GRAVITATIONAL LENSING

20 - EXTERNAL PERTURBATIONS

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Q0957+561

EXTERNAL PERTURBATIONS

Lenses are often not isolated. Therefore, it is sometimes necessary to embed the lens into an external mass distribution mimicking the presence of nearby structures. How can such perturbation be modeled?

One can think to use a potential, defined such that

$$
\begin{aligned}
\gamma_1 &= \frac{1}{2}(\Psi_{11} - \Psi_{22}) = \text{const.} \\
\gamma_2 &= \Psi_{12} = \text{const.} \\
\kappa &= \frac{1}{2}(\Psi_{11} + \Psi_{22}) = \text{const.}
\end{aligned}
$$

EXTERNAL PERTURBATIONS

If both the sum ad the difference of the 2nd derivatives must be constant, the two derivatives must be constant separately:

$$\Psi = Cx_1^2 + C'x_2^2 + Dx_1x_2 + E$$

$$\begin{aligned}
\frac{1}{2}(\Psi_{11} - \Psi_{22}) &= C - C' = \gamma_1 \\
\Psi_{12} &= D = \gamma_2 \\
\frac{1}{2}(\Psi_{11} + \Psi_{22}) &= C + C' = \kappa
\end{aligned}$$

EXTERNAL PERTURBATIONS: EXAMPLE

In the case of a constant sheet of matter, there is no shear involved:

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\end{aligned}$$

$$\Psi = Cx_1^2+C'x_2^2+Dx_1x_2+E$$

$$\vec{\alpha}=\vec{\nabla}\Psi_\kappa=\kappa\vec{x}$$

$$\vec{y}=\vec{x}-\vec{\alpha}=\vec{x}(1-\kappa)$$

A sheet of constant density will change the focussing properties of the lens. For example, if κ=1…

EXTERNAL PERTURBATIONS: EXAMPLE

In the case of a constant sheet of matter, there is no shear involved:

$$\begin{aligned}
\frac{1}{2}(\Psi_{11} - \Psi_{22}) &= C - C' = \gamma_1 = 0 \\
\Psi_{12} &= D = \gamma_2 \\
\frac{1}{2}(\Psi_{11} + \Psi_{22}) &= C + C' = \kappa
\end{aligned}$$

$$\Psi = Cx_1^2 + C'x_2^2 + Dx_1x_2 + E$$

$$\vec{\alpha} = \vec{\nabla}\Psi_\kappa = \kappa\vec{x}$$

$$\vec{y} = \vec{x} - \vec{\alpha} = \vec{x}(1-\kappa)$$

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$$C=\kappa/2$$

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$$\vec{\alpha}=\vec{\nabla}\Psi_\kappa=\kappa\vec{x}$$

$$\vec{y}=\vec{x}-\vec{\alpha}=\vec{x}(1-\kappa)$$

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EXTERNAL PERTURBATIONS

Instead, if the perturber does not contribute to the convergence:

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\frac{1}{2}(\Psi_{11}-\Psi_{22}) &= C-C'=\gamma_1 \\
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\end{aligned}
$$

$$C=-C' \quad \Rightarrow \quad C=\frac{\gamma_1}{2}$$

$$\Psi_\gamma=\frac{\gamma_1}{2}(x_1^2-x_2^2)+\gamma_2 x_1 x_2$$

$$\Psi_\gamma=\frac{\gamma}{2}x^2\cos 2(\phi-\phi_\gamma)$$

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\end{aligned}$$

$$C = -C' \quad \Rightarrow \quad C = \frac{\gamma_1}{2}$$

$$\Psi_\gamma = \frac{\gamma_1}{2}(x_1^2 - x_2^2) + \gamma_2 x_1 x_2$$

$$\Psi_\gamma = \frac{\gamma}{2}x^2 \cos 2(\phi - \phi_\gamma)$$

EXTERNAL PERTURBATIONS: EXAMPLE

We embed an isothermal sphere with a core into an external shear:

$$\Psi = \sqrt{x^2 + x_c^2} + \frac{\gamma_1}{2}(x_1^2 - x_2^2) + \gamma_2 x_1 x_2$$

EXTERNAL PERTURBATIONS: EXAMPLE

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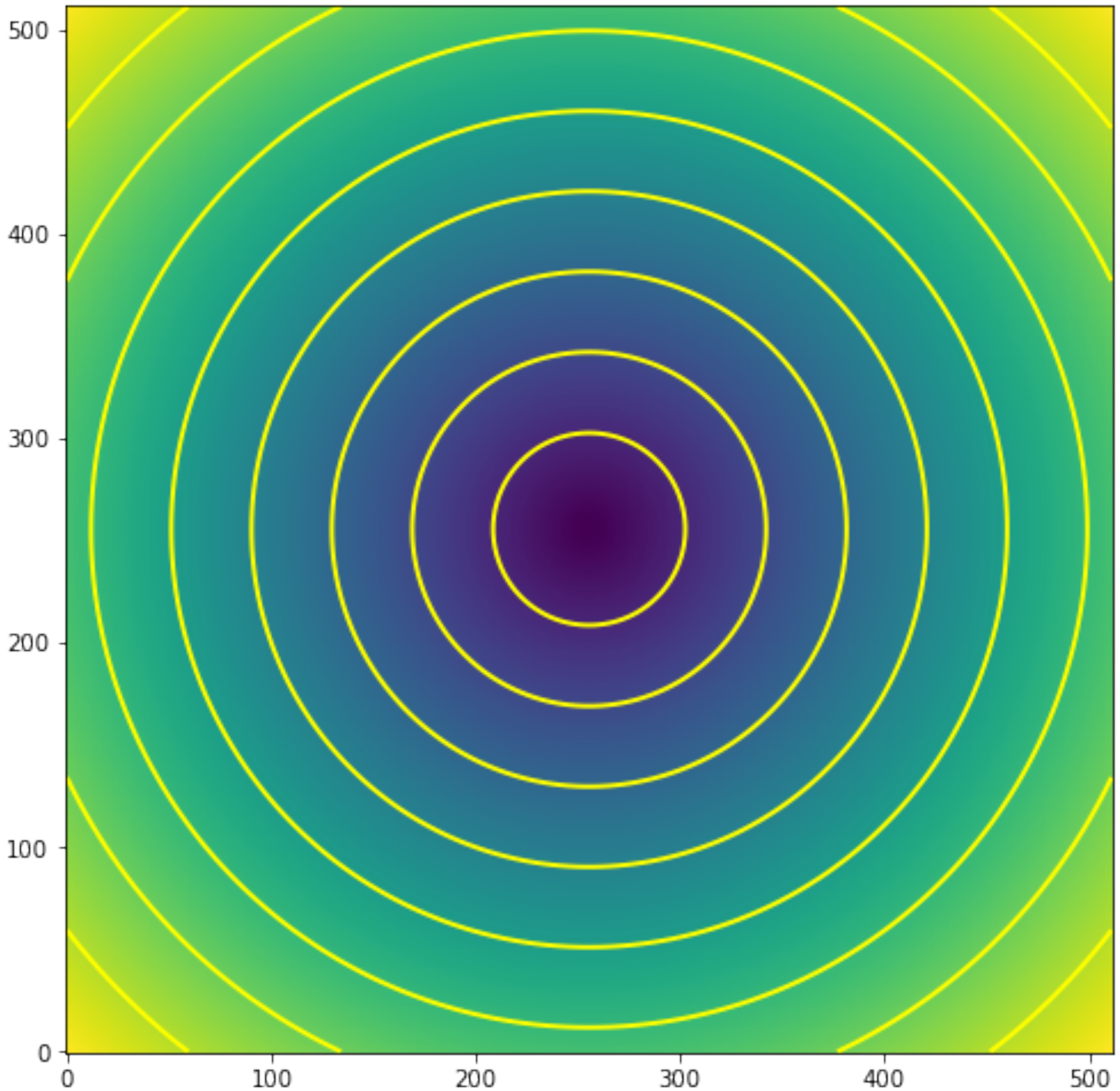
$$\Psi = \sqrt{x^2 + x_c^2} + \frac{\gamma_1}{2}(x_1^2 - x_2^2) + \gamma_2 x_1 x_2$$

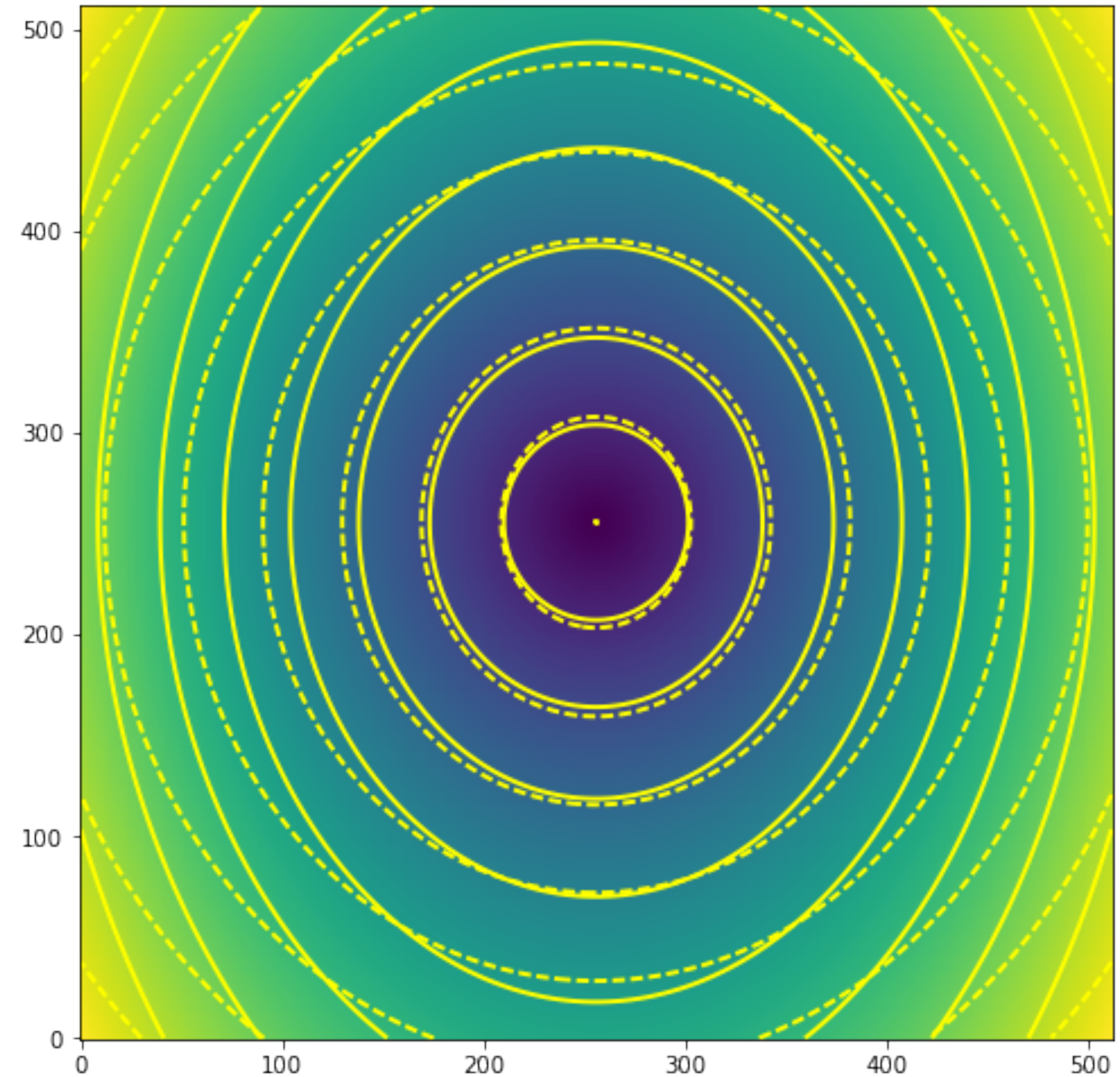
$$\begin{aligned}
\vec{\nabla}\Psi &= \frac{\vec{x}}{\sqrt{x^2 + x_c^2}} + \begin{pmatrix} \gamma_1 x_1 + \gamma_2 x_2 \\ -\gamma_1 x_2 + \gamma_2 x_1 \end{pmatrix} \\
&= \frac{\vec{x}}{\sqrt{x^2 + x_c^2}} + \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix} \vec{x}
\end{aligned}$$

EXTERNAL PERTURBATIONS: EXAMPLE

Potential



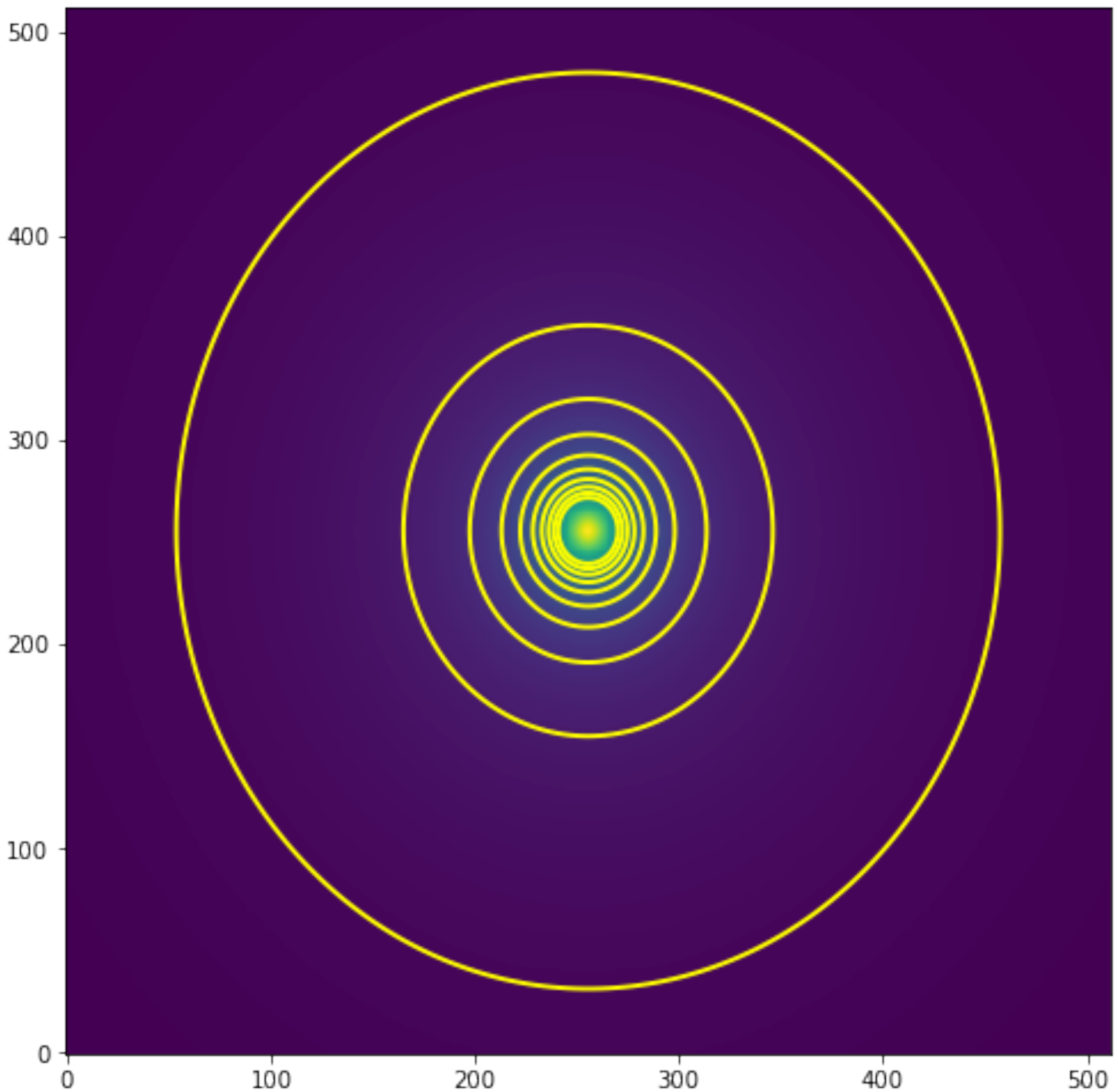
NIS



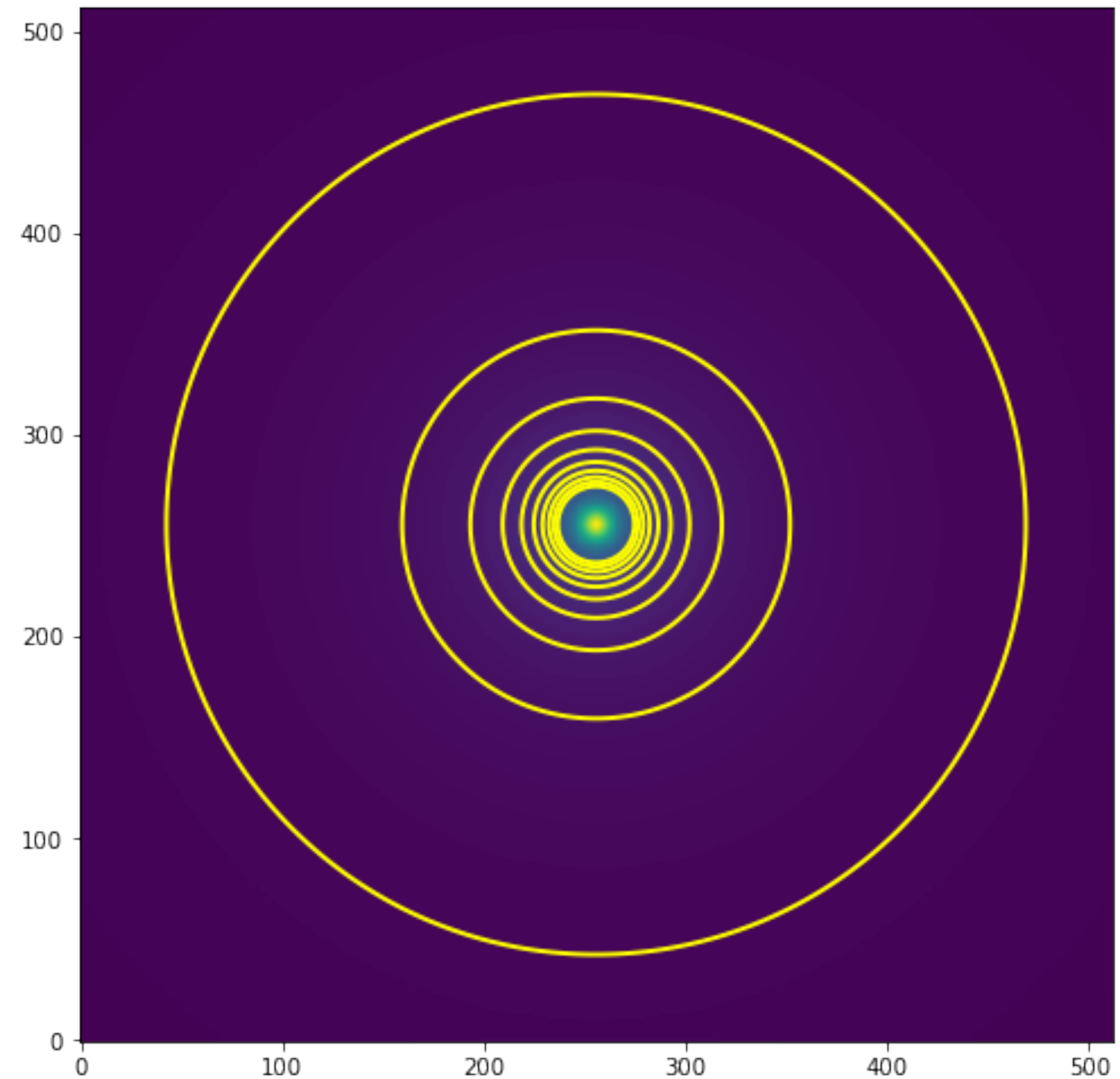
NIS+ext. shear (γ =0.1) vs f=0.9

EXTERNAL PERTURBATIONS: EXAMPLE

Convergence



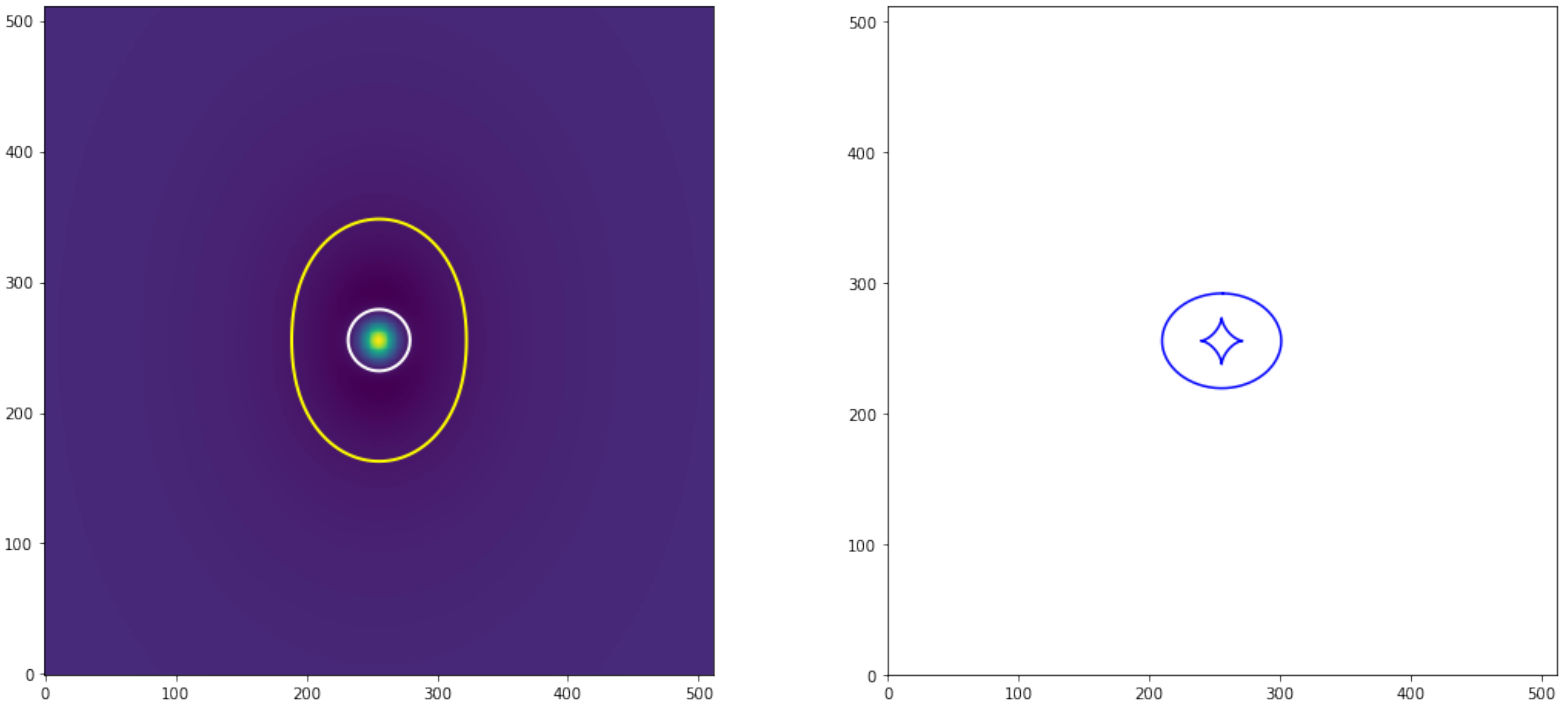
NIE (f=0.9)



NIS+ext. shear (γ =0.1)

EXTERNAL PERTURBATIONS: EXAMPLE

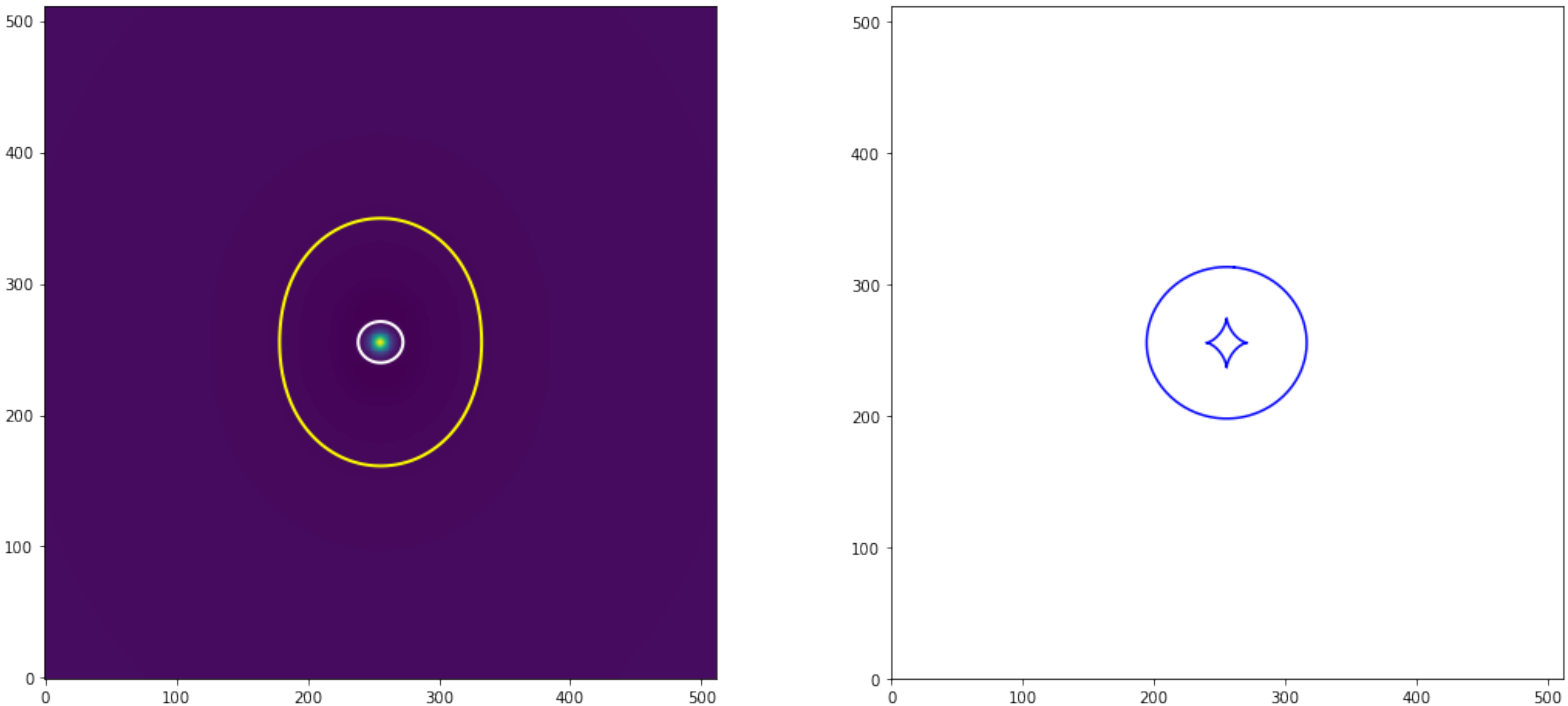
Critical lines



NIE (f=0.9)

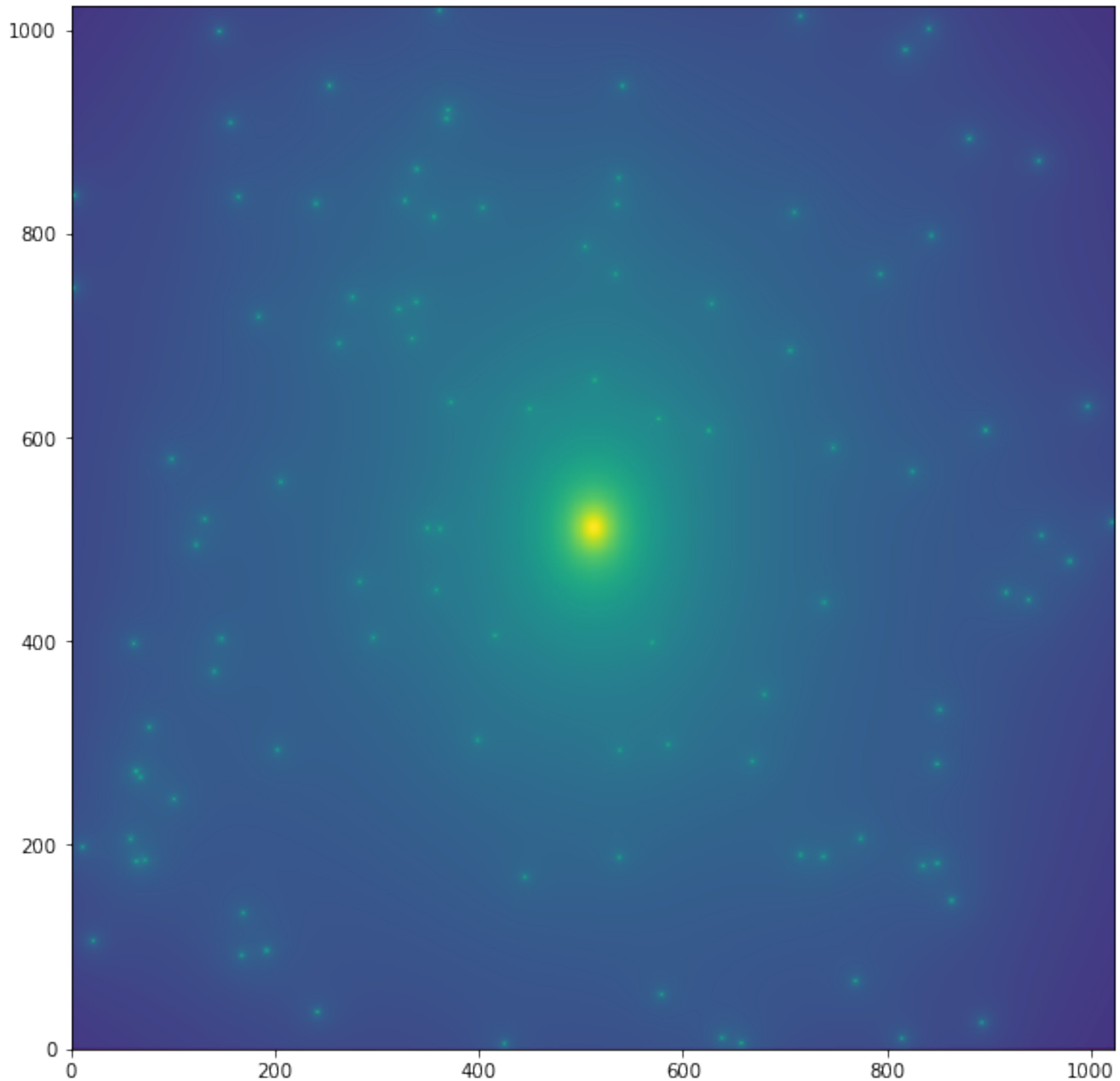
EXTERNAL PERTURBATIONS: EXAMPLE

Critical lines



NIS+ext. shear (γ =0.1)

SUBSTRUCTURES



TIME DELAYS

As seen earlier, lensing introduces a time delay:

$$t(x) = \frac{(1+z_{\rm L})}{c}\frac{D_{\rm L}D_{\rm S}}{D_{\rm LS}}\frac{\xi_0^2}{D_{\rm L}^2}\left[\frac{1}{2}(x-y)^2 - \Psi(x)\right] = \frac{(1+z_{\rm L})}{c}\frac{D_{\rm L}D_{\rm S}}{D_{\rm LS}}\tau(x)$$

If there are multiple images, each of them is probing a different line of sight…

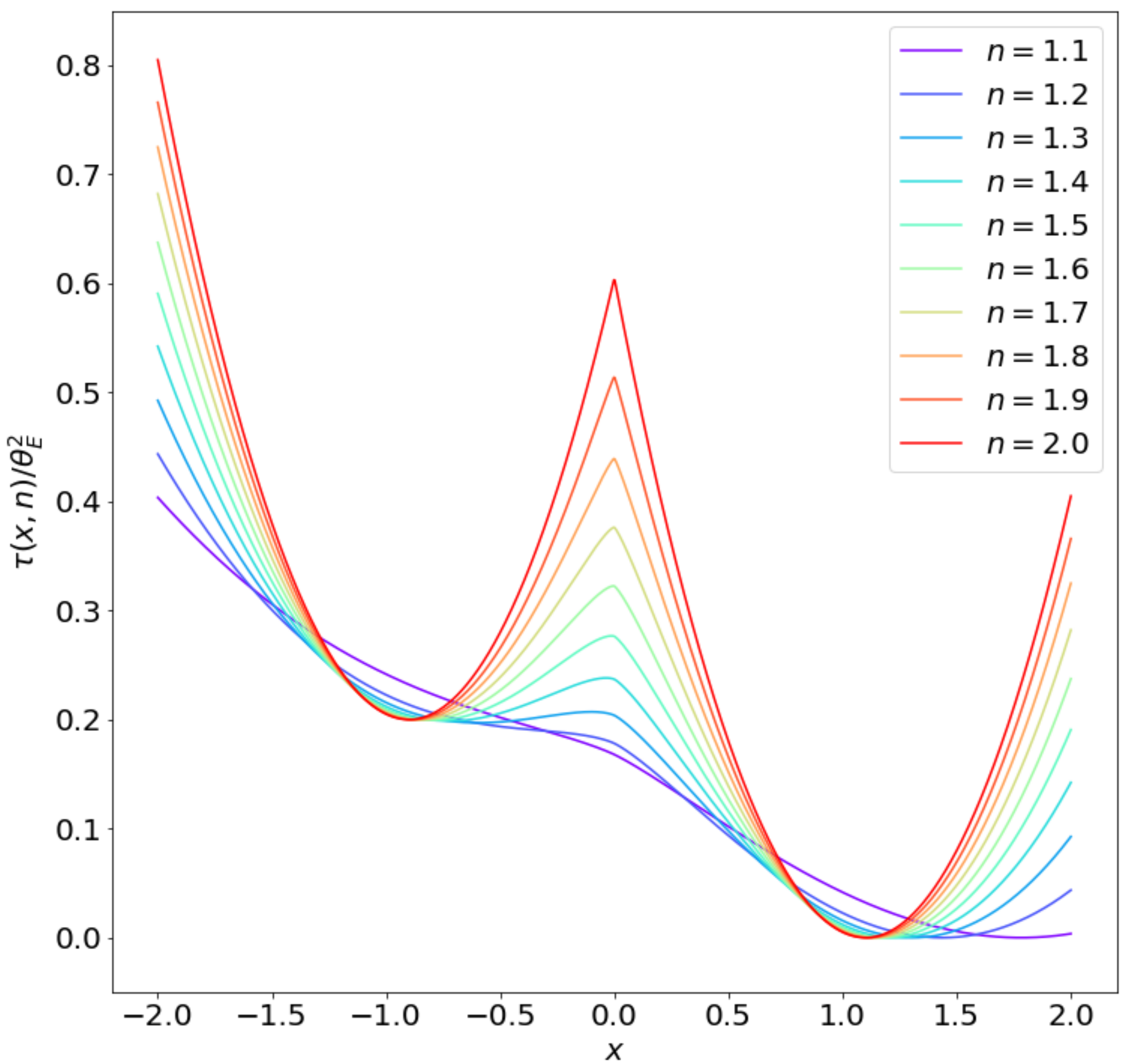
If the source is intrinsically variable, we may be able to measure a delay between the images.

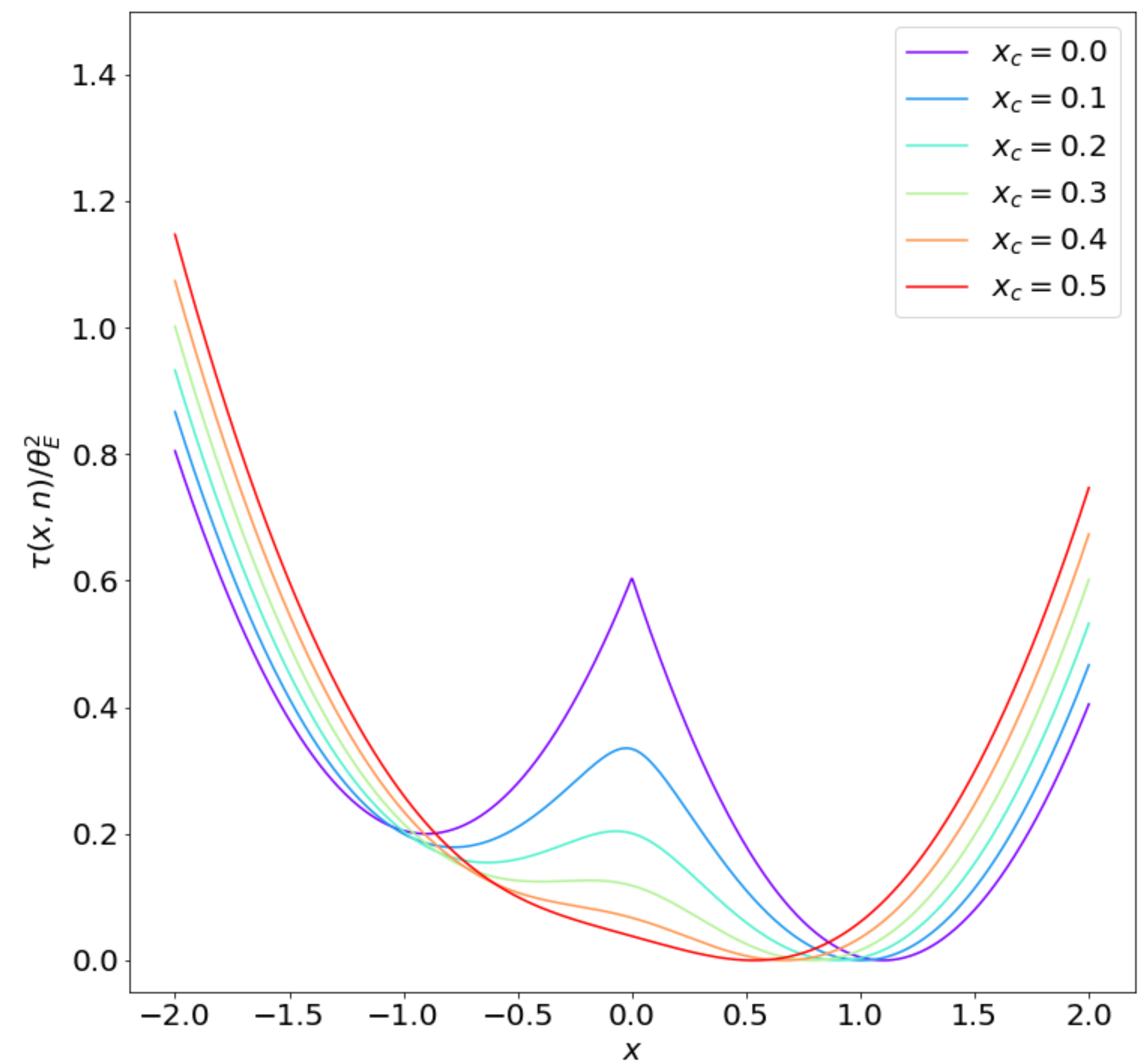
The models we have studied can be used to predict the time delay between the images. The fundamental ingredient is the lensing potential:

$$\Psi(x) = \frac{1}{3-n}x^{3-n}$$ *power-law*

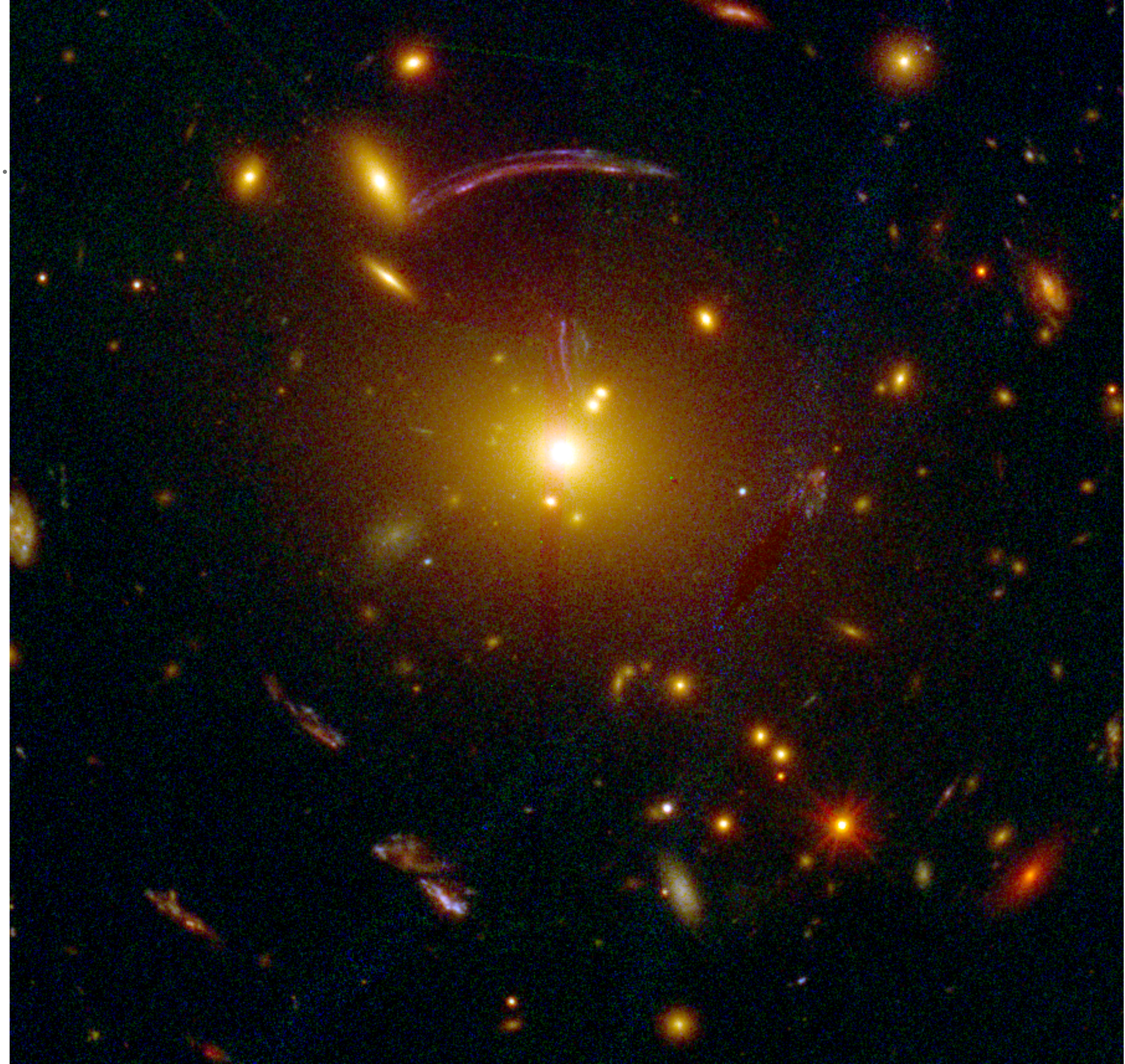
$$\Psi(x,x_c) = \sqrt{x^2+x_c^2} - x_c \ln\left(x_c + \sqrt{x^2+x_c^2}\right)$$ *NIS*

TIME DELAYS

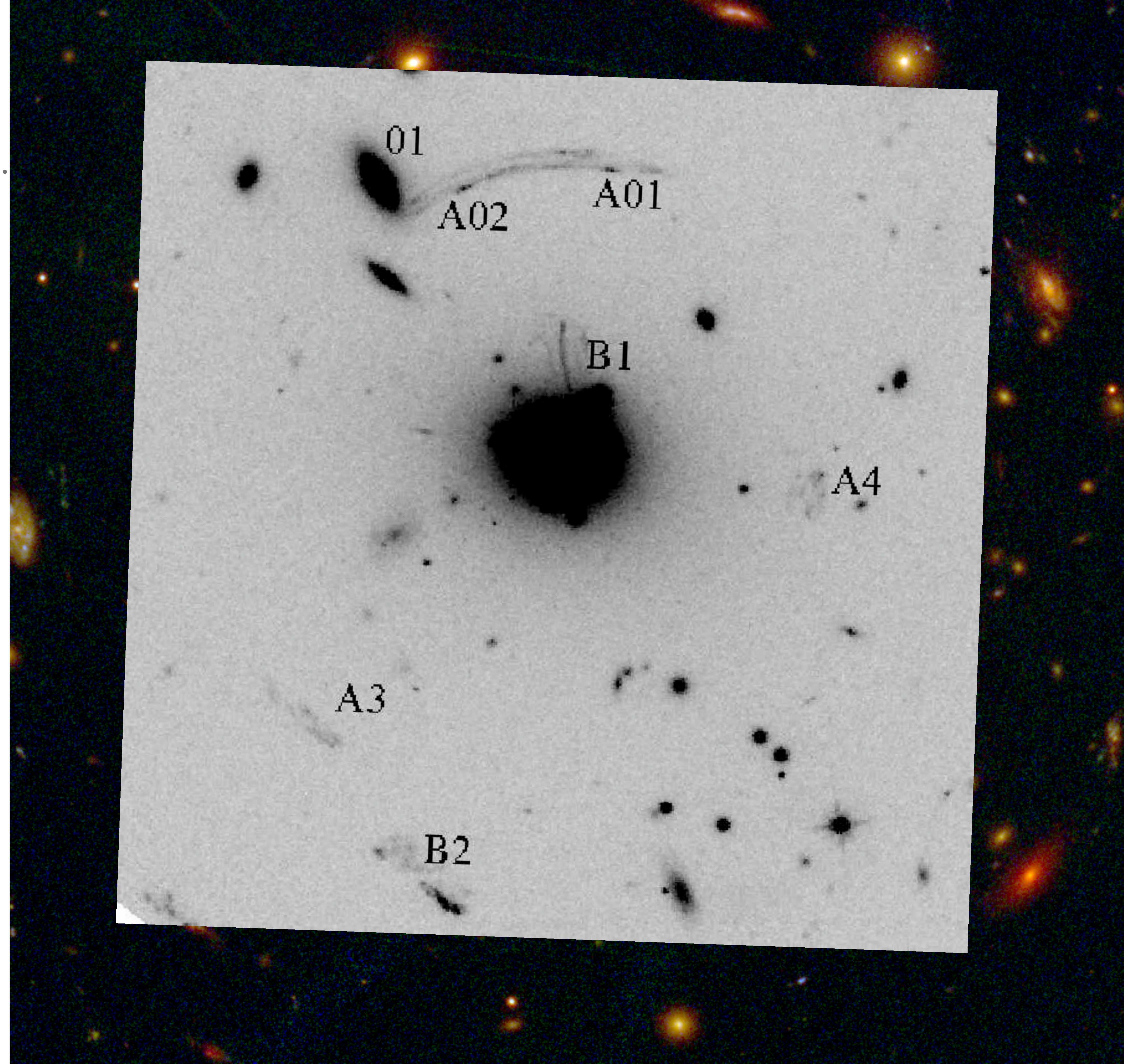

$n=1.1$
$n=1.2$
$n=1.3$
$n=1.4$
$n=1.5$
$n=1.6$
$n=1.7$
$n=1.8$
$n=1.9$
$n=2.0$
$\tau(x,n)/\theta_E^2$
x


$x_c=0.0$
$x_c=0.1$
$x_c=0.2$
$x_c=0.3$
$x_c=0.4$
$x_c=0.5$
$\tau(x,n)/\theta_E^2$
x

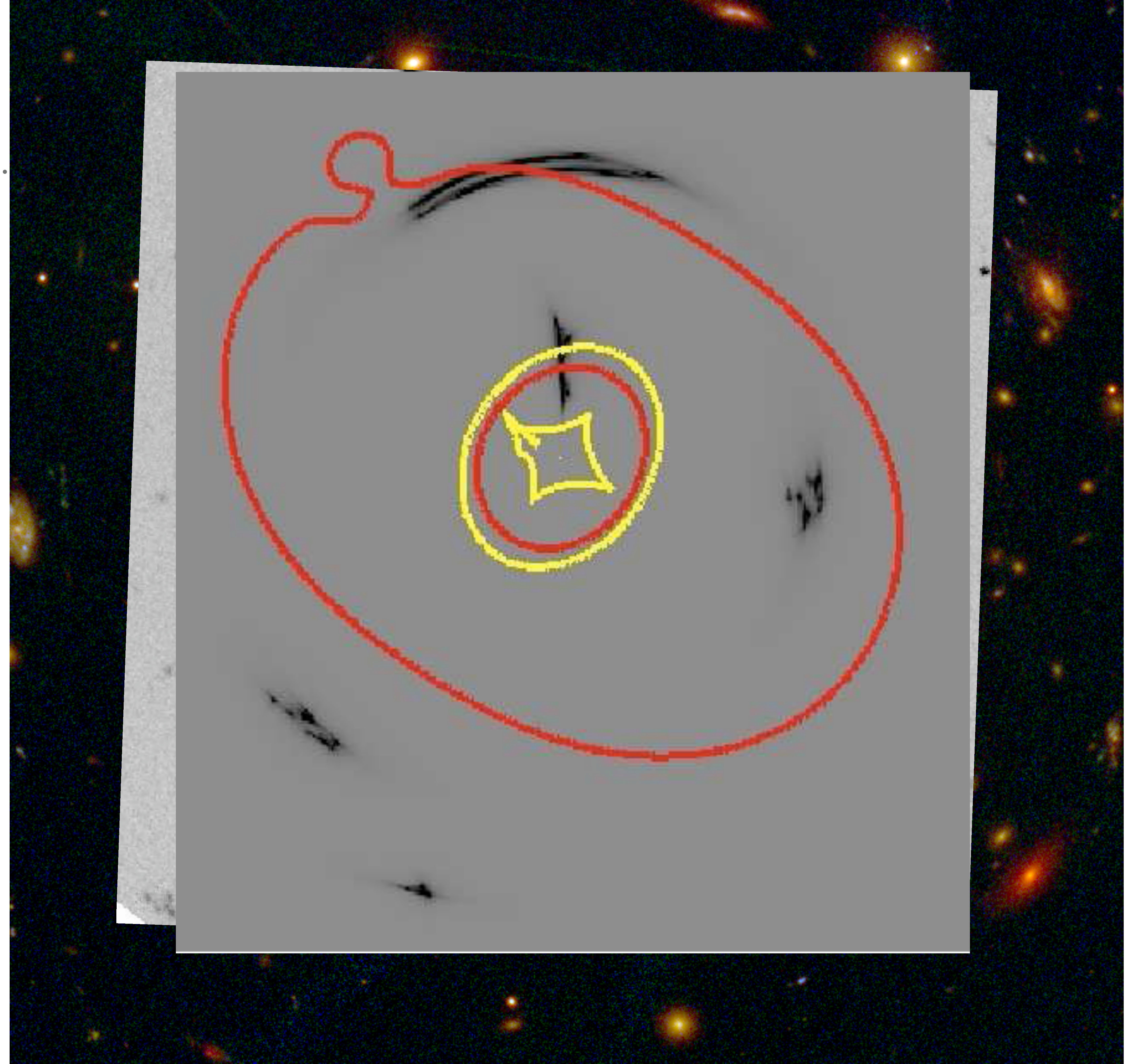
MS2137



MS2137



MS2137



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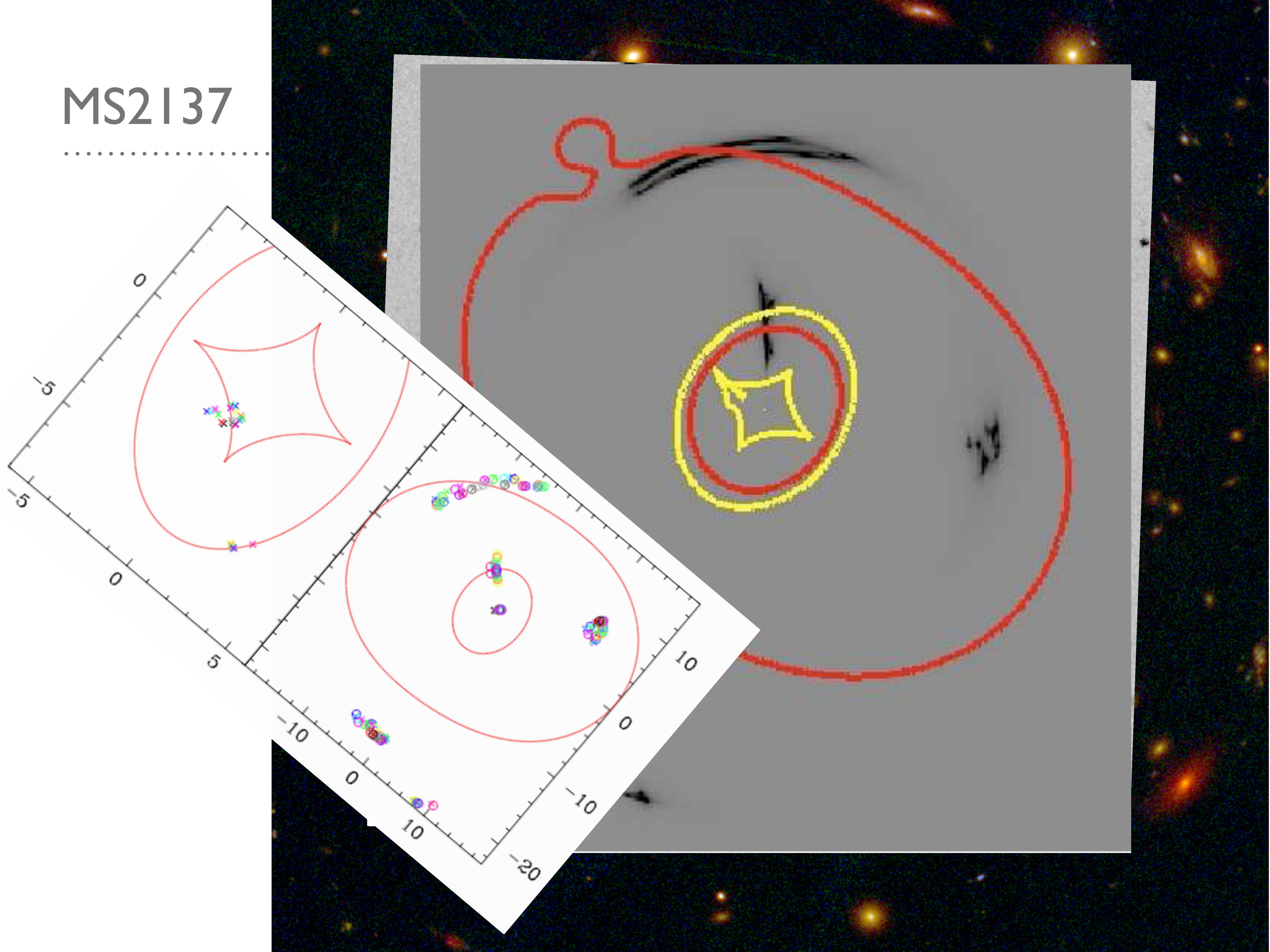
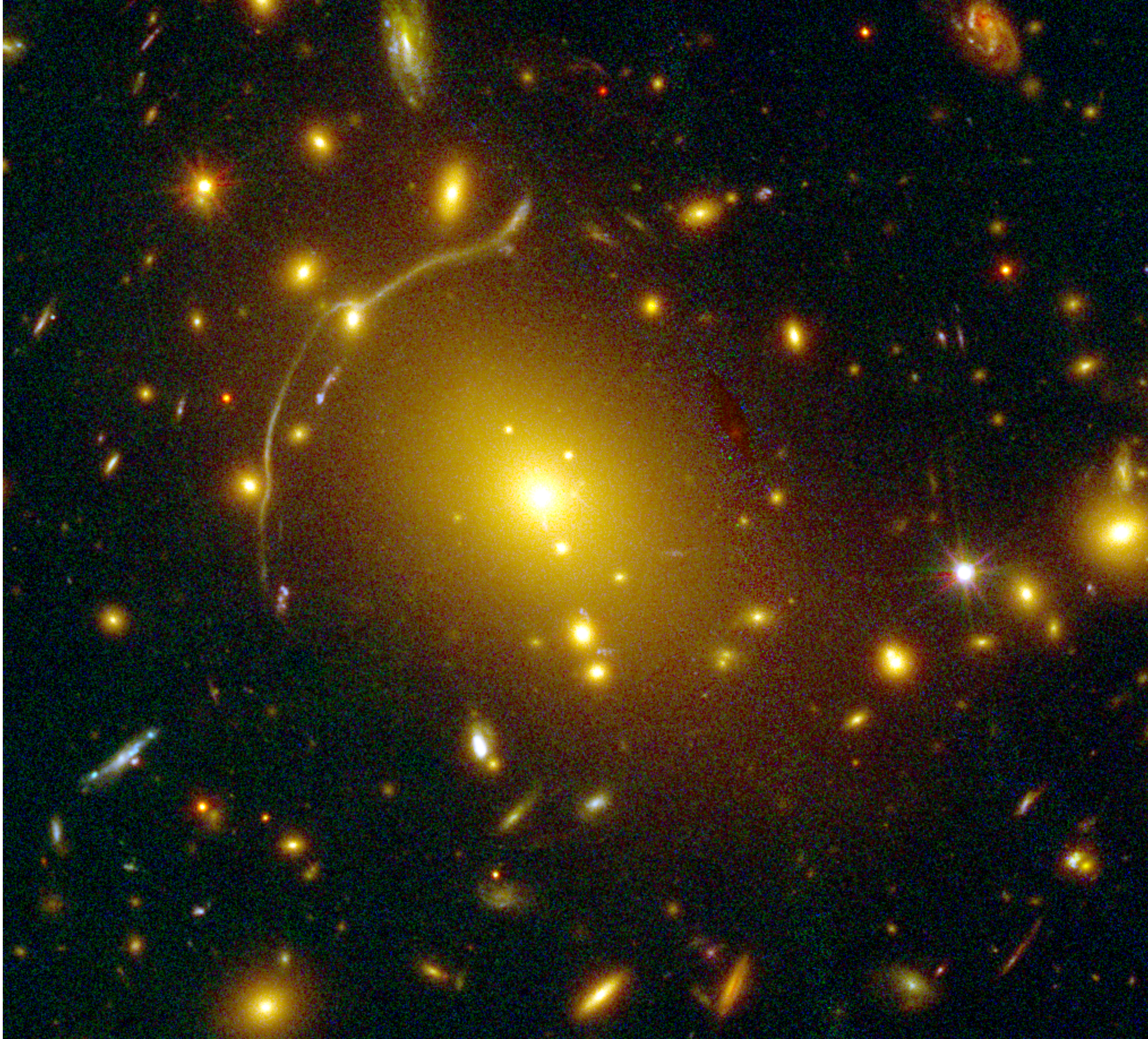
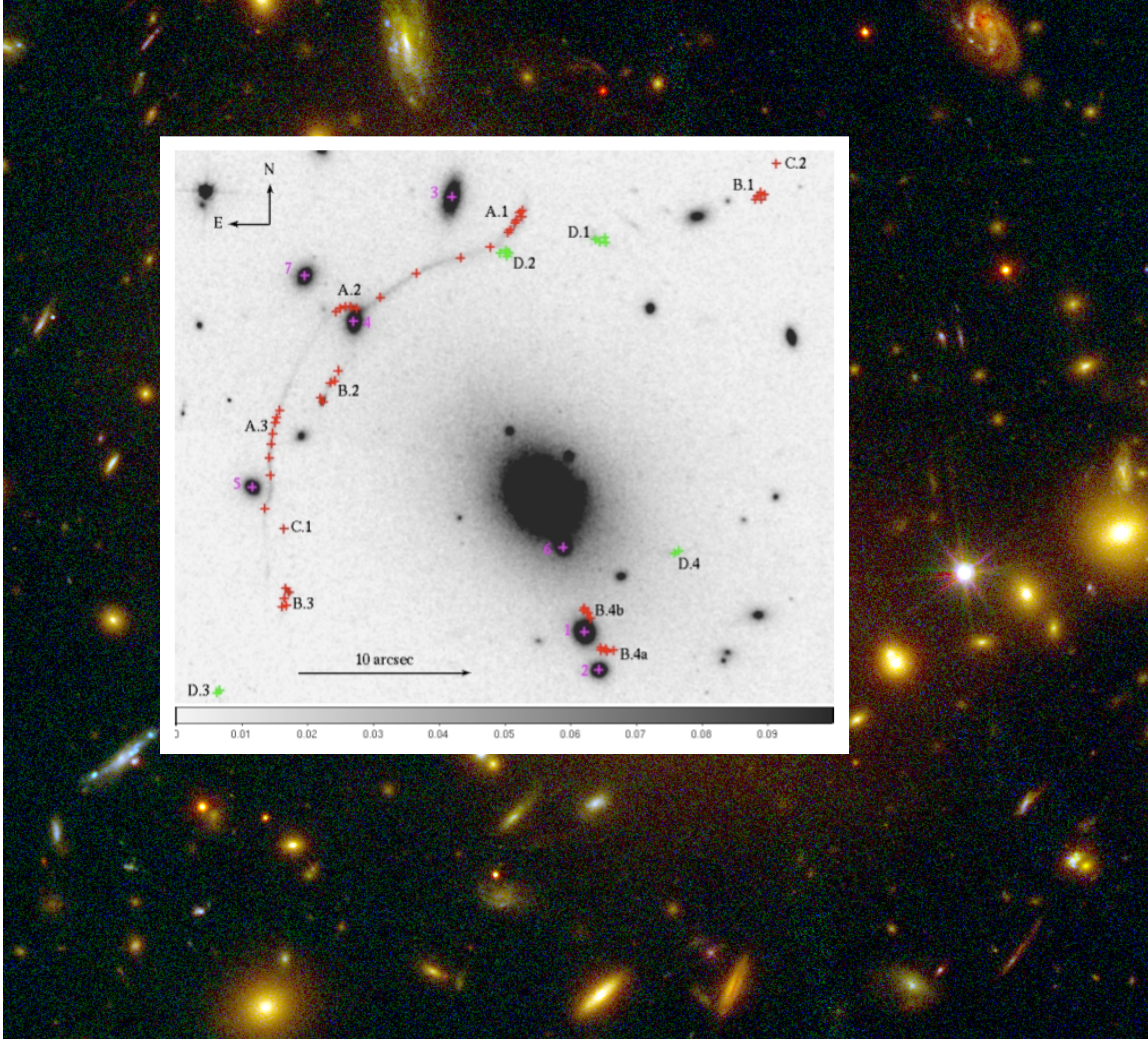


A611



A611


N
E
C.2
B.1
3
A.1
D.1
D.2
7
A.2
4
B.2
A.3
5
C.1
6
D.4
B.3
B.4b
1
B.4a
10 arcsec
2
D.3
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09

A611