GRAVITATIONAL LENSING

LECTURE 22

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AA 2016-2017

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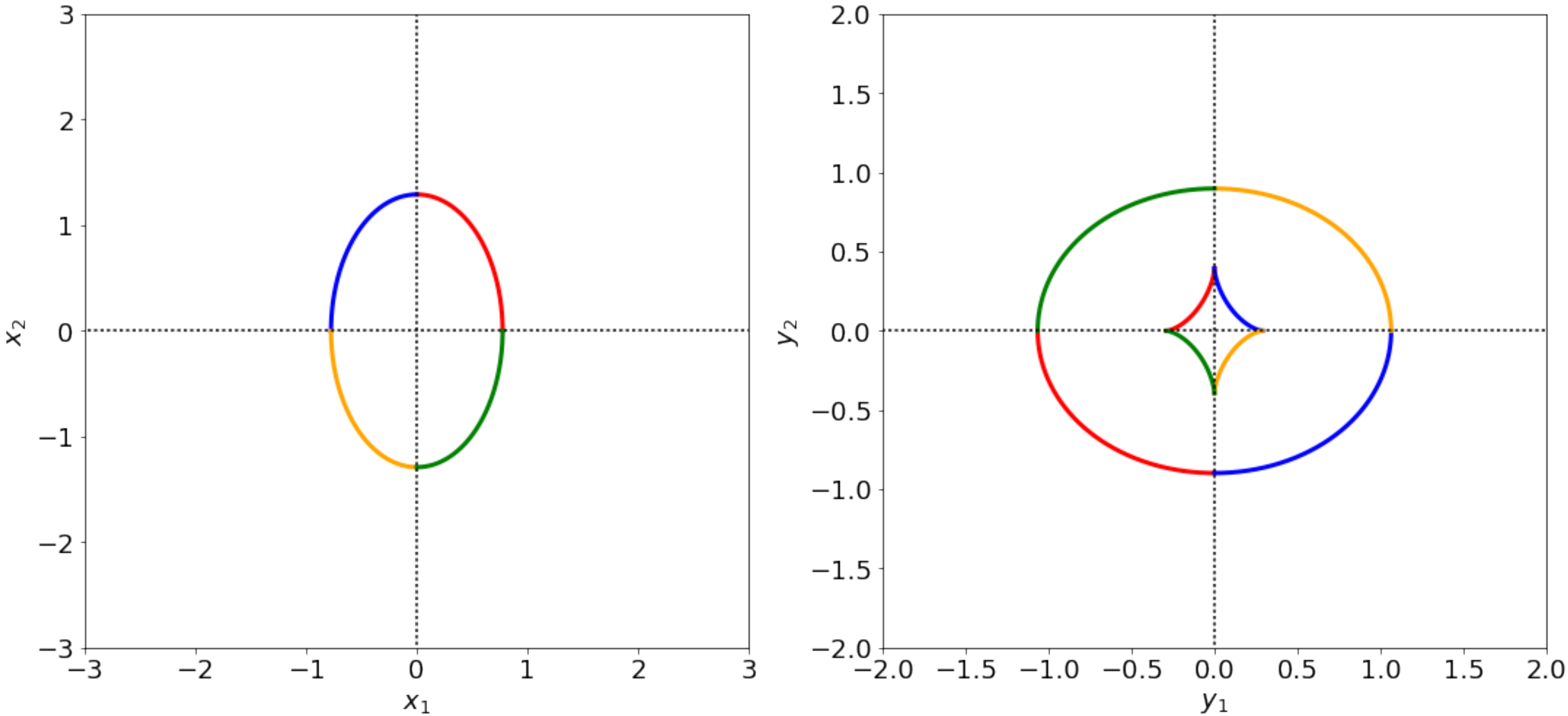
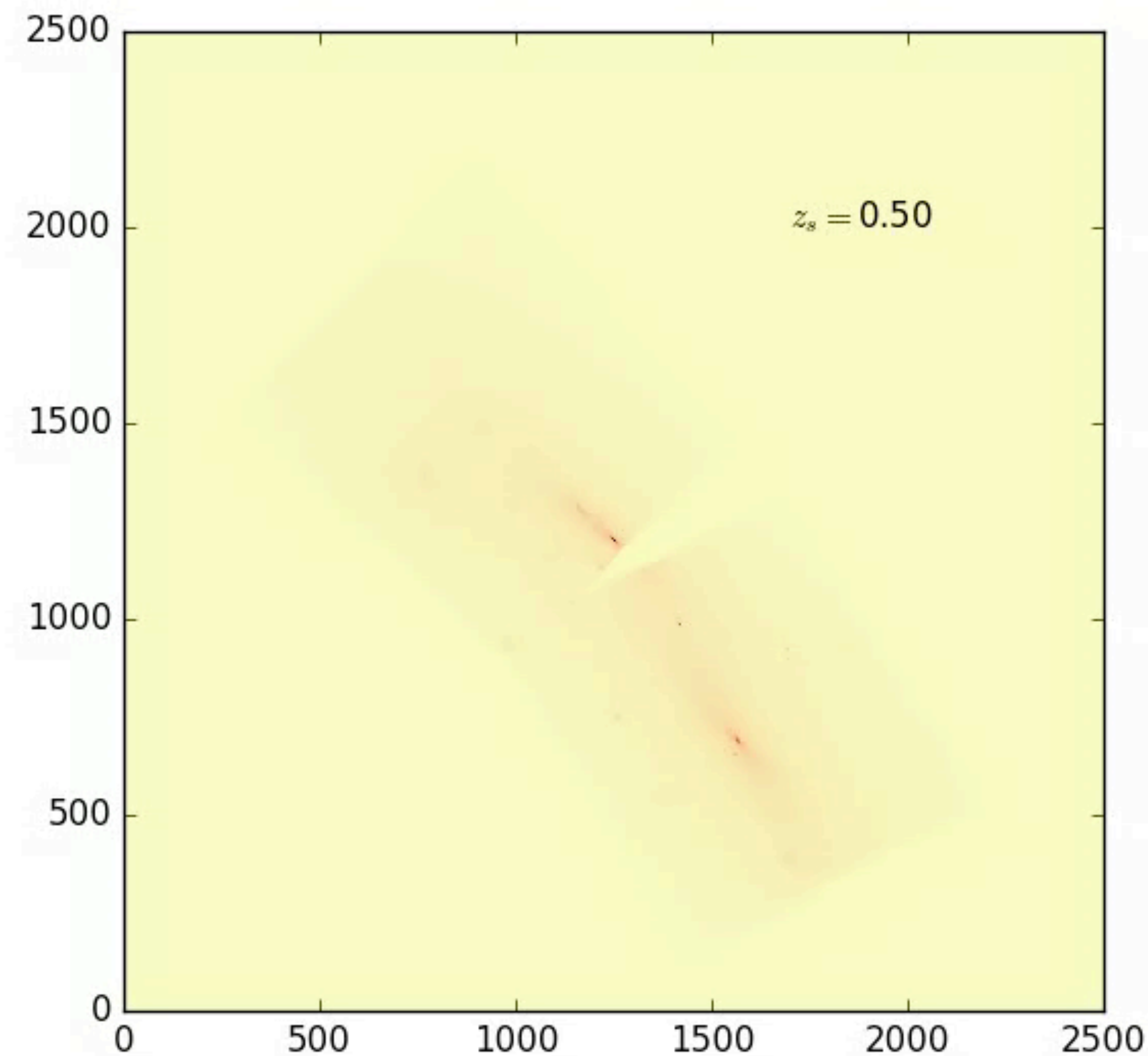
- NIE model
- mass sheet
- external shear

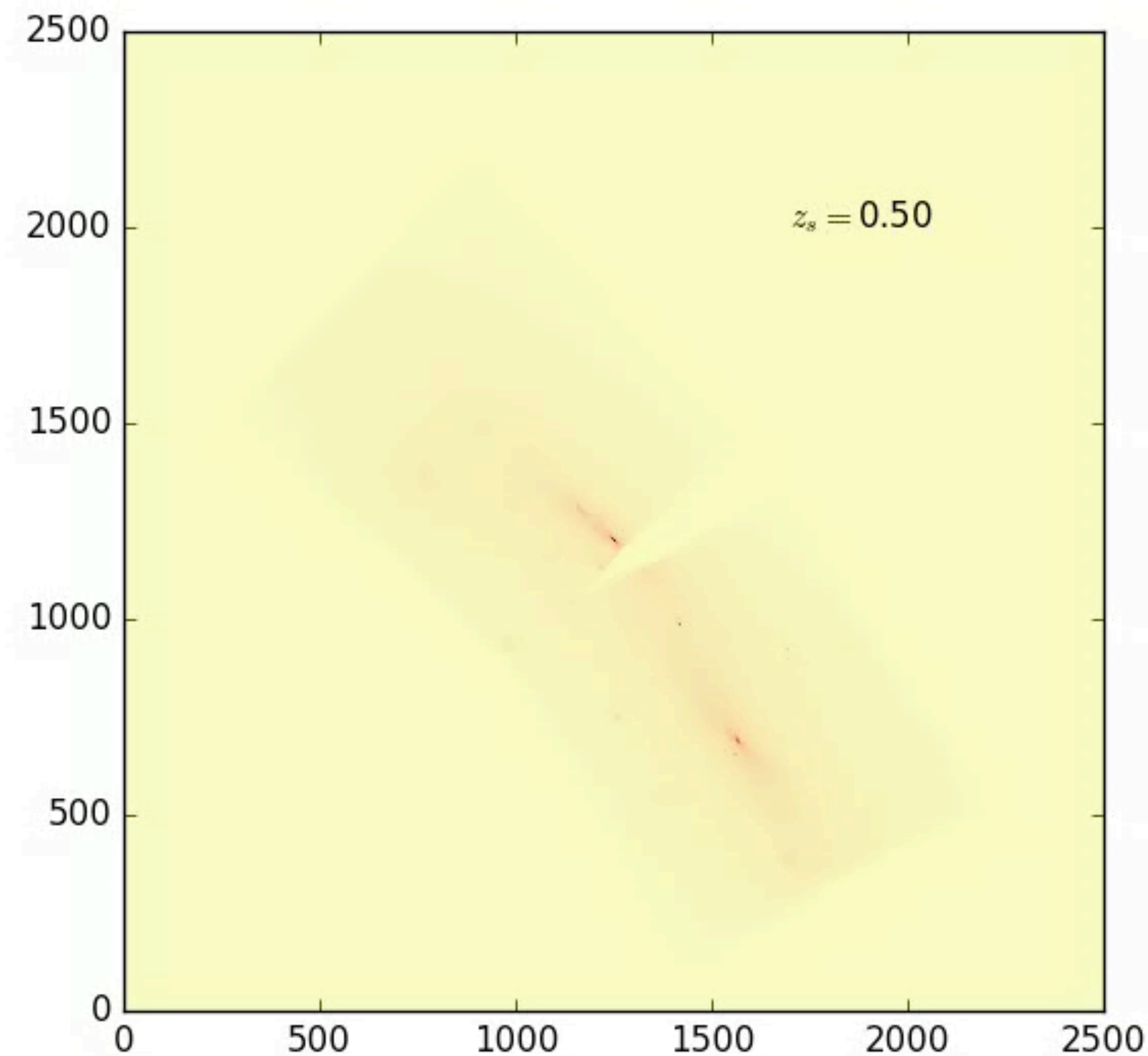
MULTIPLE IMAGES IN ELLIPTICAL MODELS

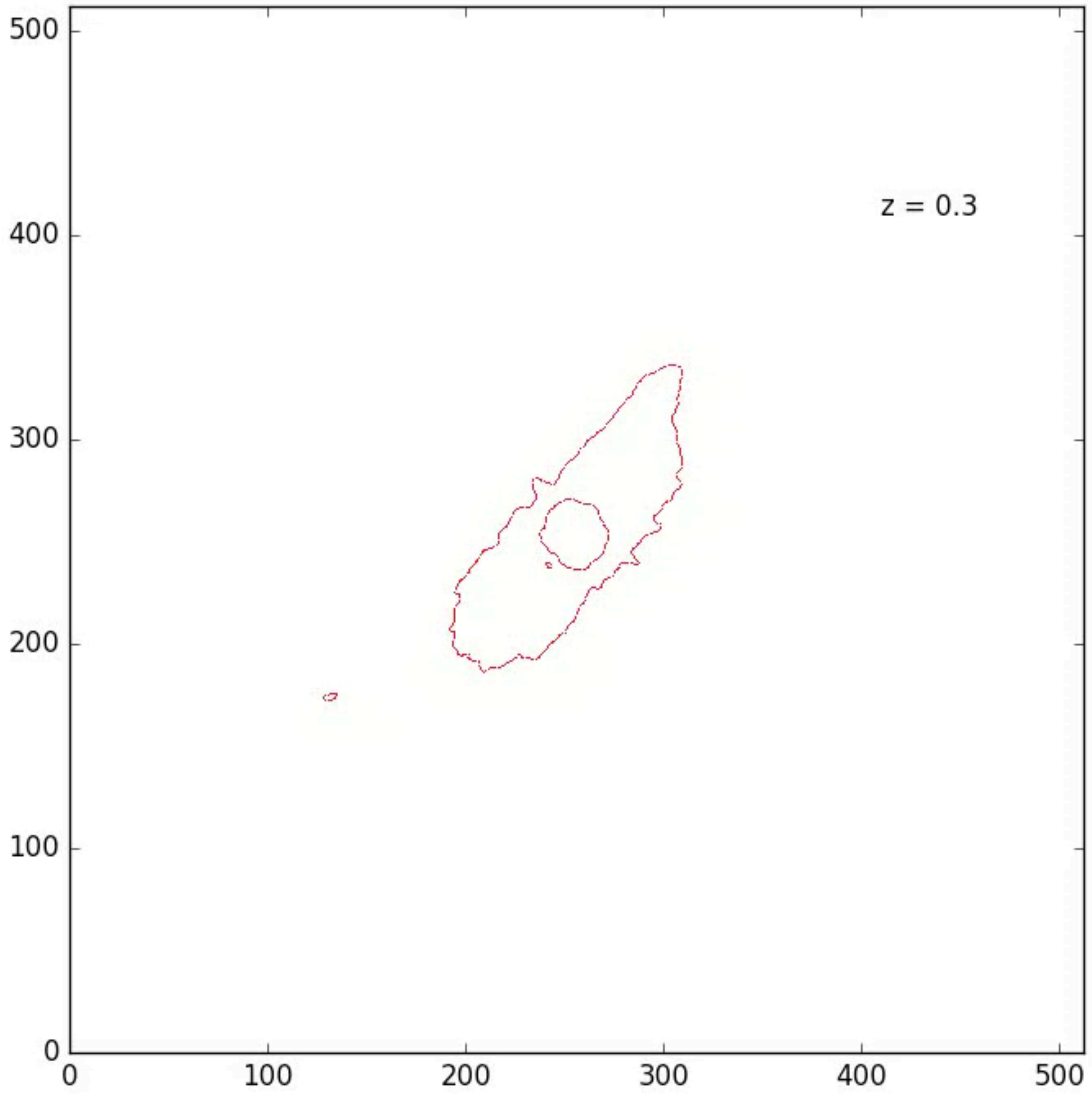


VISUALIZING THE CAUSTICS



VISUALIZING THE CAUSTICS





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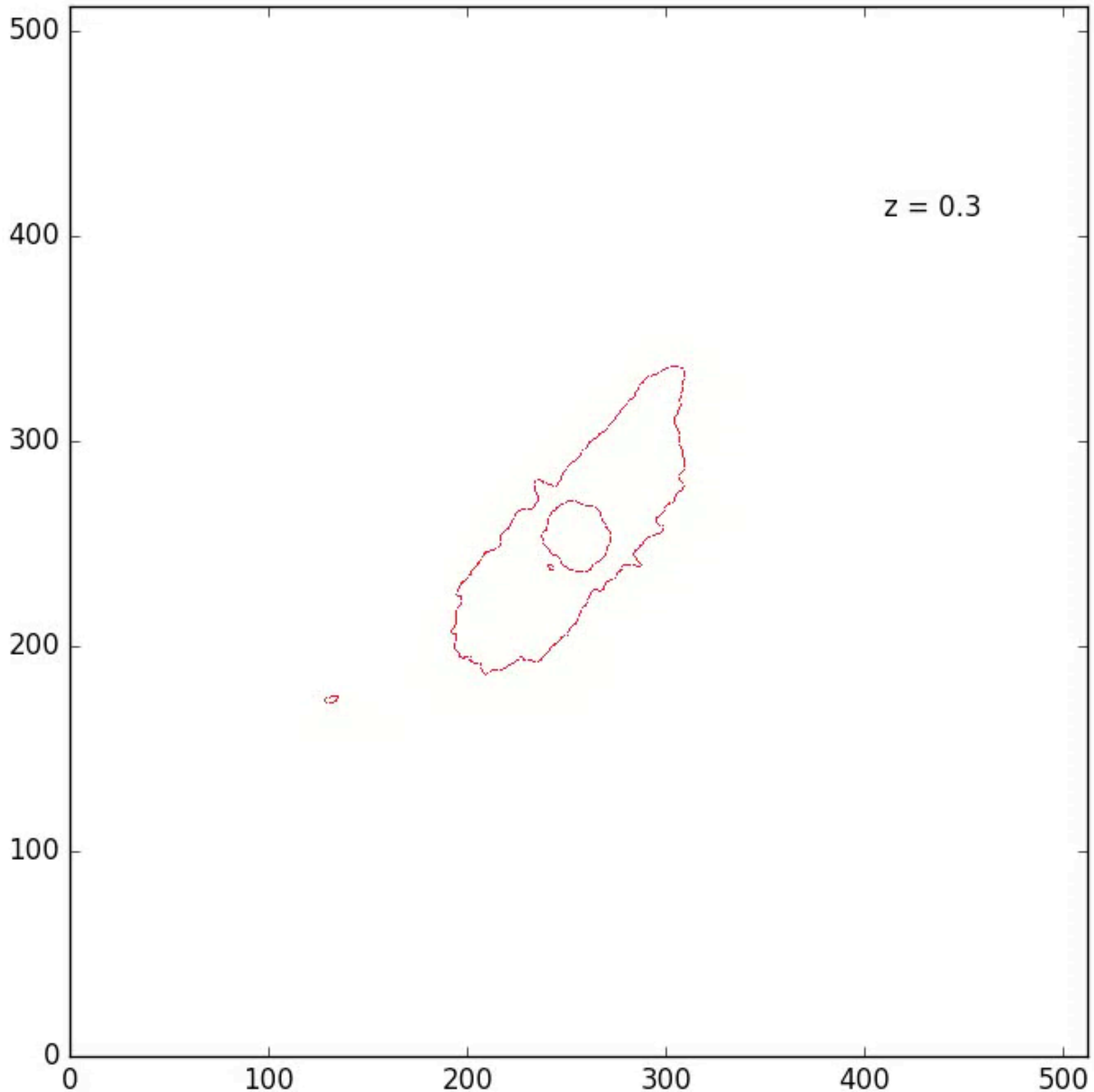
200
300
400
500


z = 0.3
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200
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0
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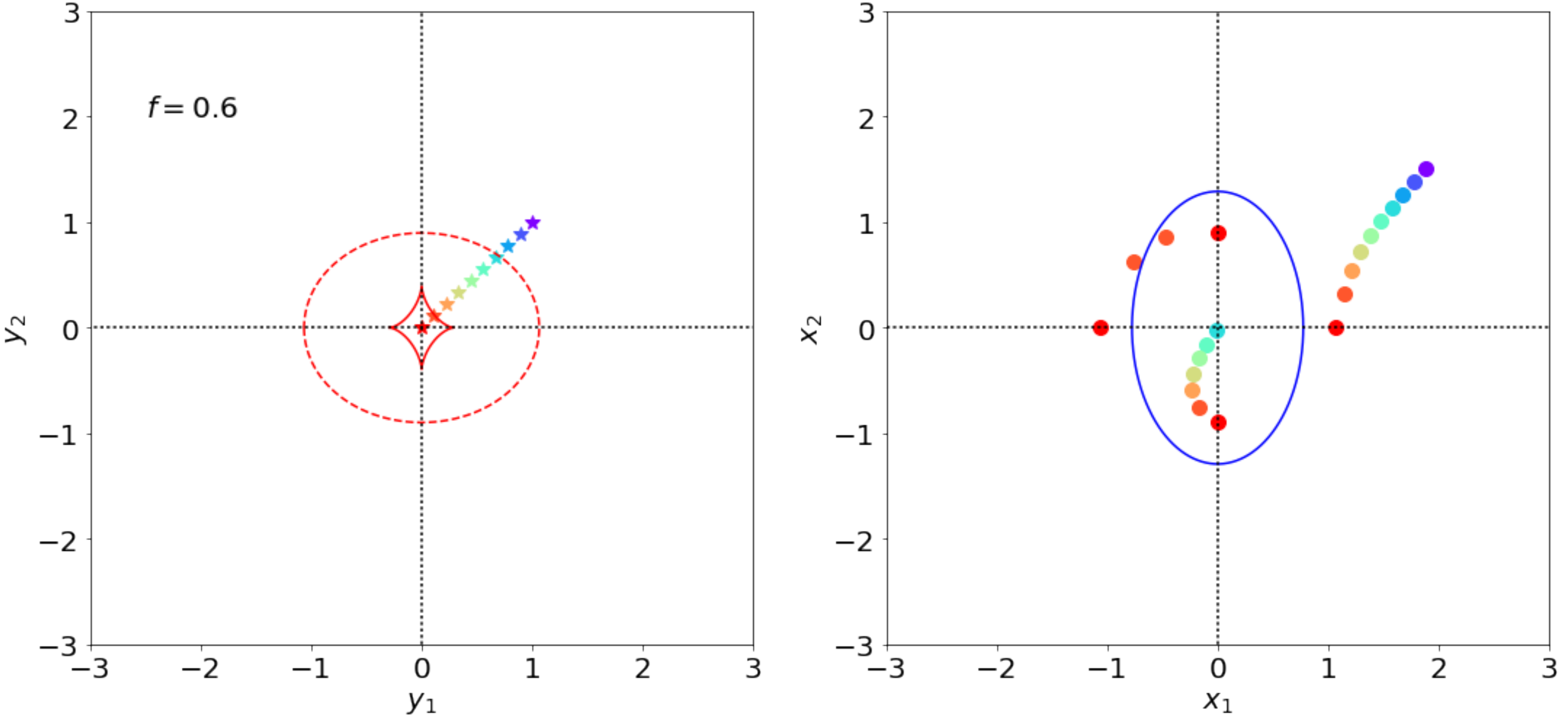
MULTIPLE IMAGES IN ELLIPTICAL MODELS



FROM SIE TO NIE

(SEE KOVNER, BARTELMANN & SCHNEIDER, 1994)

The SIE can be turned into a non-singular model by adding a core:

$$\Sigma(\vec{\xi}) = \frac{\sigma^2}{2G} \frac{\sqrt{f}}{\sqrt{\xi_1^2 + f^2\xi_2^2 + \xi_c^2}}$$

$$\kappa(\vec{x}) = \frac{\sqrt{f}}{2\sqrt{x_1^2 + f^2 x_2^2 + x_c^2}} .$$

In this case, the analytical treatment of the lens is much more complicated. We limit the discussion to the topology of the critical lines and caustics and infer information about the image multiplicities…

FROM SIE TO NIE

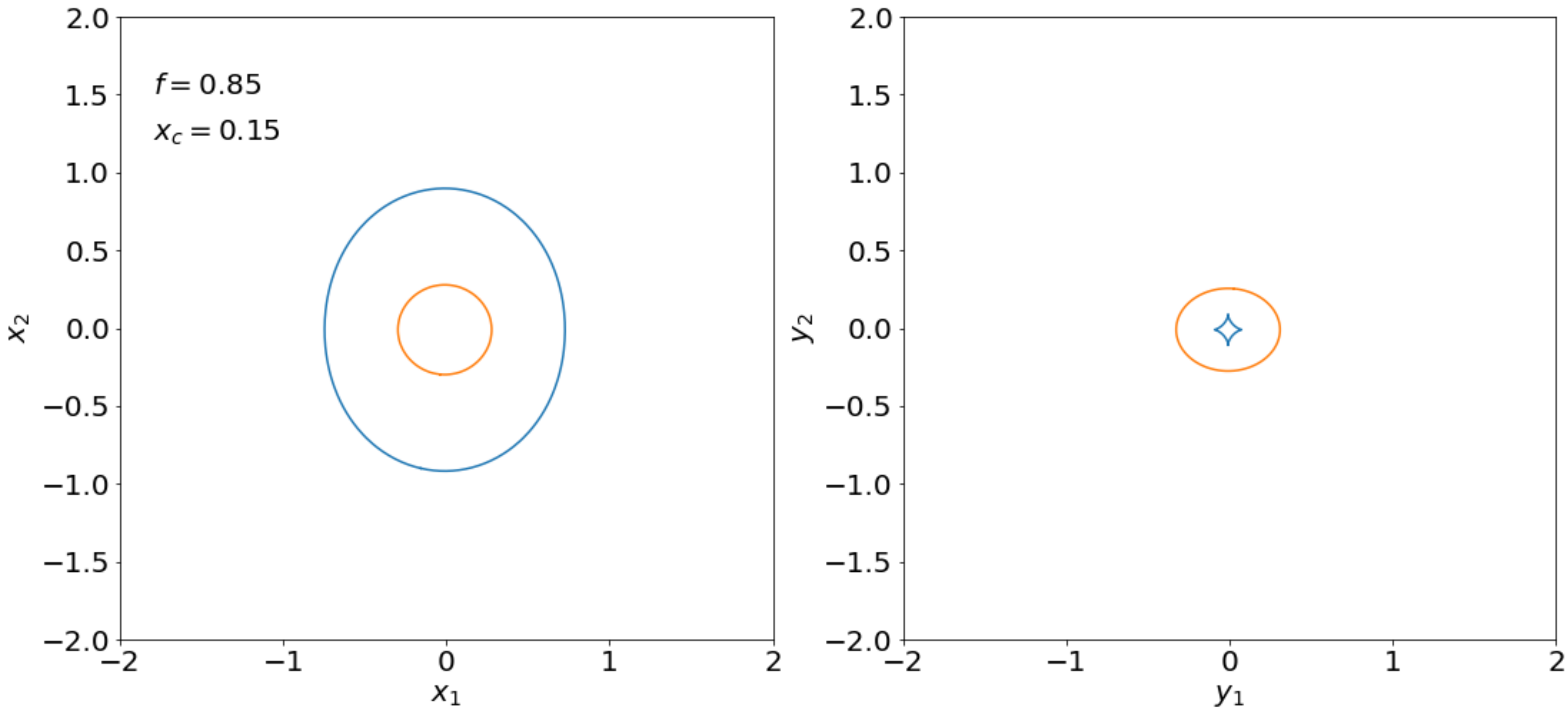
(SEE KOVNER, BARTELMANN & SCHNEIDER, 1994)

Introducing a core, the singularity is removed, thus the lens looses the CUT. Under certain circumstances, this is turned it into a regular CAUSTIC.

When this caustic exists, it corresponds to a CRITICAL LINE on the lens plane.

FROM SIE TO NIE

(SEE KOVNER, BARTELMANN & SCHNEIDER, 1994)



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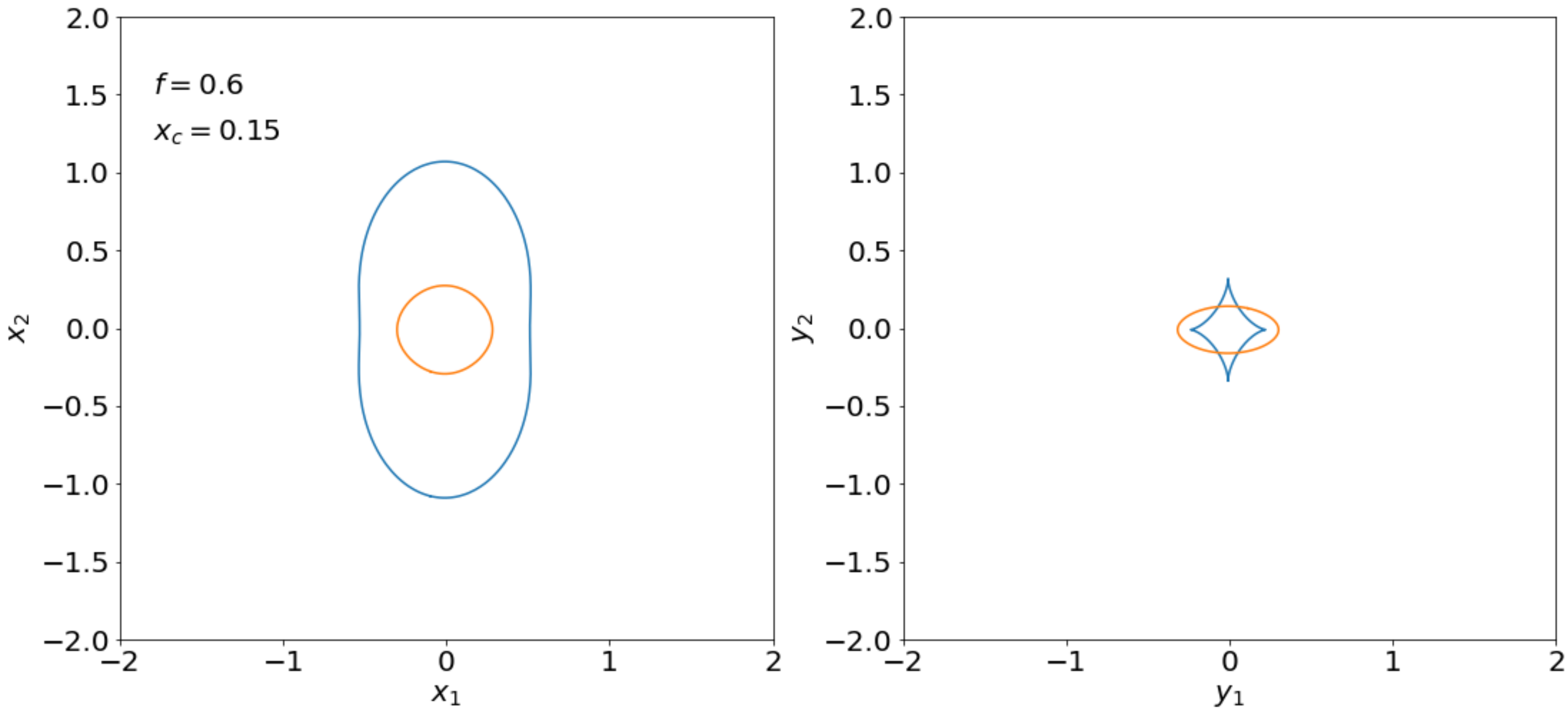

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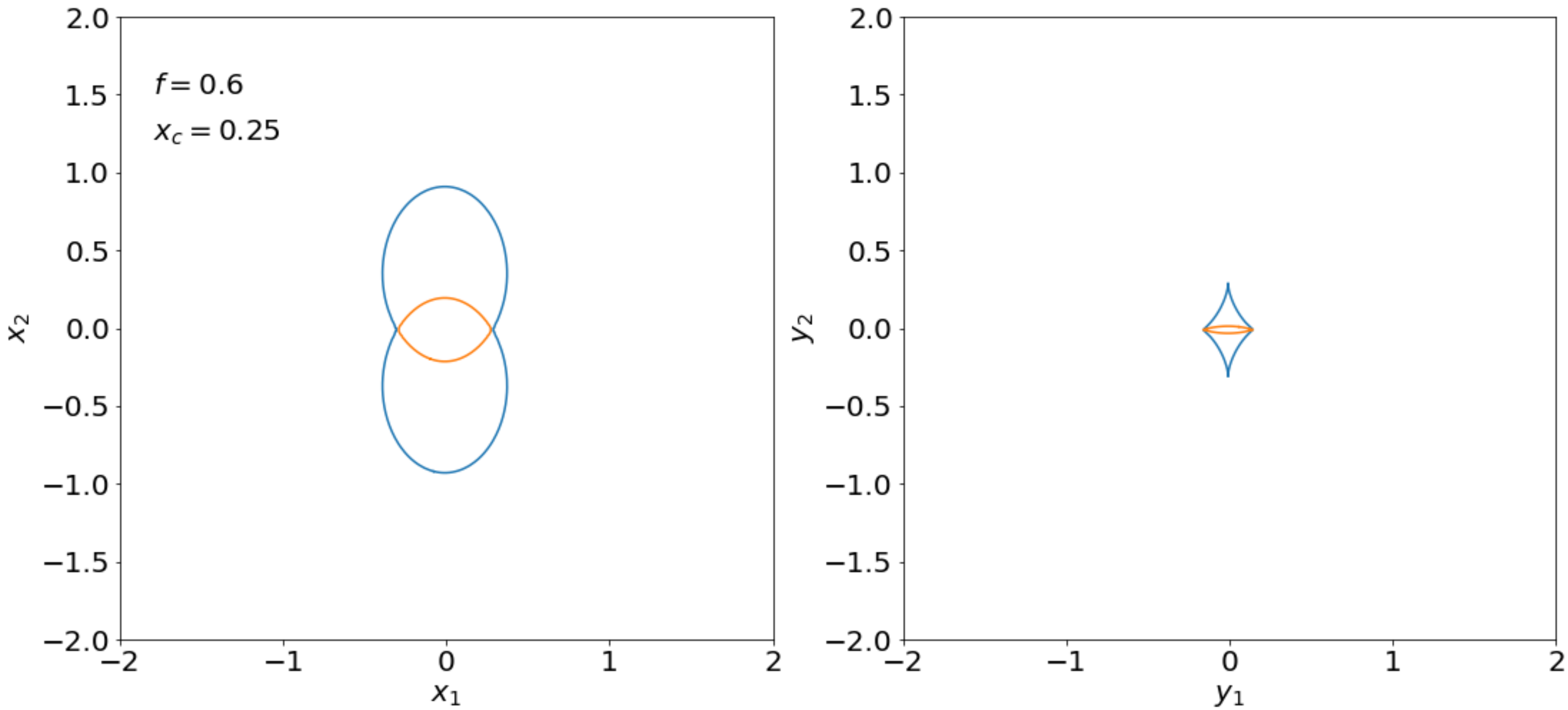

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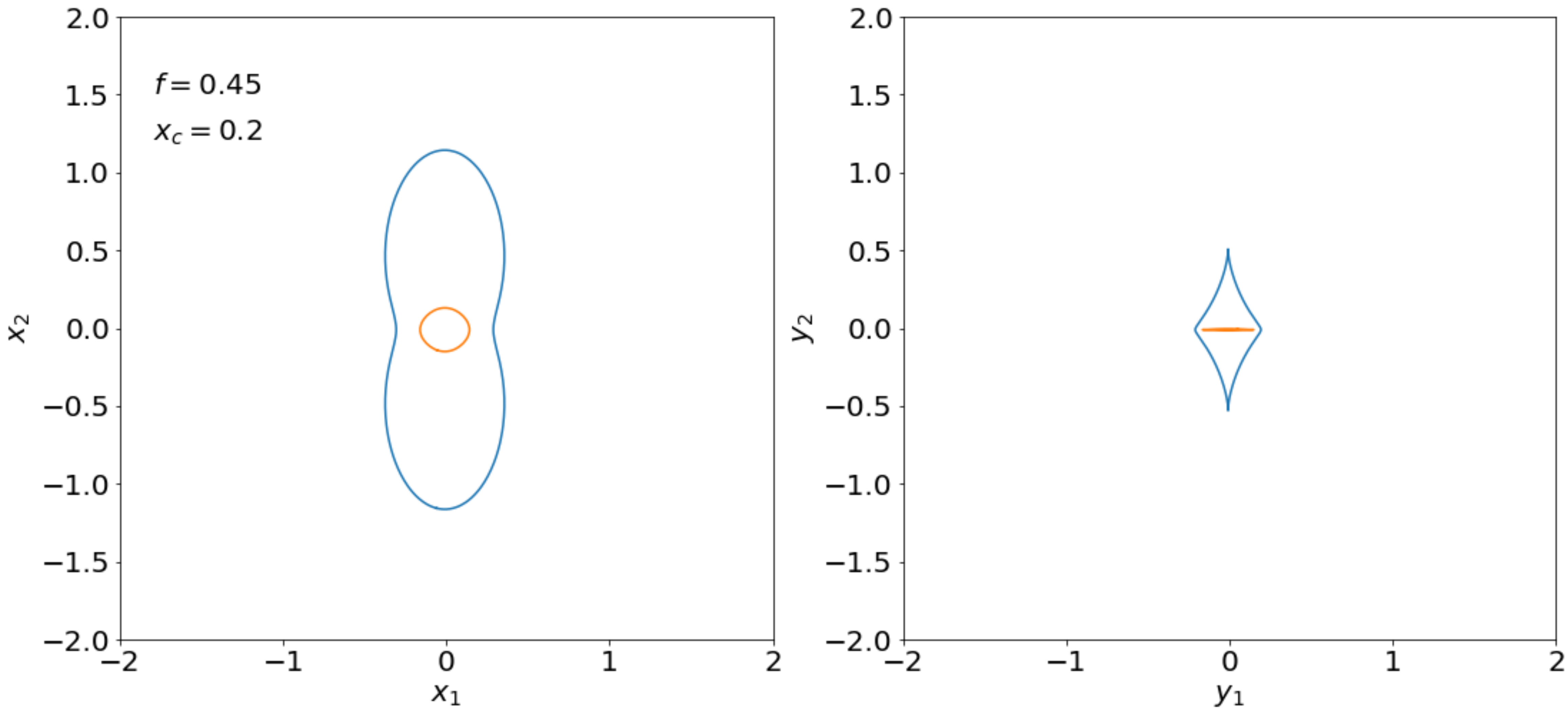

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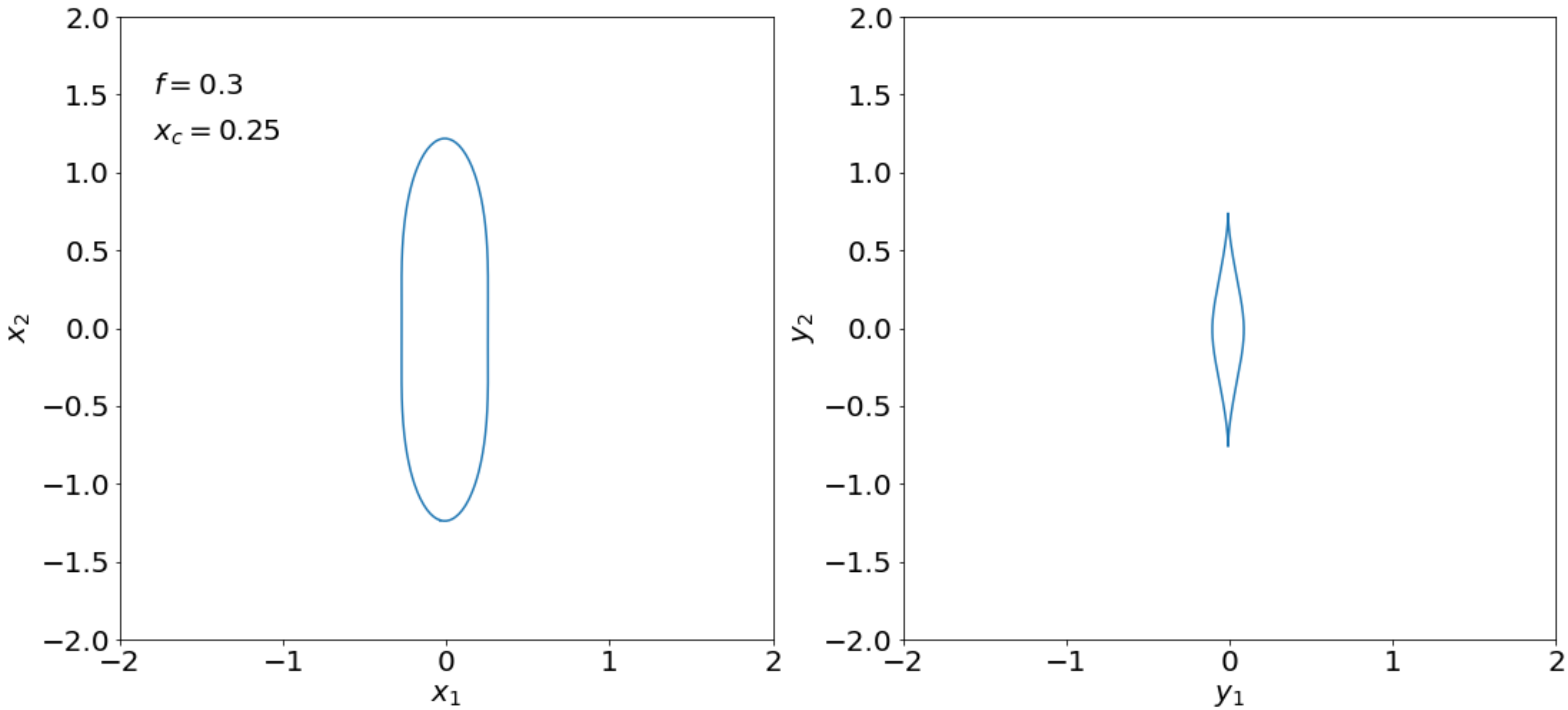

FROM SIE TO NIE

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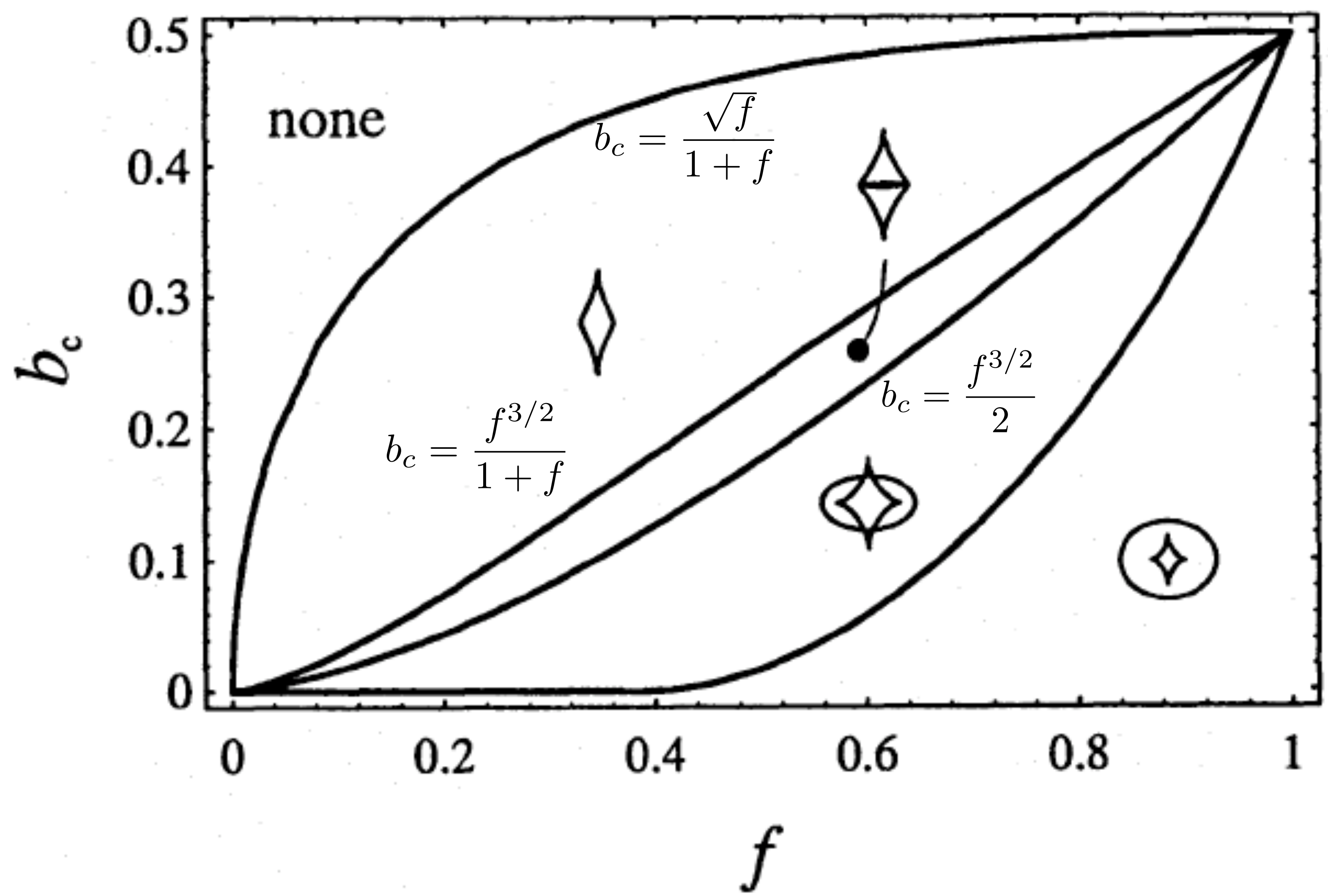
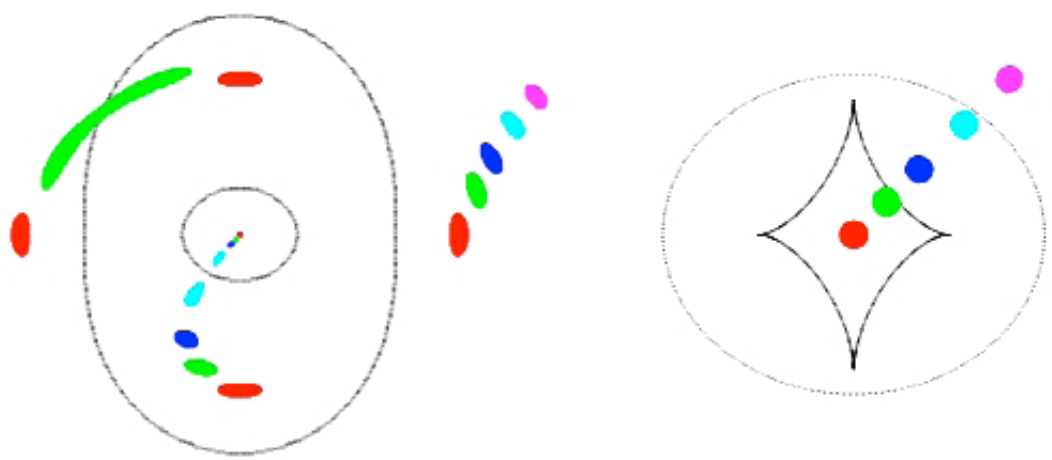
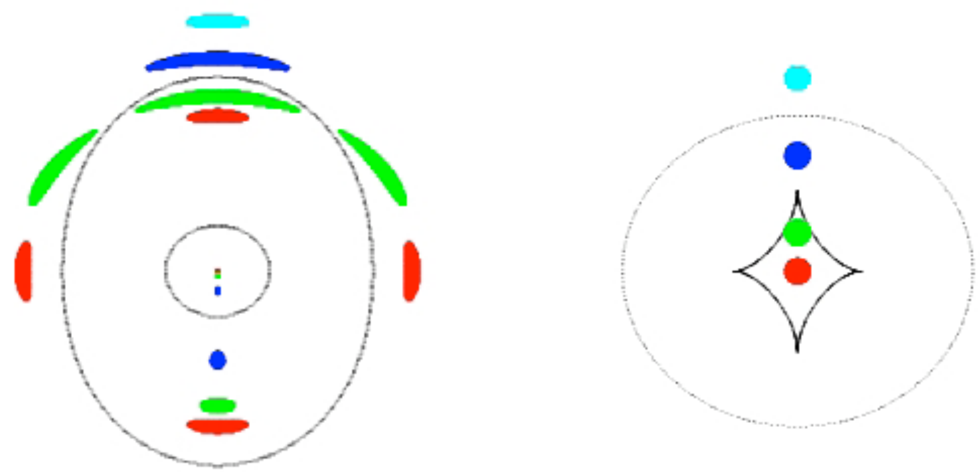


MULTIPLE IMAGES IN CORED LENSES





FROM SIE TO EPL

(SEE TESSORE & METCALF, 2015)

- Modifications of the SIE to change the slope of the density profile are discussed in Tessore & Metcalf (2015)
- Elliptical Power-Law lenses are difficult to treat analytically.
- The usage of numerical techniques is mandatory
- Here, we discuss some properties only qualitatively

FROM SIE TO EPL

(SEE TESSORE & METCALF, 2015)

$$\kappa(x) = \frac{3-n}{2} x^{1-n}$$

$$x = \sqrt{f^2 x_1^2 + x_2^2}$$

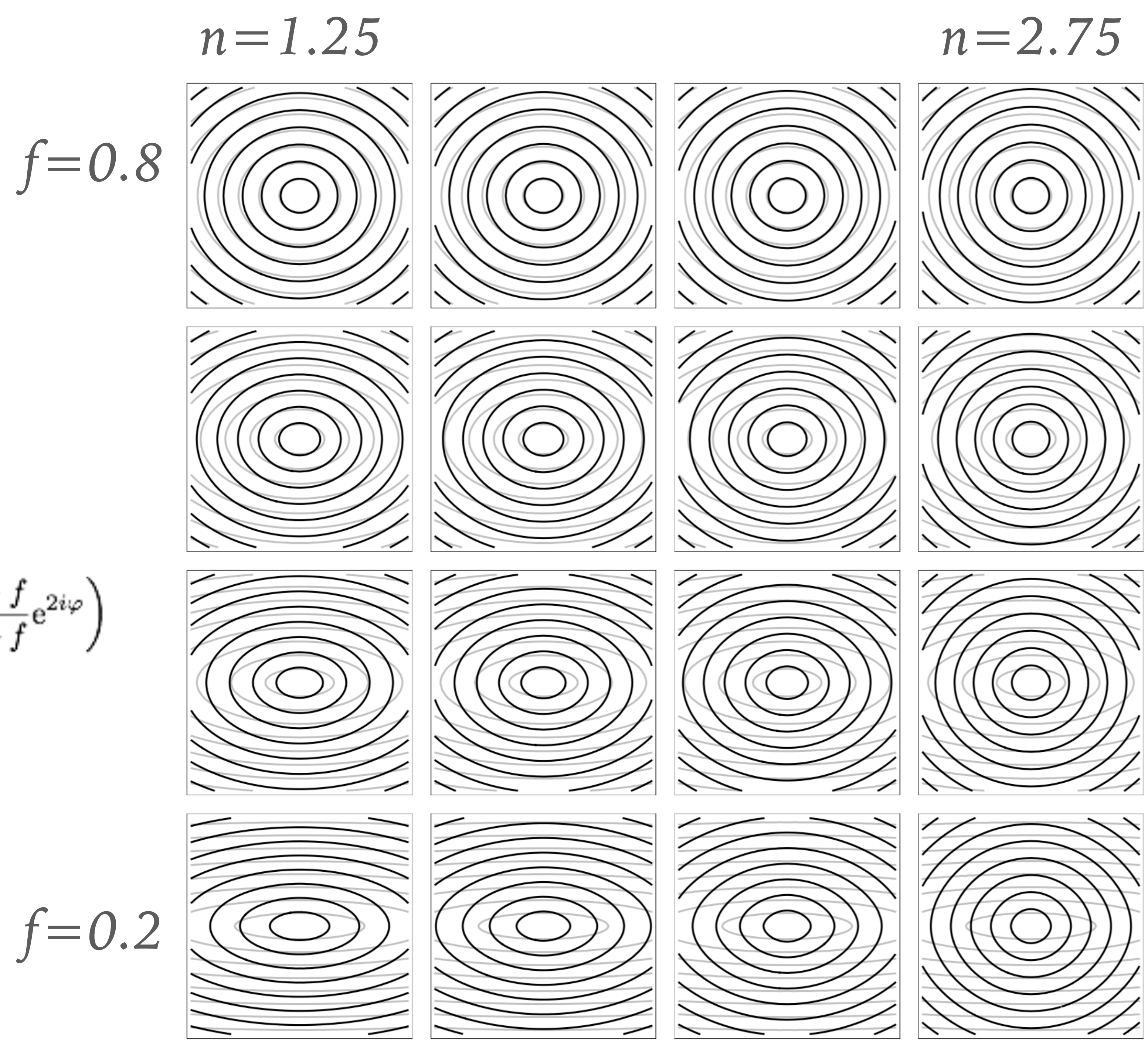
- Calculations can be better done using complex notation (e.g. Bourassa & Kantowski, 1975)
- The complex deflection angle involves the **Gauss Hypergeometric Function**:

$$\alpha(x, \varphi) = \frac{2}{1+f} x^{2-n} \mathrm{e}^{i\varphi} \; {}_2F_1\left(1, \frac{n-1}{2}; 2 - \frac{n-1}{2}; -\frac{1-f}{1+f} \mathrm{e}^{2i\varphi}\right)$$

- The potential can be found to be:

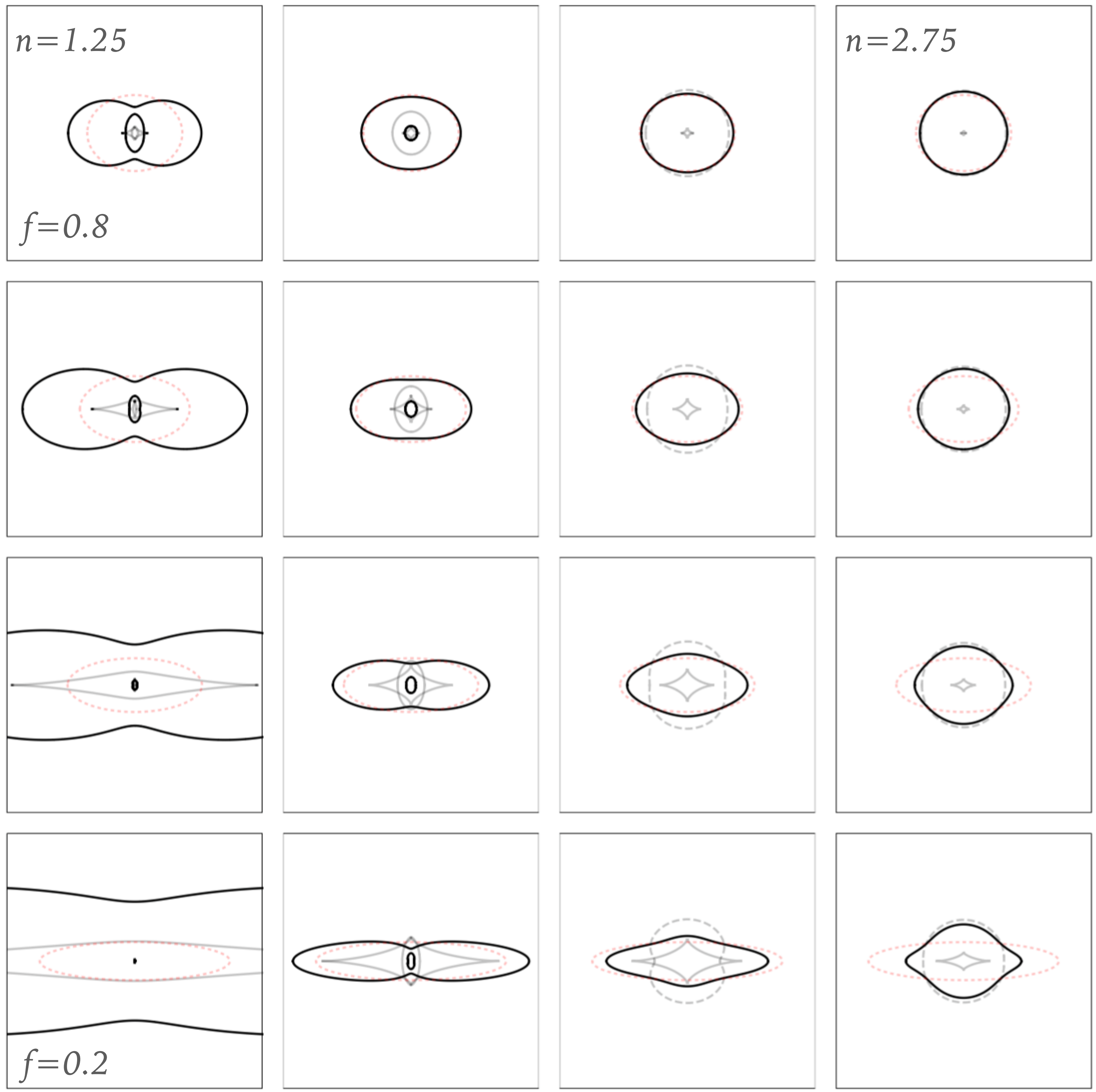
$$\psi(x_1, x_2) = \frac{x_1 \alpha_1 + x_2 \alpha_2}{3-n}$$

- Similarly, other properties such as the shear can be derived easily from the deflection angle.



FROM SIE TO EPL

(SEE TESSORE & METCALF, 2015)



EXTERNAL PERTURBATIONS

Lenses are often not isolated. Therefore, it is sometimes necessary to embed the lens into an external mass distribution mimicking the presence of nearby structures. How can such perturbation be modeled?

One can think to use a potential, defined such that

$$
\begin{aligned}
\gamma_1 &= \frac{1}{2}(\Psi_{11} - \Psi_{22}) = \text{const.} \\
\gamma_2 &= \Psi_{12} = \text{const.} \\
\kappa &= \frac{1}{2}(\Psi_{11} + \Psi_{22}) = \text{const.}
\end{aligned}
$$

EXTERNAL PERTURBATIONS

If both the sum ad the difference of the 2nd derivatives must be constant, the two derivatives must be constant separately:

$$\Psi_\gamma = Cx_1^2 + C'x_2^2 + Dx_1x_2 + E$$

$$\begin{aligned}
\frac{1}{2}(\Psi_{11} - \Psi_{22}) &= C - C' = \gamma_1 \\
\Psi_{12} &= D = \gamma_2 \\
\frac{1}{2}(\Psi_{11} + \Psi_{22}) &= C + C' = \kappa
\end{aligned}$$

EXTERNAL PERTURBATIONS

Let's assume for the moment that the perturber does not contribute to the convergence. Then:

$$C = -C' \quad \Rightarrow \quad C = \frac{\gamma_1}{2}$$

$$\Psi_\gamma = \frac{\gamma_1}{2}(x_1^2 - x_2^2) + \gamma_2 x_1 x_2$$

EXTERNAL PERTURBATIONS

If, instead, the perturber only contributes to the convergence but not to the shear:

$$\Psi_\kappa = \frac{\kappa}{2}(x_1^2 + x_2^2)$$

EXTERNAL PERTURBATIONS: EXAMPLE

We embed an isothermal sphere with a core into an external shear:

$$\Psi = \sqrt{x^2 + x_c^2} + \frac{\gamma_1}{2}(x_1^2 - x_2^2) + \gamma_2 x_1 x_2$$

EXTERNAL PERTURBATIONS: EXAMPLE

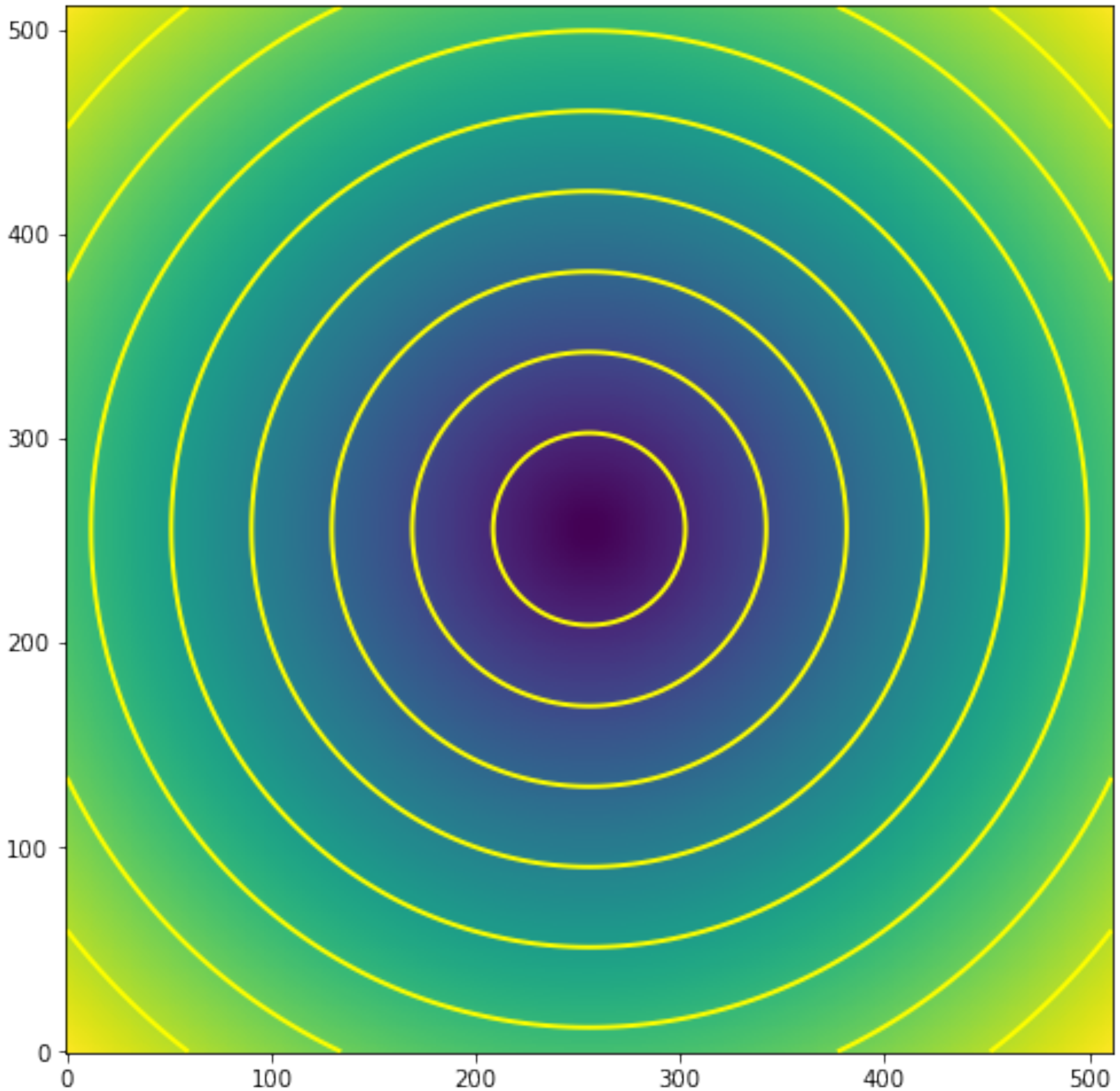
We embed an isothermal sphere with a core into an external shear:

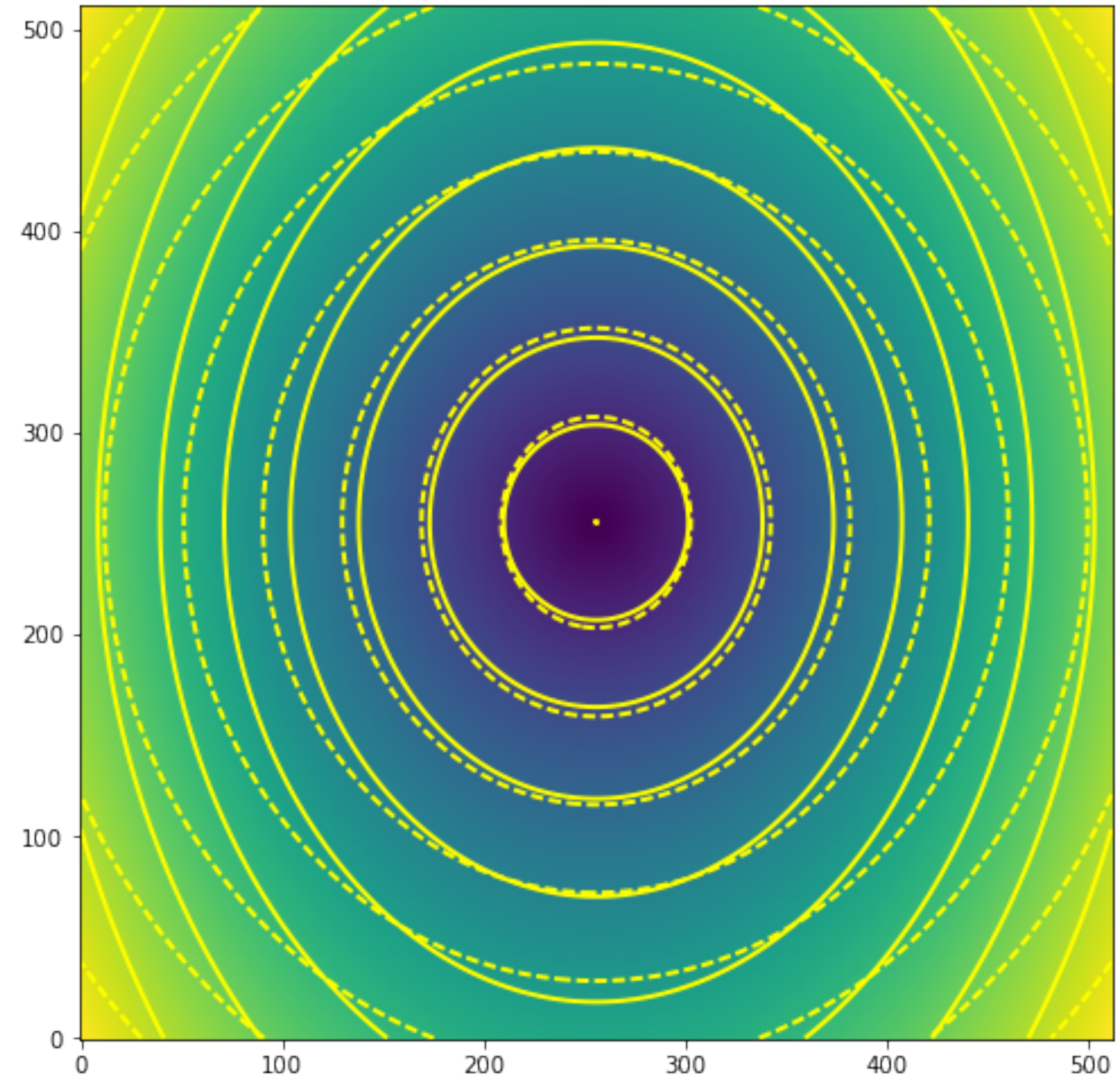
$$\Psi = \sqrt{x^2 + x_c^2} + \frac{\gamma_1}{2}(x_1^2 - x_2^2) + \gamma_2 x_1 x_2$$

$$\begin{aligned} \vec{\nabla}\Psi &= \frac{\vec{x}}{\sqrt{x^2 + x_c^2}} + \begin{pmatrix} \gamma_1 x_1 + \gamma_2 x_2 \\ -\gamma_1 x_2 + \gamma_2 x_1 \end{pmatrix} \\ &= \frac{\vec{x}}{\sqrt{x^2 + x_c^2}} + \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix} \vec{x} \end{aligned}$$

EXTERNAL PERTURBATIONS: EXAMPLE

Potential



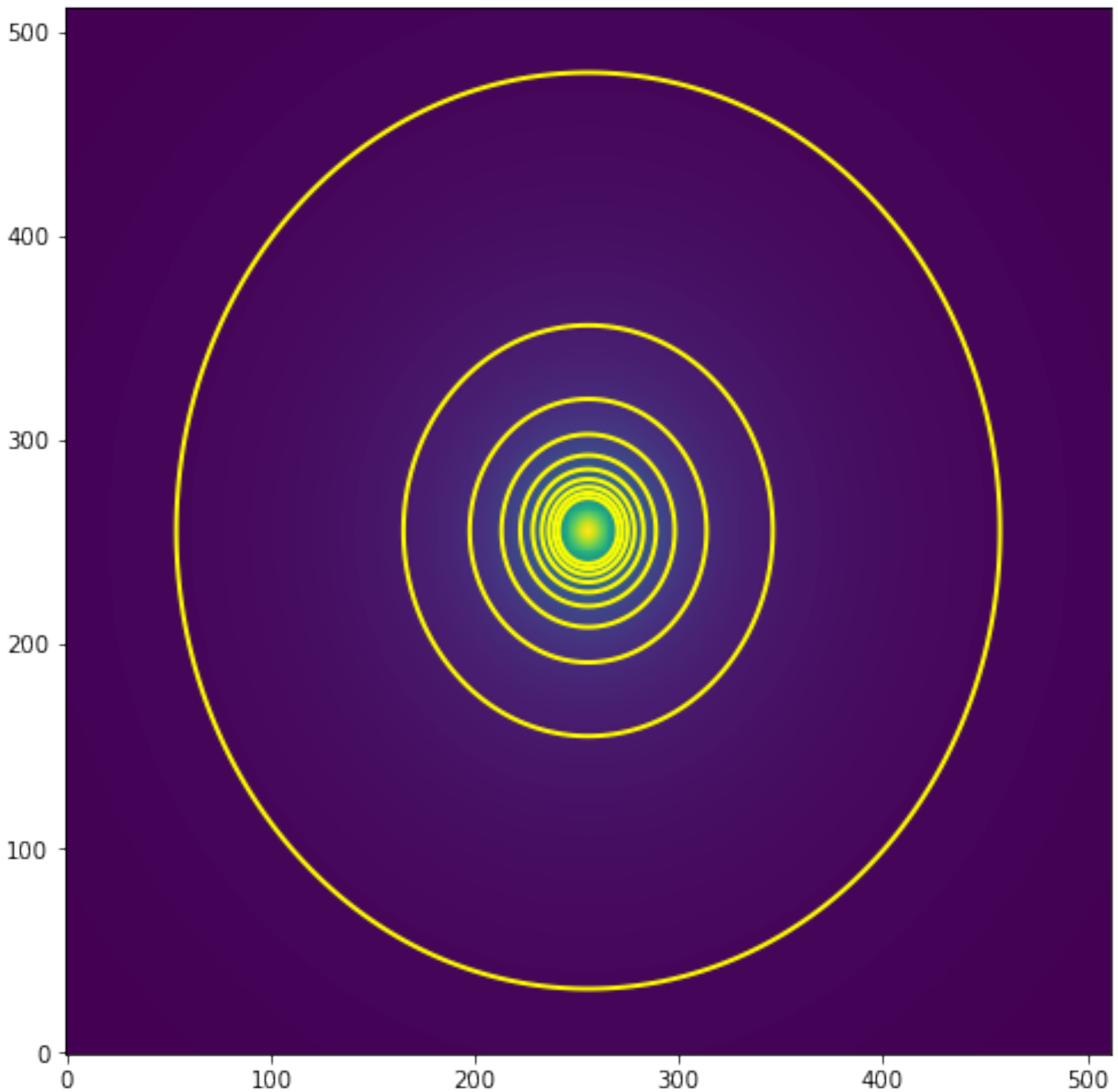


NIS

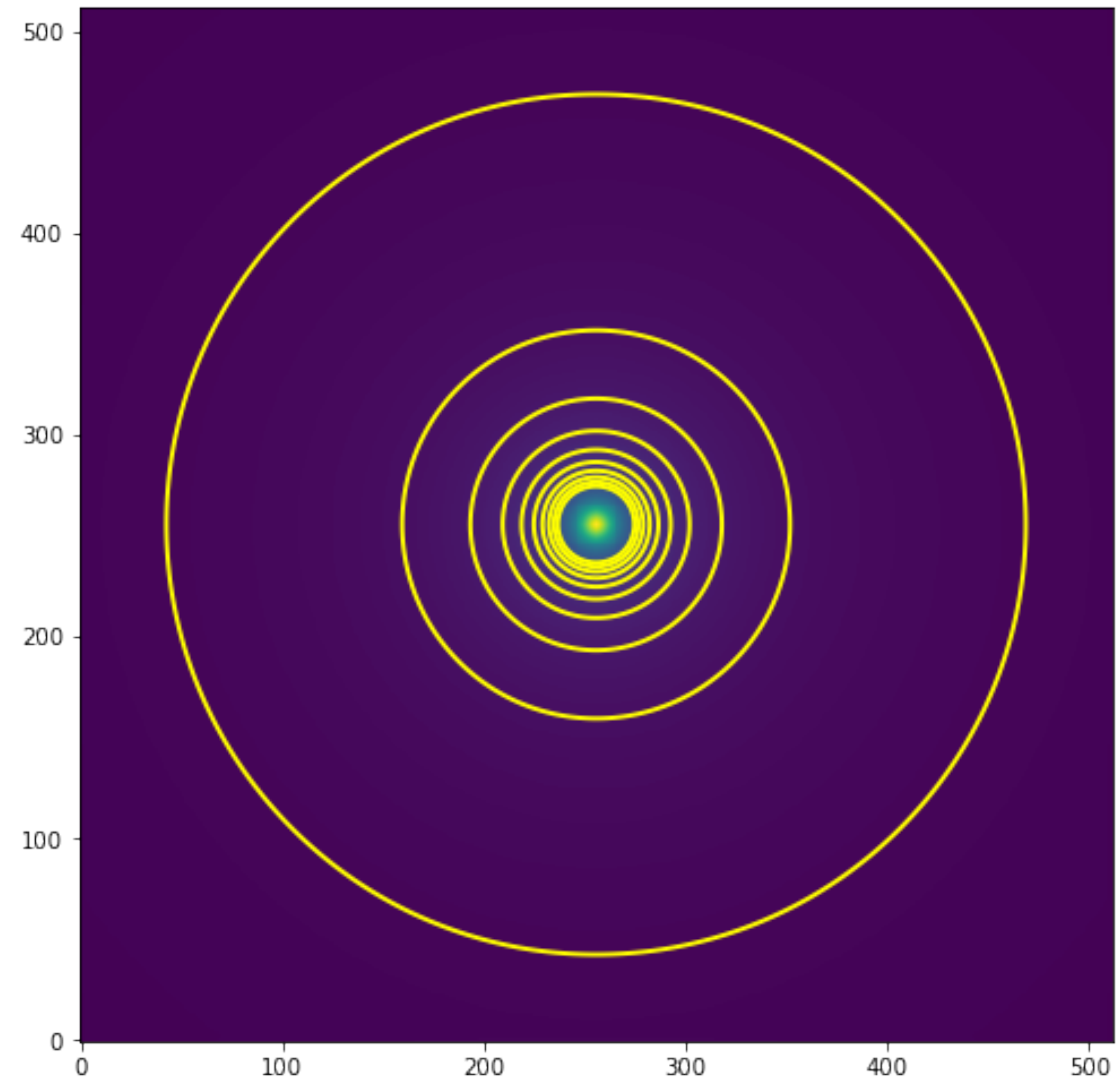
NIS+ext. shear (γ =0.1) vs f=0.9

EXTERNAL PERTURBATIONS: EXAMPLE

Convergence



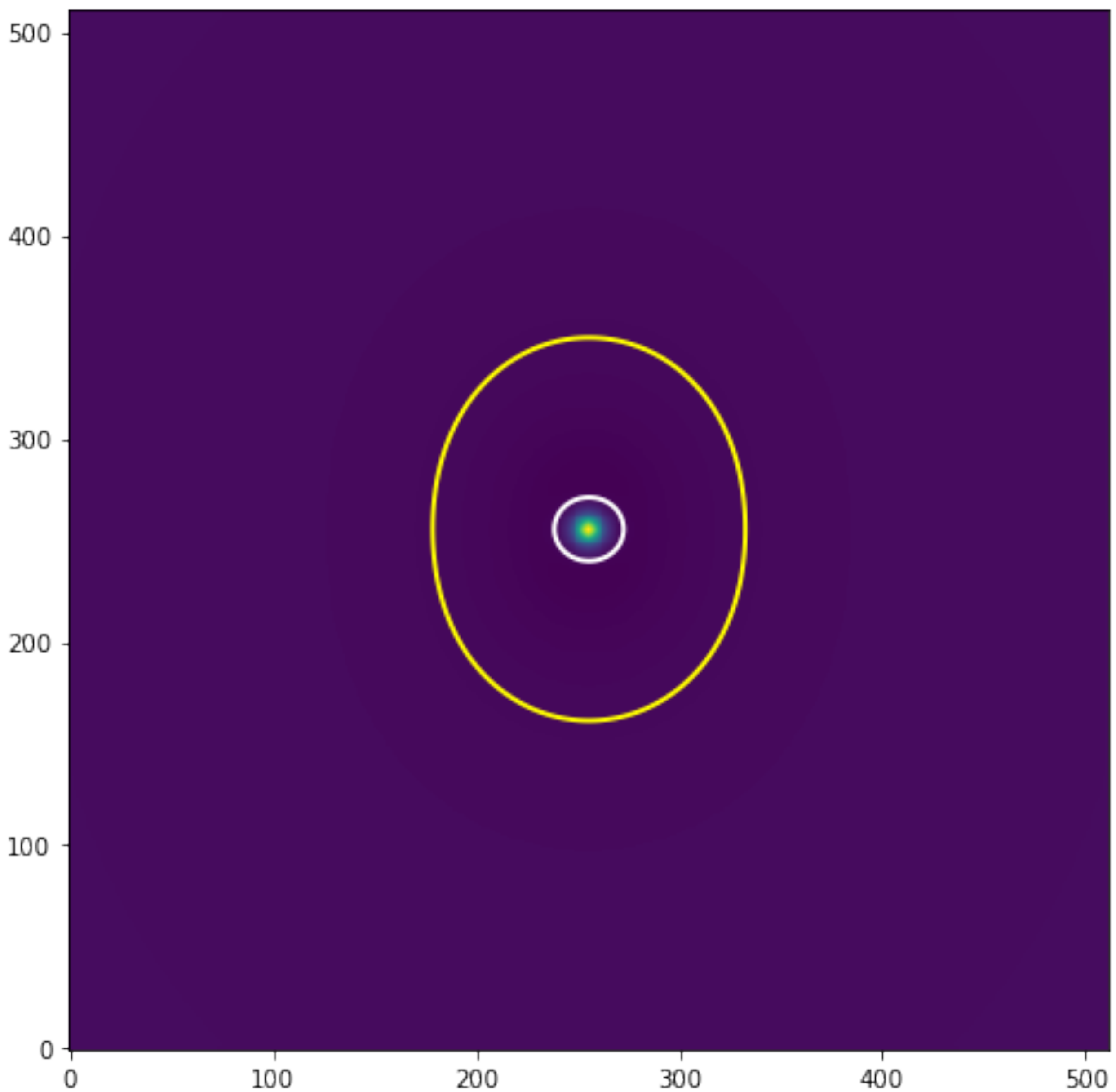
NIE (f=0.9)

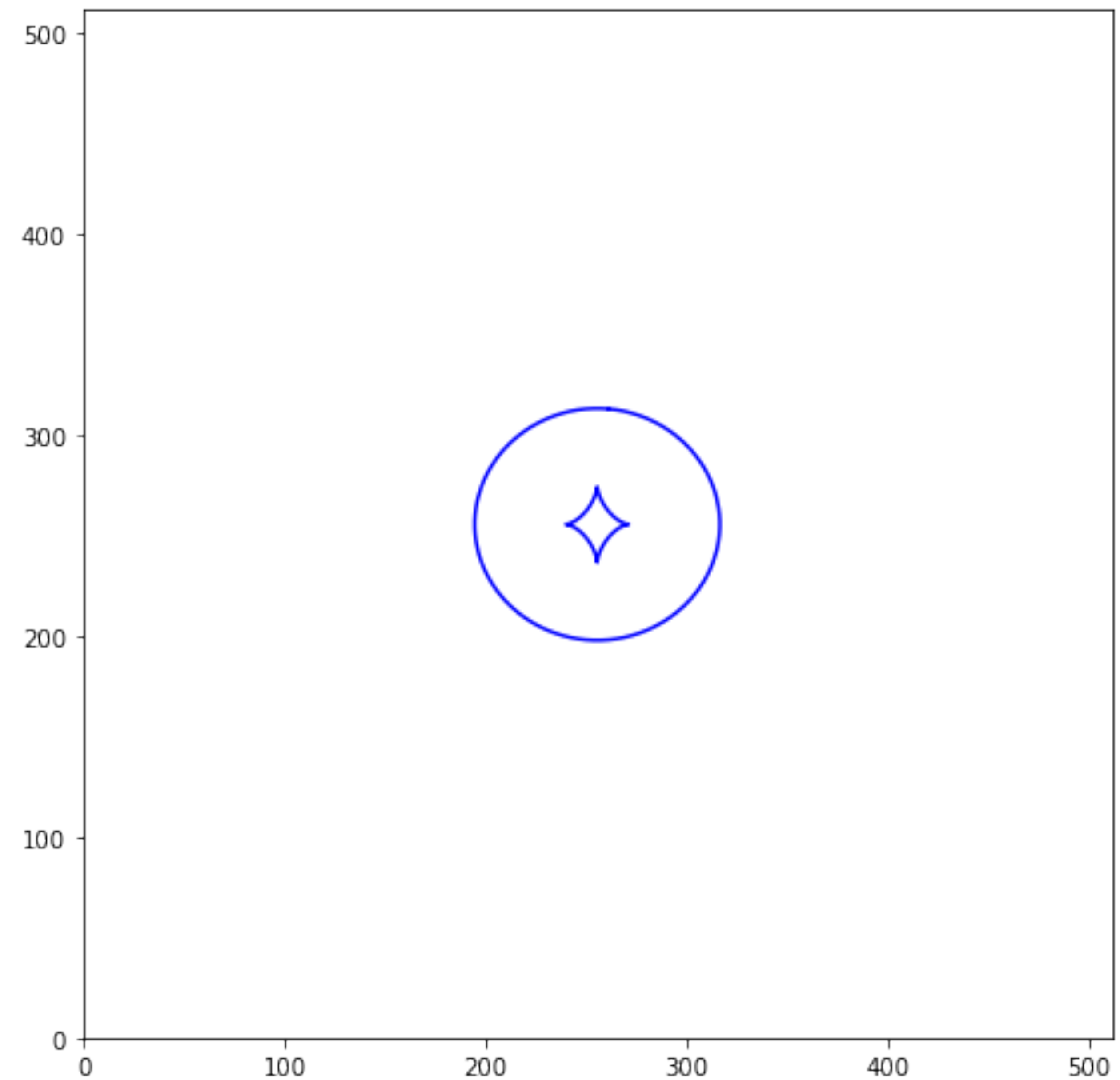


NIS+ext. shear (γ =0.1)

EXTERNAL PERTURBATIONS: EXAMPLE

Critical lines

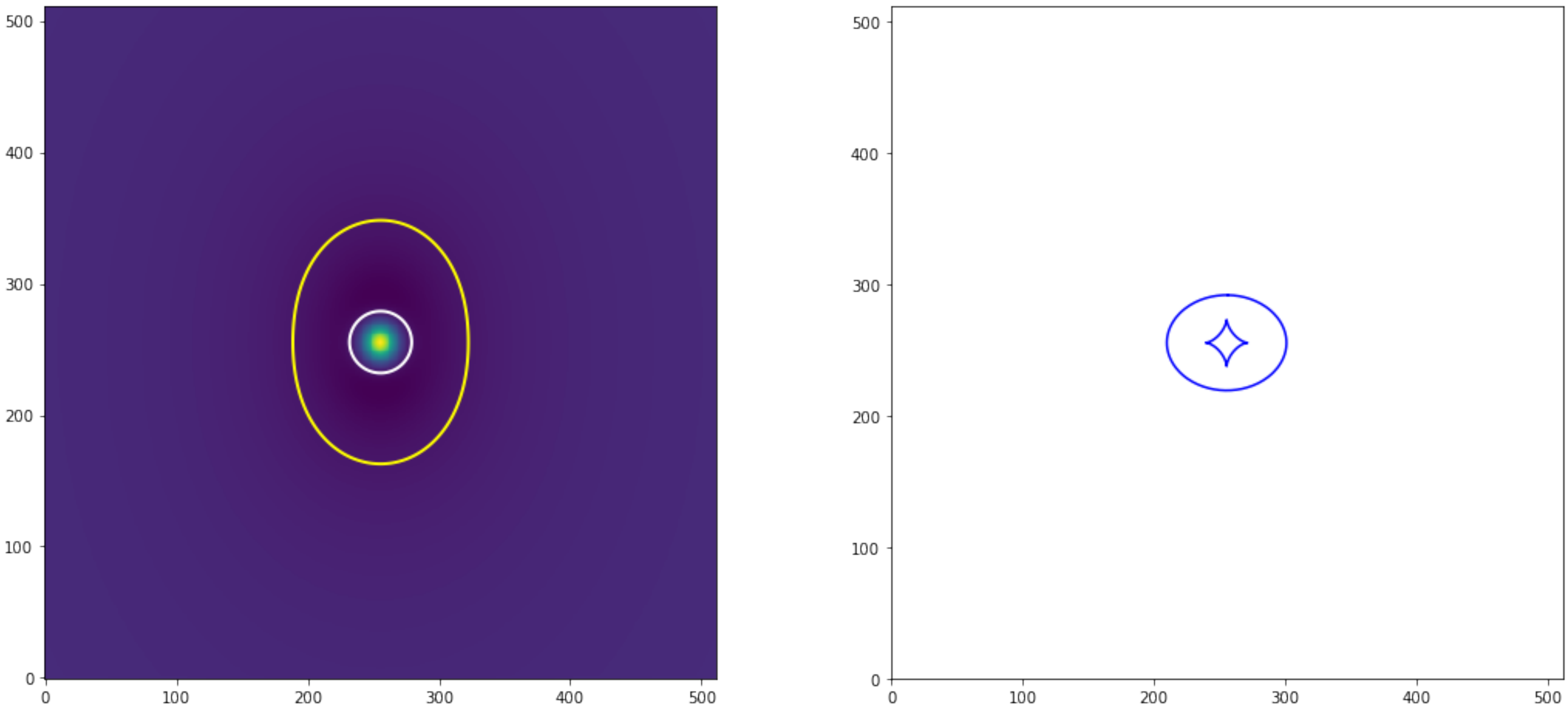




NIS+ext. shear (γ =0.1)

EXTERNAL PERTURBATIONS: EXAMPLE

Critical lines



NIE (f=0.9)

EXTERNAL PERTURBATIONS: EXAMPLE

In the case of a constant sheet of matter:

$$\vec{\alpha} = \vec{\nabla}\Psi_\kappa = \kappa\vec{x}$$

$$\vec{y} = \vec{x} - \vec{\alpha} = \vec{x}(1-\kappa)$$

A sheet of constant density will change the focussing properties of the lens. For example, if $\kappa=1$…

SUBSTRUCTURES

Lenses usually contain substructures. How do we model them?

We simply sum their potential (or the deflection field) to that of the main lens!

IMAGE SIMULATIONS