

# GRAVITATIONAL LENSING

## 7 - TIME DELAYS

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*Massimo Meneghetti*  
AA 2018-2019

# GRAVITATIONAL TIME DELAY

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- In a lensing phenomenon, light travels with an effective velocity  $c' < c$ . As seen, this implies an effective refractive index  $n > 1$
- The effective refractive index is expressed in terms of the Newtonian potential
- If we compare the travel times of two photons, one traveling at velocity  $c$  and the other at velocity  $c'$ , we notice that the second accumulates a time delay  $t_{grav}$
- This time delay is called *gravitational* time delay, or *Shapiro* time delay (Shapiro, 1964)

$$\begin{aligned} n &= 1 - \frac{2\Phi}{c^2} \\ t_{grav} &= \int \frac{dz}{c'} - \int \frac{dz}{c} \\ &= \frac{1}{c} \int (n - 1) dz \\ &= -\frac{2}{c^3} \int \Phi dz \end{aligned}$$

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$$\hat{\Psi}(\vec{\theta}) = \frac{D_{LS}}{D_L D_S} \frac{2}{c^2} \int \Phi(\vec{\theta}, z) dz$$

$$t_{grav} = -\frac{1}{c} \frac{D_S D_L}{D_{LS}} \hat{\Psi}$$

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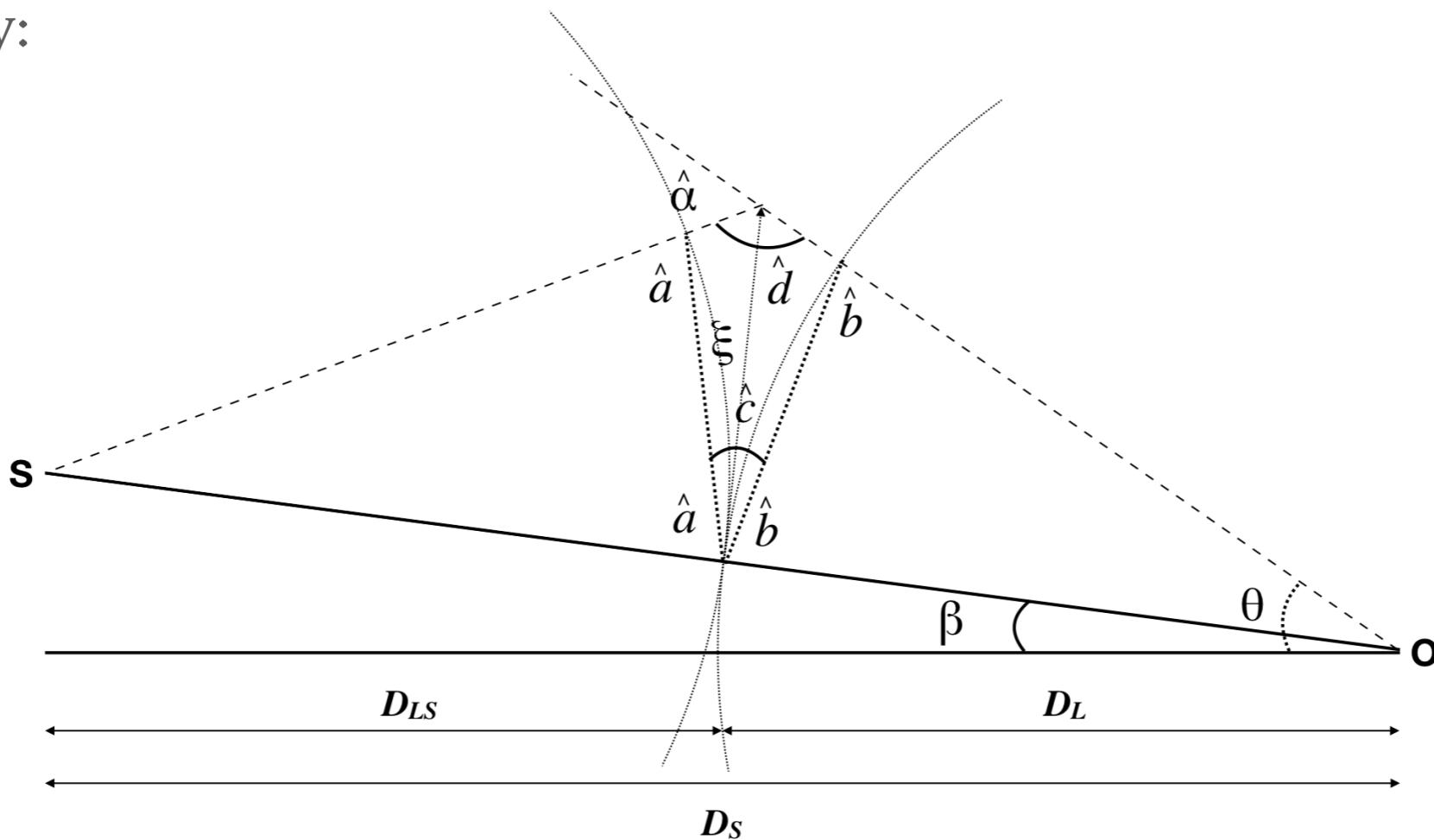
$$t_{grav} = -\frac{1}{c} \frac{D_S D_L}{D_{LS}} \hat{\Psi}$$

*This time delay does not account yet for the different path of photons!*

# GEOMETRICAL TIME DELAY

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- We need to combine the gravitational time delay to the so called *geometrical* time delay:

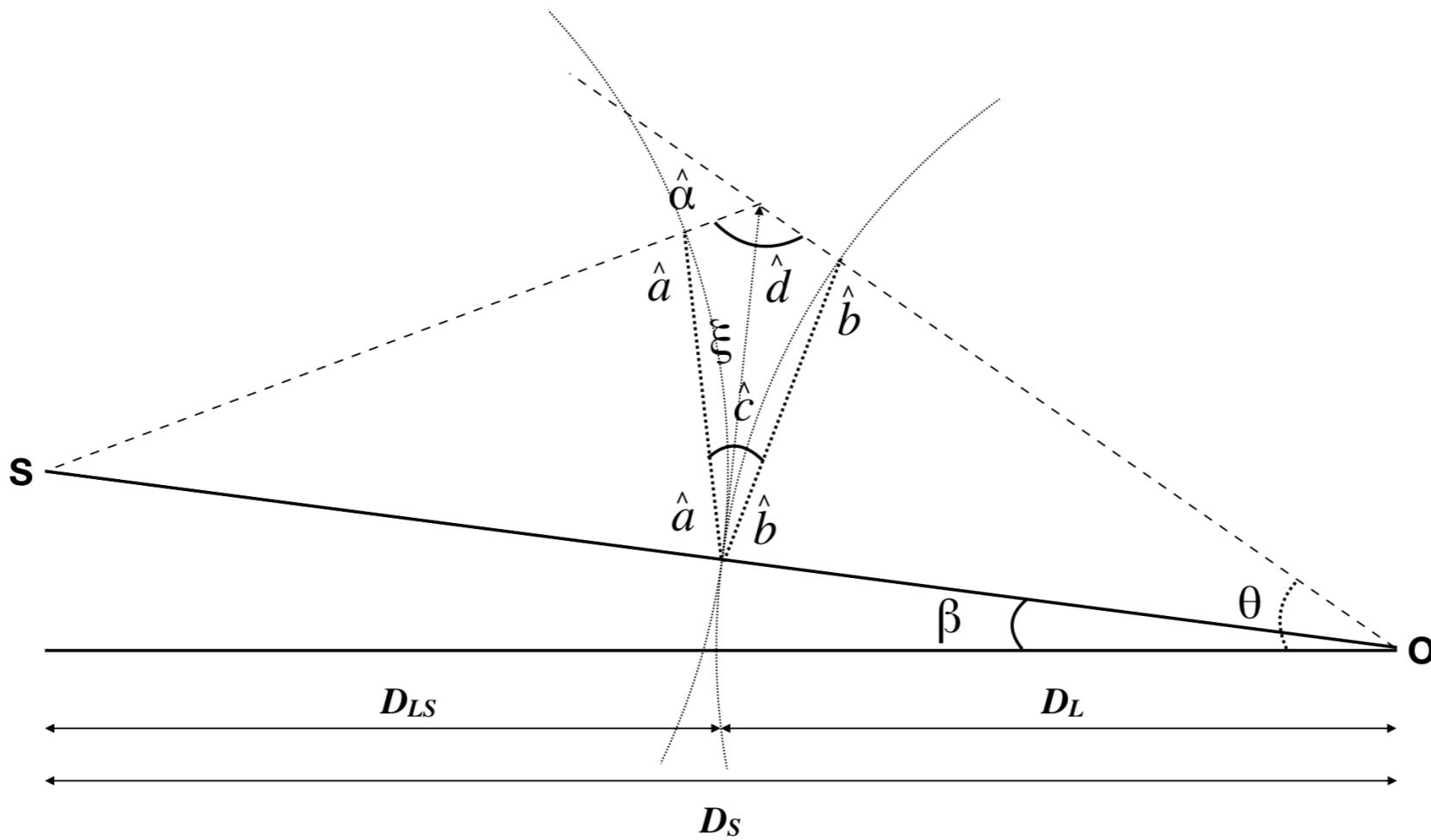


$$(\pi - \hat{a}) + (\pi - \hat{b}) + \hat{c} + \hat{d} = 2\pi \Rightarrow \hat{a} + \hat{b} = \hat{c} + \hat{d}$$

$$\hat{a} + \hat{b} + \hat{c} = \pi \Rightarrow 2\hat{c} = \pi - \hat{d}$$

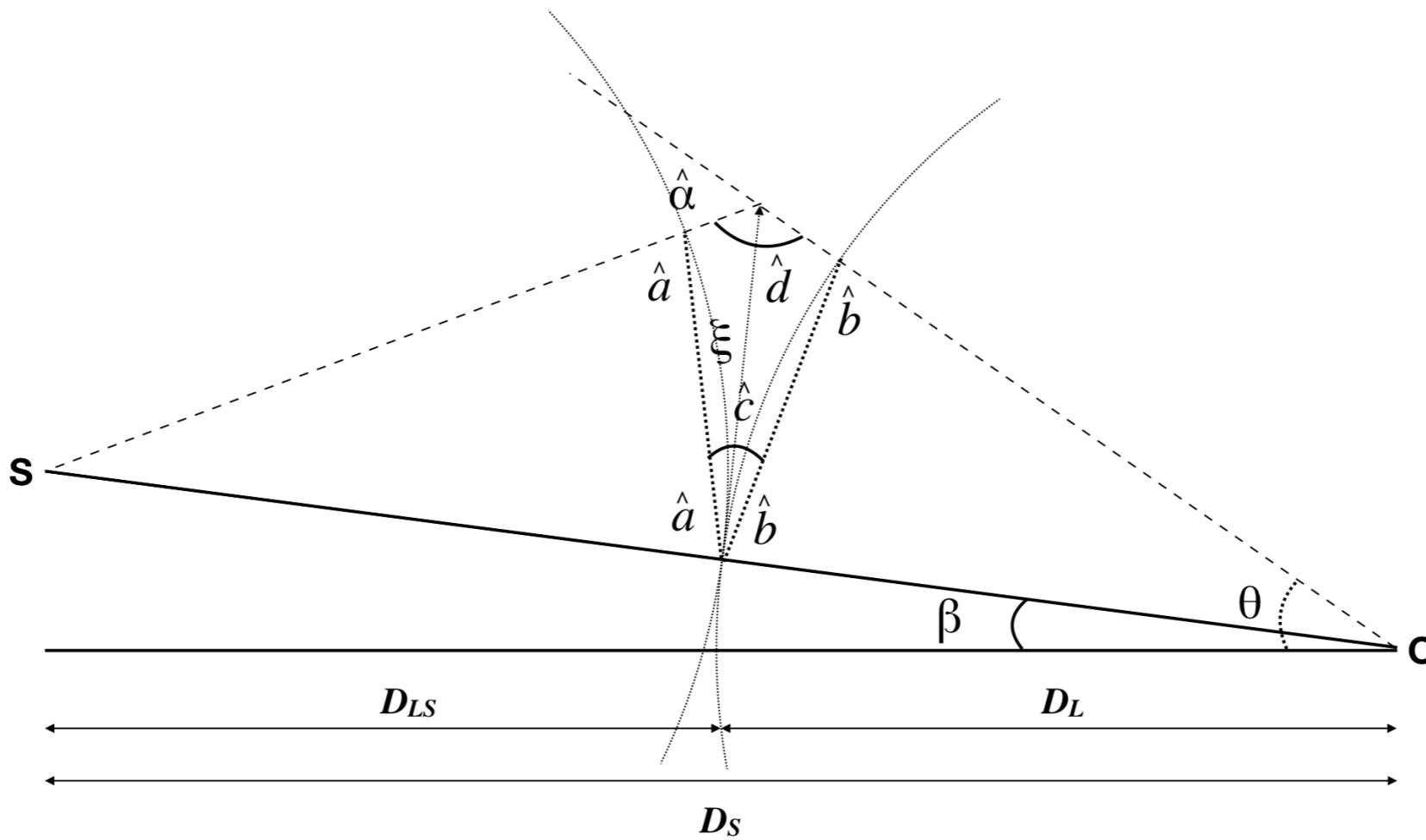
$$\hat{a} + \hat{d} = \pi \Rightarrow \hat{d} = \pi - \hat{a}$$

# GEOMETRICAL TIME DELAY



$$\begin{aligned}
 (\pi - \hat{a}) + (\pi - \hat{b}) + \hat{c} + \hat{d} &= 2\pi \Rightarrow \hat{a} + \hat{b} = \hat{c} + \hat{d} \\
 \hat{a} + \hat{b} + \hat{c} &= \pi \Rightarrow 2\hat{c} = \pi - \hat{d} \\
 \hat{\alpha} + \hat{d} &= \pi \Rightarrow \hat{d} = \pi - \hat{\alpha}
 \end{aligned}
 \quad \Rightarrow \hat{c} = \frac{\hat{\alpha}}{2} = \frac{1}{2} \frac{D_S}{D_{LS}} \alpha = \frac{1}{2} \frac{D_S}{D_{LS}} (\theta - \beta)$$

# GEOMETRICAL TIME DELAY



$$\begin{aligned}\Rightarrow \hat{c} &= \frac{\hat{\alpha}}{2} \\ &= \frac{1}{2} \frac{D_S}{D_{LS}} \alpha \\ &= \frac{1}{2} \frac{D_S}{D_{LS}} (\theta - \beta)\end{aligned}$$

$$\begin{aligned}\xi &= D_L(\theta - \beta) \\ t_{geom} &= \frac{1}{c} \xi \hat{c} \\ &= \frac{1}{2c} \frac{D_S D_L}{D_{LS}} (\theta - \beta)^2\end{aligned}$$

# TOTAL TIME DELAY

---

$$t_{geom} = \frac{1}{2c} \frac{D_S D_L}{D_{LS}} (\theta - \beta)^2$$

$$t_{grav} = -\frac{1}{c} \frac{D_S D_L}{D_{LS}} \hat{\Psi}$$

$$\begin{aligned} t_{tot} &= t_{geom} + t_{grav} \\ &= \frac{1}{2c} \frac{D_S D_L}{D_{LS}} (\theta - \beta)^2 - \frac{1}{c} \frac{D_S D_L}{D_{LS}} \hat{\Psi}(\theta) \\ &= \frac{1}{c} \frac{D_S D_L}{D_{LS}} \left[ \frac{1}{2} (\theta - \beta)^2 - \hat{\Psi}(\theta) \right] \end{aligned}$$

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*Accounting for the expansion of the universe and for the fact that this is a surface:*

$$t_{tot}(\vec{\theta}) = \frac{1+z_L}{c} \frac{D_S D_L}{D_{LS}} \left[ \frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \hat{\Psi}(\vec{\theta}) \right]$$

# TOTAL TIME DELAY

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$$t_{tot}(\vec{\theta}) = \frac{1+z_L}{c} \frac{D_S D_L}{D_{LS}} \left[ \frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \hat{\Psi}(\vec{\theta}) \right]$$

$$\tau(\vec{\theta}) = \frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \hat{\Psi}(\vec{\theta})$$

*Fermat potential*

$$D_{\Delta t} = (1+z_L) \frac{D_S D_L}{D_{LS}}$$

*Time delay distance*

# TIME DELAY SURFACE

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$$t(\vec{\theta}) = t_{geom} + t_{grav} \propto \left( \frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \hat{\Psi} \right)$$

$$\vec{\nabla} t(\vec{\theta}) \propto \left( \vec{\theta} - \vec{\beta} - \vec{\nabla} \hat{\Psi} \right)$$

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*Lens equation!*

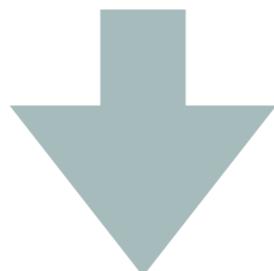
# TIME DELAY SURFACE

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*Images form at the stationary points of  $t$ !*

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*Lens equation!*



*Images form at the stationary points of  $t$ !*

$$T_{ij} = \frac{\partial^2 t(\vec{\theta})}{\partial \theta_i \partial \theta_j} \propto (\delta_{ij} - \Psi_{ij})$$

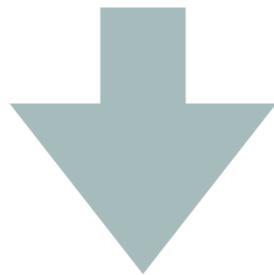
# TIME DELAY SURFACE

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*Images form at the stationary points of  $t$ !*

$$T_{ij} = \frac{\partial^2 t(\vec{\theta})}{\partial \theta_i \partial \theta_j} \propto (\delta_{ij} - \Psi_{ij}) \quad \textcolor{red}{\text{This is the Jacobian!}}$$

# TYPES OF IMAGES

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- minima (eigenvalues of  $A$  are both positive, hence  $\det A > 0$  and  $\text{Tr } A > 0$ ; positive magnification)
- saddle (eigenvalues have opposite signs, thus  $\det A < 0$ ; negative magnification)
- maxima (eigenvalues are both negative, hence  $\det A > 0$  and  $\text{Tr } A < 0$ ; positive magnification)
- Let see some examples...

# EXAMPLE OF TIME DELAY SURFACE

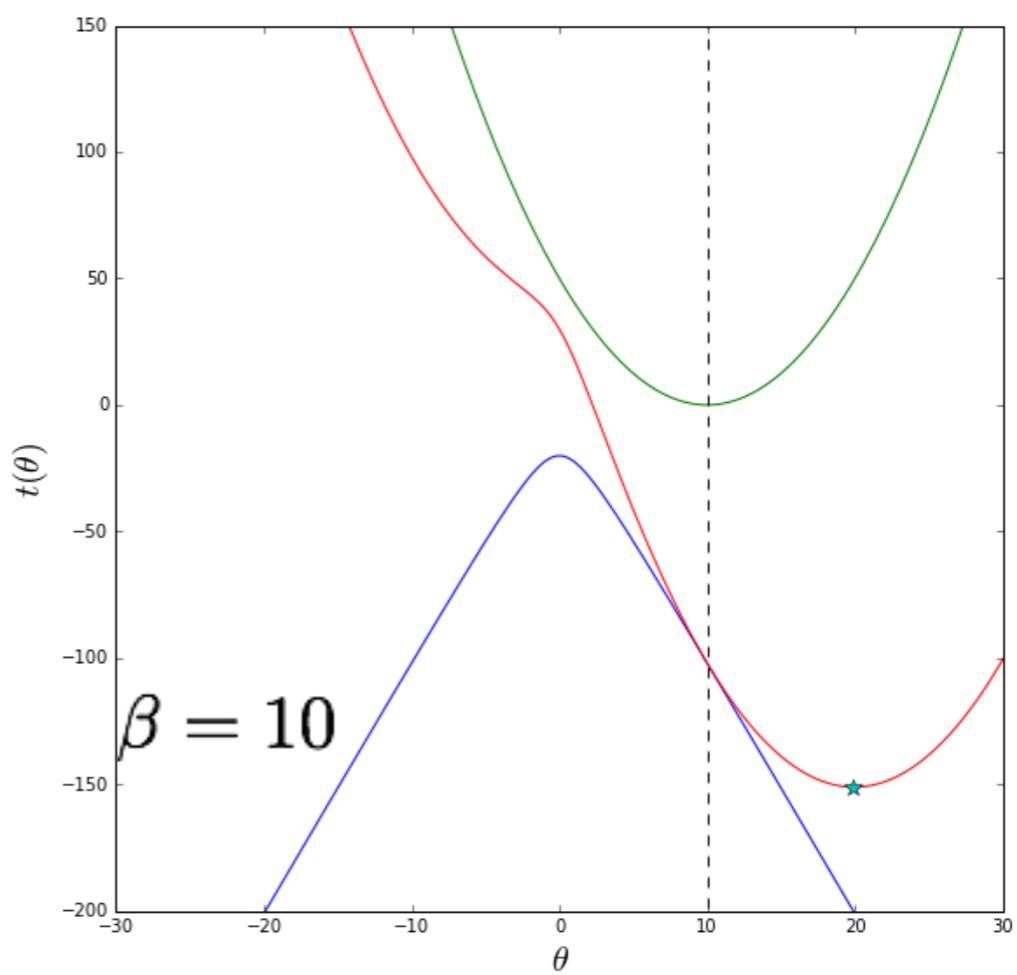
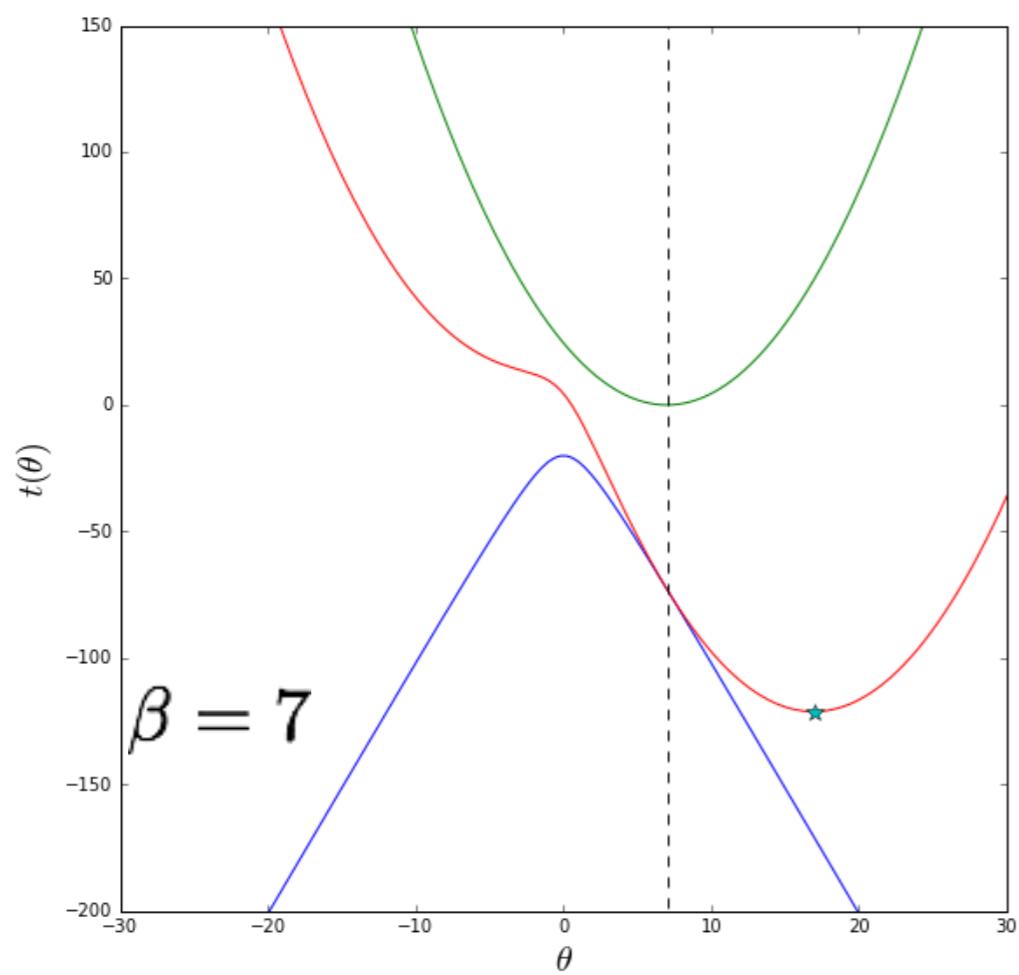
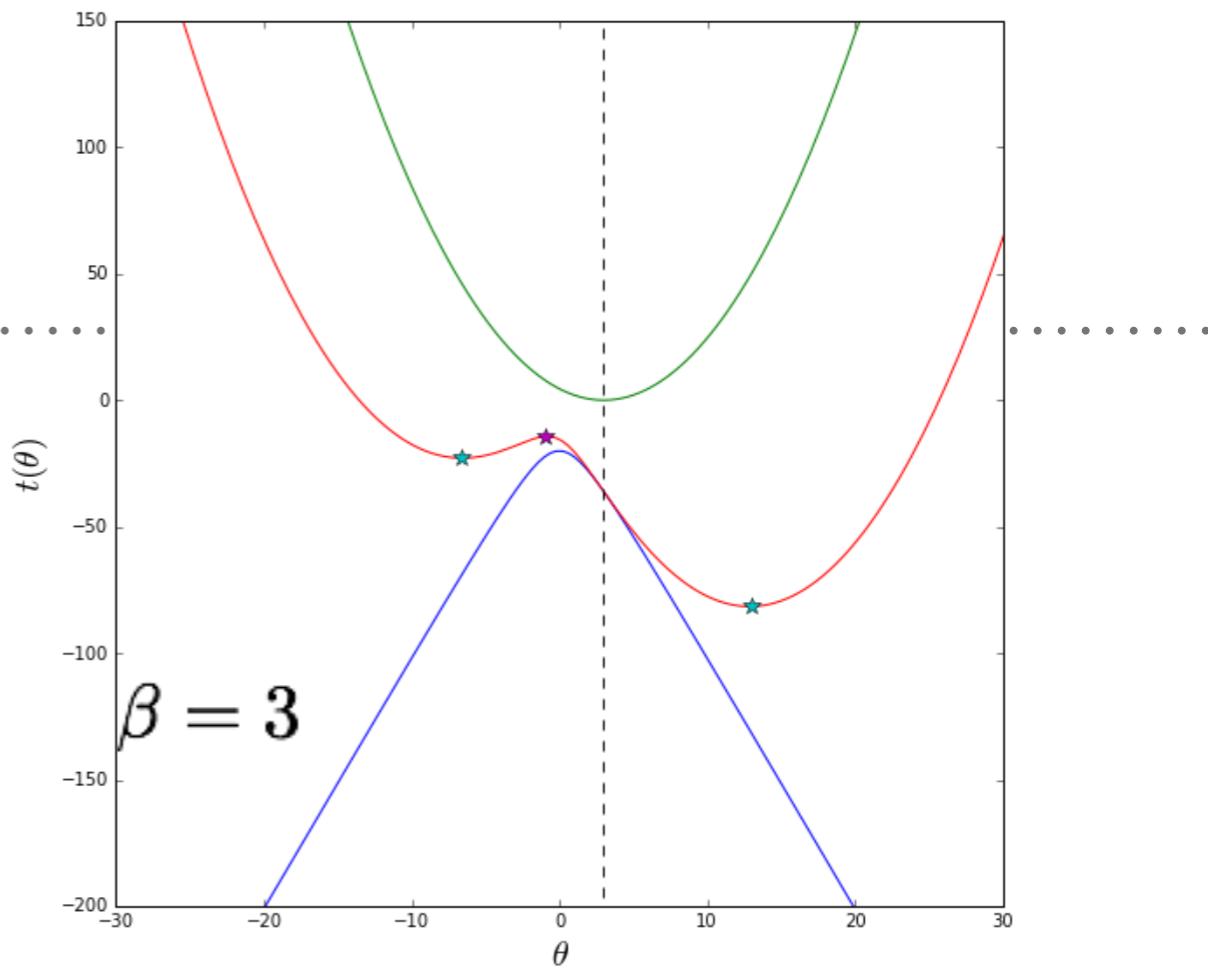
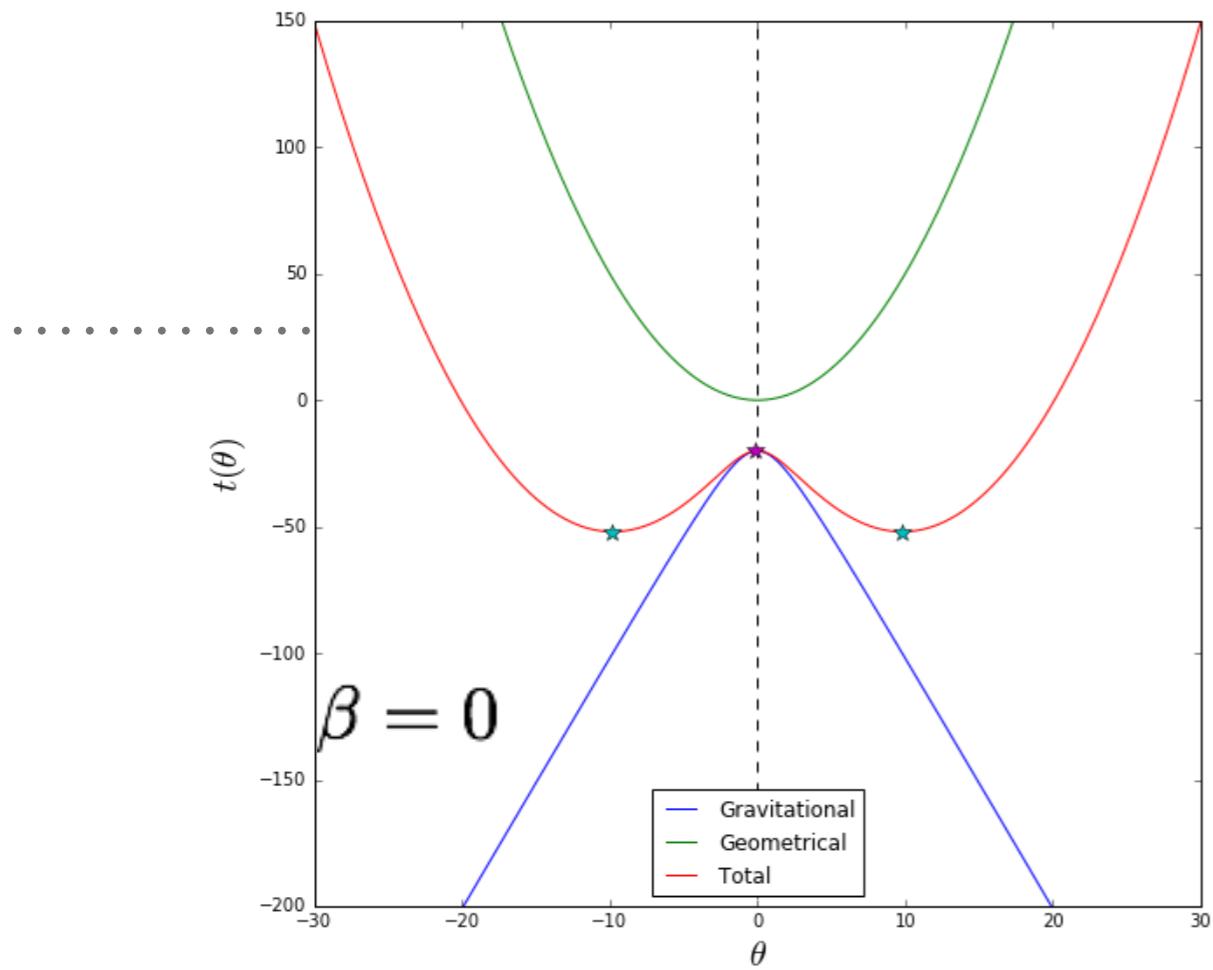
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*Toy potential:*

$$\hat{\Psi}(\theta) \propto \sqrt{\theta^2 + \theta_c^2}$$

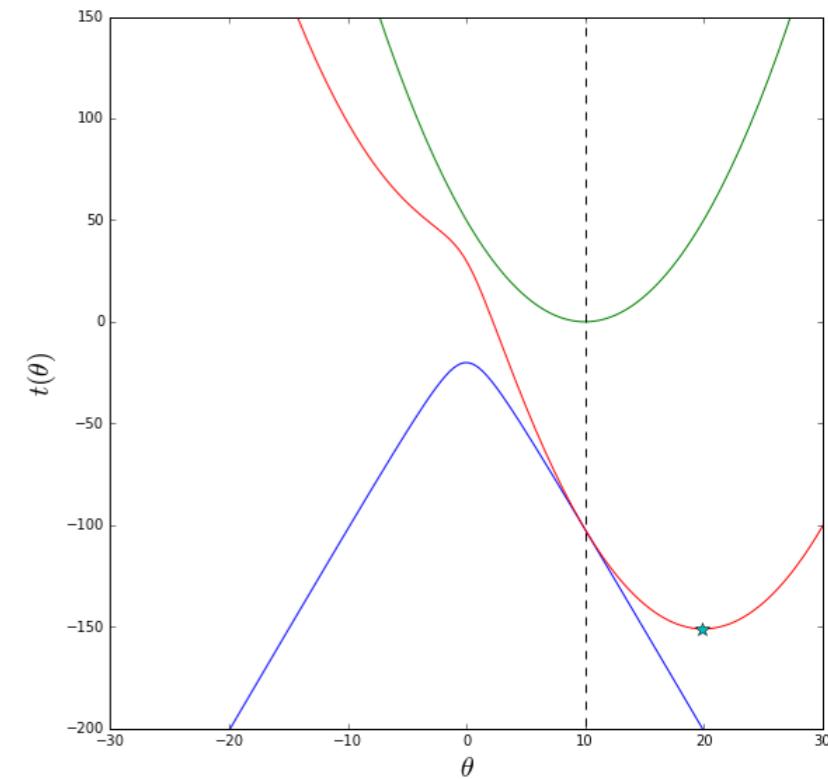
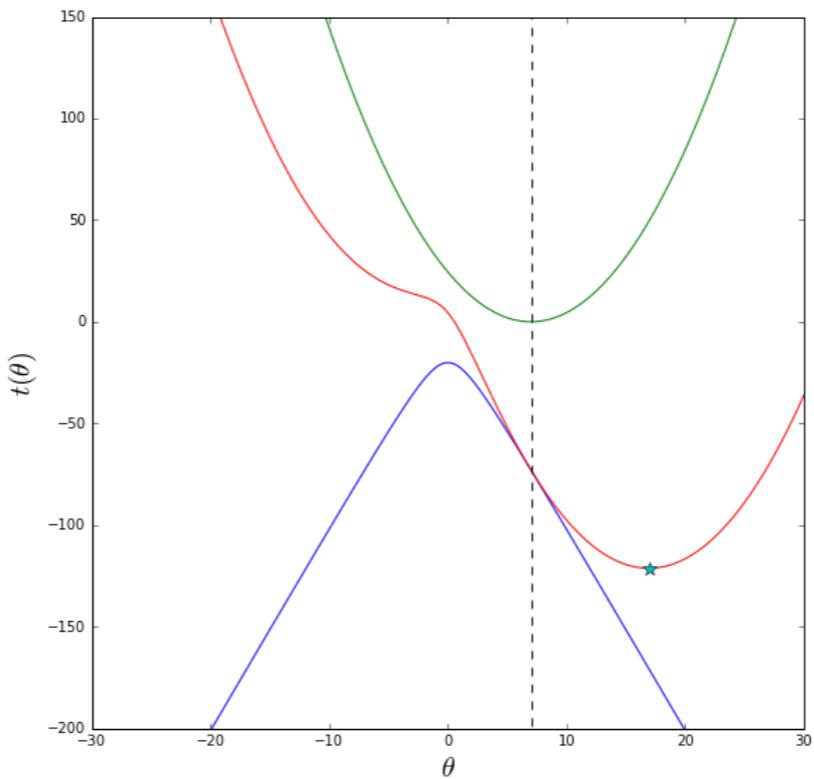
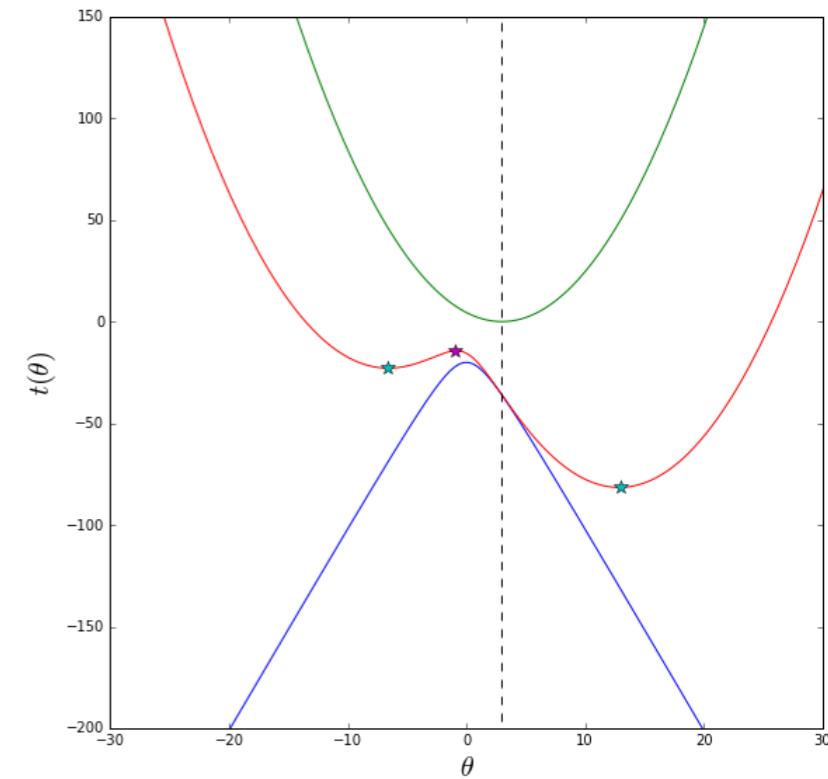
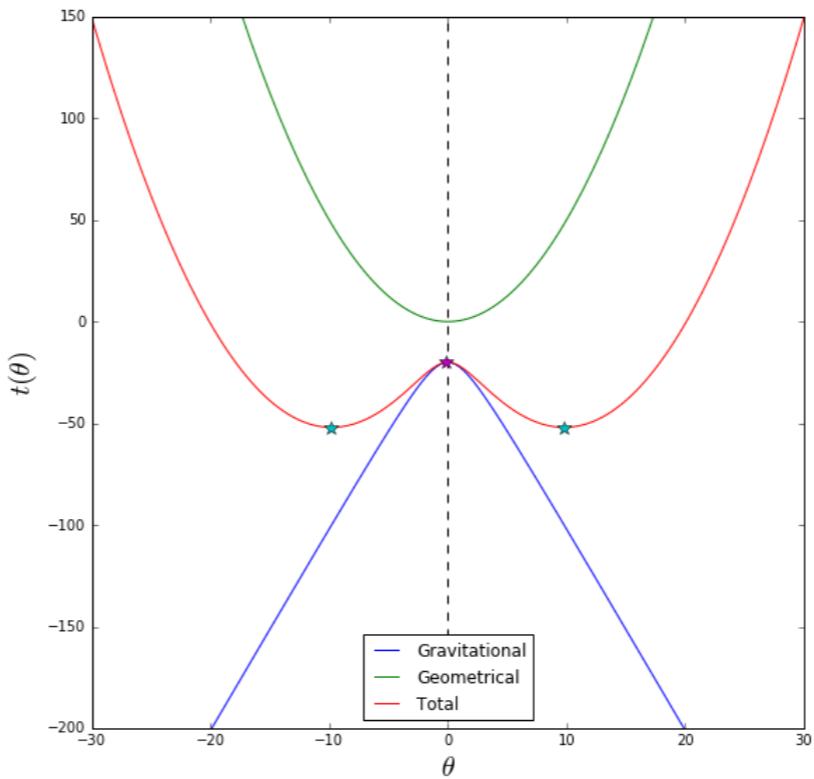
*Assuming axial-symmetry, we can discuss the time-delay function instead of the time delay surface.*

$$t(\theta) \propto \frac{1}{2}(\theta - \beta)^2 - \sqrt{\theta^2 + \theta_c^2}$$



# SOME INTERESTING PROPERTIES

- image multiplicity depends on the relative position of lens and source
- couples of images disappear after approaching each other
- the time-delay function is flat when this happens!
- $\det A=0$  means infinite magnification: the images disappear on the critical lines!
- this happens every time a source crosses a caustic!



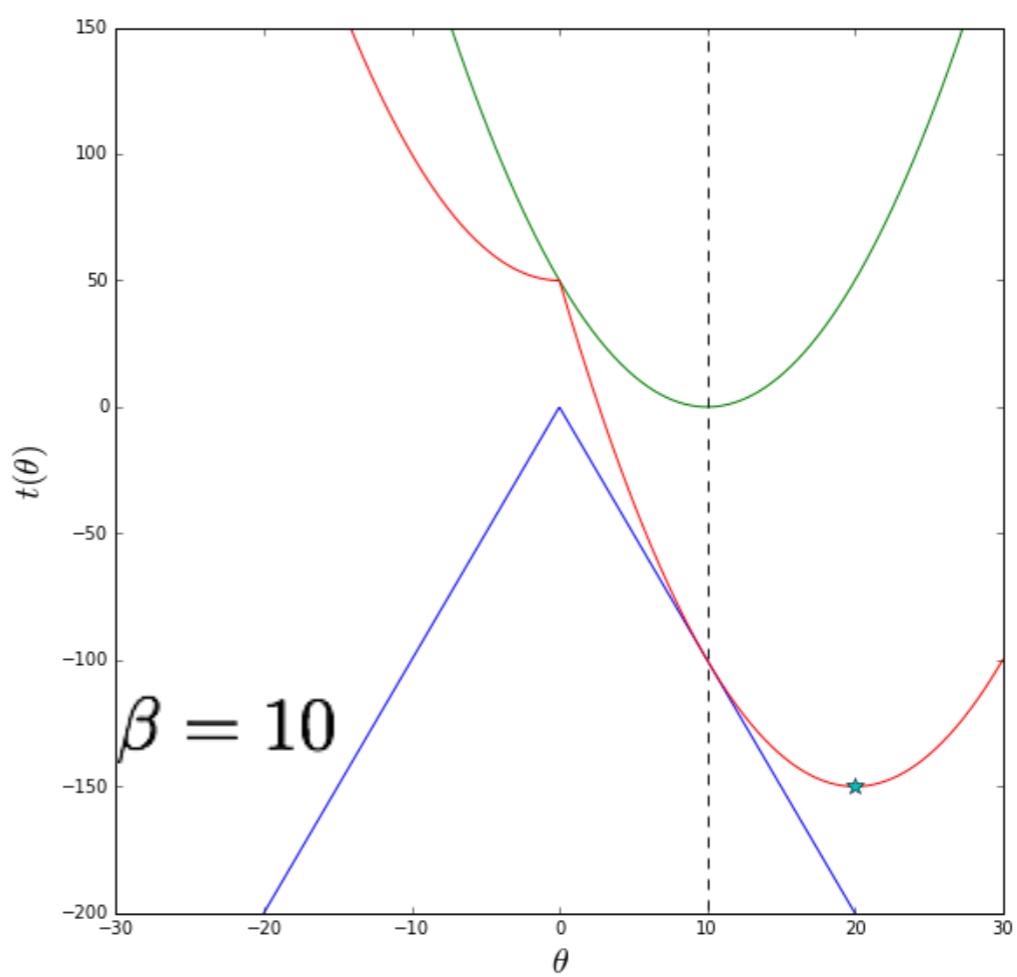
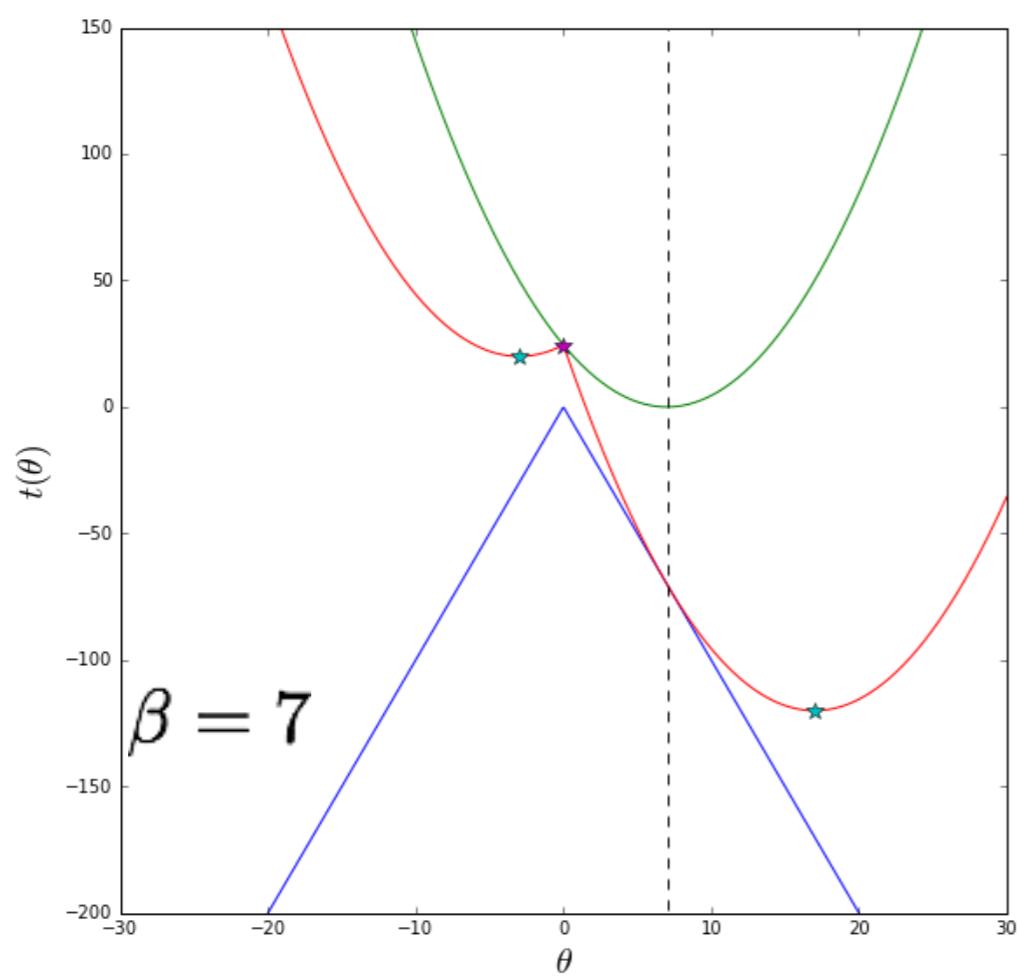
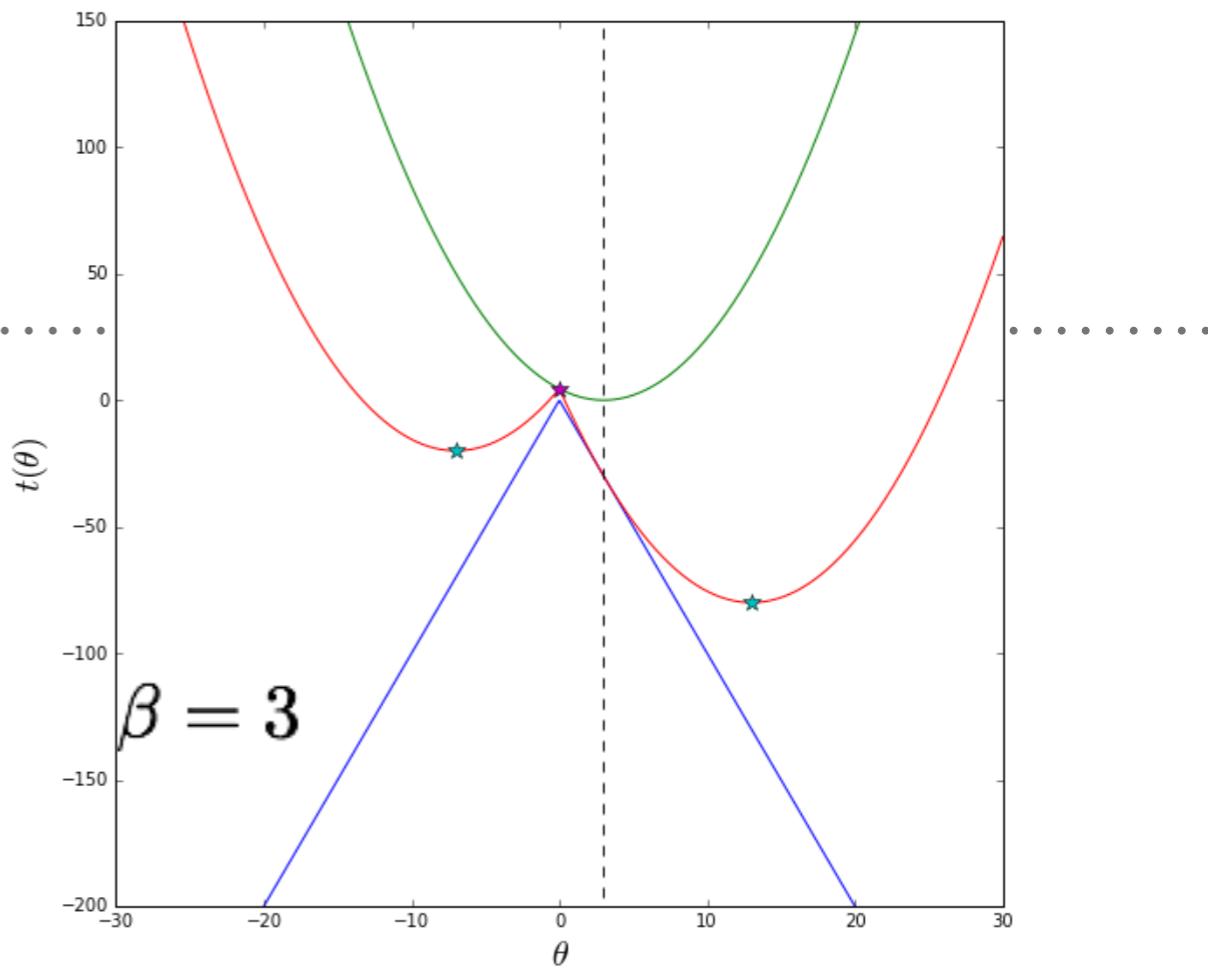
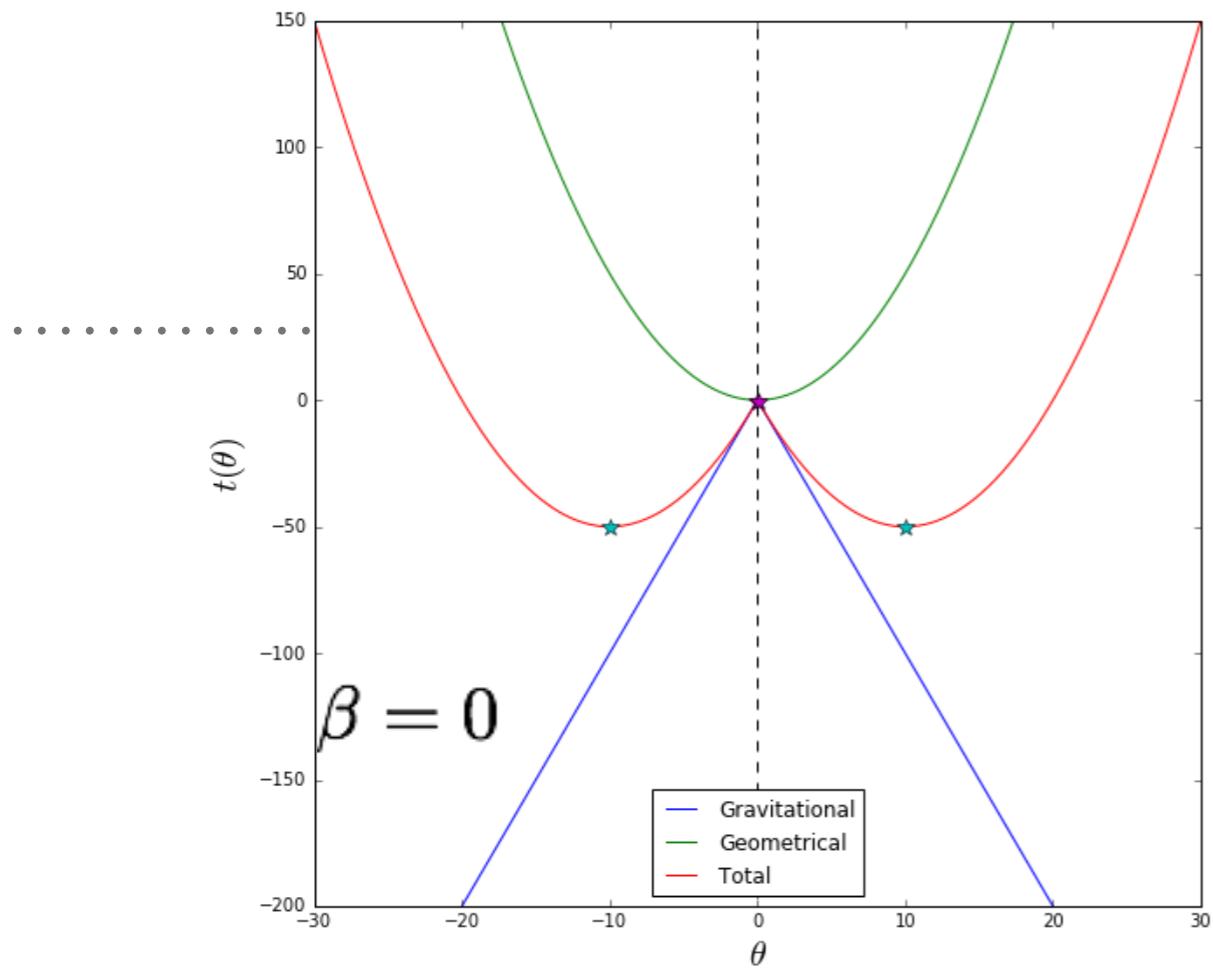
# EXAMPLE OF TIME DELAY SURFACE

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*Let's change potential:*

$$\hat{\Psi}(\theta) \propto |\theta|$$

*The lens model is the same as before, but the core has been removed*



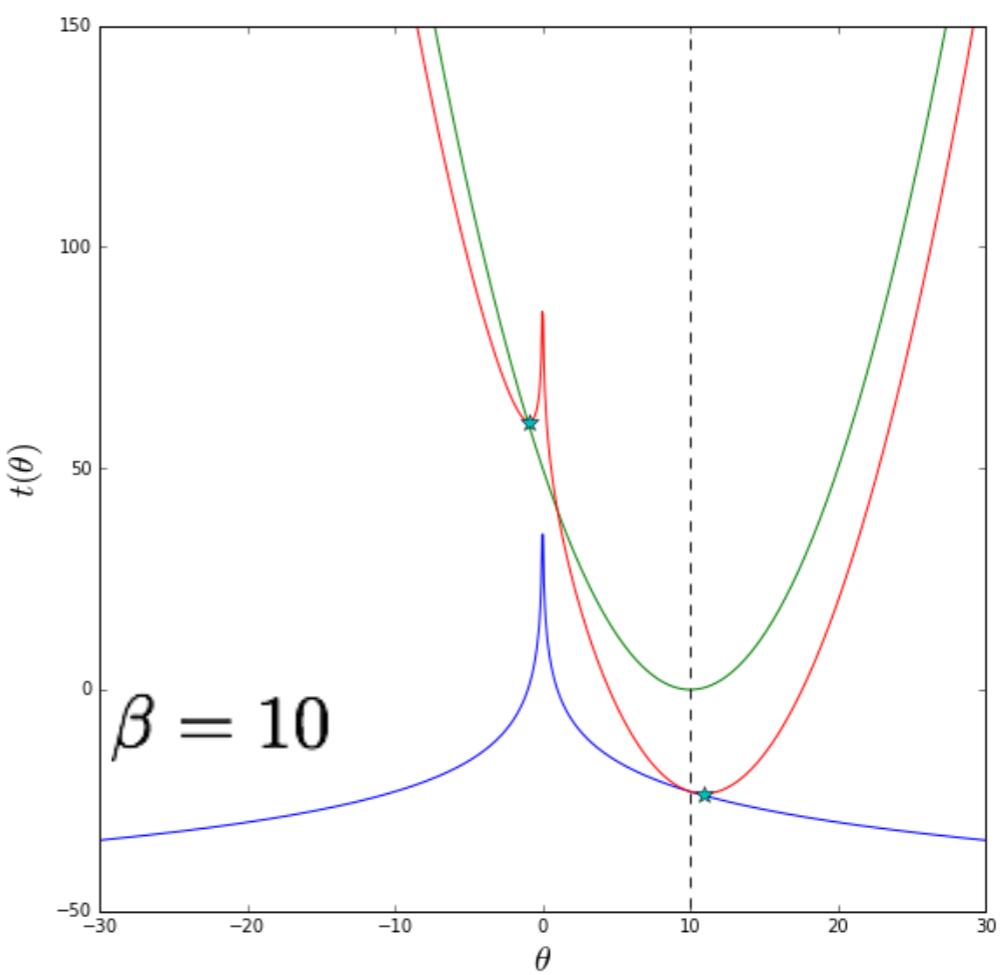
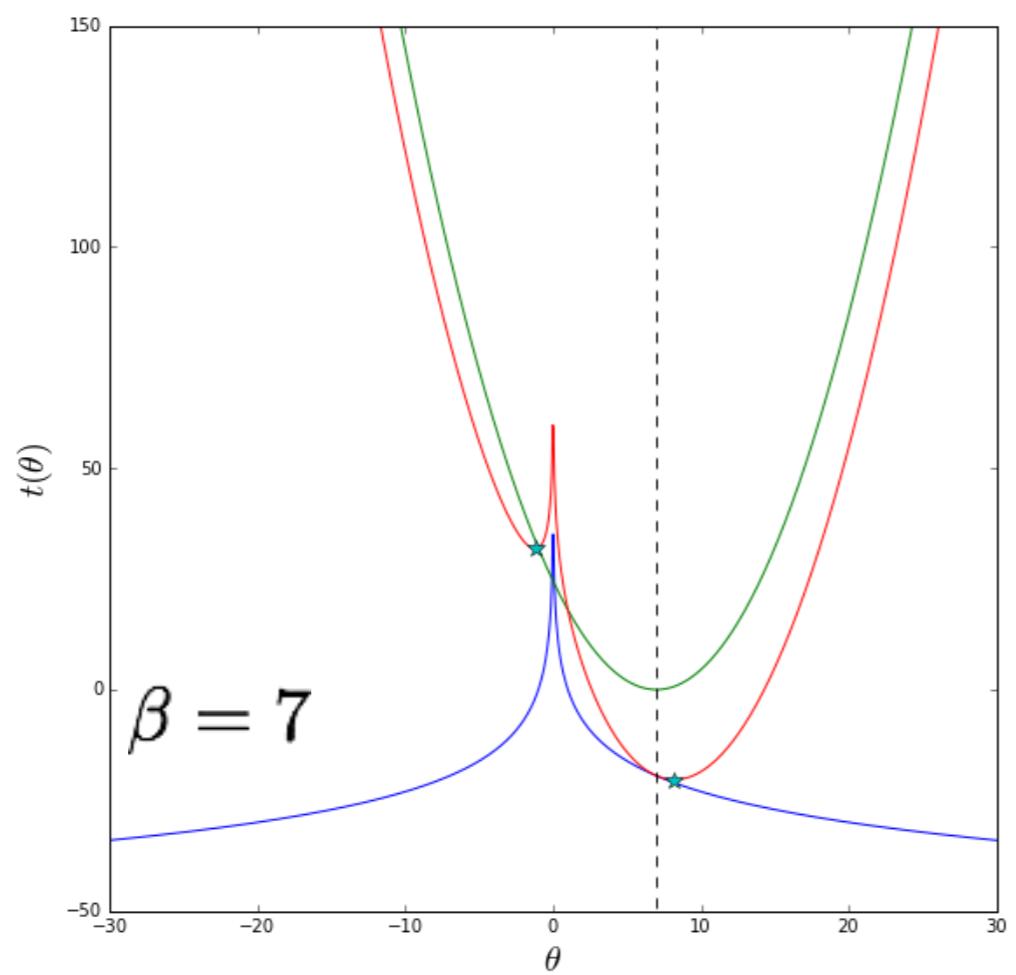
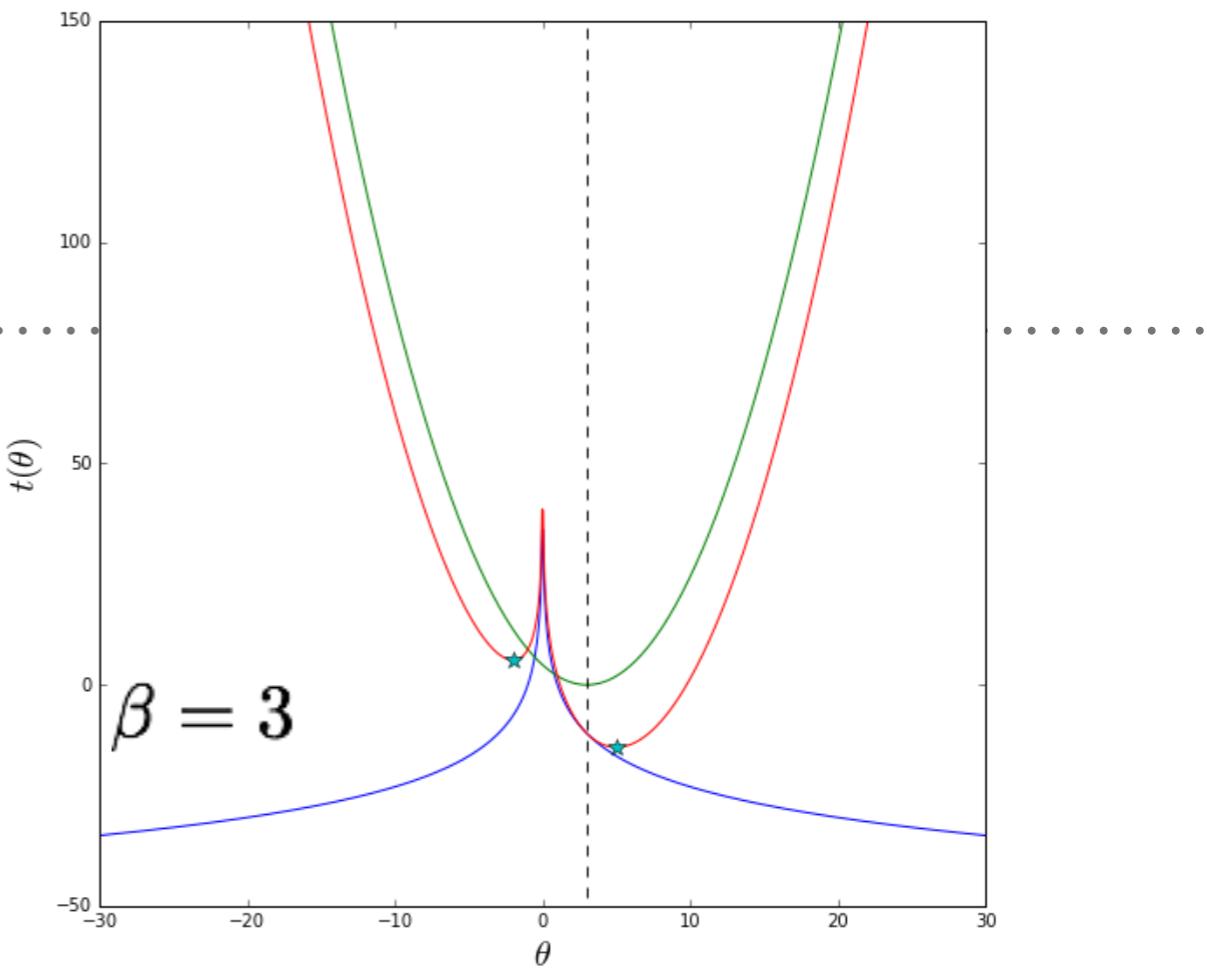
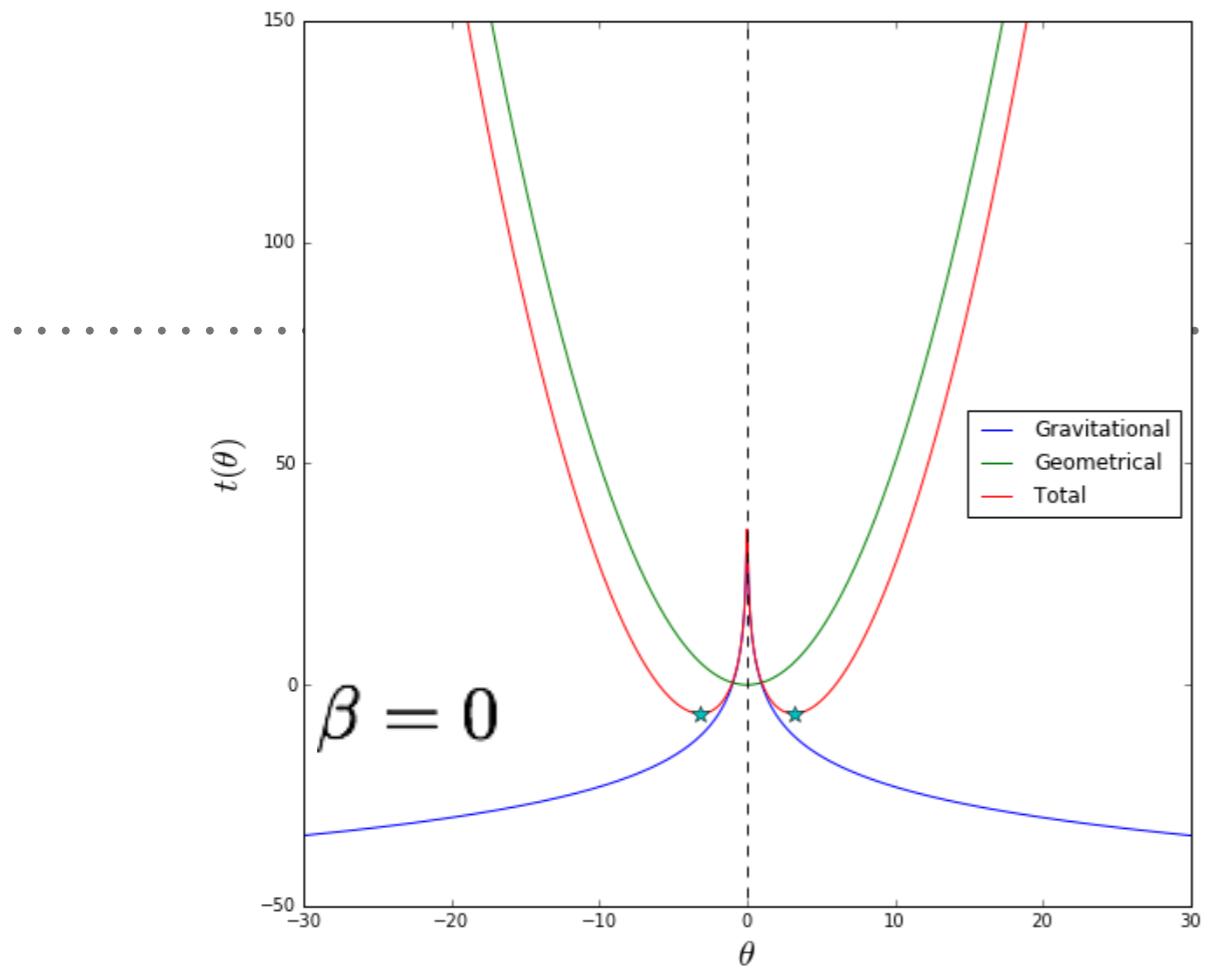
# EXAMPLE OF TIME DELAY SURFACE

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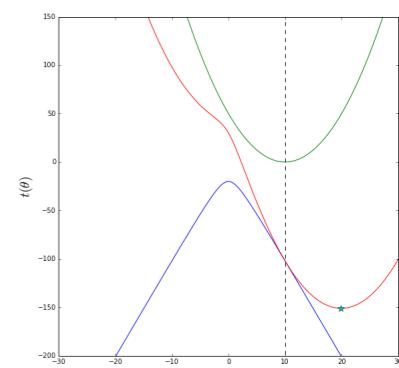
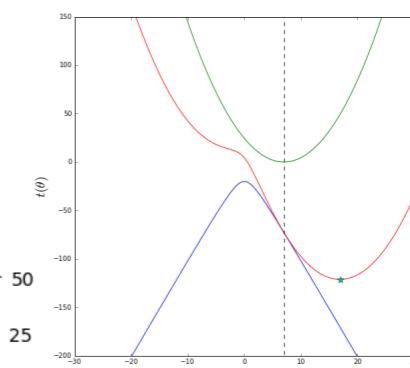
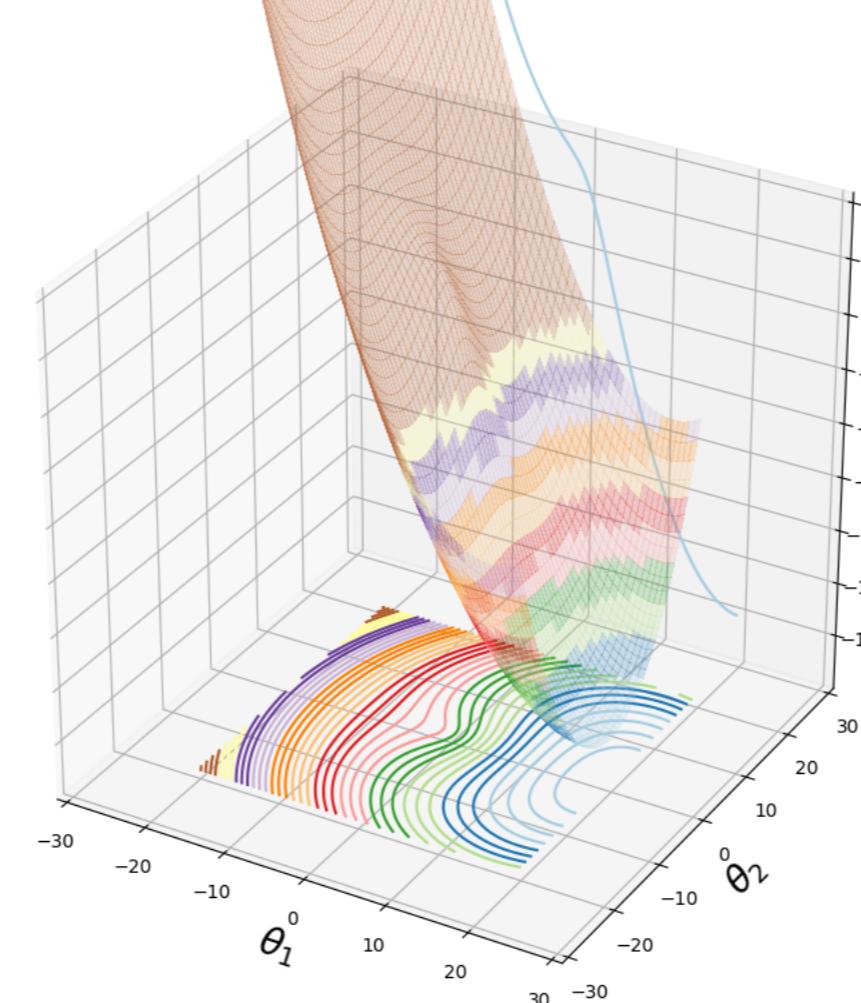
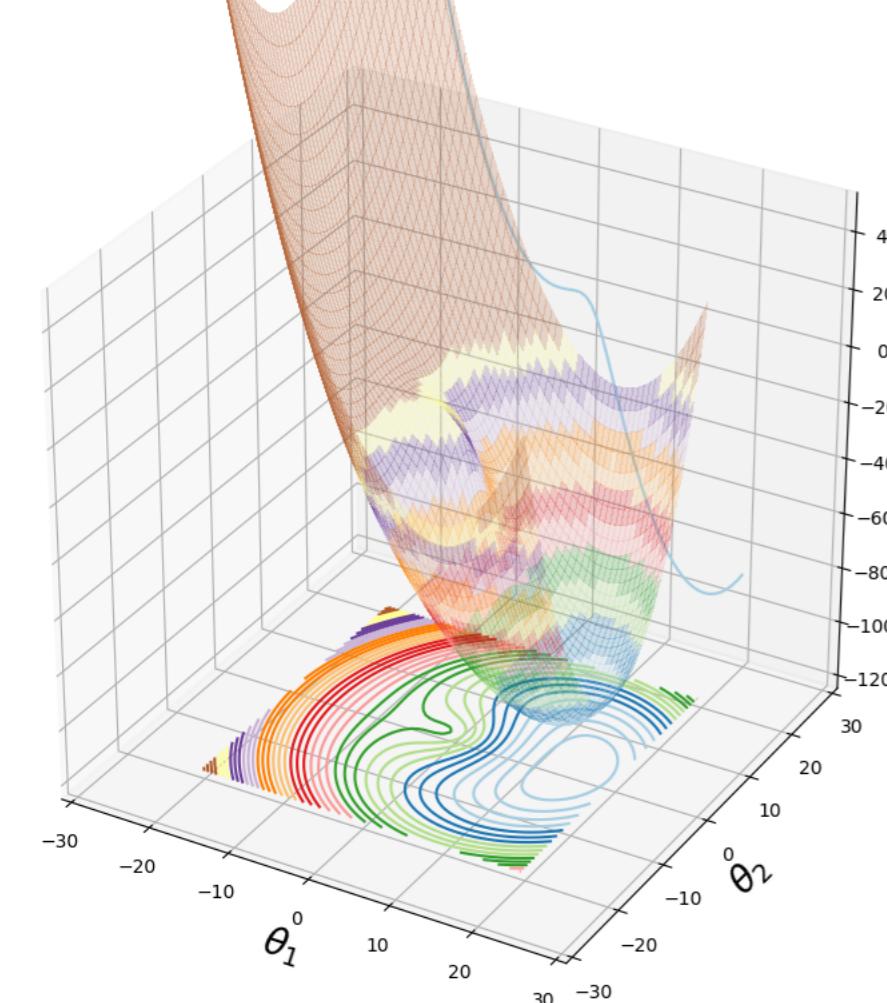
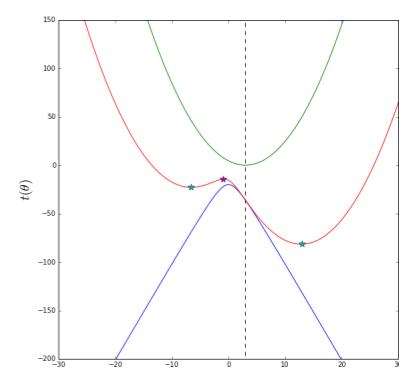
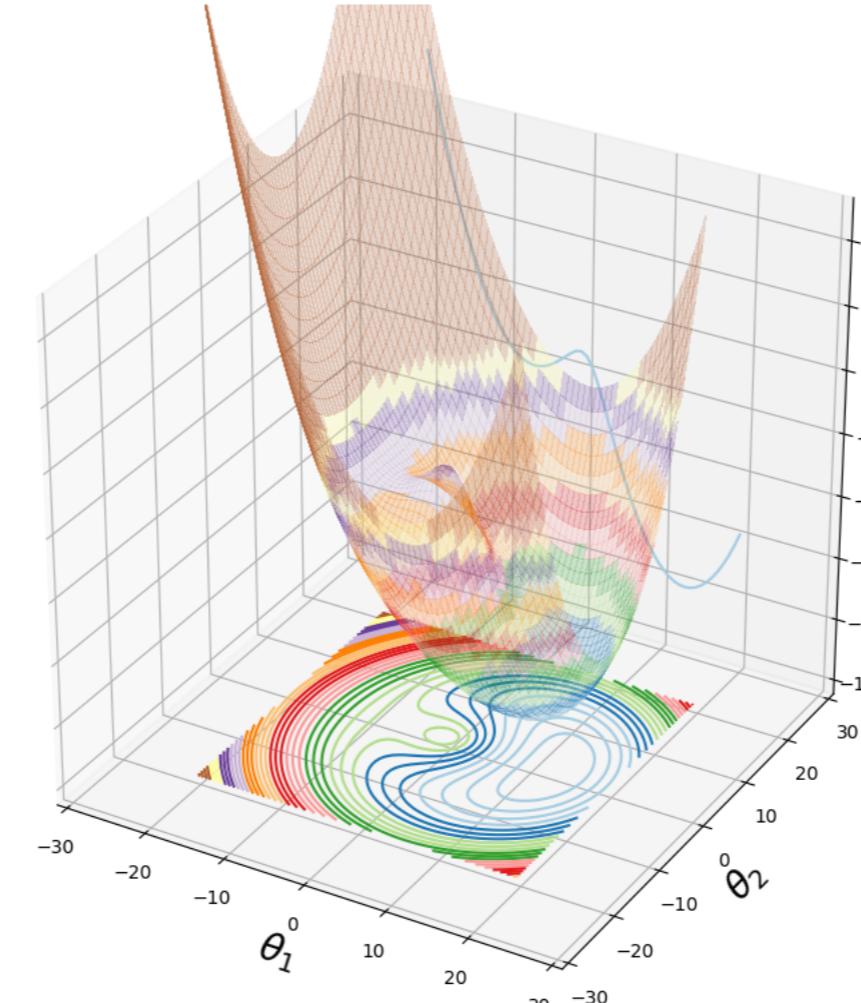
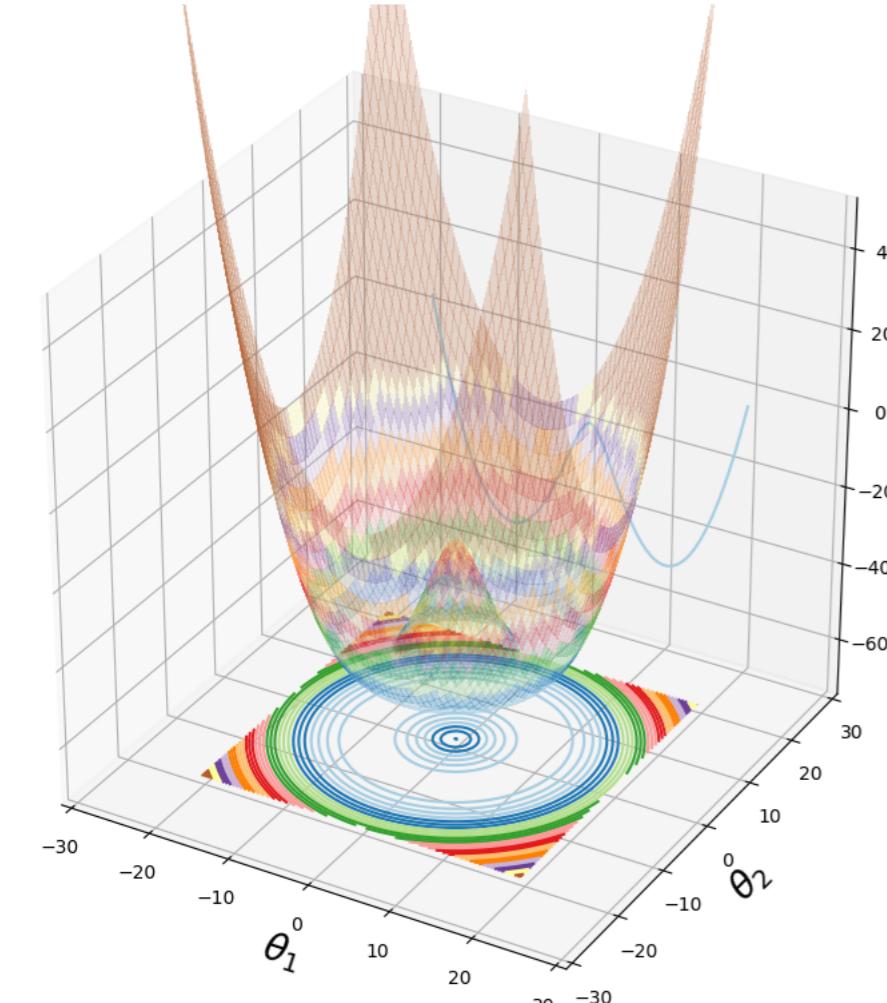
*Yet another potential*

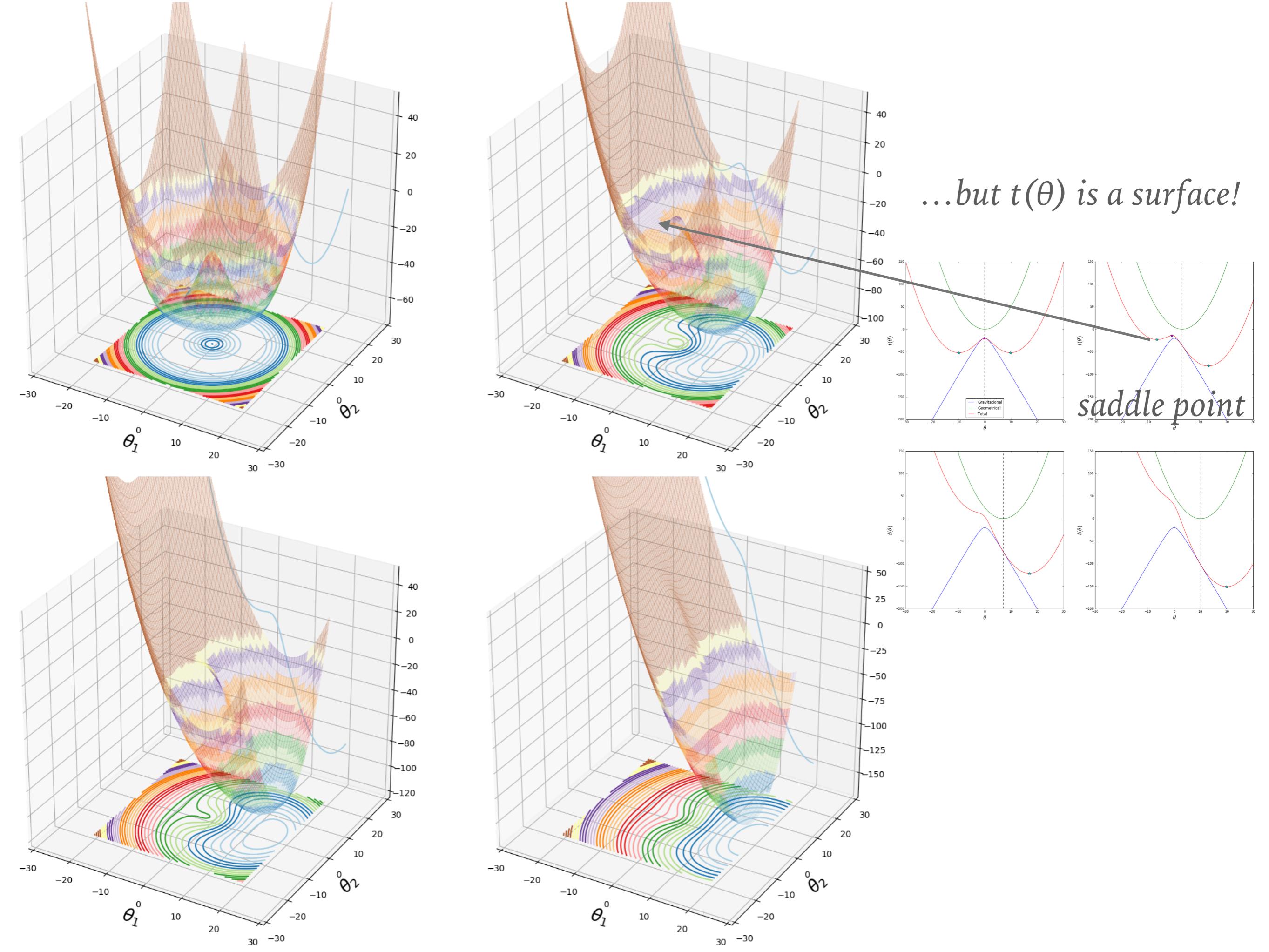
$$\hat{\Psi}(\theta) \propto \ln |\theta|$$

*This is the lensing potential of the point mass...*



*...but  $t(\theta)$  is a surface!*





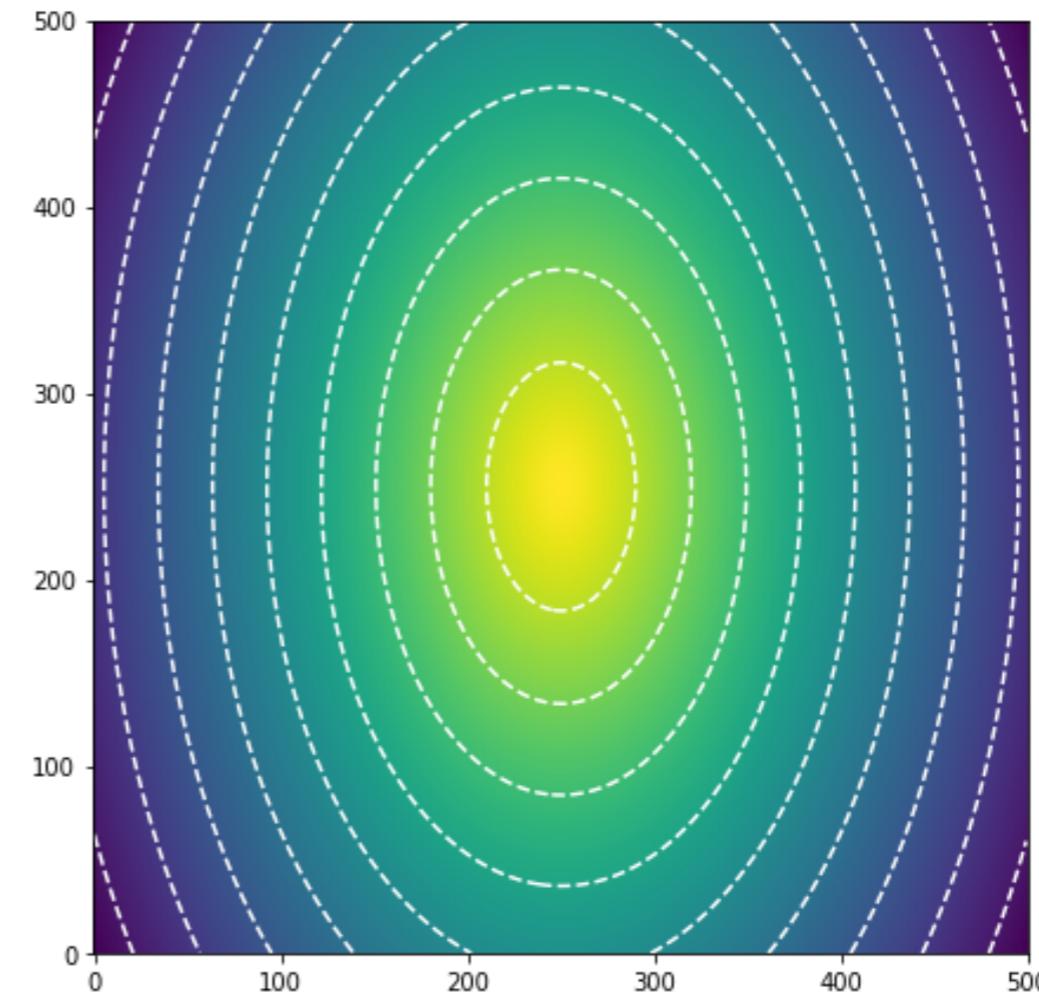
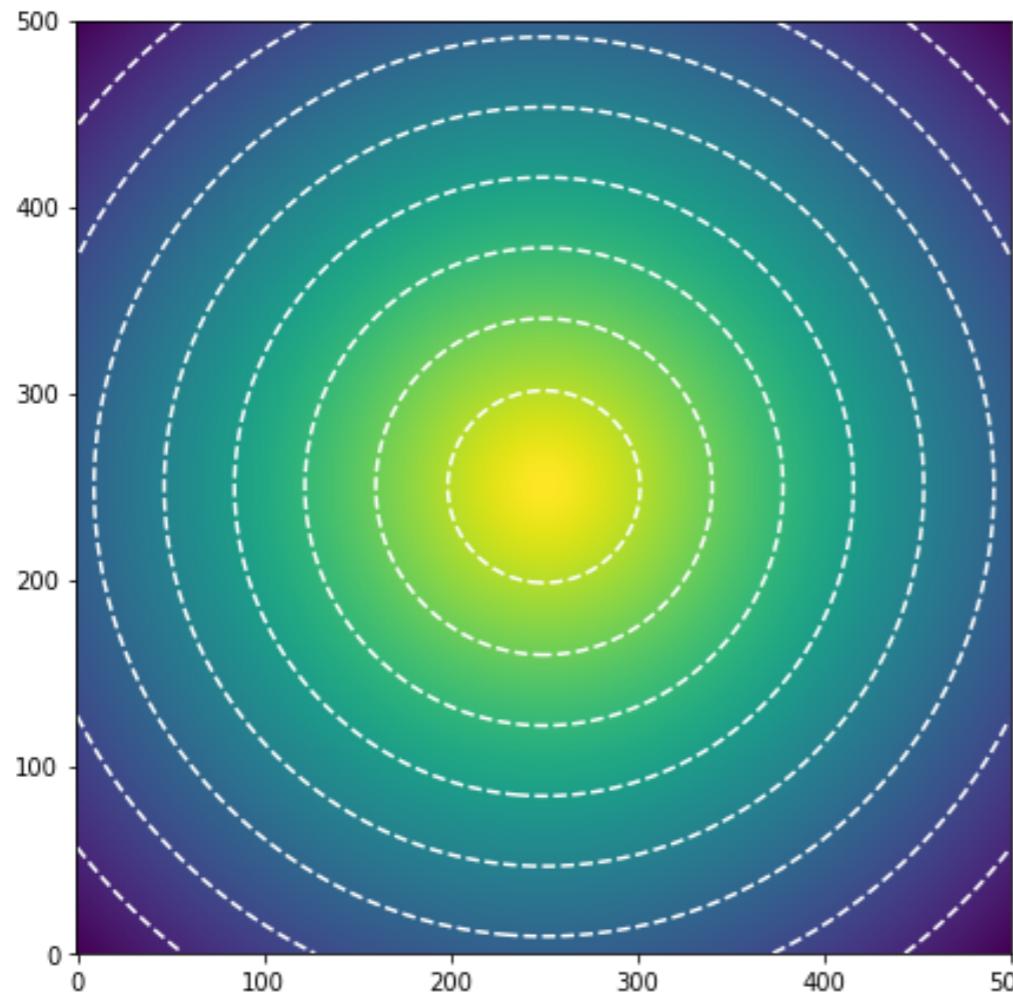
# INTRODUCING ELLIPTICITY

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*One easy way to make a lens elliptical is by modifying the potential as follows:*

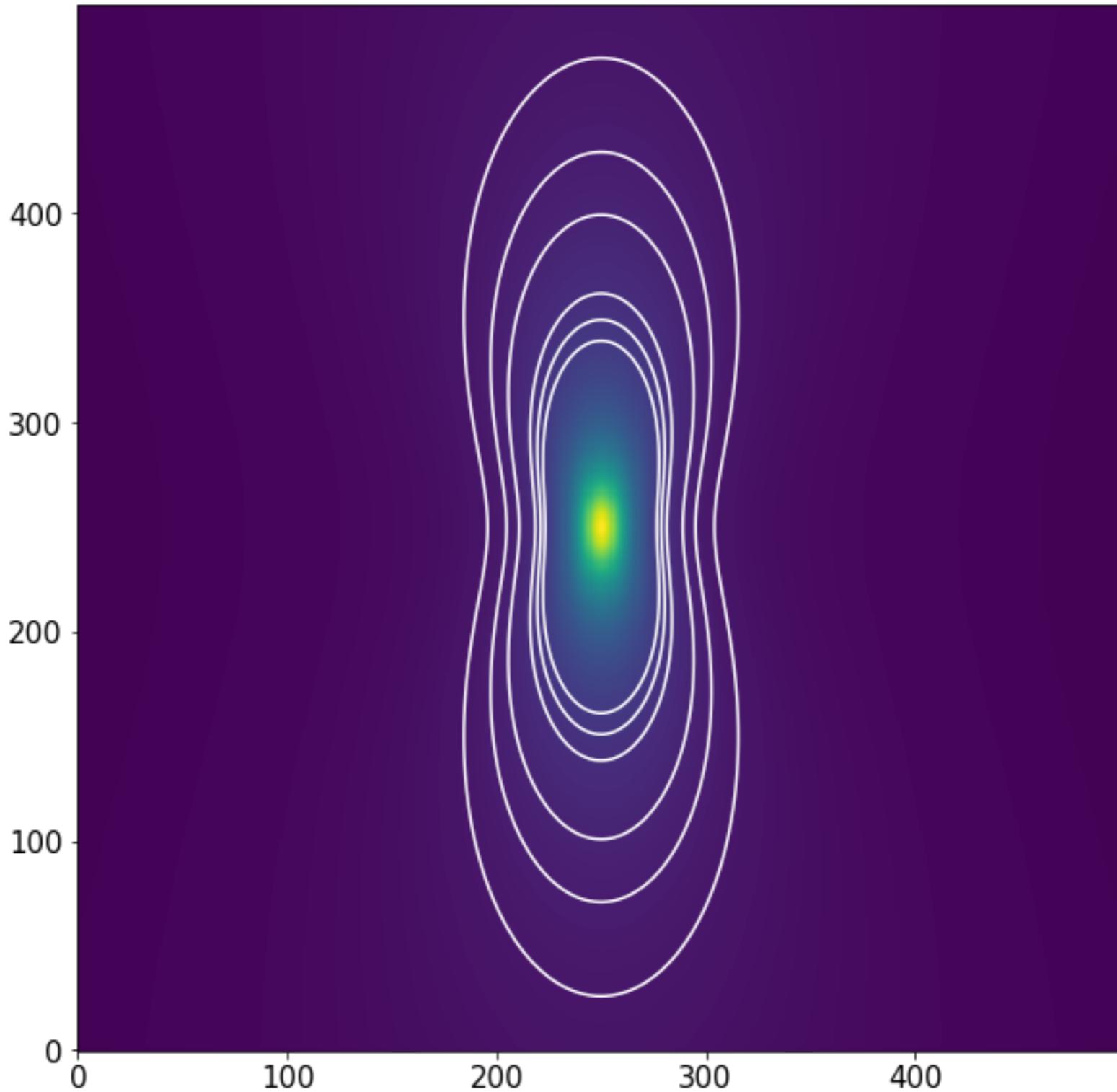
$$|\theta| \rightarrow \sqrt{\frac{\theta_1^2}{1-\epsilon} + \theta_2^2(1-\epsilon)}$$

*This makes the potential constant over ellipses rather than on circles.*



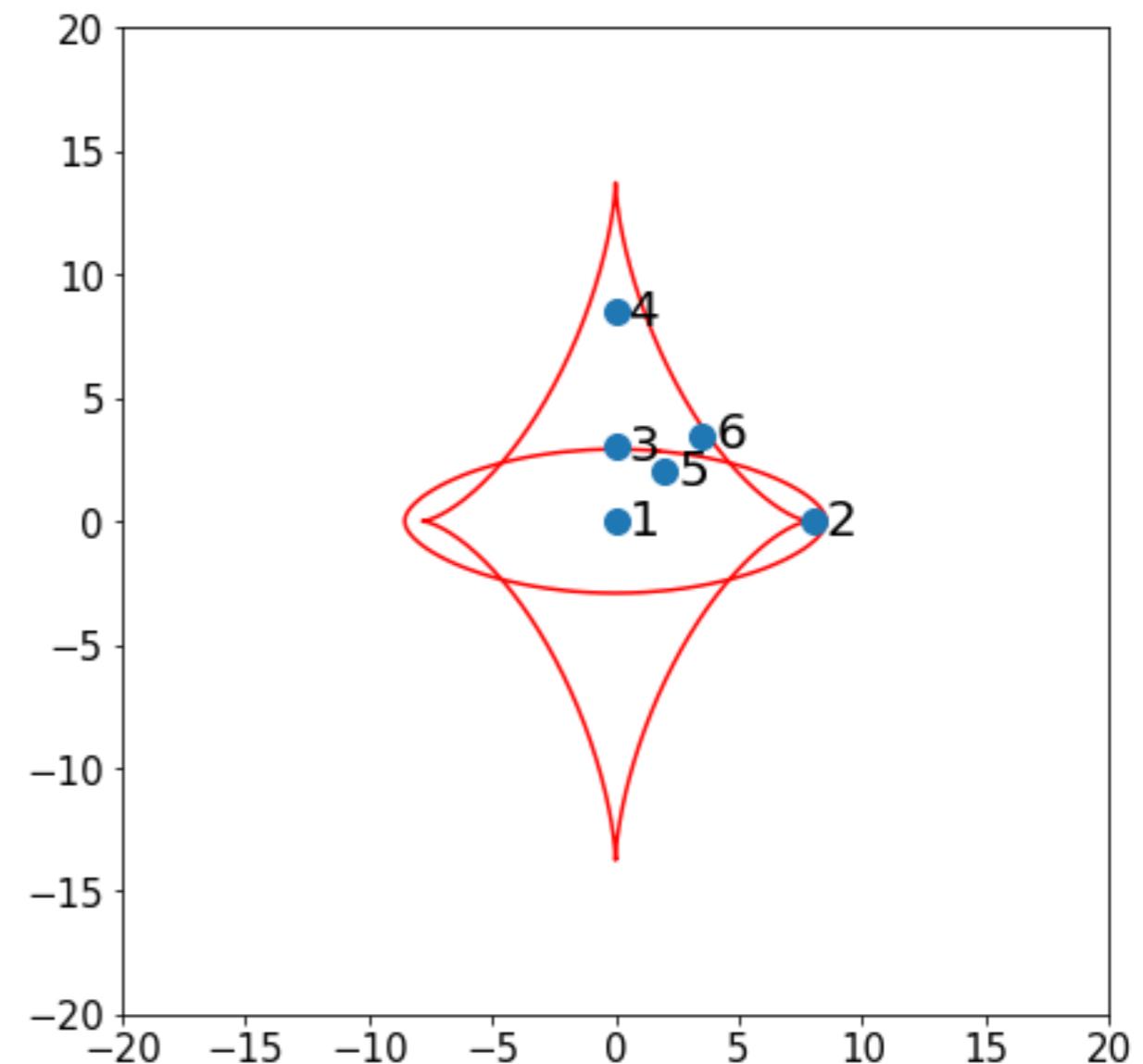
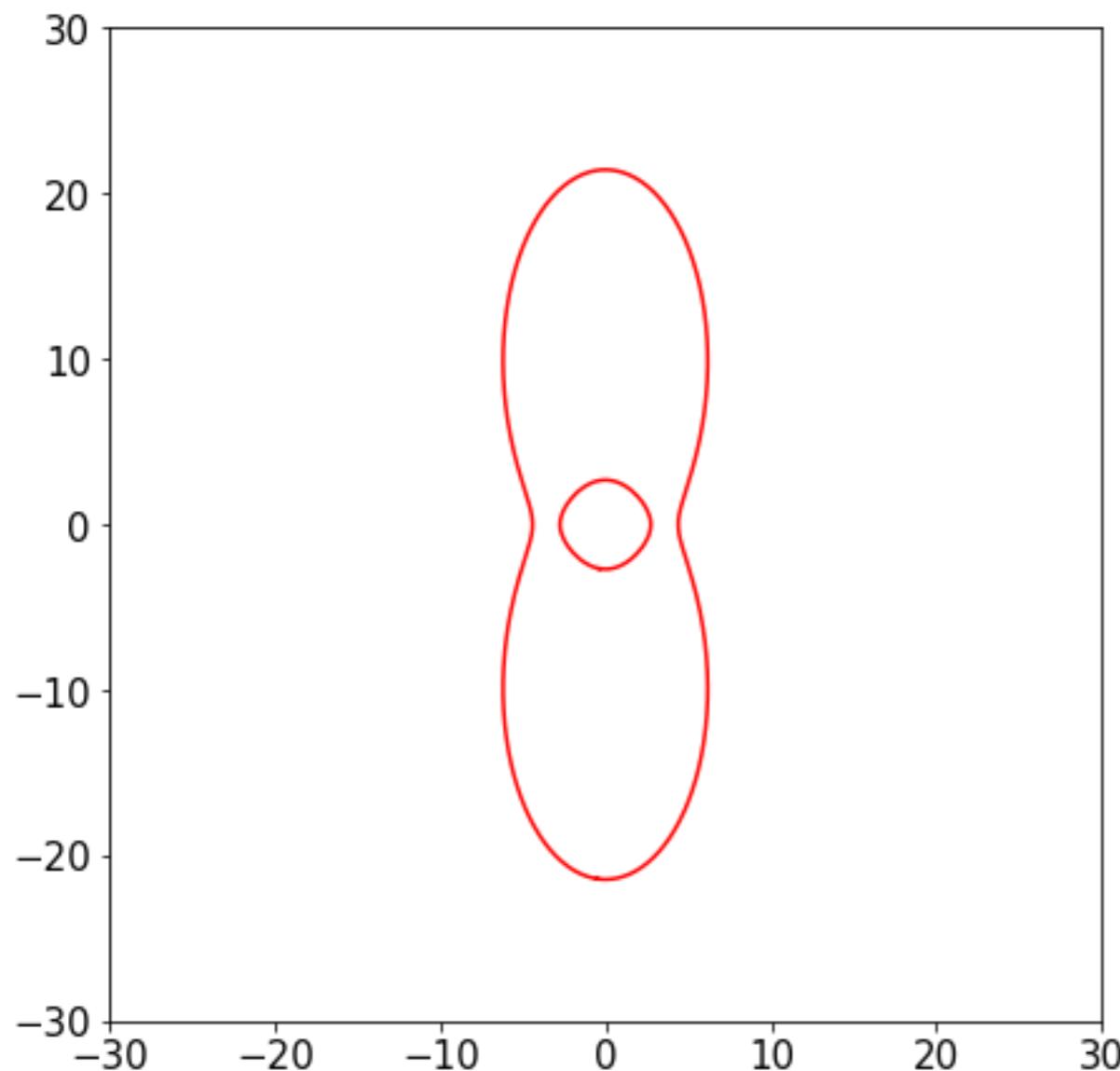
# CAUTION: PSEUDO ELLIPTICAL LENSES

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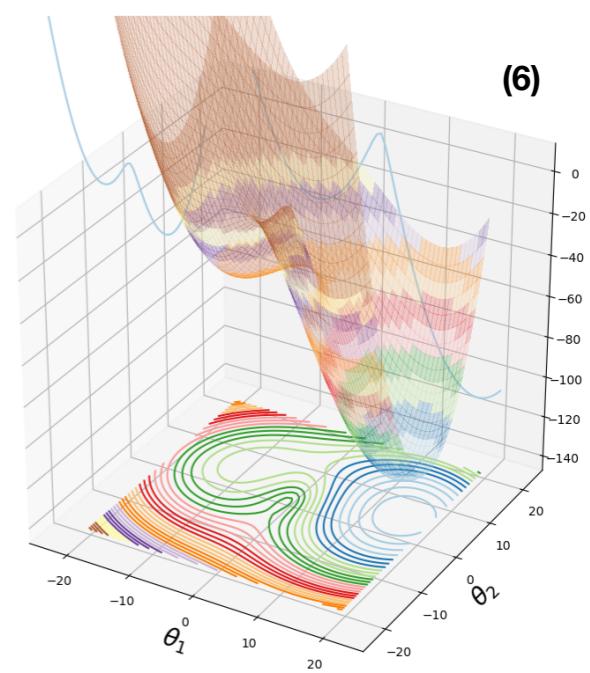
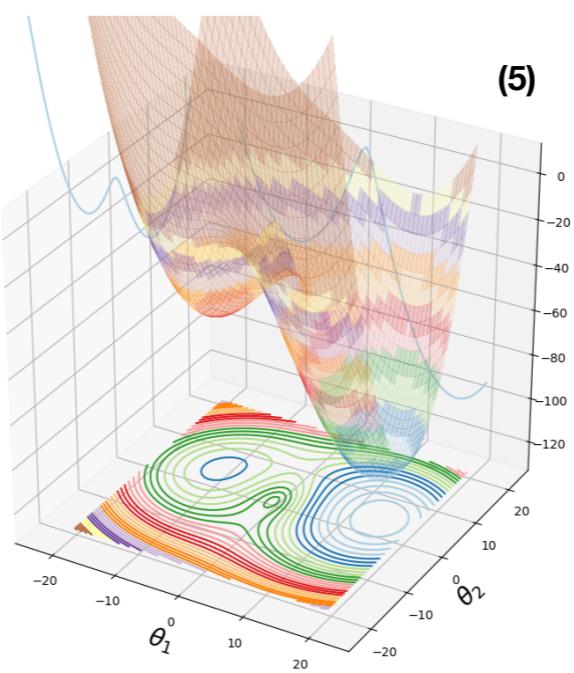
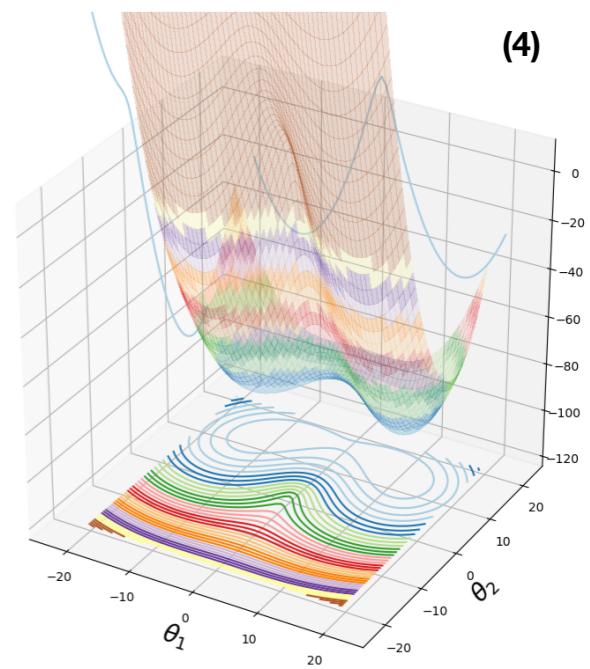
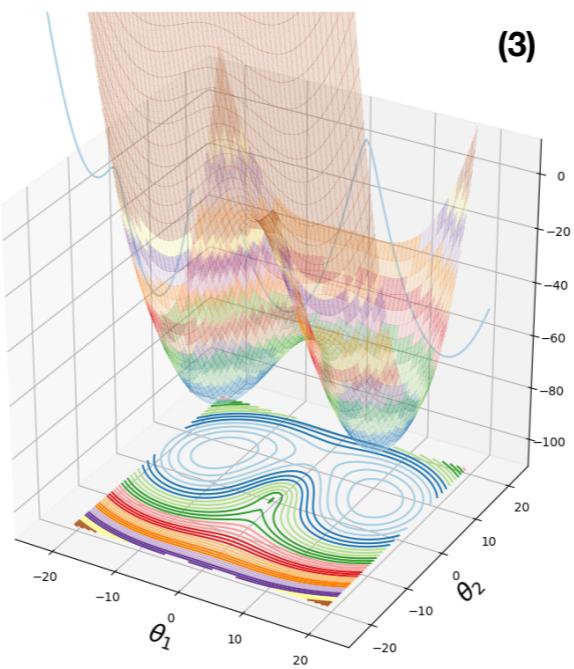
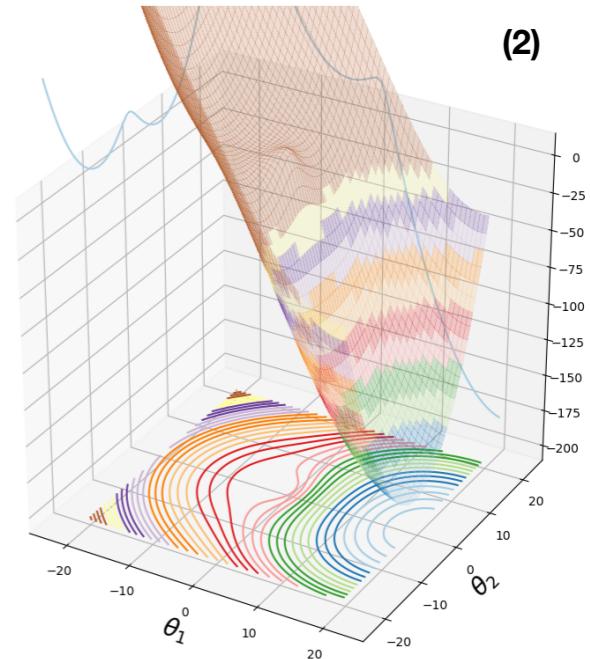
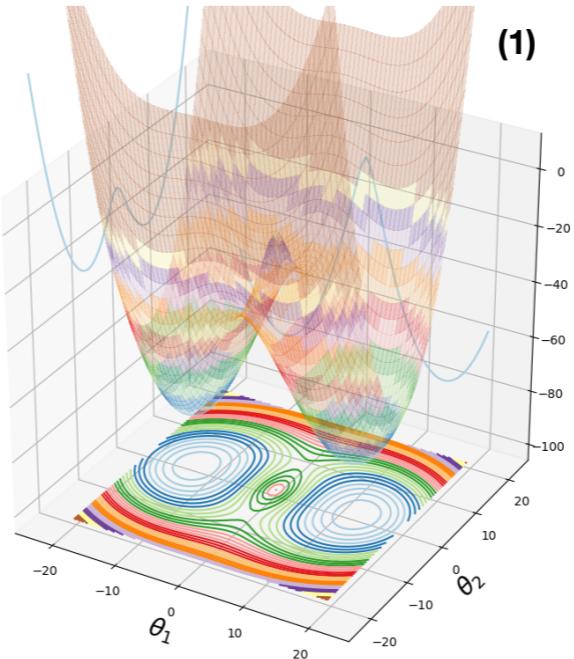
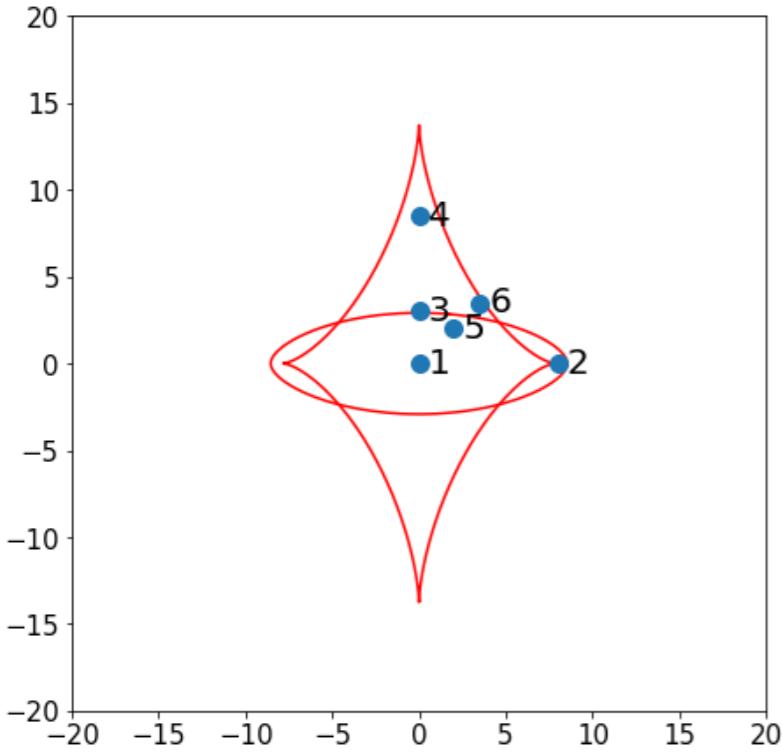
# CRITICAL LINES AND CAUSTICS

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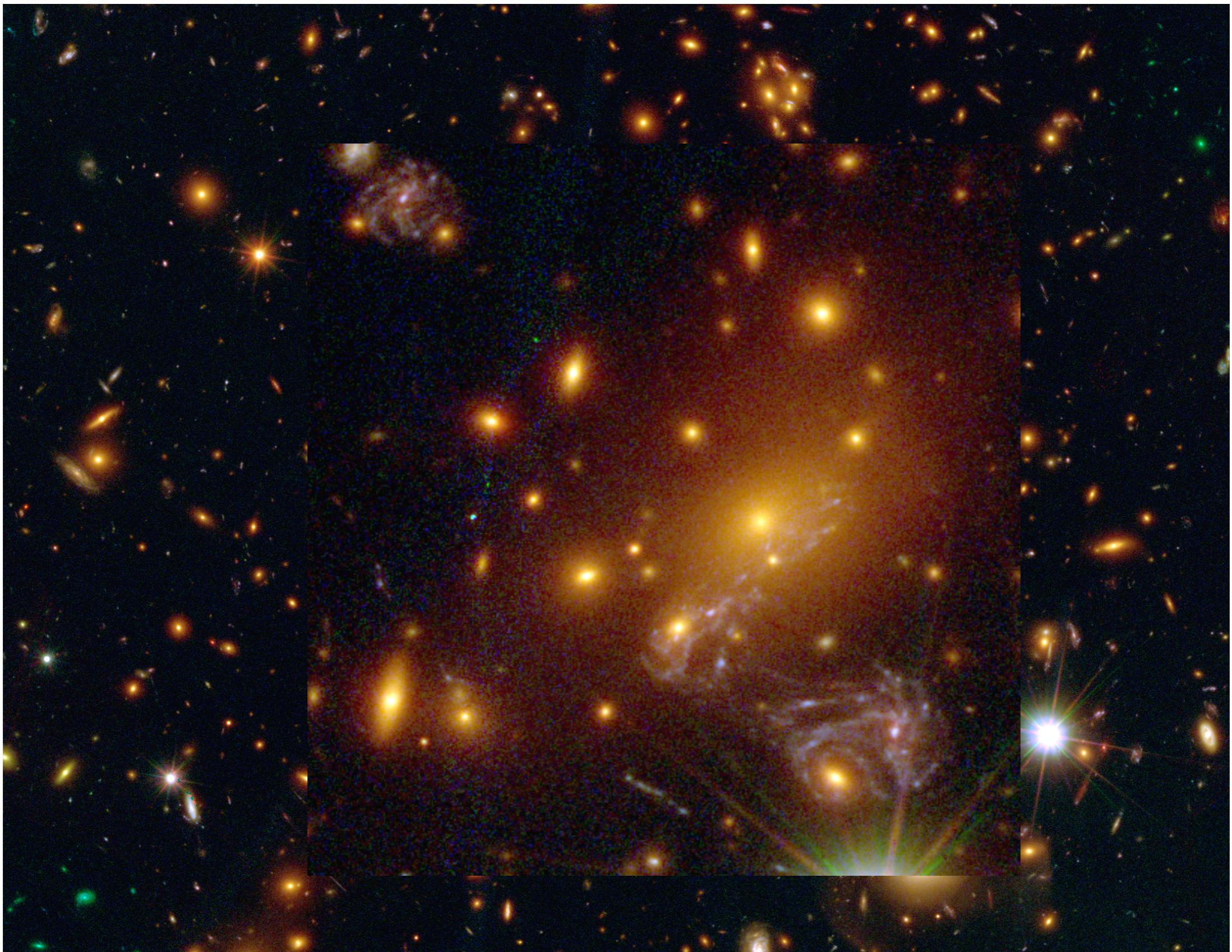
# SN REFSDAL IN MACS 1149

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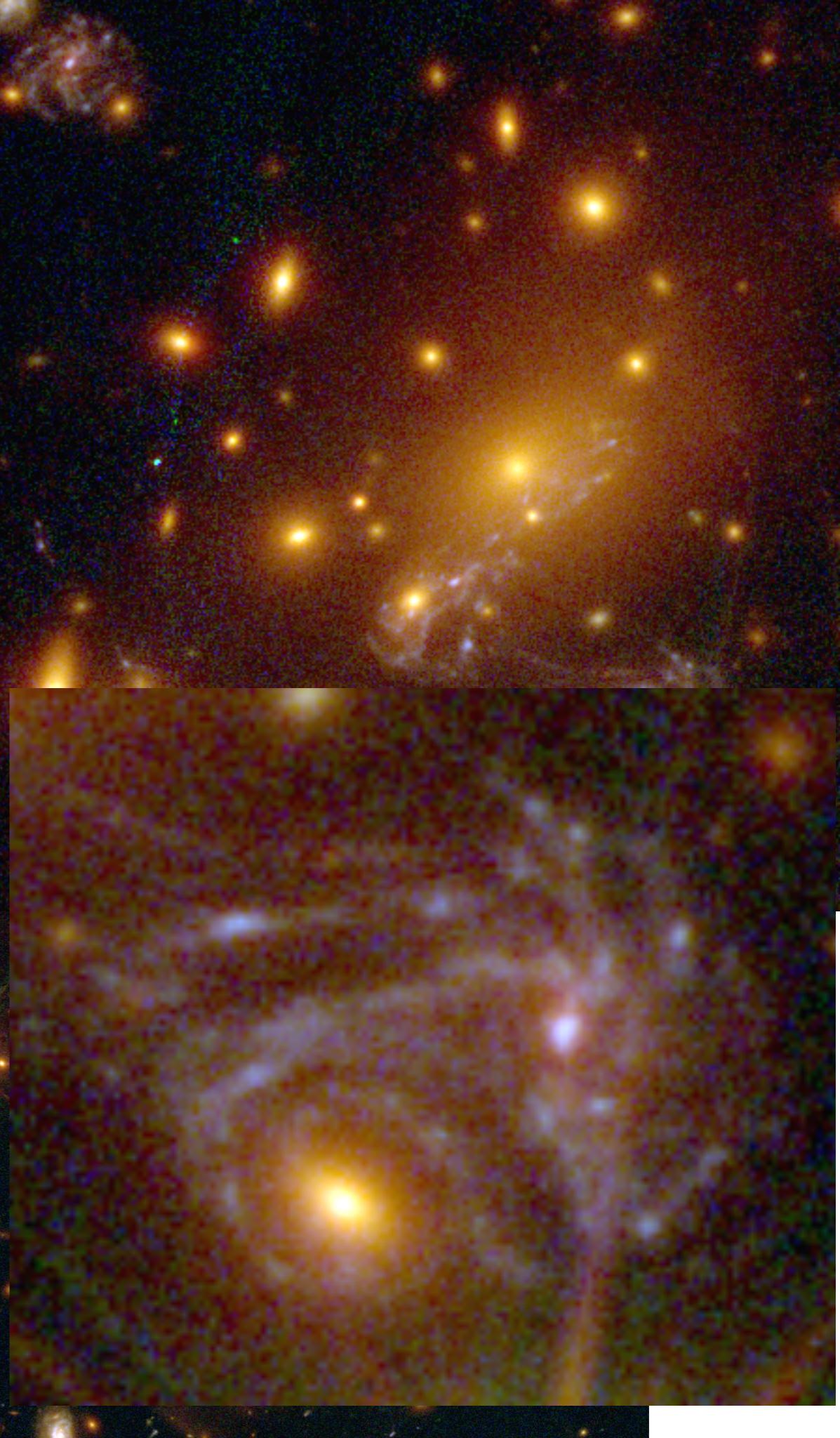
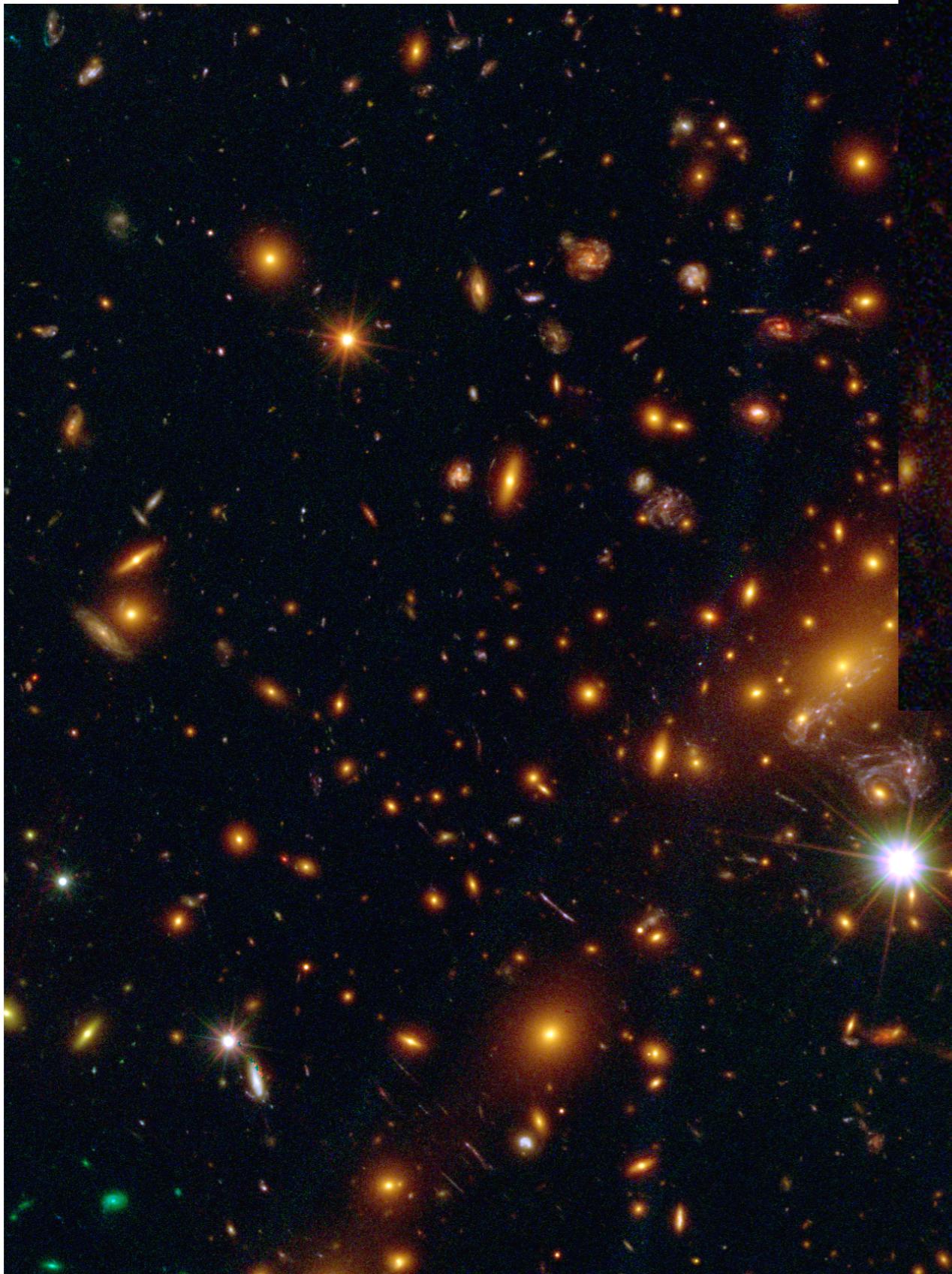
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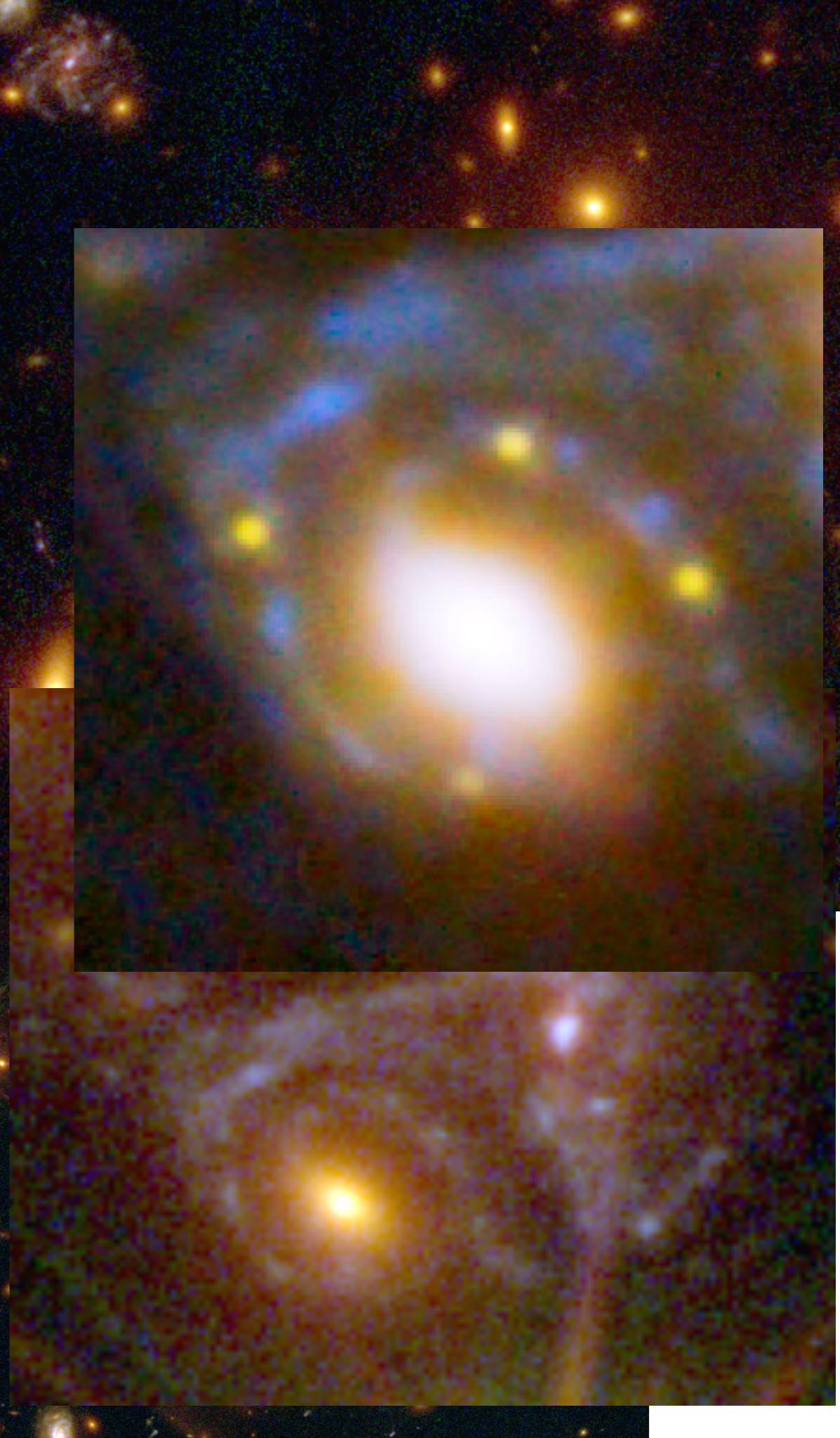
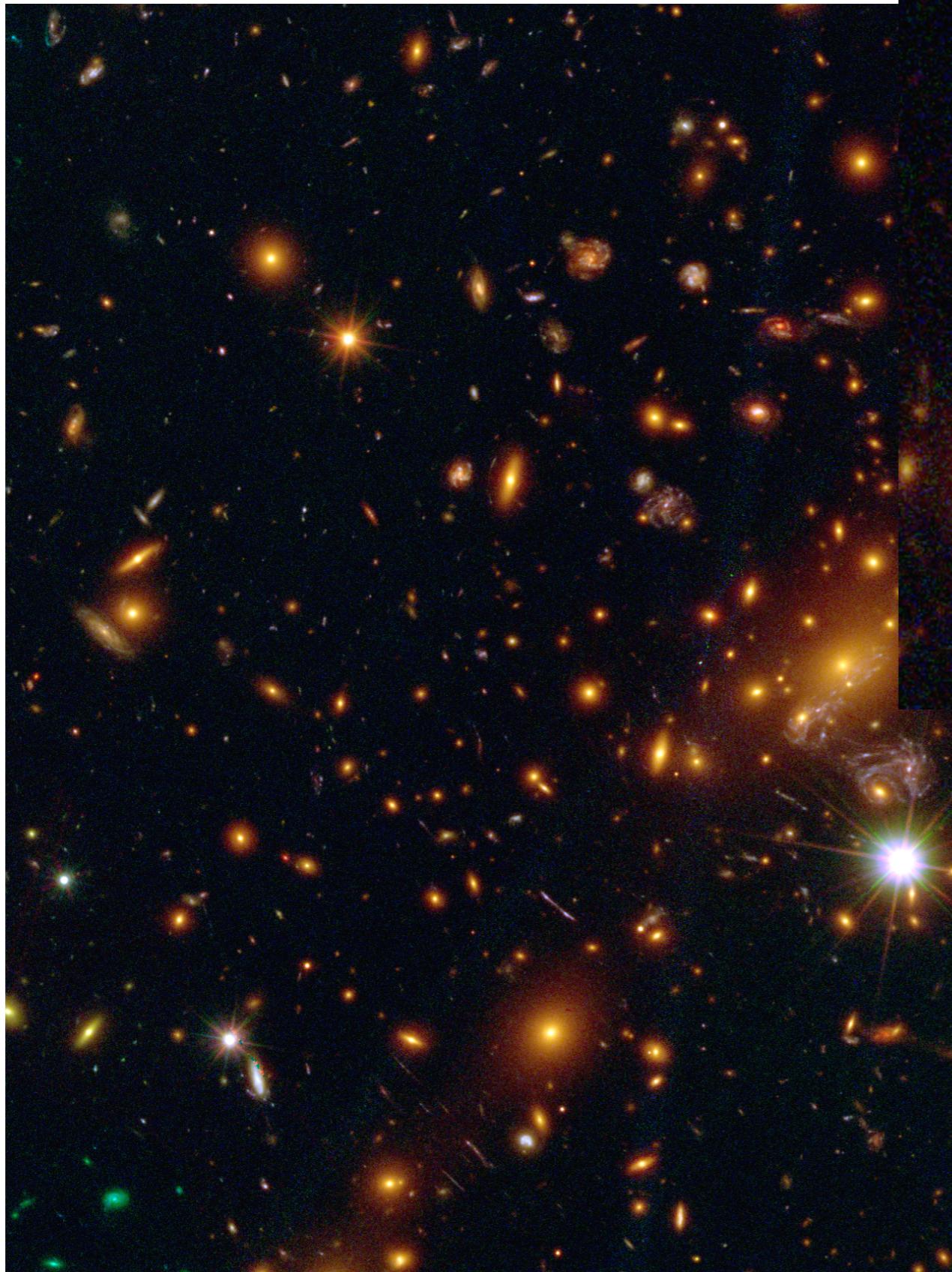


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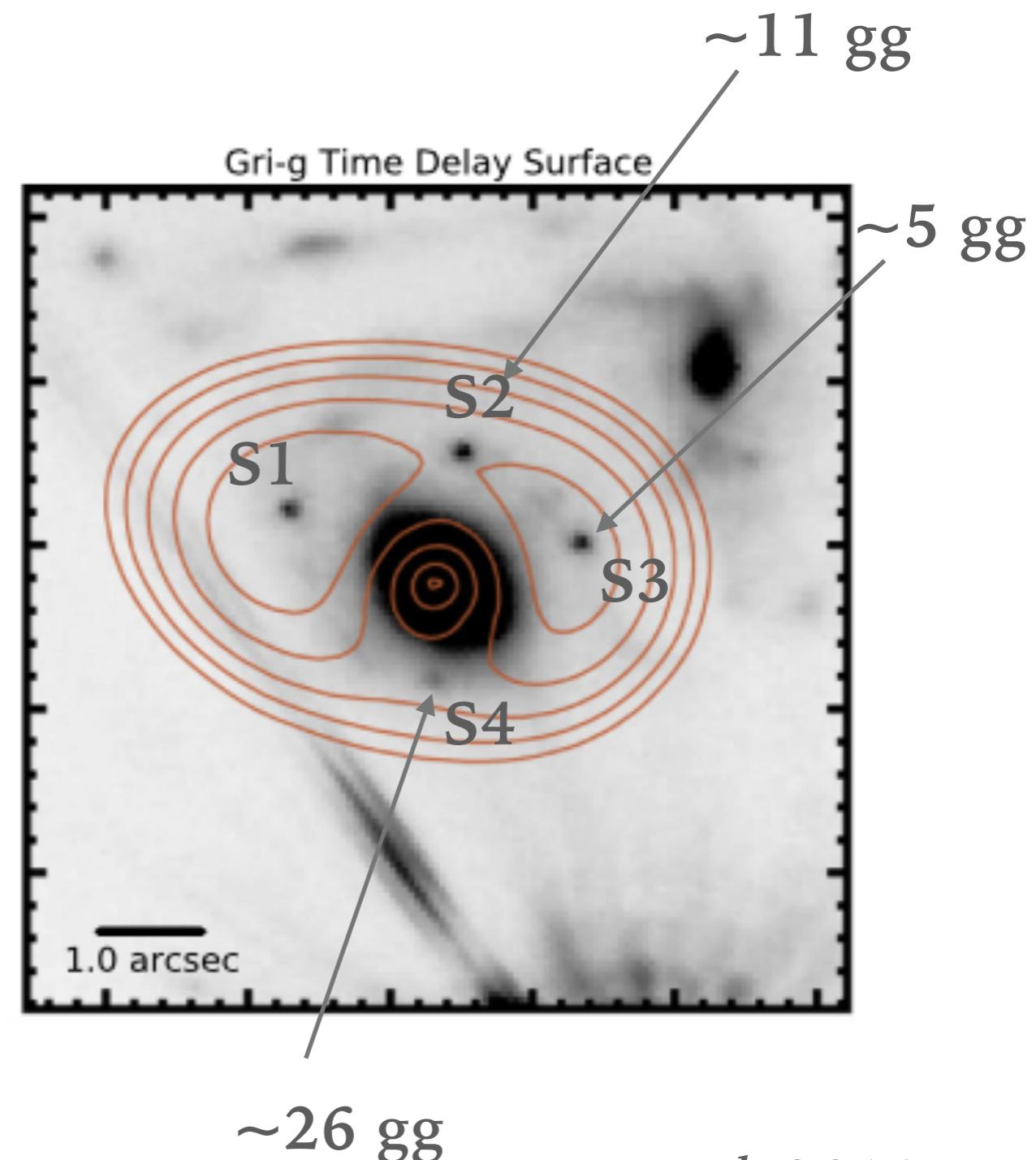
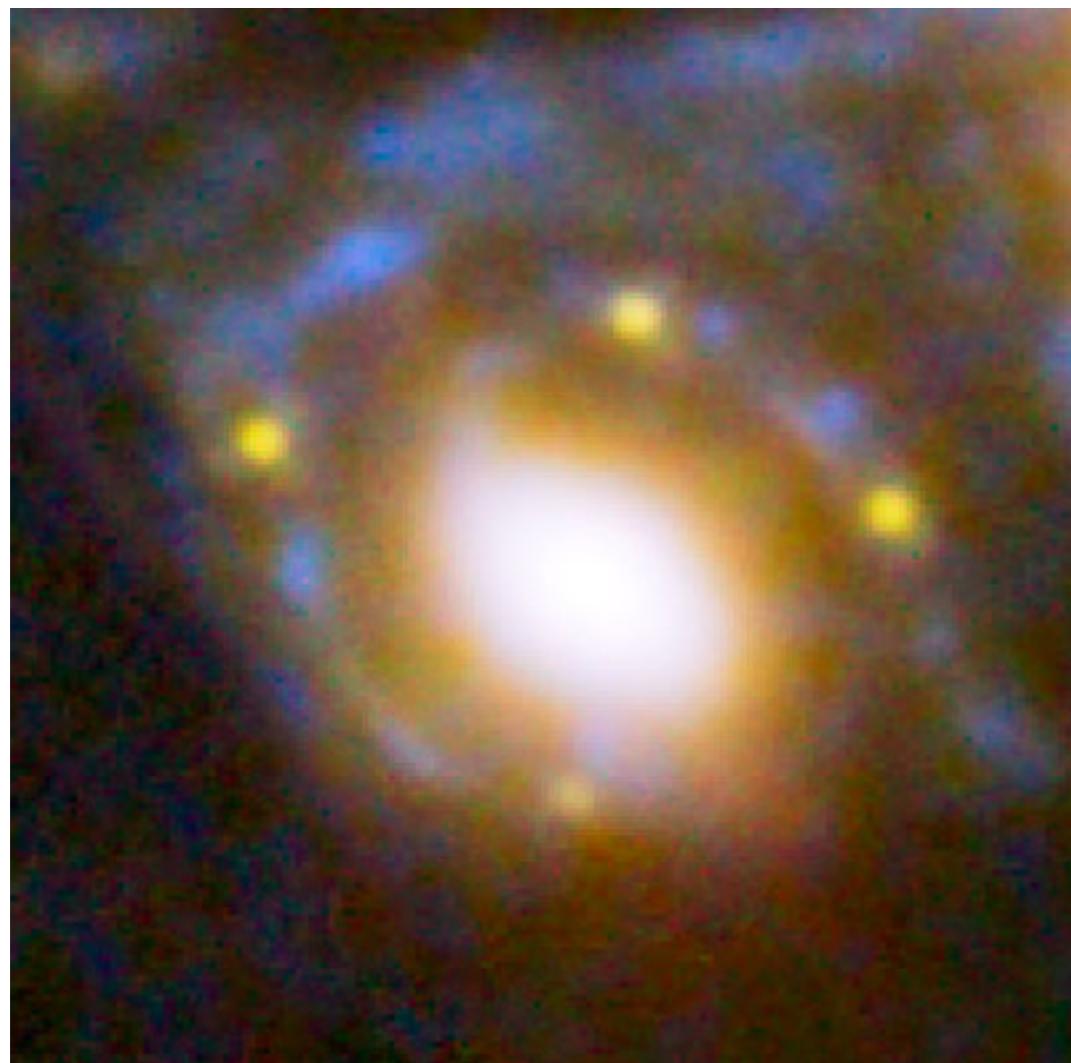
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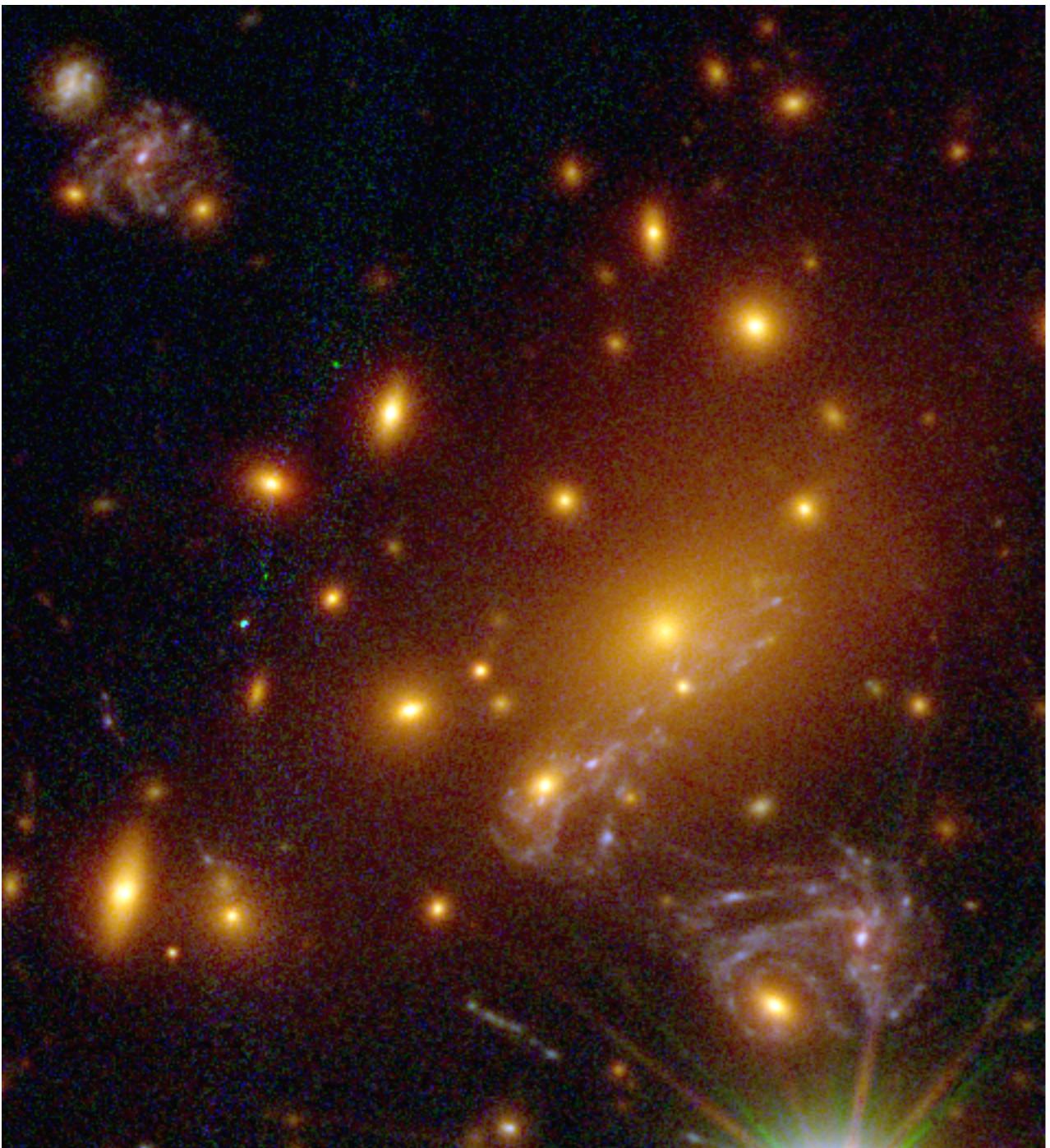
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Nov. 2014 (*Kelly et al.*)

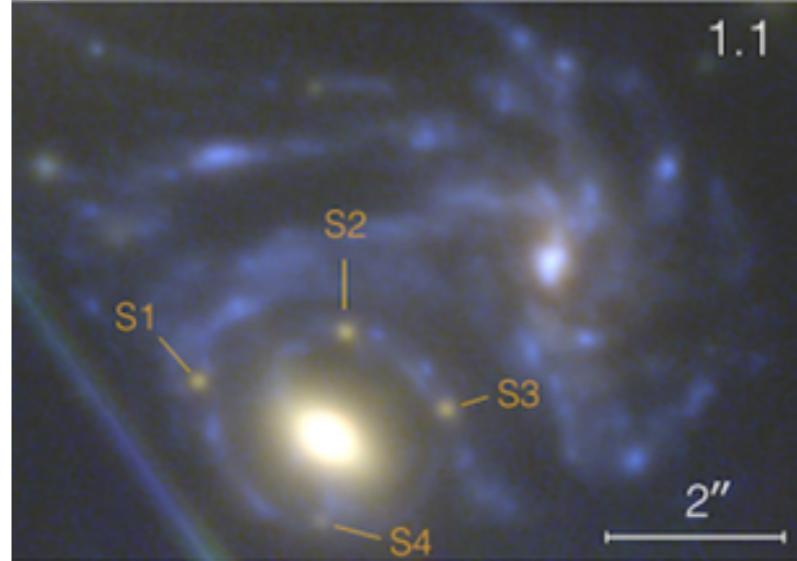
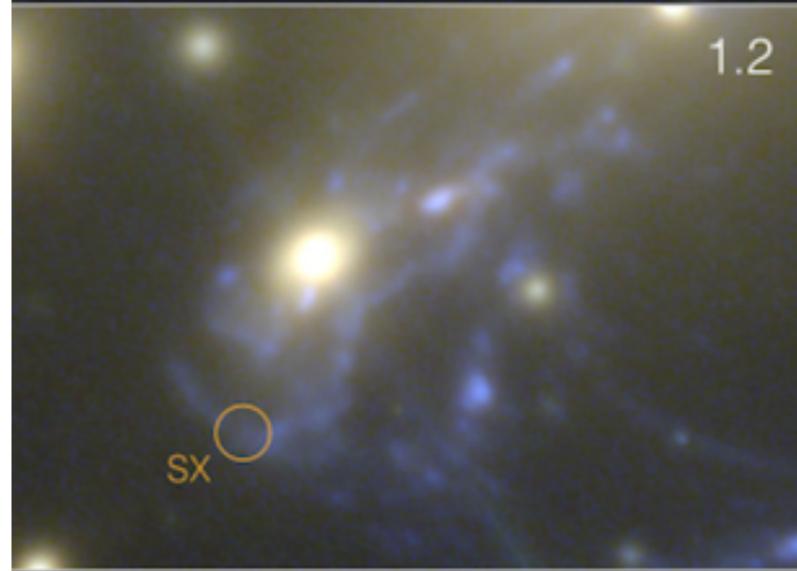
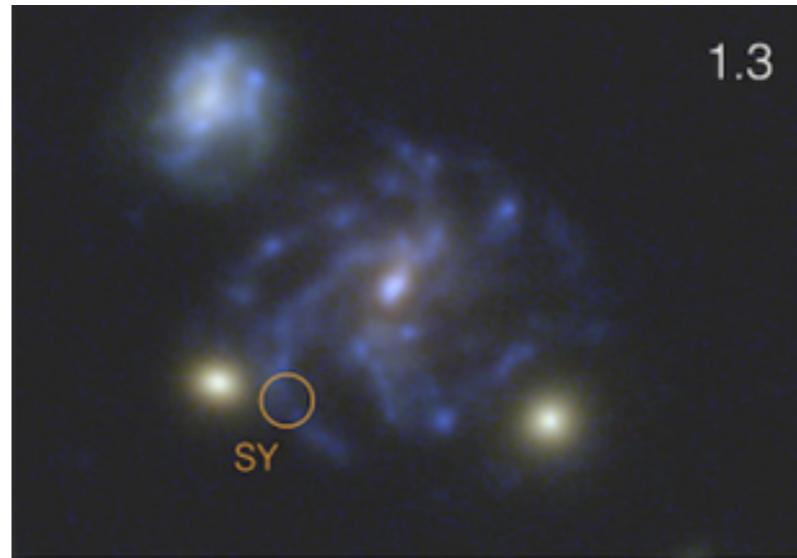
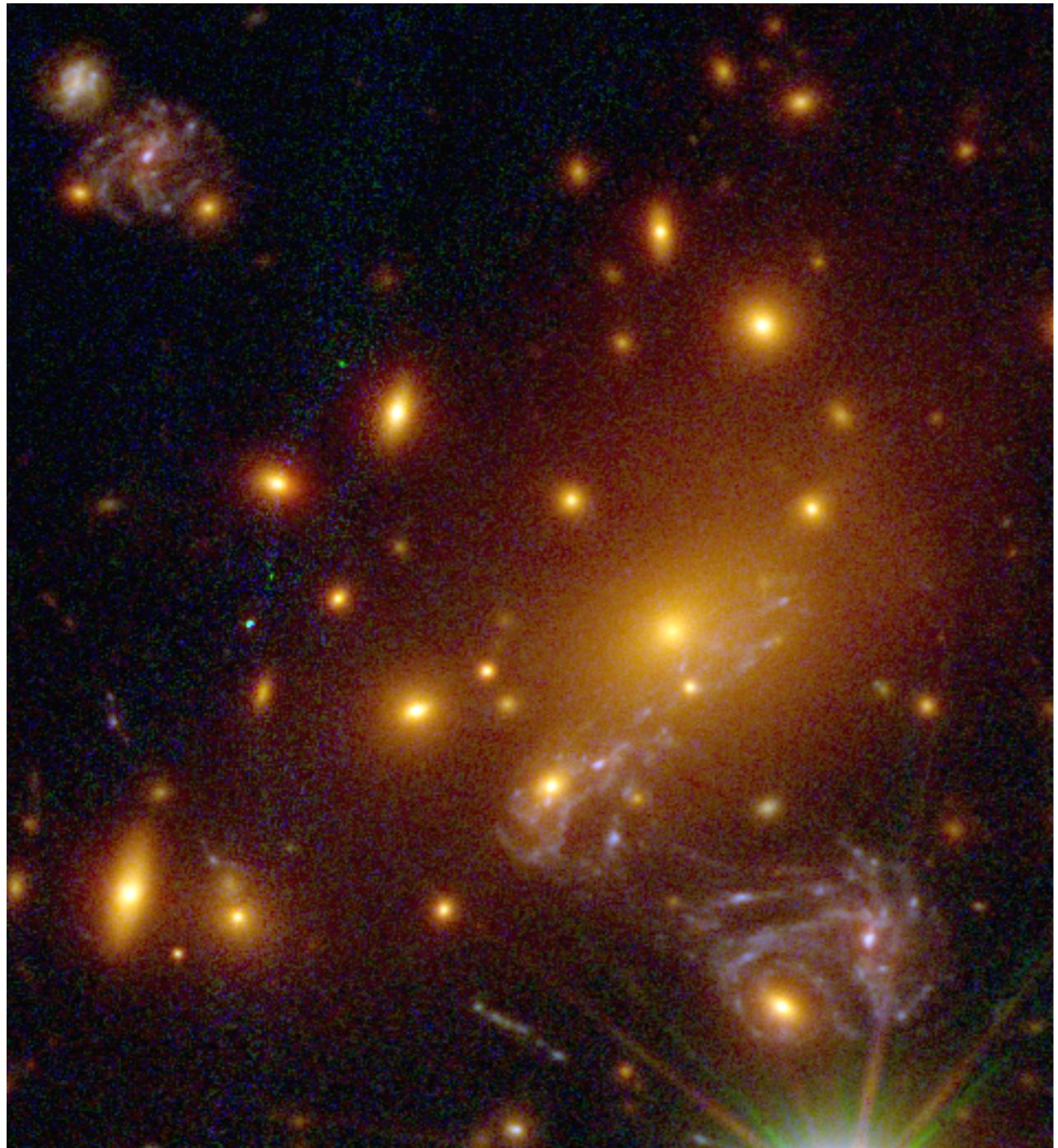


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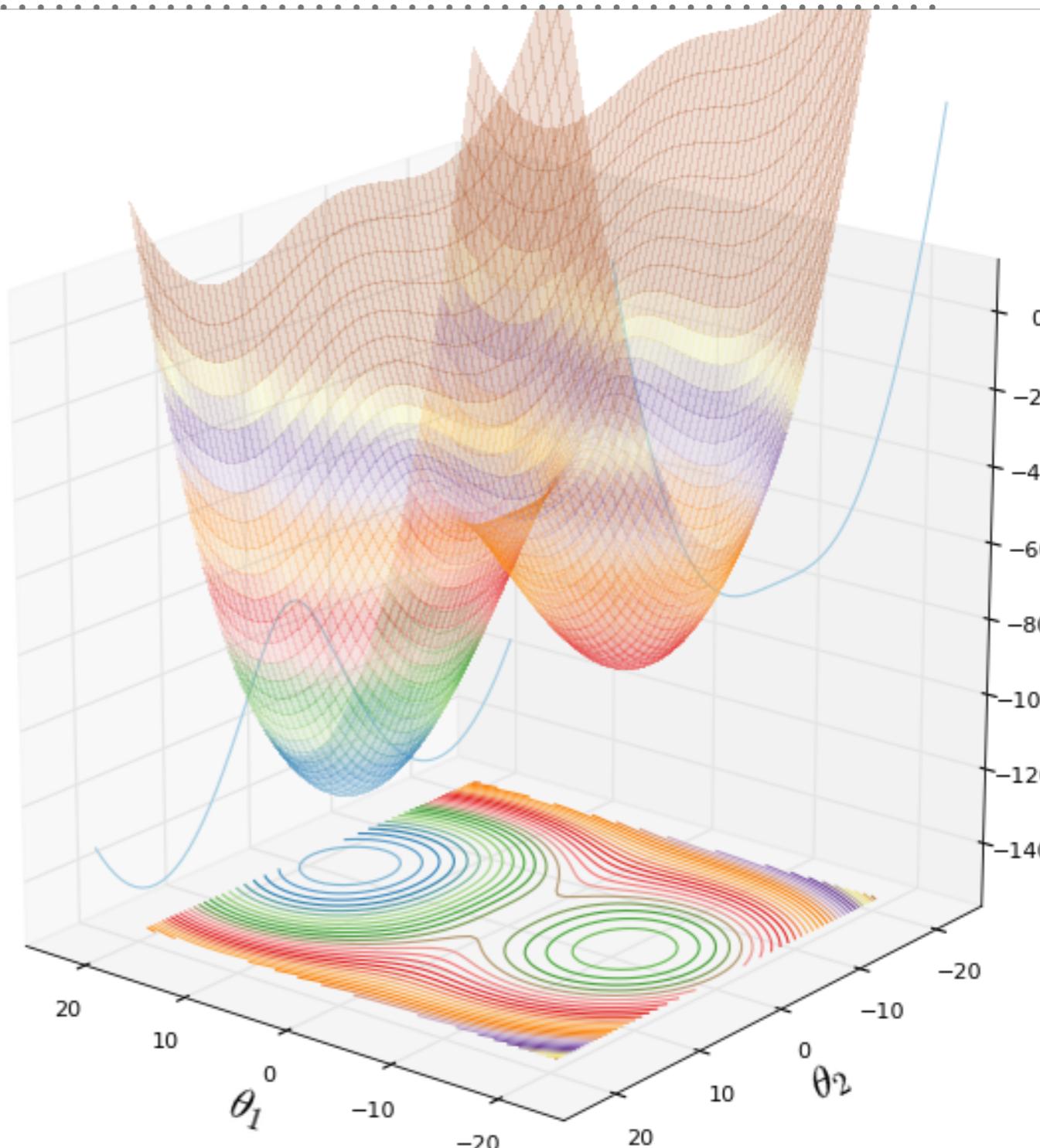
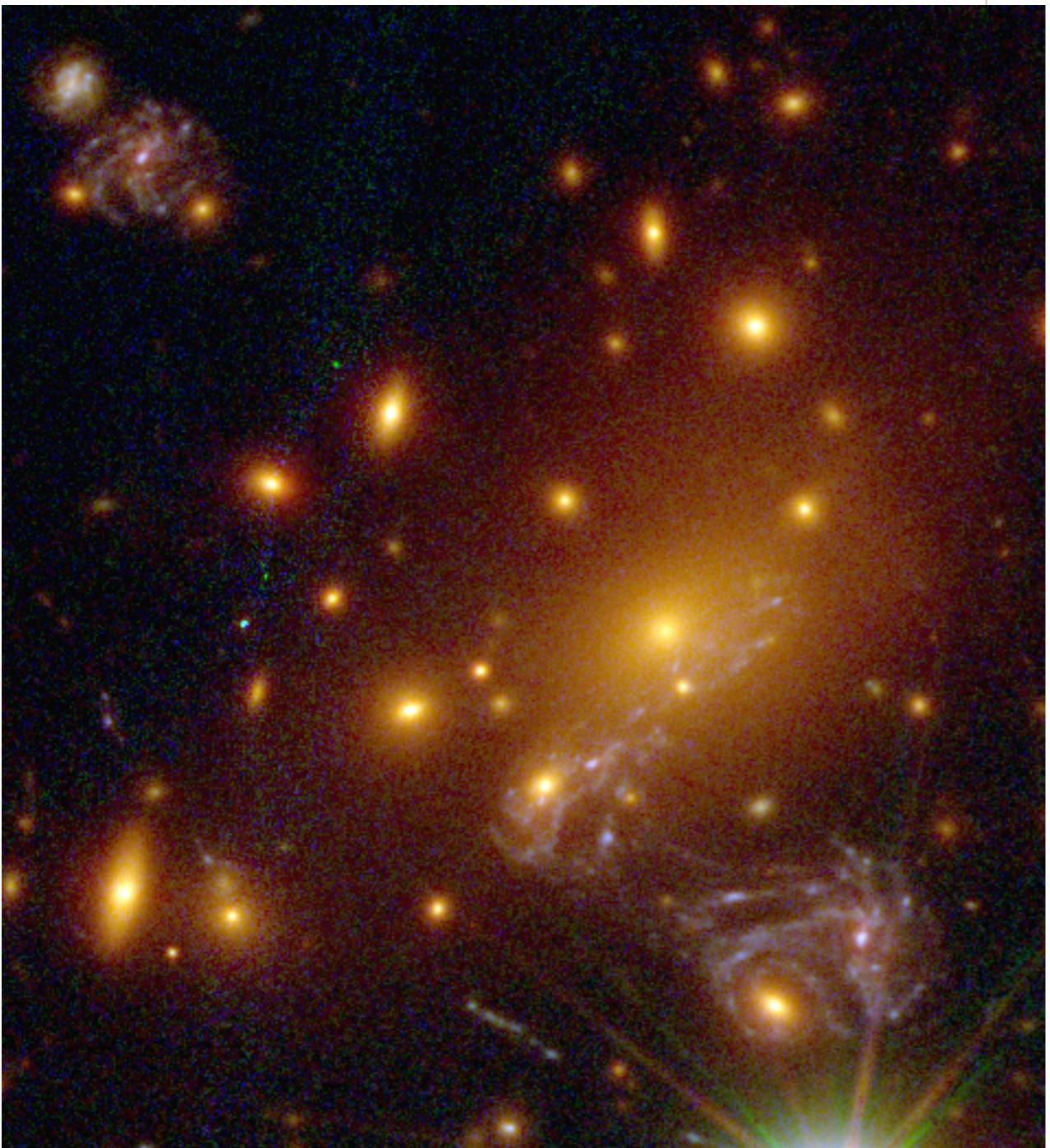


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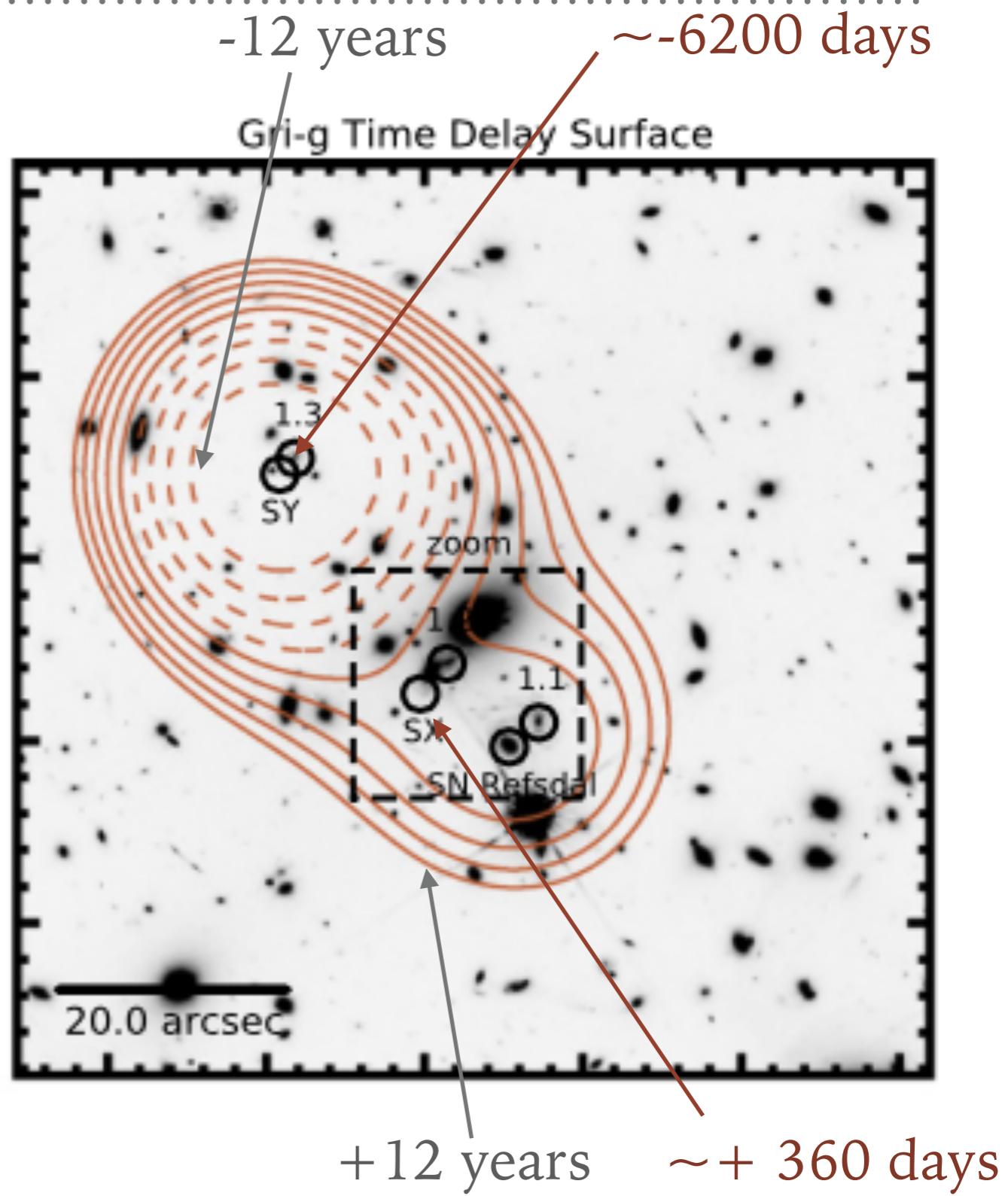


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# SN REFSDAL IN MACS 1149



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16/12/2016...

*Time delay*

(SX- S1)

$345 \pm 10$ gg

