GRAVITATIONAL LENSING

9 – POINT MASS: MICROLENSING LIGHT CURVE

Massimo Meneghetti

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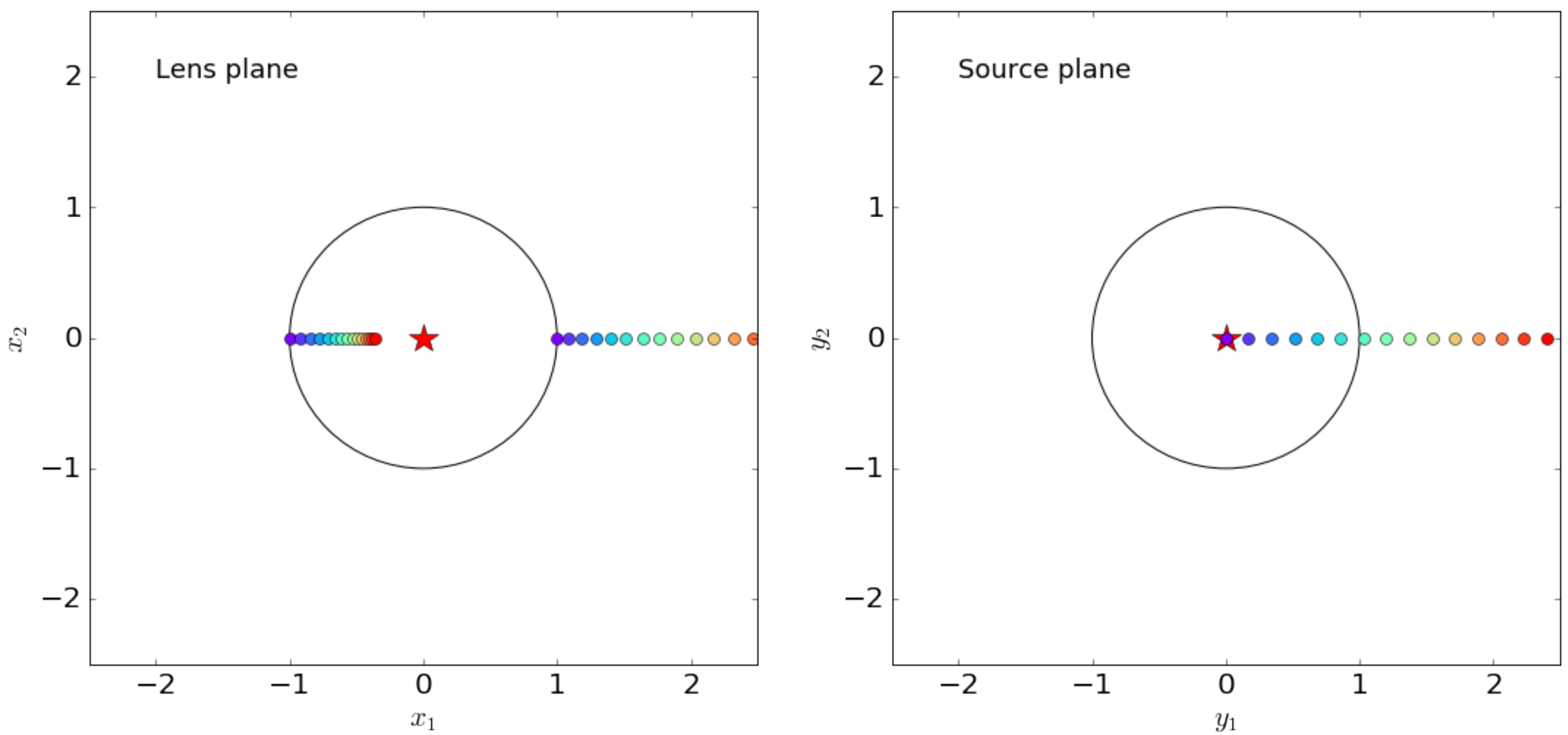
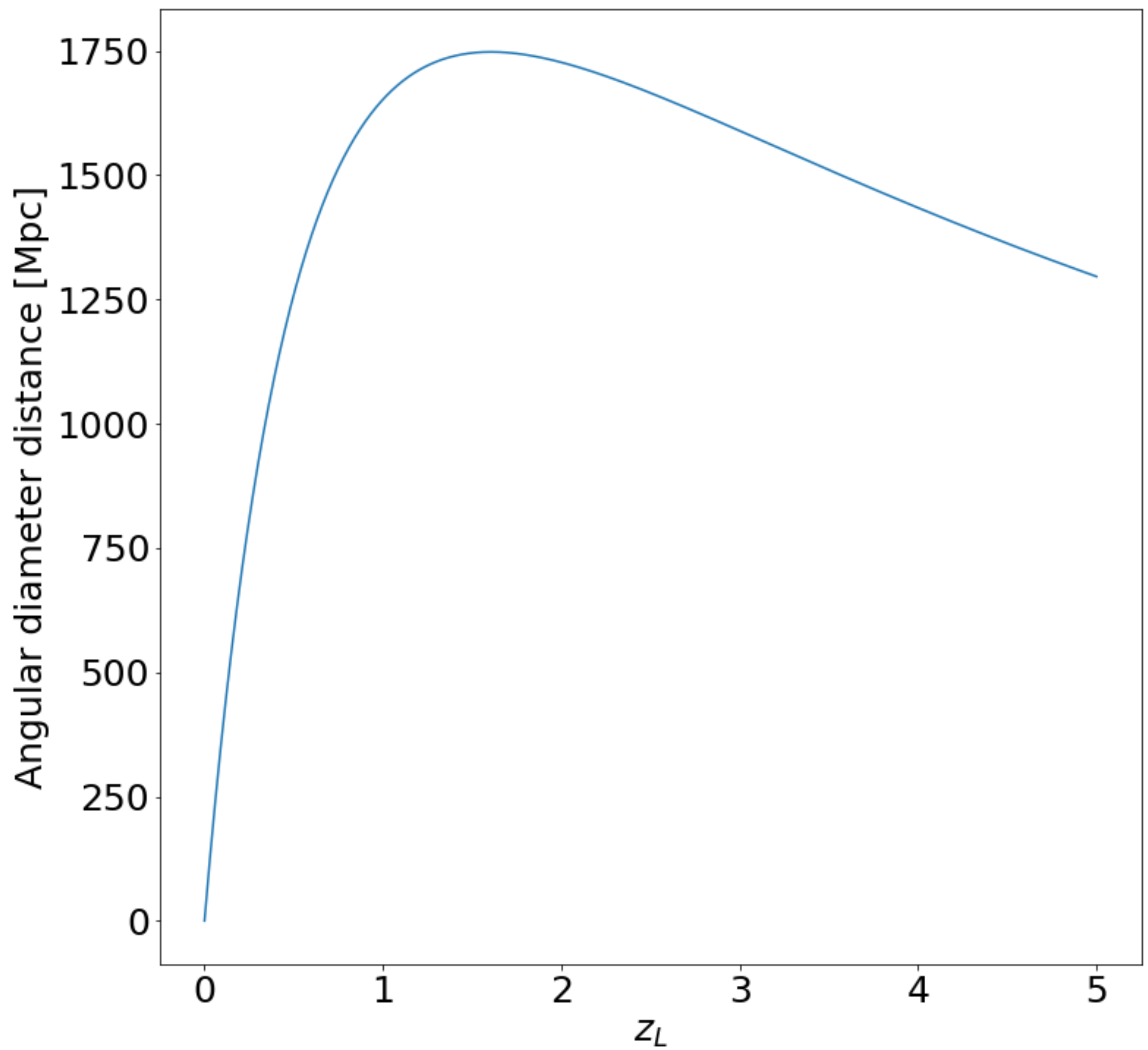
BRIEF SUMMARY

- Lens equation for point-mass lens: $y = x - \dfrac{1}{x}$
- Point-mass always produces two images: $x_{\pm} = \dfrac{1}{2}\left[y \pm \sqrt{y^2 + 4}\right]$
- The two images are one outside and one inside the Einstein ring, which is also the only critical line of this kind of lens: $\theta_E = \sqrt{\dfrac{4GM}{c^2}\dfrac{D_{LS}}{D_L D_S}}$



A FEW WORDS ON ANGULAR DIAMETER DISTANCES...

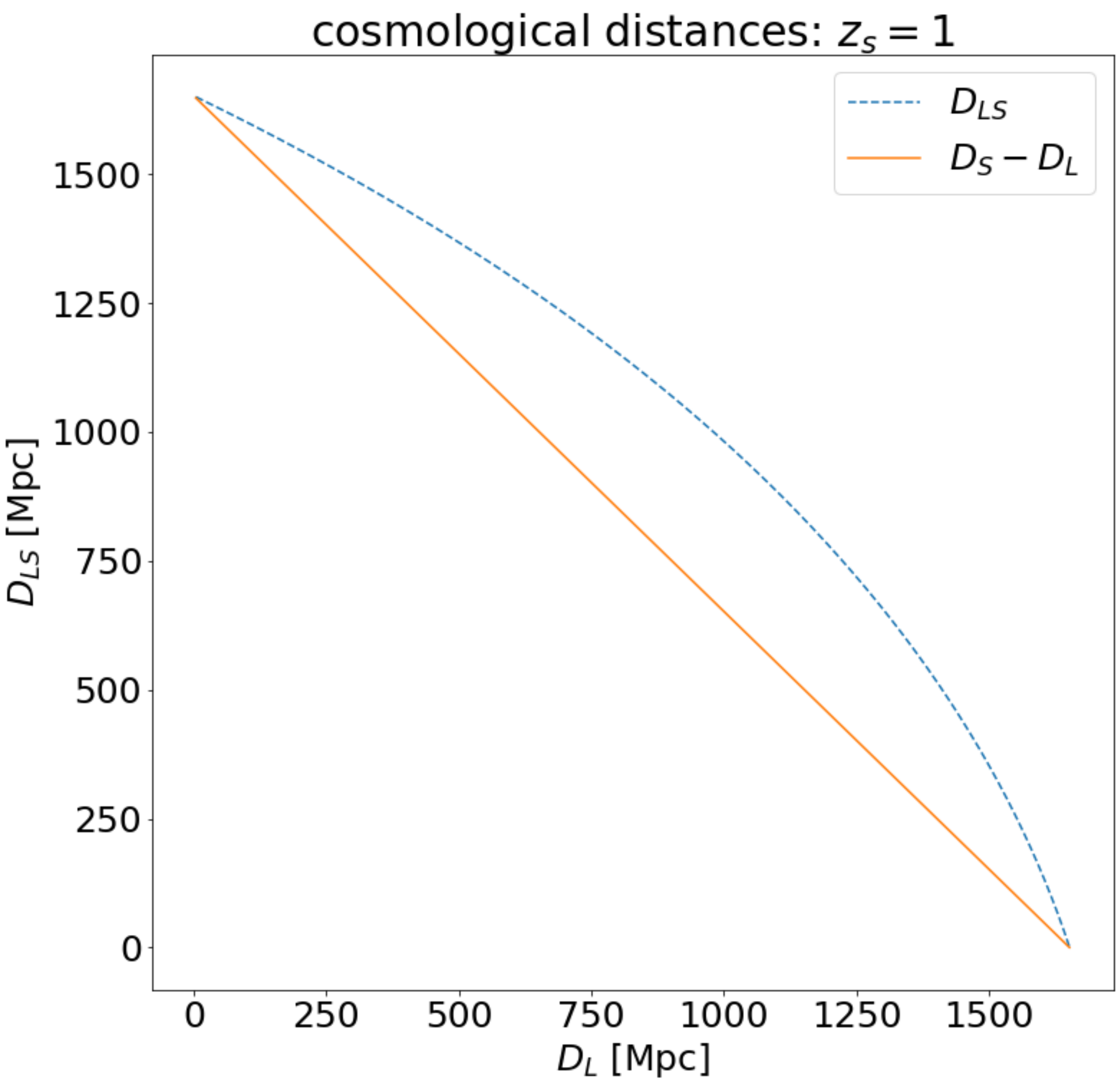
$$D_A(z) = \frac{c}{1+z}\int_0^z \frac{dz}{H(z)} \qquad H^2(z) = H_0^2[\Omega_m(z)(1+z)^3 + \Omega_{DE}(1+z)^{3(1+w)}]$$

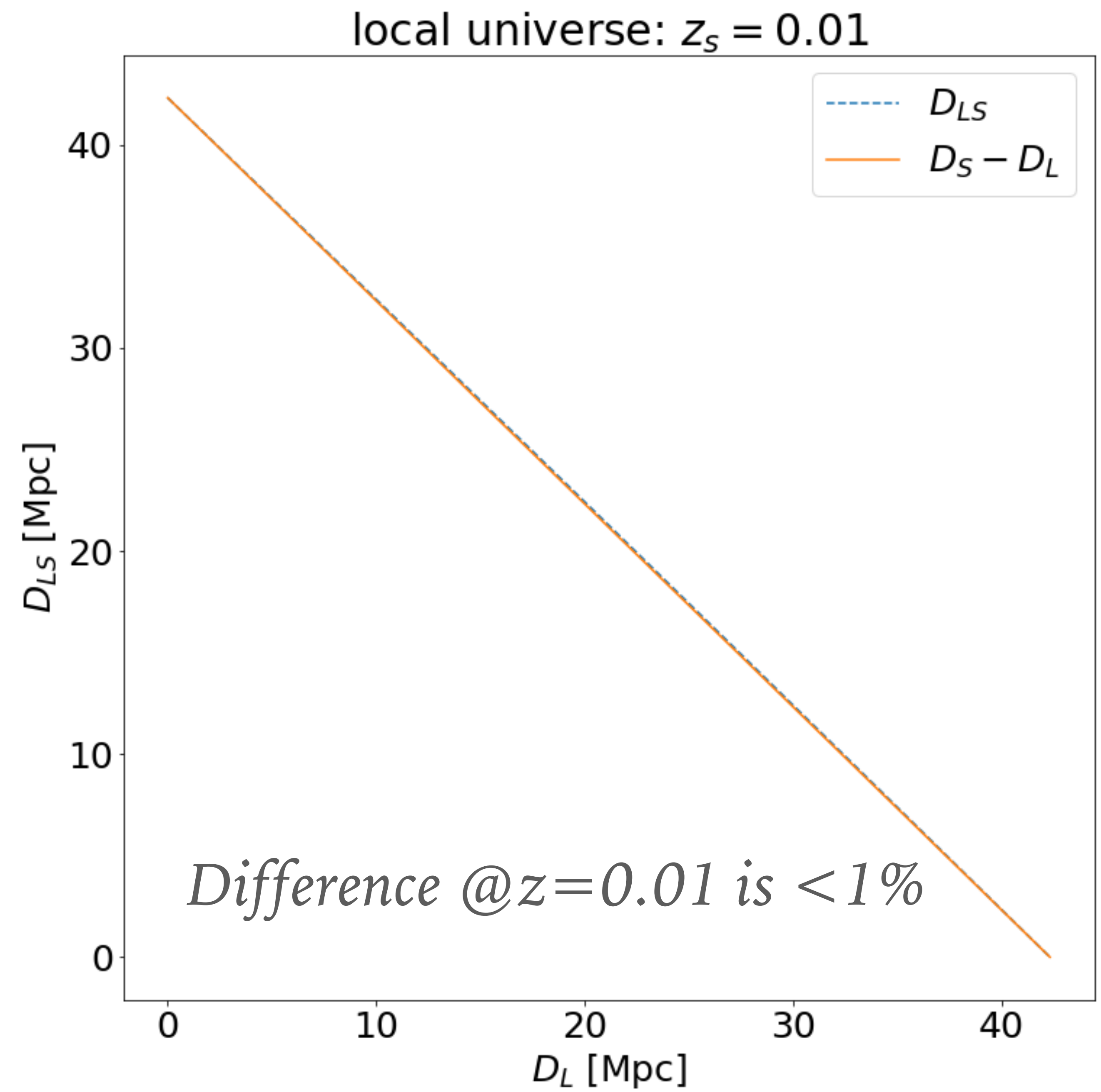


A FEW WORDS ON ANGULAR DIAMETER DISTANCES…

On cosmological scales $D_{LS} \neq D_S - D_L$

In the local universe, we can instead assume that $D_{LS} \simeq D_S - D_L$





THEREFORE…

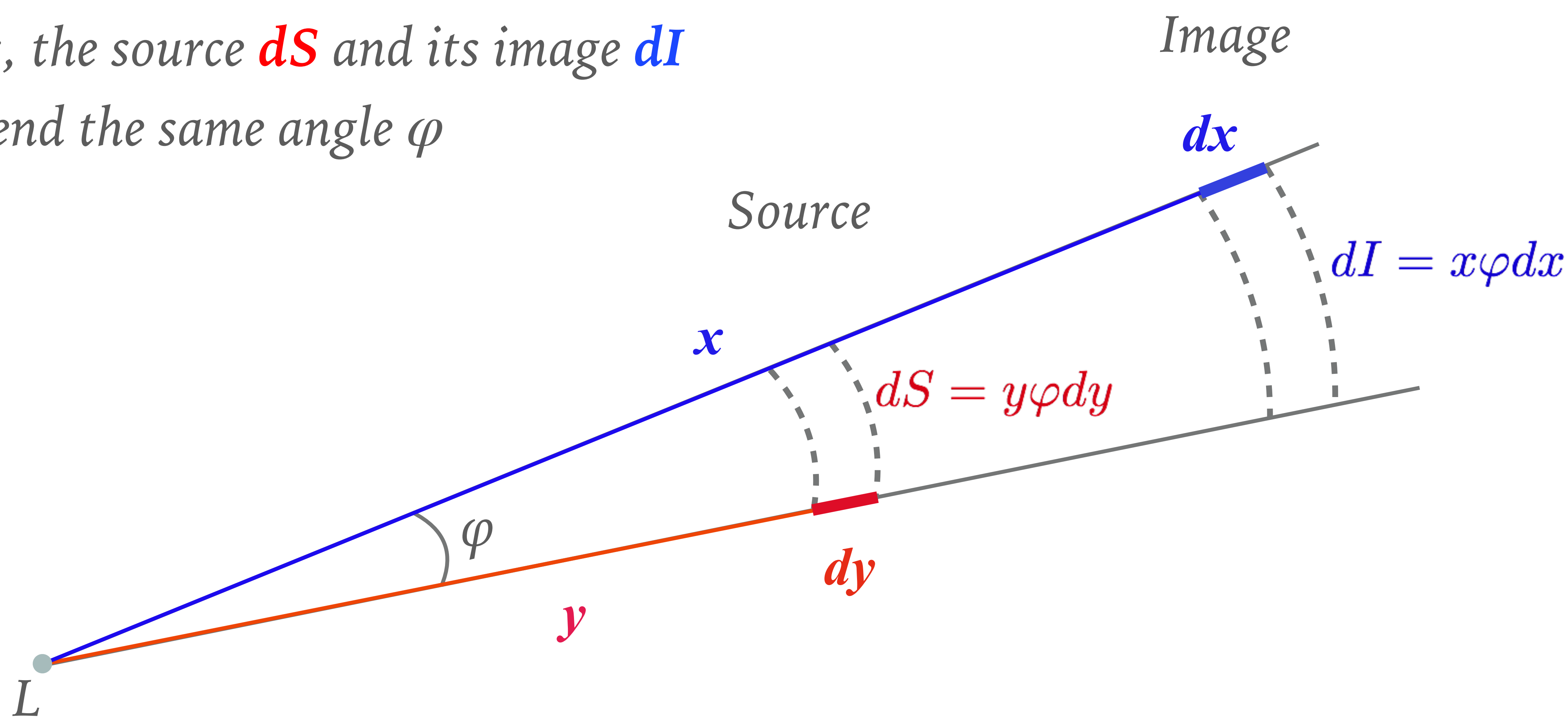
The Einstein radius can be written as $\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S}} = \sqrt{\frac{4GM}{c^2}\frac{D_S - D_L}{D_L D_S}}$

The quantity $\frac{D_S - D_L}{D_L D_S} = \frac{1}{D_L} - \frac{1}{D_S} = \pi_{rel}$ *is called **relative parallax.***

MAGNIFICATION

Remeber: $\overrightarrow{x}, \overrightarrow{y}, \overrightarrow{\alpha}(\overrightarrow{x})$ *are parallel!*

Thus, the source ***dS*** *and its image* ***dI*** *subtend the same angle* φ



The figure shows that $\mu(x) = \frac{x}{y}\frac{dx}{dy}$ *or* $\det A(x) = \frac{y}{x}\frac{dy}{dx}$

SOURCE MAGNIFICATION

Let's compute now the source magnification. This is the sum of the magnifications of the two images

$$x_\pm = \frac{1}{2}\left[y \pm \sqrt{y^2+4}\right]$$

$$\frac{x}{y} = \frac{1}{2}\left(1 \pm \frac{y^2+4}{y}\right)$$

$$\frac{dx}{dy} = \frac{1}{2}\left(1 \pm \frac{y}{y^2+4}\right)$$

Thus, the magnifications at the two image positions are

$$\begin{aligned}
\mu_\pm(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \\
&= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right) \\
&= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right)
\end{aligned}$$

SOURCE MAGNIFICATION

The total magnification is obtained by summing the magnifications of the images:

$$\begin{aligned}\mu_{\pm}(y) &= \frac{1}{4}\left(1\pm\frac{\sqrt{y^2+4}}{y}\right)\left(1\pm\frac{y}{\sqrt{y^2+4}}\right)\\ &= \frac{1}{4}\left(1\pm\frac{\sqrt{y^2+4}}{y}\pm\frac{y}{\sqrt{y^2+4}}+1\right)\\ &= \frac{1}{4}\left(2\pm\frac{2y^2+4}{y\sqrt{y^2+4}}\right)=\frac{1}{2}\left(1\pm\frac{y^2+2}{y\sqrt{y^2+4}}\right)\end{aligned}$$

$$\Rightarrow \quad \mu(y)=\mu_+(y)+|\mu_-(y)|=\frac{y^2+2}{y\sqrt{y^2+4}}$$

The sum of the signed magnification is one!

We can take a series expansion of the magnification to see that $\mu \propto 1+2/y^4$ for $y\to\infty$

Thus, the magnification drops quickly as the source moves away from the lens!

SOURCE MAGNIFICATION

In addition:

$$\left|\frac{\mu_+}{\mu_-}\right| = \frac{1+\frac{y^2+2}{y\sqrt{y^2+4}}}{\frac{y^2+2}{y\sqrt{y^2+4}}-1} = \frac{y^2+2+y\sqrt{y^2+4}}{y^2+2-y\sqrt{y^2+4}}$$

$$\frac{1}{2}\left(y+\sqrt{y^2+4}\right)^2 = y^2+2+y\sqrt{y^2+4}$$

$$\frac{1}{2}\left(y-\sqrt{y^2+4}\right)^2 = y^2+2-y\sqrt{y^2+4}$$

$$\left|\frac{\mu_+}{\mu_-}\right| = \left(\frac{y+\sqrt{y^2+4}}{y-\sqrt{y^2+4}}\right)^2 = \left(\frac{x_+}{x_-}\right)^2.$$

Laurent series expansion at infinity:

$$\left|\frac{\mu_+}{\mu_-}\right| \propto y^4$$

As we move the source away from the lens, the image in x_+ dominates the flux budget very soon.

$$\lim_{y\to\infty}\mu_- = 0$$

$$\lim_{y\to\infty}\mu_+ = 1$$

A SOURCE ON THE EINSTEIN RING

For a source on the Einstein ring:

$$x_\pm = \frac{1}{2}\left[y \pm \sqrt{y^2+4}\right] \rightarrow y = 1 \rightarrow x_\pm = \frac{1}{2}(1 \pm \sqrt{5}) \Rightarrow \mu_\pm = \left[1 - \left(\frac{2}{1 \pm \sqrt{5}}\right)^4\right]^{-1}$$

Therefore $\mu = \mu_+ + |\mu_-| = 1.17 + 0.17 = 1.34$:

$$\Delta m = -2.5 \log_{10} \mu \simeq 0.3$$

Given how quickly the magnification drops by moving the source away from the lens, we can assume that only sources within the Einstein radius are magnified in a significant way.

For this reason, the circle within the Einstein radius is assumed to be the cross section for microlensing.

SIZE OF THE EINSTEIN RADIUS

$$\theta_E \equiv \sqrt{\frac{4GM}{c^2}\frac{D_{\rm LS}}{D_{\rm L}D_{\rm S}}} \qquad\qquad D \equiv \frac{D_{\rm L}D_{\rm S}}{D_{\rm LS}}$$

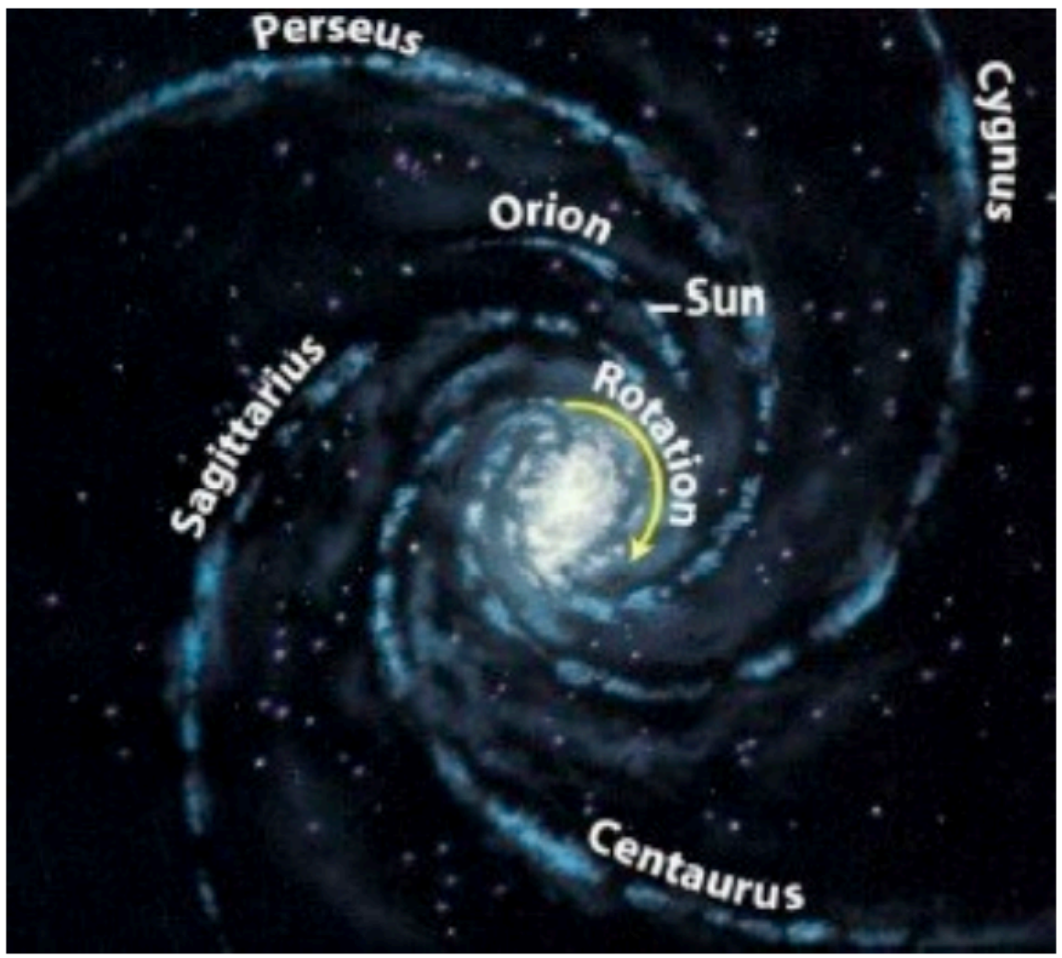
$$\begin{aligned}\theta_E &\approx (10^{-3})''\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{D}{10\mathrm{kpc}}\right)^{-1/2},\\ &\approx 1''\left(\frac{M}{10^{12}M_\odot}\right)^{1/2}\left(\frac{D}{\mathrm{Gpc}}\right)^{-1/2},\end{aligned}$$

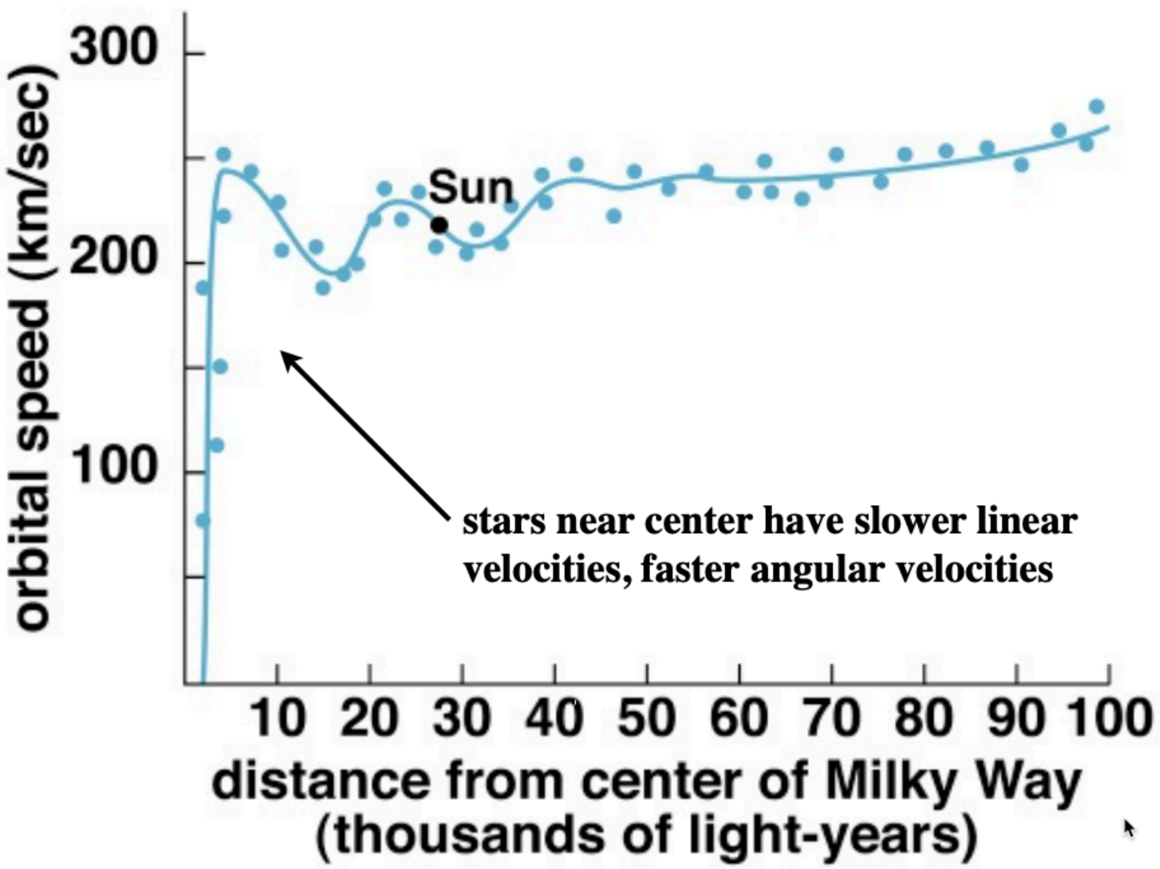
For a star like the sun within the MW, the Einstein radius is of the order of milli-arcseconds!

MICROLENSING OBSERVABLES?

- typical Einstein radii for lenses in the MW are $\sim$1 mas
- thus, the image separation is too small to resolve the images
- magnification is small also for relatively close pairs of lenses and sources
- how to detect a microlensing event?

DIFFERENTIAL ROTATION





MICROLENSING LIGHT CURVE

Assume a linear trajectory of the source relative to the lens, with impact parameter y_0

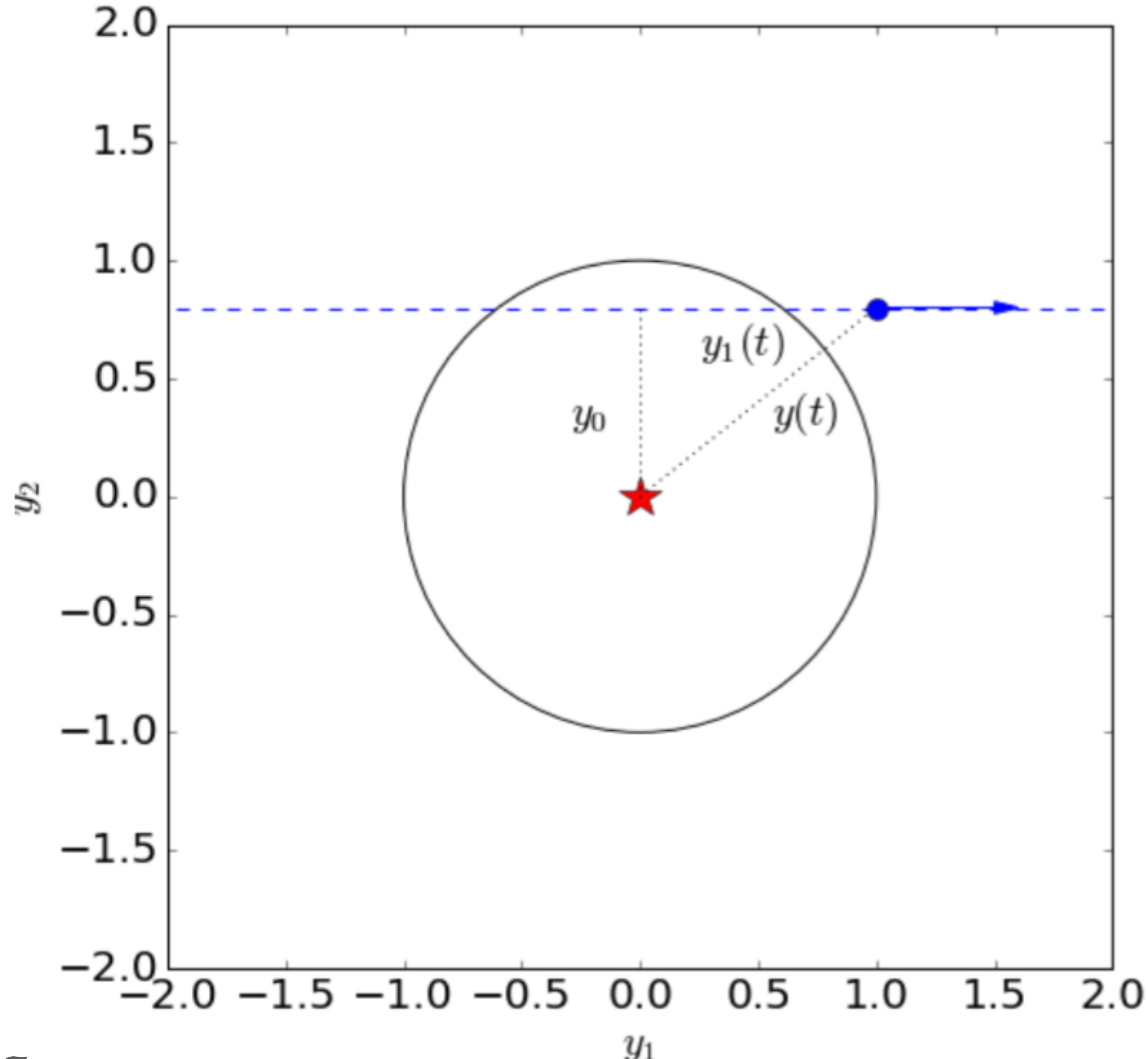
Assume also constant transverse velocity v:

$$y_1(t) = \frac{v(t - t_0)}{D_L \theta_E}$$

We can define a characteristic time of the event:

$$t_E = \frac{D_L \theta_E}{v} = \frac{\theta_E}{\mu_{rel}}$$

This is the ***Einstein radius crossing time***

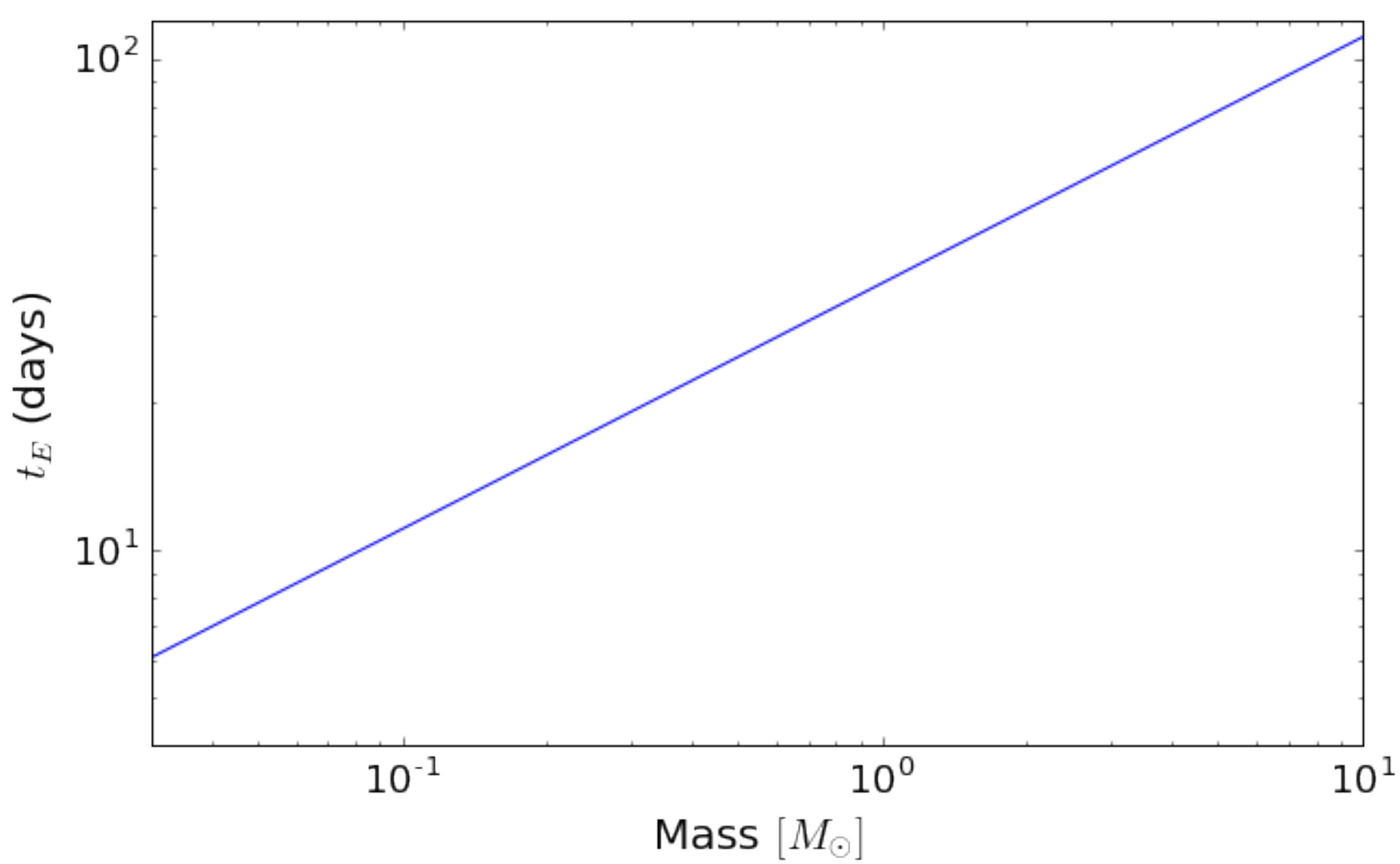


MICROLENSING LIGHT CURVE

Given the definition of Einstein radius $$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S}}$$

The order of magnitude of the t_E is

$$t_E \approx 19 \text{ days} \sqrt{4\frac{D_L}{D_S}\left(1-\frac{D_L}{D_S}\right)}\left(\frac{D_S}{8\text{kpc}}\right)^{1/2}\left(\frac{M}{0.3M_\odot}\right)^{1/2}\left(\frac{v}{200\text{km}/s}\right)^{-1}$$



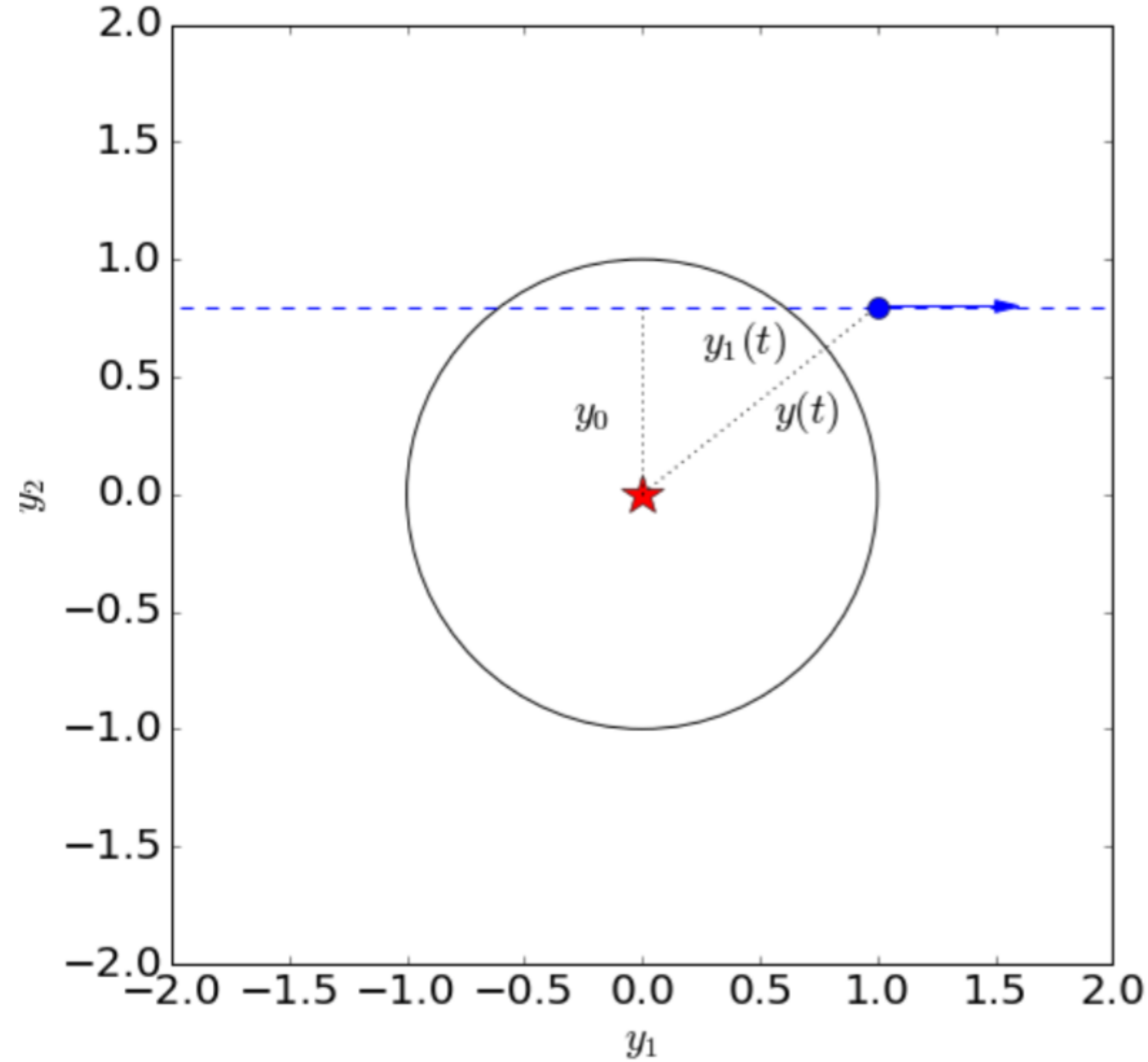
MICROLENSING LIGHT CURVE

We obtain

$$y_1(t) = \frac{v(t - t_0)}{D_L \theta_E} = \frac{t - t_0}{t_E}$$

Thus:

$$y(t) = \sqrt{y_0^2 + y_1^2(t)} = \sqrt{y_0^2 + \frac{(t - t_0)^2}{t_E^2}}$$



MICROLENSING LIGHT CURVE

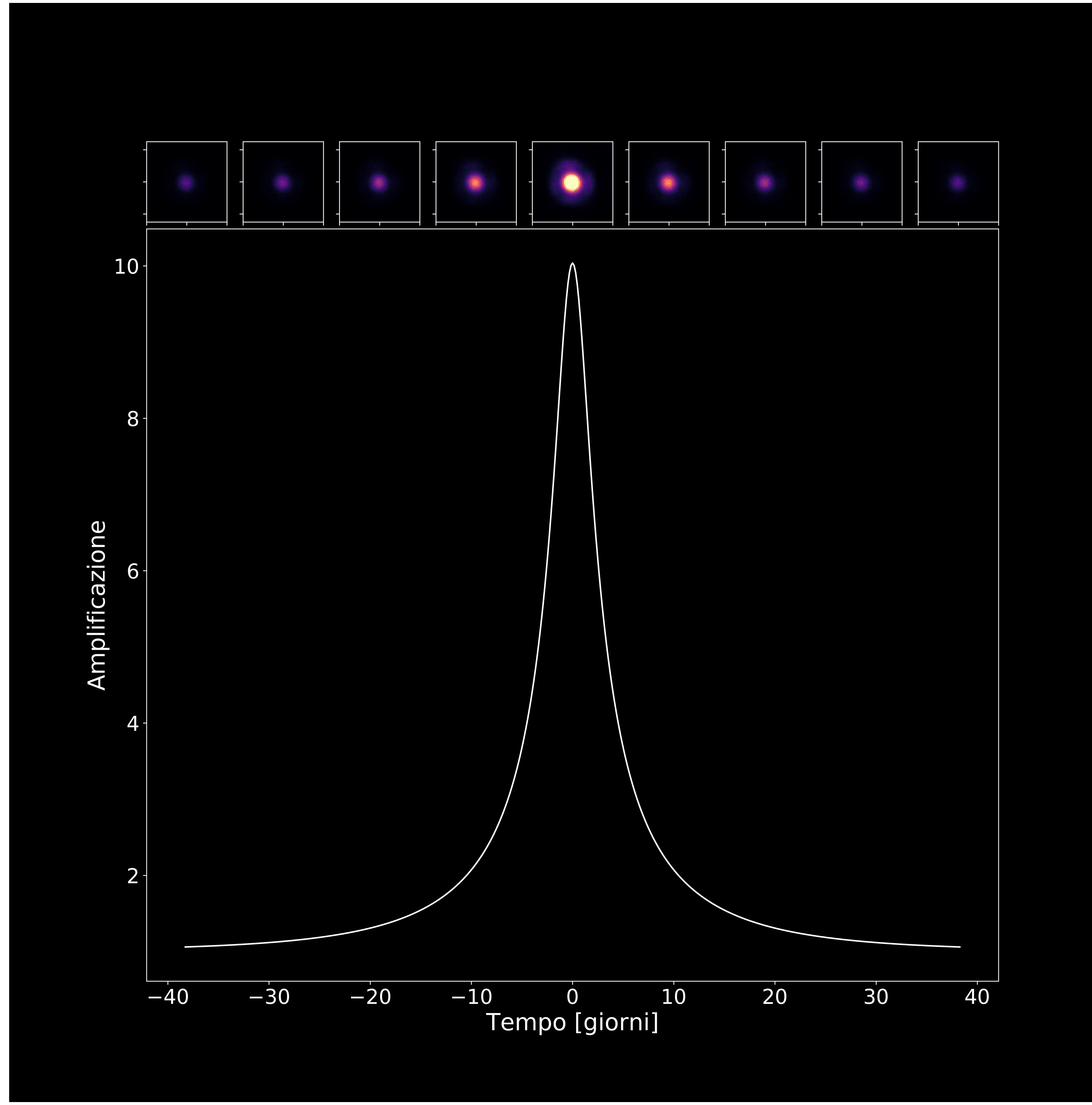
Combine

$$y(t) = \sqrt{y_0^2 + y_1^2(t)} = \sqrt{y_0^2 + \frac{(t - t_0)^2}{t_E^2}}$$

with

$$\mu(y) = \frac{y^2 + 2}{y\sqrt{y^2 + 4}}$$

And obtain $\mu(t)$

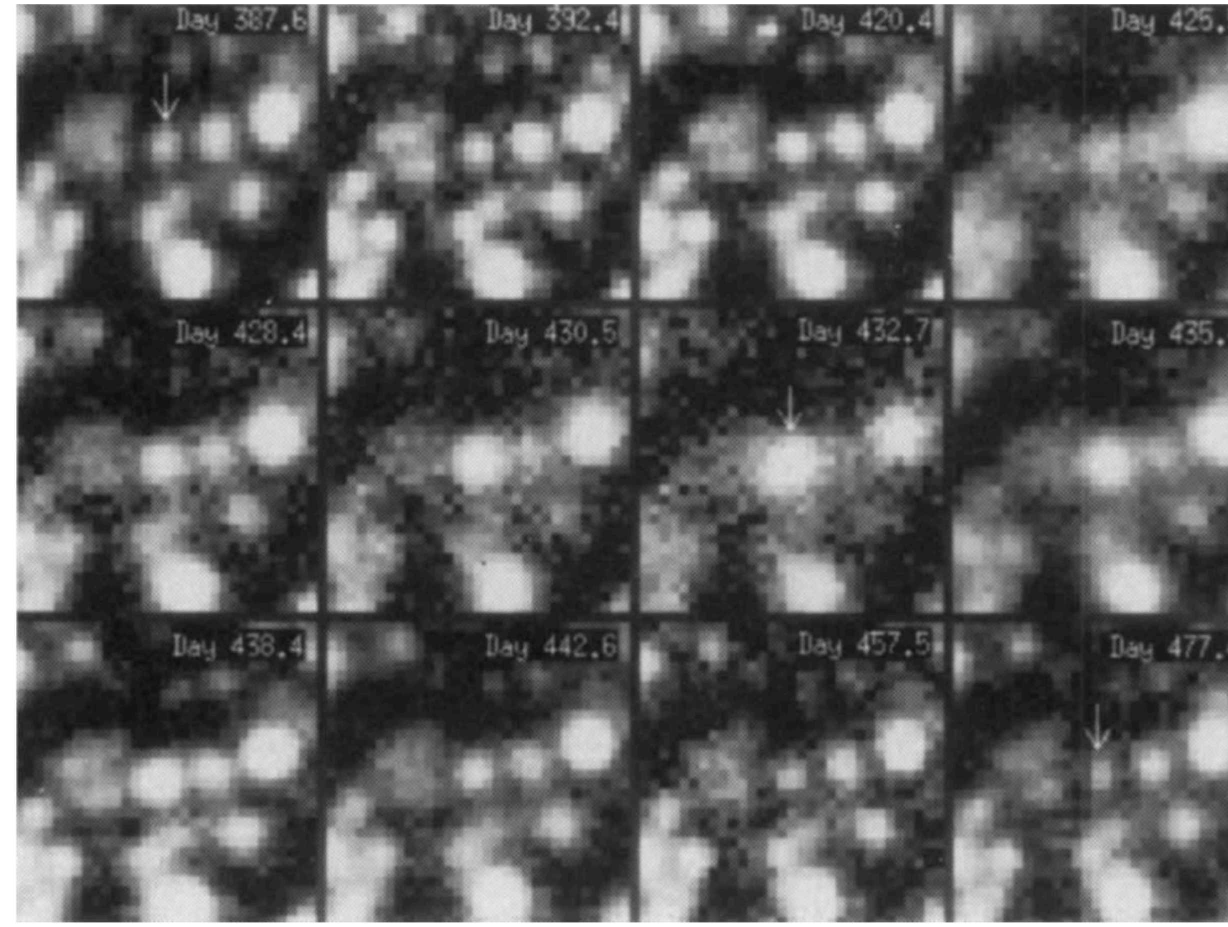


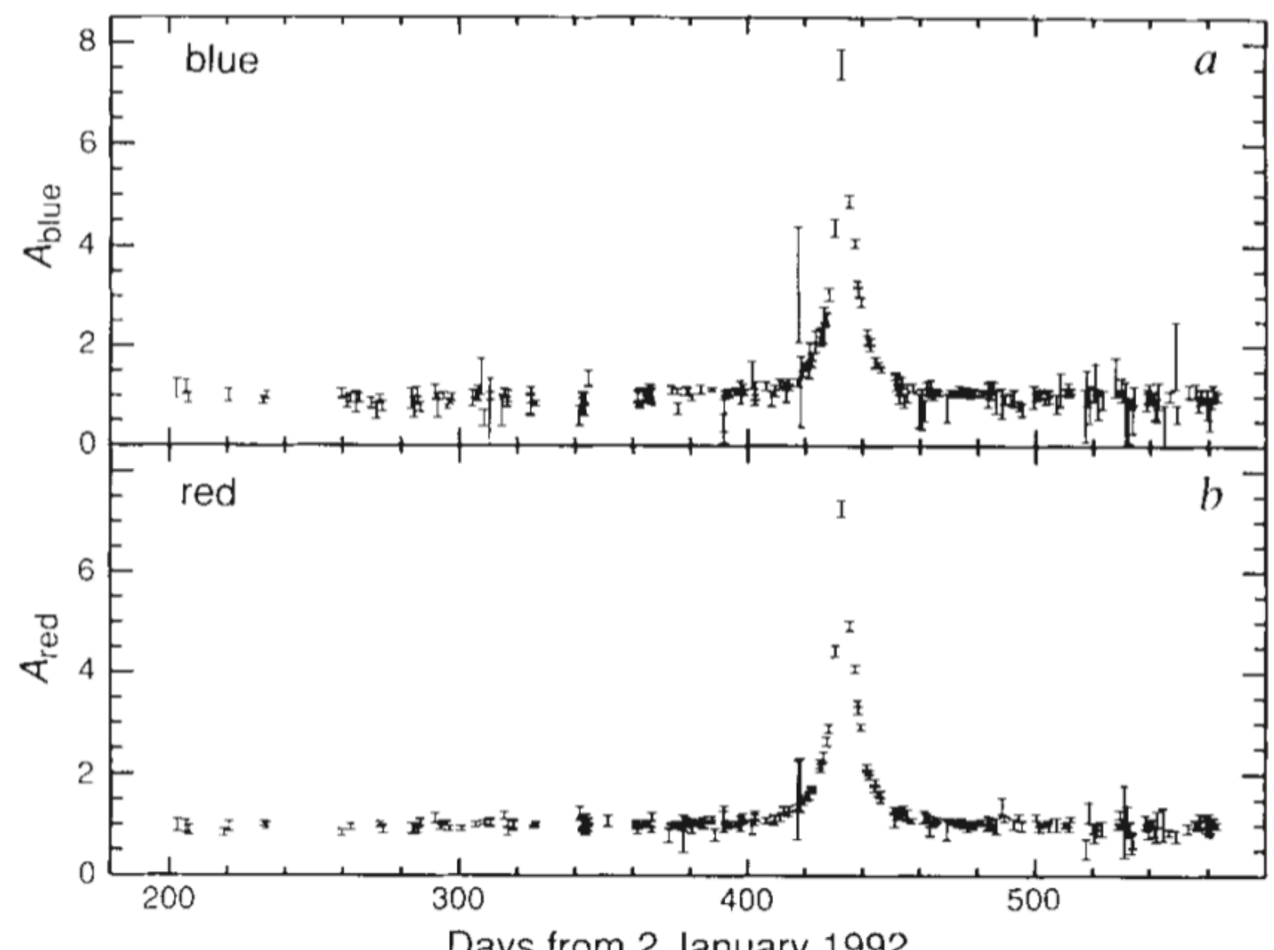
Possible gravitational microlensing of a star in the Large Magellanic Cloud

C. Alcock*†, C. W. Akerlof†¶, R. A. Allsman*, T. S. Axelrod*, D. P. Bennett*†, S. Chan‡, K. H. Cook*†, K. C. Freeman‡, K. Griest†‖, S. L. Marshall†§, H-S. Park*, S. Perlmutter†, B. A. Peterson‡, M. R. Pratt†§, P. J. Quinn‡, A. W. Rodgers‡, C. W. Stubbs†§ & W. Sutherland†

* Lawrence Livermore National Laboratory, Livermore, California 94550, USA
† Center for Particle Astrophysics, University of California, Berkeley, California 94720, USA
‡ Mt Stromlo and Siding Spring Observatories, Australian National University, Weston, ACT 2611, Australia
§ Department of Physics, University of California, Santa Barbara, California 93106, USA
‖ Department of Physics, University of California, San Diego, California 92039, USA
¶ Department of Physics, University of Michigan, Ann Arbor, Michigan 48109, USA

THERE is now abundant evidence for the presence of large quantities of unseen matter surrounding normal galaxies, including our own[1,2]. The nature of this 'dark matter' is unknown, except that it cannot be made of normal stars, dust or gas, as they would be easily detected. Exotic particles such as axions, massive neutrinos or other weakly interacting massive particles (collectively known as WIMPs) have been proposed[3,4], but have yet to be detected. A less exotic alternative is normal matter in the form of bodies with masses ranging from that of a large planet to a few solar masses. Such objects, known collectively as massive compact halo objects[5] (MACHOs), might be brown dwarfs or 'jupiters' (bodies too small to produce their own energy by fusion), neutron stars, old white dwarfs or black holes. Paczynski[6] suggested that MACHOs might act as gravitational microlenses, temporarily amplifying the apparent brightness of background stars in nearby galaxies. We are conducting a microlensing experiment to determine whether the dark matter halo of our Galaxy is made up of MACHOs. Here we report a candidate for such a microlensing event, detected by monitoring the light curves of 1.8 million stars in the Large Magellanic Cloud for one year. The light curve shows no variation for most of the year of data taking, and an upward excursion lasting over 1 month, with a maximum increase of ~2 mag. The most probable lens mass, inferred from the duration of the candidate lensing event, is ~0.1 solar mass.





Alcock et al. 1993, Nature, 365, 621

Please, check out notebook 9_2020