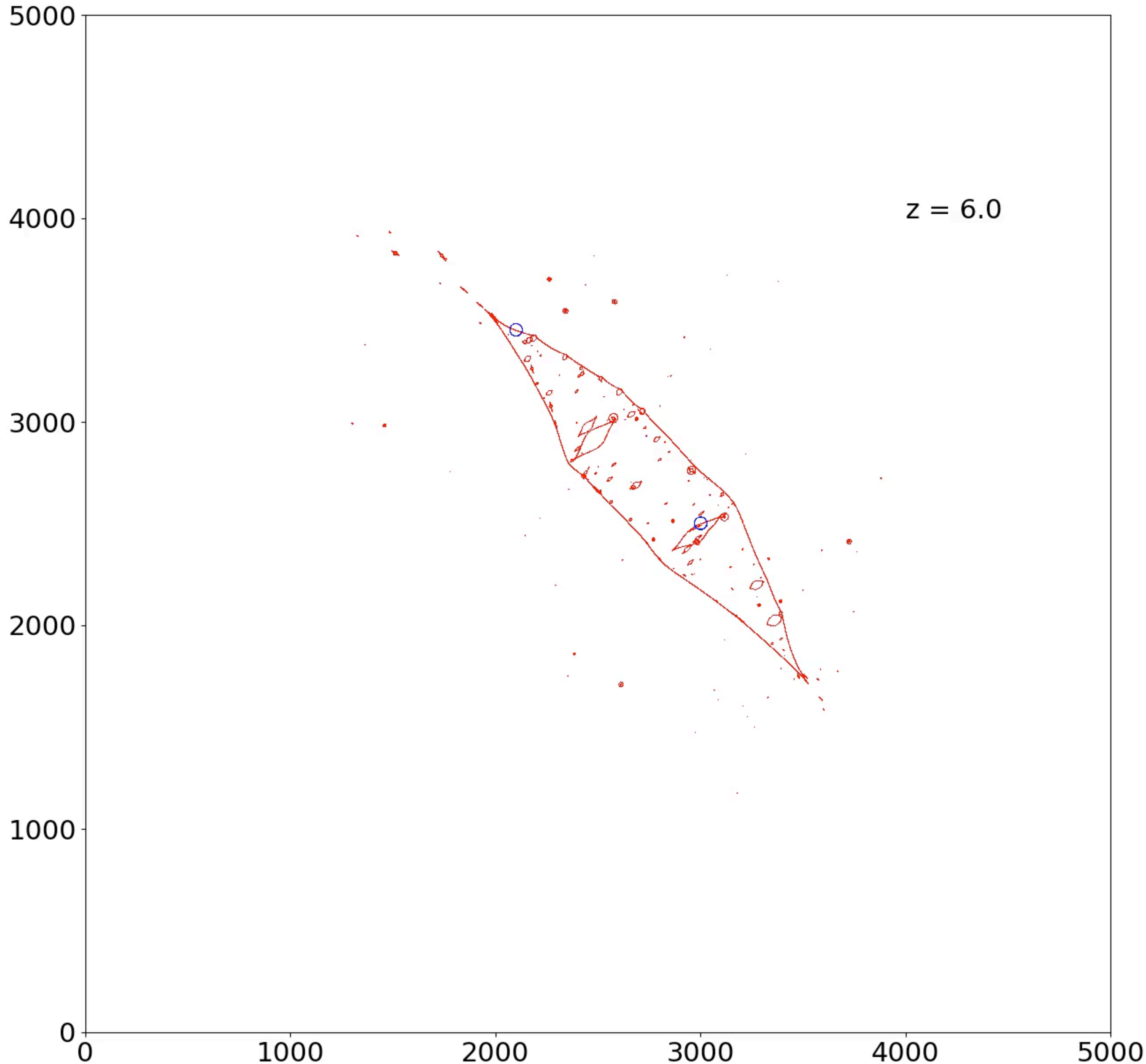
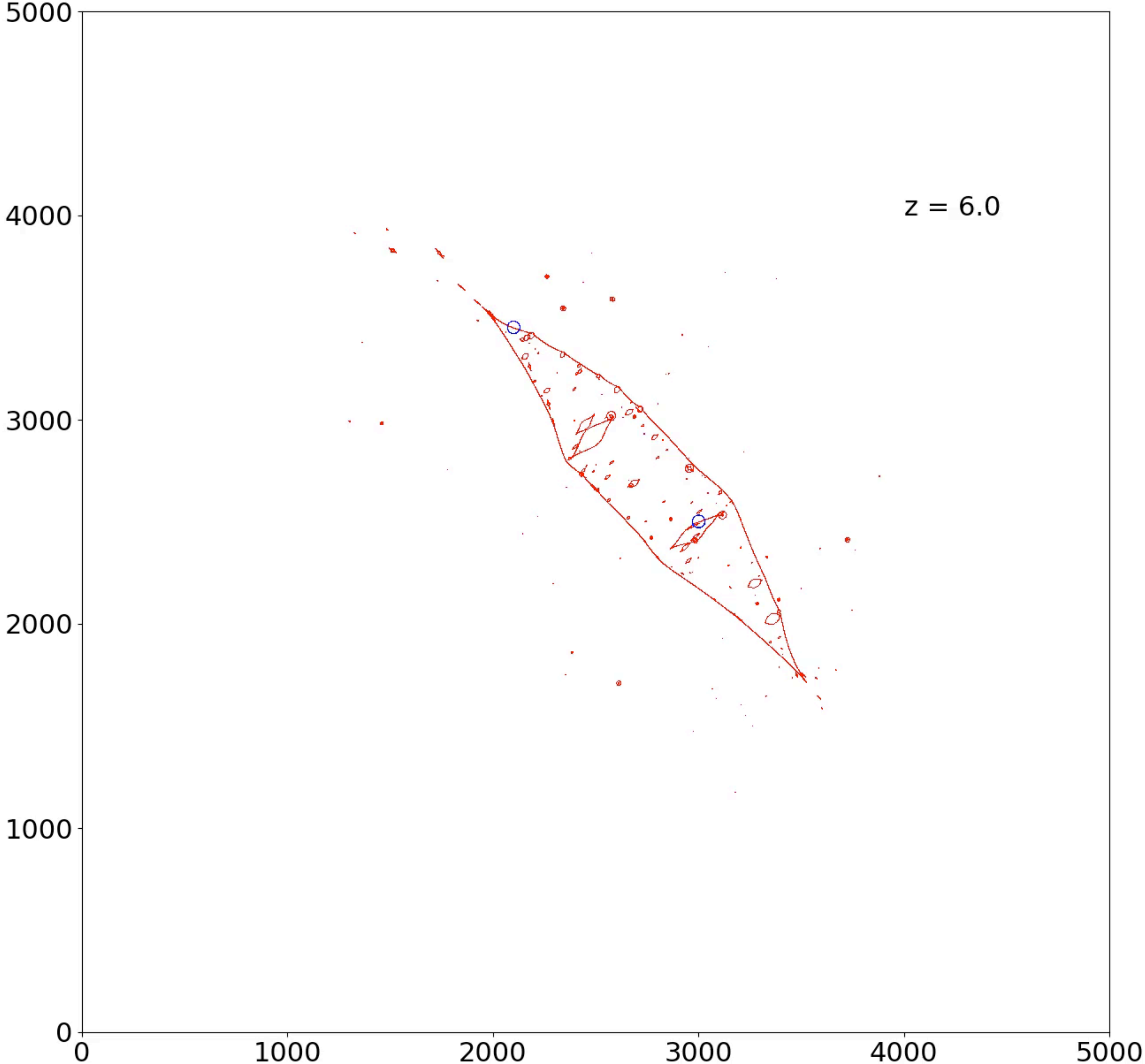


z = 6.0
0
1000
2000
3000
4000
5000
0
1000
2000
3000
4000
5000



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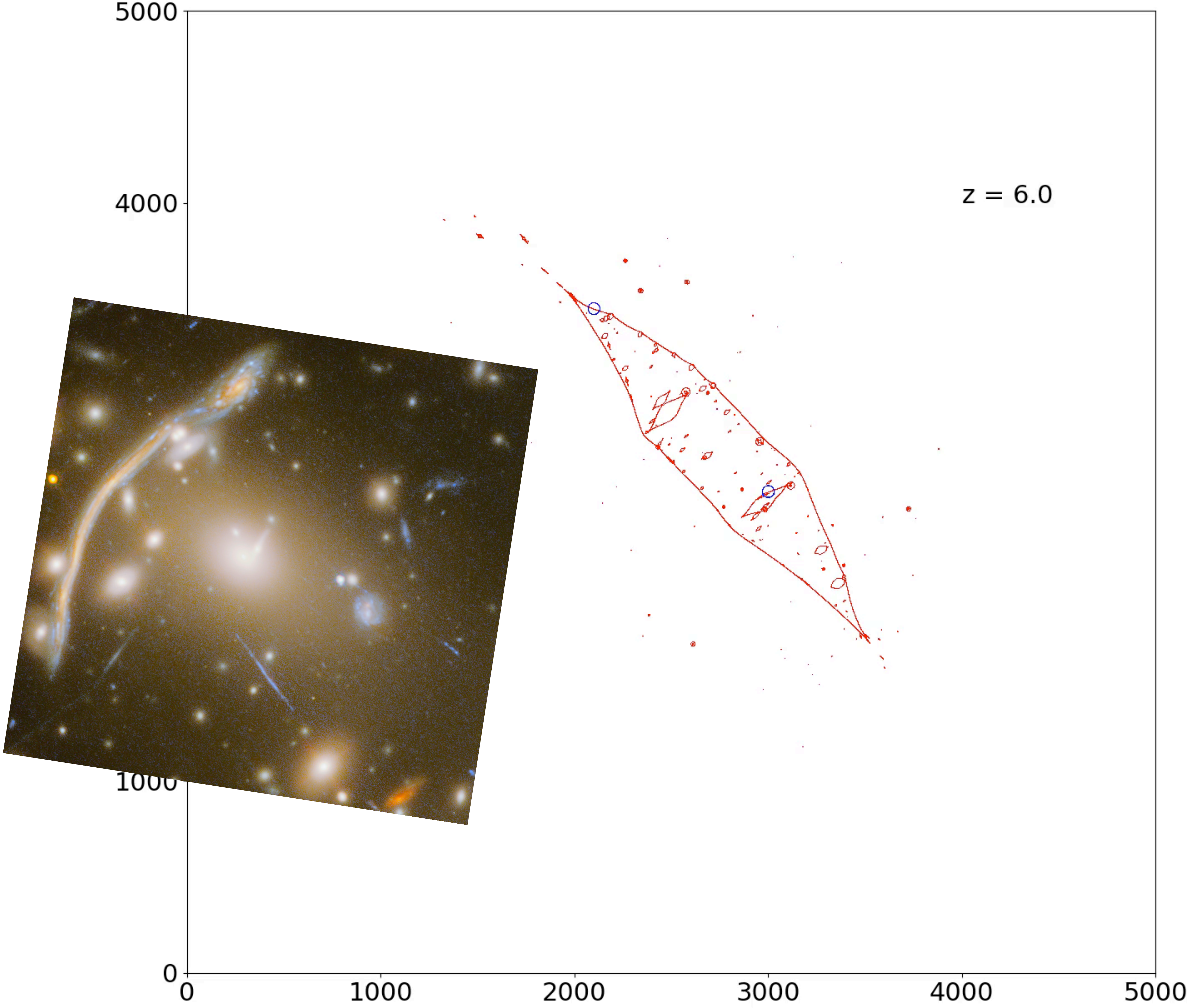
1000
2000
3000
4000
5000
0
1000
2000
3000
4000
5000



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4000
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2000
3000
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5000

SECOND ORDER LENS EQUATION

$$\beta_i \simeq \frac{\partial \beta_i}{\partial \theta_j} \theta_j$$

$$A_{ij}$$

SECOND ORDER LENS EQUATION

$$\beta_i \simeq \frac{\partial \beta_i}{\partial \theta_j}\theta_j + \frac{1}{2}\frac{\partial^2 \beta_i}{\partial \theta_j \partial \theta_k}\theta_j\theta_k$$

$$A_{ij}$$

$$\frac{\partial A_{ij}}{\partial \theta_k} = D_{ijk}$$

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$$\frac{\partial A_{ij}}{\partial \theta_k} = D_{ijk}$$

SECOND ORDER LENS EQUATION

$$\beta_i \simeq \underbrace{\frac{\partial \beta_i}{\partial \theta_j}}_{A_{ij}} \theta_j + \frac{1}{2} \underbrace{\frac{\partial^2 \beta_i}{\partial \theta_j \partial \theta_k}}_{\frac{\partial A_{ij}}{\partial \theta_k} = D_{ijk}} \theta_j \theta_k$$

$$D_{ij1} = \begin{pmatrix} -2\gamma_{1,1} - \gamma_{2,2} & -\gamma_{2,1} \\ -\gamma_{2,1} & -\gamma_{2,2} \end{pmatrix} \qquad D_{ij2} = \begin{pmatrix} -\gamma_{2,1} & -\gamma_{2,2} \\ -\gamma_{2,2} & 2\gamma_{1,2} - \gamma_{2,1} \end{pmatrix}$$

COMPLEX NOTATION

$$v = (v_1, v_2) \longrightarrow v = v_1 + iv_2$$

therefore,

$$\alpha = \alpha_1 + i\alpha_2$$

$$\gamma = \gamma_1 + i\gamma_2$$

we can also define complex differential operators:

$$\partial = \partial_1 + i\partial_2$$

$$\partial^\dagger = \partial_1 - i\partial_2$$

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we can also define complex differential operators:

$$\partial = \partial_1 + i\partial_2$$ *Spin raising operator*

$$\partial^\dagger = \partial_1 - i\partial_2$$ *Spin lowering operator*

COMPLEX NOTATION

$$\partial\hat{\Psi} = \partial_1\hat{\Psi} + i\partial_2\hat{\Psi} = \alpha_1 + i\alpha_2 = \alpha$$

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From spin-0 scalar field to spin-1 vector field (deflection angle)

COMPLEX NOTATION

$$\partial\hat{\Psi} = \partial_1\hat{\Psi} + i\partial_2\hat{\Psi} = \alpha_1 + i\alpha_2 = \alpha$$

From spin-0 scalar field to spin-1 vector field (deflection angle)

$$\partial^\dagger\partial = \partial_1^2 + \partial_2^2 = \triangle$$

$$\partial^\dagger\partial\hat{\Psi} = \triangle\hat{\Psi} = 2\kappa$$

COMPLEX NOTATION

$$\partial\hat{\Psi} = \partial_1\hat{\Psi} + i\partial_2\hat{\Psi} = \alpha_1 + i\alpha_2 = \alpha$$

From spin-0 scalar field to spin-1 vector field (deflection angle)

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$$\partial^\dagger\partial\hat{\Psi} = \triangle\hat{\Psi} = 2\kappa$$

From spin-1 vector field to spin-0 scalar field

COMPLEX NOTATION

$$\partial\hat{\Psi} = \partial_1\hat{\Psi} + i\partial_2\hat{\Psi} = \alpha_1 + i\alpha_2 = \alpha$$

From spin-0 scalar field to spin-1 vector field (deflection angle)

$$\partial^\dagger\partial = \partial_1^2 + \partial_2^2 = \triangle$$

$$\partial^\dagger\partial\hat{\Psi} = \triangle\hat{\Psi} = 2\kappa$$

From spin-1 vector field to spin-0 scalar field

$$\frac{1}{2}\partial\partial\hat{\Psi} = \frac{1}{2}\partial\alpha = \gamma$$

The shear is a spin-2 field

COMPLEX NOTATION

$$
\begin{aligned}
F &= \frac{1}{2}\partial\partial^{\dagger}\partial\hat{\Psi} = \partial\kappa \\
G &= \frac{1}{2}\partial\partial\partial\hat{\Psi} = \partial\gamma
\end{aligned}
$$

$$
\begin{gathered}
D_{111} = -2\gamma_{11} - \gamma_{22} = -\frac{1}{2}(3F_1 + G_1) \\
D_{211} = D_{121} = D_{112} = -\gamma_{21} = -\frac{1}{2}(F_2 + G_2) \\
D_{122} = D_{212} = D_{221} = -\gamma_{22} = -\frac{1}{2}(F_1 - G_1) \\
D_{222} = 2\gamma_{12} - \gamma_{21} = -\frac{1}{2}(3F_2 - G_2)
\end{gathered}
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$$F = F_1 + iF_2 = (\gamma_{1,1} + \gamma_{2,2}) + i(\gamma_{2,1} - \gamma_{1,2})$$

$$G = G_1 + iG_2 = (\gamma_{1,1} - \gamma_{2,2}) + i(\gamma_{2,1} + \gamma_{1,2})$$

$$D_{ij1} = \begin{pmatrix} -2\gamma_{1,1} - \gamma_{2,2} & -\gamma_{2,1} \\ -\gamma_{2,1} & -\gamma_{2,2} \end{pmatrix} \qquad D_{ij2} = \begin{pmatrix} -\gamma_{2,1} & -\gamma_{2,2} \\ -\gamma_{2,2} & 2\gamma_{1,2} - \gamma_{2,1} \end{pmatrix}$$

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\end{aligned}
$$

COMPLEX NOTATION

$$F = \frac{1}{2}\partial\partial^\dagger\partial\hat{\Psi} = \partial\kappa$$ *Spin-1*

$$G = \frac{1}{2}\partial\partial\partial\hat{\Psi} = \partial\gamma$$ *Spin-3*

$$F = F_1 + iF_2 = (\gamma_{1,1} + \gamma_{2,2}) + i(\gamma_{2,1} - \gamma_{1,2})$$

$$G = G_1 + iG_2 = (\gamma_{1,1} - \gamma_{2,2}) + i(\gamma_{2,1} + \gamma_{1,2})$$

$$D_{ij1} = \begin{pmatrix} -2\gamma_{1,1} - \gamma_{2,2} & -\gamma_{2,1} \\ -\gamma_{2,1} & -\gamma_{2,2} \end{pmatrix} \qquad D_{ij2} = \begin{pmatrix} -\gamma_{2,1} & -\gamma_{2,2} \\ -\gamma_{2,2} & 2\gamma_{1,2} - \gamma_{2,1} \end{pmatrix}$$

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