

GRAVITATIONAL LENSING

8 - TIME DELAYS

Massimo Meneghetti
AA 2018-2019

TOTAL TIME DELAY

$$t_{tot}(\vec{\theta}) = \frac{1+z_L}{c} \frac{D_S D_L}{D_{LS}} \left[\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \hat{\Psi}(\vec{\theta}) \right]$$

$$\tau(\vec{\theta}) = \frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \hat{\Psi}(\vec{\theta})$$

Fermat potential

$$D_{\Delta t} = (1+z_L) \frac{D_S D_L}{D_{LS}}$$

Time delay distance

TIME DELAY SURFACE

$$t(\vec{\theta}) = t_{geom} + t_{grav} \propto \left(\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \hat{\Psi} \right)$$

$$\vec{\nabla} t(\vec{\theta}) \propto \left(\vec{\theta} - \vec{\beta} - \vec{\nabla} \hat{\Psi} \right)$$

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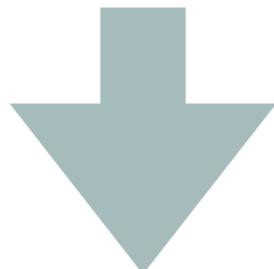
Lens equation!

TIME DELAY SURFACE

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Lens equation!



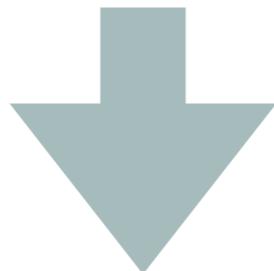
Images form at the stationary points of t !

TIME DELAY SURFACE

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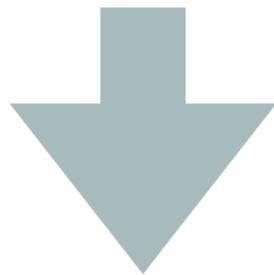
$$T_{ij} = \frac{\partial^2 t(\vec{\theta})}{\partial \theta_i \partial \theta_j} \propto (\delta_{ij} - \Psi_{ij})$$

TIME DELAY SURFACE

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Lens equation!



Images form at the stationary points of t !

$$T_{ij} = \frac{\partial^2 t(\vec{\theta})}{\partial \theta_i \partial \theta_j} \propto (\delta_{ij} - \Psi_{ij}) \quad \textcolor{red}{\text{This is the Jacobian!}}$$

TYPES OF IMAGES

- minima (eigenvalues of A are both positive, hence $\det A > 0$ and $\text{Tr } A > 0$; positive magnification)
- saddle (eigenvalues have opposite signs, thus $\det A < 0$; negative magnification)
- maxima (eigenvalues are both negative, hence $\det A > 0$ and $\text{Tr } A < 0$; positive magnification)
- Let see some examples...

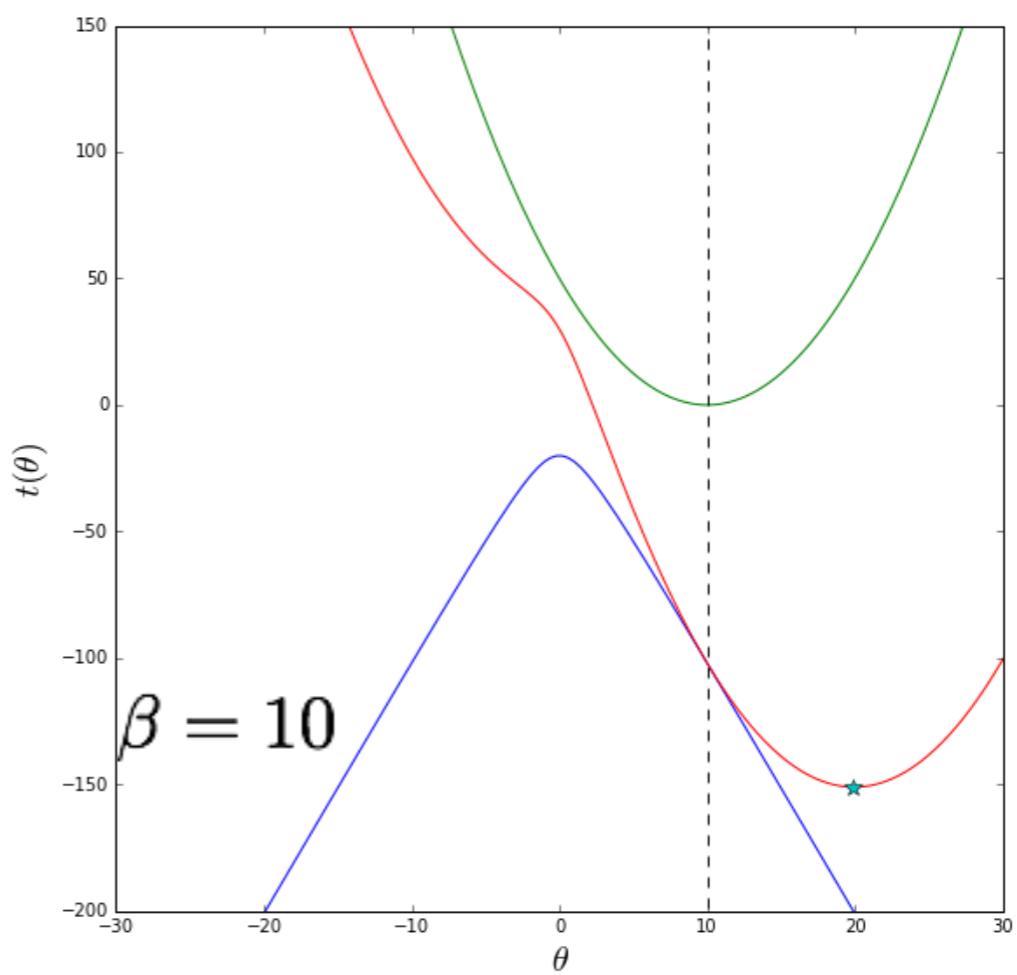
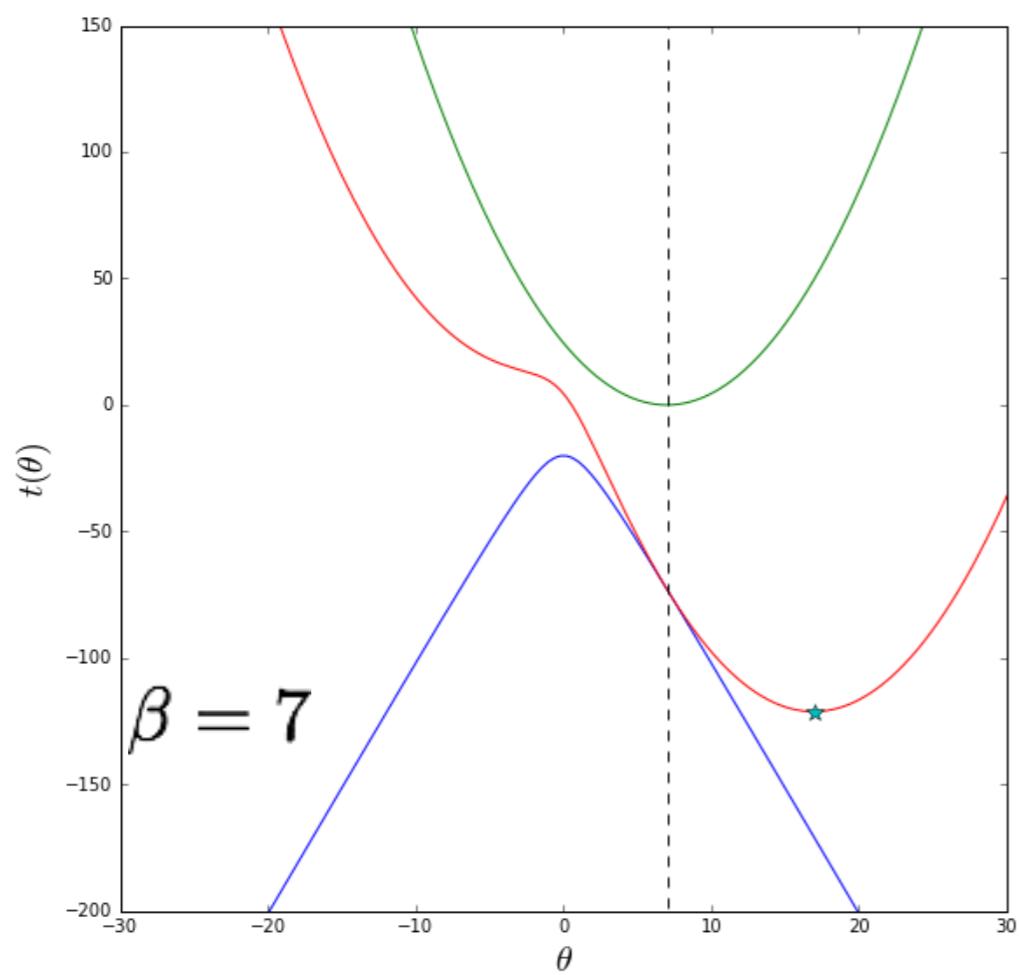
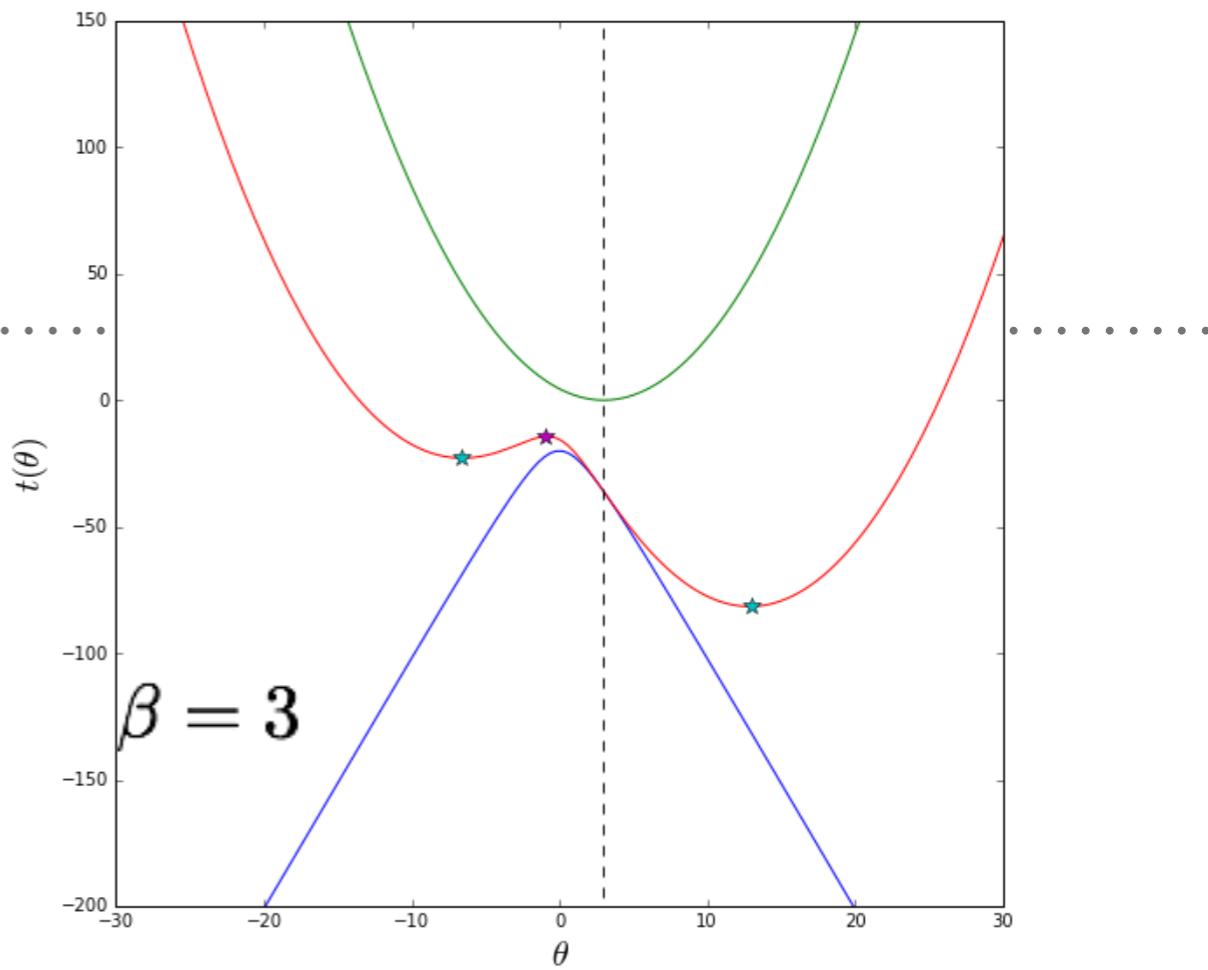
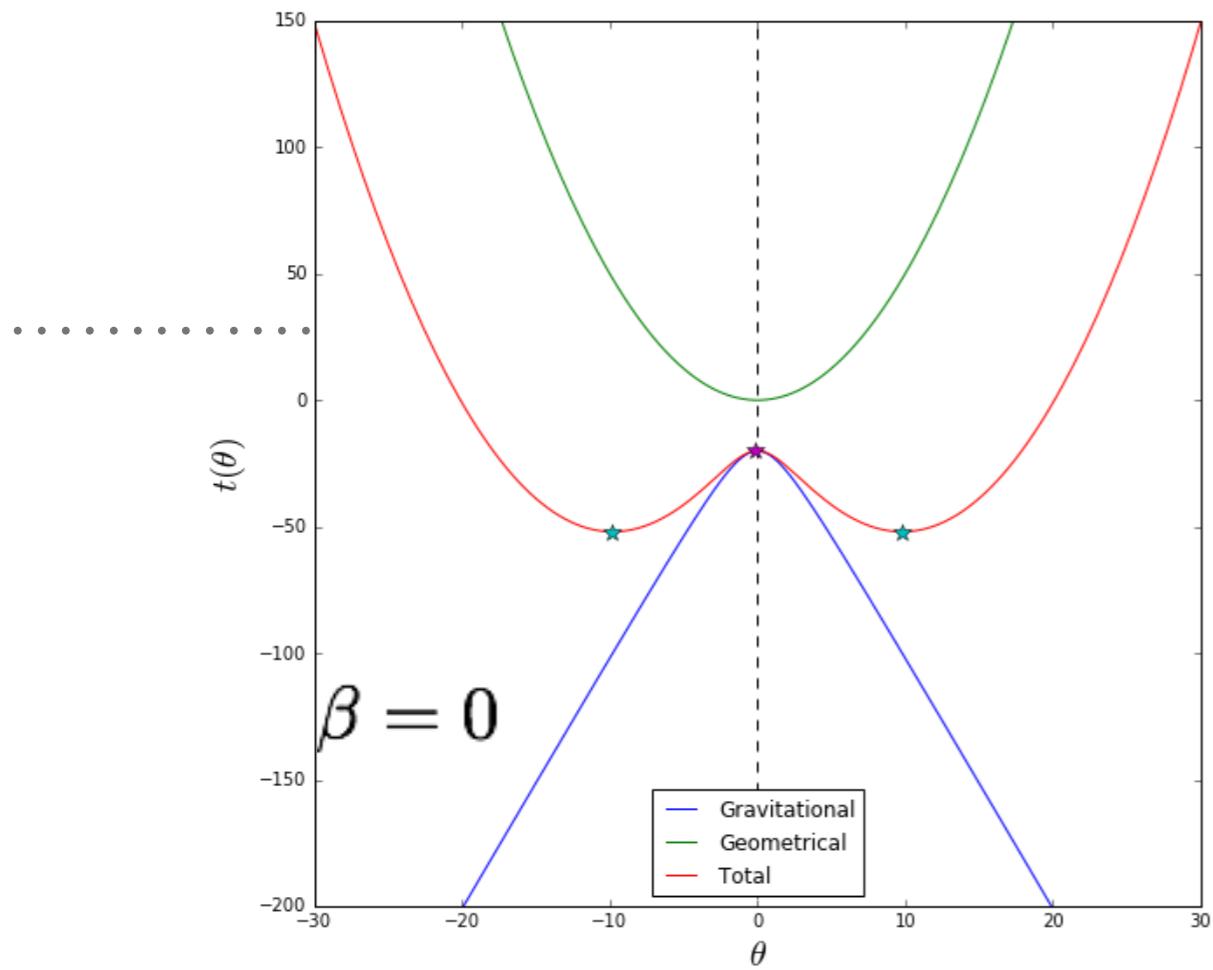
EXAMPLE OF TIME DELAY SURFACE

Toy potential:

$$\hat{\Psi}(\theta) \propto \sqrt{\theta^2 + \theta_c^2}$$

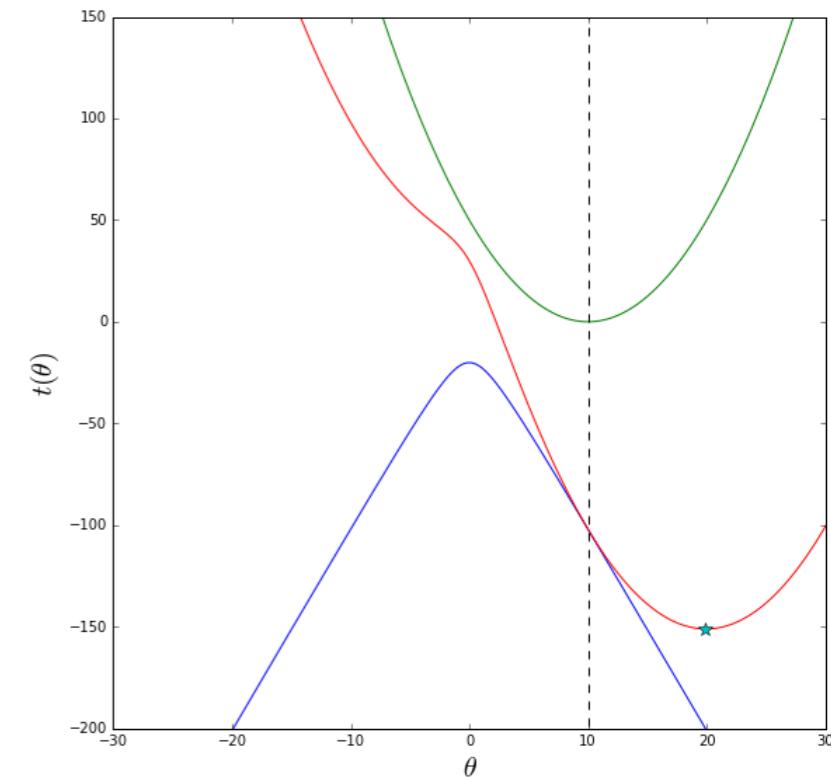
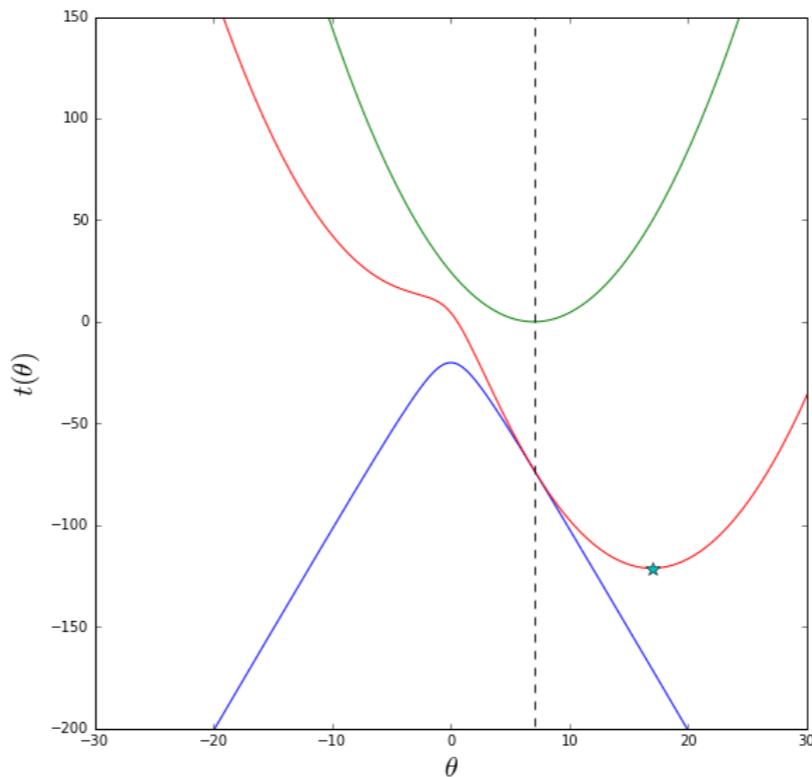
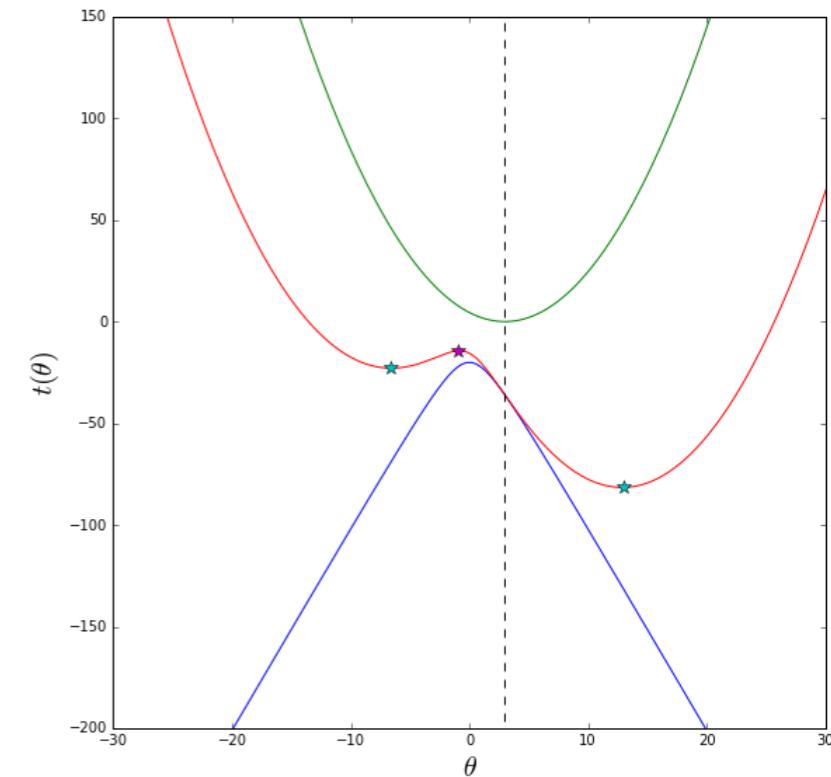
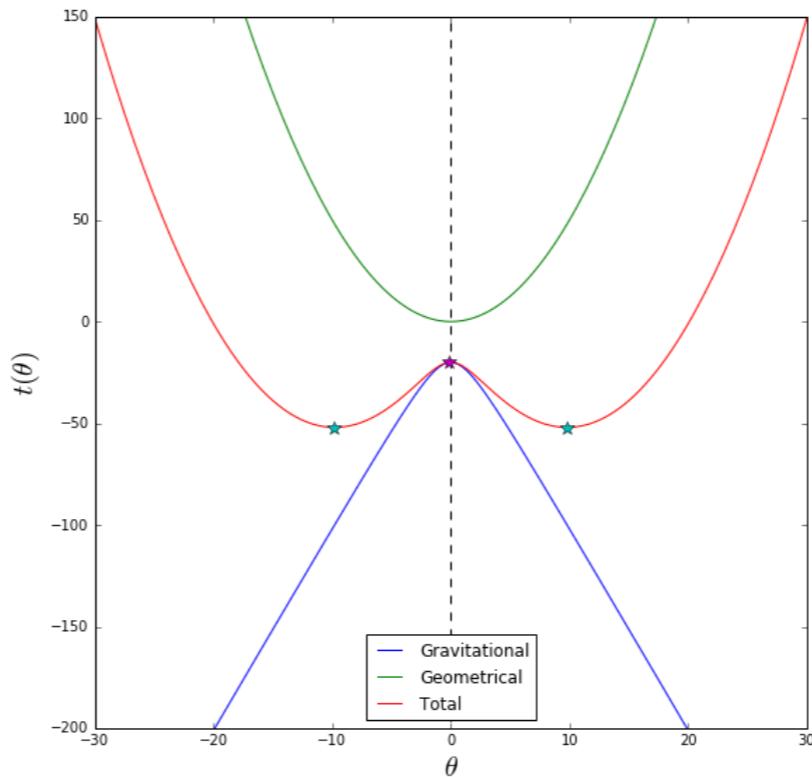
Assuming axial-symmetry, we can discuss the time-delay function instead of the time delay surface.

$$t(\theta) \propto \frac{1}{2}(\theta - \beta)^2 - \sqrt{\theta^2 + \theta_c^2}$$



SOME INTERESTING PROPERTIES

- image multiplicity depends on the relative position of lens and source
- couples of images disappear after approaching each other
- the time-delay function is flat when this happens!
- $\det A=0$ means infinite magnification: the images disappear on the critical lines!
- this happens every time a source crosses a caustic!

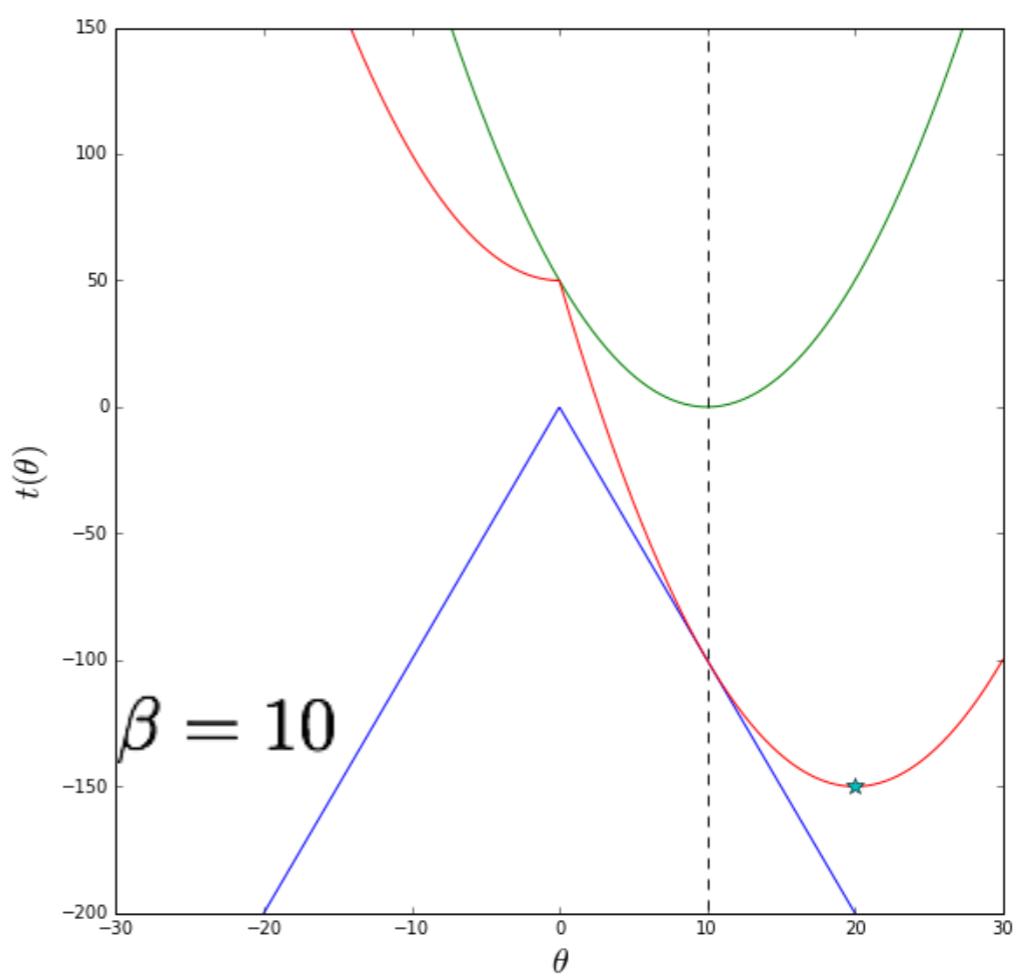
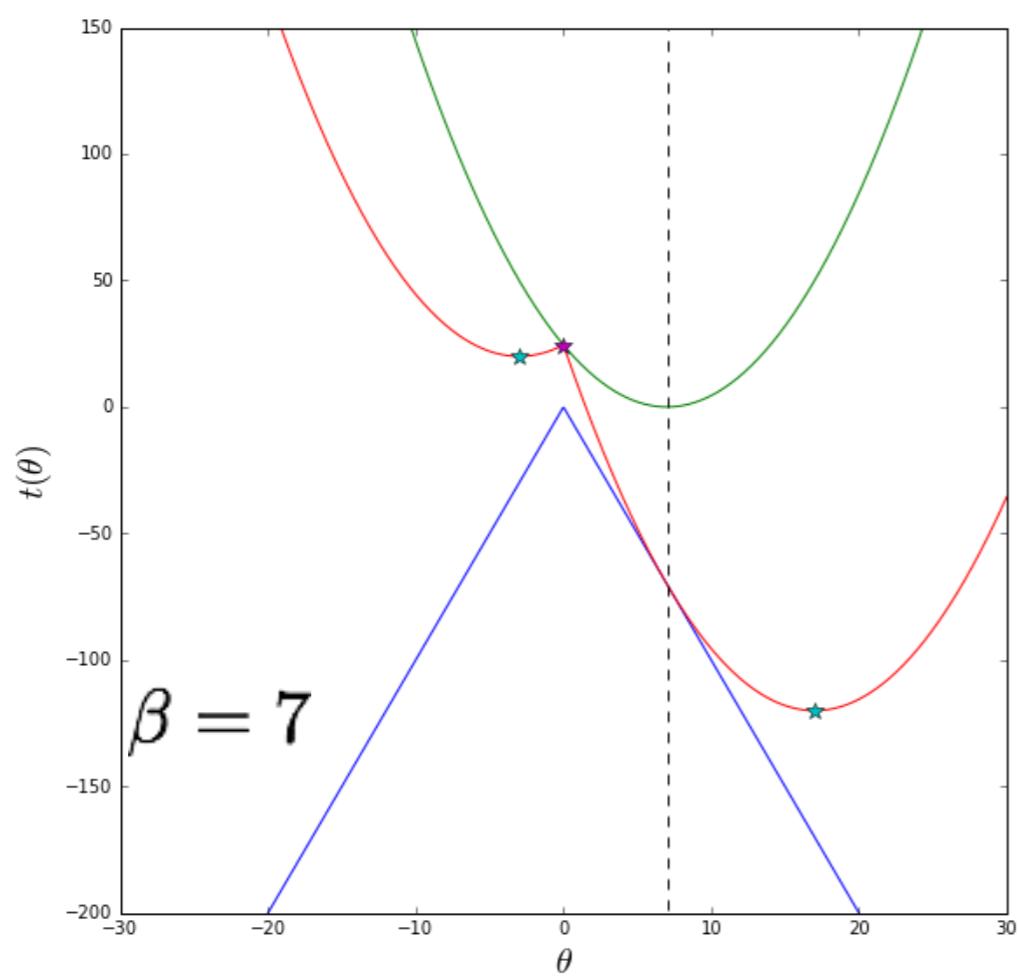
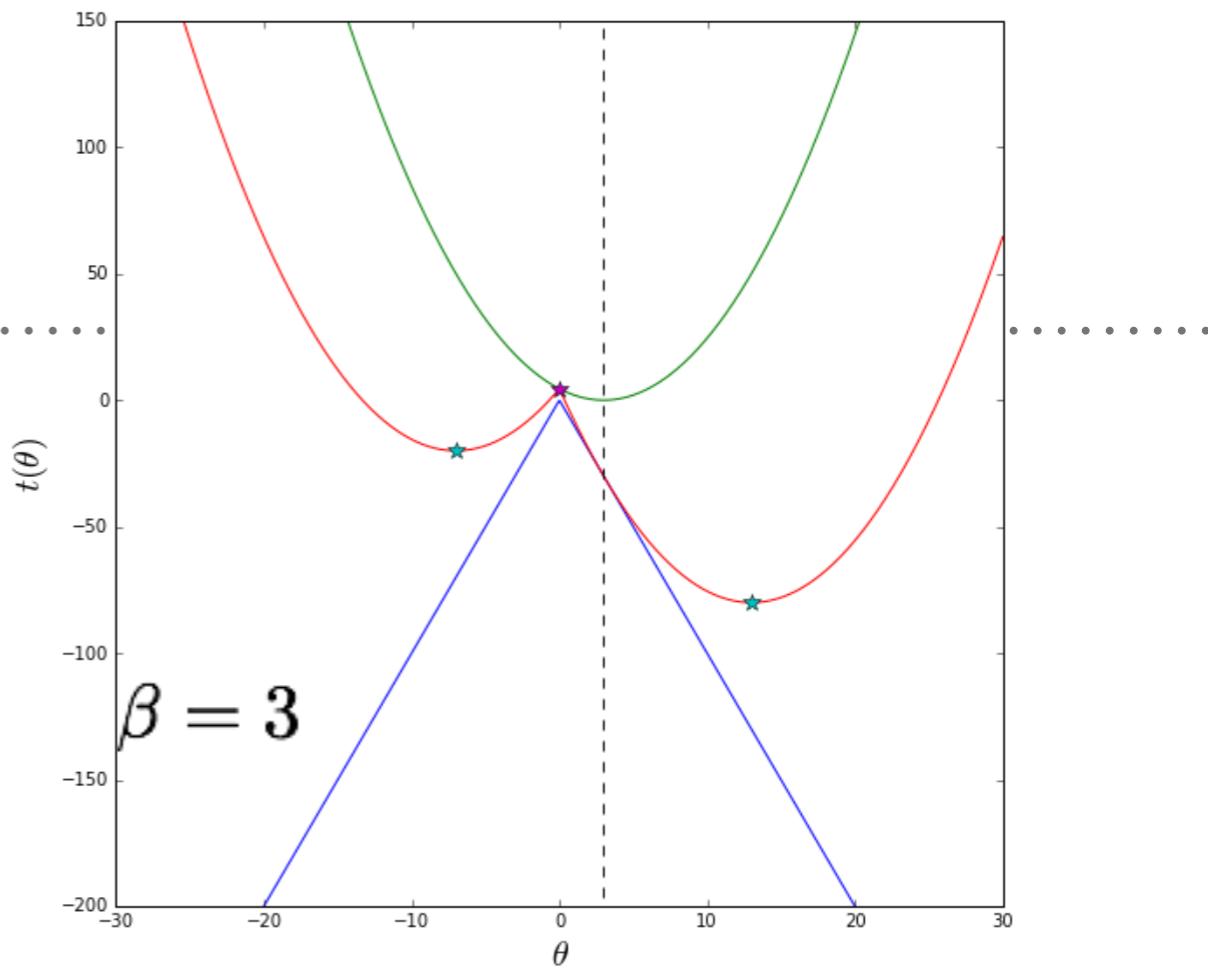
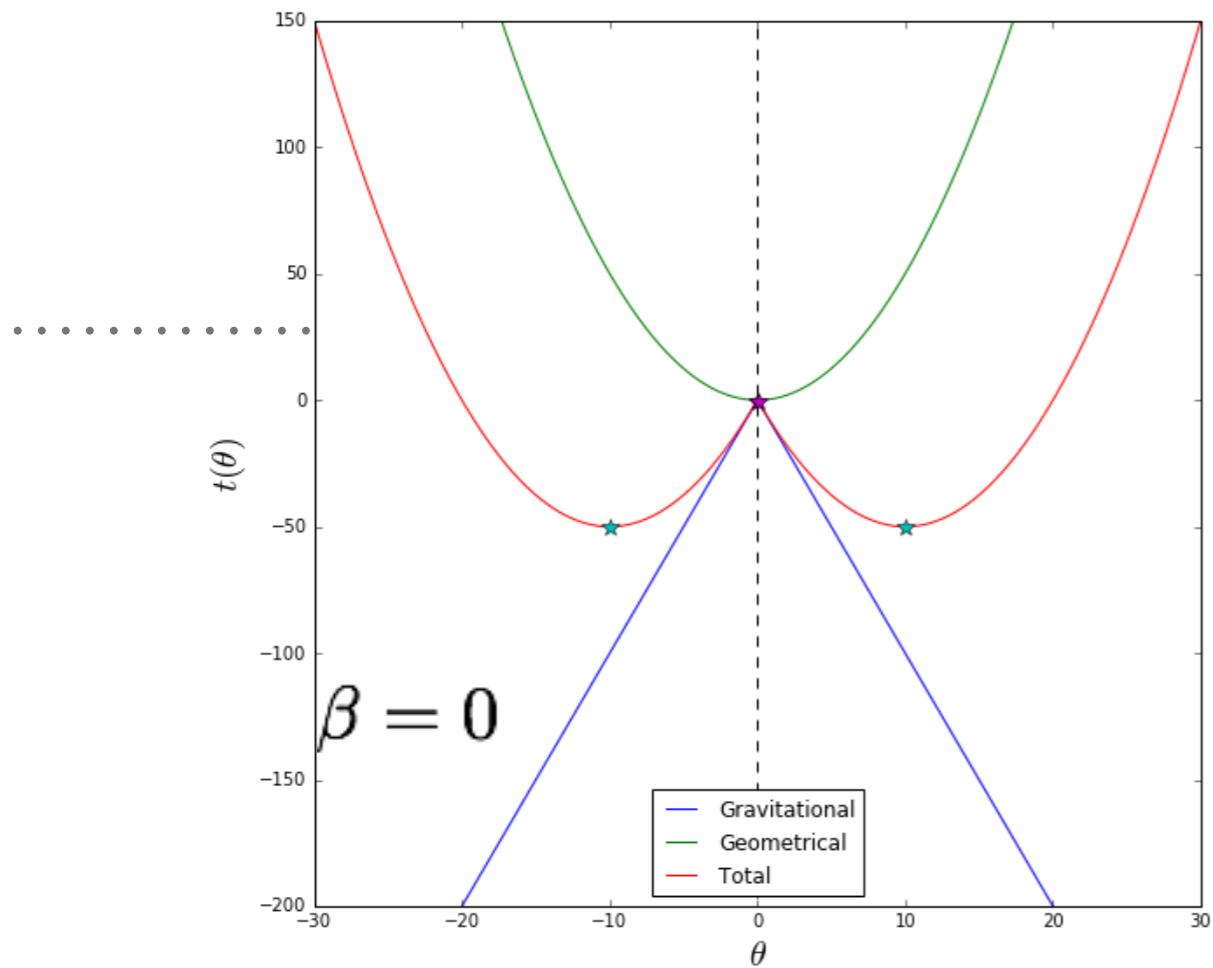


EXAMPLE OF TIME DELAY SURFACE

Let's change potential:

$$\hat{\Psi}(\theta) \propto |\theta|$$

The lens model is the same as before, but the core has been removed

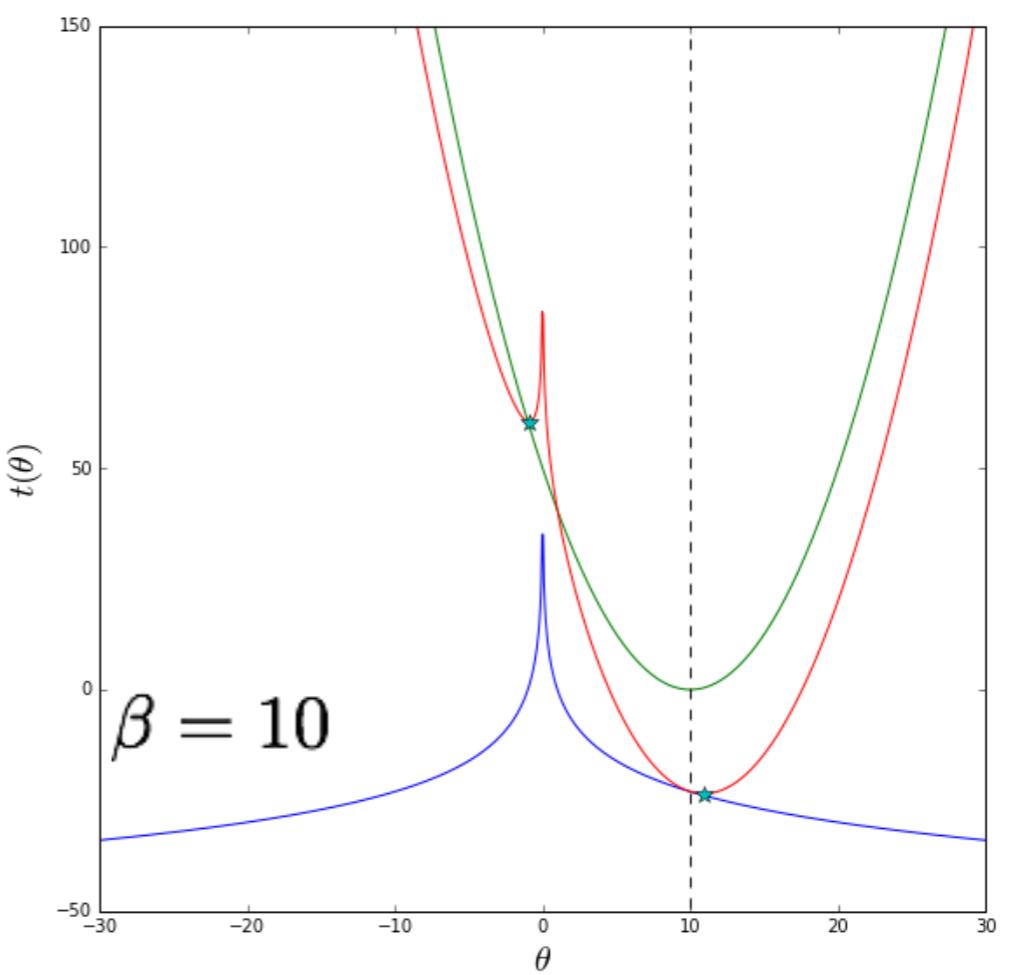
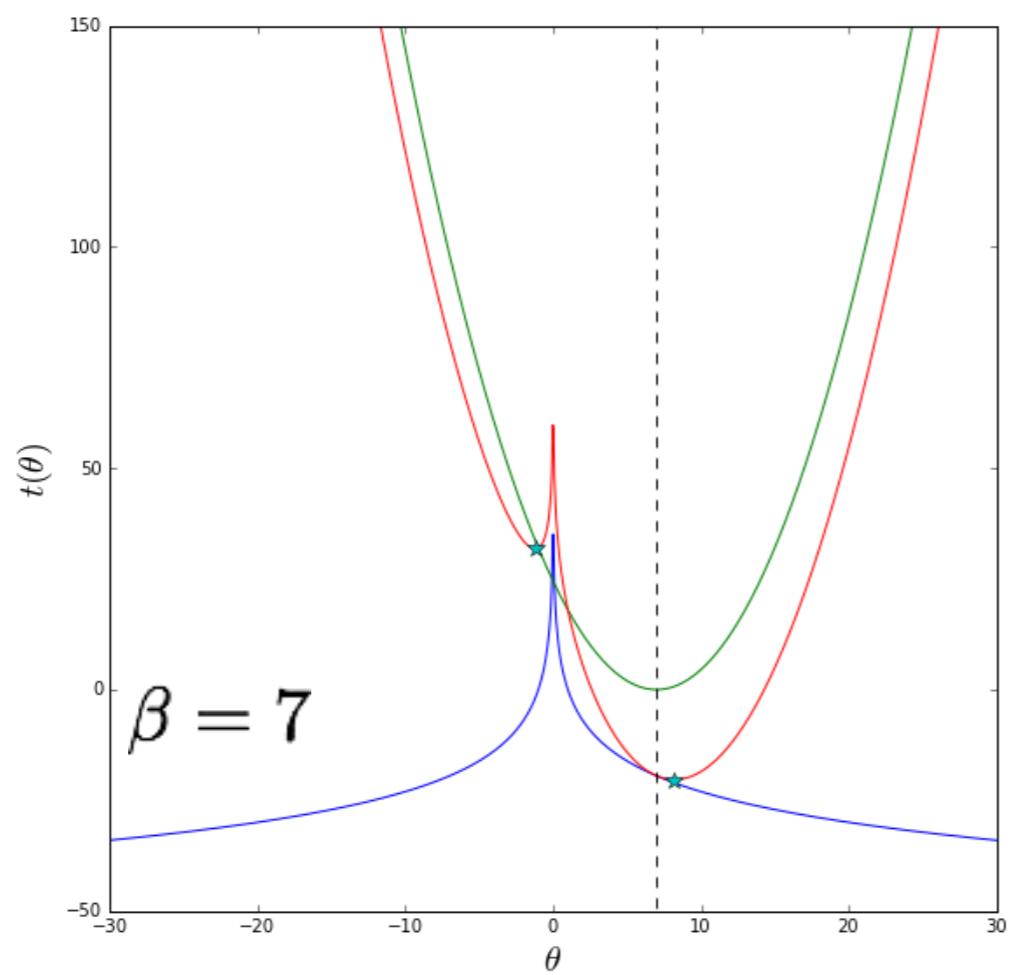
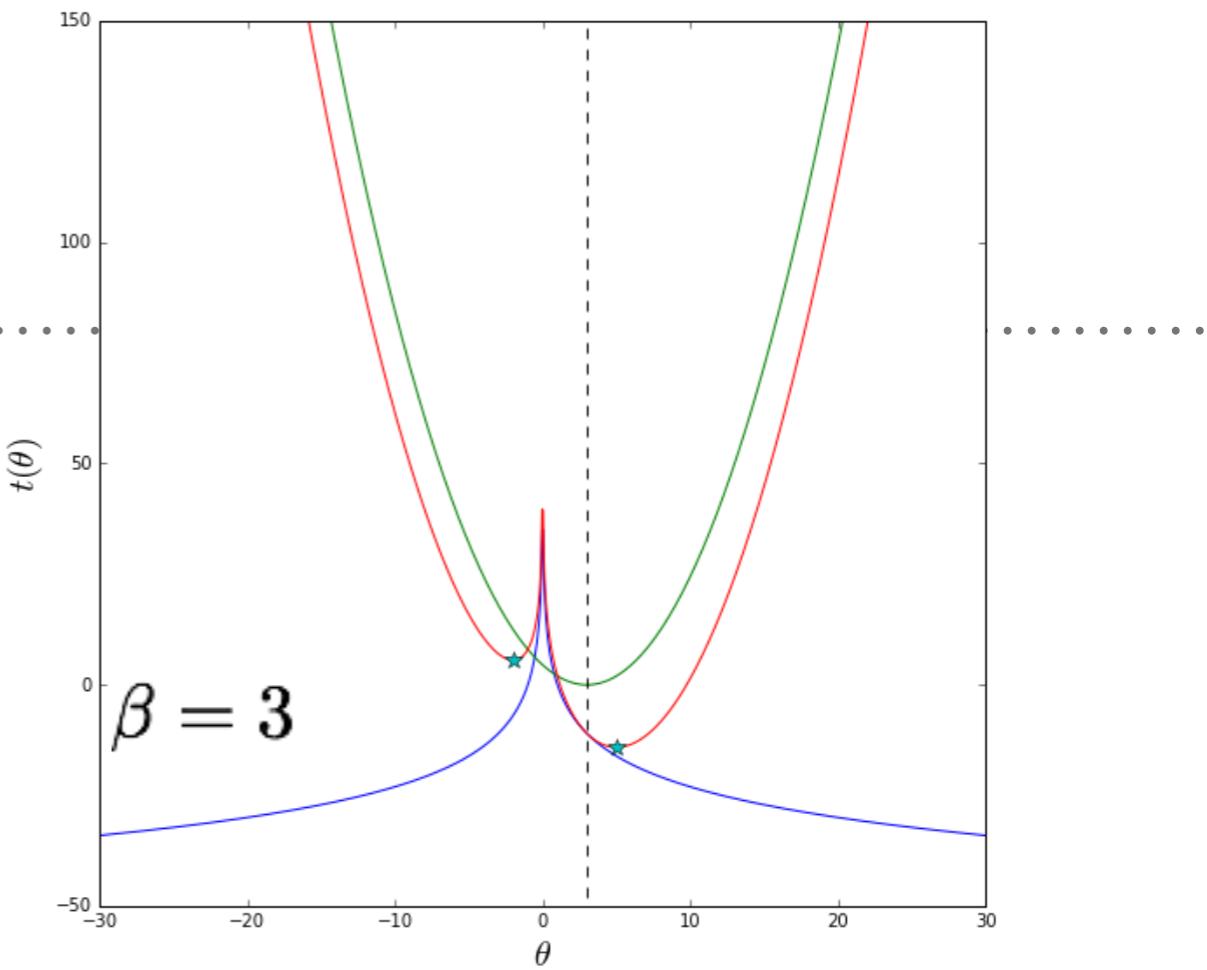
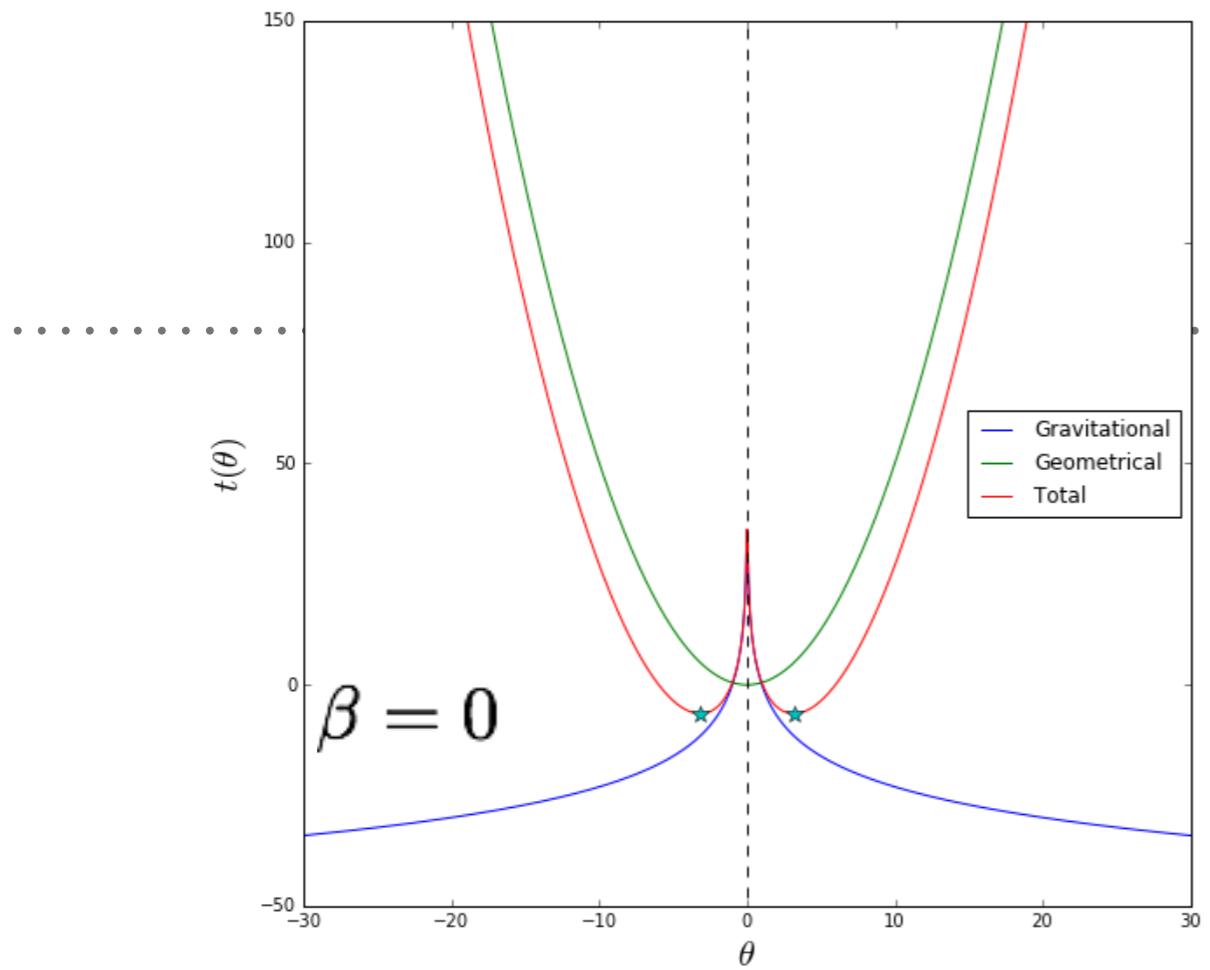


EXAMPLE OF TIME DELAY SURFACE

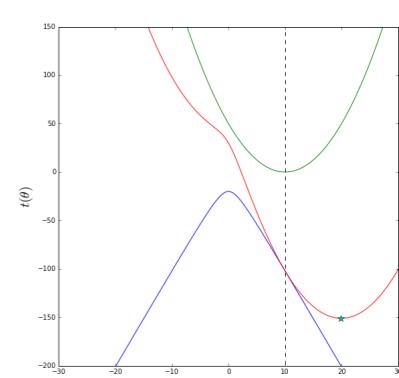
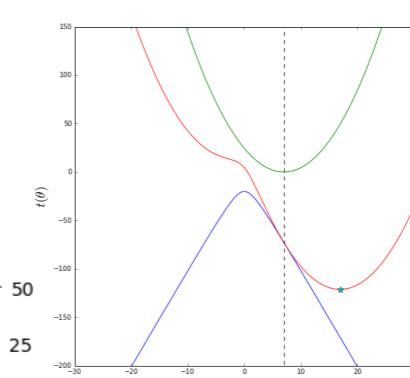
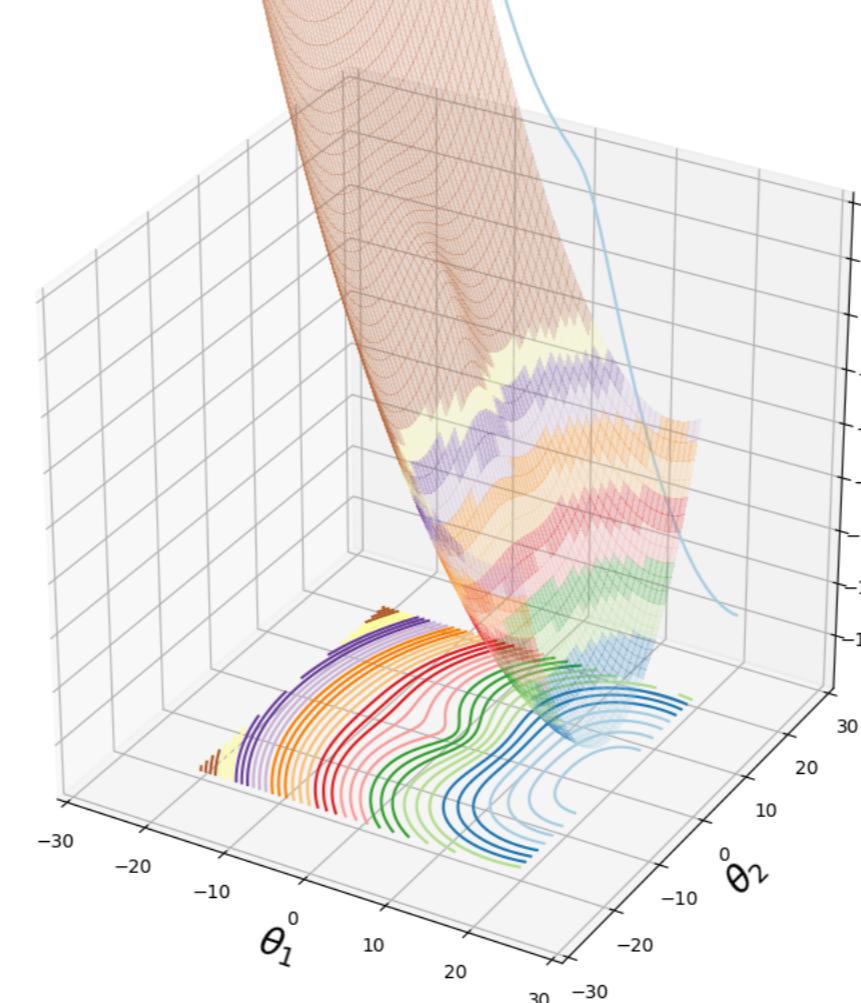
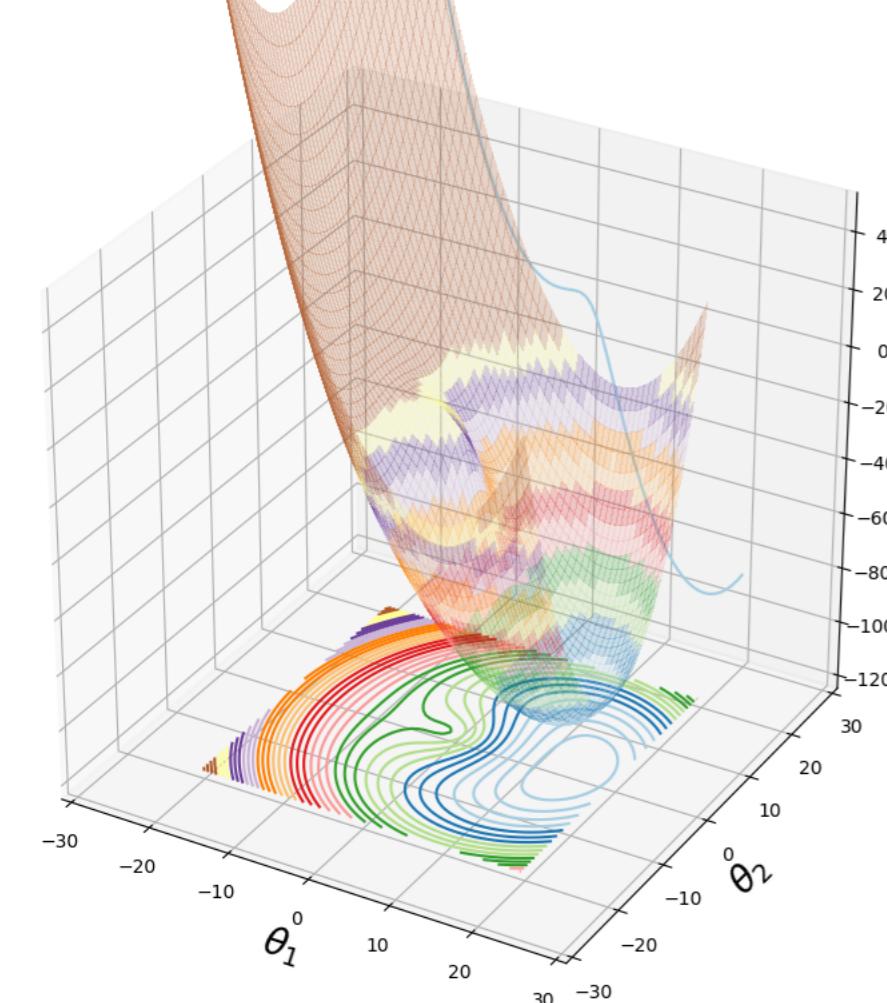
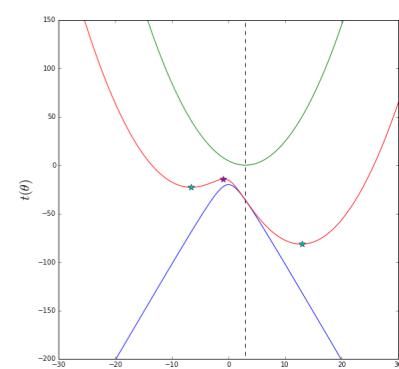
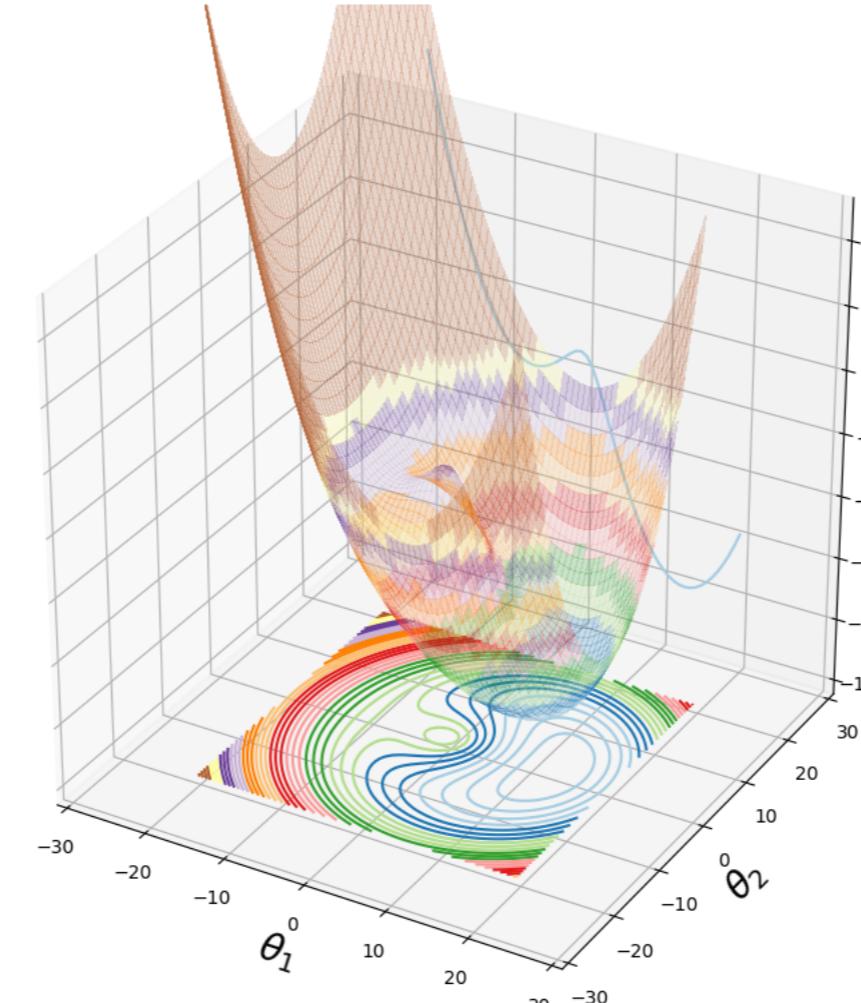
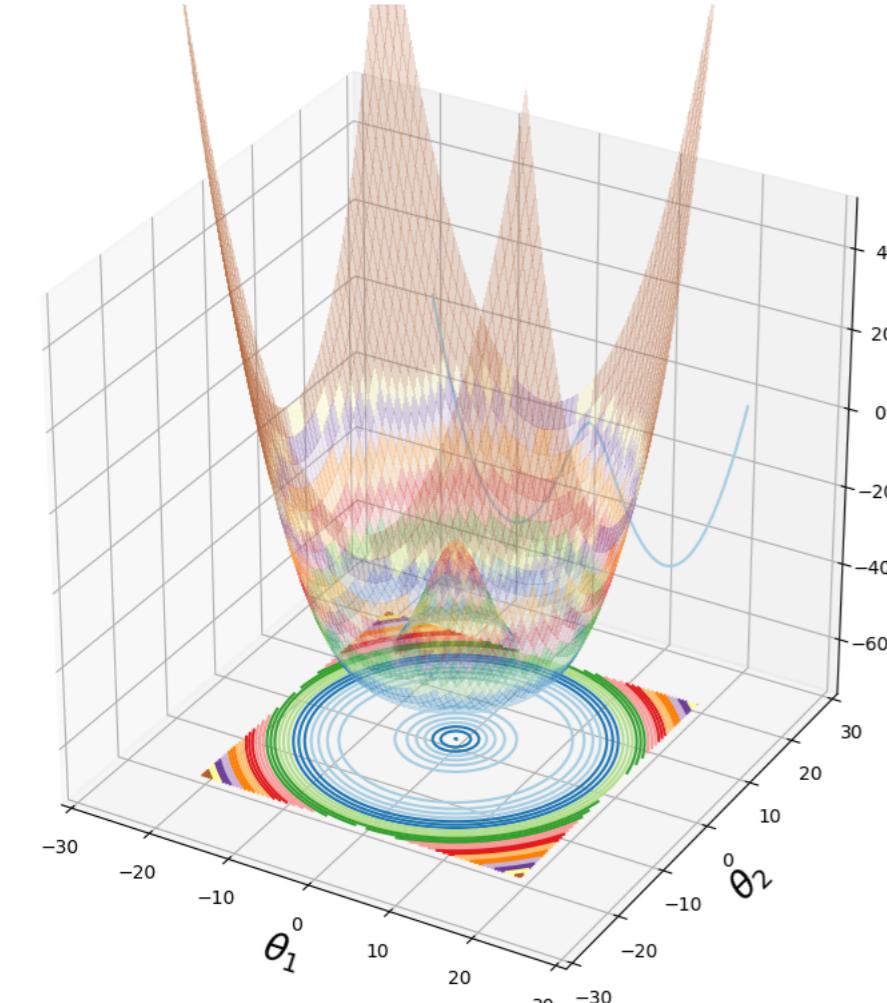
Yet another potential

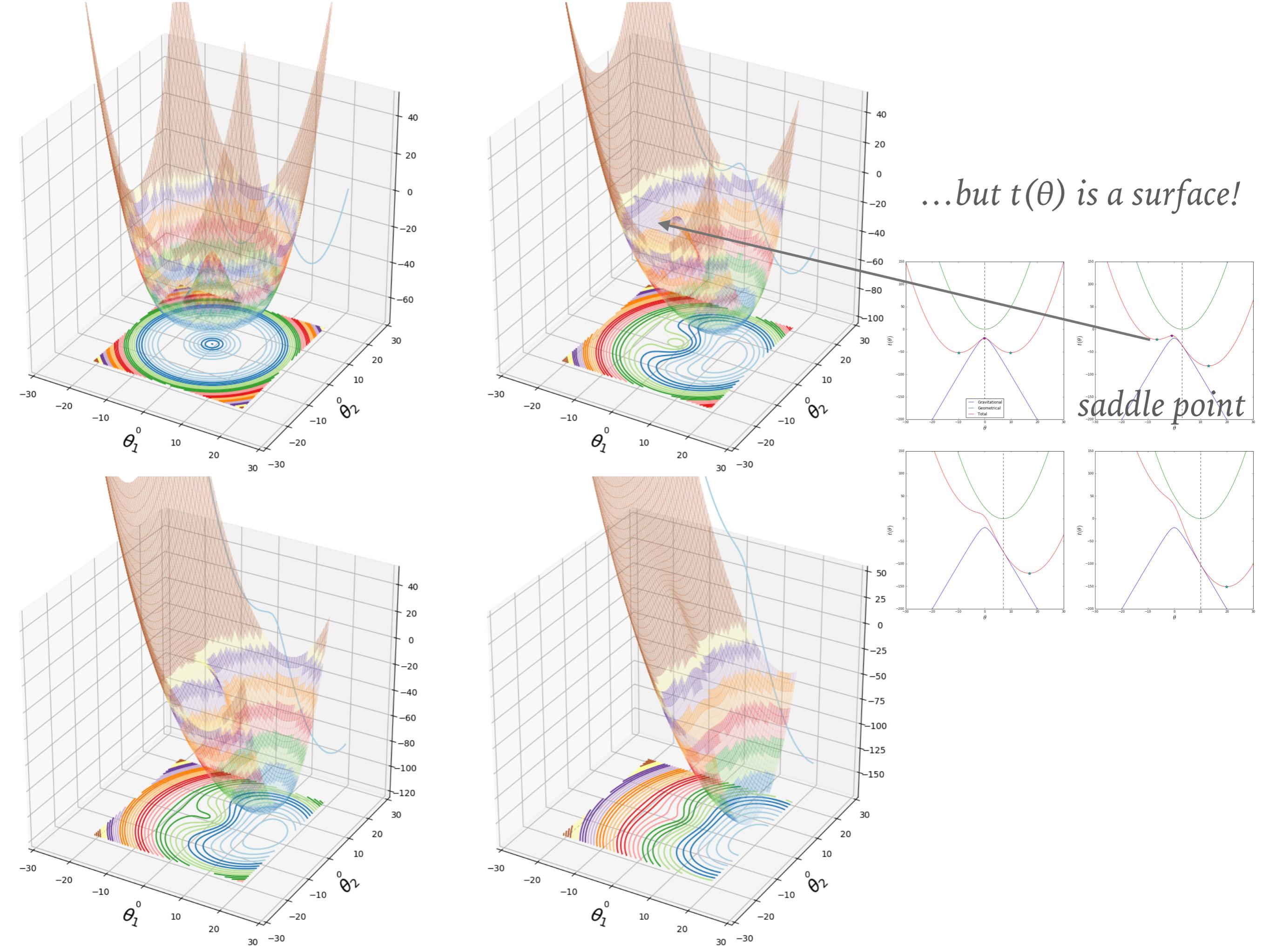
$$\hat{\Psi}(\theta) \propto \ln |\theta|$$

This is the lensing potential of the point mass...



...but $t(\theta)$ is a surface!



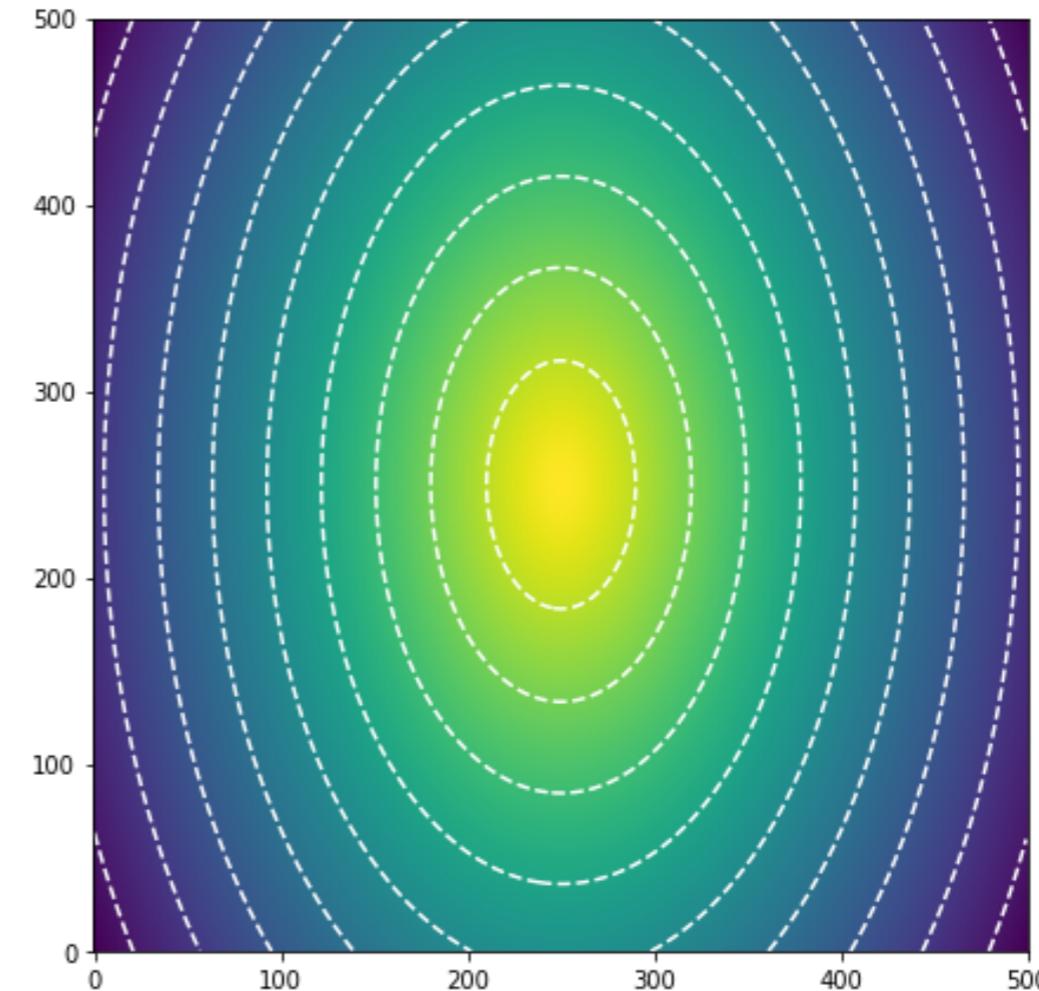
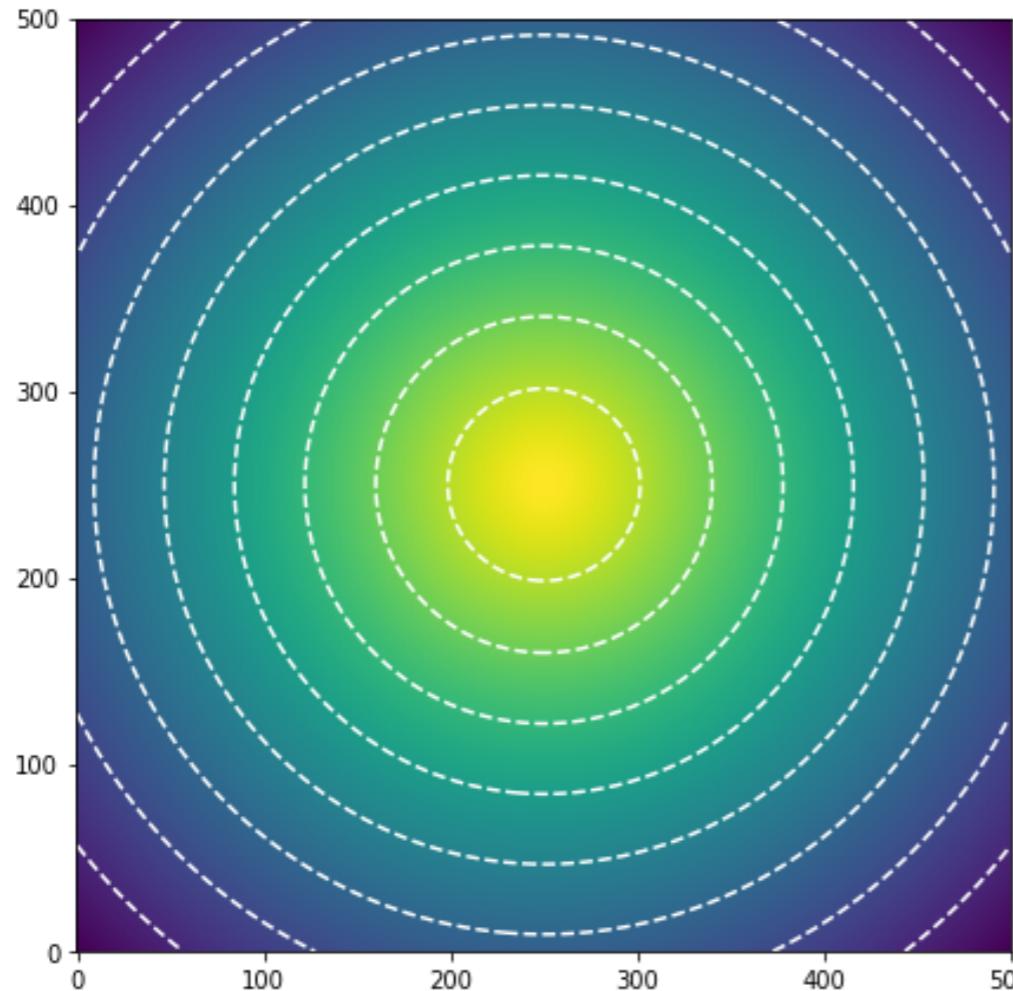


INTRODUCING ELLIPTICITY

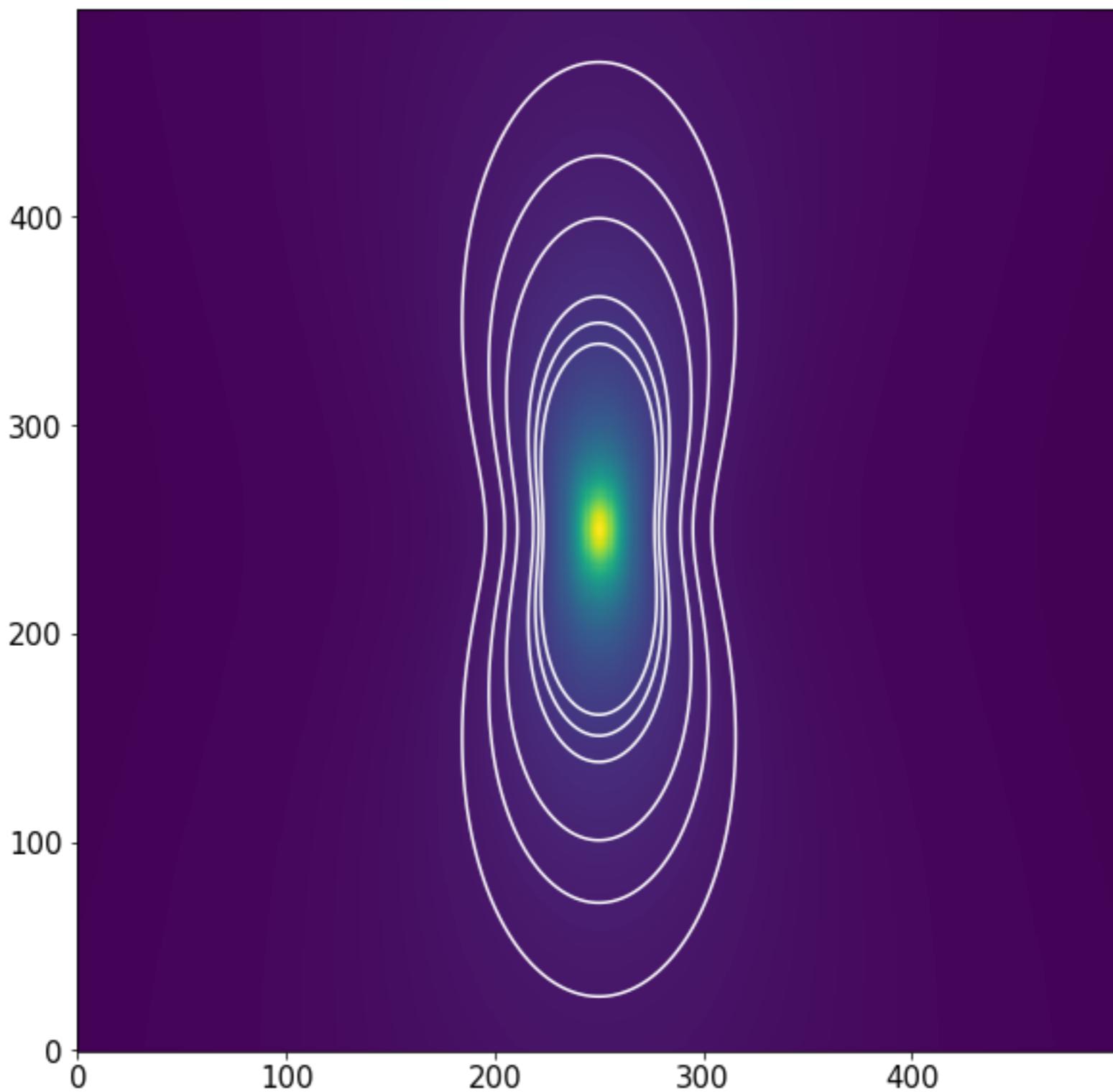
One easy way to make a lens elliptical is by modifying the potential as follows:

$$|\theta| \rightarrow \sqrt{\frac{\theta_1^2}{1-\epsilon} + \theta_2^2(1-\epsilon)}$$

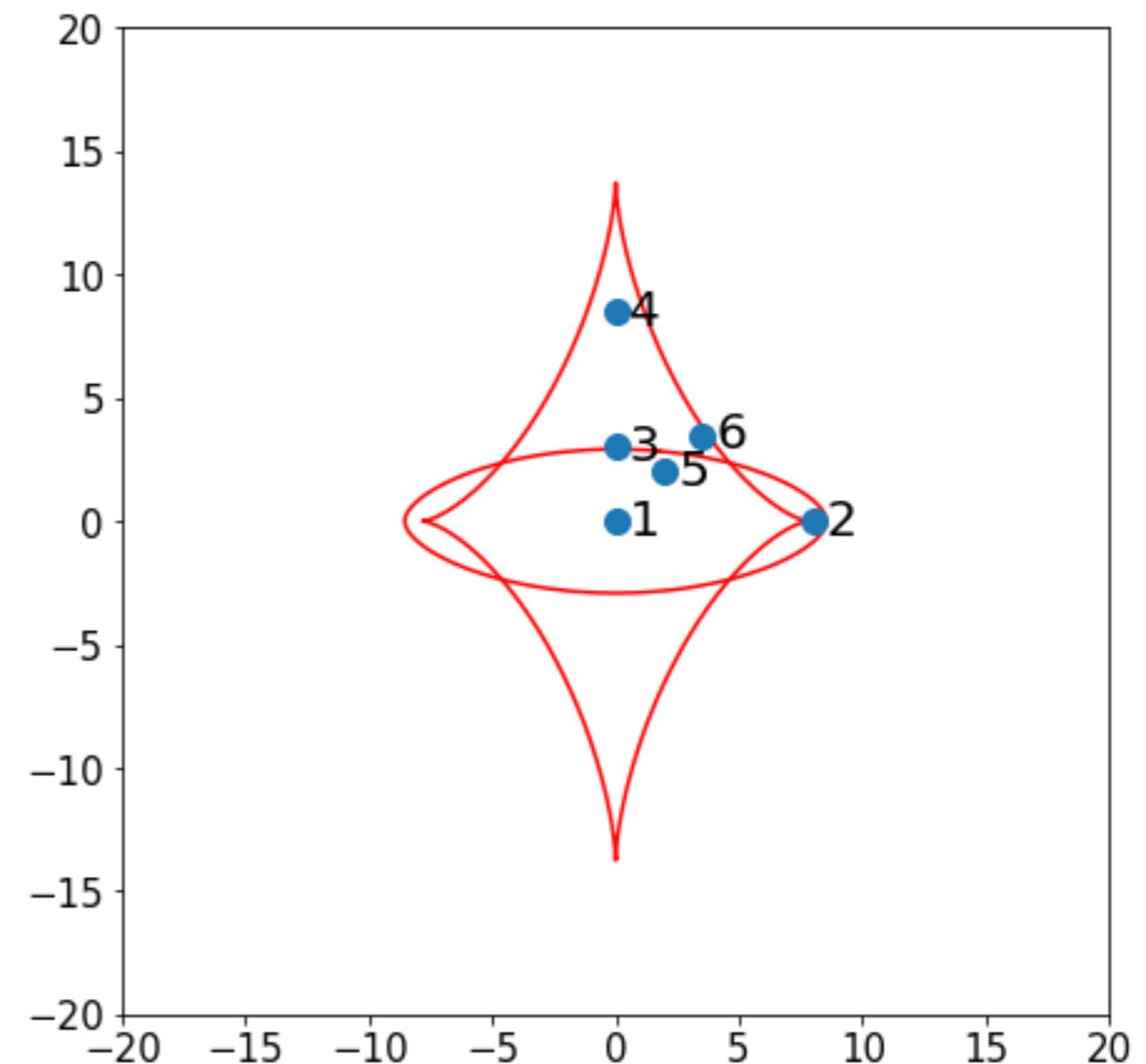
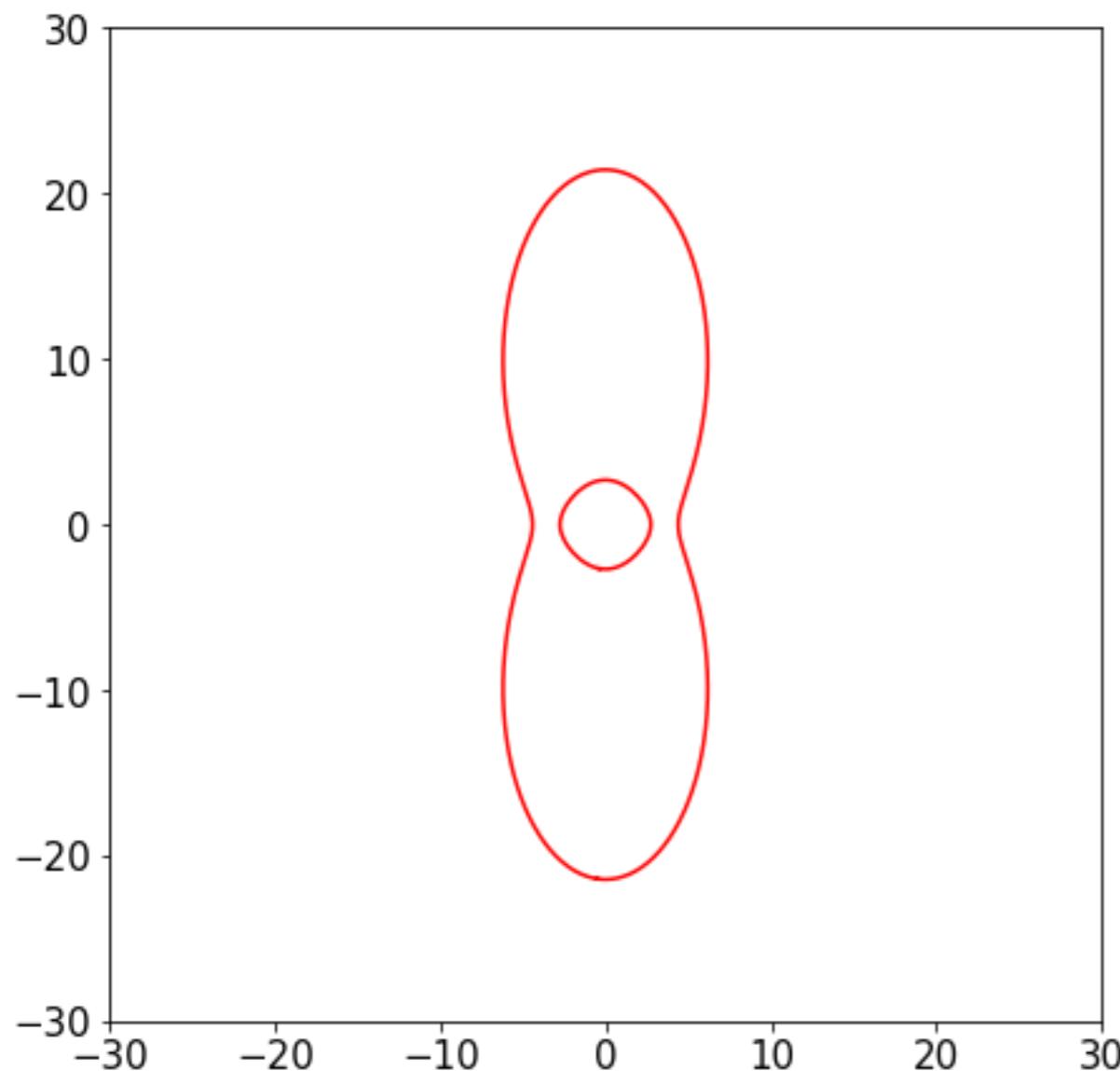
This makes the potential constant over ellipses rather than on circles.



CAUTION: PSEUDO ELLIPTICAL LENSES

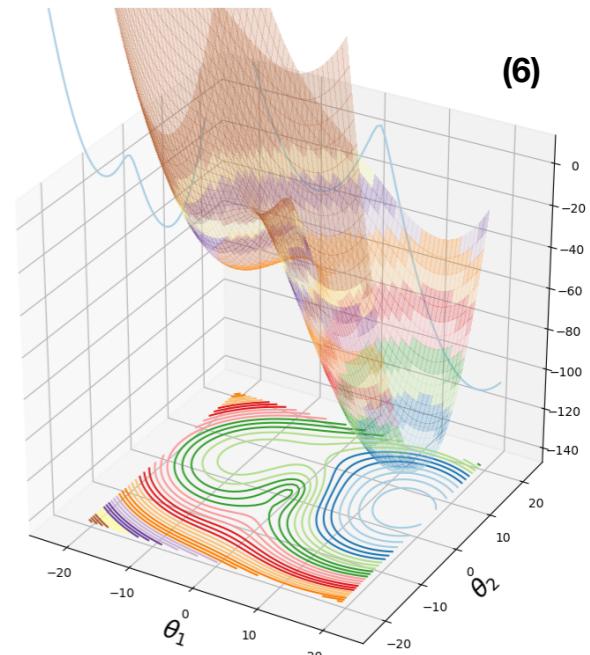
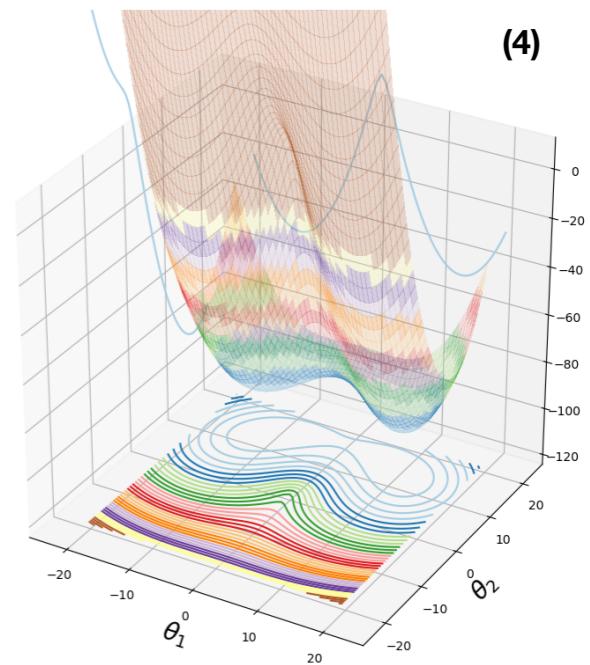
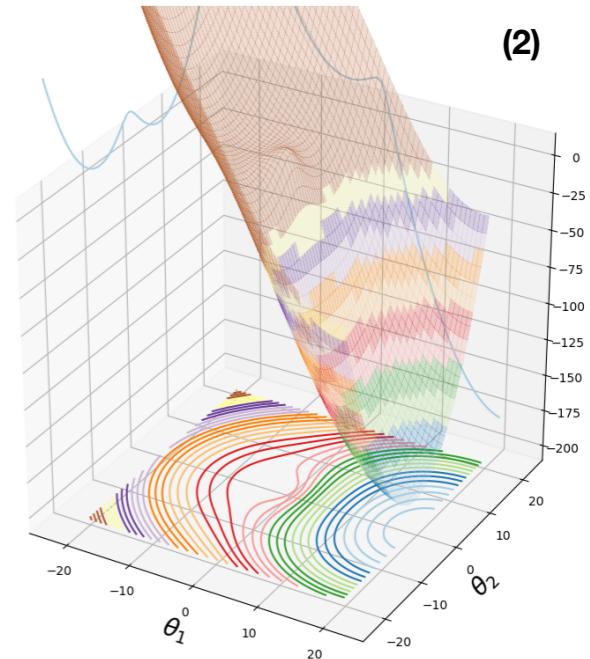
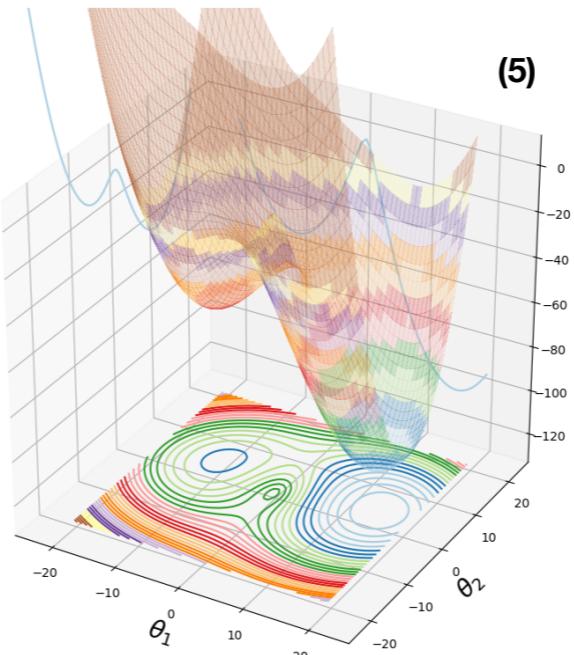
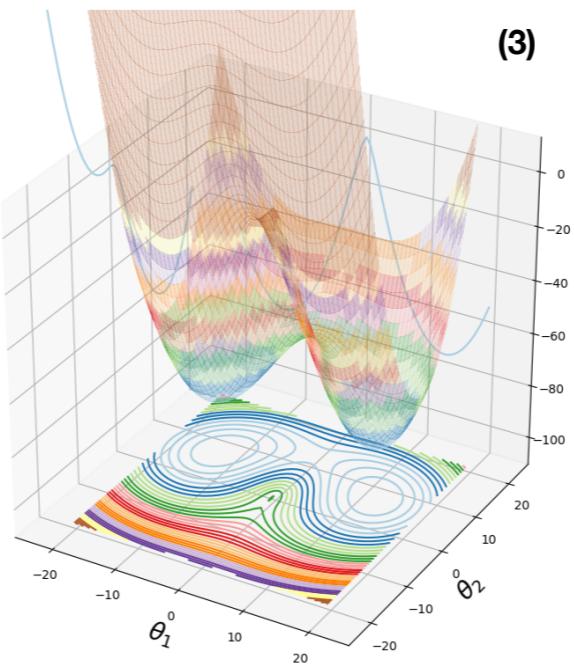
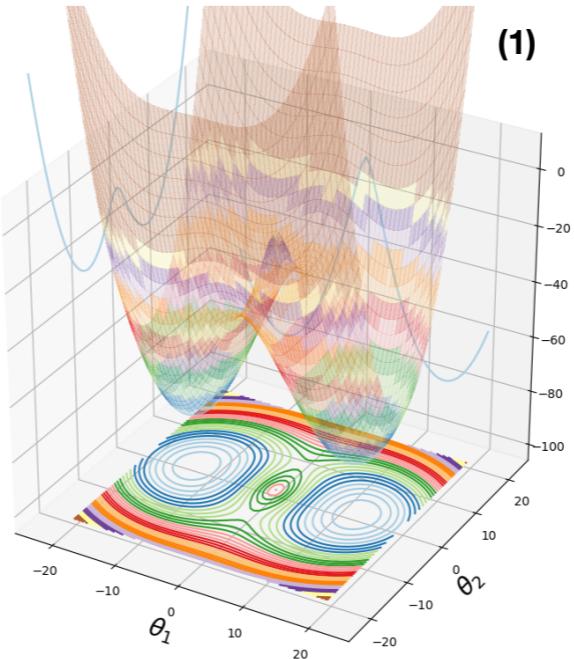
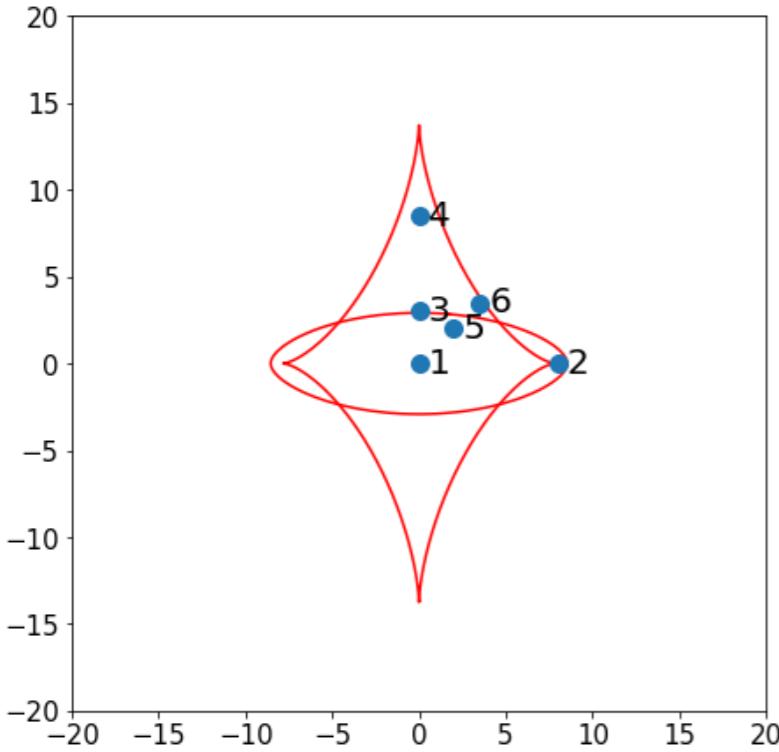


CRITICAL LINES AND CAUSTICS



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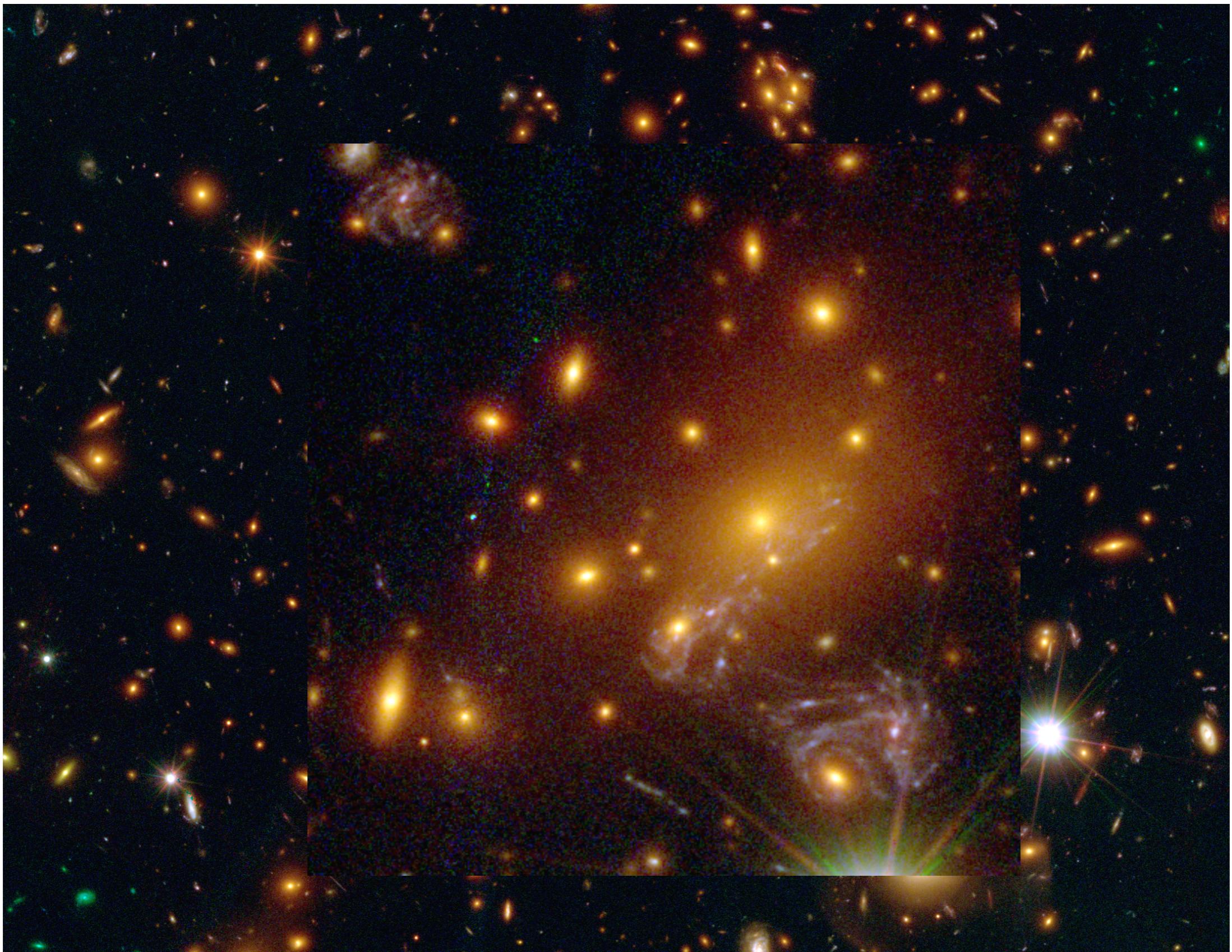
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SN REFSDAL IN MACS 1149



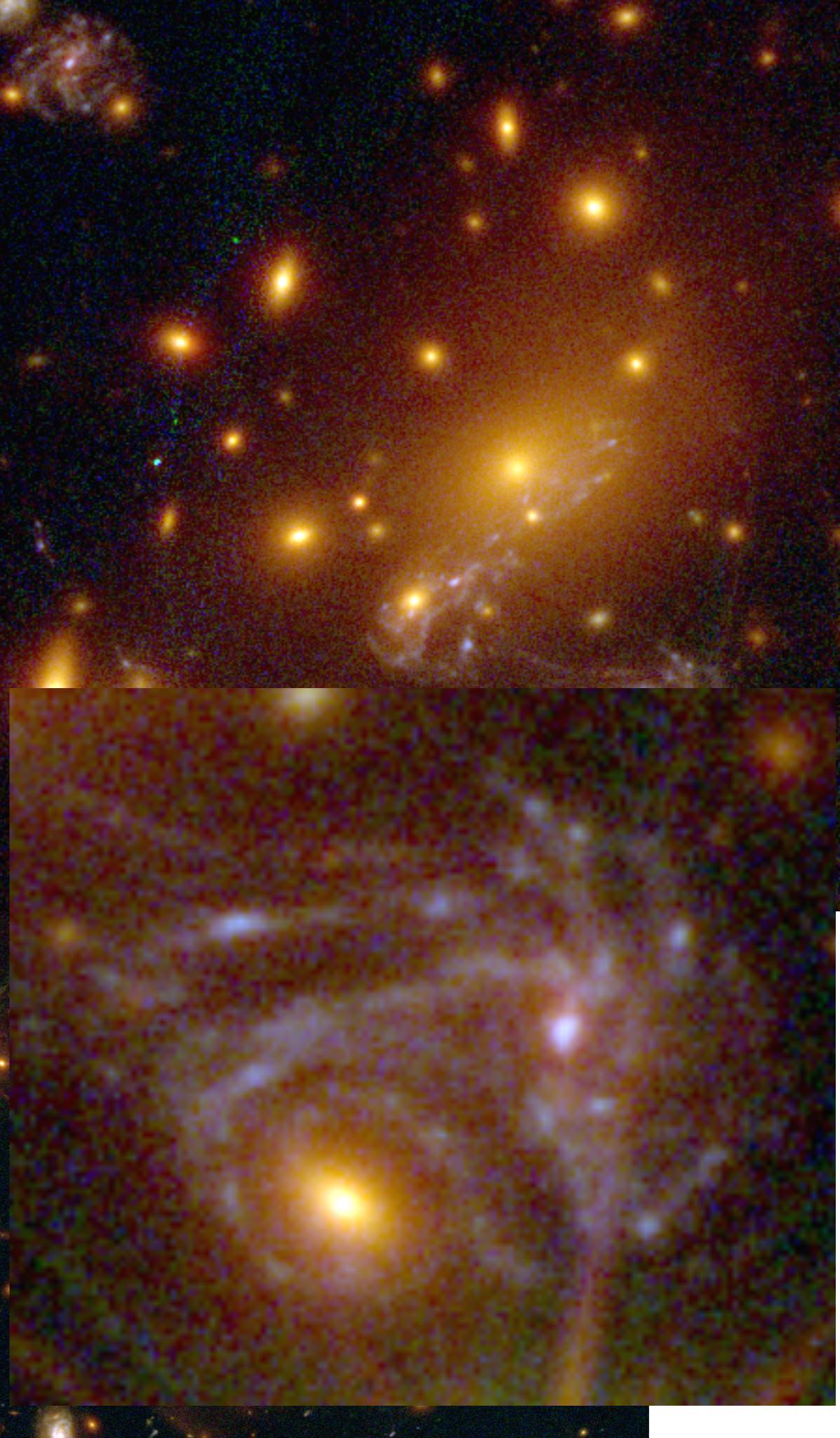
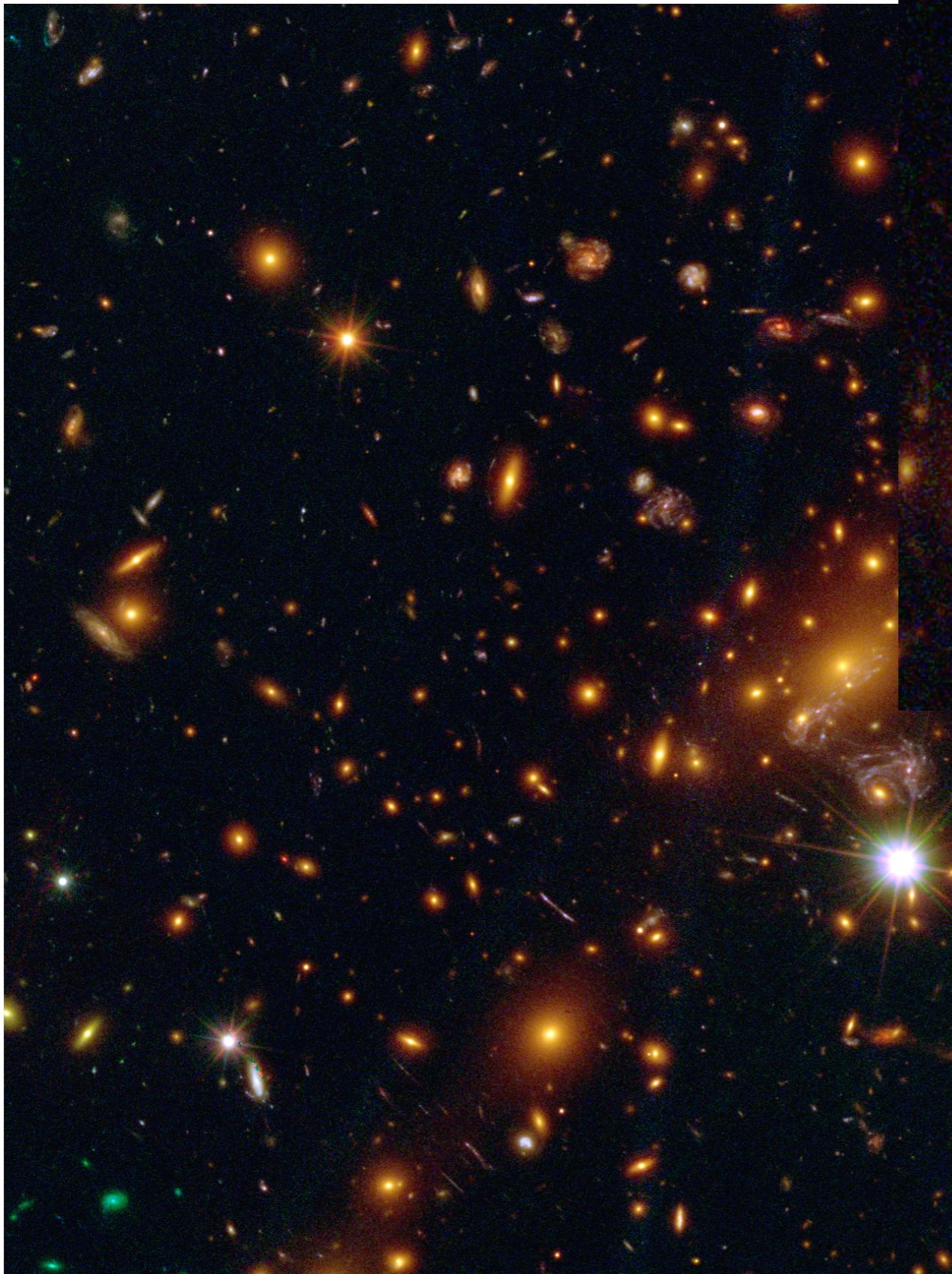
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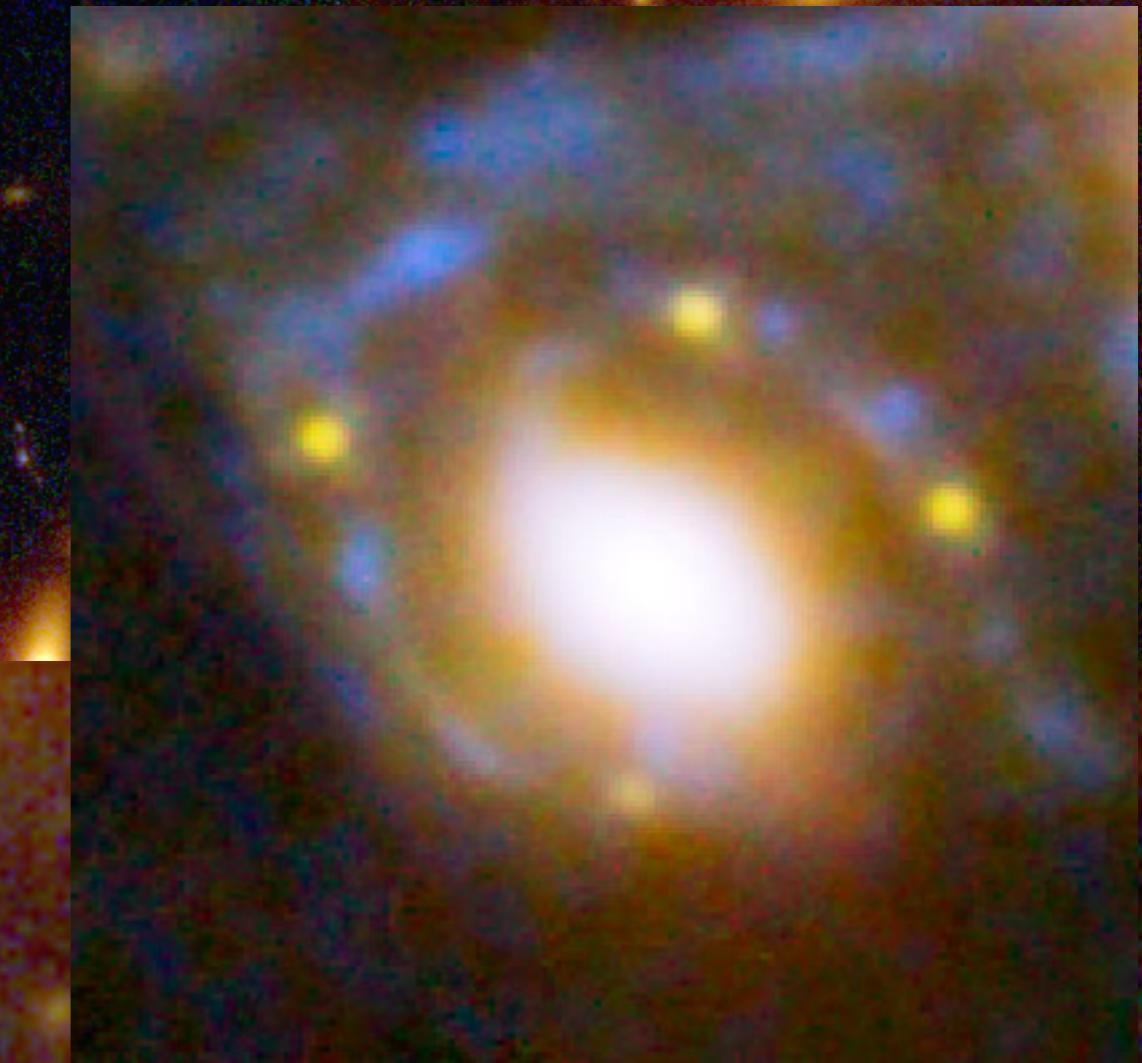
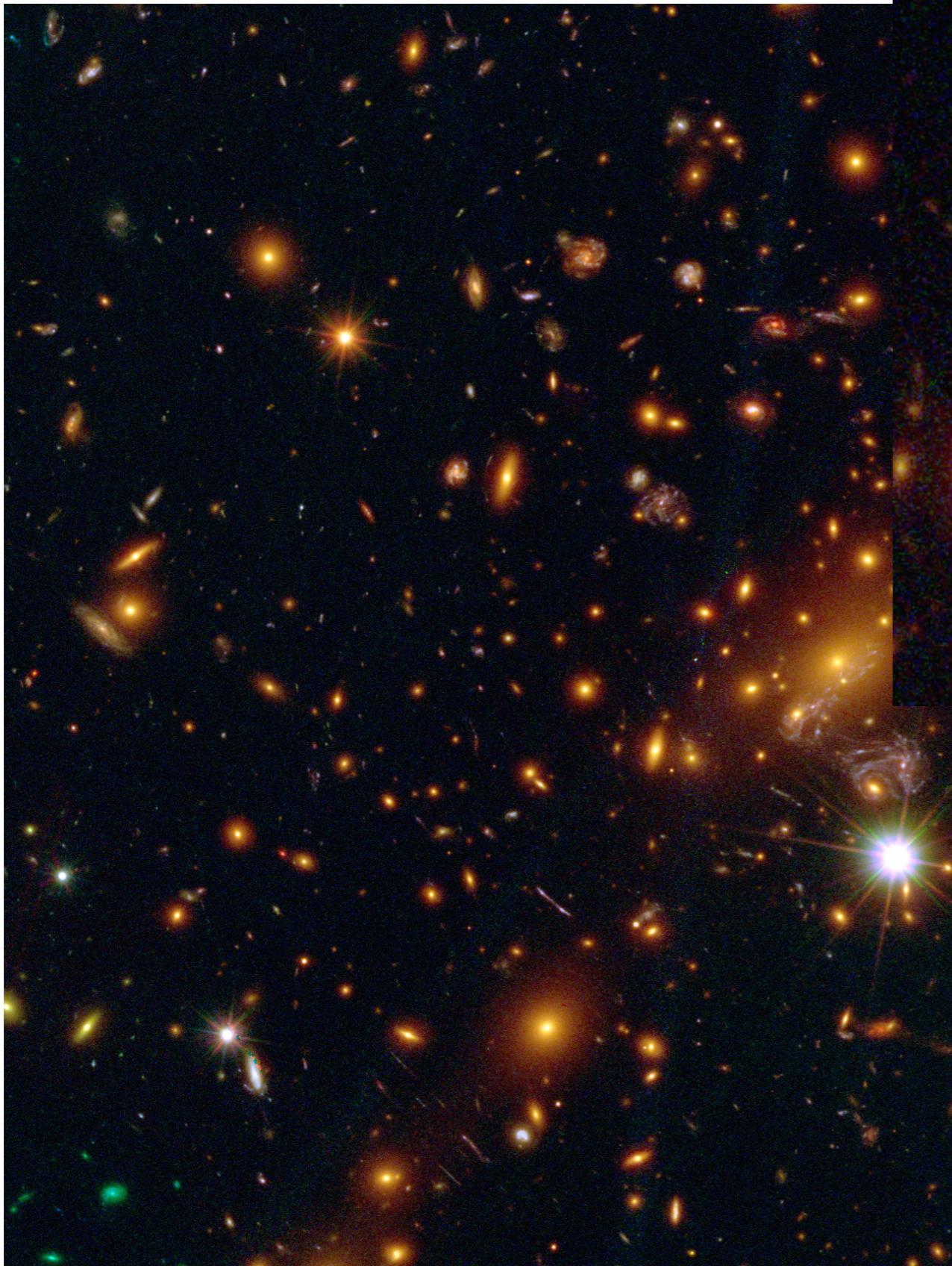
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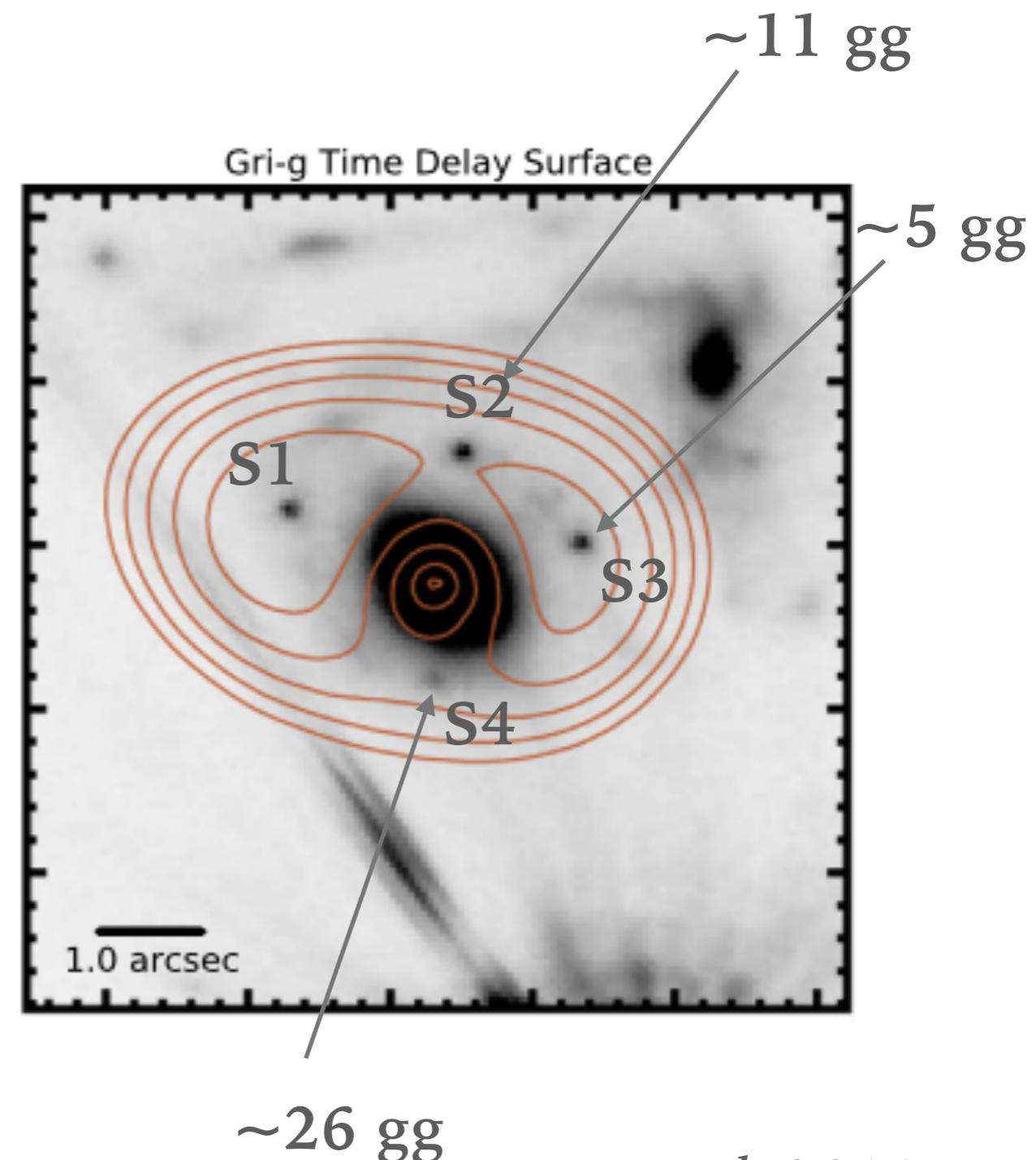
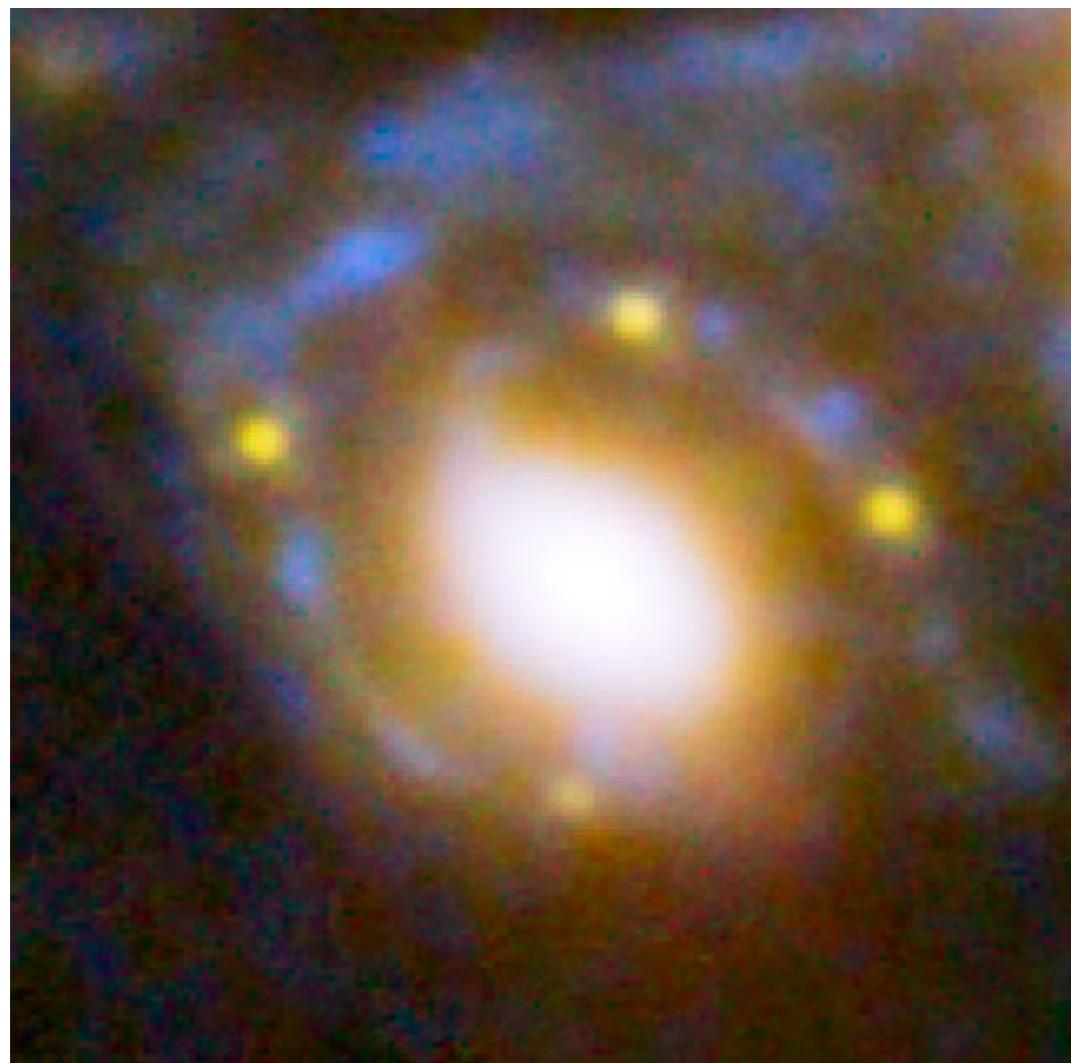


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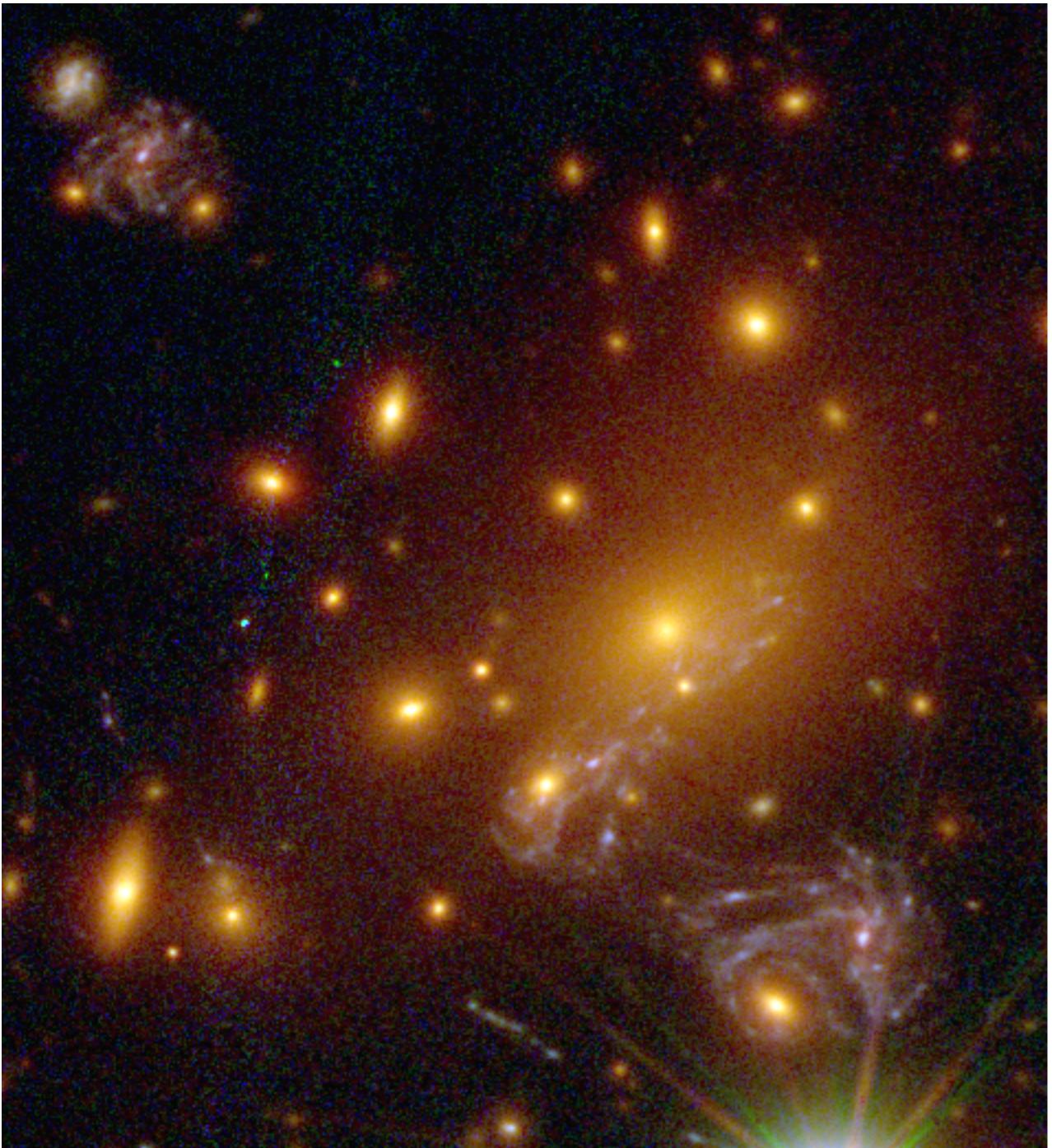


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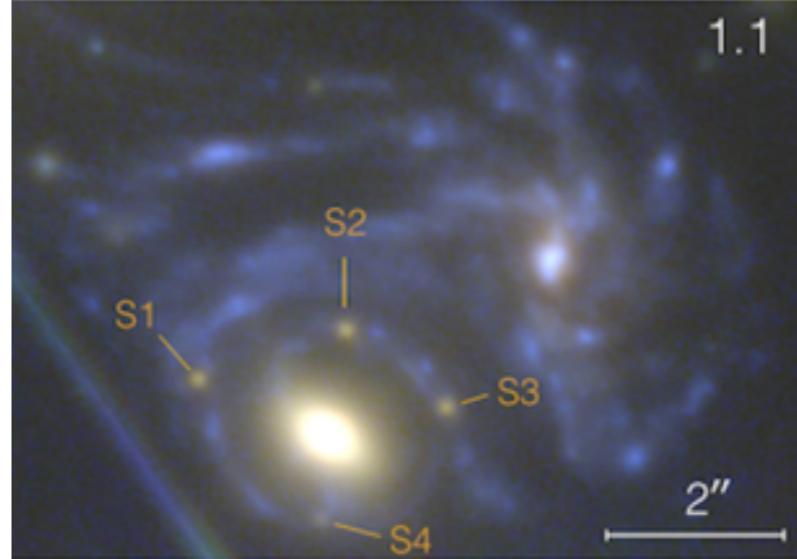
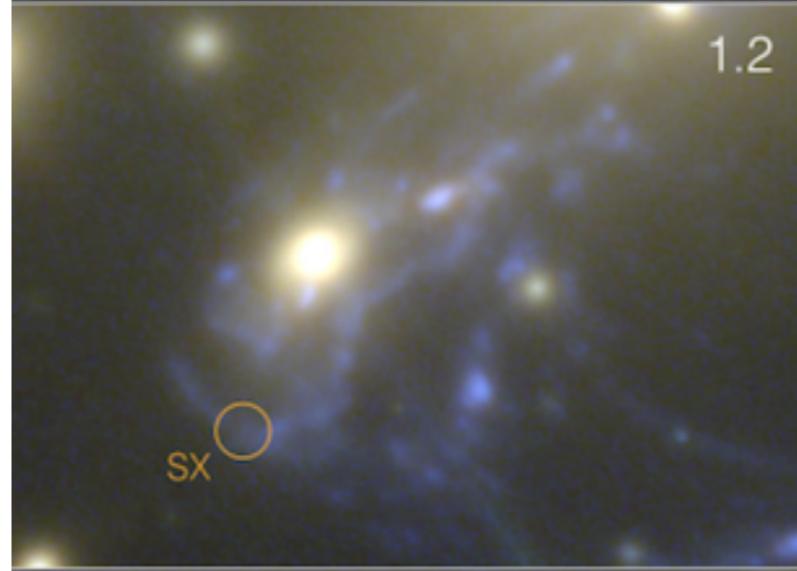
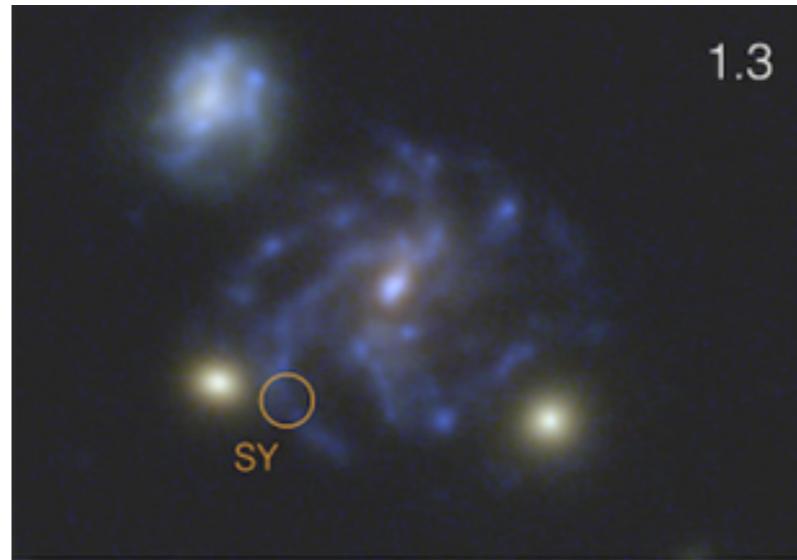
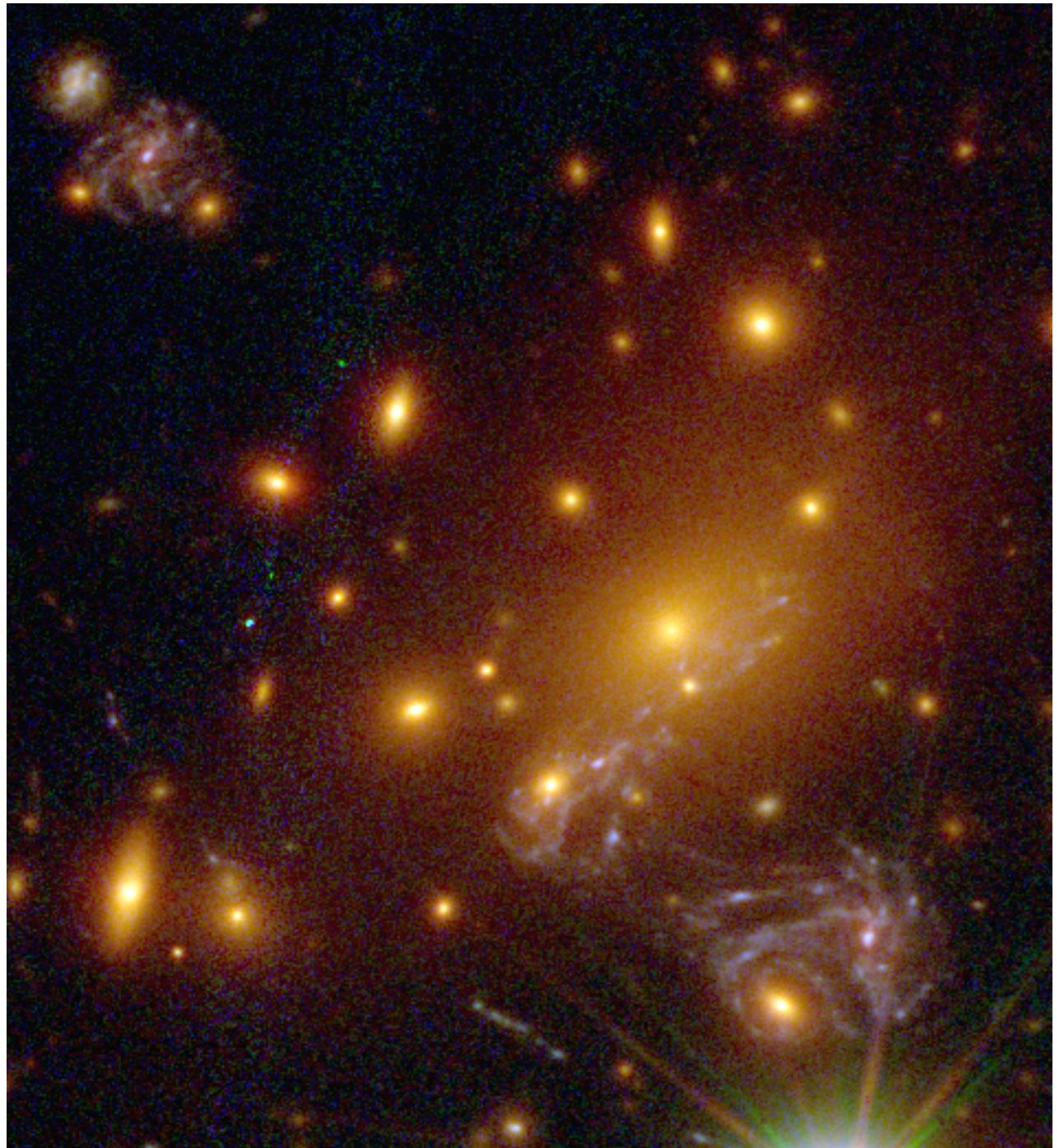
Nov. 2014 (*Kelly et al.*)



SN REFSDAL IN MACS 1149

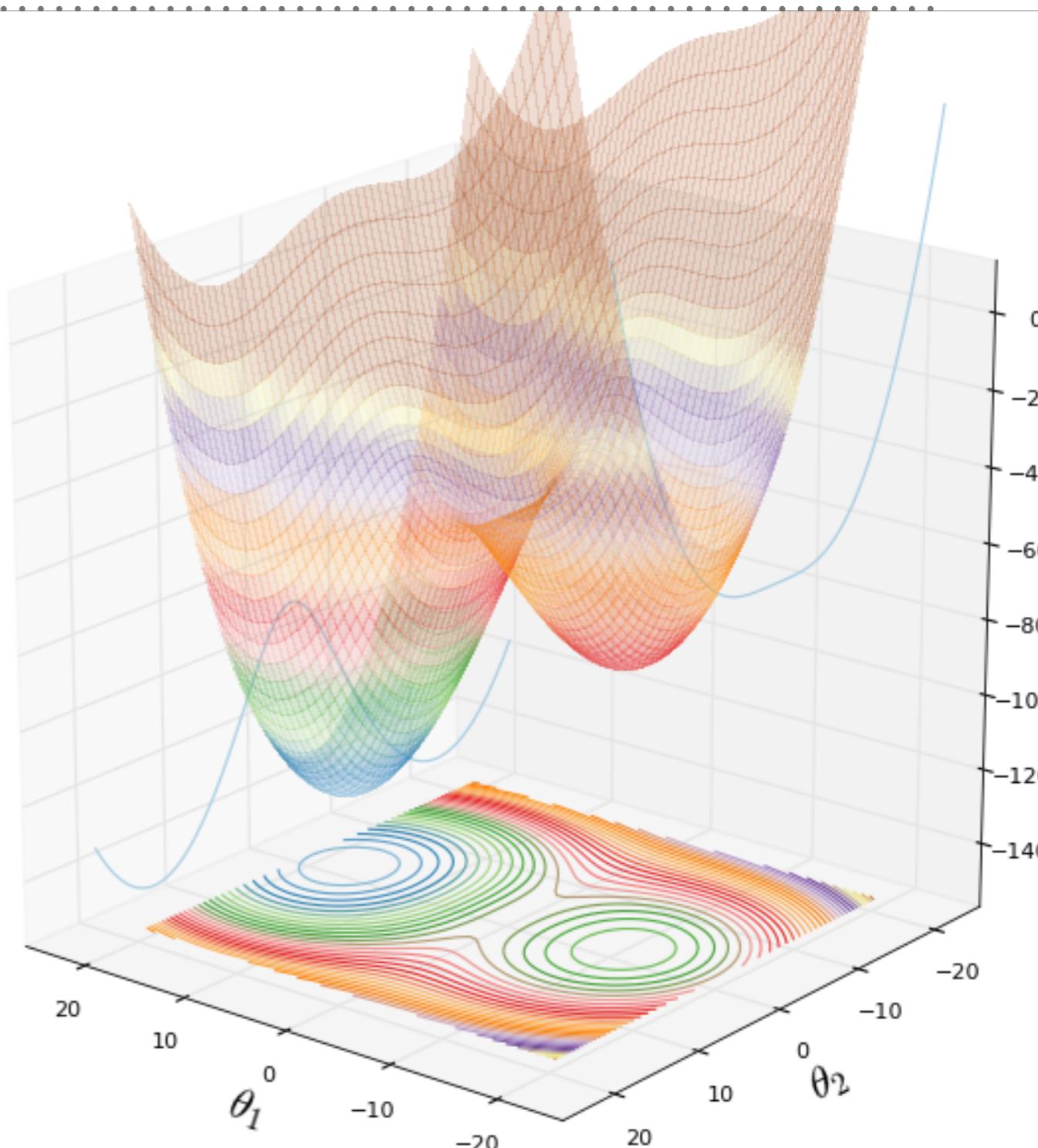
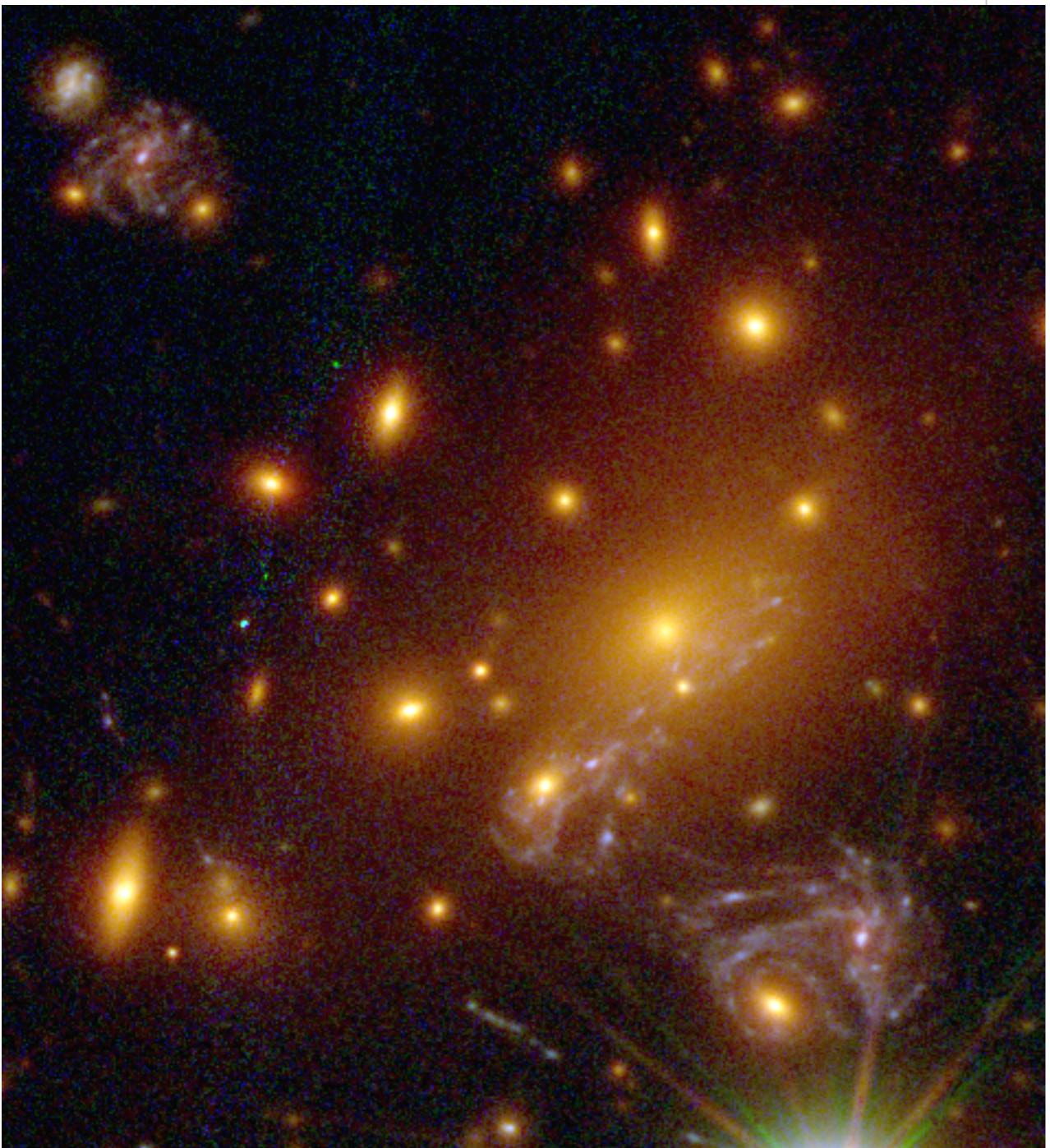


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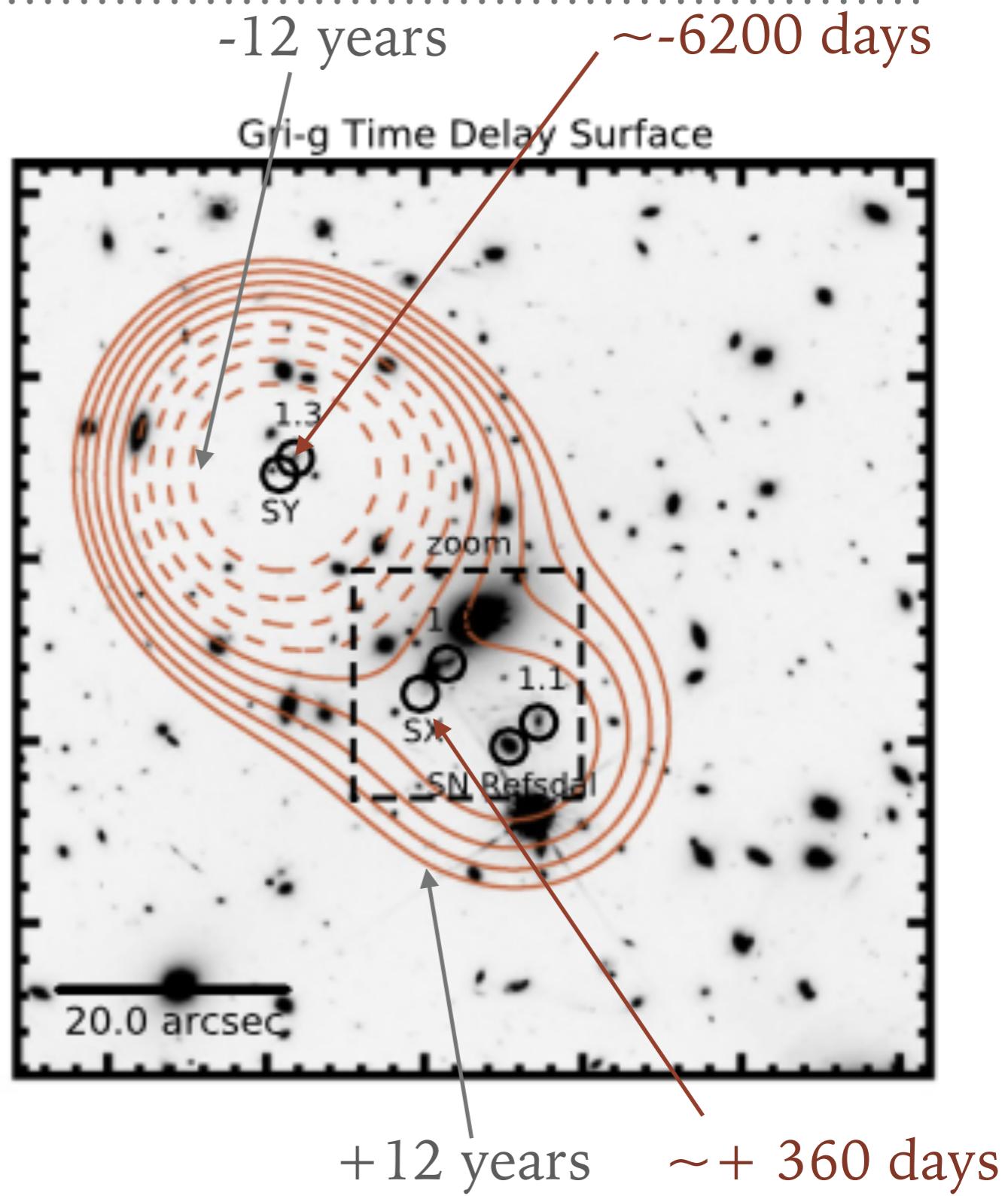


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SN REFSDAL IN MACS 1149



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16/12/2016...

Time delay

(SX- S1)

345 ± 10 gg

