GRAVITATIONAL LENSING

5 - MAGNIFICATION, FLEXION, TIME DELAYS

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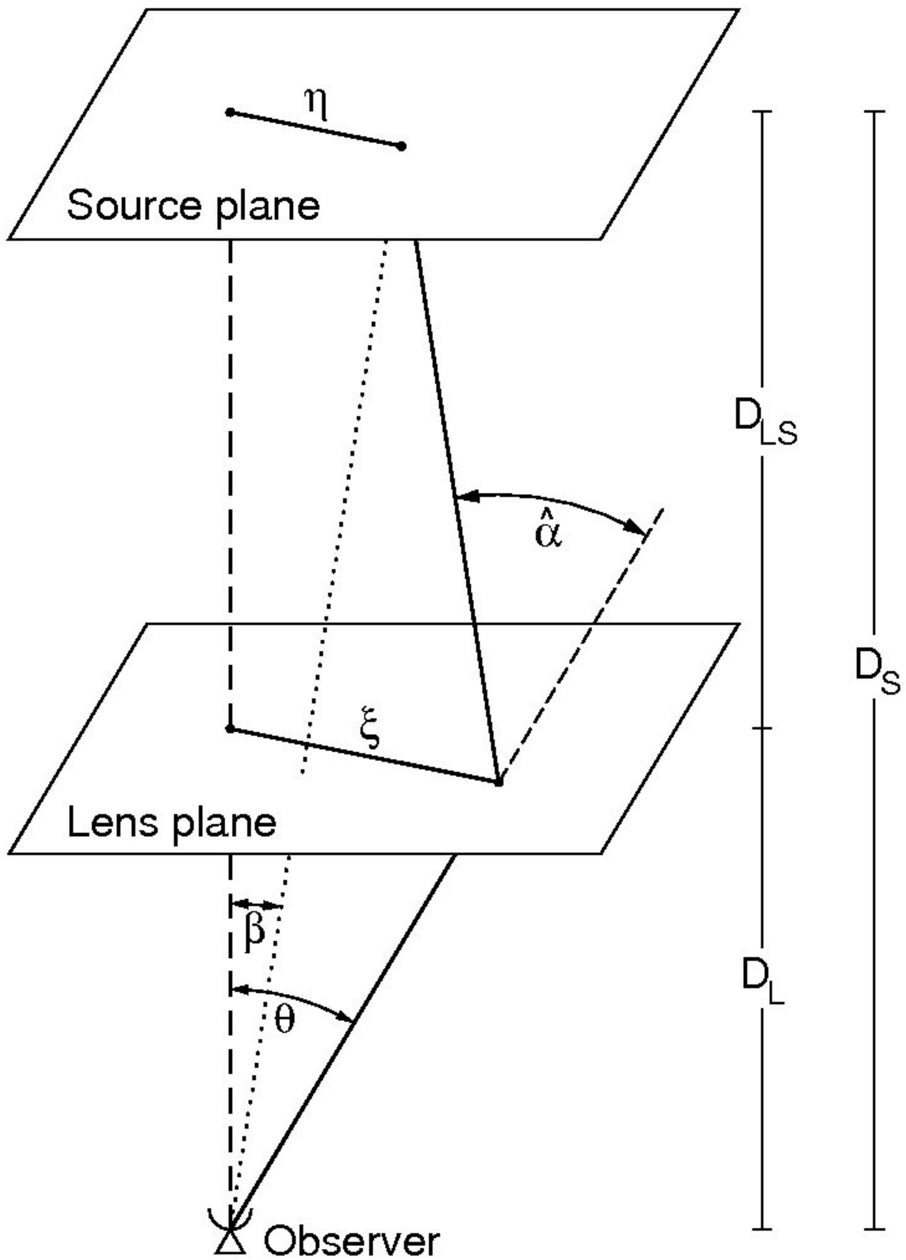
AA 2018-2019

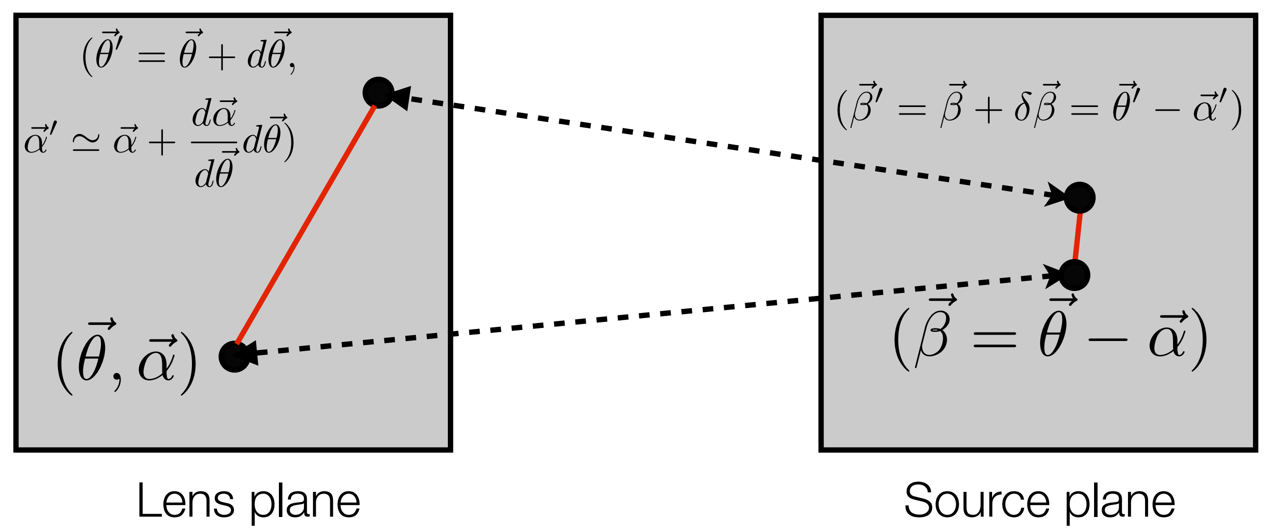
RESULTS FROM THE PAST LESSON



- *we derived the lens equation*

$$\vec{\beta} = \vec{\theta} - \frac{D_{LS}}{D_S}\hat{\vec{\alpha}}(\vec{\theta}) = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

- *Assuming that the d.a. does not vary significantly over the scale dΘ:*



$$(\vec{\beta}' - \vec{\beta}) = \left(I - \frac{d\vec{\alpha}}{d\vec{\theta}}\right)(\vec{\theta}' - \vec{\theta}) = A(\vec{\theta}' - \vec{\theta})$$

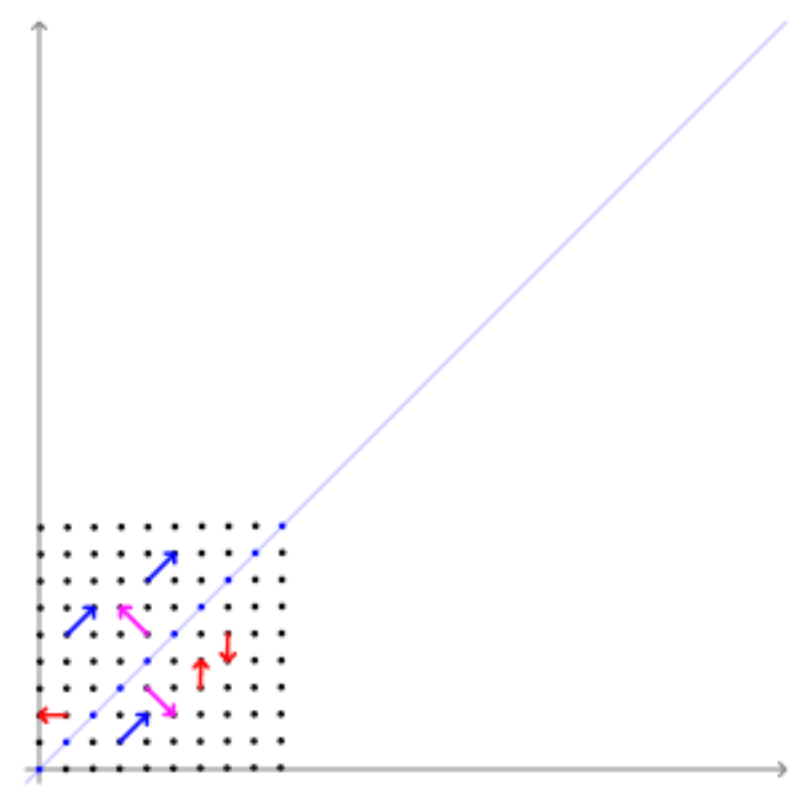
RESULTS FROM THE PAST LESSON

$$A = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$
$$= (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix}$$

Lens mapping at first order is a linear application, distorting areas.

Distortion directions are given by the ***eigenvectors*** *of A.*

Distortion amplitudes in these directions are given by the ***eigenvalues****.*



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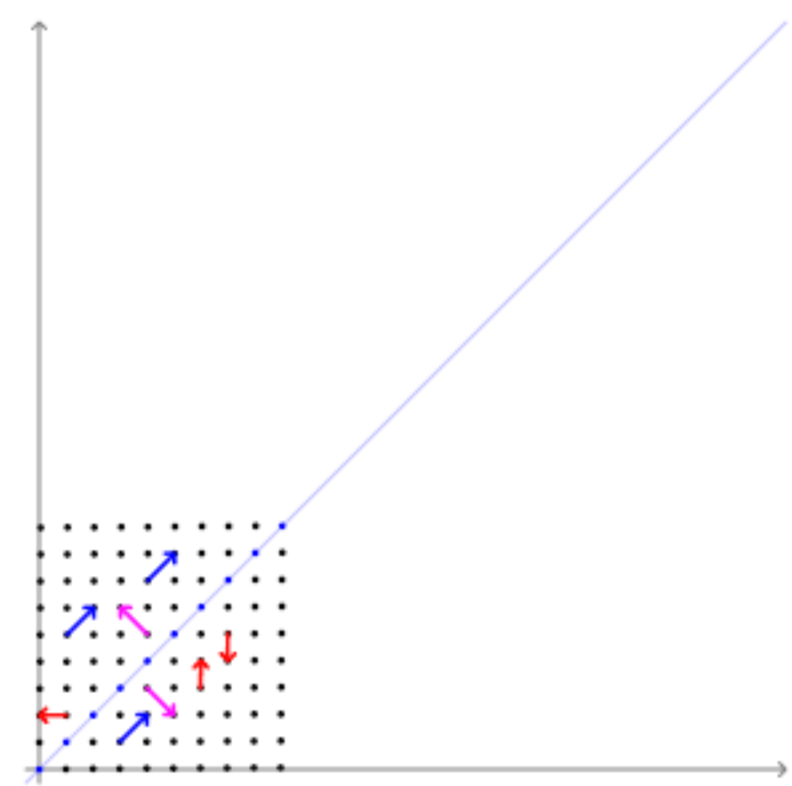


RESULTS FROM THE PAST LESSON

$$\beta_1^2 + \beta_2^2 = \beta^2$$

In the reference frame where A is diagonal:

$$\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 1-\kappa-\gamma & 0 \\ 0 & 1-\kappa+\gamma \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

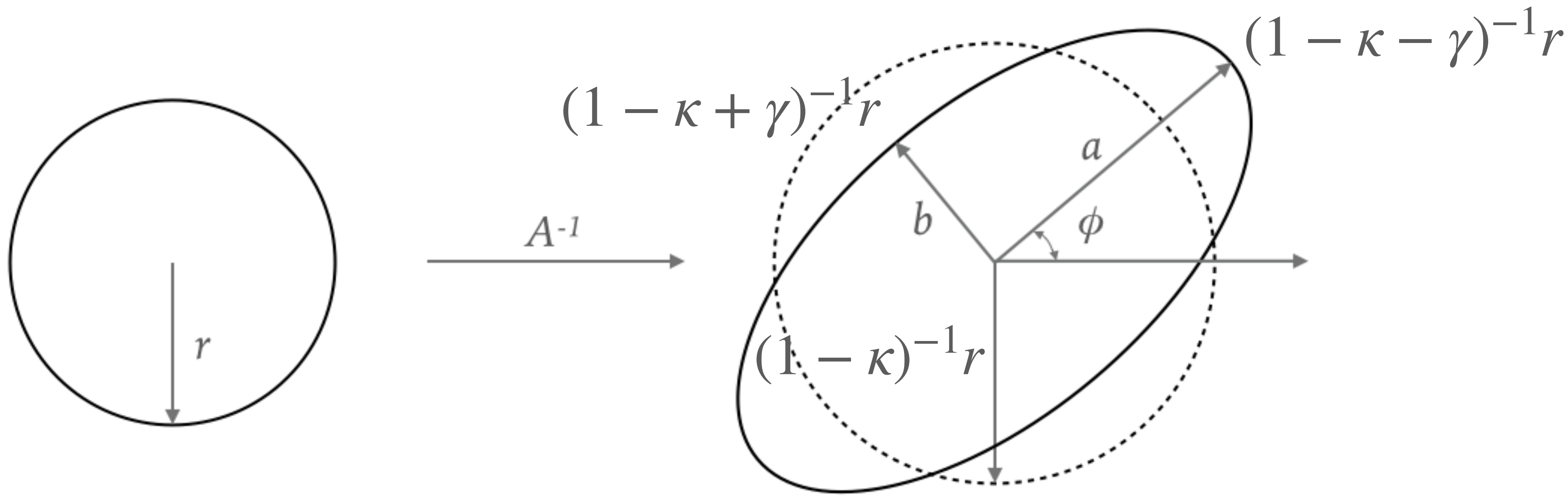
$$\begin{aligned} \beta_1 &= (1-\kappa-\gamma)\theta_1 \\ \beta_2 &= (1-\kappa+\gamma)\theta_2 \end{aligned}$$

$$\beta^2 = \beta_1^2 + \beta_2^2 = (1-\kappa-\gamma)^2\theta_1^2 + (1-\kappa+\gamma)^2\theta_2^2$$

This is the equation of an ellipse with semi-axes:

$$a = \frac{\beta}{1-\kappa-\gamma} \qquad b = \frac{\beta}{1-\kappa+\gamma}$$

FIRST ORDER DISTORTION OF A CIRCULAR SOURCE

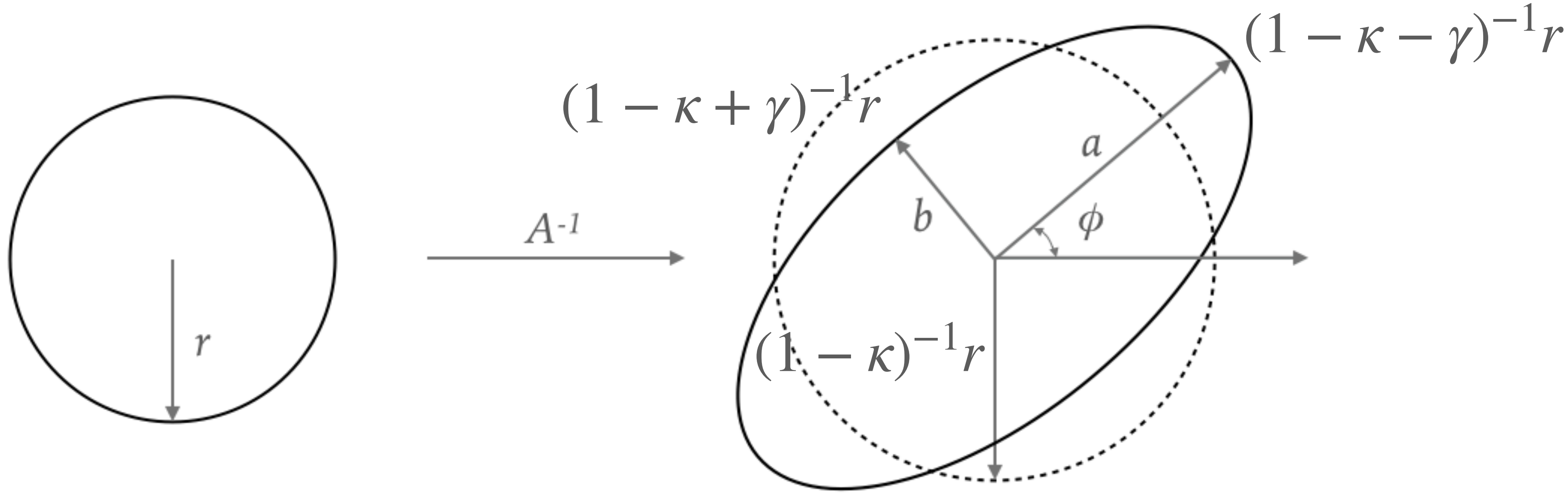


convergence: *responsible for isotropic expansion or contraction*

shear: *responsible for anisotropic distortion*

Ellipticity: $$e = \frac{a-b}{a+b} = \frac{\gamma}{1-\kappa} = g$$

BY HOW MUCH DOES THE AREA OF THE IMAGE DIFFER FROM THAT OF THE SOURCE?



$$I = \pi ab = \frac{\pi r^2}{(1-\kappa-\gamma)(1-\kappa+\gamma)} = \pi r^2 \det A^{-1} = S \det A^{-1}$$

$$\frac{I}{S} = \det A^{-1}$$

CONSERVATION OF SURFACE BRIGHTNESS

The source ***surface brightness*** *is*

$$I_\nu = \frac{dE}{dt dA d\Omega d\nu}$$

In phase space, the radiation emitted is characterized by the density

$$f(\vec{x}, \vec{p}, t) = \frac{dN}{d^3x d^3p}$$

In absence of photon creations or absorptions, f is conserved (Liouville theorem)

$$dN = \frac{dE}{h\nu} = \frac{dE}{cp}$$

$$d^3x = c dt dA$$

$$d^3\vec{p} = p^2 dp d\Omega$$

$$f(\vec{x}, \vec{p}, t) = \frac{dN}{d^3x d^3p} = \frac{dE}{hcp^3 dA dt d\nu d\Omega} = \frac{I_\nu}{hcp^3}$$

Since GL does not involve creation or absorption of photons, neither it changes the photon momenta (achromatic!), surface brightness is conserved!

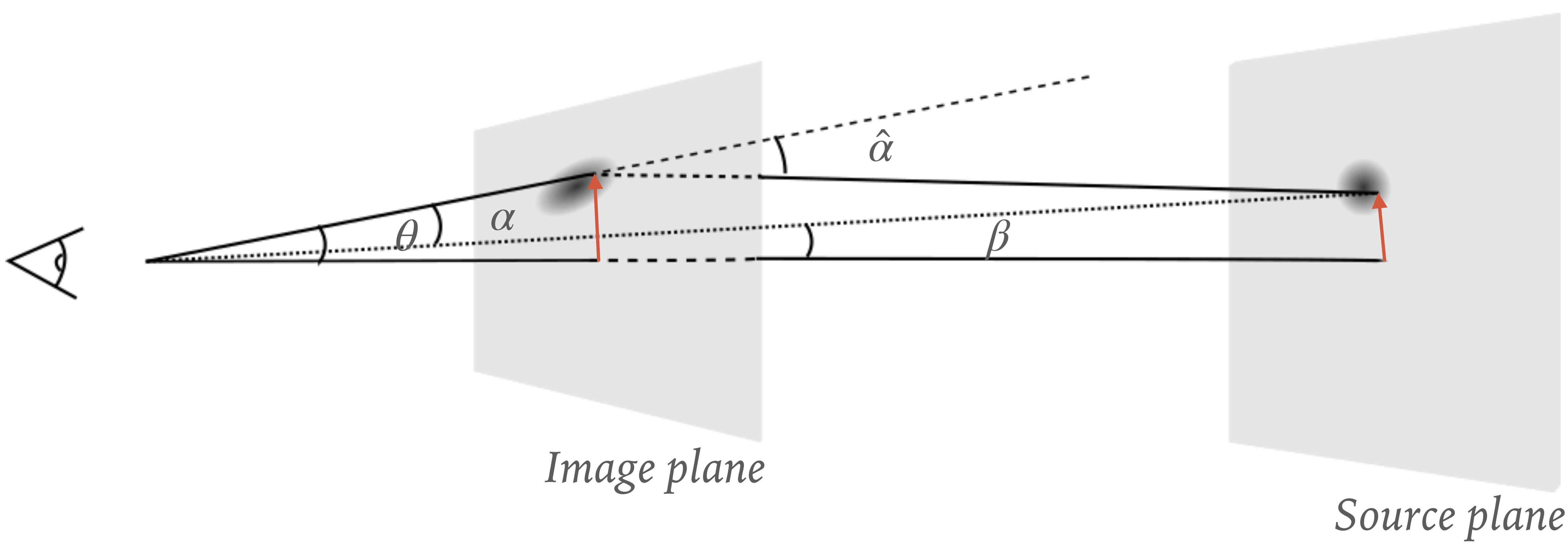
CONSERVATION OF SURFACE BRIGHTNESS

This result implies that the surface brightness at the observed position is related to the surface brightness at the intrinsic position as

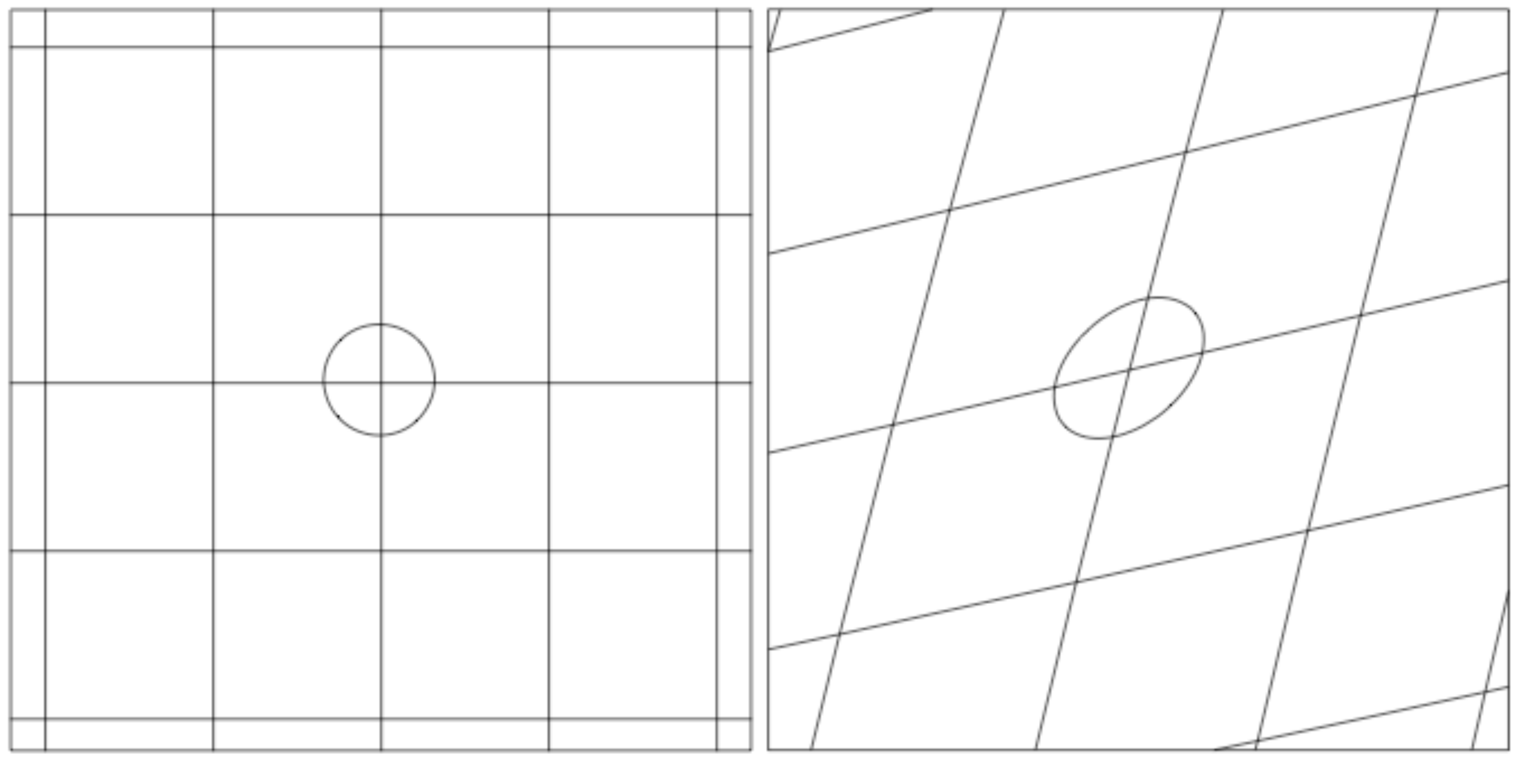
$$I_\nu(\vec{\theta}) = I_\nu^S[\vec{\beta}(\vec{\theta})]$$

where

$$\vec{\beta}(\vec{\theta}) = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

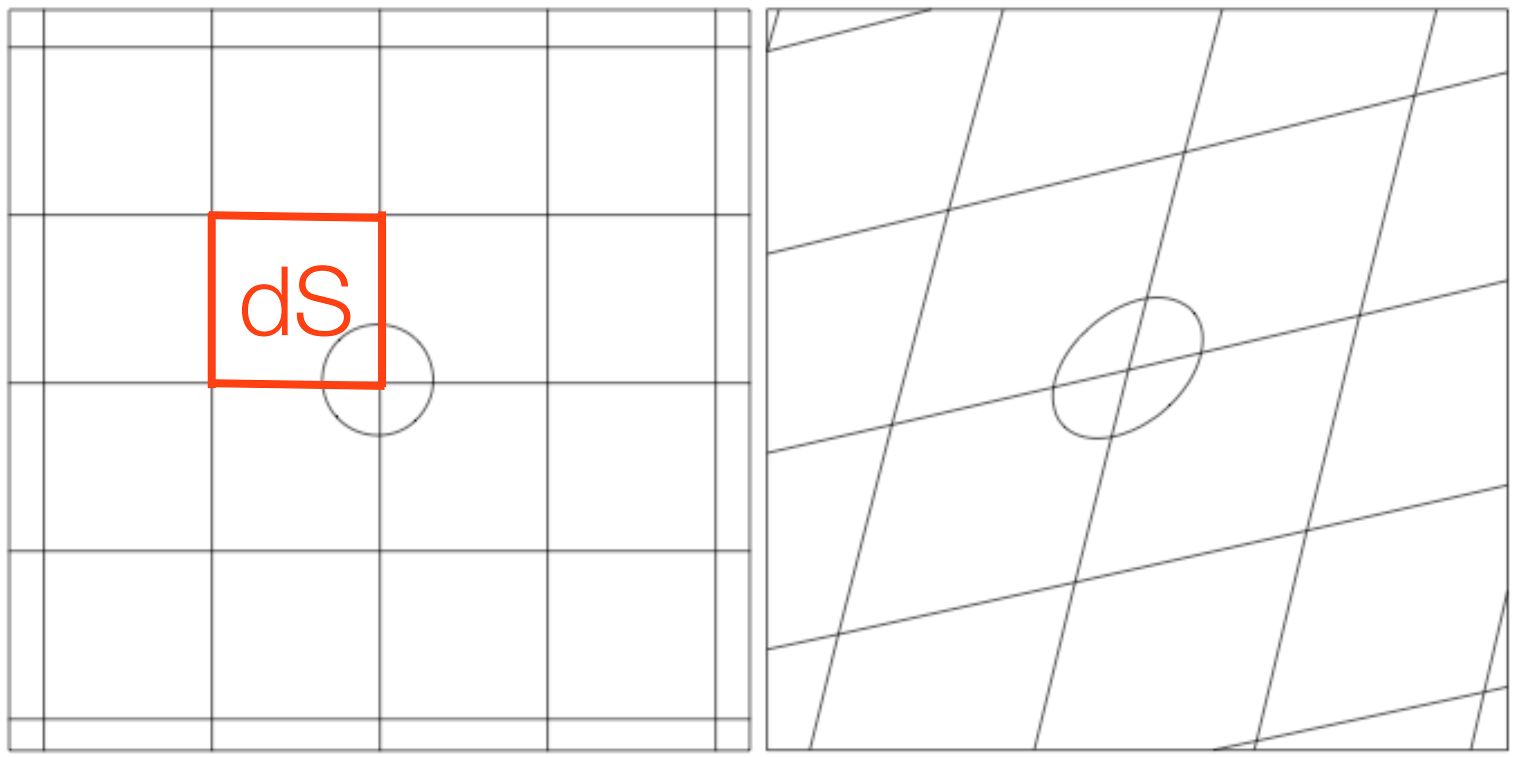


MAGNIFICATION



Kneib & Natarajan (2012)

MAGNIFICATION



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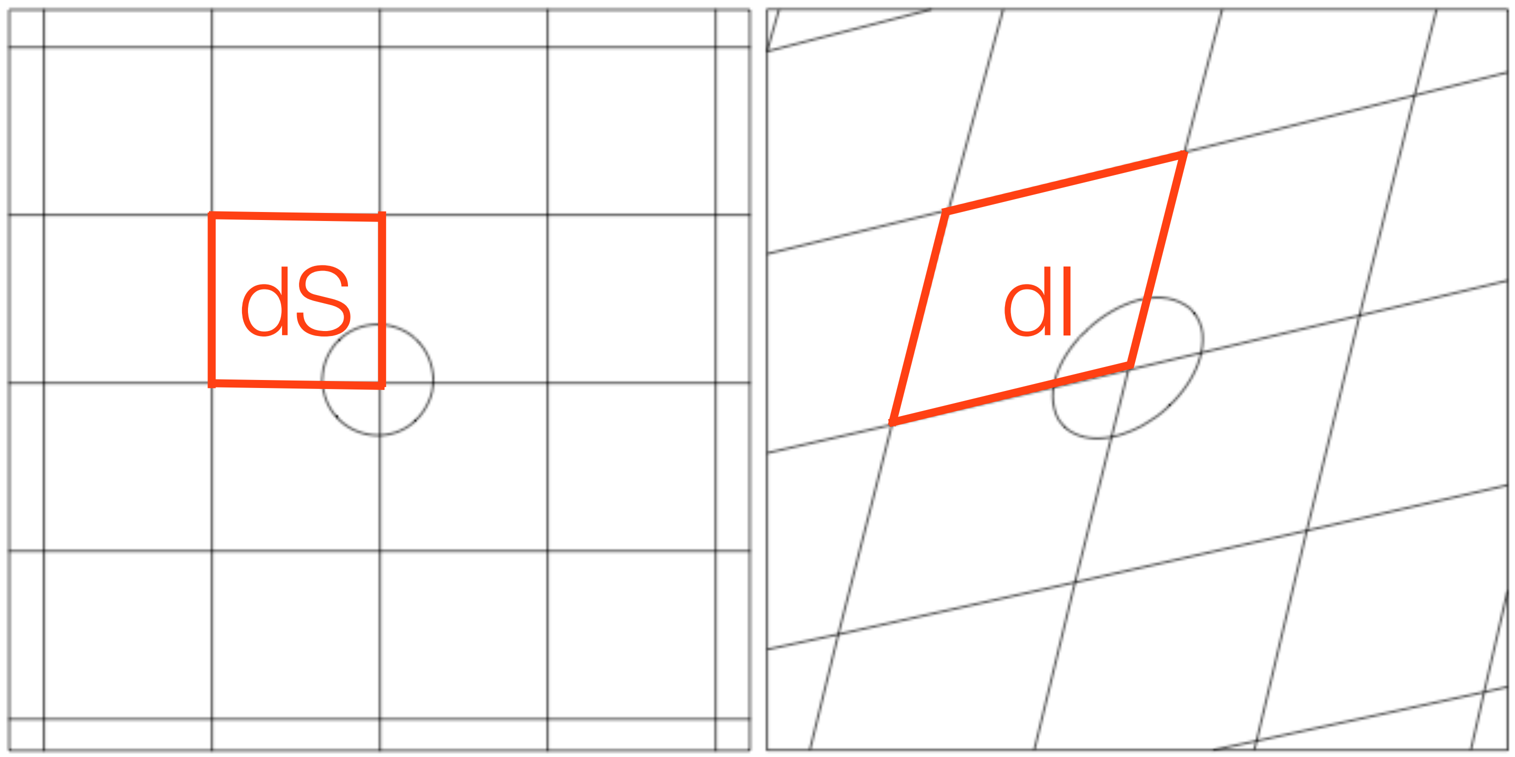

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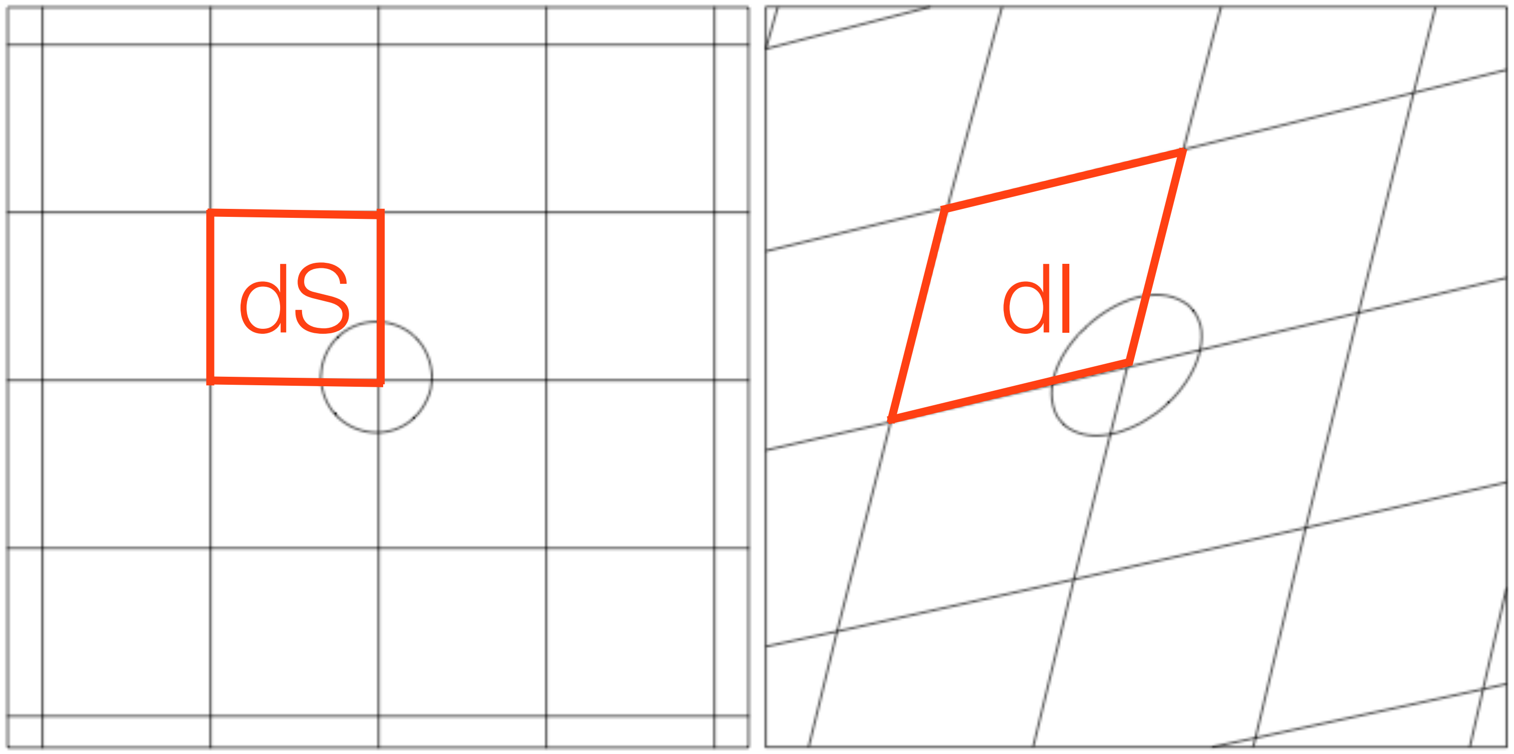

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$$\boxed{\mu = \frac{dI}{dS} = \frac{\delta\theta^2}{\delta\beta^2} = \det A^{-1}}$$

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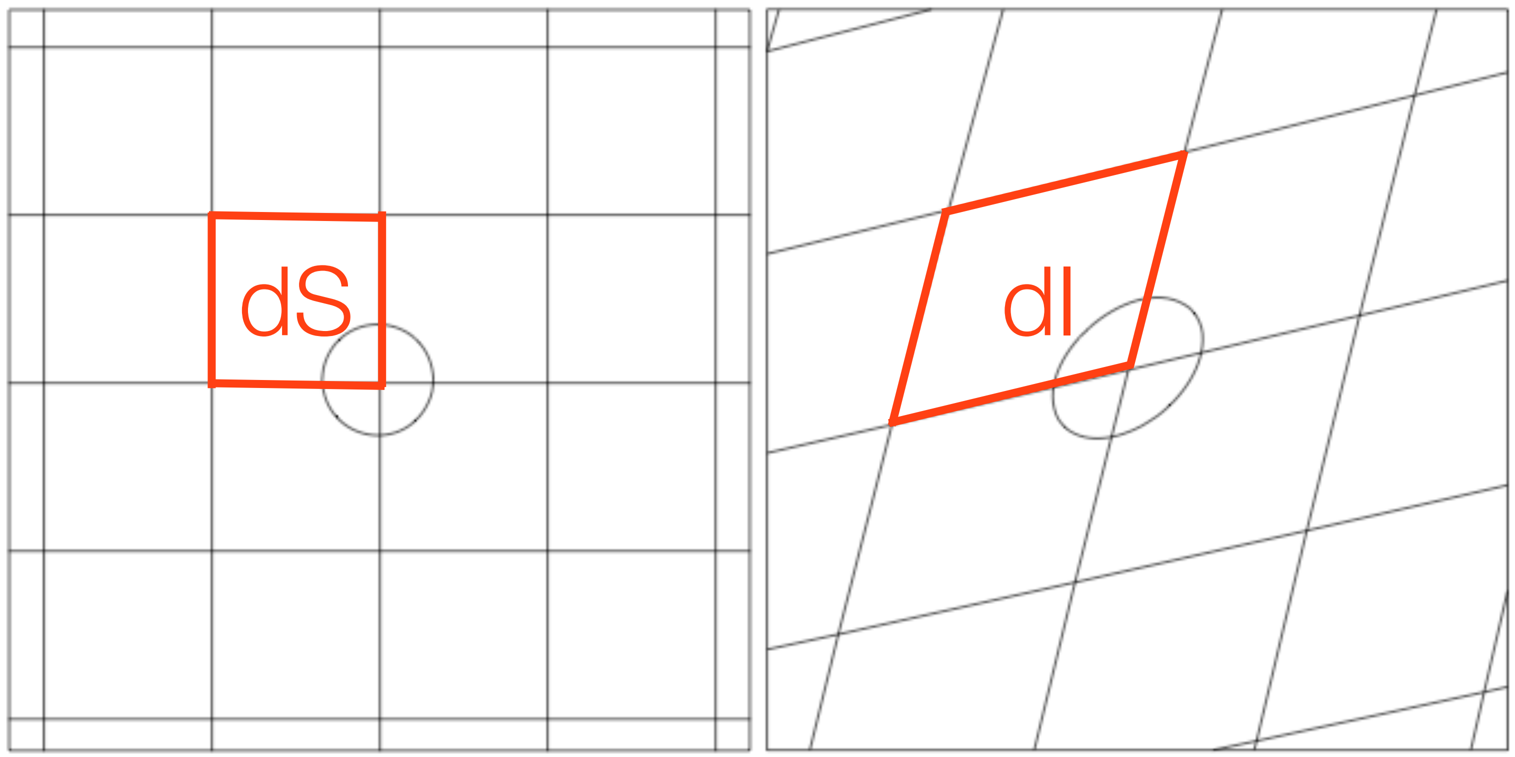

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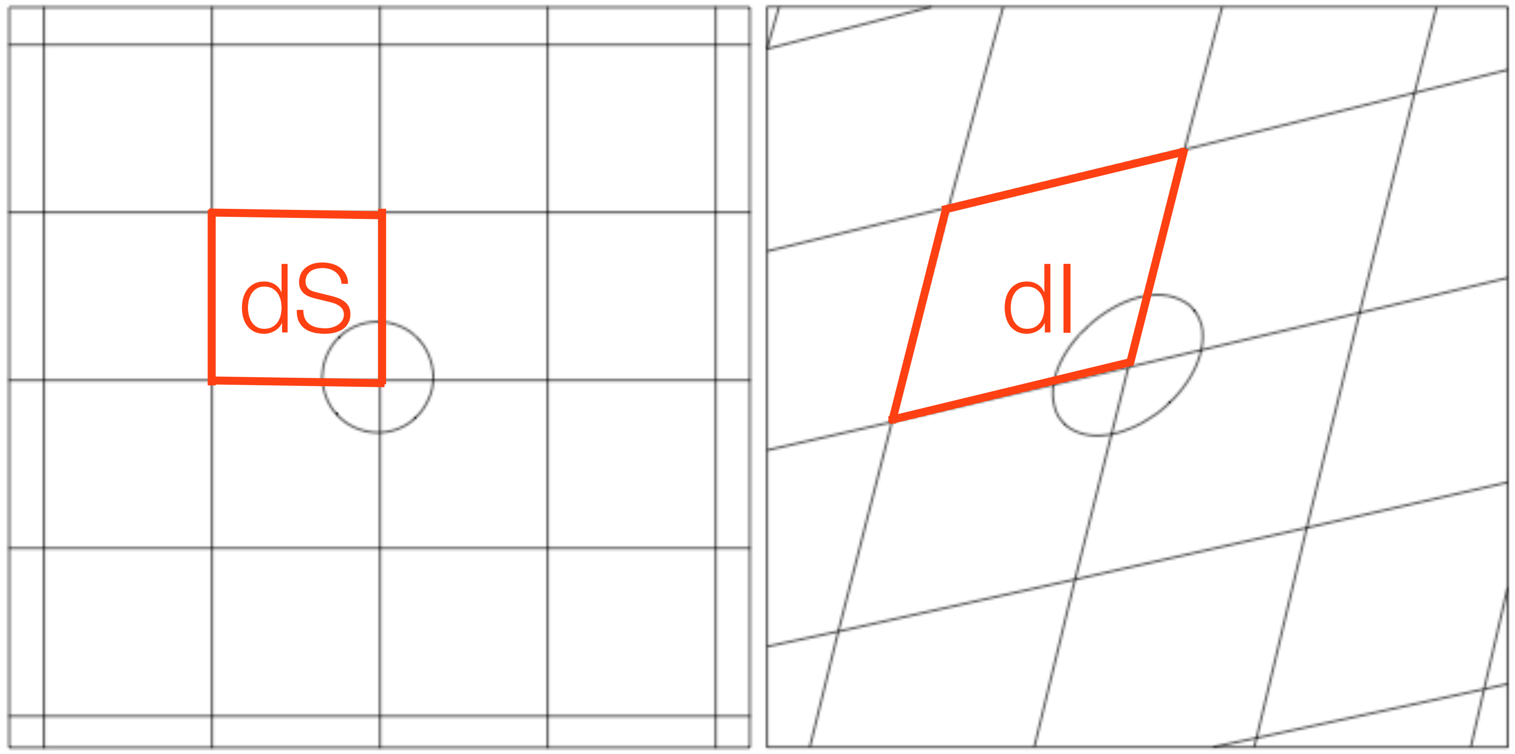
$$\boxed{\mu = \frac{dI}{dS} = \frac{\delta\theta^2}{\delta\beta^2} = \det A^{-1}}$$

$$F_\nu = \int_I I_\nu(\vec{\theta})d^2\theta = \int_S I_\nu^S[\vec{\beta}(\vec{\theta})]\mu d^2\beta$$

MAGNIFICATION



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Lensing changes the amount of photons (flux) we receive from the source by changing the solid angle the source subtends

CRITICAL LINES AND CAUSTICS

Both convergence and shear are functions of position on the lens plane:

$\kappa = \kappa(\vec{\theta})$

$\gamma = \gamma(\vec{\theta})$

The determinant of the lensing Jacobian is

$$\det A = (1 - \kappa - \gamma)(1 - \kappa + \gamma) = \mu^{-1}$$

*The **critical lines** are the lines where the eigenvalues of the Jacobian are zero:*

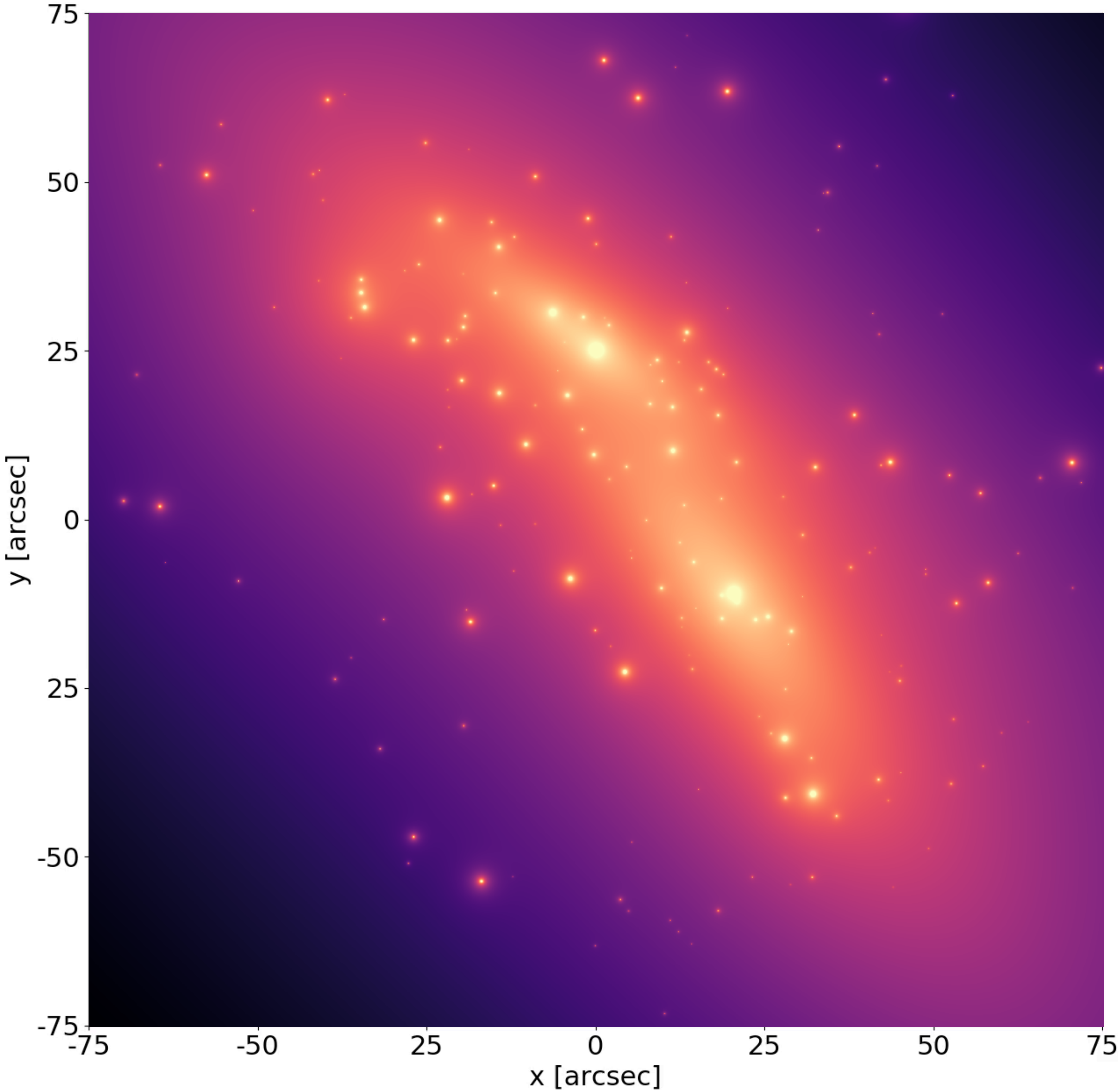
$(1 - \kappa - \gamma) = 0$ *tangential critical line*

$(1 - \kappa + \gamma) = 0$ *radial critical line*

Along these lines the magnification diverges!

*Via the lens equations, they are mapped into the **caustics**…*

Model of MACS0416 by Caminha, MM, et al. (2016)



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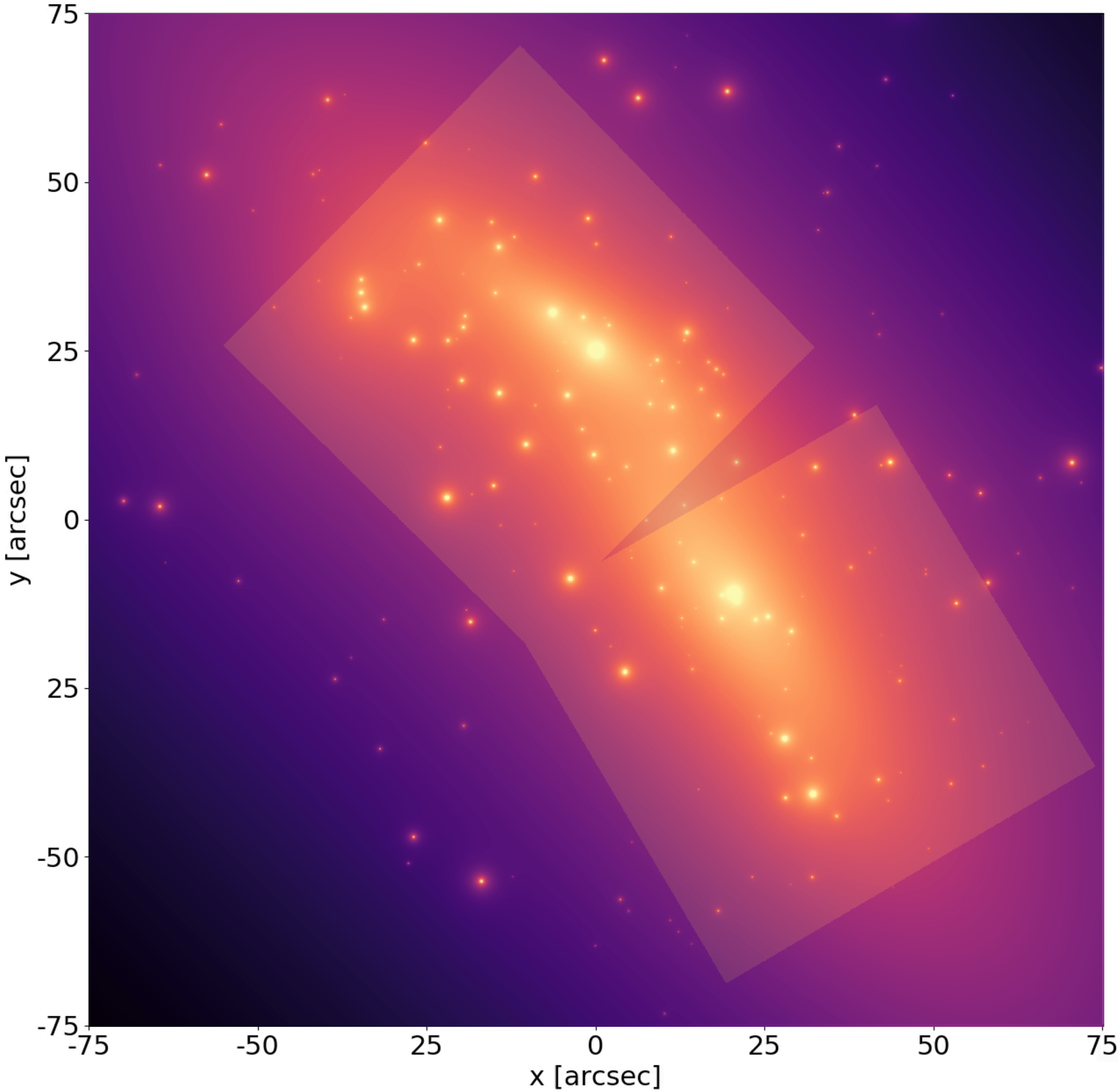

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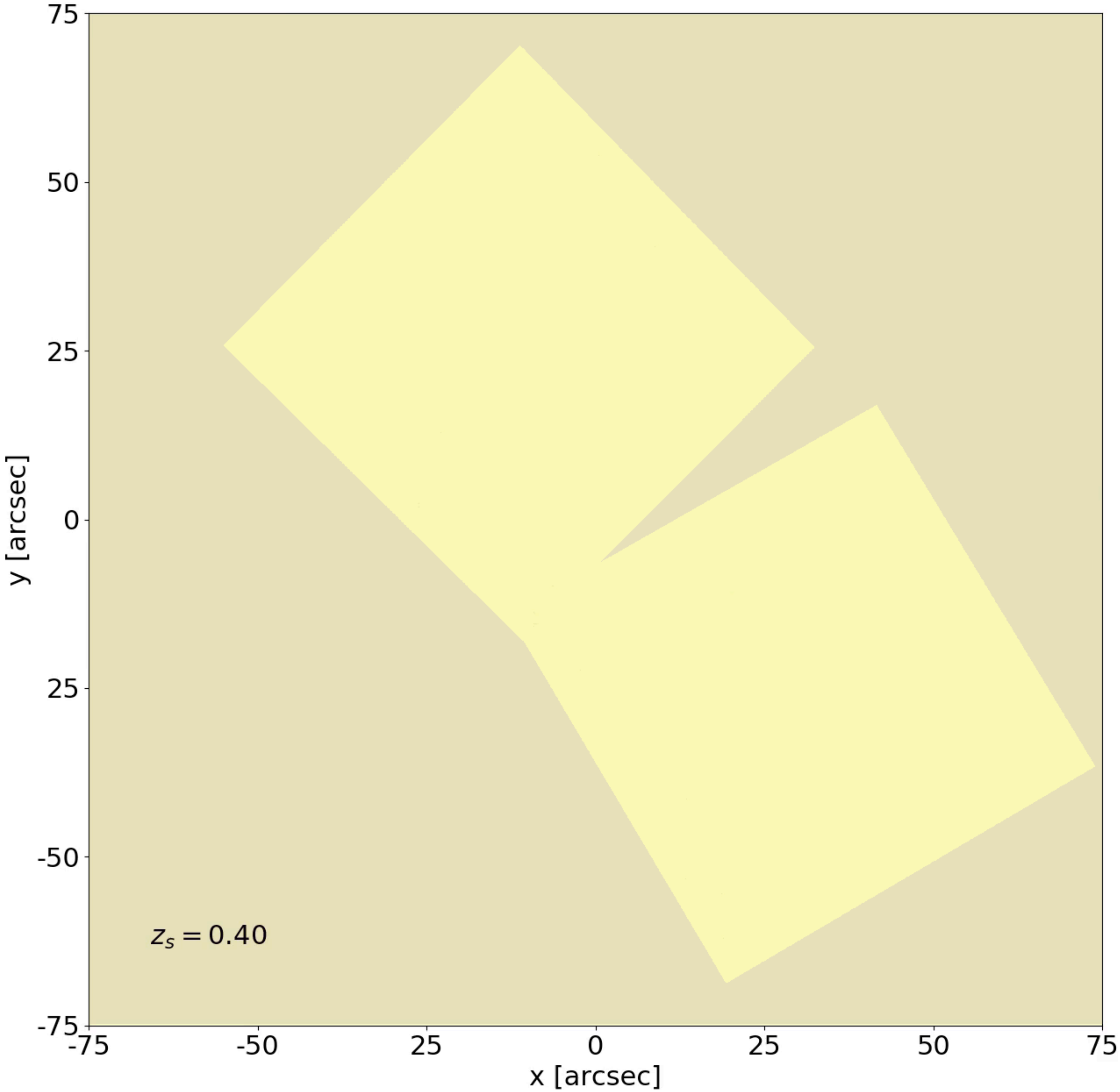

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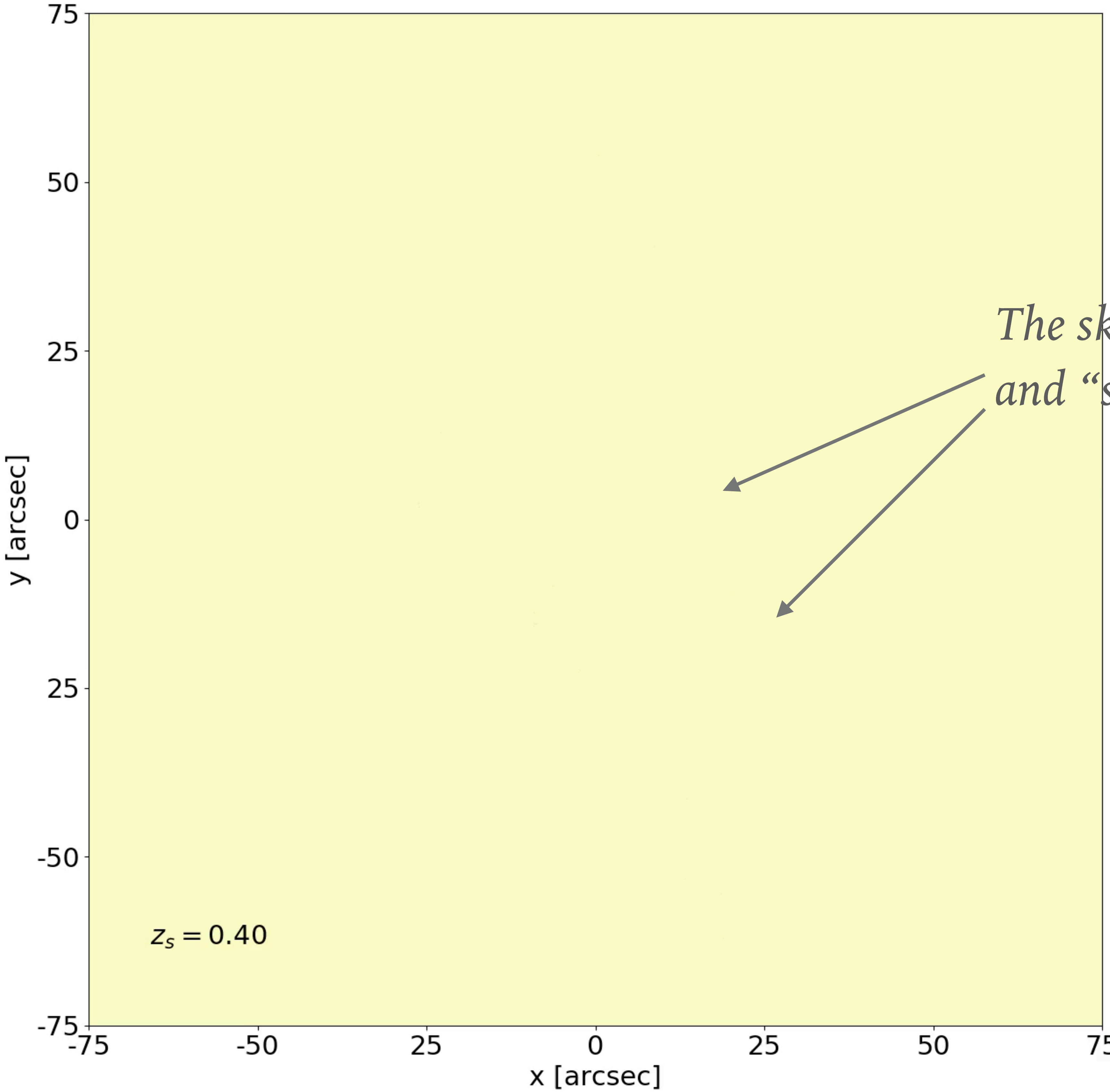
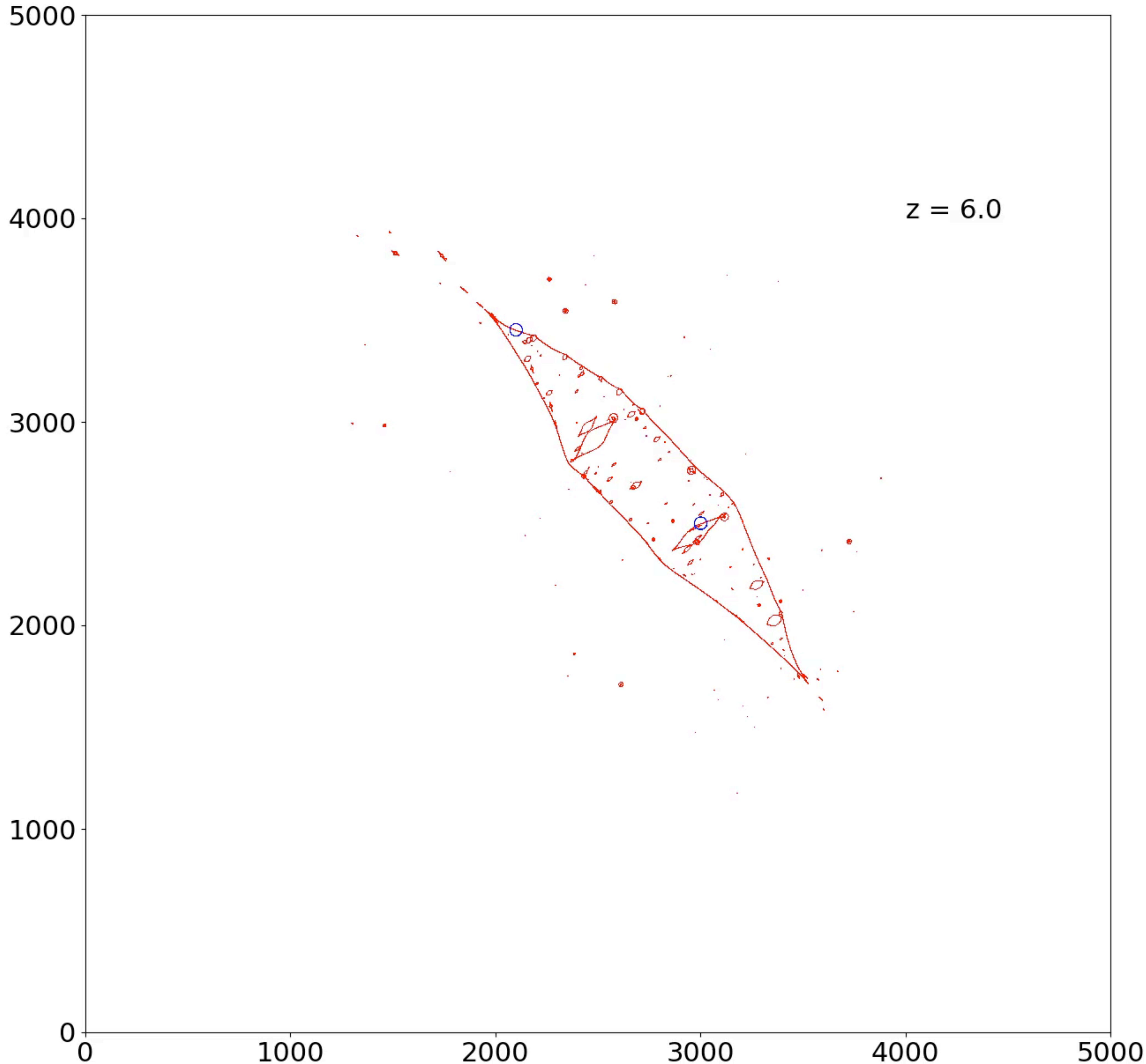
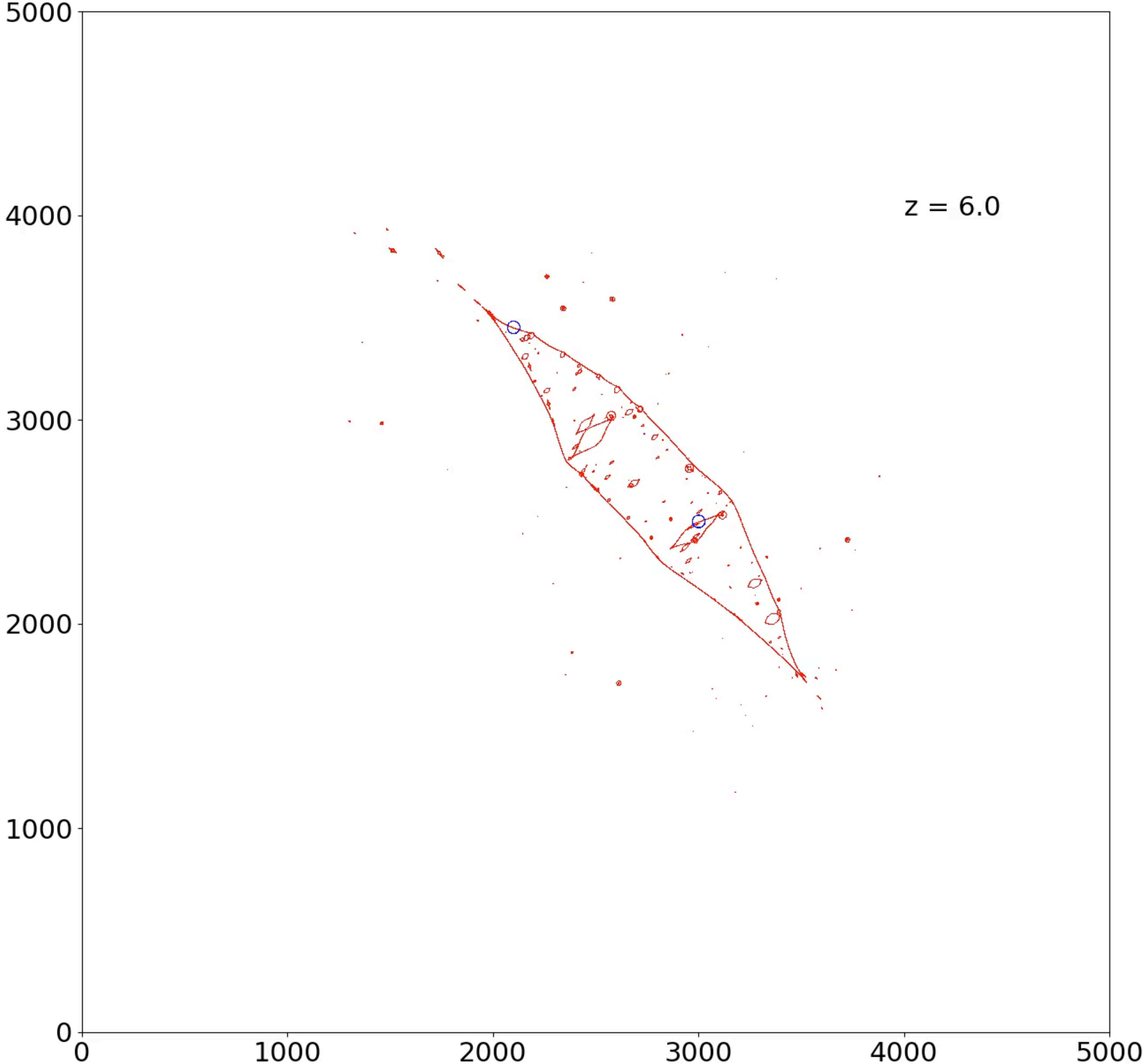


The sky is "folded" and "squeezed"



z = 6.0
5000
4000
3000
2000
1000
0
0
1000
2000
3000
4000
5000



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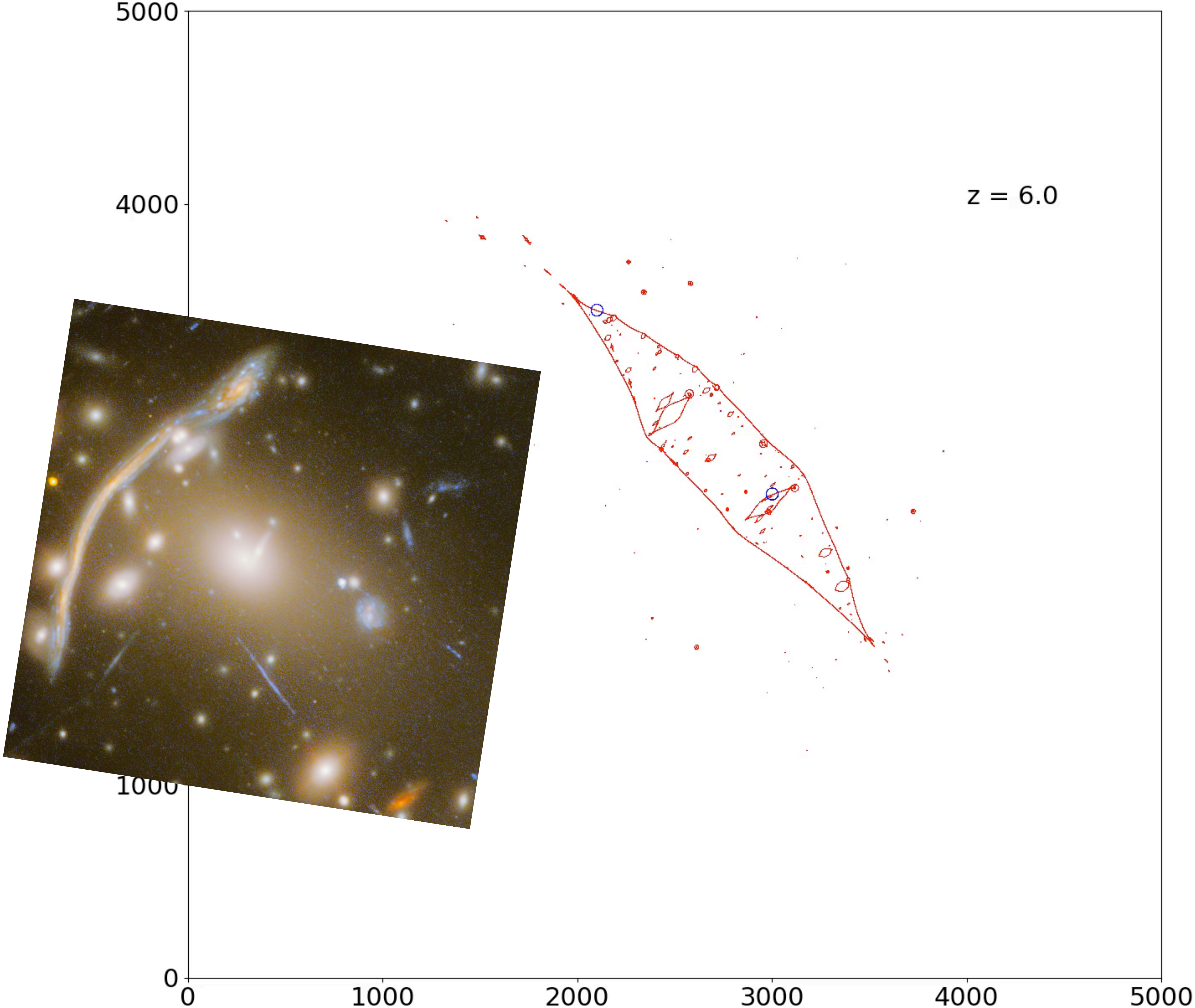

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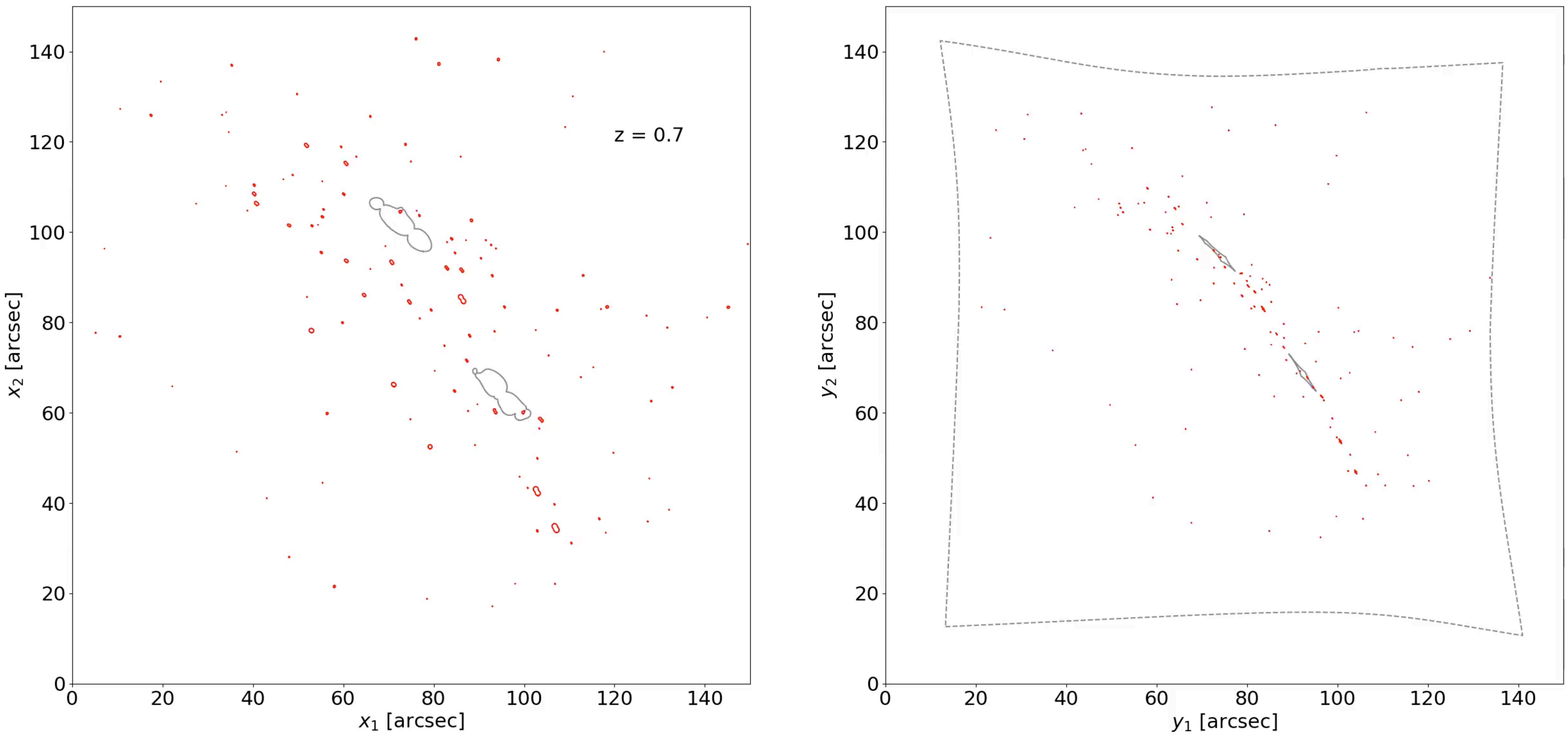
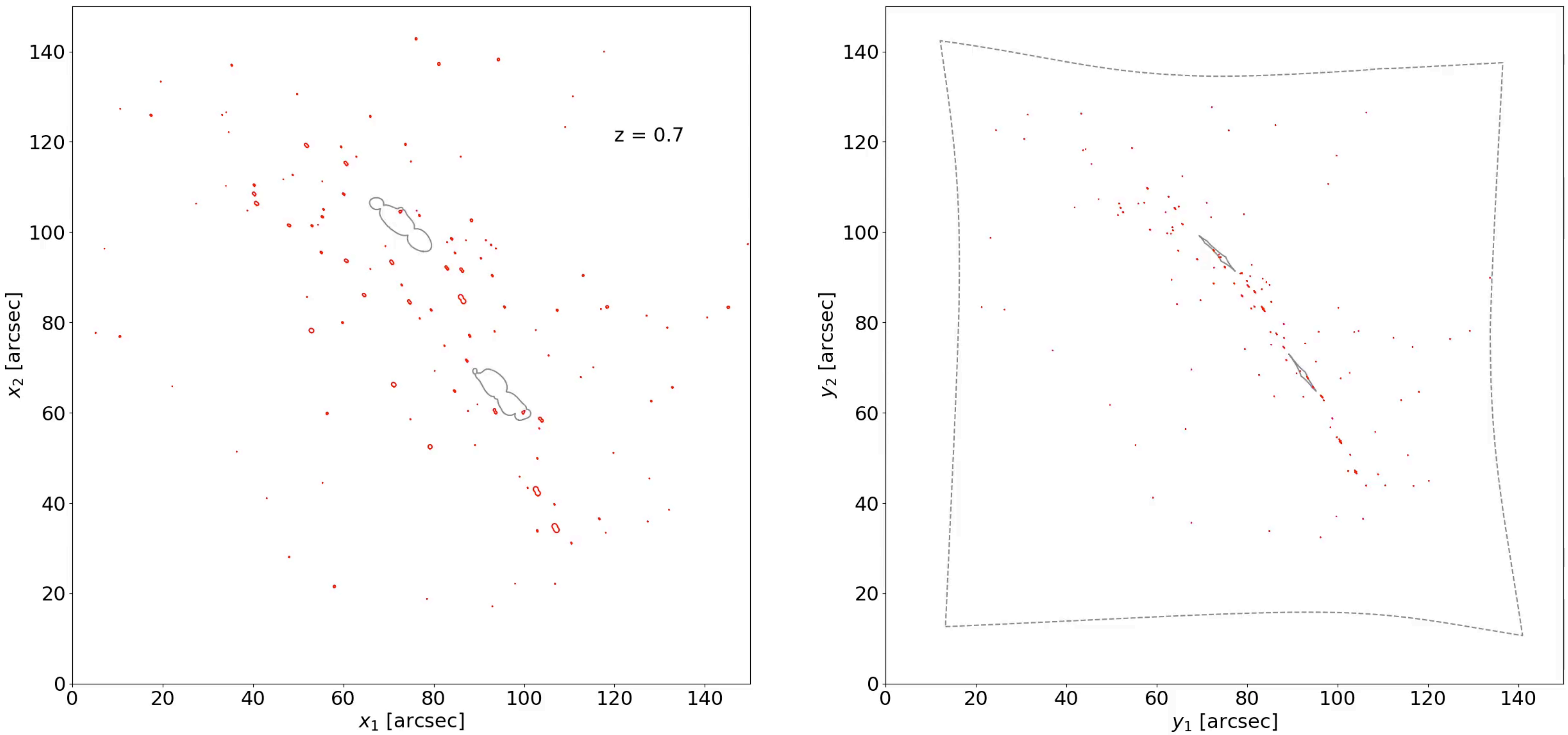


The size of the critical lines and caustics changes as a function of the source redshift! This is because the deflection angle between two redshifts changes by a factor

$$\Xi(z_L, z_{S_1}, z_{S_2}) = \frac{D_{LS_1} D_{S_2}}{D_{S_1} D_{LS_2}}$$



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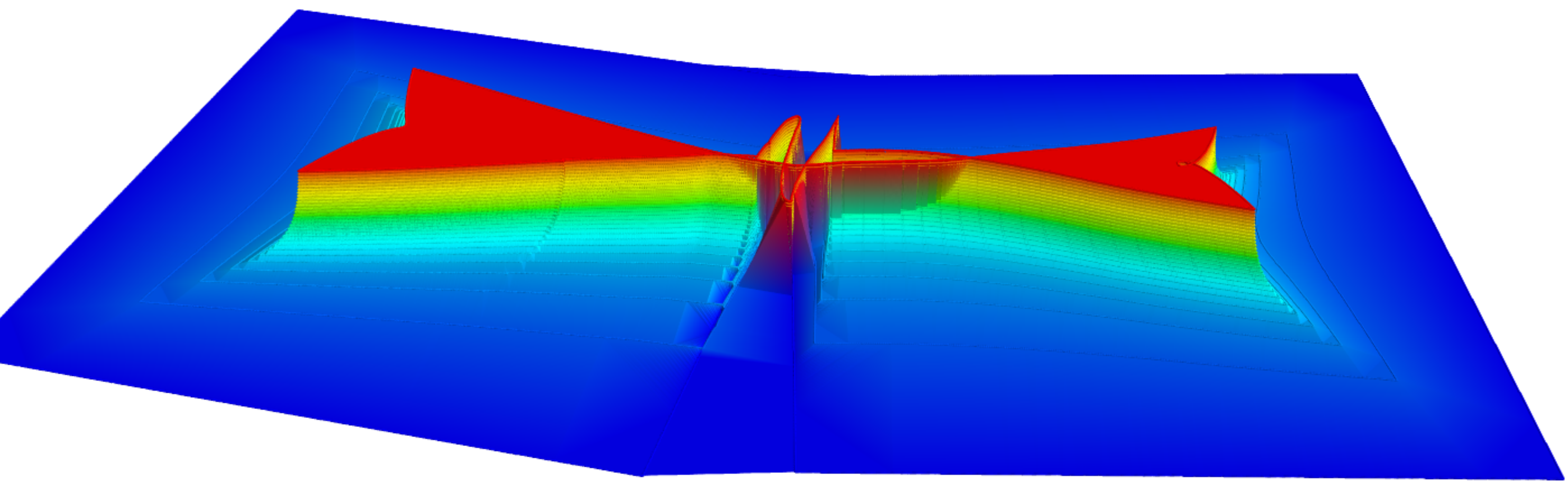
SAMPLED VOLUME



SECOND ORDER LENS EQUATION

$$\beta_i \simeq \frac{\partial \beta_i}{\partial \theta_j} \theta_j$$

$$A_{ij}$$

SECOND ORDER LENS EQUATION

$$\beta_i \simeq \frac{\partial \beta_i}{\partial \theta_j}\theta_j + \frac{1}{2}\frac{\partial^2 \beta_i}{\partial \theta_j \partial \theta_k}\theta_j\theta_k$$

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$$\frac{\partial A_{ij}}{\partial \theta_k} = D_{ijk}$$

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$$D_{ij1} = \begin{pmatrix} -2\gamma_{1,1} - \gamma_{2,2} & -\gamma_{2,1} \\ -\gamma_{2,1} & -\gamma_{2,2} \end{pmatrix} \qquad D_{ij2} = \begin{pmatrix} -\gamma_{2,1} & -\gamma_{2,2} \\ -\gamma_{2,2} & 2\gamma_{1,2} - \gamma_{2,1} \end{pmatrix}$$

COMPLEX NOTATION

$$v = (v_1, v_2) \longrightarrow v = v_1 + iv_2$$

therefore,

$$\alpha = \alpha_1 + i\alpha_2$$

$$\gamma = \gamma_1 + i\gamma_2$$

we can also define complex differential operators:

$$\partial = \partial_1 + i\partial_2$$

$$\partial^\dagger = \partial_1 - i\partial_2$$

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we can also define complex differential operators:

$\partial = \partial_1 + i\partial_2$ *Spin raising operator*

$\partial^\dagger = \partial_1 - i\partial_2$ *Spin lowering operator*

COMPLEX NOTATION

$$\partial \hat{\Psi} = \partial_1 \hat{\Psi} + i\partial_2 \hat{\Psi} = \alpha_1 + i\alpha_2 = \alpha$$

COMPLEX NOTATION

$$\partial\hat{\Psi} = \partial_1\hat{\Psi} + i\partial_2\hat{\Psi} = \alpha_1 + i\alpha_2 = \alpha$$

From spin-0 scalar field to spin-1 vector field (deflection angle)

COMPLEX NOTATION

$$\partial\hat{\Psi} = \partial_1\hat{\Psi} + i\partial_2\hat{\Psi} = \alpha_1 + i\alpha_2 = \alpha$$

From spin-0 scalar field to spin-1 vector field (deflection angle)

$$\partial^\dagger\partial = \partial_1^2 + \partial_2^2 = \triangle$$

$$\partial^\dagger\partial\hat{\Psi} = \triangle\hat{\Psi} = 2\kappa$$

COMPLEX NOTATION

$$\partial\hat{\Psi} = \partial_1\hat{\Psi} + i\partial_2\hat{\Psi} = \alpha_1 + i\alpha_2 = \alpha$$

From spin-0 scalar field to spin-1 vector field (deflection angle)

$$\partial^\dagger\partial = \partial_1^2 + \partial_2^2 = \triangle$$

$$\partial^\dagger\partial\hat{\Psi} = \triangle\hat{\Psi} = 2\kappa$$

From spin-1 vector field to spin-0 scalar field

COMPLEX NOTATION

$$\partial\hat{\Psi} = \partial_1\hat{\Psi} + i\partial_2\hat{\Psi} = \alpha_1 + i\alpha_2 = \alpha$$

From spin-0 scalar field to spin-1 vector field (deflection angle)

$$\partial^\dagger\partial = \partial_1^2 + \partial_2^2 = \Delta$$

$$\partial^\dagger\partial\hat{\Psi} = \Delta\hat{\Psi} = 2\kappa$$

From spin-1 vector field to spin-0 scalar field

$$\frac{1}{2}\partial\partial\hat{\Psi} = \frac{1}{2}\partial\alpha = \gamma$$

The shear is a spin-2 field

COMPLEX NOTATION

$$
\begin{aligned}
F &= \frac{1}{2}\partial\partial^{\dagger}\partial\hat{\Psi} = \partial\kappa \\
G &= \frac{1}{2}\partial\partial\partial\hat{\Psi} = \partial\gamma
\end{aligned}
$$

COMPLEX NOTATION

$$F = \frac{1}{2}\partial\partial^{\dagger}\partial\hat{\Psi} = \partial\kappa$$

Spin-1

$$G = \frac{1}{2}\partial\partial\partial\hat{\Psi} = \partial\gamma$$

Spin-3

COMPLEX NOTATION

$$F = \frac{1}{2}\partial\partial^\dagger\partial\hat{\Psi} = \partial\kappa$$ *Spin-1*

$$G = \frac{1}{2}\partial\partial\partial\hat{\Psi} = \partial\gamma$$ *Spin-3*

$$F = F_1 + iF_2 = (\gamma_{1,1} + \gamma_{2,2}) + i(\gamma_{2,1} - \gamma_{1,2})$$

$$G = G_1 + iG_2 = (\gamma_{1,1} - \gamma_{2,2}) + i(\gamma_{2,1} + \gamma_{1,2})$$

$$D_{ij1} = \begin{pmatrix} -2\gamma_{1,1} - \gamma_{2,2} & -\gamma_{2,1} \\ -\gamma_{2,1} & -\gamma_{2,2} \end{pmatrix} \qquad D_{ij2} = \begin{pmatrix} -\gamma_{2,1} & -\gamma_{2,2} \\ -\gamma_{2,2} & 2\gamma_{1,2} - \gamma_{2,1} \end{pmatrix}$$

$$D_{111} = -2\gamma_{11} - \gamma_{22} = -\frac{1}{2}(3F_1 + G_1)$$

$$D_{211} = D_{121} = D_{112} = -\gamma_{21} = -\frac{1}{2}(F_2 + G_2)$$

$$D_{122} = D_{212} = D_{221} = -\gamma_{22} = -\frac{1}{2}(F_1 - G_1)$$

$$D_{222} = 2\gamma_{12} - \gamma_{21} = -\frac{1}{2}(3F_2 - G_2)$$