

GRAVITATIONAL LENSING

4 - LENSING POTENTIAL, LENS MAPPING

Massimo Meneghetti
AA 2018-2019

LENSING POTENTIAL

$$\hat{\vec{\alpha}} = \frac{2}{c^2} \int_{-\infty}^{+\infty} \vec{\nabla}_{\perp} \Phi dz$$

This formula tells us that the deflection is caused by the projection of the Newtonian gravitational potential on the lens plane.

$$\hat{\Psi}(\vec{\theta}) = \frac{D_{LS}}{D_L D_S} \frac{2}{c^2} \int \Phi(D_L \vec{\theta}, z) dz \quad \text{We introduce the effective lensing potential}$$

LENSING POTENTIAL

$$\hat{\vec{\alpha}} = \frac{2}{c^2} \int_{-\infty}^{+\infty} \vec{\nabla}_{\perp} \Phi dz$$

This formula tells us that the deflection is caused by the projection of the Newtonian gravitational potential on the lens plane.

$$\hat{\Psi}(\vec{\theta}) = \frac{D_{LS}}{D_L D_S} \frac{2}{c^2} \int \Phi(D_L \vec{\theta}, z) dz$$

1

We introduce the effective lensing potential

the lensing potential is the projection of the 3D potential

LENSING POTENTIAL

$$\hat{\vec{\alpha}} = \frac{2}{c^2} \int_{-\infty}^{+\infty} \vec{\nabla}_{\perp} \Phi dz$$

This formula tells us that the deflection is caused by the projection of the Newtonian gravitational potential on the lens plane.

$$\hat{\Psi}(\vec{\theta}) = \frac{D_{LS}}{D_L D_S} \frac{2}{c^2} \int \Phi(D_L \vec{\theta}, z) dz$$

We introduce the effective lensing potential

the lensing potential is the projection of the 3D potential

2

the lensing potential scales with distances

1

OTHER PROPERTIES OF THE LENSING POTENTIAL

$$\vec{\nabla}_\theta \hat{\Psi}(\vec{\theta}) = \vec{\alpha}(\vec{\theta})$$

The reduced deflection angle is the gradient of the lensing potential

$$\begin{aligned}\vec{\nabla}_\theta \hat{\Psi}(\vec{\theta}) &= D_L \vec{\nabla}_\perp \hat{\Psi} = \vec{\nabla}_\perp \left(\frac{D_{LS}}{D_S} \frac{2}{c^2} \int \hat{\Phi}(\vec{\theta}, z) dz \right) \\ &= \frac{D_{LS}}{D_S} \frac{2}{c^2} \int \vec{\nabla}_\perp \Phi(\vec{\theta}, z) dz \\ &= \vec{\alpha}(\vec{\theta})\end{aligned}$$

NOTE THAT...

... the same result holds if we use the dimension-less notation:

$$\vec{\nabla}_x = \frac{\xi_0}{D_L} \vec{\nabla}_\theta$$



$$\vec{\nabla}_x \hat{\Psi} = \frac{\xi_0}{D_L} \vec{\nabla}_\theta \hat{\Psi} = \frac{\xi_0}{D_L} \vec{\alpha}$$

By multiplying both sides of the equation by D_L^2/ξ_0^2 we obtain:

$$\frac{D_L^2}{\xi_0^2} \vec{\nabla}_x \hat{\Psi} = \frac{D_L}{\xi_0} \vec{\alpha} \quad \rightarrow \quad \Psi = \frac{D_L^2}{\xi_0^2} \hat{\Psi} \quad \rightarrow \quad \vec{\nabla}_x \Psi(\vec{x}) = \vec{\alpha}(\vec{x})$$

We have introduced the dimensionless counter-part of the lensing potential!

OTHER PROPERTIES OF THE LENSING POTENTIAL

$$\Delta_\theta \hat{\Psi}(\vec{\theta}) = 2\kappa(\vec{\theta})$$

The laplacian of the lensing potential is twice the convergence:

$$\kappa(\vec{\theta}) \equiv \frac{\Sigma(\vec{\theta})}{\Sigma_{\text{cr}}} \quad \text{with} \quad \Sigma_{\text{cr}} = \frac{c^2}{4\pi G} \frac{D_S}{D_L D_{LS}}$$

OTHER PROPERTIES OF THE LENSING POTENTIAL

$$\Delta_\theta \hat{\Psi}(\vec{\theta}) = 2\kappa(\vec{\theta})$$

The laplacian of the lensing potential is twice the convergence:

$$\kappa(\vec{\theta}) \equiv \frac{\Sigma(\vec{\theta})}{\Sigma_{\text{cr}}} \quad \text{with} \quad \Sigma_{\text{cr}} = \frac{c^2}{4\pi G} \frac{D_S}{D_L D_{LS}}$$

$$[G] = L^3/M/T^2$$

$$[c^2] = L^2/T^2$$

$$[D_X] = L$$

OTHER PROPERTIES OF THE LENSING POTENTIAL

$$\Delta_\theta \hat{\Psi}(\vec{\theta}) = 2\kappa(\vec{\theta})$$

The laplacian of the lensing potential is twice the convergence:

$$\kappa(\vec{\theta}) \equiv \frac{\Sigma(\vec{\theta})}{\Sigma_{\text{cr}}} \quad \text{with} \quad \Sigma_{\text{cr}} = \frac{c^2}{4\pi G} \frac{D_S}{D_L D_{LS}}$$

$$[G] = L^3/M/T^2$$

$$[c^2] = L^2/T^2$$

$$[D_X] = L$$

The critical surface density is a characteristic density to distinguish between strong and weak gravitational lenses!

OTHER PROPERTIES OF THE LENSING POTENTIAL

$$\Delta_\theta \hat{\Psi}(\vec{\theta}) = 2\kappa(\vec{\theta})$$

The laplacian of the lensing potential is twice the convergence:

We start from the poisson equation

$$\Delta_\theta \Phi = 4\pi G \rho$$

The surface mass density is then:

$$\Sigma(\vec{\theta}) = \frac{1}{4\pi G} \int_{-\infty}^{+\infty} \Delta_\theta \Phi dz$$

$$\kappa(\vec{\theta}) = \frac{1}{c^2} \frac{D_L D_{LS}}{D_S} \int_{-\infty}^{+\infty} \Delta_\theta \Phi dz$$

Let's introduce the Laplacian operator on the lens plane:

$$\Delta_\theta = \frac{\partial^2}{\partial \theta_1^2} + \frac{\partial^2}{\partial \theta_2^2} = D_L^2 \left(\frac{\partial^2}{\partial \xi_1^2} + \frac{\partial^2}{\partial \xi_2^2} \right) = D_L^2 \left(\Delta - \frac{\partial^2}{\partial z^2} \right)$$

Then:

$$\Delta_\theta \Phi = \frac{1}{D_L^2} \Delta_\theta \Phi + \frac{\partial^2 \Phi}{\partial z^2}$$

OTHER PROPERTIES OF THE LENSING POTENTIAL

With this substitution:

$$\kappa(\vec{\theta}) = \frac{1}{c^2} \frac{D_{\text{LS}}}{D_{\text{S}} D_{\text{L}}} \left[\Delta_\theta \int_{-\infty}^{+\infty} \Phi dz + D_{\text{L}}^2 \int_{-\infty}^{+\infty} \frac{\partial^2 \Phi}{\partial z^2} dz \right]$$

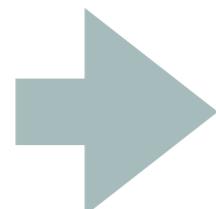
where the second term in the sum is zero, if the lens is gravitationally bound!

Given the definition of lensing potential:

$$\kappa(\theta) = \frac{1}{2} \Delta_\theta \hat{\Psi}$$

Note that:

$$\Delta_\theta = D_{\text{L}}^2 \Delta_\xi = \frac{D_{\text{L}}^2}{\xi_0^2} \Delta_x \quad \kappa(\theta) = \frac{1}{2} \Delta_\theta \hat{\Psi} = \frac{1}{2} \frac{\xi_0^2}{D_{\text{L}}^2} \Delta_\theta \Psi$$



$$\kappa(\vec{x}) = \frac{1}{2} \Delta_x \Psi(\vec{x})$$

DIMENSIONLESS NOTATION

From

$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}') \Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2} d^2 \xi'$$

we obtain

$$\vec{\alpha}(\vec{x}) = \frac{1}{\pi} \int_{\mathbf{R}^2} d^2 x' \kappa(\vec{x}') \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|}$$

Using

$$\vec{\nabla}_x \Psi(\vec{x}) = \vec{\alpha}(\vec{x})$$

$$\Psi(\vec{x}) = \frac{1}{\pi} \int_{\mathbf{R}^2} \kappa(\vec{x}') \ln |\vec{x} - \vec{x}'| d^2 x'$$

DIMENSIONLESS NOTATION

From

$$\vec{\alpha}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}') \Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2} d^2 \xi'$$

we obtain

$$\vec{\alpha}(\vec{x}) = \frac{1}{\pi} \int_{\mathbf{R}^2} d^2 x' \kappa(\vec{x}') \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|}$$

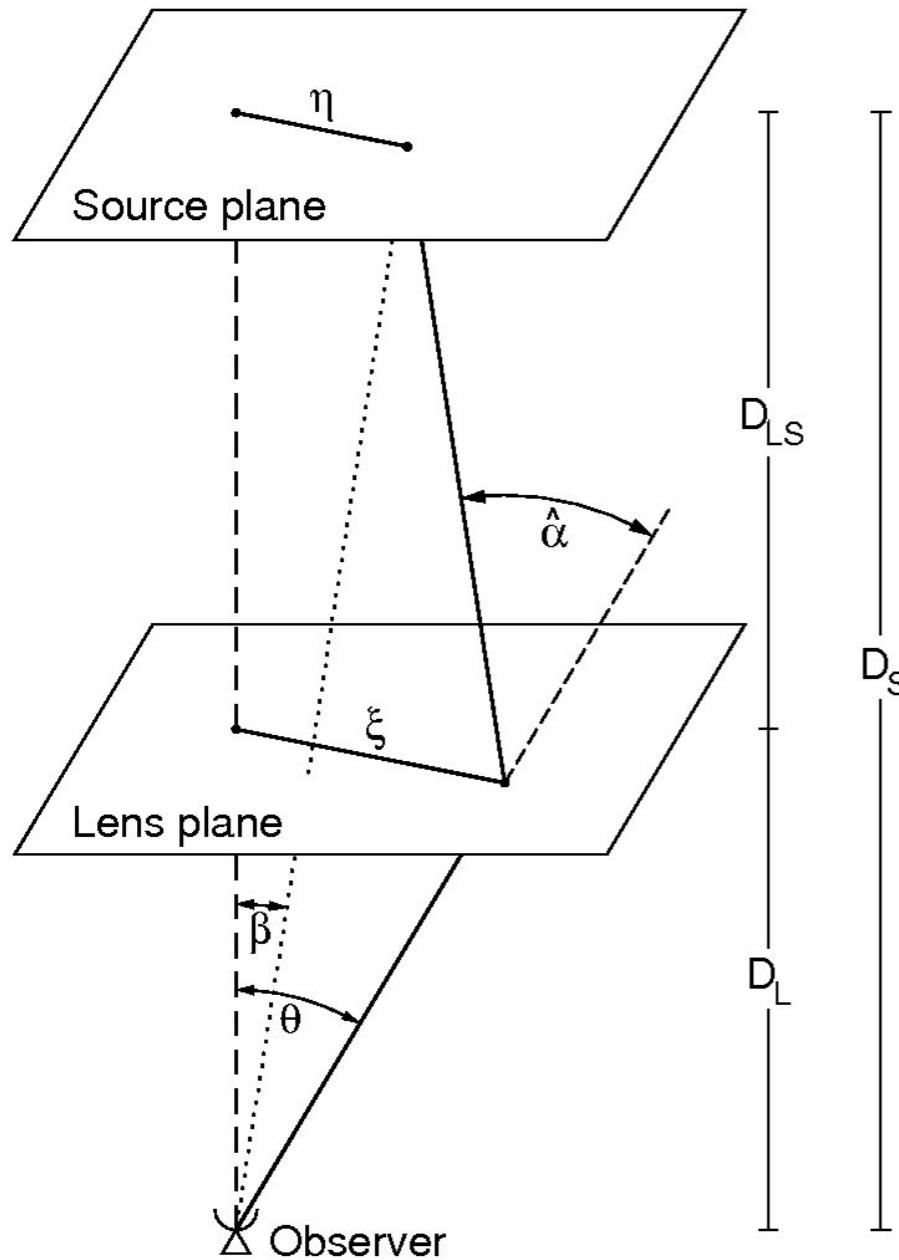
Using

$$\vec{\nabla}_{\vec{x}} \Psi(\vec{x}) = \vec{\alpha}(\vec{x})$$

$$\Psi(\vec{x}) = \frac{1}{\pi} \int_{\mathbf{R}^2} \kappa(\vec{x}') \ln |\vec{x} - \vec{x}'| d^2 x'$$

Convolution kernels

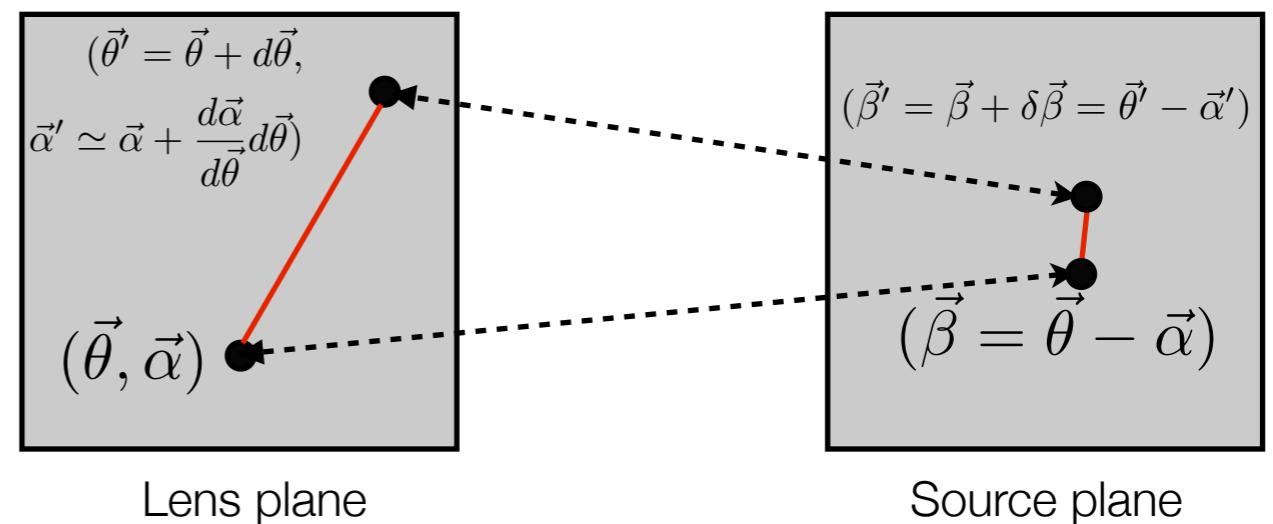
LENS MAPPING (FIRST ORDER)



- we derived the lens equation

$$\vec{\beta} = \vec{\theta} - \frac{D_{LS}}{D_S} \hat{\vec{\alpha}}(\vec{\theta}) = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

- Assuming that the d.a. does not vary significantly over the scale $d\Theta$:



$$(\vec{\beta}' - \vec{\beta}) = \left(I - \frac{d\vec{\alpha}}{d\vec{\theta}} \right) (\vec{\theta}' - \vec{\theta}) = A(\vec{\theta}' - \vec{\theta})$$

LENS MAPPING (FIRST ORDER)

$$A \equiv \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \left(\delta_{ij} - \frac{\partial \alpha_i(\vec{\theta})}{\partial \theta_j} \right) = \left(\delta_{ij} - \frac{\partial^2 \hat{\Psi}(\vec{\theta})}{\partial \theta_i \partial \theta_j} \right)$$

A is called “the lensing Jacobian”: it is a symmetric second rank tensor describing the first order mapping between lens and source planes.

This tensor can be written as the sum of an isotropic part, proportional to its trace, and an anisotropic, traceless part.

$$A_{iso,i,j} = \frac{1}{2} \text{Tr} A \delta_{i,j}$$

$$A_{aniso,i,j} = A_{i,j} - \frac{1}{2} \text{Tr} A \delta_{i,j}$$

ANISOTROPIC PART

$$\begin{aligned}
 \left(A - \frac{1}{2} \text{tr}A \cdot I \right)_{ij} &= \delta_{ij} - \hat{\Psi}_{ij} - \frac{1}{2}(1 - \hat{\Psi}_{11} + 1 - \hat{\Psi}_{22})\delta_{ij} \\
 &= -\hat{\Psi}_{ij} + \frac{1}{2}(\hat{\Psi}_{11} + \hat{\Psi}_{22})\delta_{ij} \\
 &= \begin{pmatrix} -\frac{1}{2}(\hat{\Psi}_{11} - \hat{\Psi}_{22}) & -\hat{\Psi}_{12} \\ -\hat{\Psi}_{12} & \frac{1}{2}(\hat{\Psi}_{11} - \hat{\Psi}_{22}) \end{pmatrix}
 \end{aligned}$$

$$\frac{\partial^2 \hat{\Psi}(\vec{\theta})}{\partial \theta_i \partial \theta_j} \equiv \hat{\Psi}_{ij}$$

Introducing the shear:

$$\begin{aligned}
 \gamma_1 &= \frac{1}{2}(\hat{\Psi}_{11} - \hat{\Psi}_{22}) \\
 \gamma_2 &= \hat{\Psi}_{12} = \hat{\Psi}_{21},
 \end{aligned}$$



$$\Gamma = \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix}$$

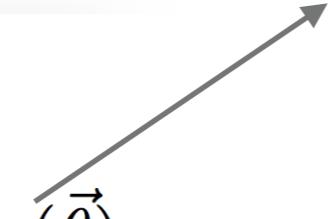
*Symmetric, trace-less tensor
with eigenvalues:*

$$\pm \sqrt{\gamma_1^2 + \gamma_2^2} = \pm \gamma$$

ISOTROPIC PART

$$\begin{aligned}\frac{1}{2} \text{tr} A \cdot I &= \left[1 - \frac{1}{2}(\hat{\Psi}_{11} + \hat{\Psi}_{22}) \right] \delta_{ij} \\ &= \left(1 - \frac{1}{2} \Delta \hat{\Psi} \right) \delta_{ij} = (1 - \kappa) \delta_{ij}\end{aligned}$$

Remember: $\Delta_\theta \Psi(\vec{\theta}) = 2\kappa(\vec{\theta})$



THE SHEAR IS NOT A VECTOR!

$$\det A = (1 - \kappa - \gamma)(1 - \kappa + \gamma)$$
$$\gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$$

There is thus an orthogonal coordinate transformation $R(\varphi)$, a rotation by an angle φ , which brings the Jacobian matrix into diagonal form.

Generally, the Jacobian matrix transforms as

$$A \rightarrow A' = R(\varphi)^T A R(\varphi)$$

This shows that the shear components transform under coordinate rotations as

$$\gamma_1 \rightarrow \gamma'_1 = \gamma_1 \cos(2\varphi) + \gamma_2 \sin(2\varphi)$$

$$\gamma_2 \rightarrow \gamma'_2 = -\gamma_1 \sin(2\varphi) + \gamma_2 \cos(2\varphi)$$

i.e. unlike a vector! Since the shear components are mapped onto each other after rotations of $\varphi = \pi$ rather than $\varphi = 2\pi$, they form a so-called spin-2 field.

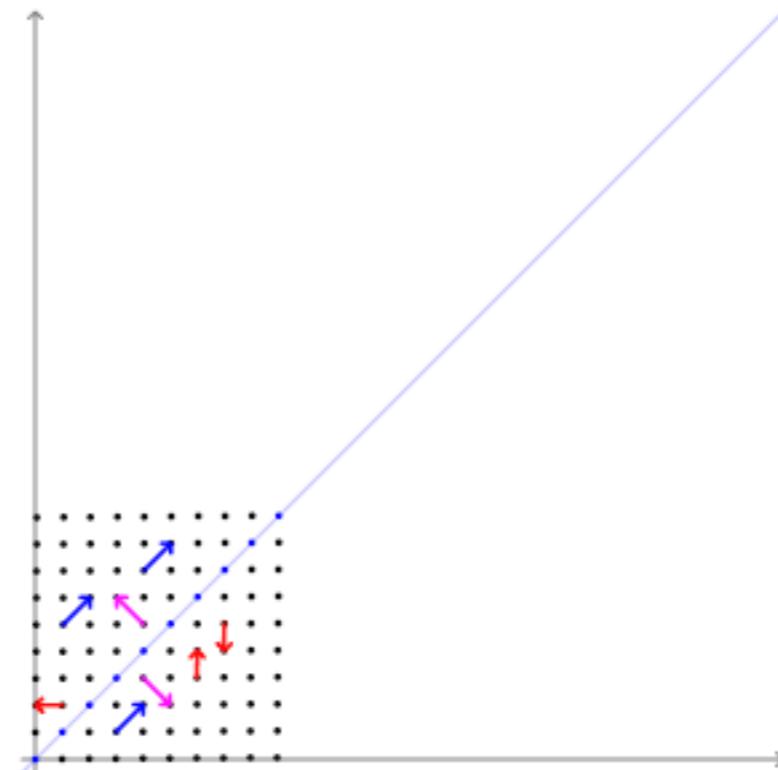
LENSING JACOBIAN

$$\begin{aligned} A &= \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix} \\ &= (1 - \kappa) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix} \end{aligned}$$

Lens mapping at first order is a linear application, distorting areas.

Distortion directions are given by the eigenvectors of A .

Distortion amplitudes in these directions are given by the eigenvalues.



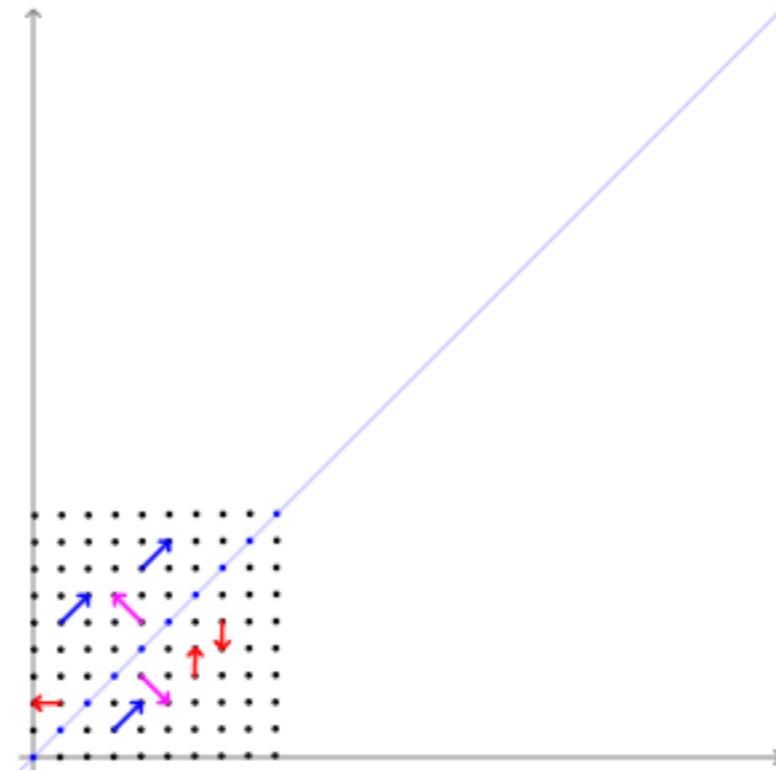
LENSING JACOBIAN

$$\begin{aligned} A &= \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix} \\ &= (1 - \kappa) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix} \end{aligned}$$

Lens mapping at first order is a linear application, distorting areas.

Distortion directions are given by the eigenvectors of A .

Distortion amplitudes in these directions are given by the eigenvalues.



EXAMPLE: FIRST ORDER DISTORTION OF A CIRCULAR SOURCE

$$\beta_1^2 + \beta_2^2 = \beta^2$$

In the reference frame where A is diagonal:

$$\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 1 - \kappa - \gamma & 0 \\ 0 & 1 - \kappa + \gamma \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

$$\beta_1 = (1 - \kappa - \gamma)\theta_1$$

$$\beta_2 = (1 - \kappa + \gamma)\theta_2$$

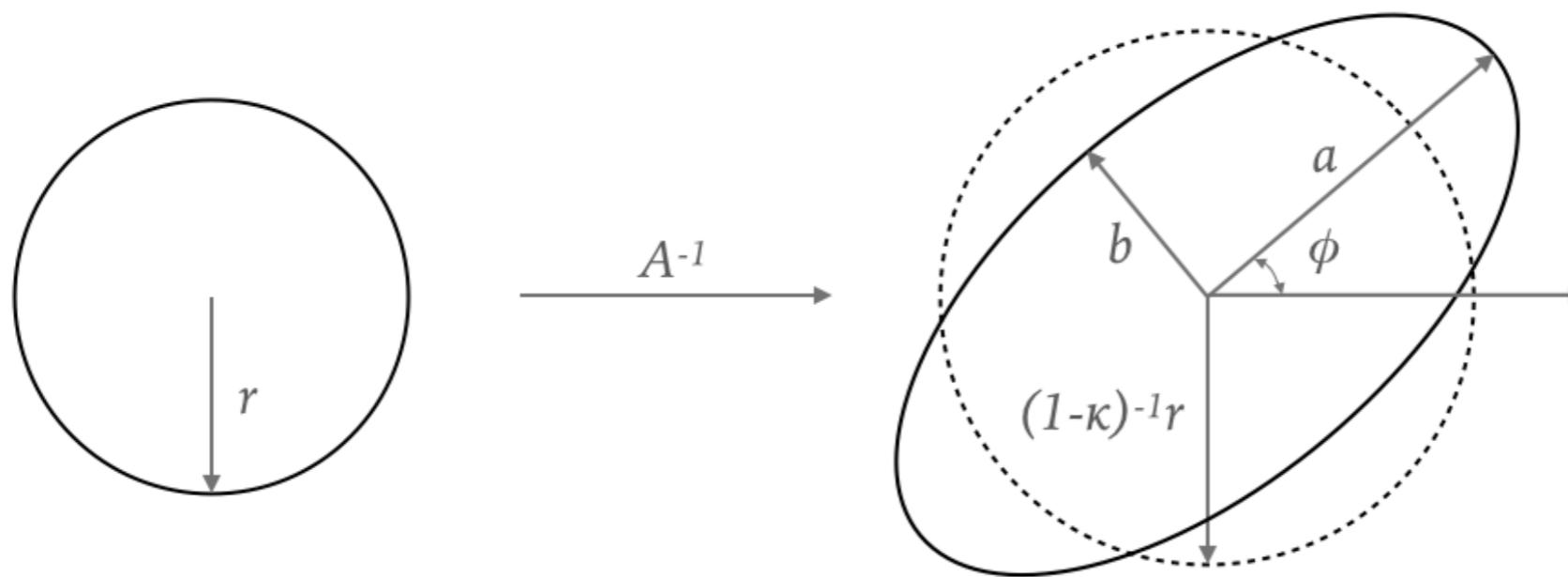
$$\beta^2 = \beta_1^2 + \beta_2^2 = (1 - \kappa - \gamma)^2\theta_1^2 + (1 - \kappa + \gamma)^2\theta_2^2$$

This is the equation of an ellipse with semi-axes:

$$a = \frac{\beta}{1 - \kappa - \gamma}$$

$$b = \frac{\beta}{1 - \kappa + \gamma}$$

EXAMPLE: FIRST ORDER DISTORTION OF A CIRCULAR SOURCE



convergence: responsible for isotropic expansion or contraction

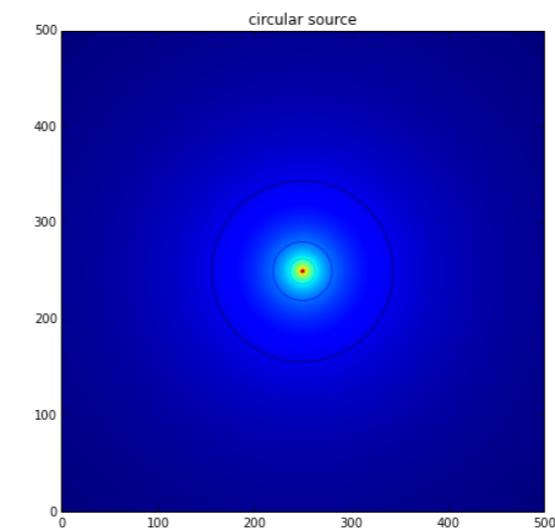
shear: responsible for anisotropic distortion

$$\text{Ellipticity: } e = \frac{a - b}{a + b} = \frac{\gamma}{1 - \kappa} = g$$

ON THE SPIN-2 NATURE OF SHEAR: QUIZ

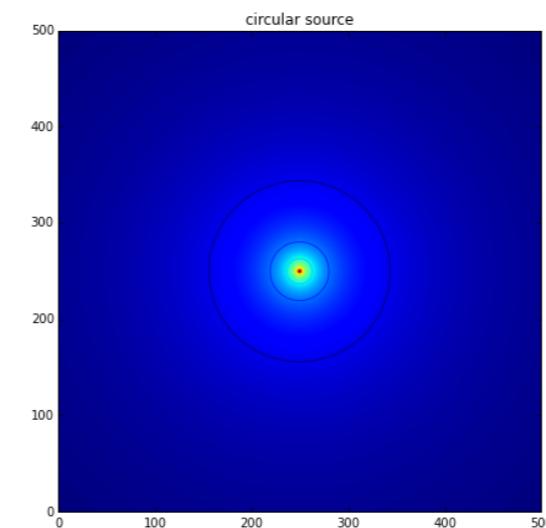
ON THE SPIN-2 NATURE OF SHEAR: QUIZ

- Let's consider a circular source



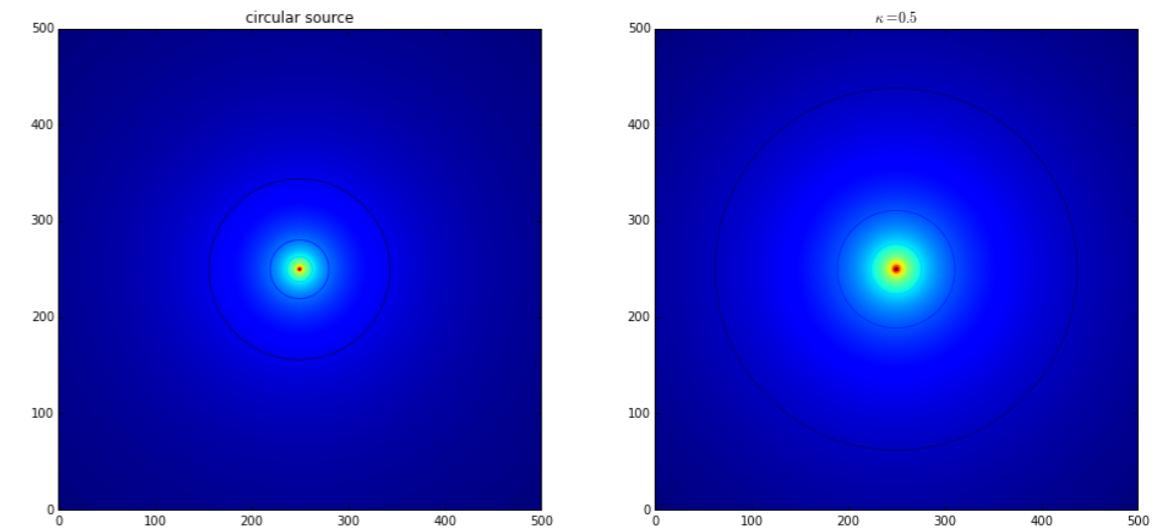
ON THE SPIN-2 NATURE OF SHEAR: QUIZ

- Let's consider a circular source
- How is it distorted if we apply a pure convergence transformation?



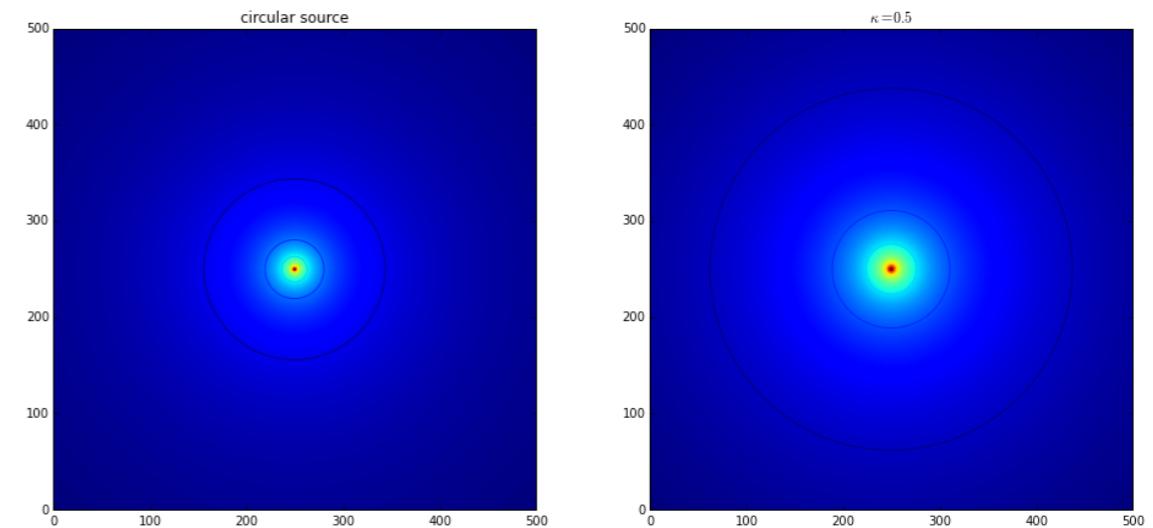
ON THE SPIN-2 NATURE OF SHEAR: QUIZ

- Let's consider a circular source
- How is it distorted if we apply a pure convergence transformation?



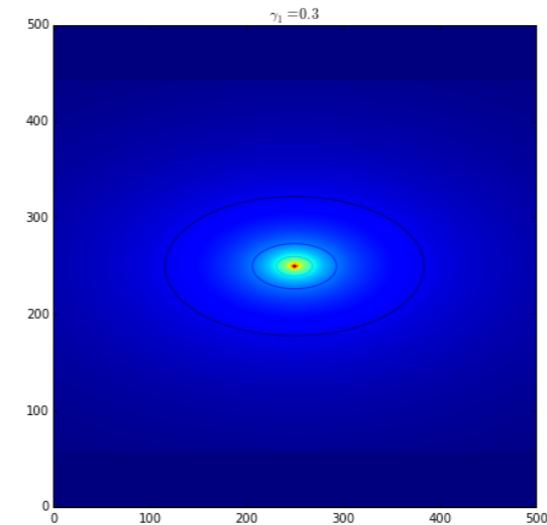
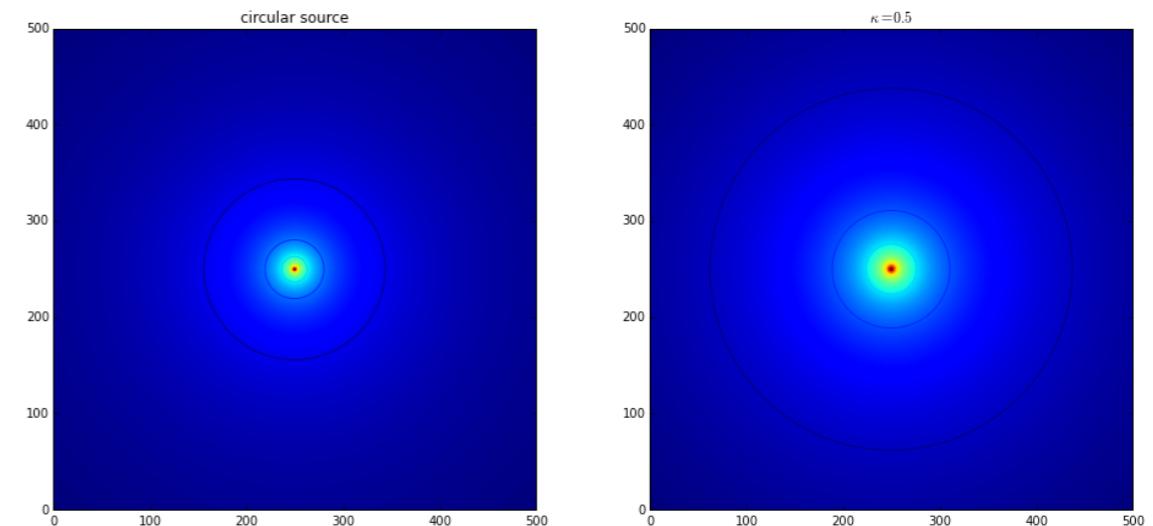
ON THE SPIN-2 NATURE OF SHEAR: QUIZ

- Let's consider a circular source
- How is it distorted if we apply a pure convergence transformation?
- Now, assume that $\gamma_1 > 0$ and $\gamma_2 = 0$. How is the image distorted?



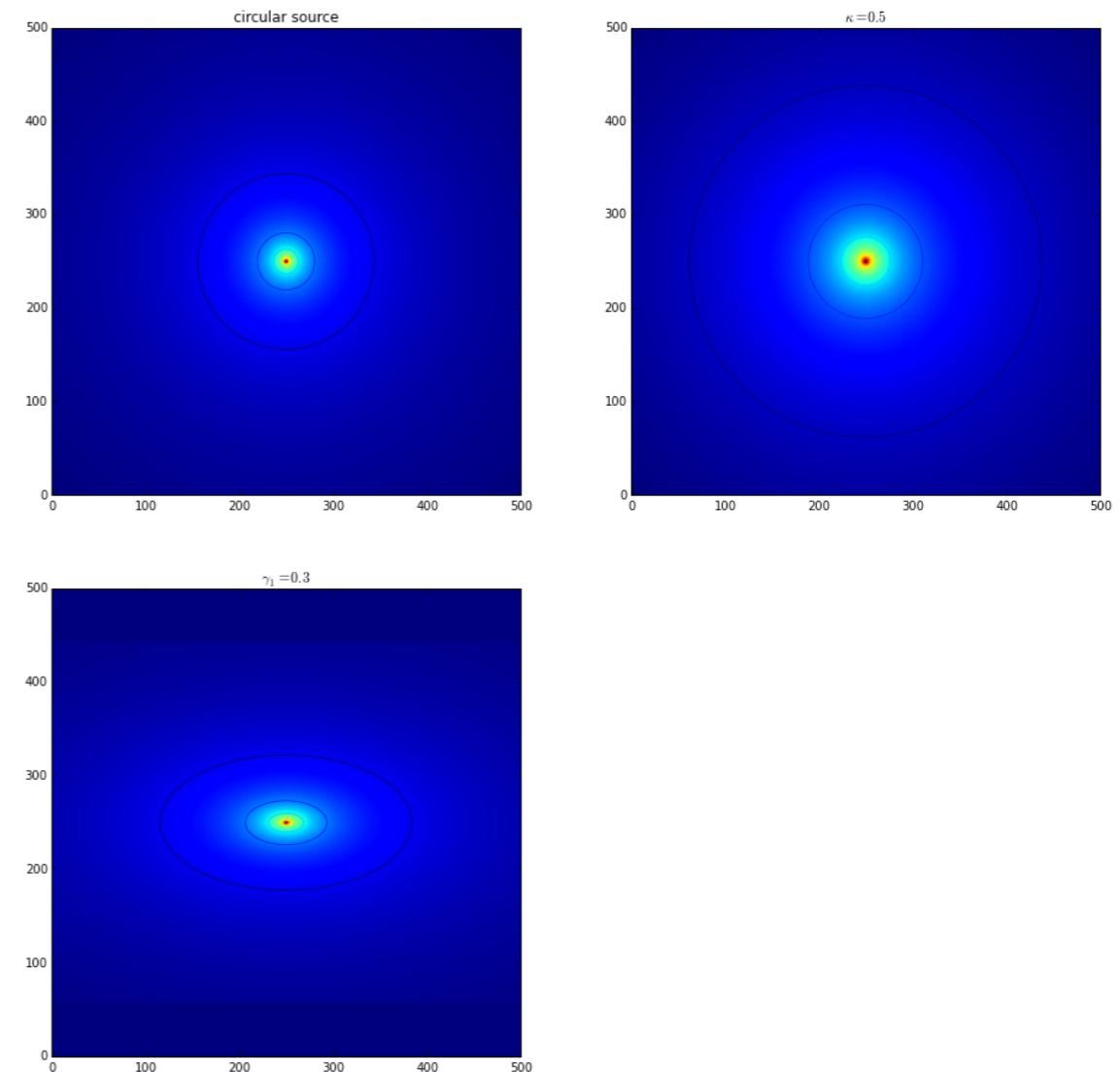
ON THE SPIN-2 NATURE OF SHEAR: QUIZ

- Let's consider a circular source
- How is it distorted if we apply a pure convergence transformation?
- Now, assume that $\gamma_1 > 0$ and $\gamma_2 = 0$. How is the image distorted?



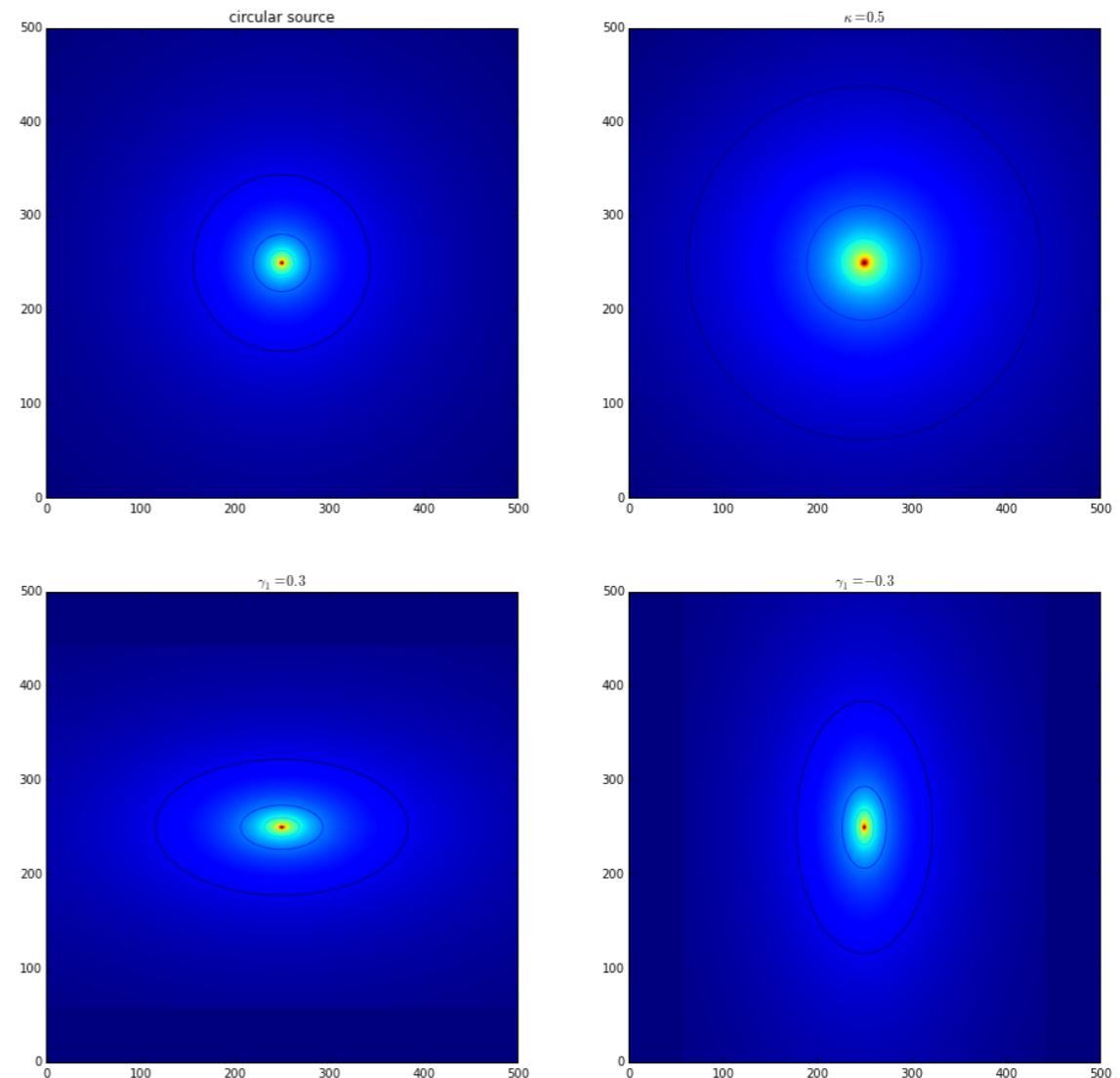
ON THE SPIN-2 NATURE OF SHEAR: QUIZ

- Let's consider a circular source
- How is it distorted if we apply a pure convergence transformation?
- Now, assume that $\gamma_1 > 0$ and $\gamma_2 = 0$. How is the image distorted?
- And what if $\gamma_1 < 0$ and $\gamma_2 = 0$?



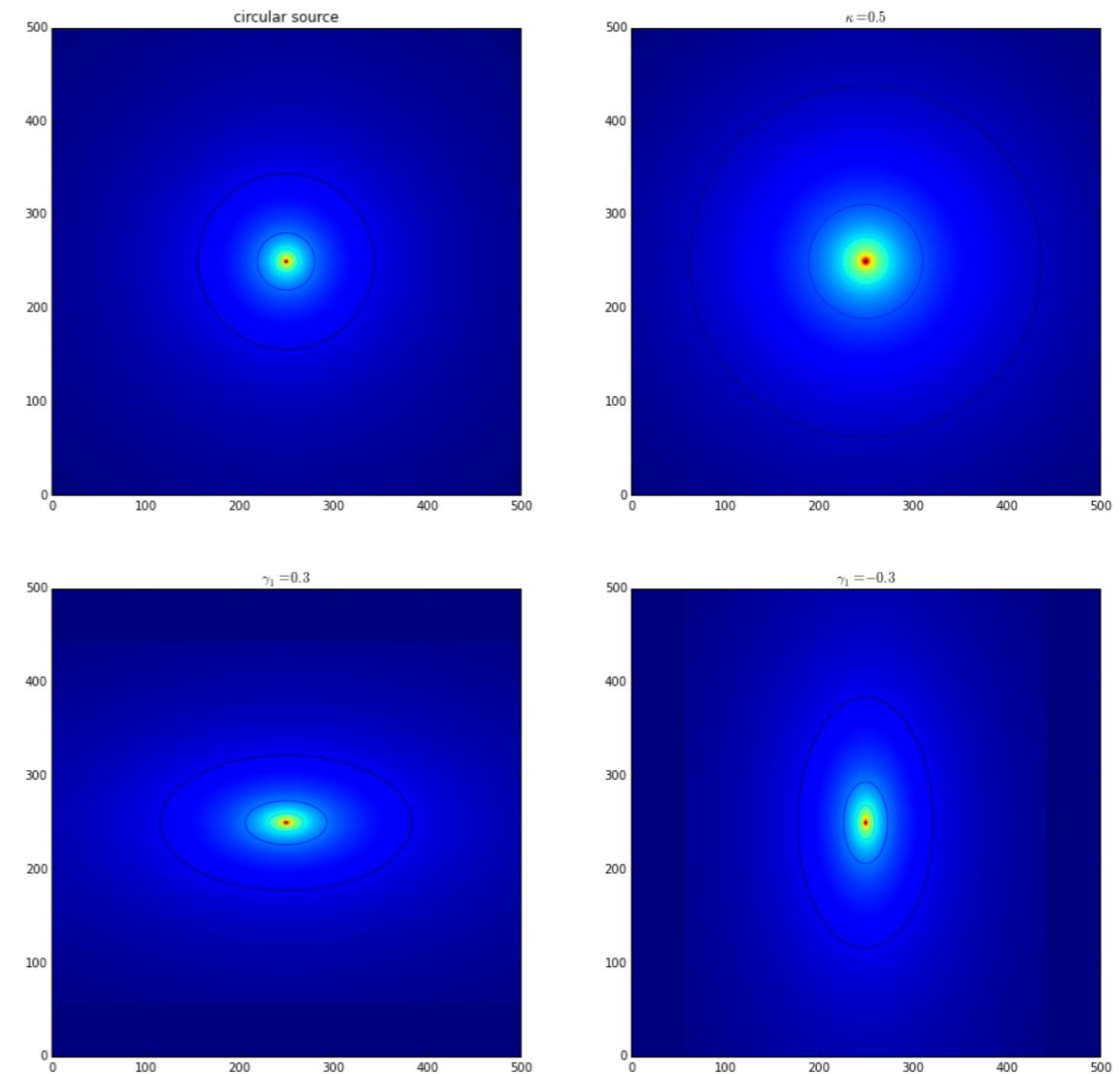
ON THE SPIN-2 NATURE OF SHEAR: QUIZ

- Let's consider a circular source
- How is it distorted if we apply a pure convergence transformation?
- Now, assume that $\gamma_1 > 0$ and $\gamma_2 = 0$. How is the image distorted?
- And what if $\gamma_1 < 0$ and $\gamma_2 = 0$?



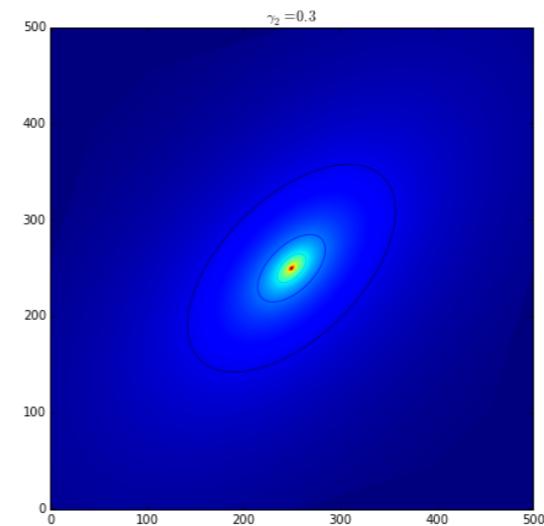
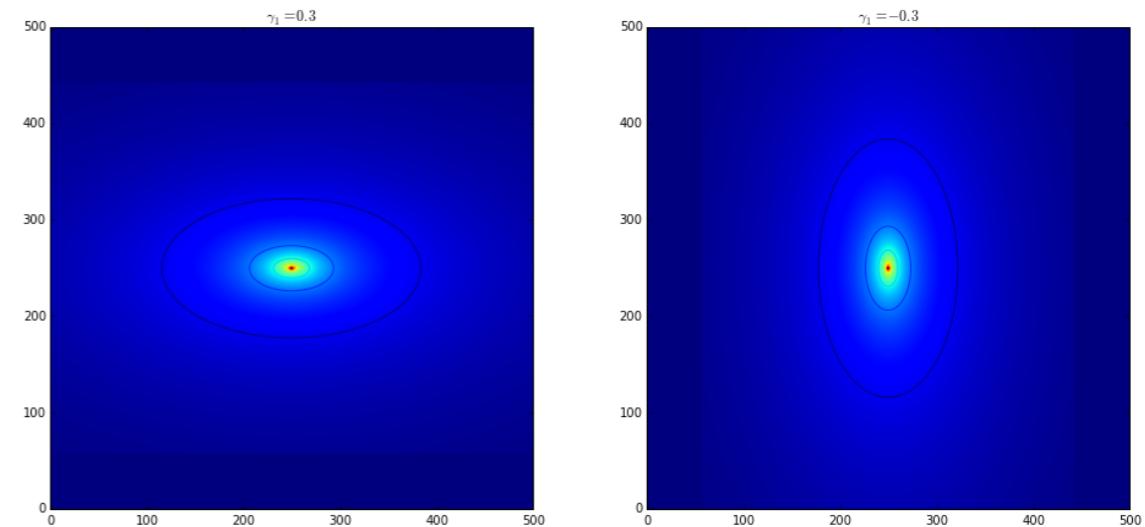
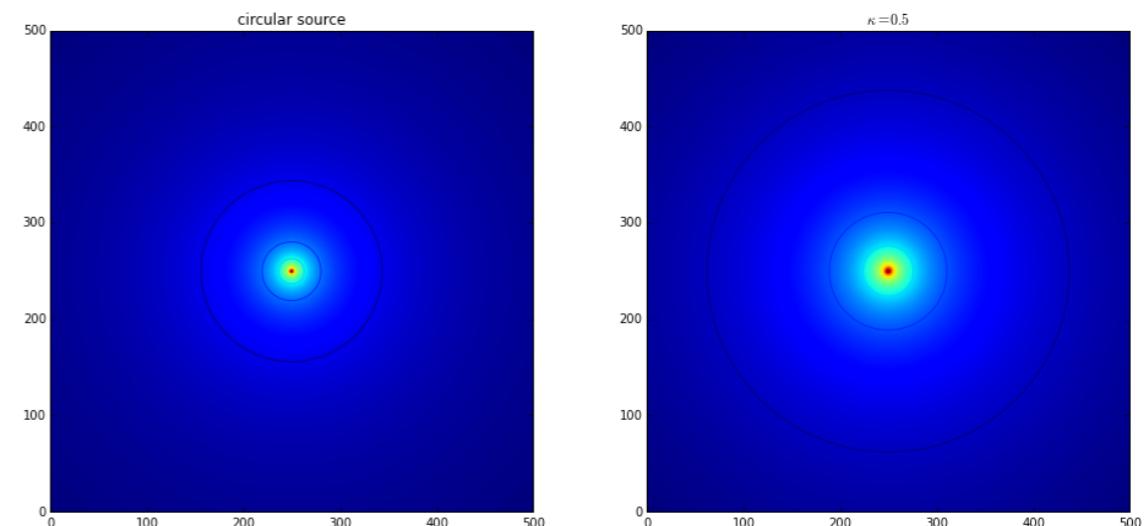
ON THE SPIN-2 NATURE OF SHEAR: QUIZ

- Let's consider a circular source
- How is it distorted if we apply a pure convergence transformation?
- Now, assume that $\gamma_1 > 0$ and $\gamma_2 = 0$. How is the image distorted?
- And what if $\gamma_1 < 0$ and $\gamma_2 = 0$?
- Let's set $\gamma_1 = 0$. How is the image distorted if $\gamma_2 > 0$?



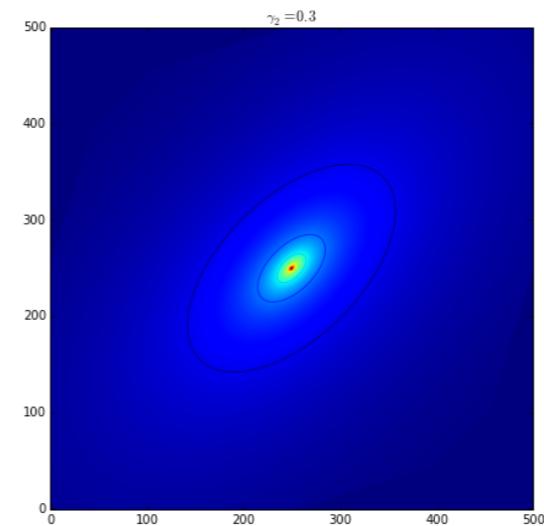
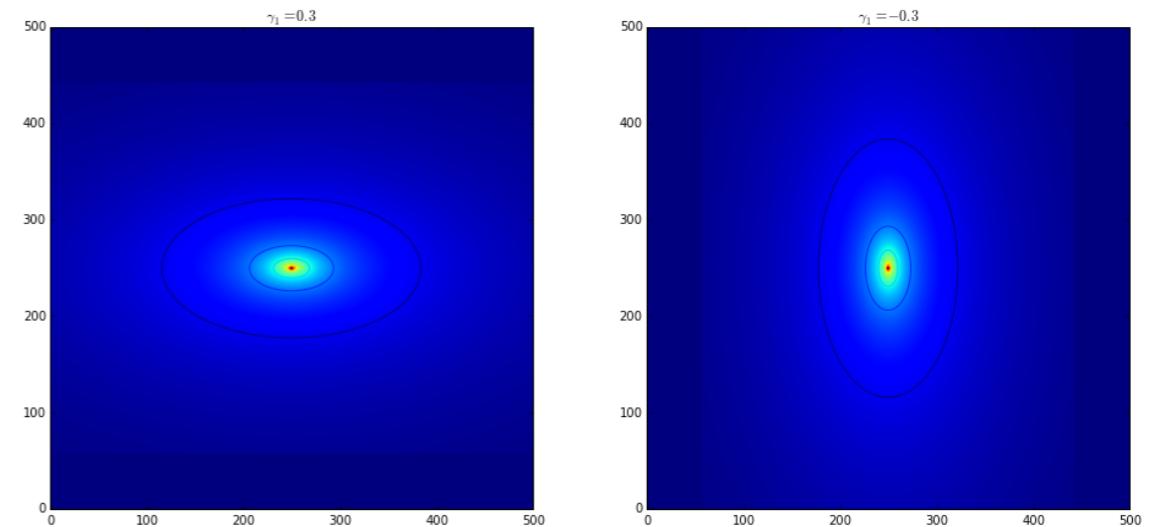
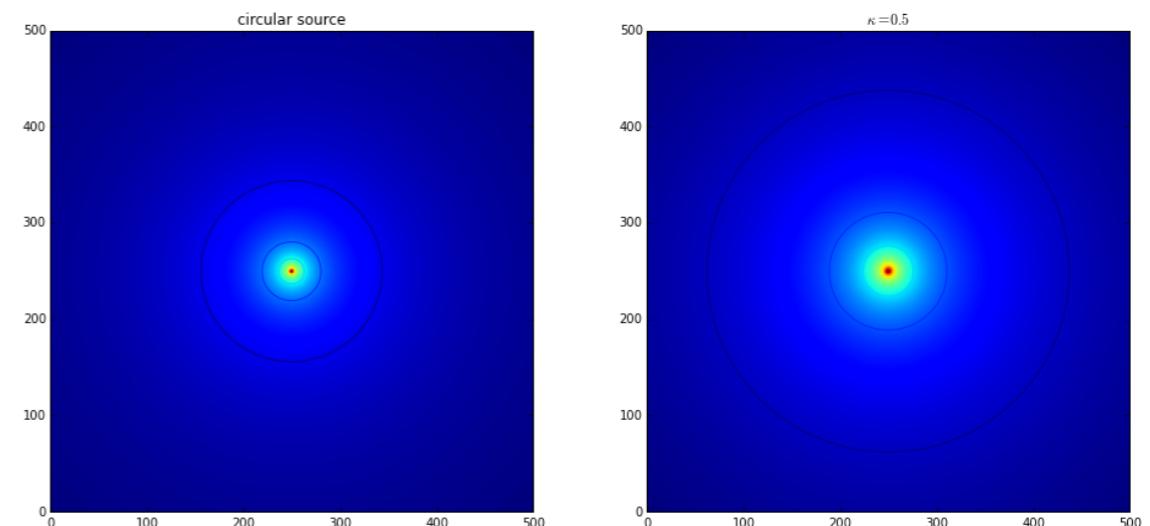
ON THE SPIN-2 NATURE OF SHEAR: QUIZ

- Let's consider a circular source
- How is it distorted if we apply a pure convergence transformation?
- Now, assume that $\gamma_1 > 0$ and $\gamma_2 = 0$. How is the image distorted?
- And what if $\gamma_1 < 0$ and $\gamma_2 = 0$?
- Let's set $\gamma_1 = 0$. How is the image distorted if $\gamma_2 > 0$?



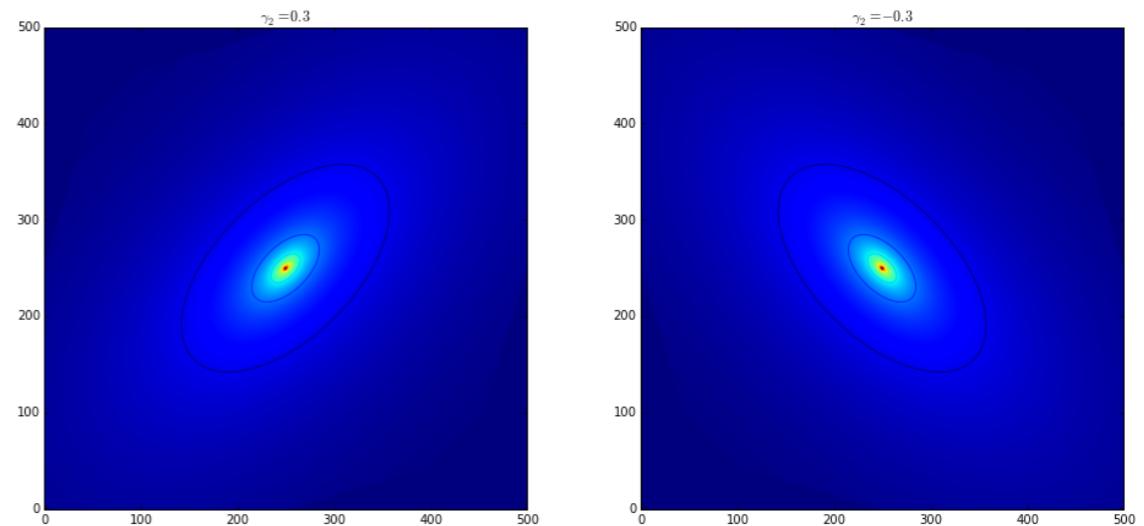
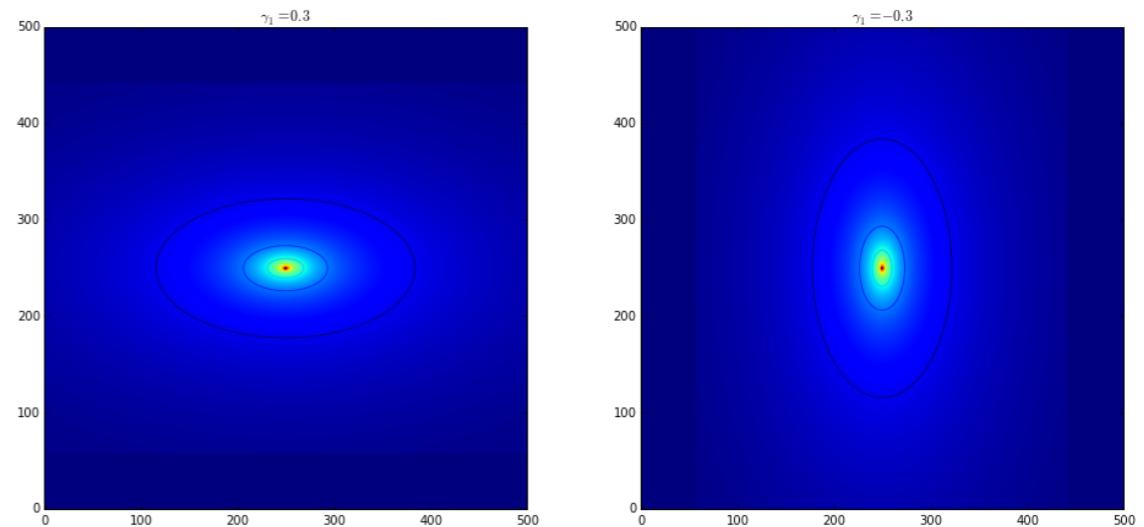
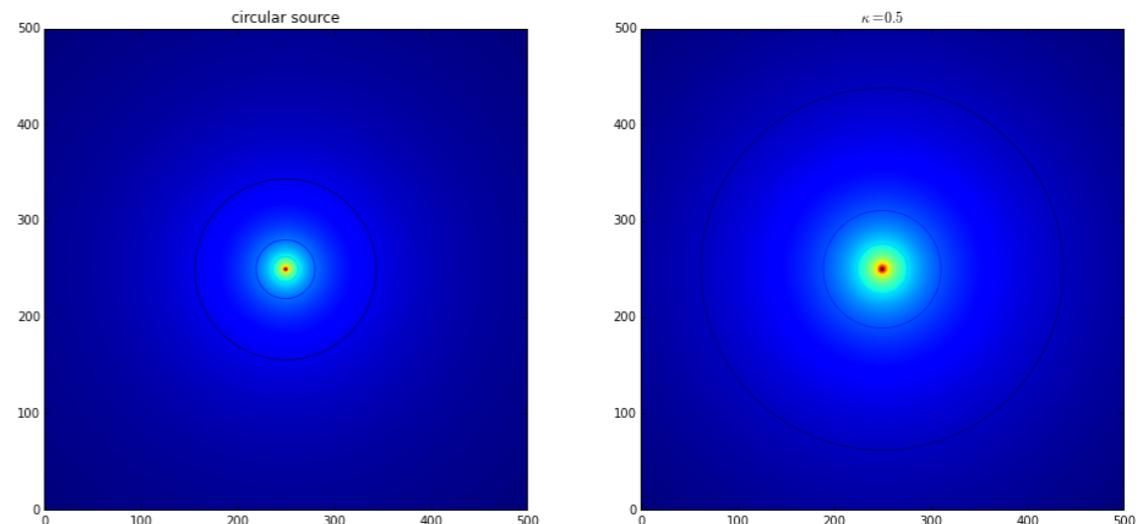
ON THE SPIN-2 NATURE OF SHEAR: QUIZ

- Let's consider a circular source
- How is it distorted if we apply a pure convergence transformation?
- Now, assume that $\gamma_1 > 0$ and $\gamma_2 = 0$. How is the image distorted?
- And what if $\gamma_1 < 0$ and $\gamma_2 = 0$?
- Let's set $\gamma_1 = 0$. How is the image distorted if $\gamma_2 > 0$?
- And if $\gamma_2 < 0$?

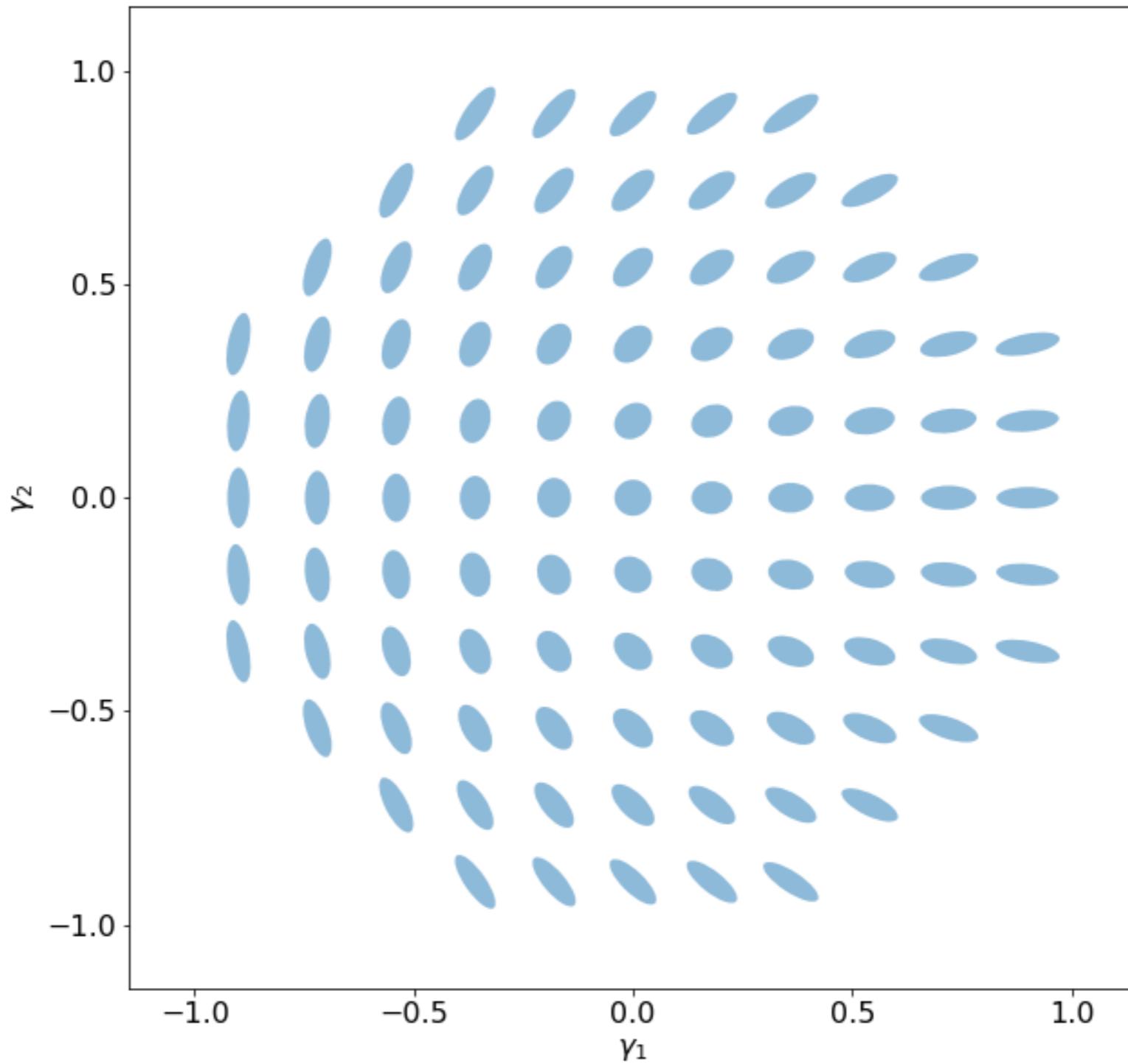


ON THE SPIN-2 NATURE OF SHEAR: QUIZ

- Let's consider a circular source
- How is it distorted if we apply a pure convergence transformation?
- Now, assume that $\gamma_1 > 0$ and $\gamma_2 = 0$. How is the image distorted?
- And what if $\gamma_1 < 0$ and $\gamma_2 = 0$?
- Let's set $\gamma_1 = 0$. How is the image distorted if $\gamma_2 > 0$?
- And if $\gamma_2 < 0$?



SHEAR DISTORTIONS



DEPENDENCE ON REDSHIFT

We have seen that the lensing potential, the deflection angle, the convergence, the shear... depend on a combination of distances.

For example, the lensing potential is:

$$\hat{\Psi}(\vec{\theta}) = \frac{D_{LS}}{D_L D_S} \frac{2}{c^2} \int \Phi(D_L \vec{\theta}, z) dz$$

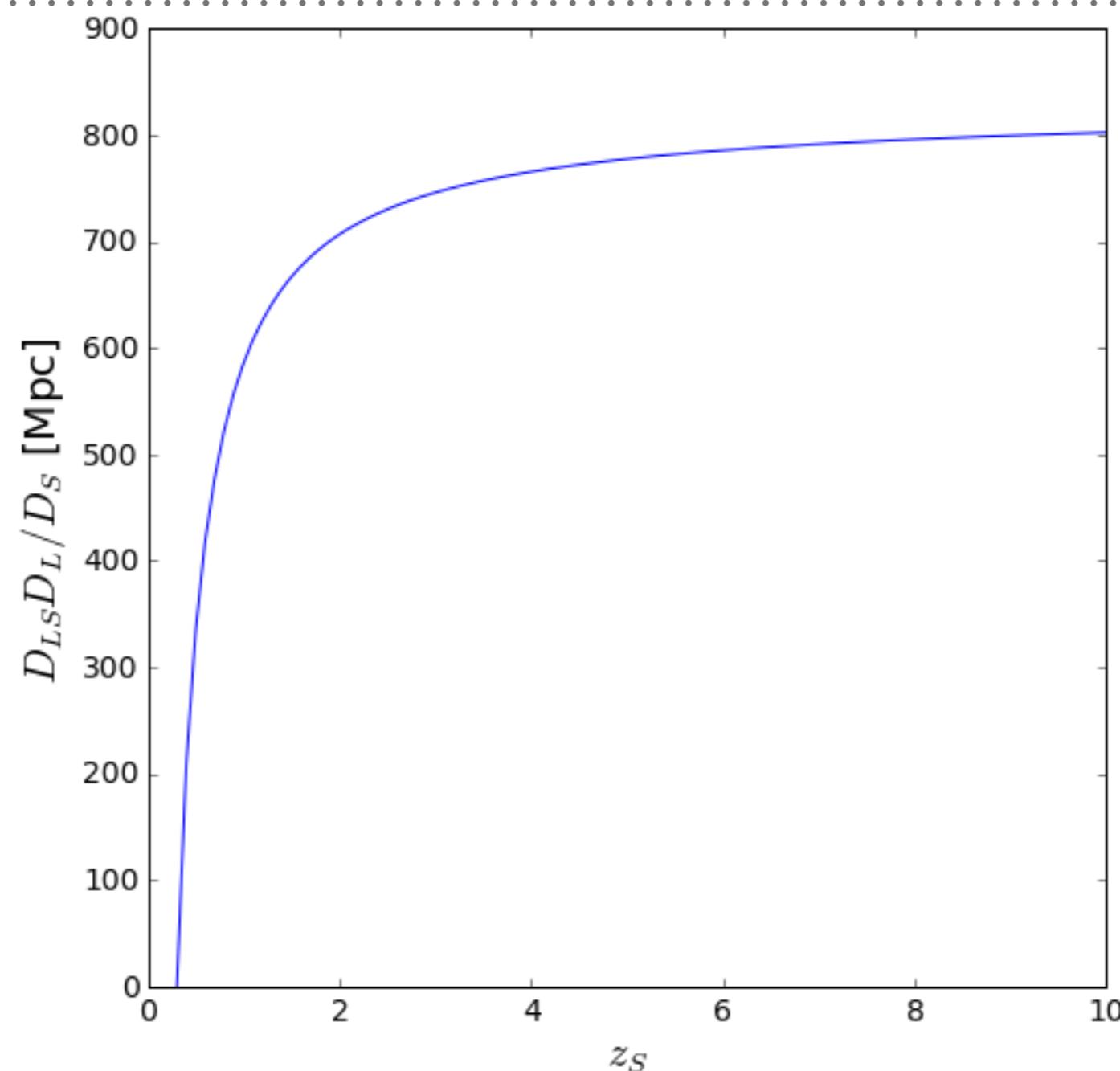
Every spatial derivative of Ψ introduces a factor D_L .

The distance ratio $D_{LS} D_L / D_S$ is called “lensing distance”.

Both the shear and the convergence, being second derivatives of the lensing potential, scale as the lensing distance

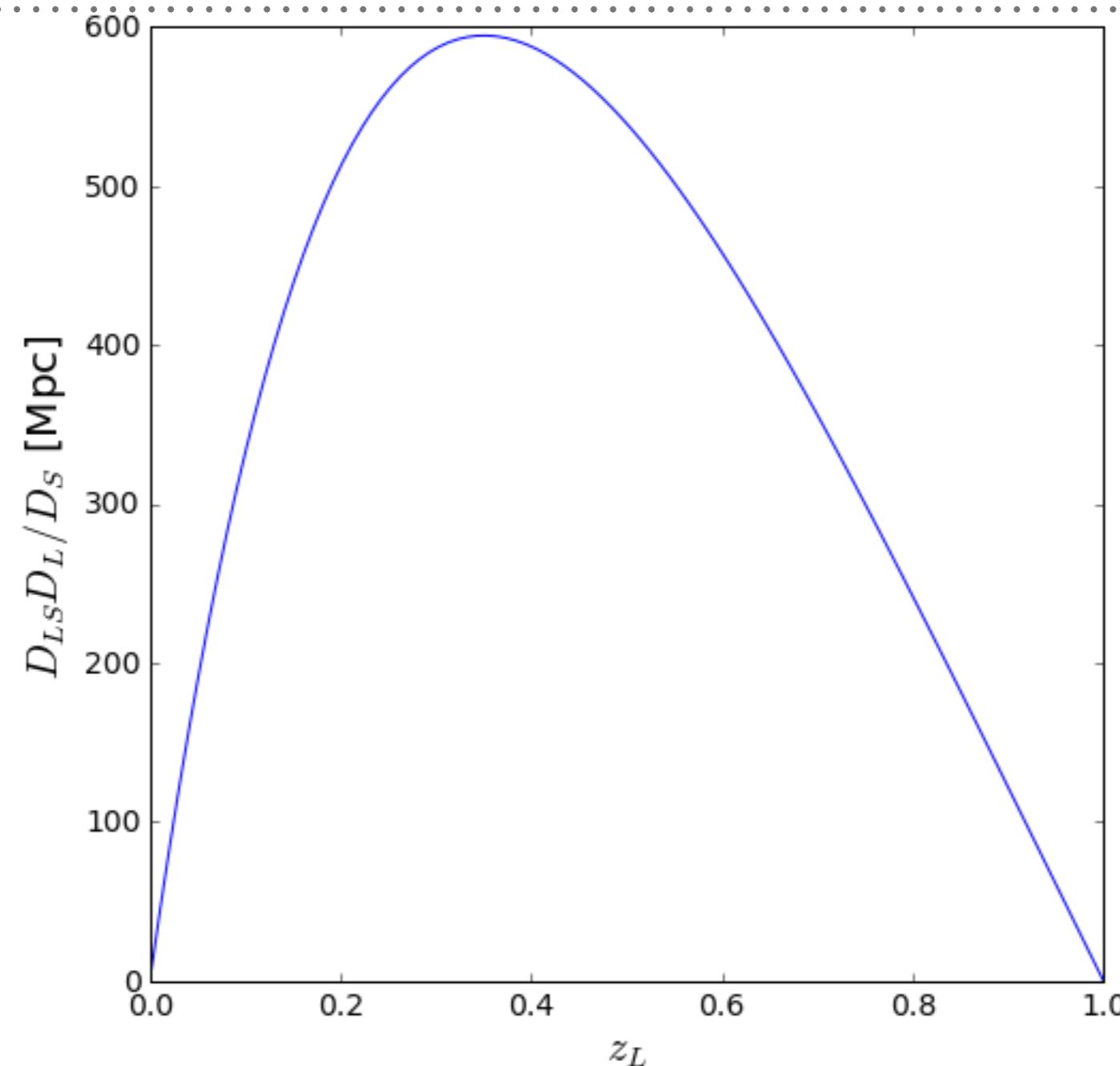
$$\Sigma_{cr} = \frac{c^2}{4\pi G} \frac{D_S}{D_{LS} D_L}$$

HOW DOES THE LENSING DISTANCE SCALE WITH SOURCE REDSHIFT?



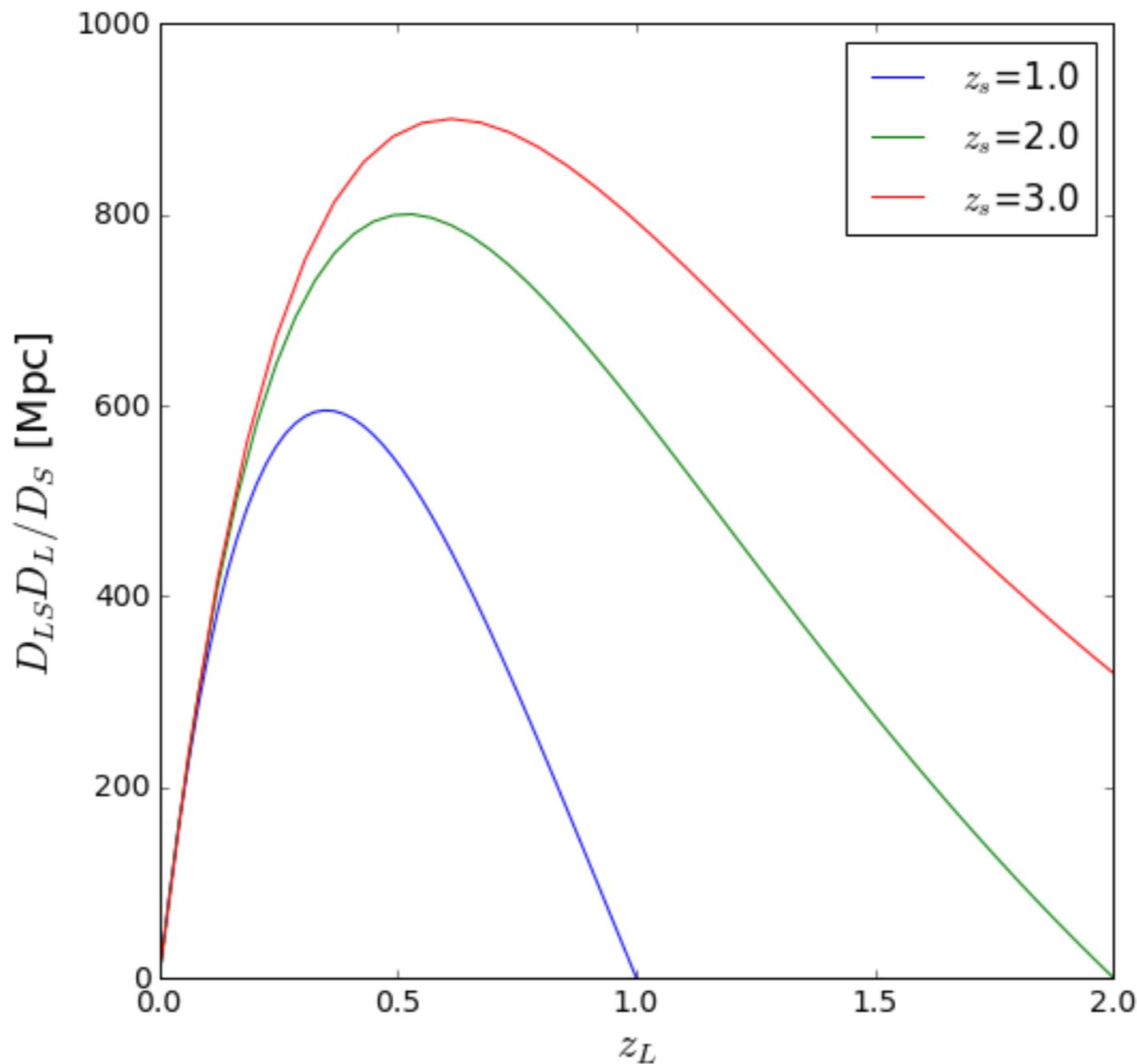
Note that if the lensing distance grows, the critical surface density decreases, the convergence and the shear grow!

HOW DOES THE LENSING DISTANCE SCALE WITH LENS REDSHIFT?



The lensing distance peaks at \sim half way between the source and the observer, meaning that there is an optimal distance where the lens produces its largest effects.

HOW DOES THE LENSING DISTANCE SCALE WITH LENS REDSHIFT?



Of course, the peak moves to larger distances as the distance to the source increases.

CONSERVATION OF SURFACE BRIGHTNESS

*The source surface
brightness is*

$$I_\nu = \frac{dE}{dtdAd\Omega d\nu}$$

In phase space, the radiation emitted is characterized by the density

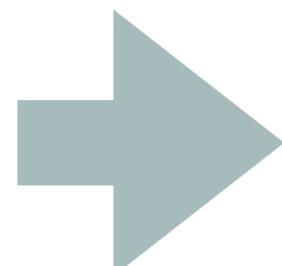
$$f(\vec{x}, \vec{p}, t) = \frac{dN}{d^3x d^3p}$$

In absence of photon creations or absorptions, f is conserved (Liouville theorem)

$$dN = \frac{dE}{h\nu} = \frac{dE}{cp}$$

$$d^3x = cdtdA$$

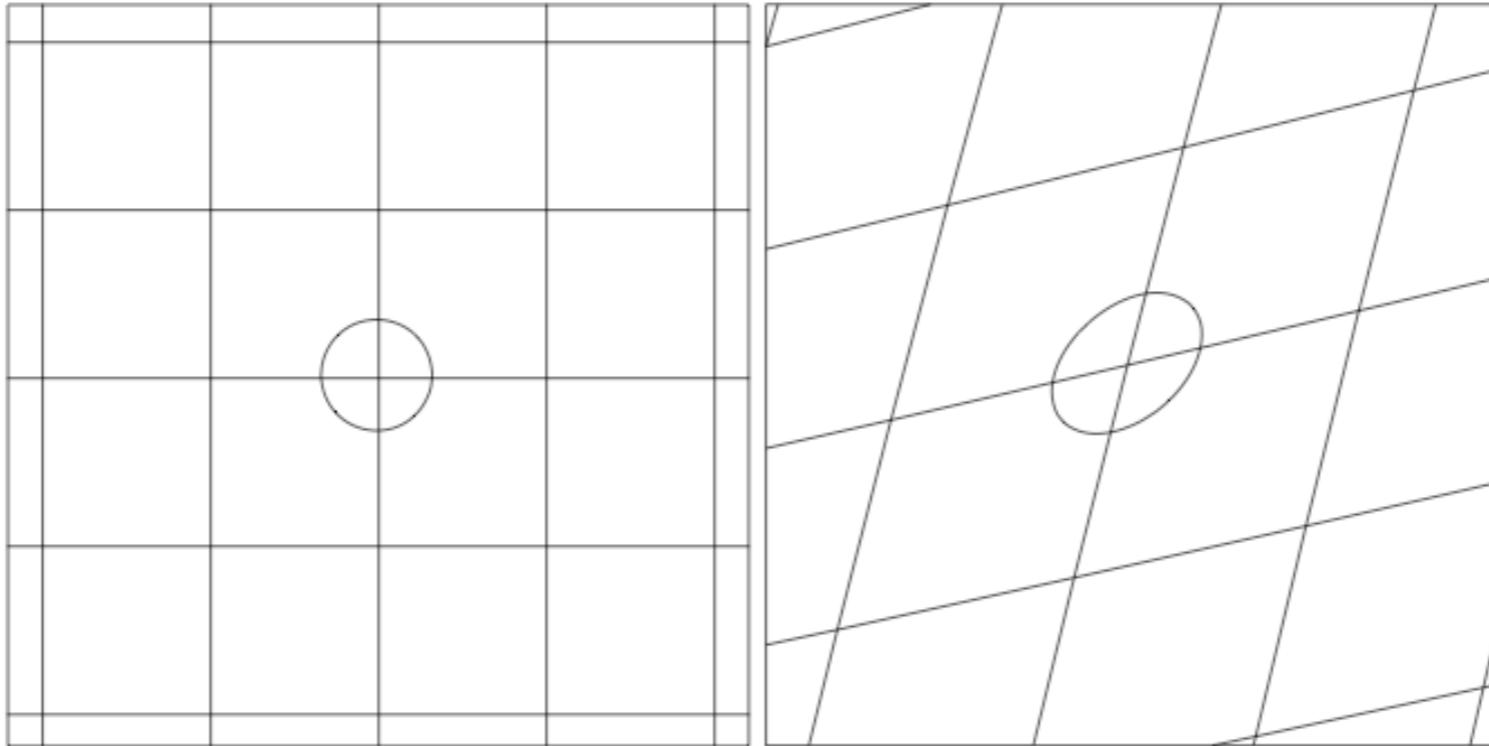
$$d^3\vec{p} = p^2 dp d\Omega$$



$$f(\vec{x}, \vec{p}, t) = \frac{dN}{d^3x d^3p} = \frac{dE}{hcp^3 dAdtd\nu d\Omega} = \frac{I_\nu}{hcp^3}$$

Since GL does not involve creation or absorption of photons, neither it changes the photon momenta (achromatic!), surface brightness is conserved!

MAGNIFICATION

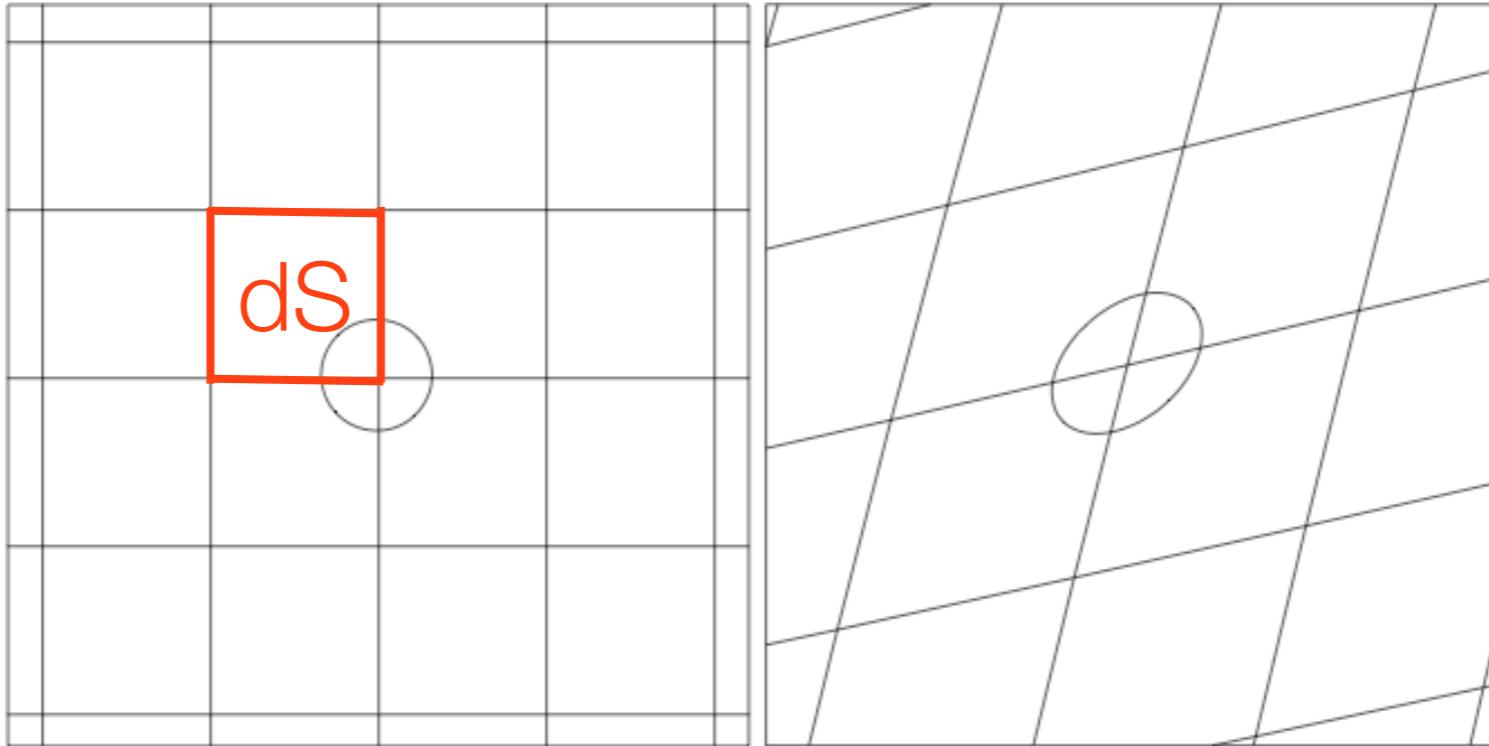


Kneib & Natarajan (2012)

$$F_\nu = \int_I I_\nu(\vec{\theta}) d^2\theta = \int_S I_\nu^S[\vec{\beta}(\vec{\theta})] \mu d^2\beta$$

Lensing changes the amount of photons (flux) we receive from the source by changing the solid angle the source subtends

MAGNIFICATION

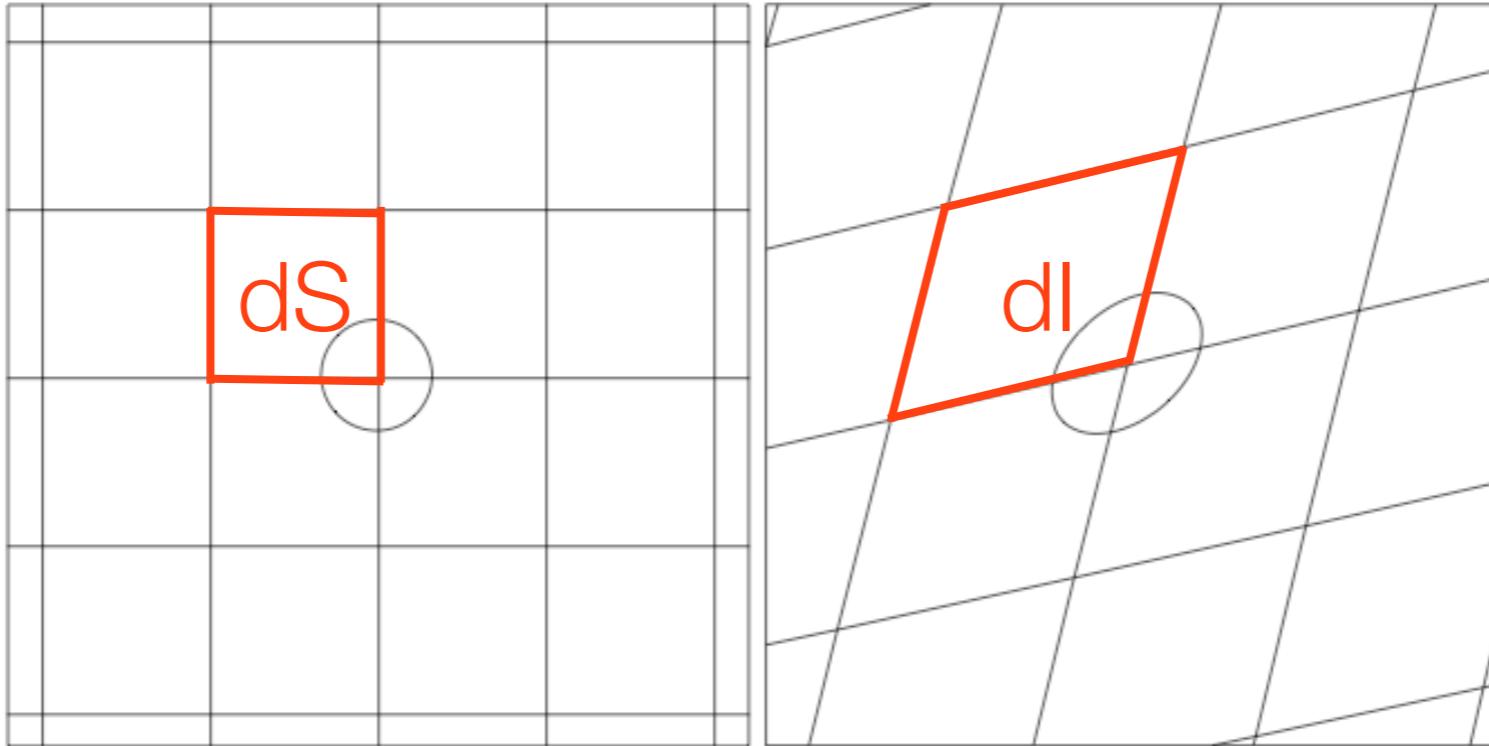


Kneib & Natarajan (2012)

$$F_\nu = \int_I I_\nu(\vec{\theta}) d^2\theta = \int_S I_\nu^S[\vec{\beta}(\vec{\theta})] \mu d^2\beta$$

Lensing changes the amount of photons (flux) we receive from the source by changing the solid angle the source subtends

MAGNIFICATION

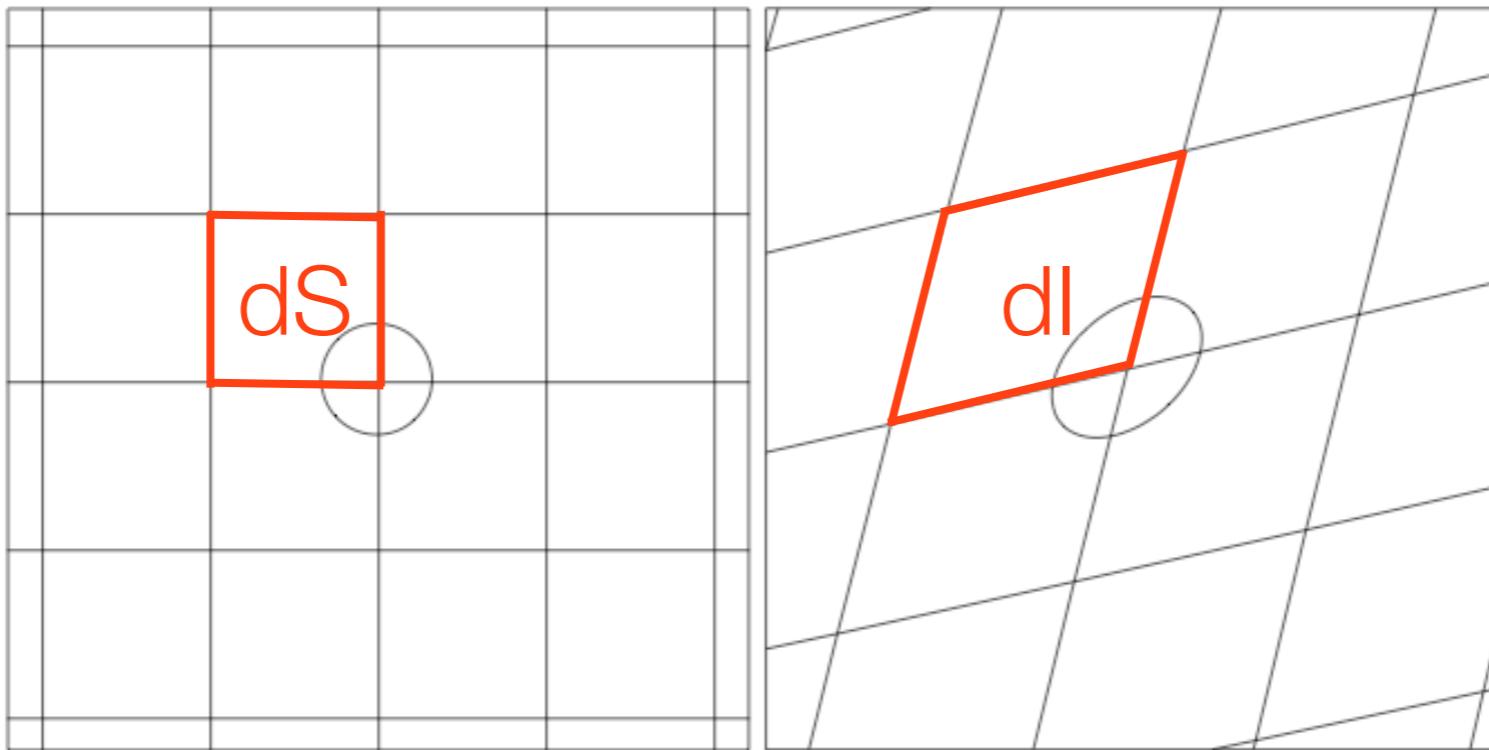


Kneib & Natarajan (2012)

$$F_\nu = \int_I I_\nu(\vec{\theta}) d^2\theta = \int_S I_\nu^S[\vec{\beta}(\vec{\theta})] \mu d^2\beta$$

Lensing changes the amount of photons (flux) we receive from the source by changing the solid angle the source subtends

MAGNIFICATION



Kneib & Natarajan (2012)

$$\mu = \frac{dI}{dS} = \frac{\delta\theta^2}{\delta\beta^2} = \det A^{-1}$$

$$F_\nu = \int_I I_\nu(\vec{\theta}) d^2\theta = \int_S I_\nu^S[\vec{\beta}(\vec{\theta})] \mu d^2\beta$$

Lensing changes the amount of photons (flux) we receive from the source by changing the solid angle the source subtends

CRITICAL LINES AND CAUSTICS

Both convergence and shear are functions of position on the lens plane:

$$\kappa = \kappa(\vec{\theta})$$

$$\gamma = \gamma(\vec{\theta})$$

The determinant of the lensing Jacobian is

$$\det A = (1 - \kappa - \gamma)(1 - \kappa + \gamma) = \mu^{-1}$$

The critical lines are the lines where the eigenvalues of the Jacobian are zero:

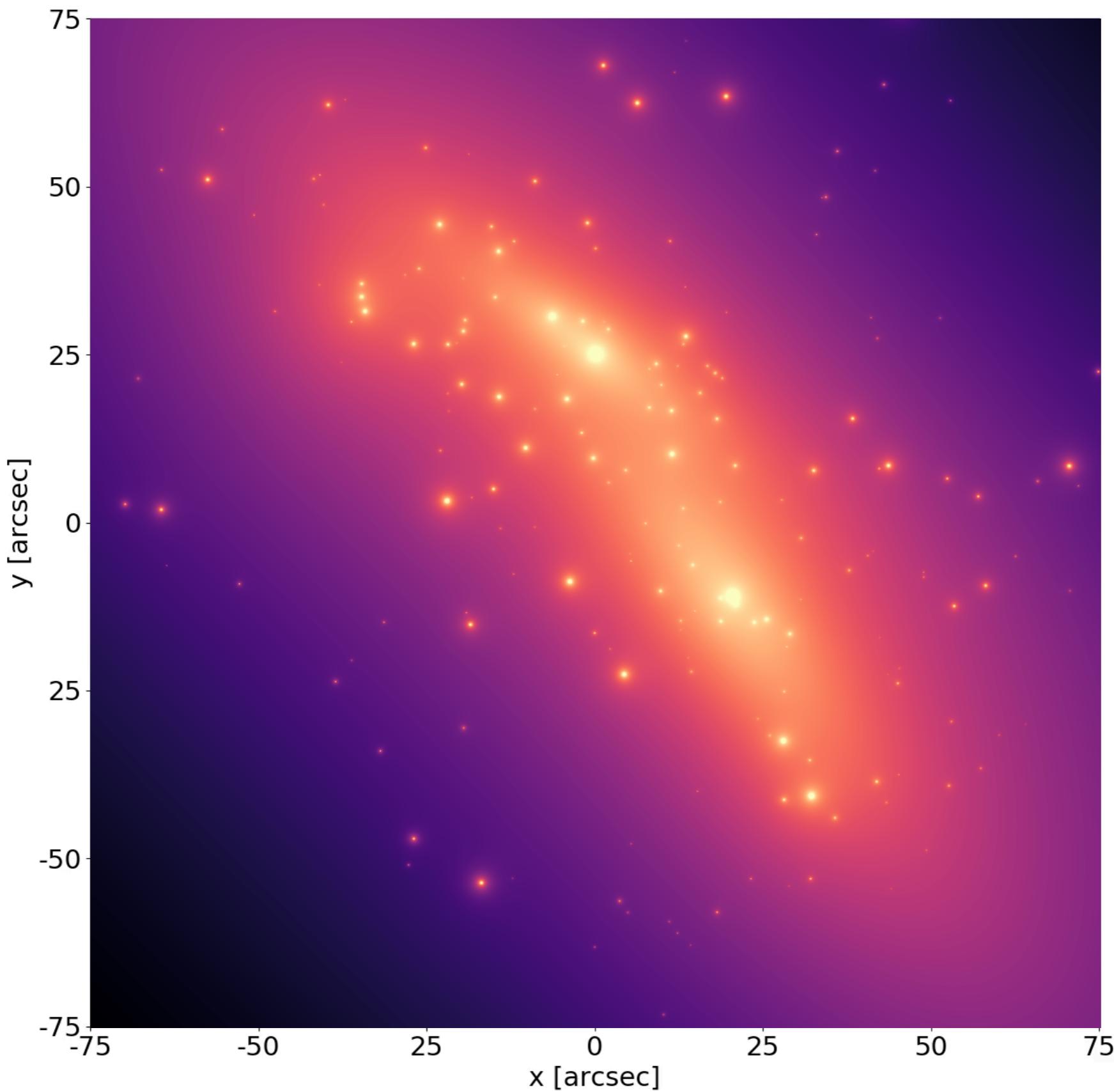
$$(1 - \kappa - \gamma) = 0 \quad \text{tangential critical line}$$

$$(1 - \kappa + \gamma) = 0 \quad \text{radial critical line}$$

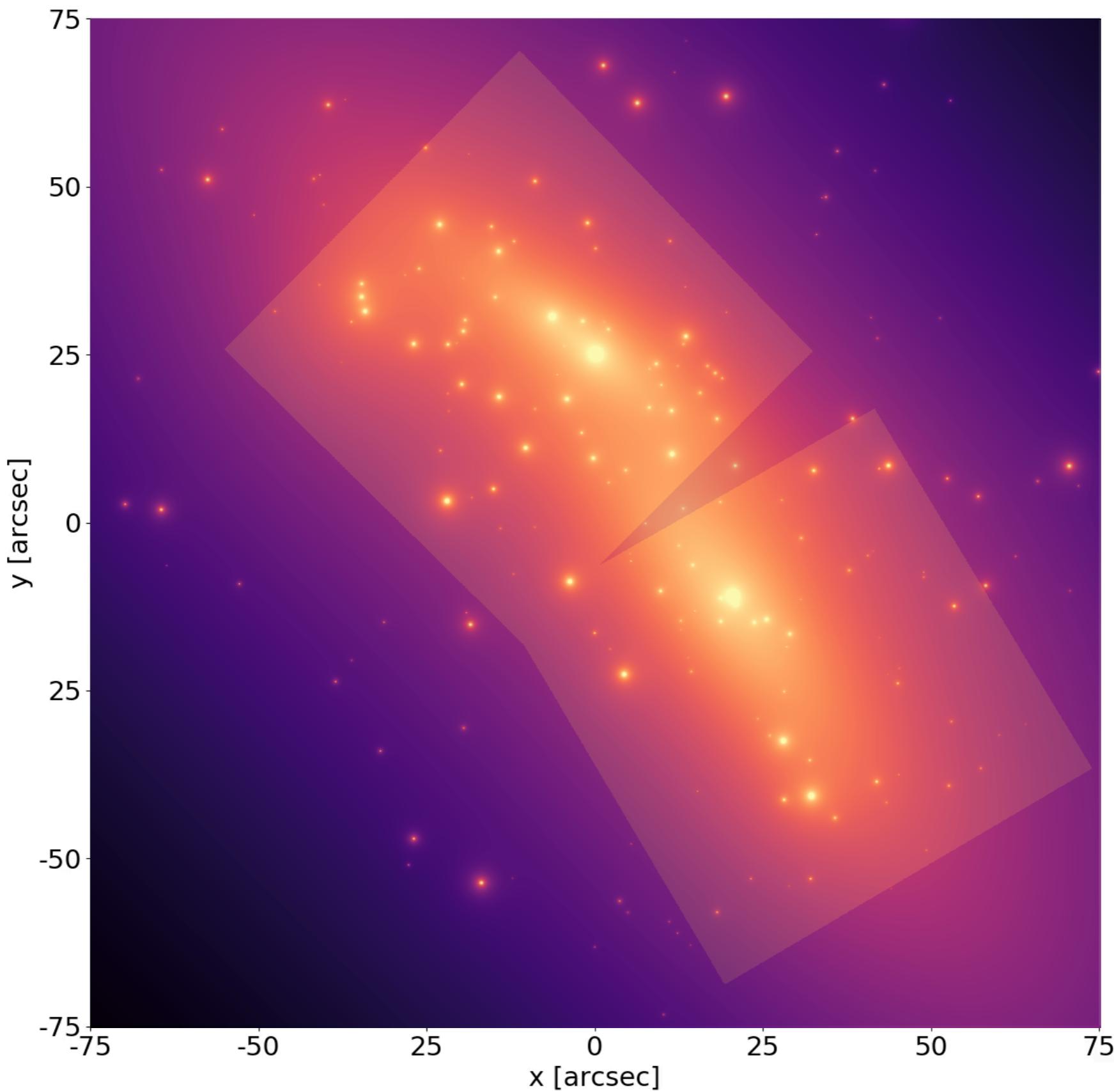
Along these lines the magnification diverges!

Via the lens equations, they are mapped into the caustics...

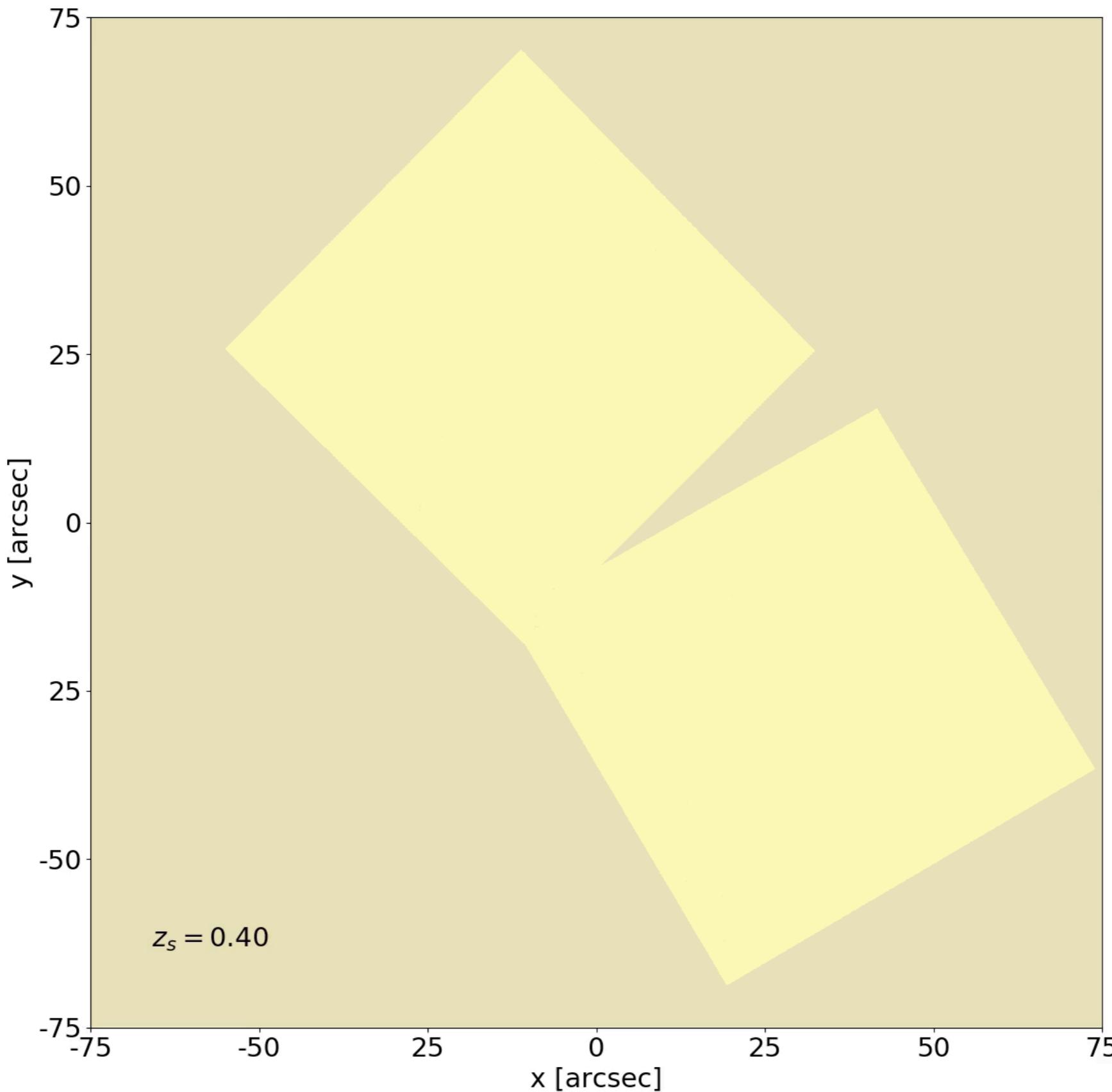
Model of MACS0416 by Caminha, MM, et al. (2016)



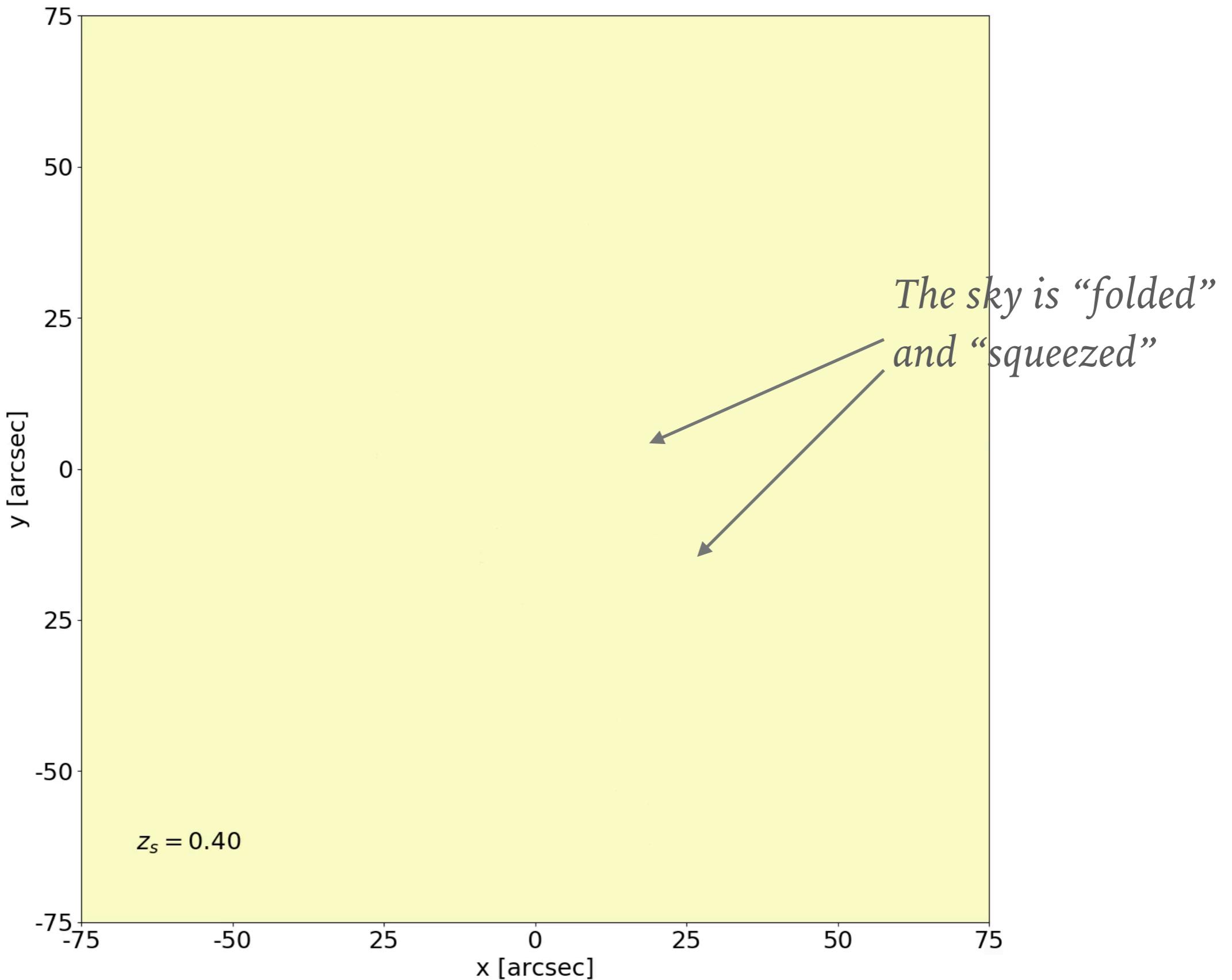
Model of MACS0416 by Caminha, MM, et al. (2016)

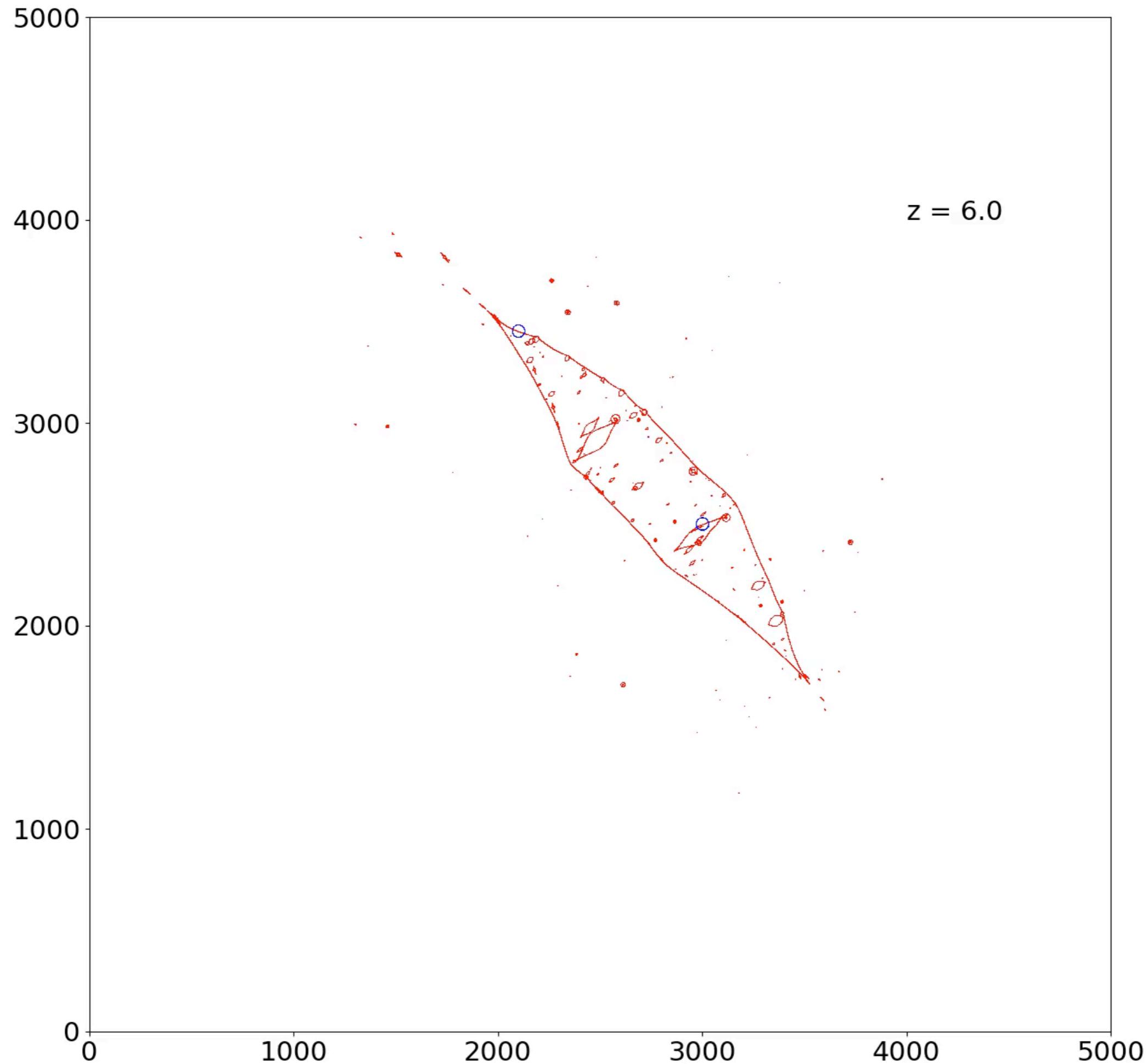


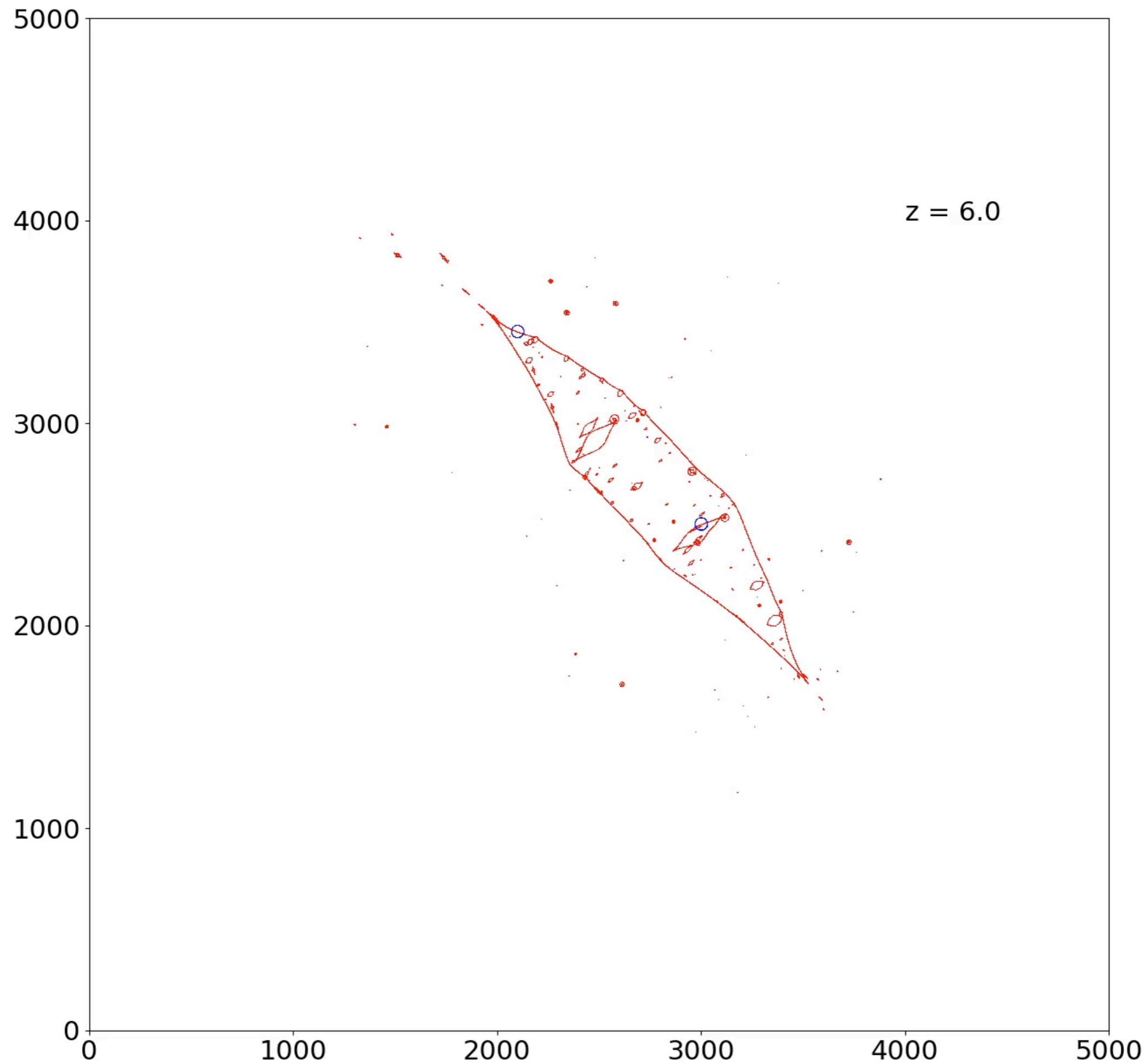
Model of MACS0416 by Caminha, MM, et al. (2016)

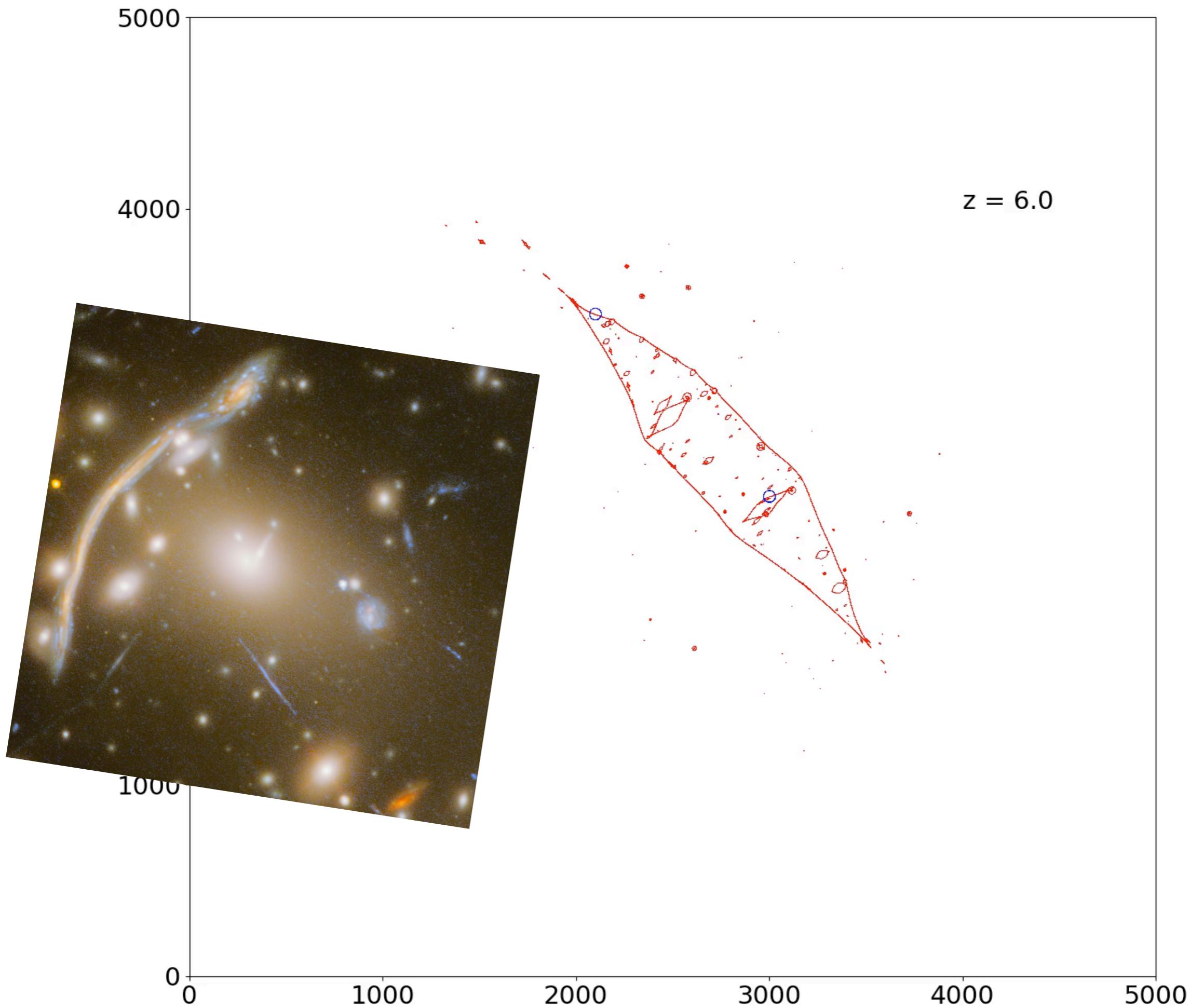


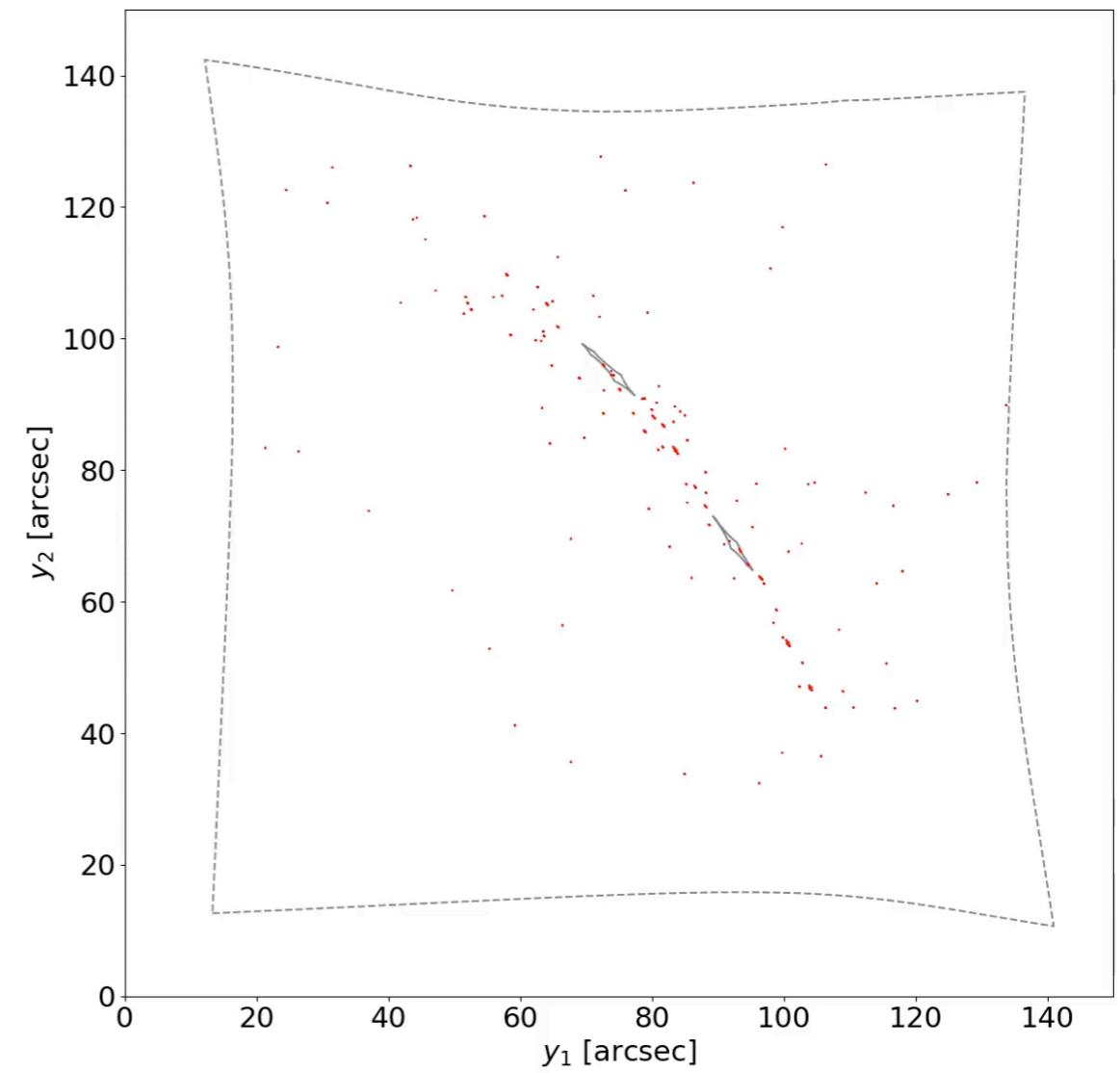
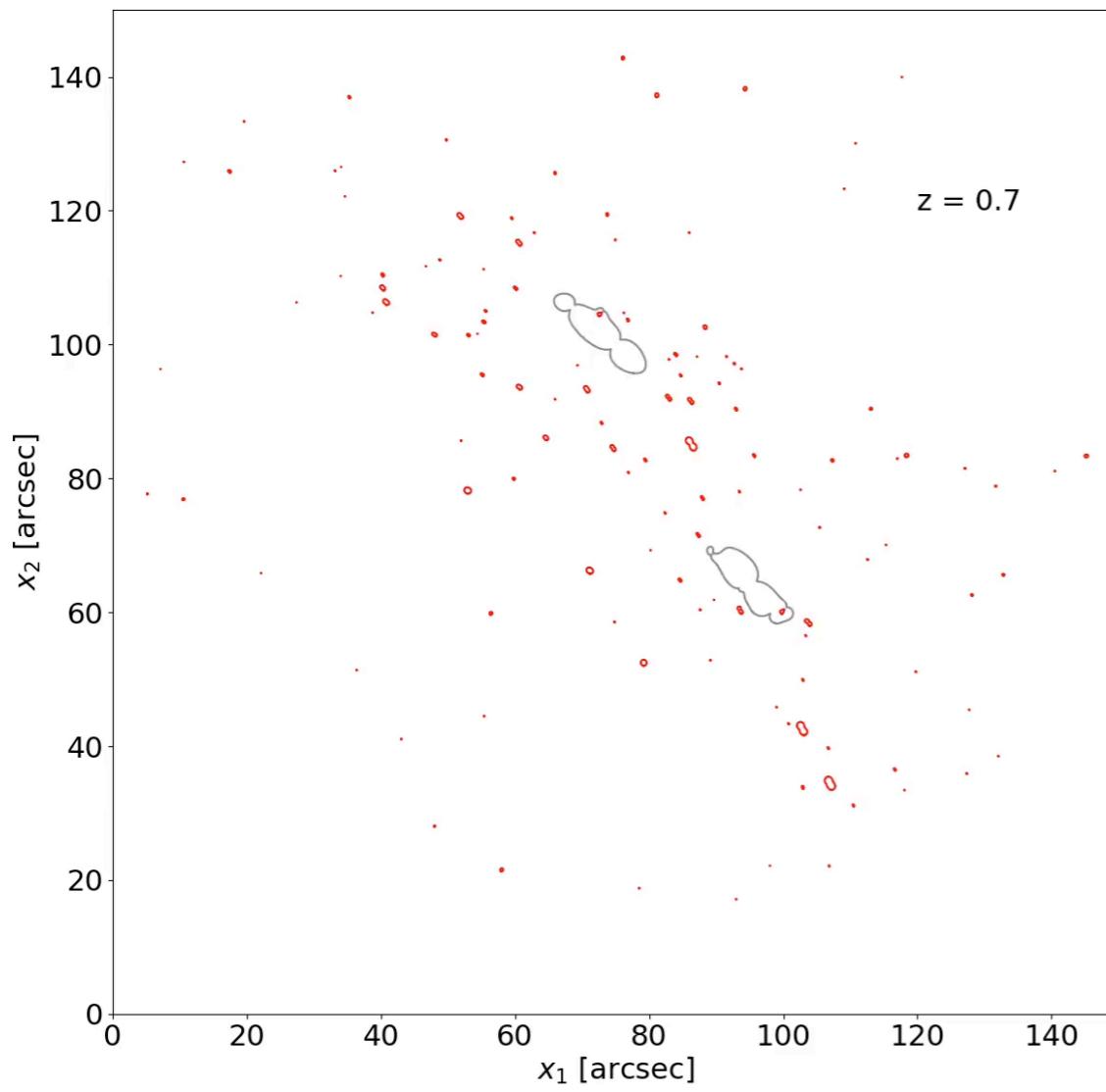
Model of MACS0416 by Caminha, MM, et al. (2016)





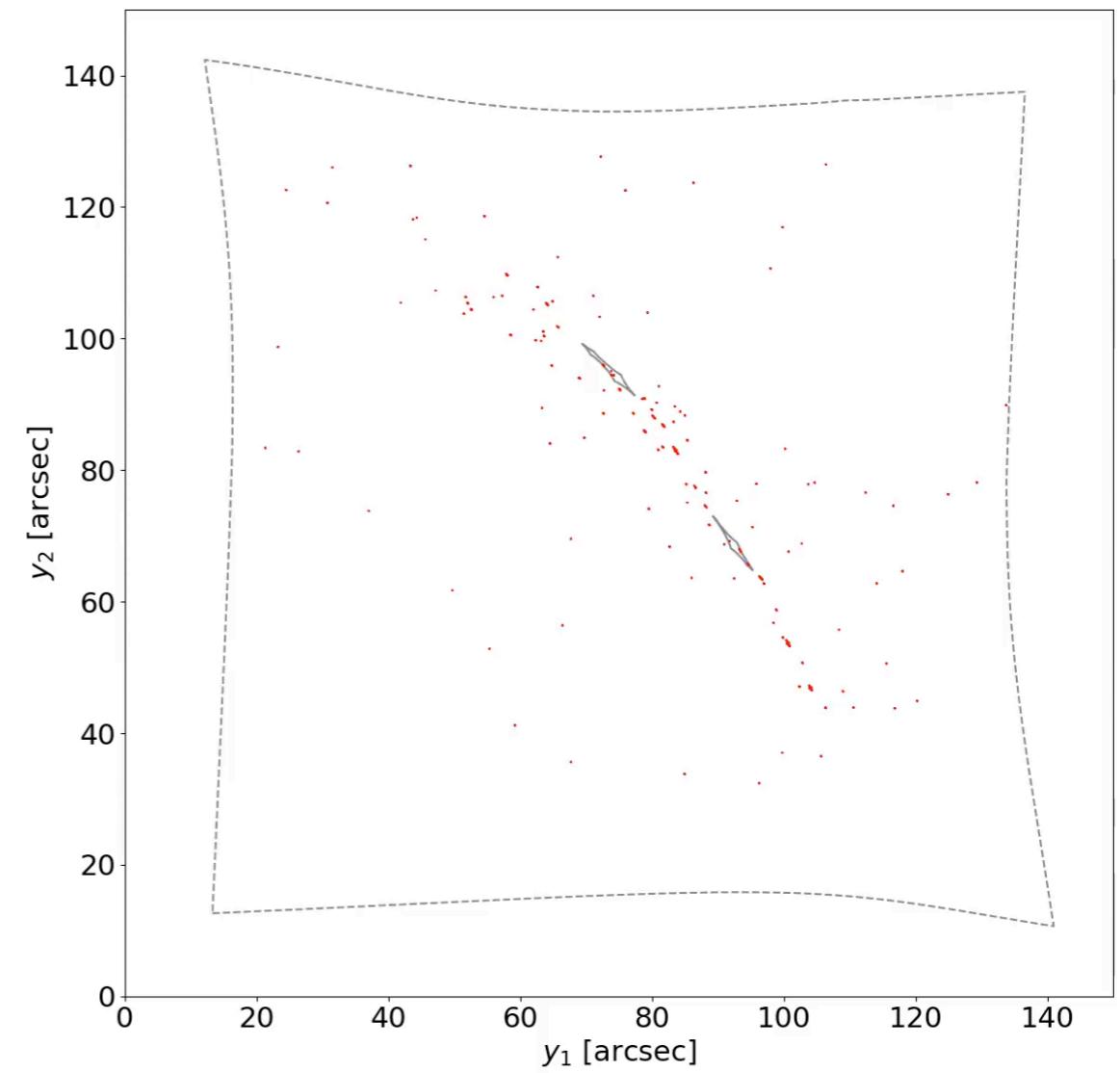
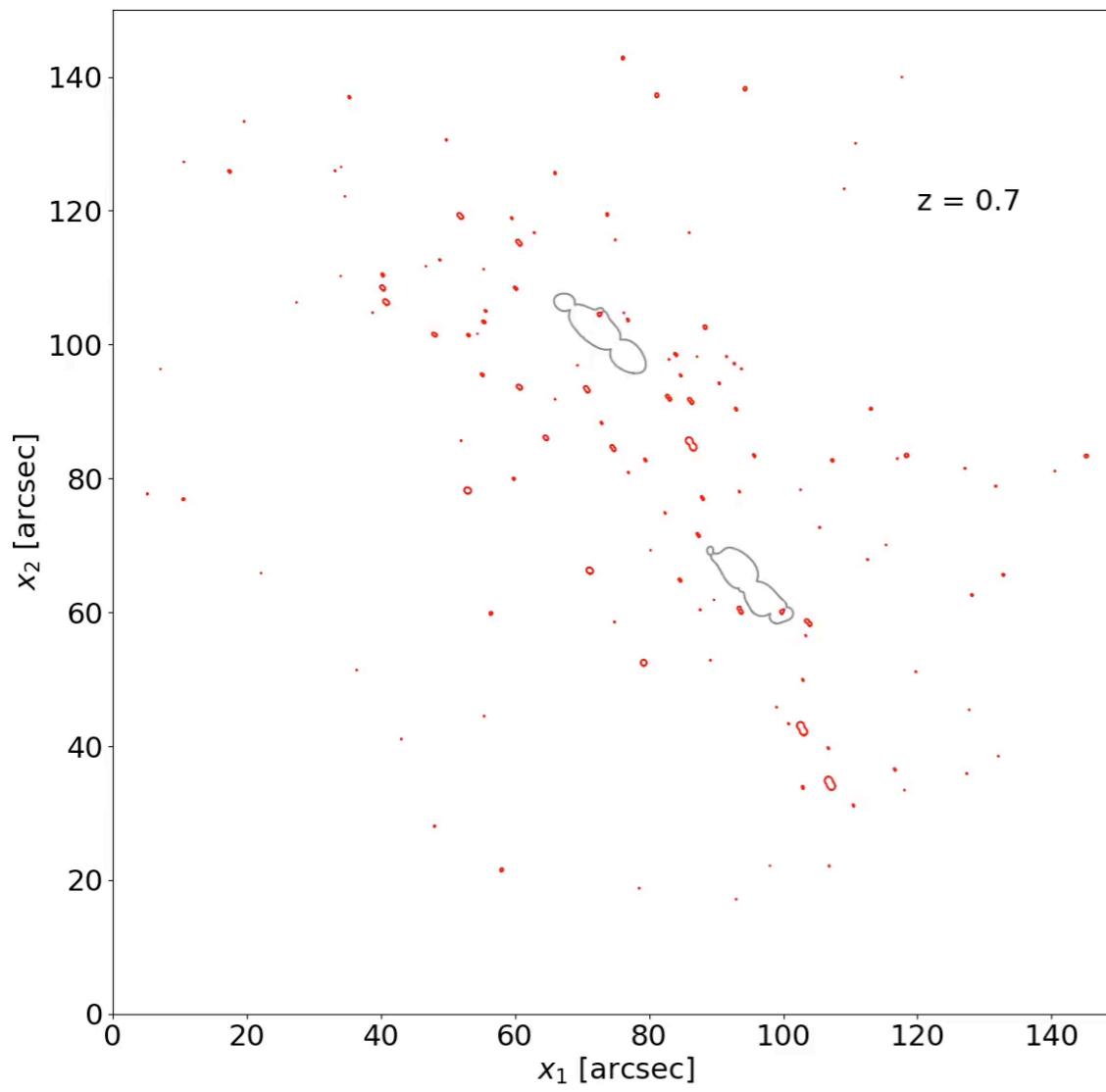






The size of the critical lines and caustics changes as a function of the source redshift! This is because the deflection angle between two redshifts changes by a factor

$$\Xi(z_d, z_{s_1}, z_{s_2}) = \frac{D_{ds_1} D_{s_2}}{D_{s_1} D_{ds_2}}$$



The size of the critical lines and caustics changes as a function of the source redshift! This is because the deflection angle between two redshifts changes by a factor

$$\Xi(z_d, z_{s_1}, z_{s_2}) = \frac{D_{ds_1} D_{s_2}}{D_{s_1} D_{ds_2}}$$

SAMPLED VOLUME

