GRAVITATIONAL LENSING

12 - MICROLENSING LIGHT CURVES: BREAKING THE MICROLENSING DEGENERACIES

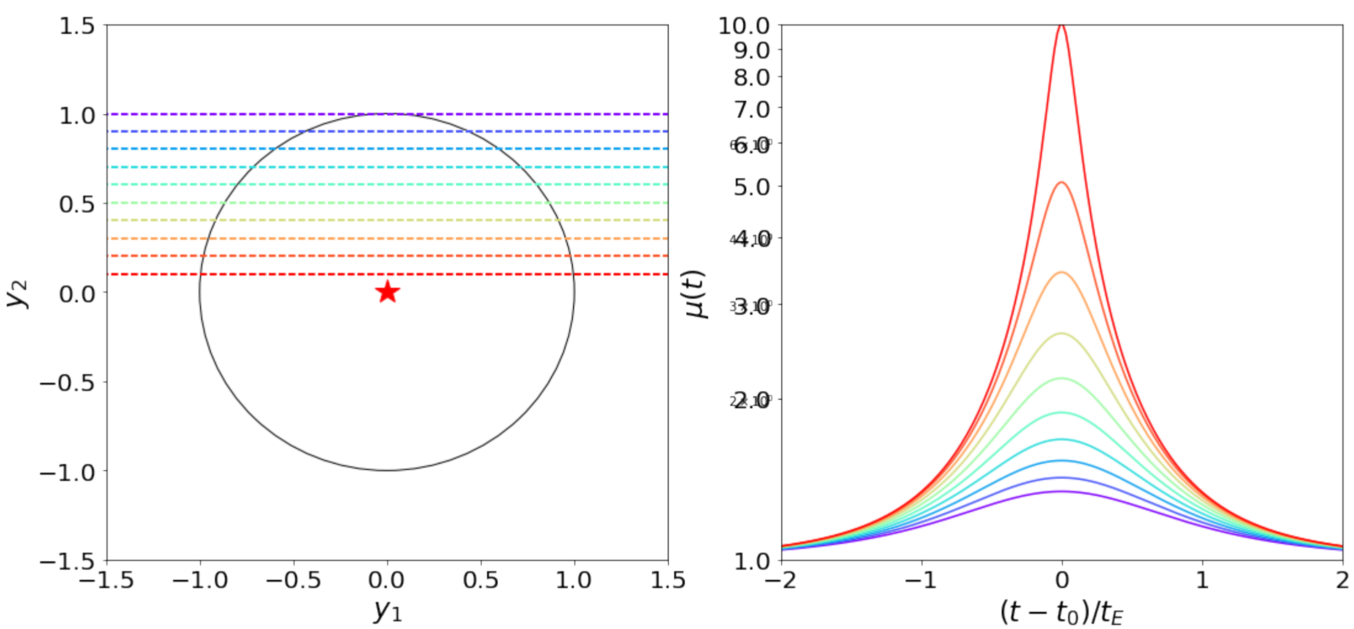
Massimo Meneghetti

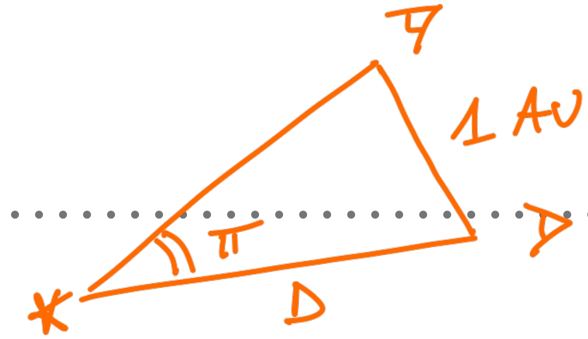
AA 2018-2019

CHANGES IN PERSPECTIVE...

$$y(t) = \sqrt{y_0^2 + y_1^2(t)} = \sqrt{y_0^2 + \frac{(t-t_0)^2}{t_E^2}}$$

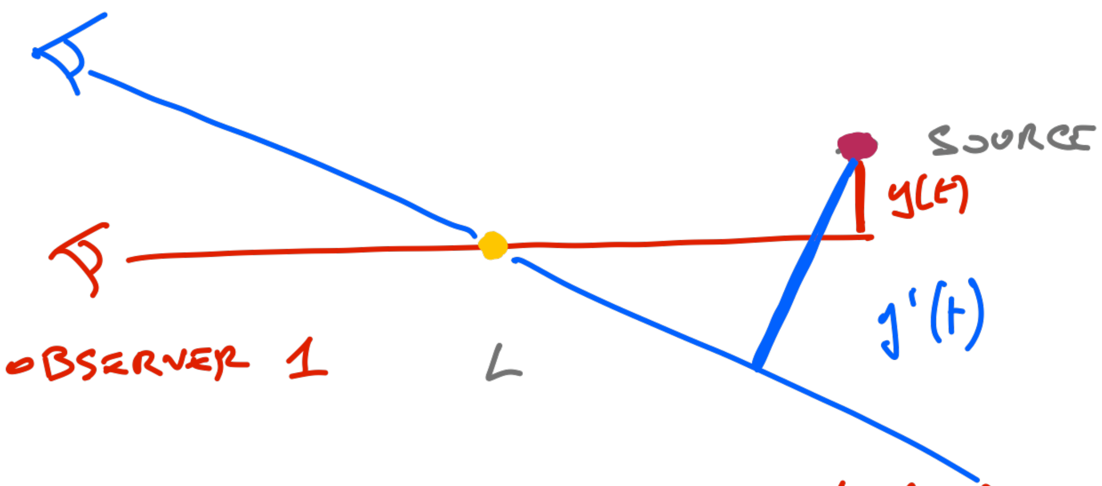
$$\mu(t) = \frac{y^2(t) + 2}{y(t)\sqrt{y^2(t) + 4}}$$





$\pi = \frac{1\,AU}{D}$ PARALLAX

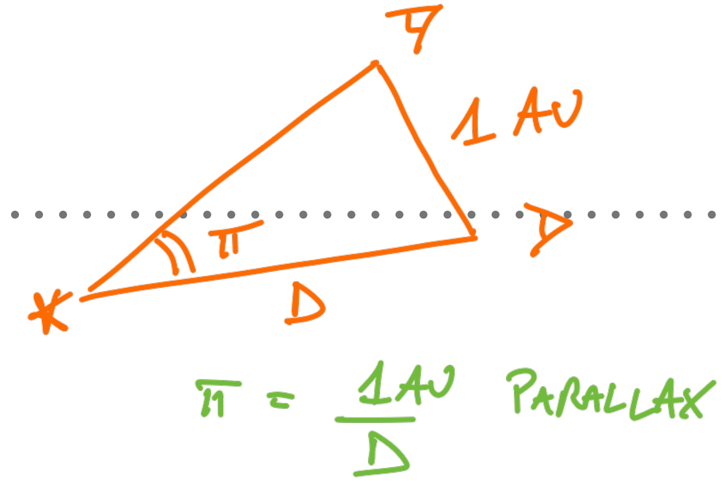
OBSERVER 2



OBSERVER 1

$\mu(y'(t)) \neq \mu(y(t))$

CHANGES IN PERSPECTIVE…



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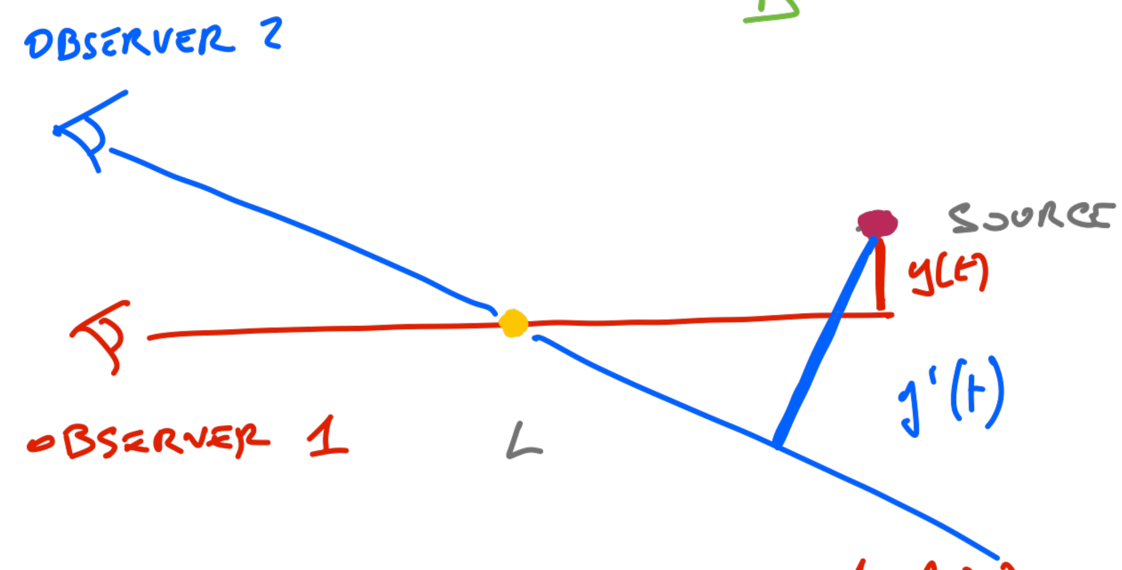
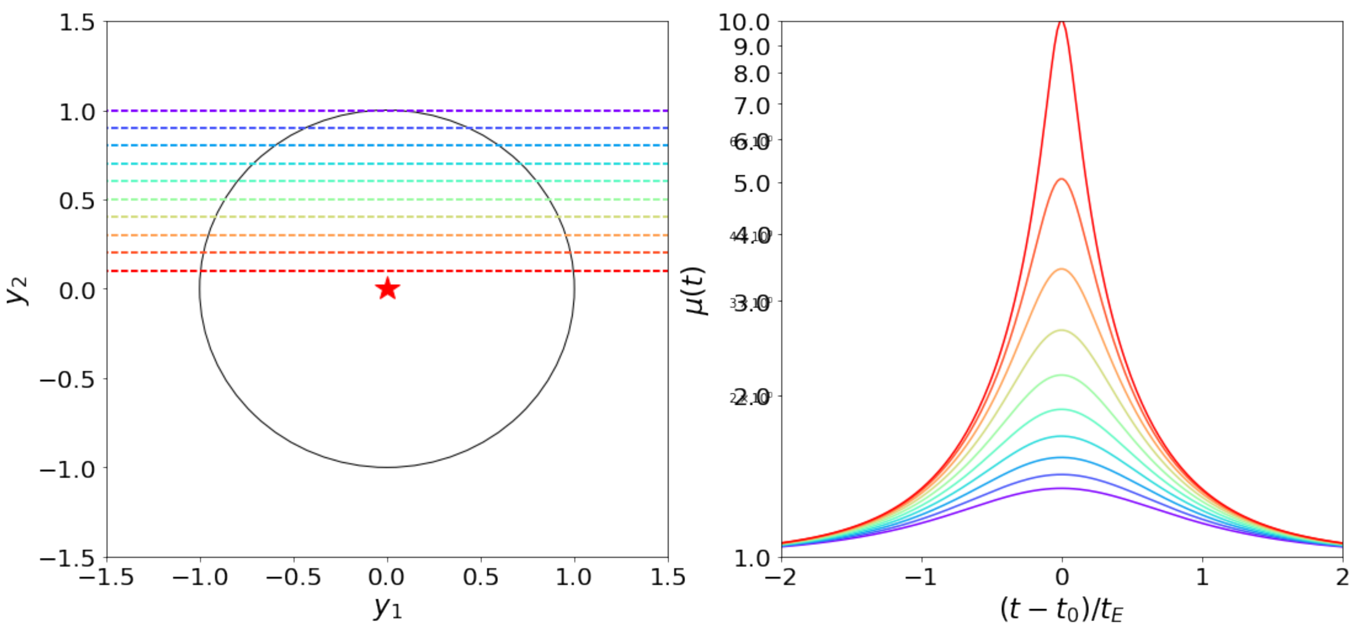
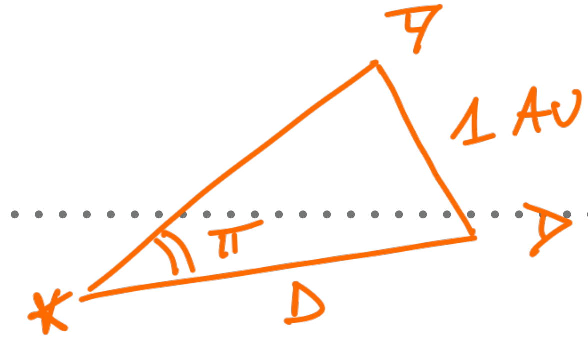


$$\mu(y'(t)) \neq \mu(y(t))$$



Two observers looking at the same Microlensing event will see two different light curves (under some circumstances).

CHANGES IN PERSPECTIVE...



$$\pi = \frac{1\,AU}{D} \quad \text{PARALLAX}$$

$$y(t) = \sqrt{y_0^2 + y_1^2(t)} = \sqrt{y_0^2 + \frac{(t-t_0)^2}{t_E^2}}$$

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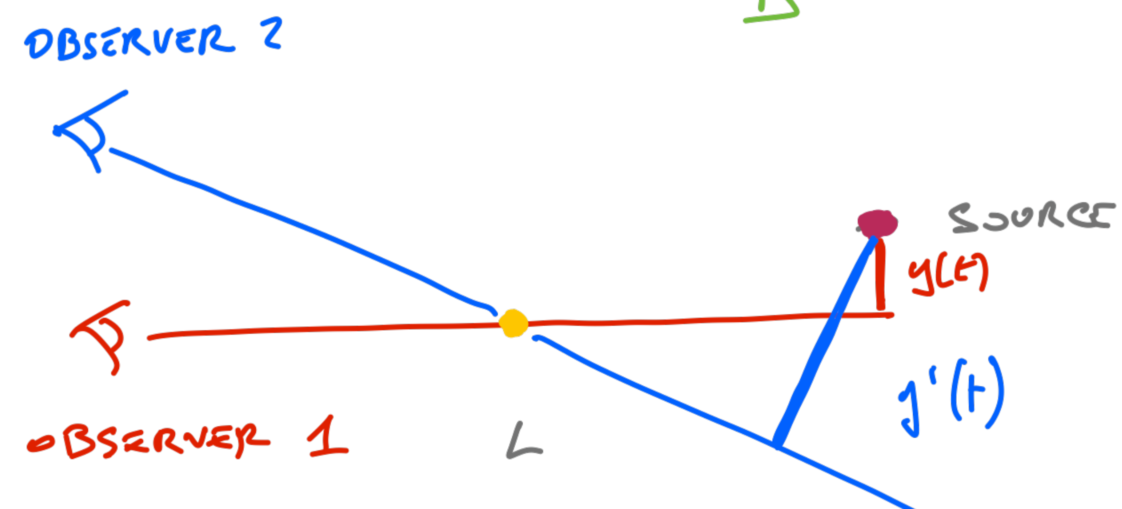

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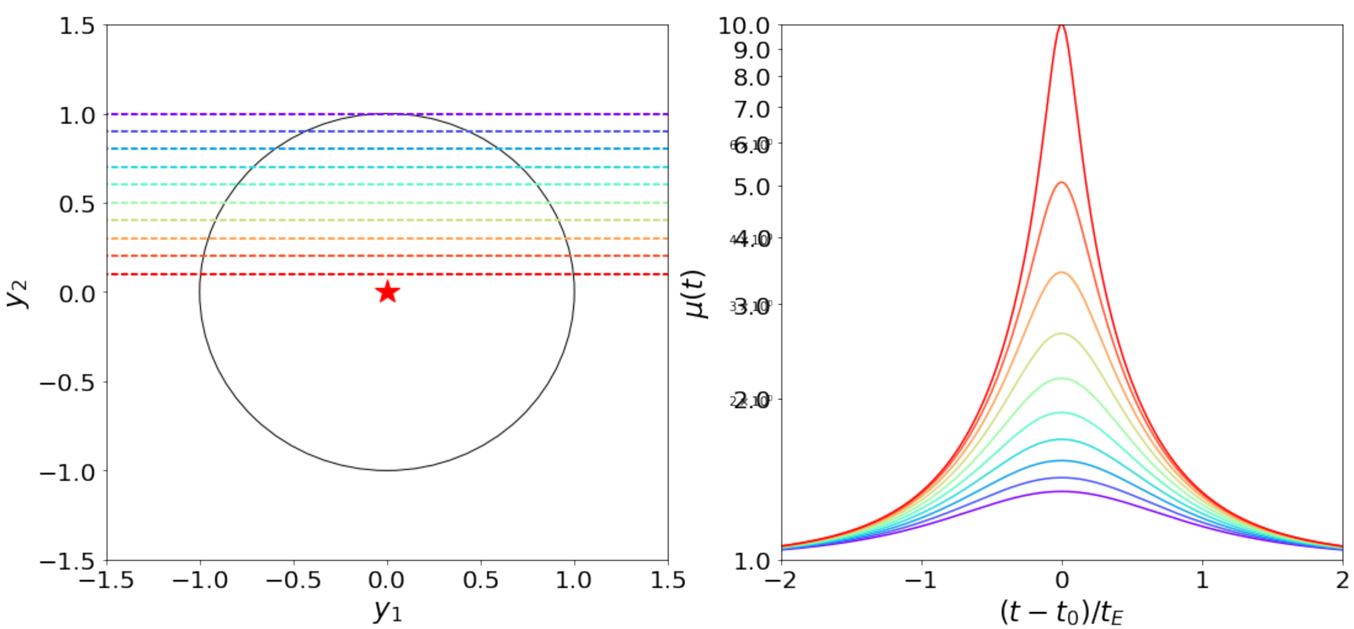


Two observers looking at the same Microlensing event will see two different light curves (under some circumstances).

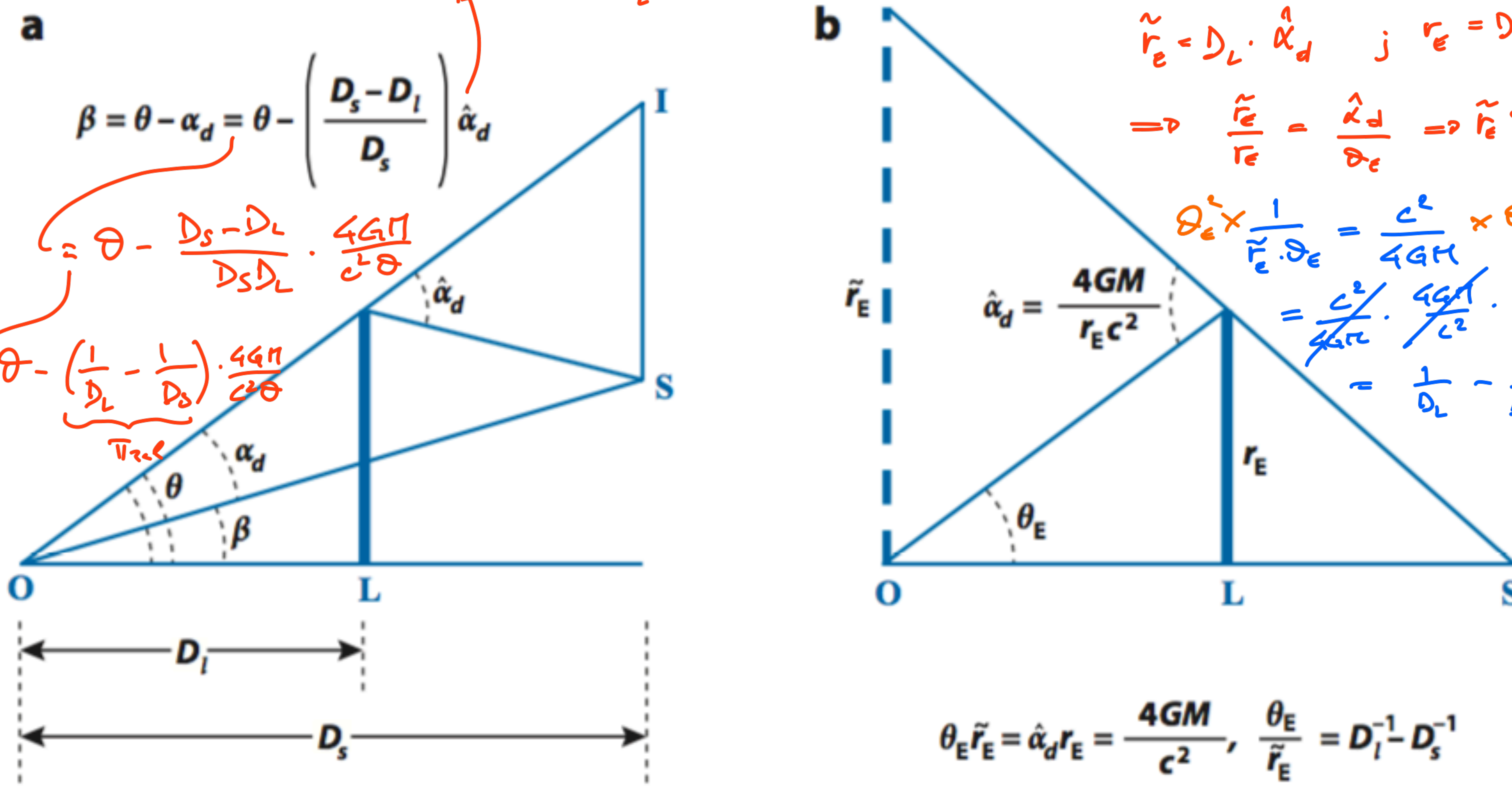
This is a parallax effect! But what is the relevant baseline???

MICROLENSING PARALLAX

$$\theta_E = \sqrt{\frac{4GM}{c^2}\,\frac{D_{LS}}{D_L\cdot D_S}}$$

Gaudi (2012)

for galactic microlensing: $\underline{D_{LS} = D_S - D_L}$



$$\hat{\alpha}_d = \frac{4GM}{c^2\theta\cdot D_L}$$

$$= \theta - \frac{D_S - D_L}{D_S D_L}\cdot\frac{4GM}{c^2\theta}$$

$$= \theta - \underbrace{\left(\frac{1}{D_L} - \frac{1}{D_S}\right)}_{\pi_{rel}}\cdot\frac{4GM}{c^2\theta}$$

$\tilde{r}_E = D_L\cdot\hat{\alpha}_d$; $r_E = D_L\cdot\theta_E$

$$\Rightarrow \frac{\tilde{r}_E}{r_E} = \frac{\hat{\alpha}_d}{\theta_E} \Rightarrow \tilde{r}_E\,\theta_E = r_E\,\hat{\alpha}_d$$

$$\theta_E^2 \times \frac{1}{\tilde{r}_E\cdot\theta_E} = \frac{c^2}{4GM}\times\theta_E^2$$

$$= \cancel{\frac{c^2}{4GM}}\cdot\cancel{\frac{4GM}{c^2}}\cdot\frac{D_S - D_L}{D_L D_S}$$

$$= \frac{1}{D_L} - \frac{1}{D_S}$$

$$\pi_{rel} = \frac{1}{D_L} - \frac{1}{D_S}$$

$$\boxed{\frac{1}{\tilde{r}_E} = \frac{\pi_{rel}}{\theta_E} \equiv \pi_E}$$

MICROLENSING PARALLAX

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$$\theta_E = \frac{4G}{c^2} M \frac{\pi_{rel}}{\theta_E} = kM\pi_E \qquad k = 8.15\text{mas } M_\odot^{-1}$$

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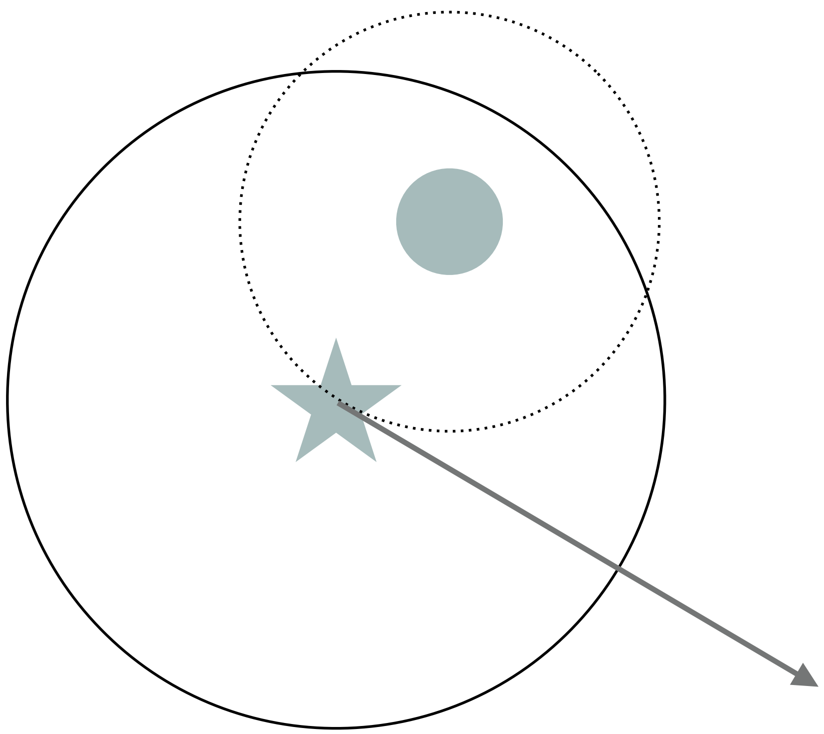
$$M = \frac{\theta_E}{k\pi_E}$$

If we can measure the microlensing parallax, then we measure the mass as a function of the distance. If we can measure θ_E, then we break the lensing degeneracy and measure the lens mass (without even seeing the lens light!)

MICROLENSING PARALLAX

- This may happen for different reasons (several kind of parallax effects).
- Due to the motion of the observer:
  - annual/orbital parallax: the earth moves around the sun
- Due to the separation between observers
  - satellite parallax: when we can look at the same microlensing event from two positions simultaneously from a space telescope and from the earth
  - terrestrial parallax: when we can look at the same microlensing event from two positions on the earth

IMPORTANT: MICROLENSING PARALLAX IS A VECTOR!



Magnification does not depend on the direction of proper motion

$$\mu(t) = \frac{y^2(t) + 2}{y(t)\sqrt{y^2(t) + 4}}$$

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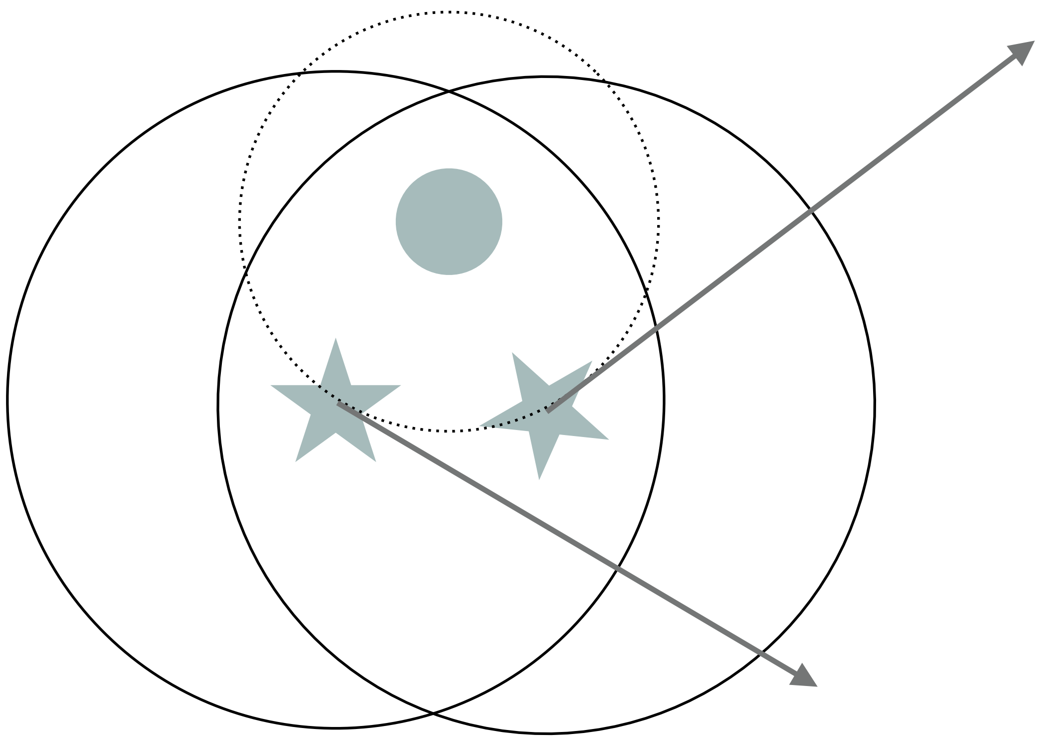

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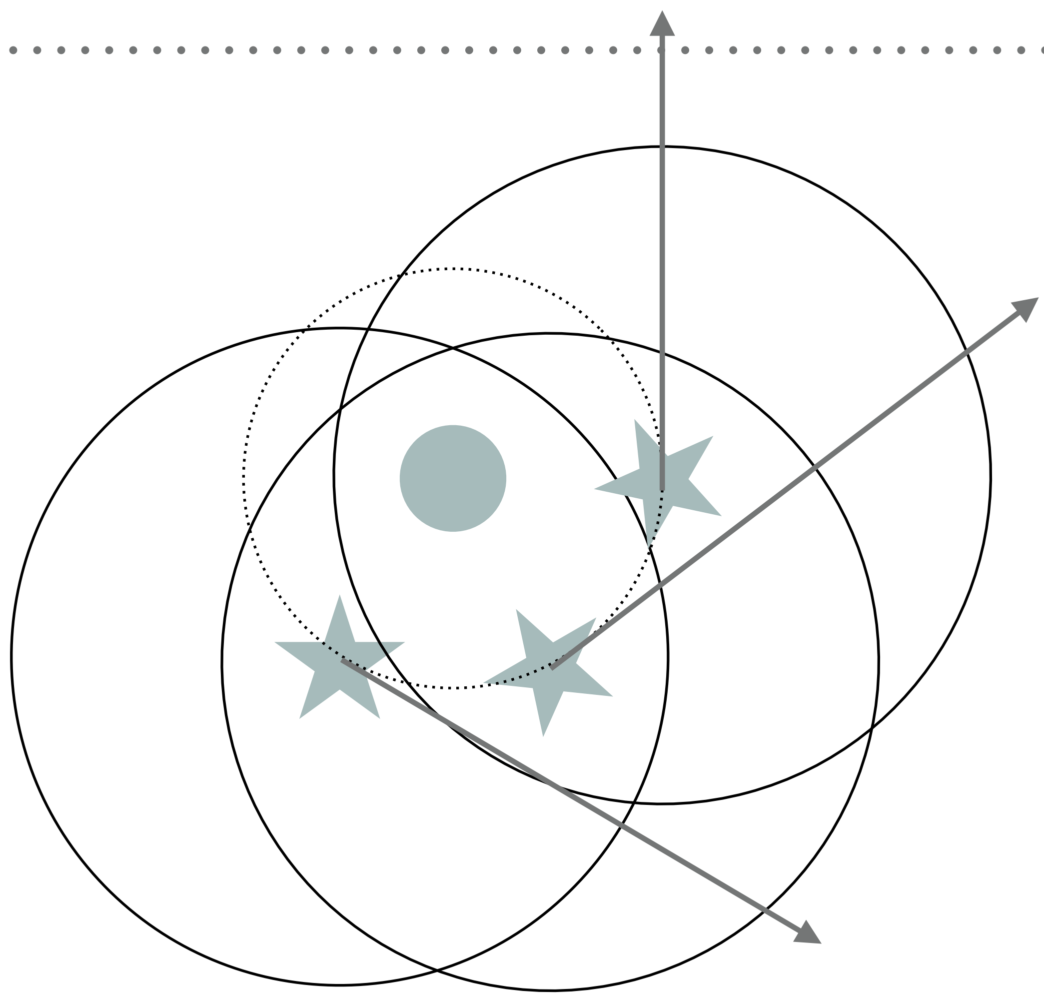

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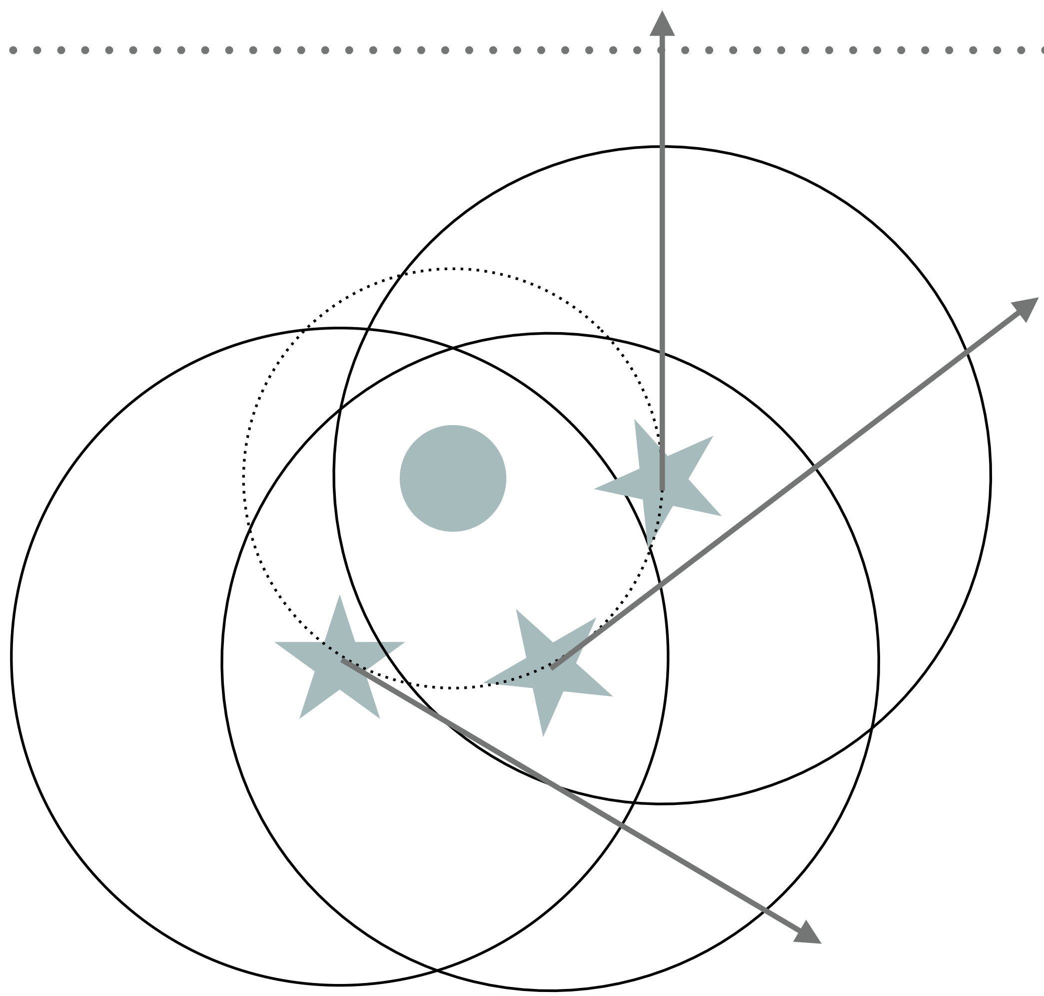

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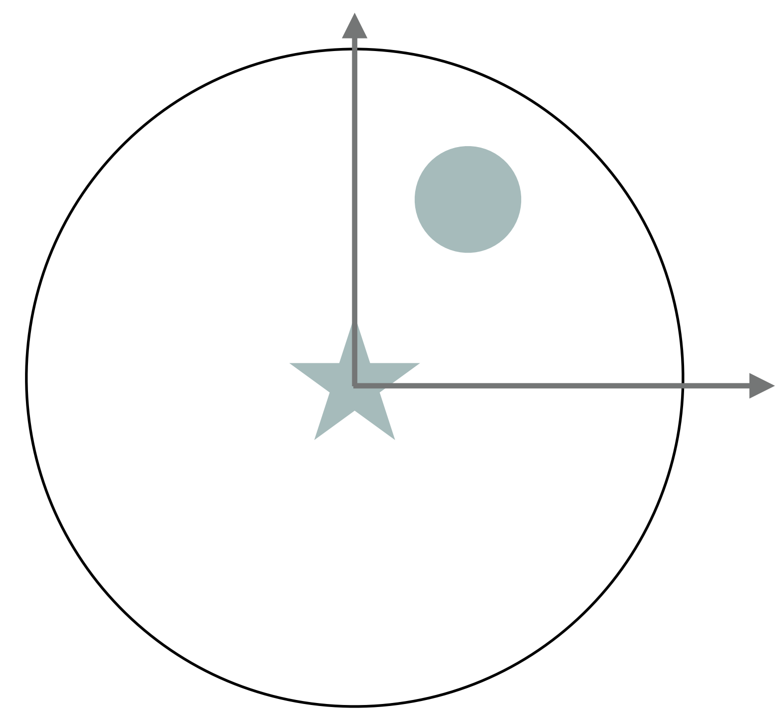
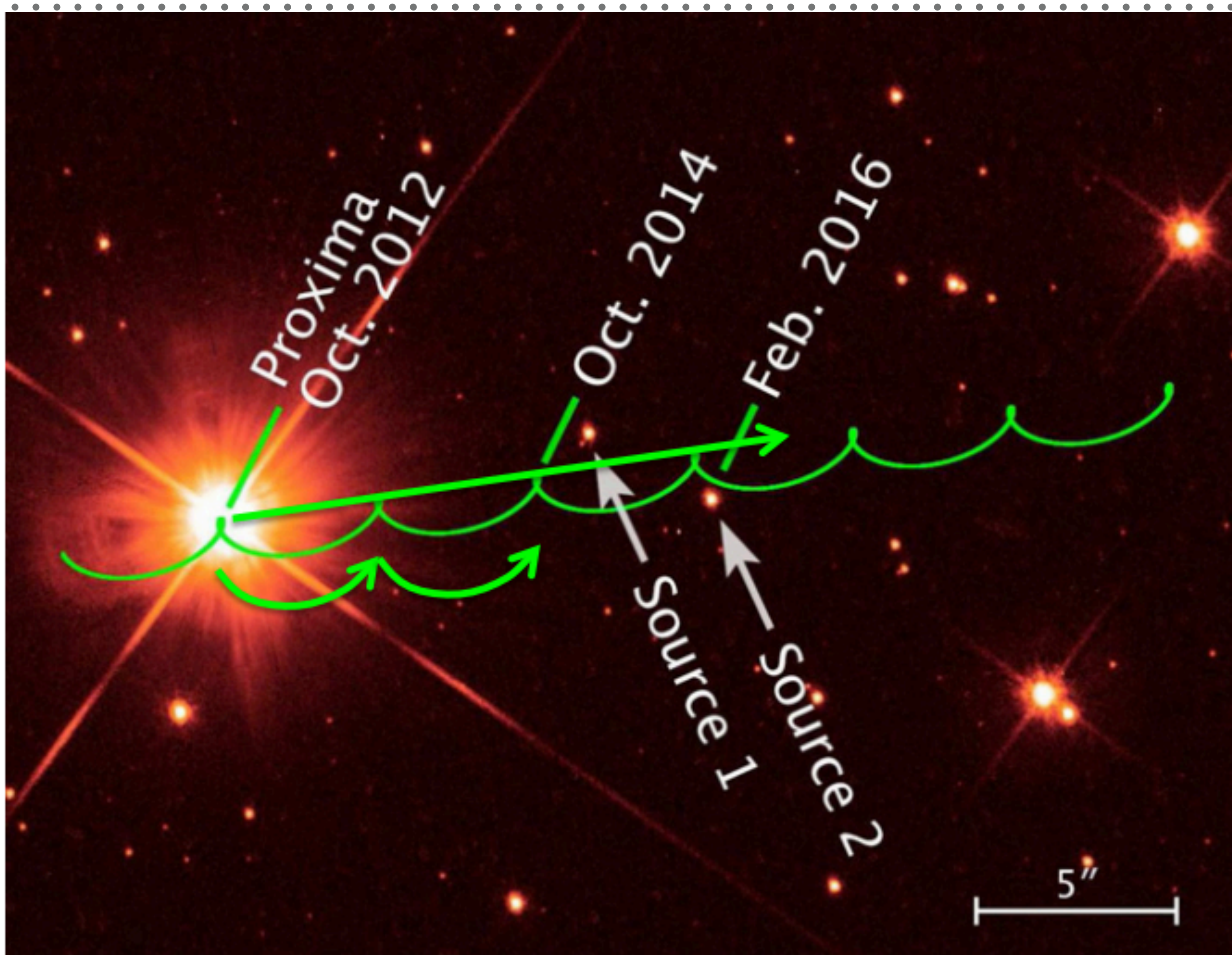
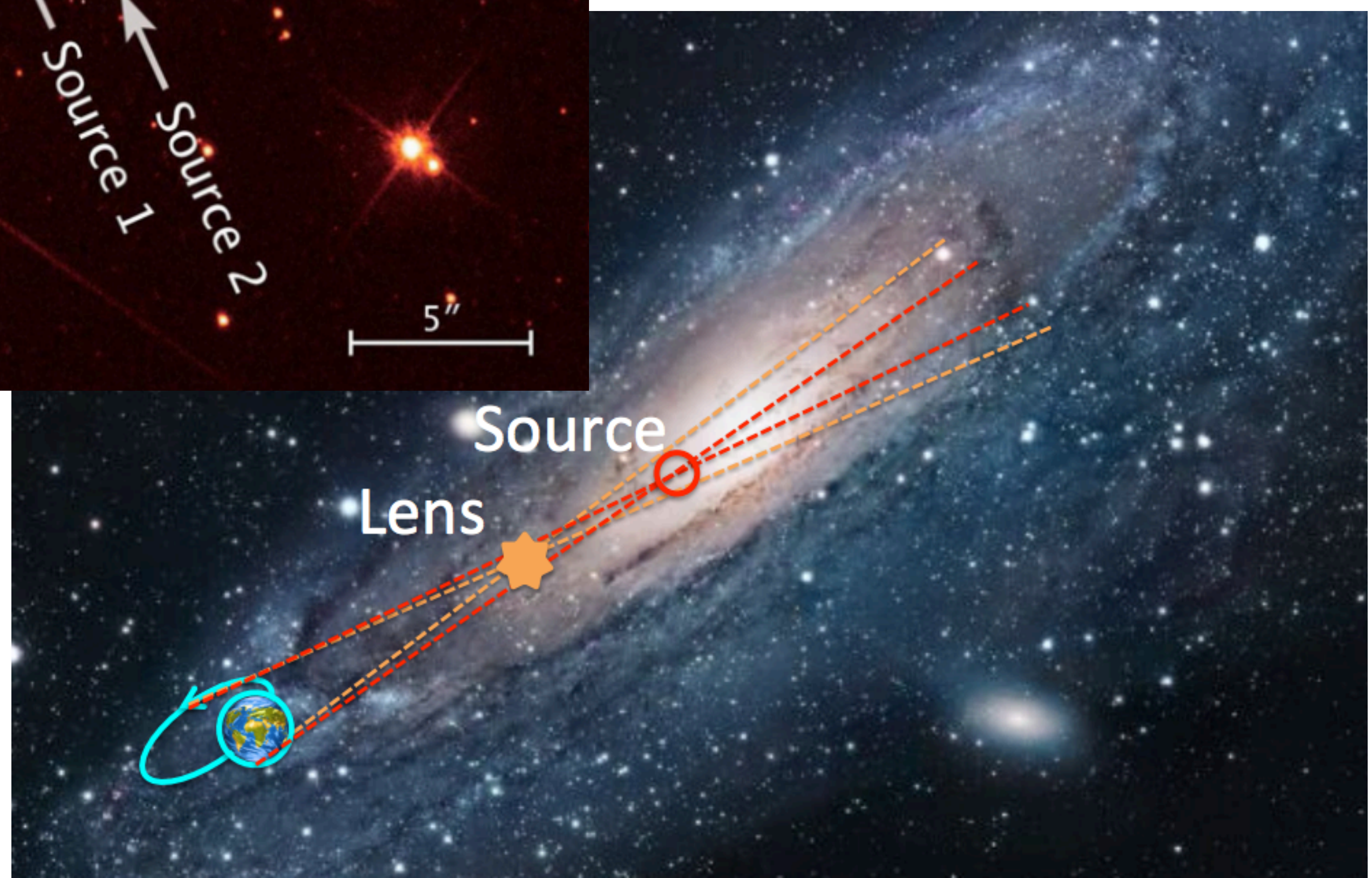


…but microlensing parallax does!

Depending on the lens displacement relative to the source (parallel or perpendicular to the proper motion), we will see different effects

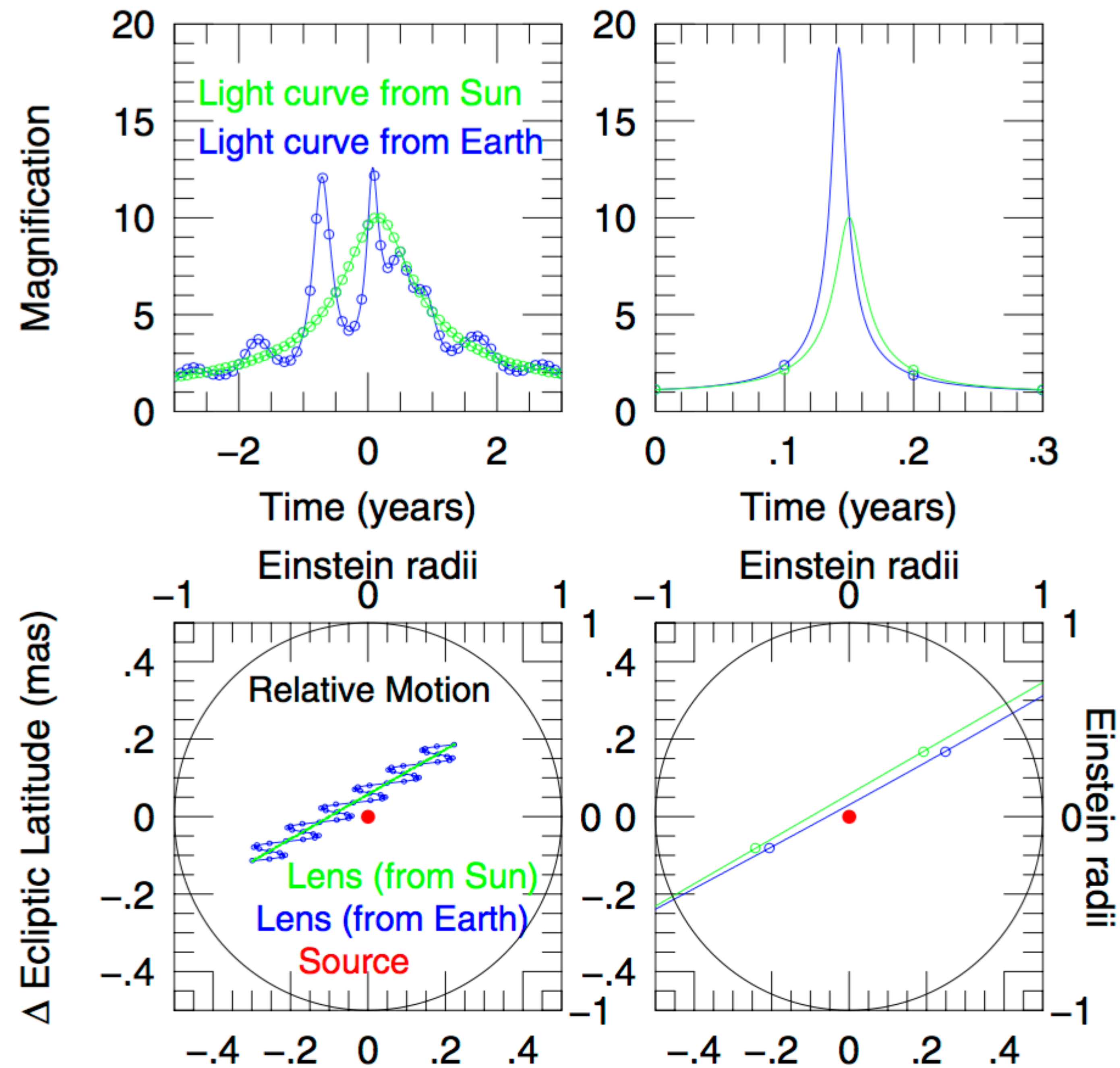
ORBITAL PARALLAX





ORBITAL PARALLAX

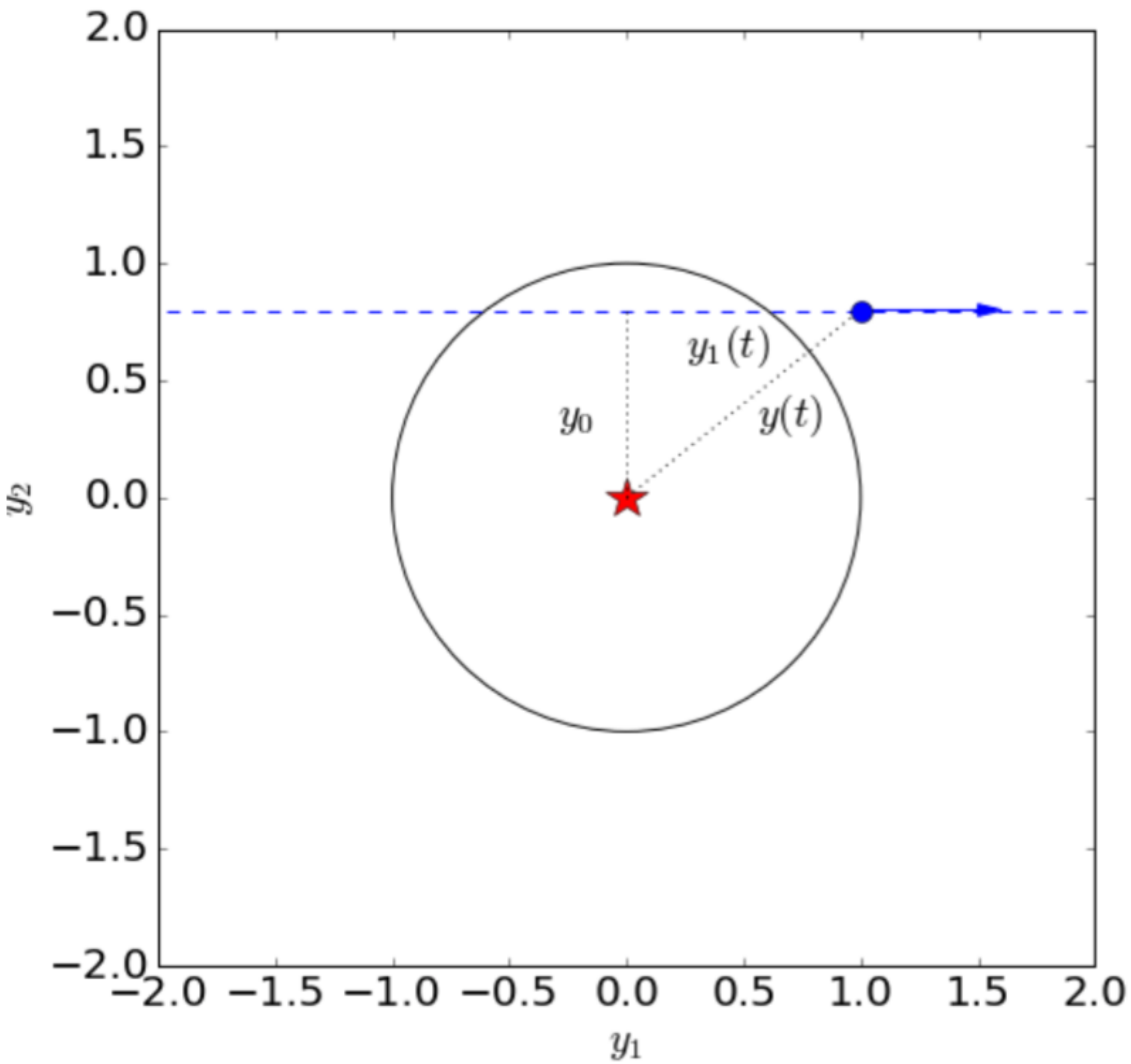
- on the left: what we would see if the μ_{hel}=0.1 mas/year
- on the right: the typical μ_{hel}=5 mas/year

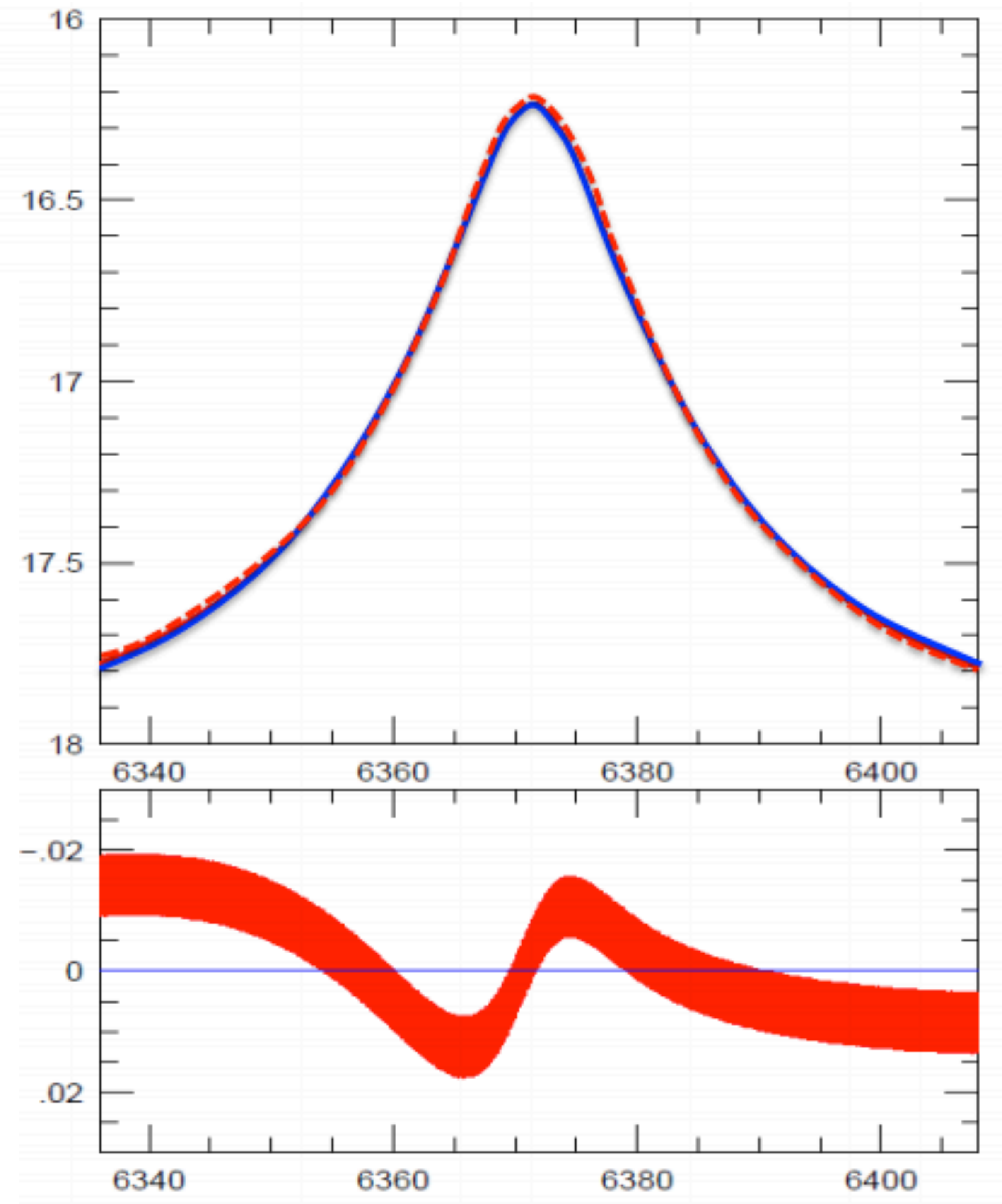


Gould & Horne, 2013

COMPONENT PARALLEL TO THE LENS TRAJECTORY

Asymmetric distortion of the light curve due to acceleration of the lens





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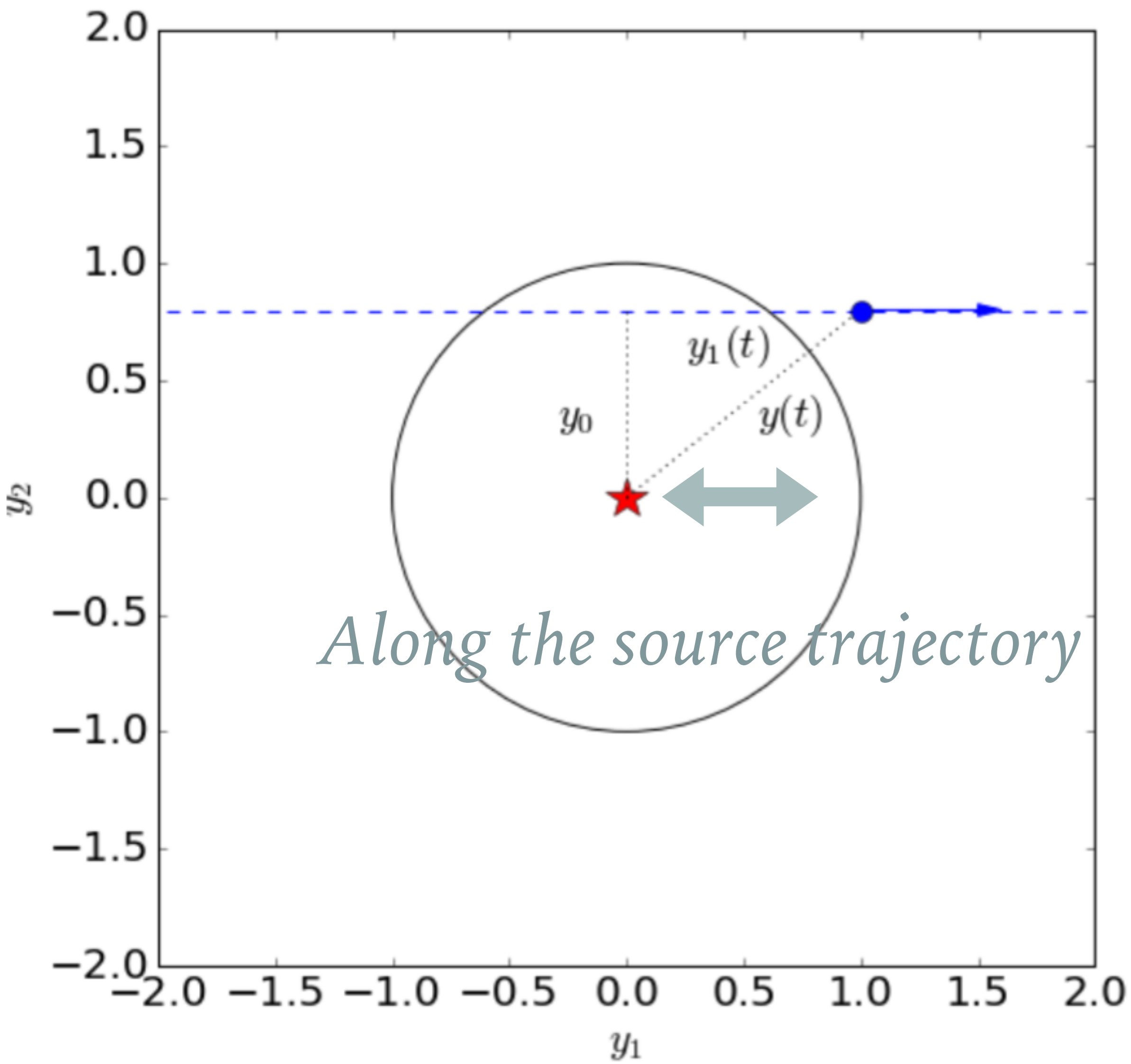
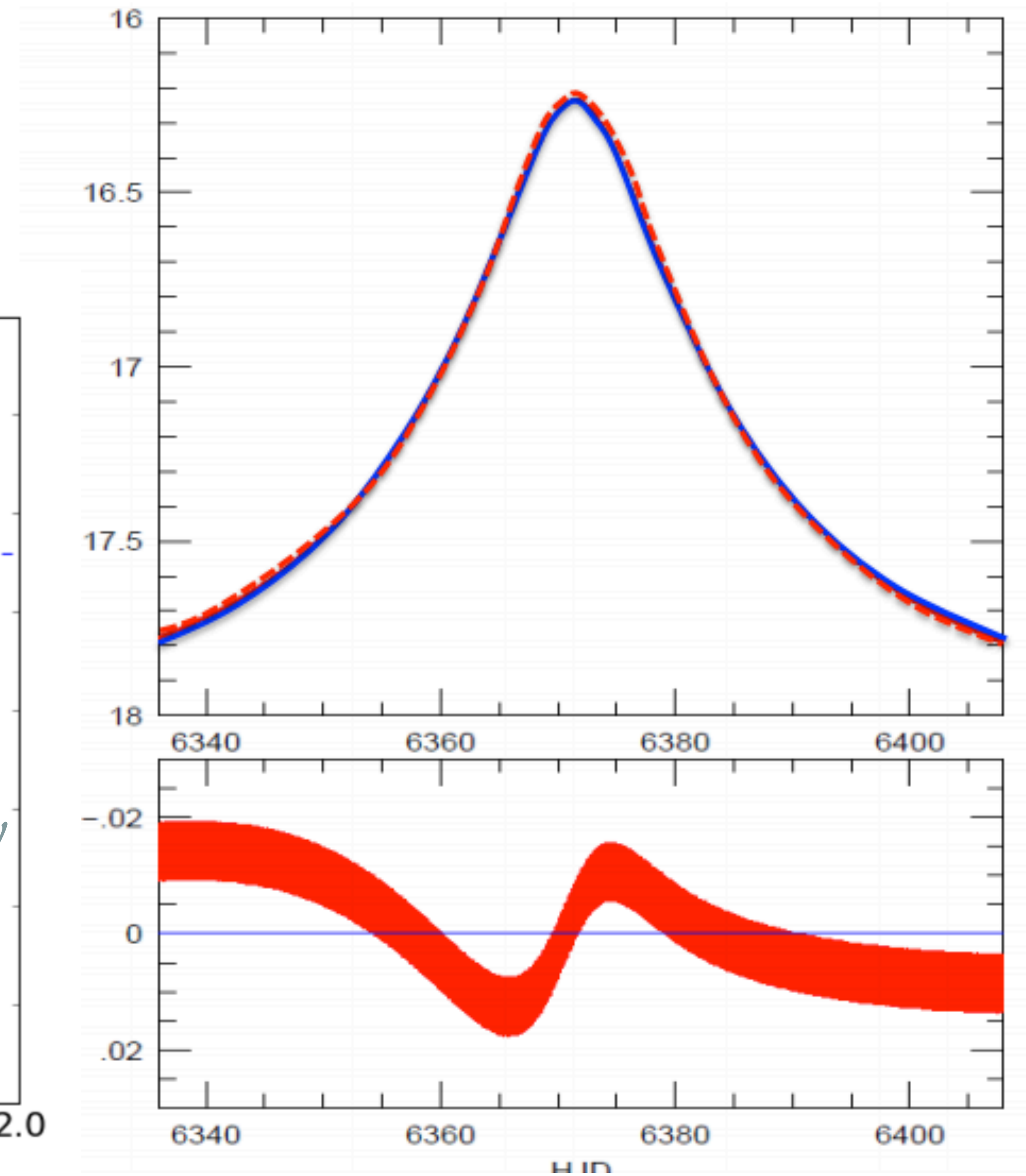
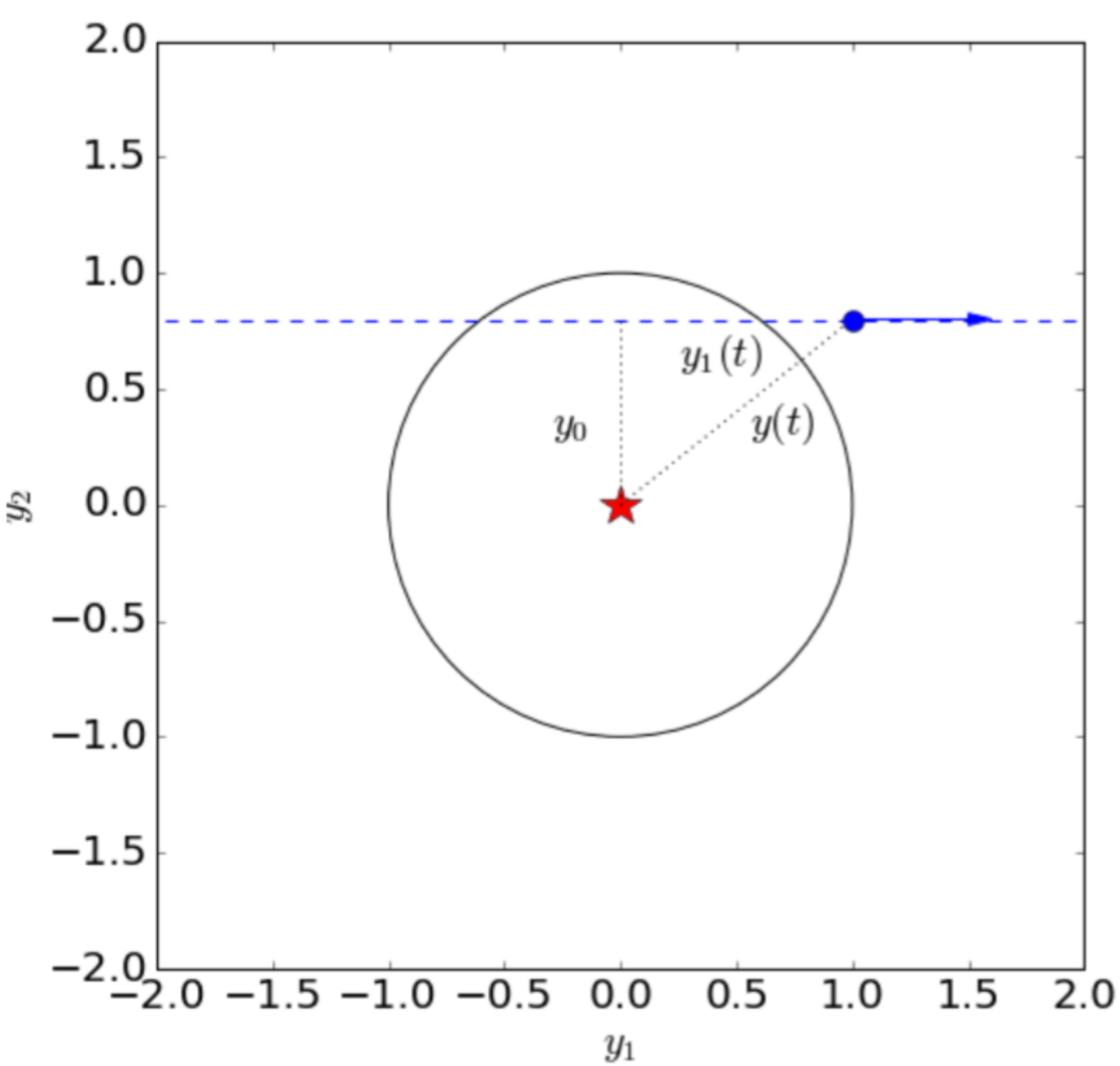
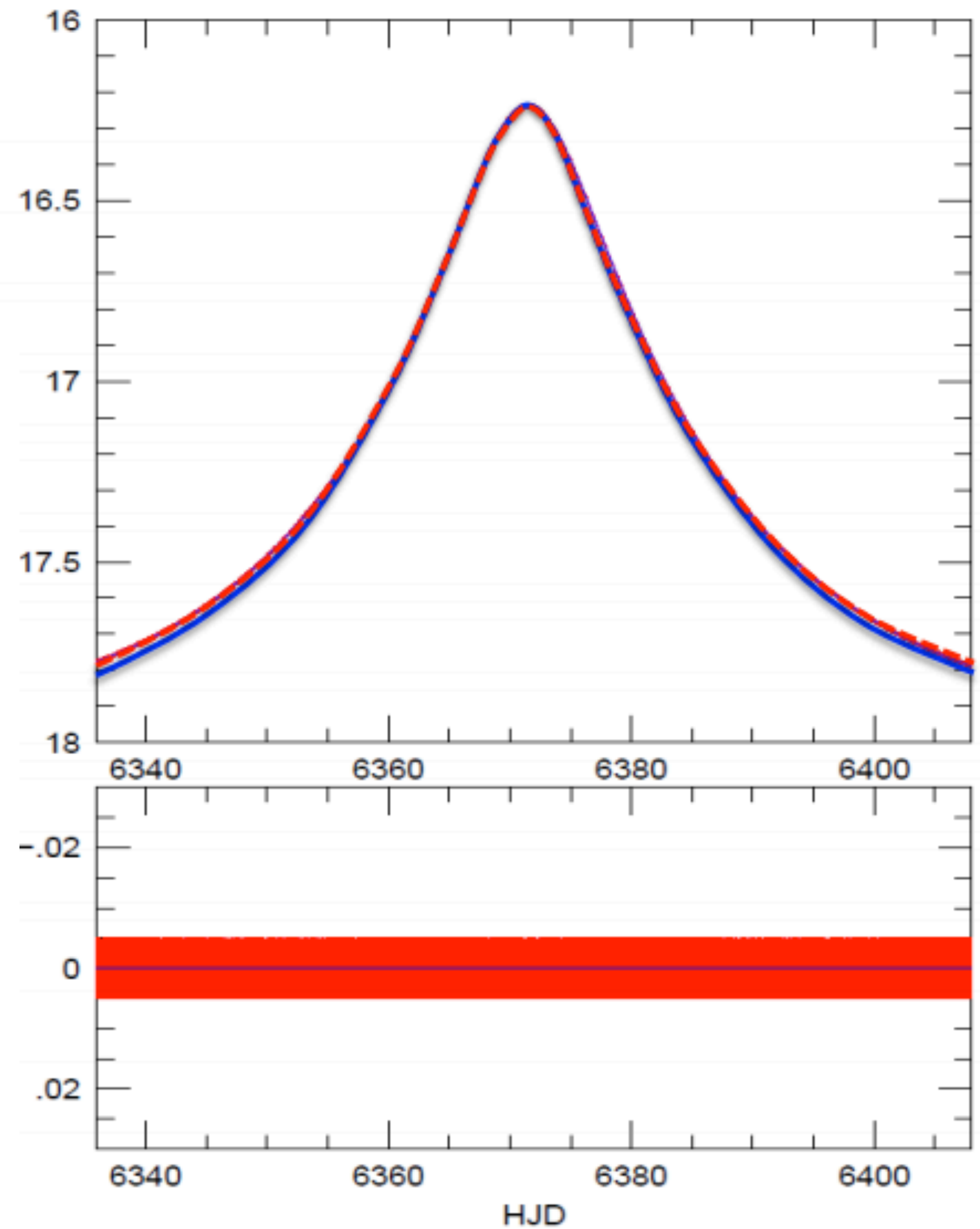




COMPONENT PERPENDICULAR TO THE LENS TRAJECTORY

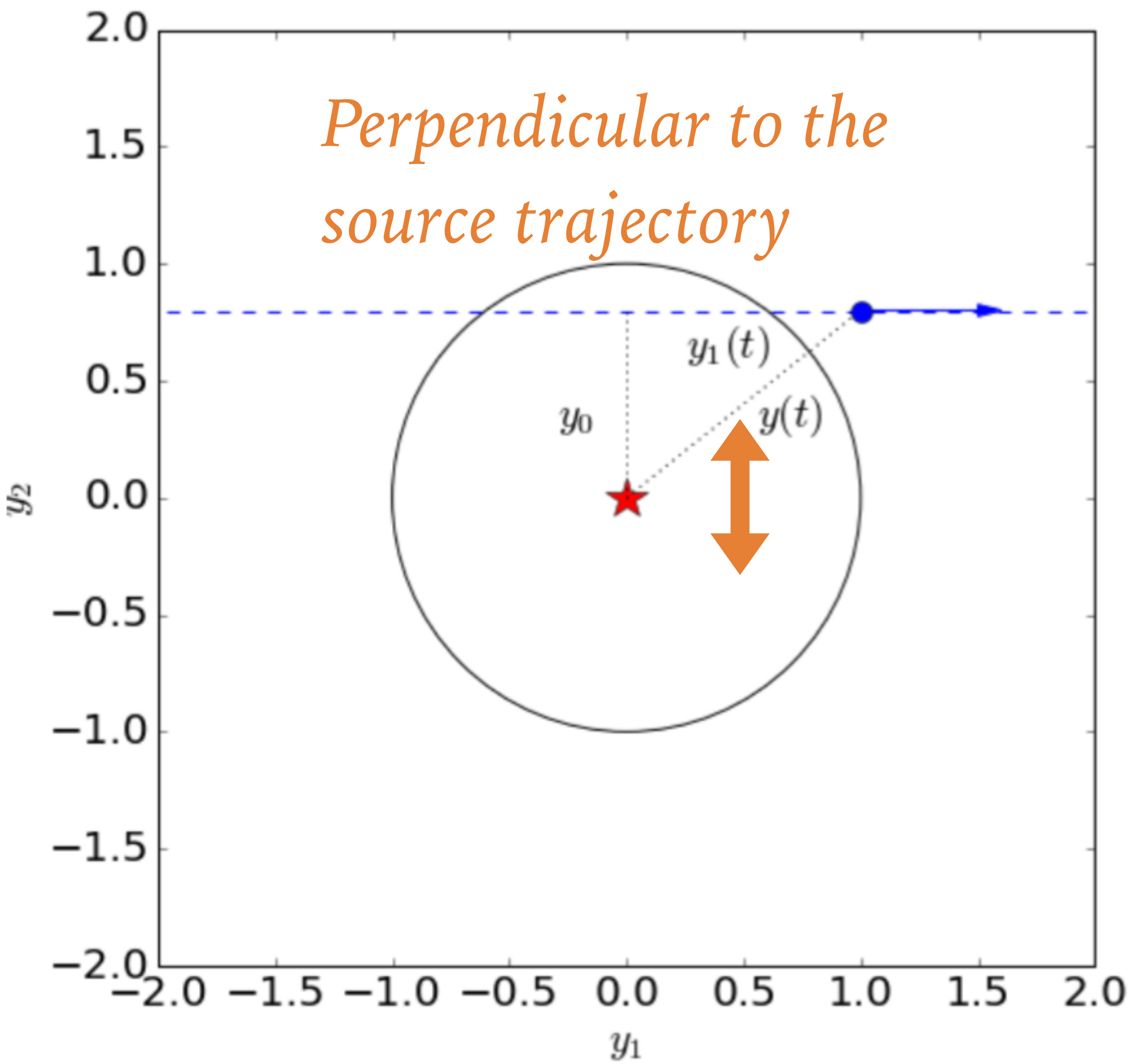
Symmetric distortion of the light curve due to motion perpendicular to lens trajectory

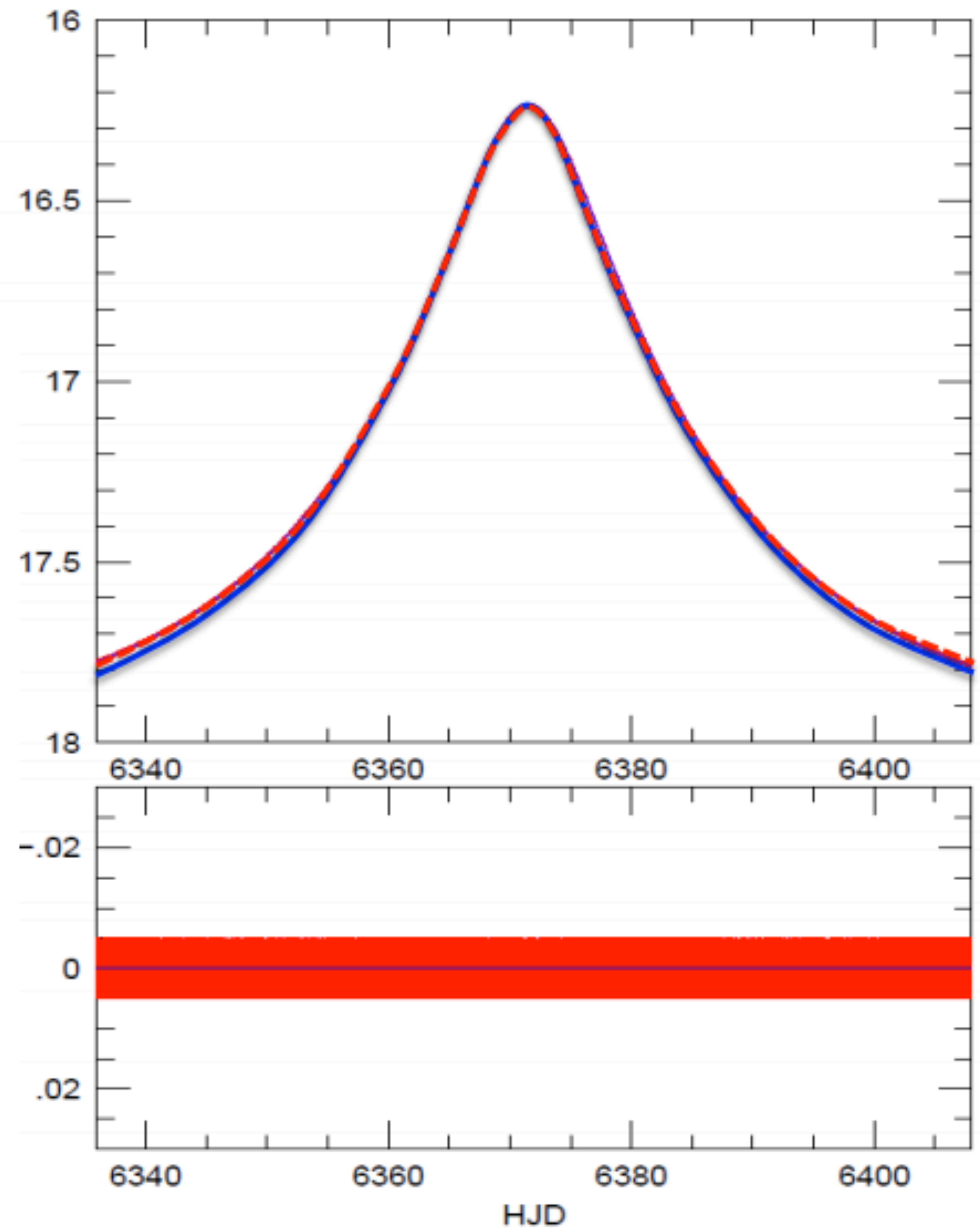




COMPONENT PERPENDICULAR TO THE LENS TRAJECTORY

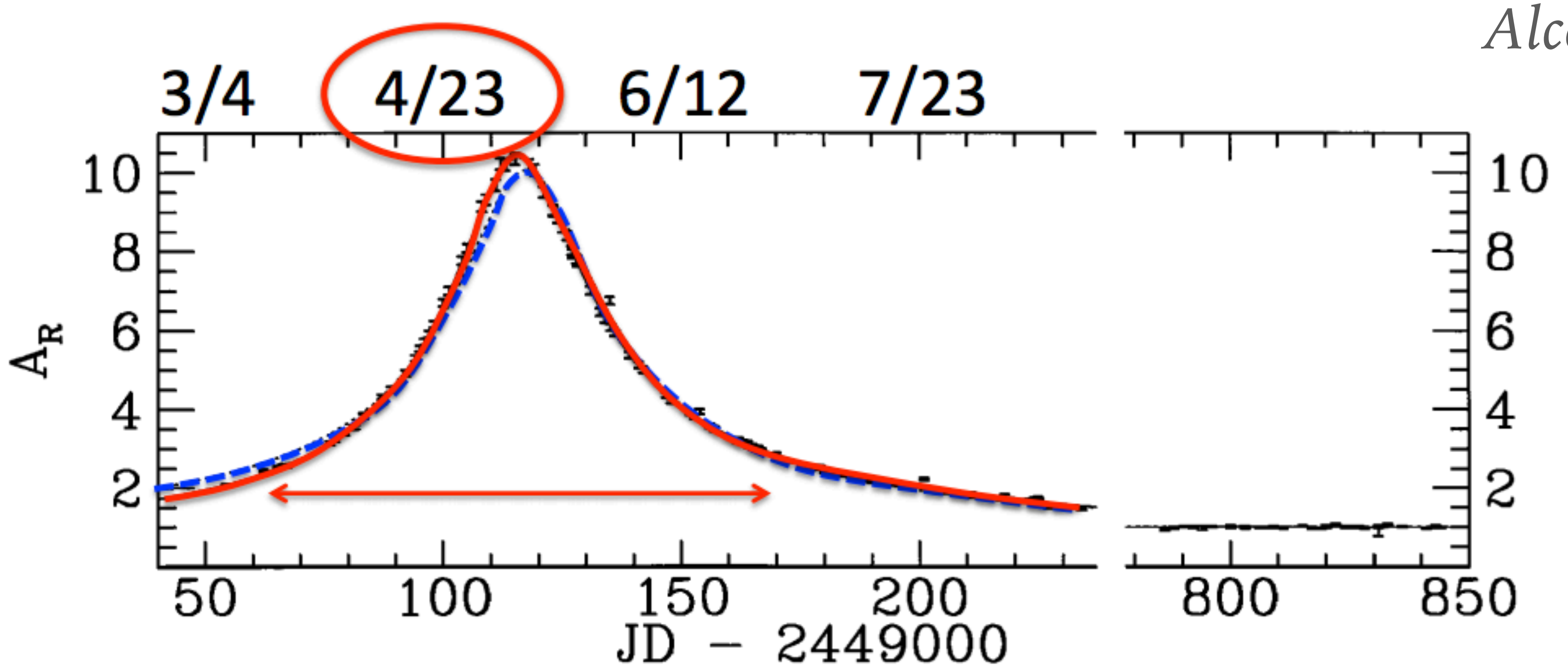
Symmetric distortion of the light curve due to motion perpendicular to lens trajectory



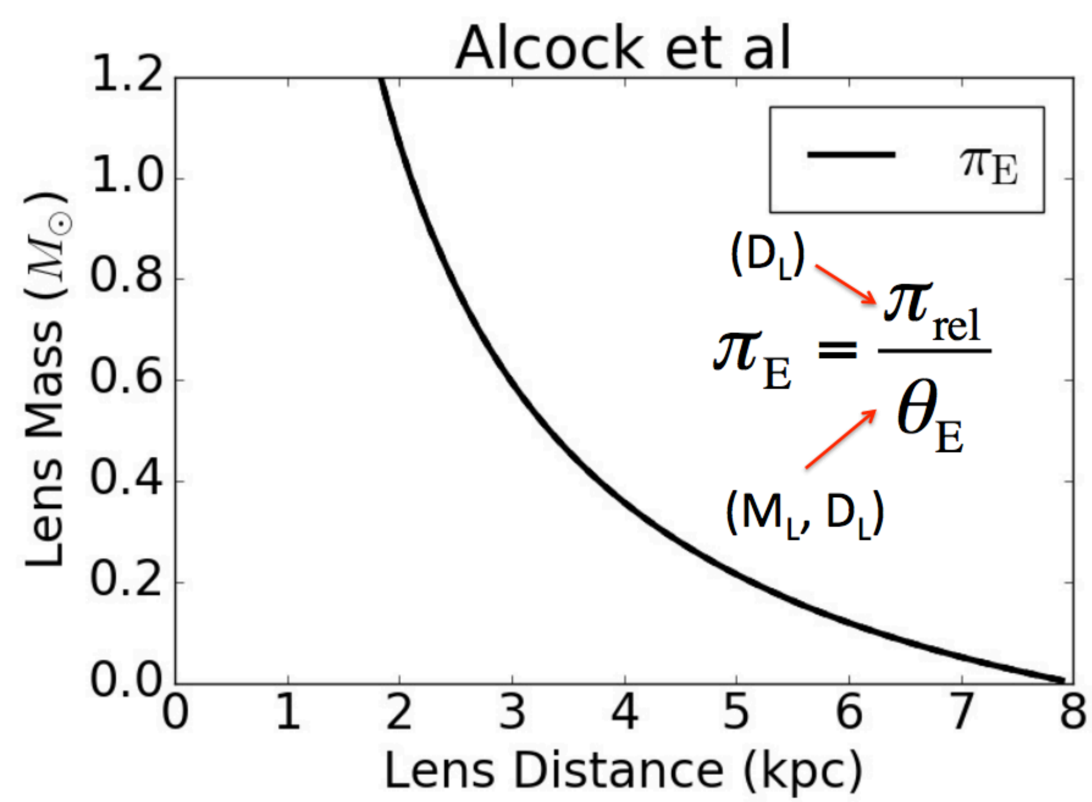


FIRST DETECTION OF MICROLENS PARALLAX

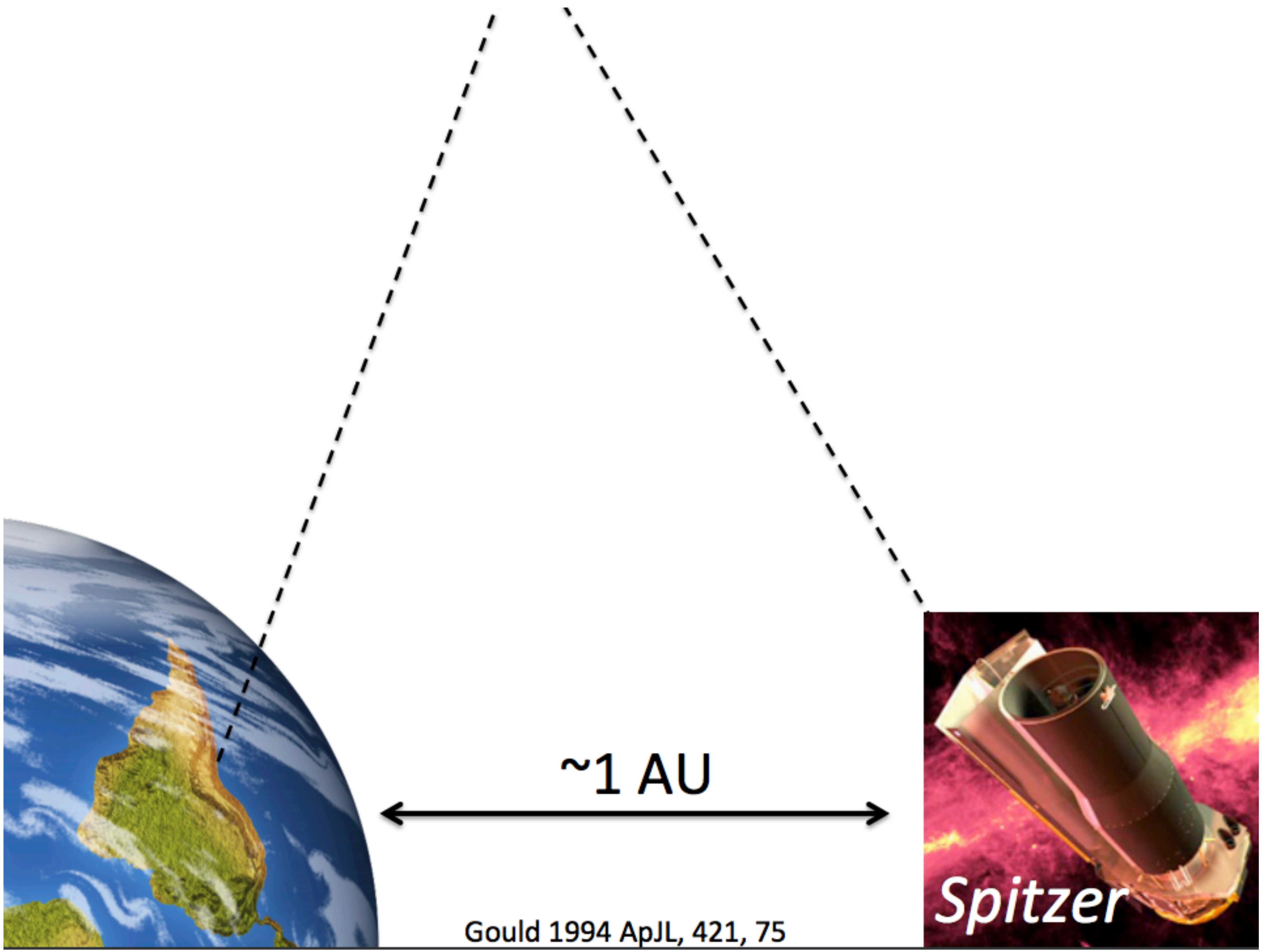
Alcock et al. 1995



Even without an estimate of θ_E, measuring the parallax still allows to measure the mass vs distance



SATELLITE PARALLAX



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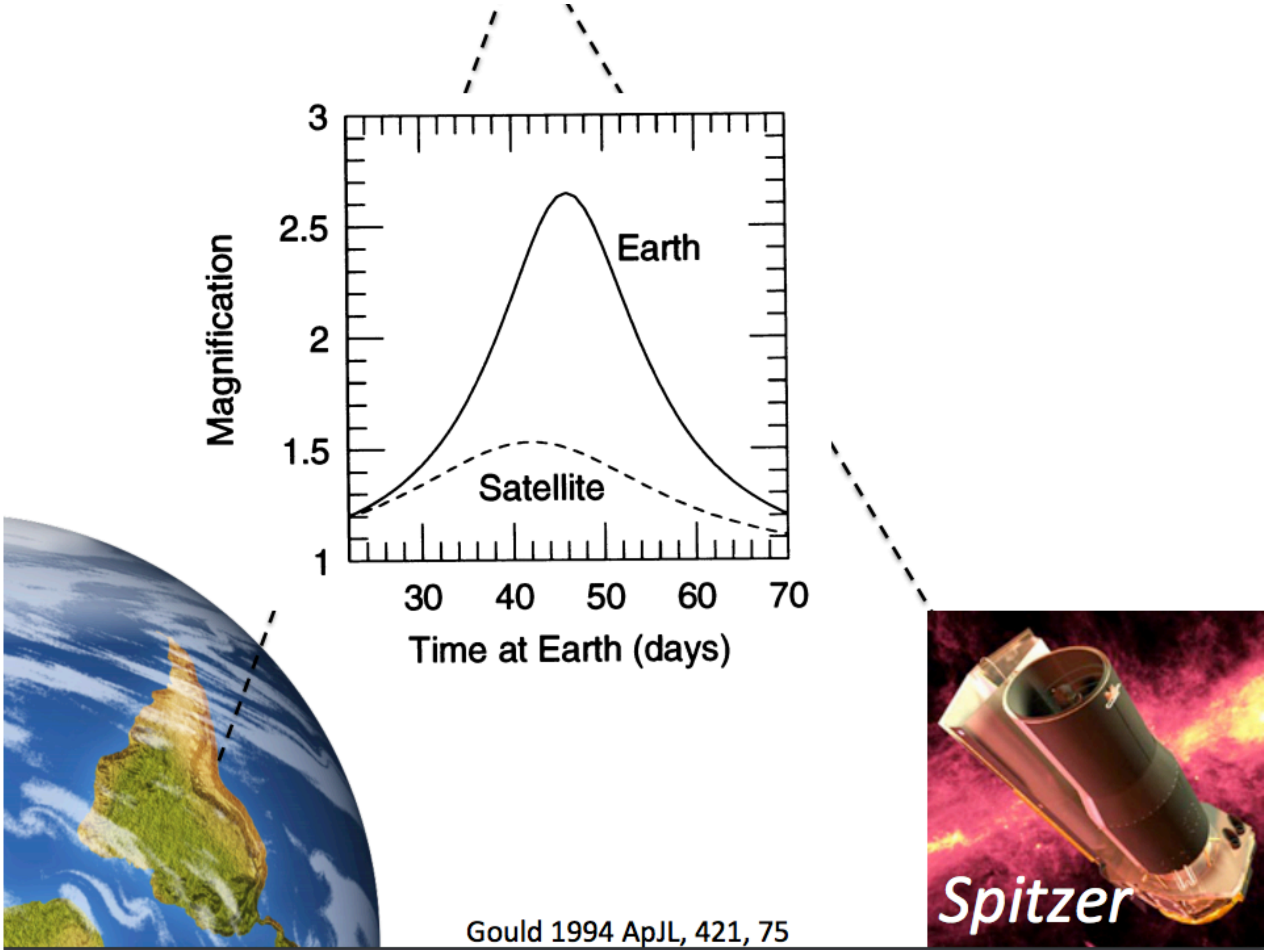

Gould 1994 ApJL, 421, 75

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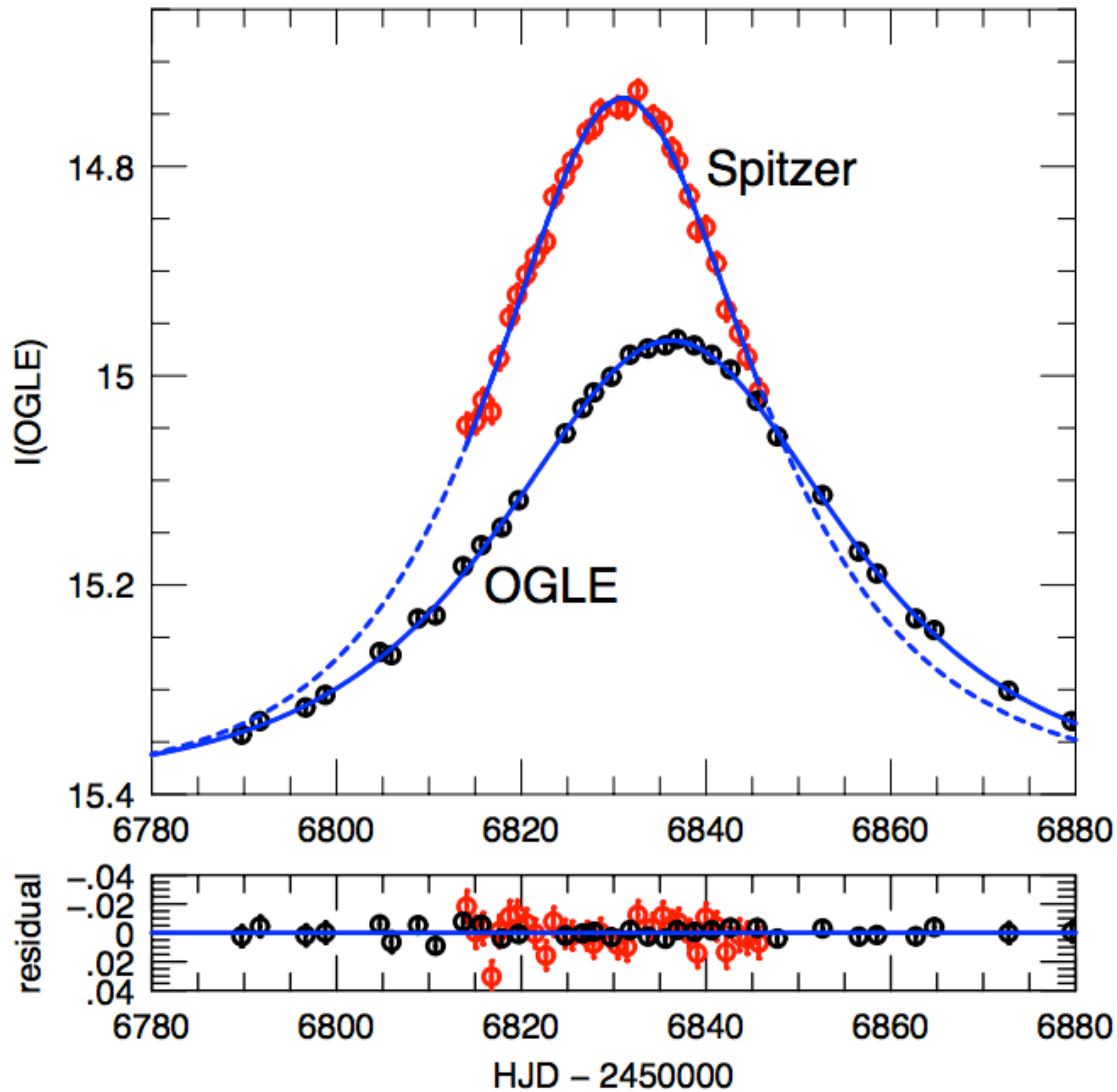
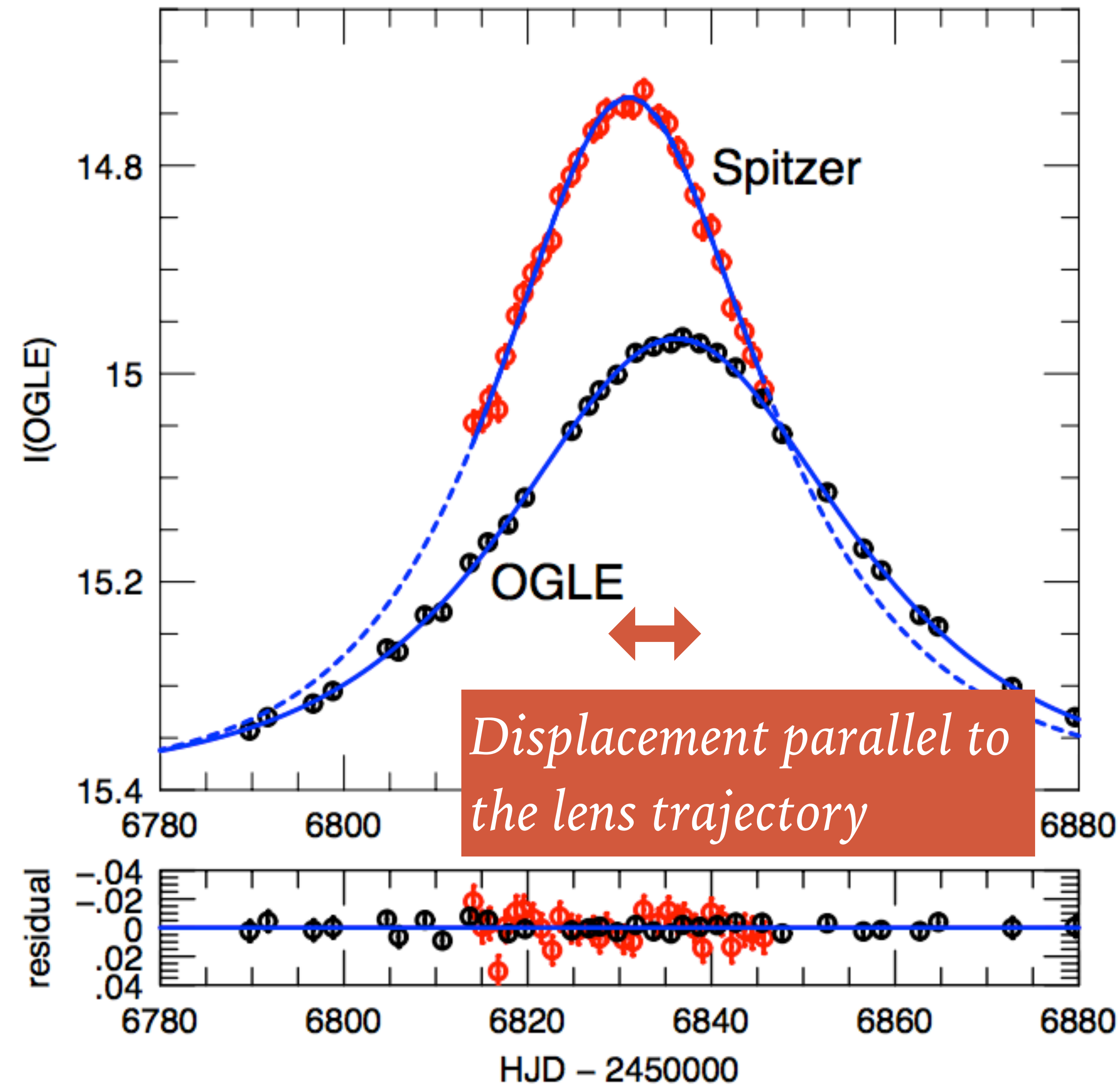


Yee et al. 2015, ApJ, 802, 76

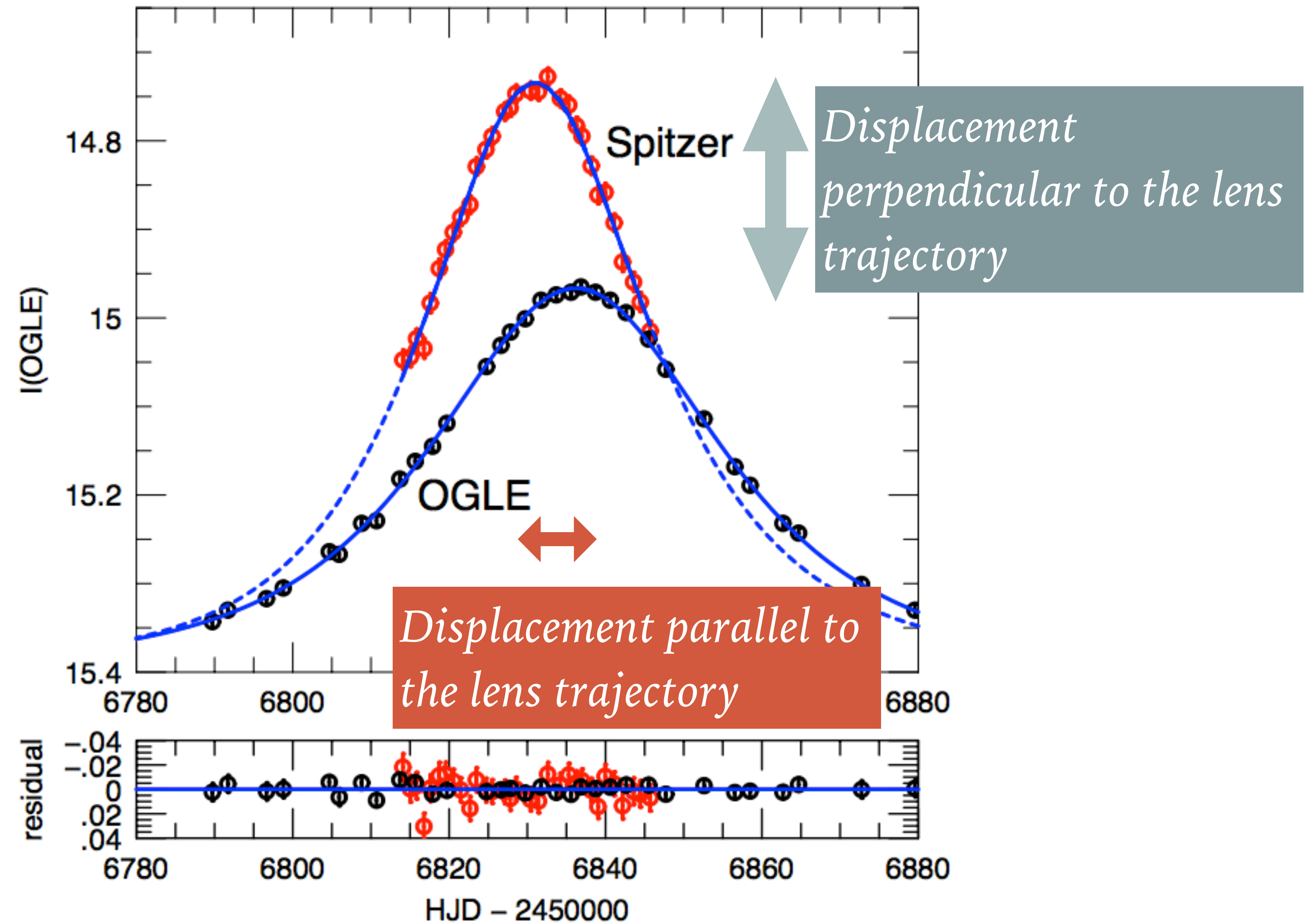
SATELLITE PARALLAX



Displacement parallel to the lens trajectory

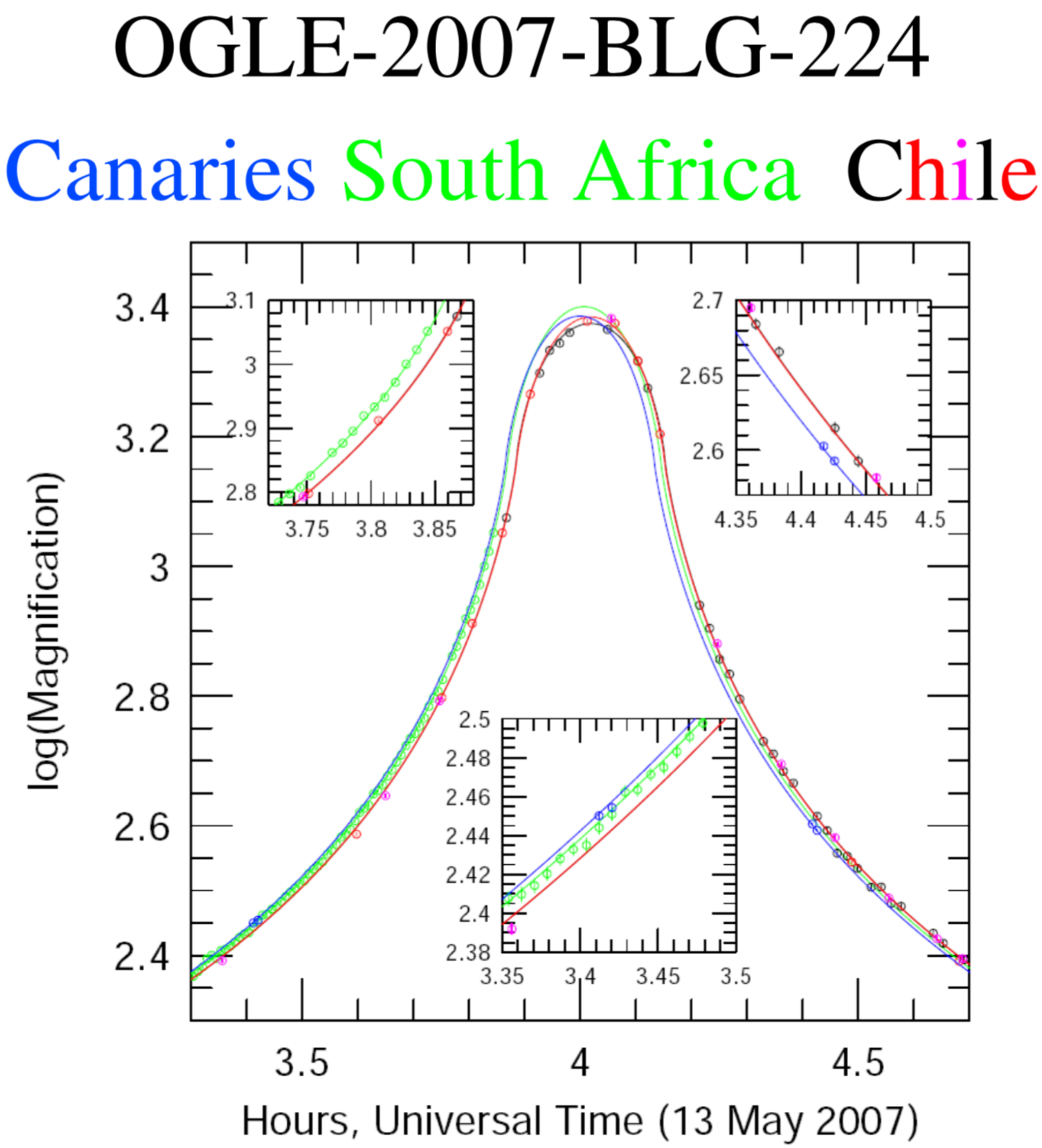
Yee et al. 2015, ApJ, 802, 76

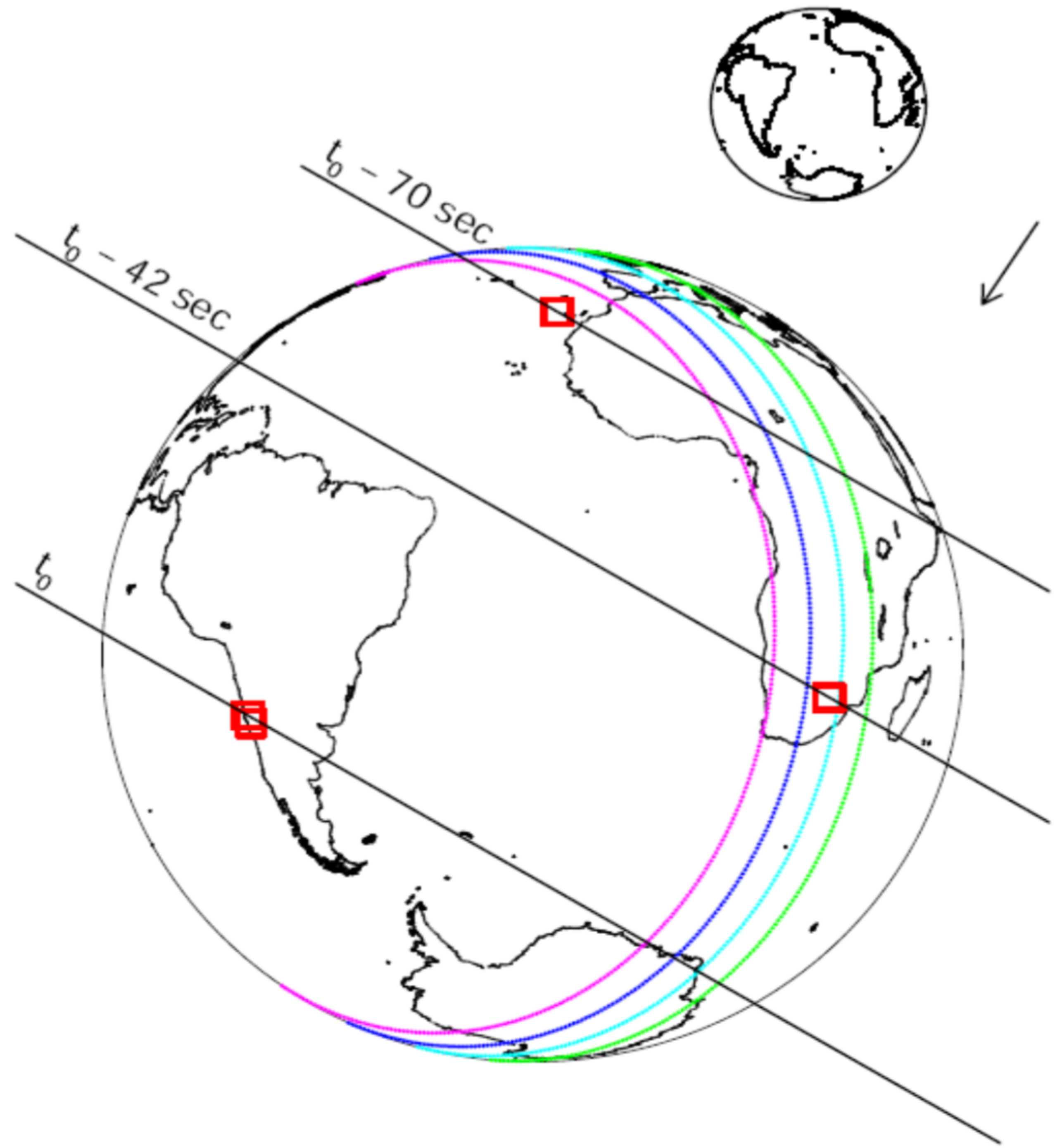
SATELLITE PARALLAX



Yee et al. 2015, ApJ, 802, 76

MICROLENS PARALLAX (TERRESTRIAL)



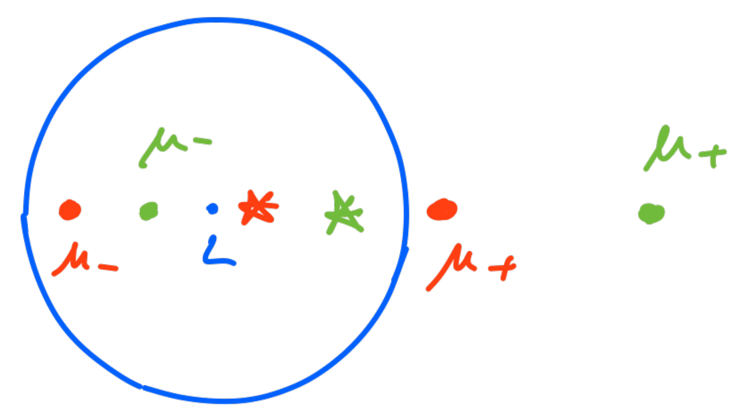


Gould & Horne, 2013

WHAT IS ASTROMETRIC MICROLENSING?

- during a microlensing event, the two images of the source cannot be resolved ($\theta_E \sim 1$mas)
- their positions and the magnifications change as a function of time
- in particular, the image forming outside the Einstein ring dominates, in terms of flux for most of the time
- what an observer will see is one source at the light centroid, which will move as a function of time depending on where the two images form and on how much flux they emit

$$\begin{aligned}
\mu_{\pm}(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \\
&= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right) \\
&= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right)
\end{aligned}$$



THE EQUATIONS

$$x_\pm = \frac{1}{2}\left[y \pm \sqrt{y^2+4}\right]$$

$$\begin{aligned} x_{\pm,\parallel} &= \frac{1}{2}(1 \pm Q)y_\parallel \\ x_{\pm,\perp} &= \frac{1}{2}(1 \pm Q)y_\perp \end{aligned}$$

$$Q = \frac{\sqrt{y^2+4}}{y}$$

$$\begin{aligned} \mu_\pm(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \\ &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right) \\ &= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right) \end{aligned}$$

$$\vec{x}_c = \frac{\vec{x}_+|\mu_+| + \vec{x}_-|\mu_-|}{|\mu_+| + |\mu_-|}$$

$$\delta\vec{x}_c = \vec{x}_c - \vec{y}$$

LIGHT CENTROID SHIFT AMPLITUDE

$$
\begin{aligned}
\delta x_c &= \frac{\frac{1}{4}\left[(y+\sqrt{y^2+4})\left(1+\frac{y^2+2}{y\sqrt{y^2+4}}\right)-(y-\sqrt{y^2+4})\left(1-\frac{y^2+2}{y\sqrt{y^2+4}}\right)\right]}{\frac{y^2+2}{y\sqrt{y^2+4}}}-y \\
&= \frac{\frac{1}{4}\left(y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}+\frac{y^2+2}{y}-y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}-\frac{y^2+2}{y}\right)}{\frac{y^2+2}{y\sqrt{y^2+4}}}-y \\
&= \frac{y}{y^2+2}\,.
\end{aligned}
$$

Given the sign, the shift points in the same direction of y.

Note that $y \gg \sqrt{2}$, $\delta x_c \approx \frac{1}{y}$

Thus, the shift decreases relatively slow with y… remember the scaling of μ?

LIGHT CENTROID SHIFT AMPLITUDE

$$
\begin{aligned}
\delta x_c &= \frac{\frac{1}{4}\left[(y+\sqrt{y^2+4})\left(1+\frac{y^2+2}{y\sqrt{y^2+4}}\right)-(y-\sqrt{y^2+4})\left(1-\frac{y^2+2}{y\sqrt{y^2+4}}\right)\right]}{\frac{y^2+2}{y\sqrt{y^2+4}}}-y \\
&= \frac{\frac{1}{4}\left(y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}+\frac{y^2+2}{y}-y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}-\frac{y^2+2}{y}\right)}{\frac{y^2+2}{y\sqrt{y^2+4}}}-y \\
&= \frac{y}{y^2+2}\,.
\end{aligned}
$$

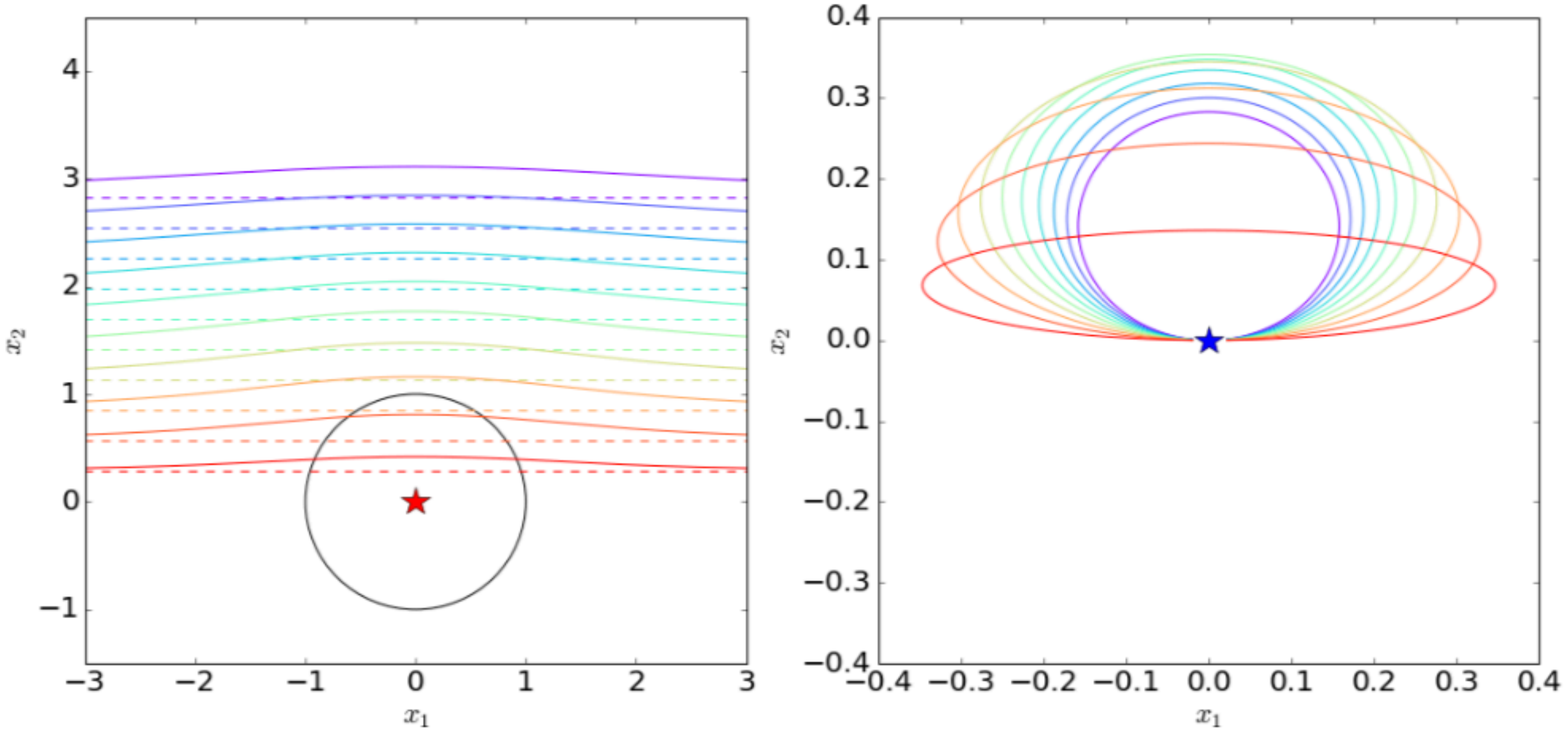
In addition

$$\frac{d(\delta x_c)}{dy}=\frac{2-y^2}{(y^2+2)^2}$$

the shift is maximum at $y=\sqrt{2}$, $\delta x_c=\delta x_{c,max}=(2\sqrt{2})^{-1}$

This corresponds to $\sim 0.354\theta_E$ *which is above the accuracy of GAIA*

VISUALISATION OF THE EFFECT



WHAT IS THE PROBABILITY TO OBSERVE A MICROLENSING EVENT?

- microlensing event: a variation of the source flux which follows the law we derived in the last lecture:

$$\mu(y) = \mu_+(y) + |\mu_-(y)| = \frac{y^2+2}{y\sqrt{y^2+4}}$$

- two important things to remember:
  - the total magnification drops as $\mu \propto 1 + 2/y^4$ for $y \to \infty$
  - the ratio of image magnification grows as $\left|\frac{\mu_+}{\mu_-}\right| \propto y^4$
- for these reasons, we anticipated that the microlensing cross section is

$$\sigma = \pi\theta_E^2$$

- more generally, **the cross section is the area (or the solid angle) within which a source has to be located in order to undergo a microlensing event.** In our case this $\mu \gtrsim 1.34$

OPTICAL DEPTH

The number of lenses in the solid angle Ω between D_L to D_L+dD_L is

$$dN_L = \Omega D_{\mathrm{L}}^2 n(D_{\mathrm{L}}) dD_{\mathrm{L}}$$

where $n(D_L)$ is the number density of lenses.

OPTICAL DEPTH

The number of lenses in the solid angle Ω *between* D_L *to* $D_L + dD_L$ *is*

$$dN_L = \Omega D_{\mathrm{L}}^2 n(D_{\mathrm{L}}) dD_{\mathrm{L}}$$

where $n(D_L)$ *is the number density of lenses.*

Each of these lenses has a cross section $\sigma = \pi\theta_E^2$

OPTICAL DEPTH

The number of lenses in the solid angle Ω between D_L to D_L+dD_L is

$$dN_L = \Omega D_{\mathrm{L}}^2 n(D_{\mathrm{L}}) dD_{\mathrm{L}}$$

where $n(D_L)$ is the number density of lenses.

Each of these lenses has a cross section $\sigma = \pi\theta_E^2$

Summing all the cross sections and dividing by the solid angle, we measure the probability that a source at distance D_S is lensed by a lens at distance D_L

OPTICAL DEPTH

The number of lenses in the solid angle Ω between D_L to D_L+dD_L is

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*Integrating over D_L, we obtain the **optical depth**:*

$$\tau(D_{\mathrm{S}}) = \frac{1}{\Omega}\int_0^{D_{\mathrm{S}}} [\Omega D_{\mathrm{L}}^2 n(D_{\mathrm{L}})](\pi\theta_E^2) dD_{\mathrm{L}}$$

OPTICAL DEPTH

If all lenses have the same mass: $n(D_\mathrm{L}) = \rho(D_\mathrm{L})/M$

On the other hand, $\theta_E \equiv \sqrt{\frac{4GM}{c^2}\frac{D_\mathrm{LS}}{D_\mathrm{L}D_\mathrm{S}}}$

Therefore, the optical depth does not depend on the mass of the lenses, but only on the mass density!

Note that if not all masses are equal we should integrate over M

OPTICAL DEPTH

$$\tau(D_{\mathrm{S}}) = \frac{1}{\Omega}\int_0^{D_{\mathrm{S}}} [\Omega D_{\mathrm{L}}^2 n(D_{\mathrm{L}})](\pi\theta_E^2)\,dD_{\mathrm{L}}$$

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$$\begin{aligned}\tau(D_{\mathrm{S}}) &= \frac{4\pi G}{c^2}\int_0^{D_{\mathrm{S}}}\rho(D_{\mathrm{L}})D_{\mathrm{L}}^2\frac{D_{\mathrm{LS}}}{D_{\mathrm{L}}D_{\mathrm{S}}}dD_{\mathrm{L}}\\ &= \frac{4\pi G}{c^2}\int_0^{D_{\mathrm{S}}}\rho(D_{\mathrm{L}})D_{\mathrm{L}}\frac{D_{\mathrm{S}}-D_{\mathrm{L}}}{D_{\mathrm{S}}}dD_{\mathrm{L}}\end{aligned}$$

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\tau(D_\mathrm{S}) &= \frac{4\pi G}{c^2}\int_0^{D_\mathrm{S}} \rho(D_\mathrm{L})D_\mathrm{L}^2 \frac{D_\mathrm{LS}}{D_\mathrm{L}D_\mathrm{S}}dD_\mathrm{L} \\
&= \frac{4\pi G}{c^2}\int_0^{D_\mathrm{S}} \rho(D_\mathrm{L})D_\mathrm{L} \frac{D_\mathrm{S}-D_\mathrm{L}}{D_\mathrm{S}}dD_\mathrm{L} \\
&= \frac{4\pi G}{c^2}\int_0^{D_\mathrm{S}} \rho(D_\mathrm{L})\frac{D_\mathrm{L}}{D_\mathrm{S}}\left(1-\frac{D_\mathrm{L}}{D_\mathrm{S}}\right)D_\mathrm{S}dD_\mathrm{L}
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\end{aligned}$$

with the substitution $x = D_{\mathrm{L}}/D_{\mathrm{S}},\, dx = dD_{\mathrm{L}}/D_{\mathrm{S}}$

$$\tau(D_{\mathrm{S}}) = \frac{4\pi G}{c^2}D_{\mathrm{S}}^2\int_0^1 \rho(x)x(1-x)dx$$

OPTICAL DEPTH DENSITY

$$\frac{d\tau}{dx} \propto \rho(x)x(1-x)$$

The function which modulates the lens contribution to the optical depth has a peak at x=0.5 (half way between the observer and the source).

Then, whether most of the optical depth is accumulated near the observer or near the source, depends on the mass density profile…

OPTICAL DEPTH (SIMPLEST CASE)

Uniform distribution of the lenses

$$\rho(x) = \rho_0 = \text{const.}$$

$$\tau(D_S) = \frac{4\pi G}{c^2}\rho_0 D_S^2 \int_0^1 x(1-x)dx = \frac{2}{3}\frac{\pi G}{c^2} D_S^2 \rho_0$$

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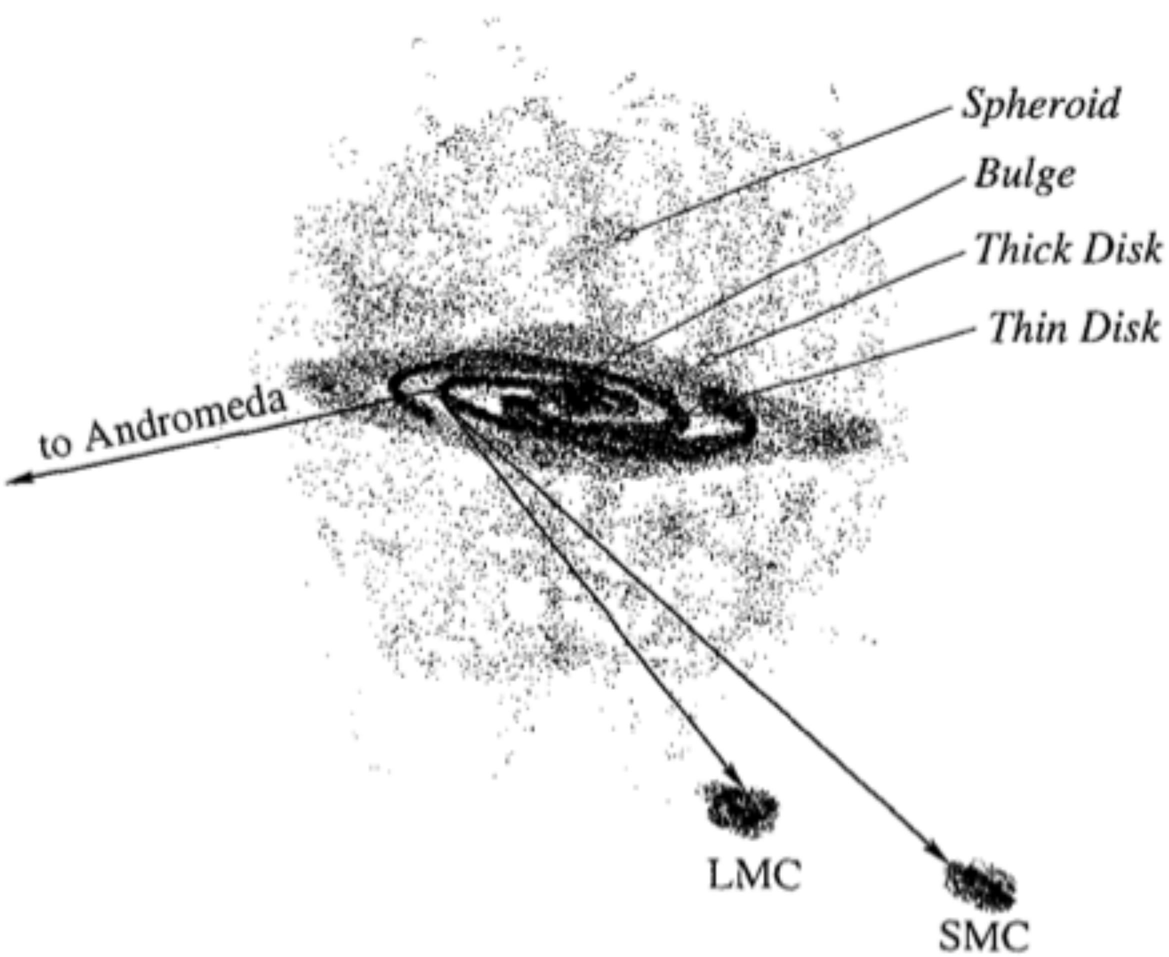
$$\tau \approx 2.6 \times 10^{-7}$$

The true optical depth towards the galactic center is 3-10 times larger…

A MODEL FOR OUR GALAXY



All these components have their own optical depths...

I) thin & thick disk (young stars & gas)

$$\rho^D(R,z) = \rho_0^D \exp\left(-\frac{R-R_0}{h_R} - \frac{|z|}{h_z}\right)$$

$$\sigma^D \simeq 20 \text{ km/s} \qquad v_{rot}^D \simeq 220 \text{ km/s}$$

$$\sigma^{TD} \simeq 40 \text{ km/s} \qquad v_{rot}^{TD} \simeq 180 \text{ km/s}$$

II) Spheroid (old star halo)

$$\rho^S \propto r^{-3.5} \qquad \sigma^S \simeq 120 \text{ km/s}$$

III) Bulge (contains a bar)

$$\rho^B(s) = \frac{M_0}{8\pi abc} \exp\left[-\frac{s^2}{2}\right]$$

$$s^4 \equiv [(x'/a)^2 + (y'/b)^2]^2 + (z'/c)^4$$

OPTICAL DEPTH (JUST A BIT MORE REALISTIC)

Lenses in the galactic disk:

scale of the disk

$$\rho(R) = \rho_0 \exp\left(-(R - R_0)/R_D\right)$$

density in the sun neighborhood

distance of earth from the galactic center

distance of lenses from the galactic center

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$$R - R_0 = D_{LS} - D_S = D_S - D_L - D_S = -D_L$$

$$\rho(D_L) = \rho_0 \exp\left(D_L/R_D\right)$$

OPTICAL DEPTH (JUST A BIT MORE REALISTIC)

Making the substitutions $x = D_L/D_S \quad x' = R_D/D_S$

$$\tau(D_S) = \frac{4\pi G}{c^2}\rho_0 D_S^2 \int_0^1 \exp{(x/x')} x(1-x)dx$$

$$\tau(D_S) = \frac{4\pi G}{c^2}\rho_0 D_S^2 x'^2 [2x' - 1 + \exp{(1/x')}(2x' - 1)]$$

$$D_S = 8 \text{ kpc} \qquad R_D = 3 \text{ kpc} \qquad \rho_0 = 0.1 M_\odot \text{ pc}^{-3}$$

$$\tau \approx 2.9 \times 10^{-6}$$