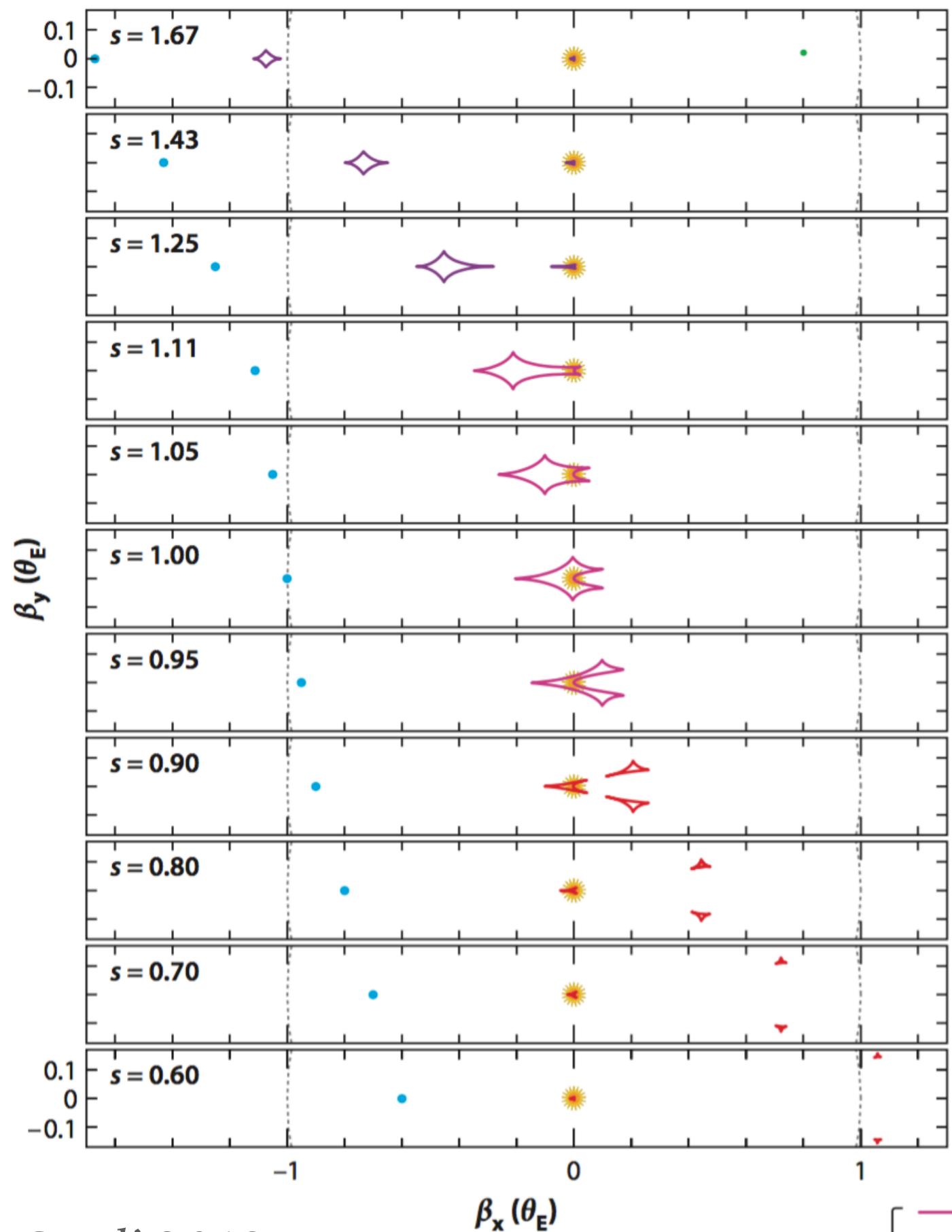
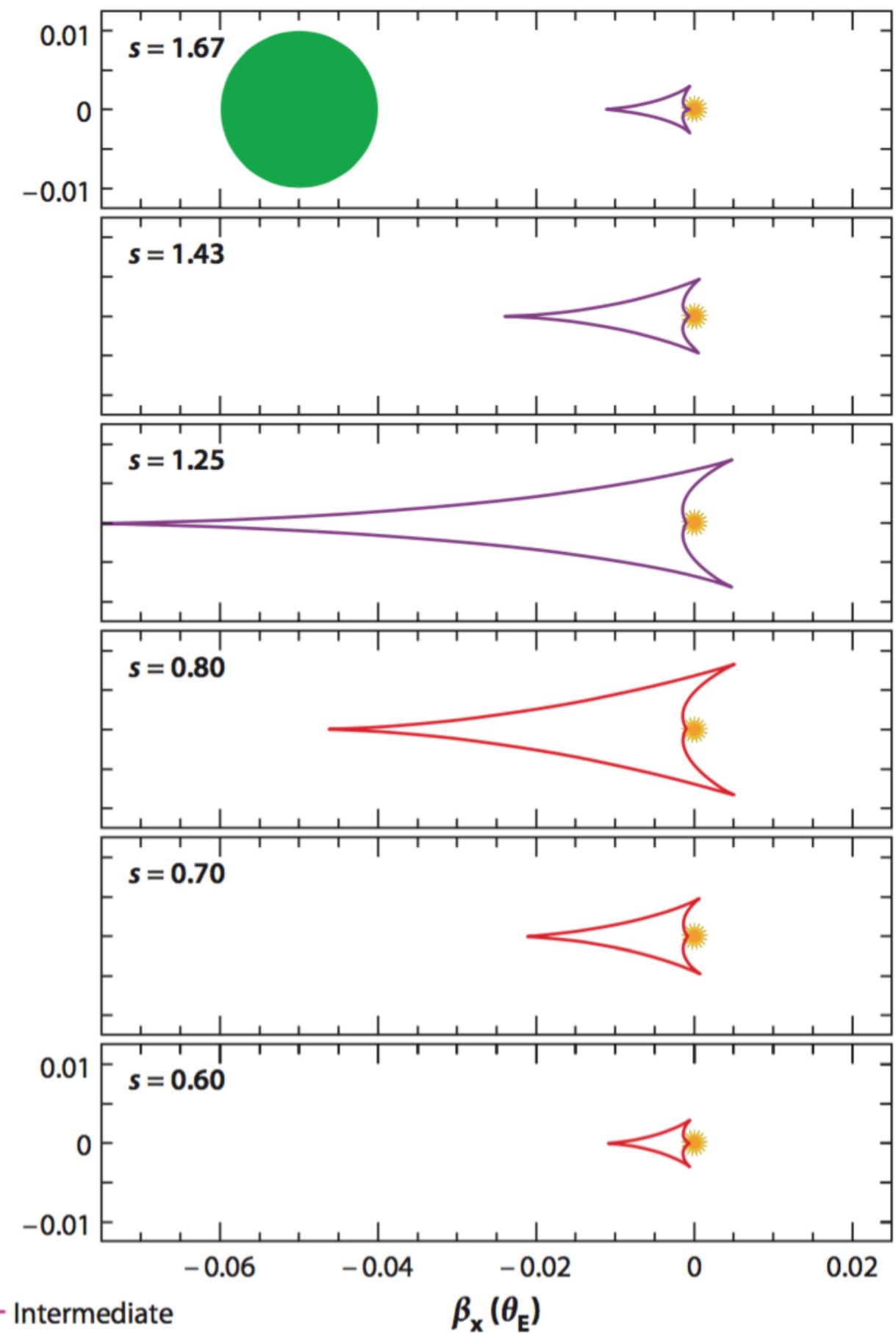


# GRAVITATIONAL LENSING

## 17 - BINARY LENSES: PLANETARY MICROLENSING

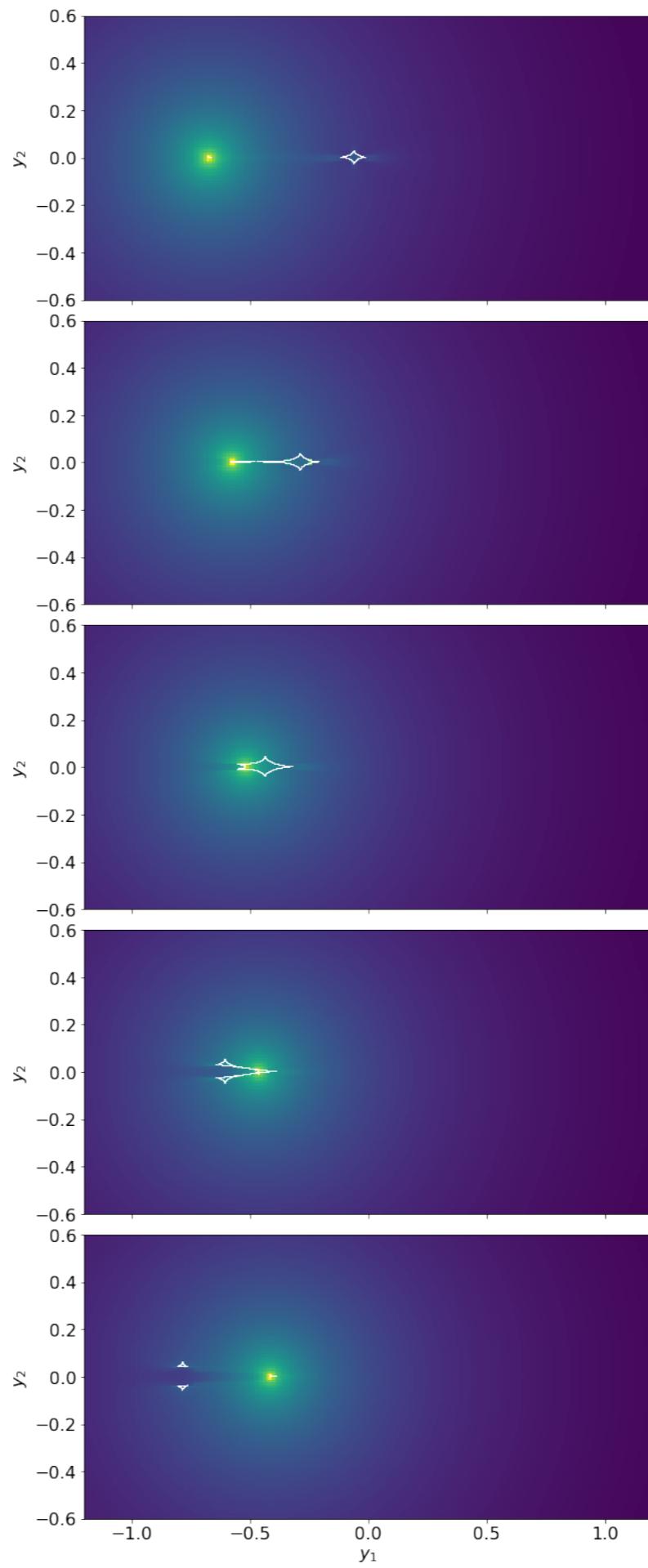
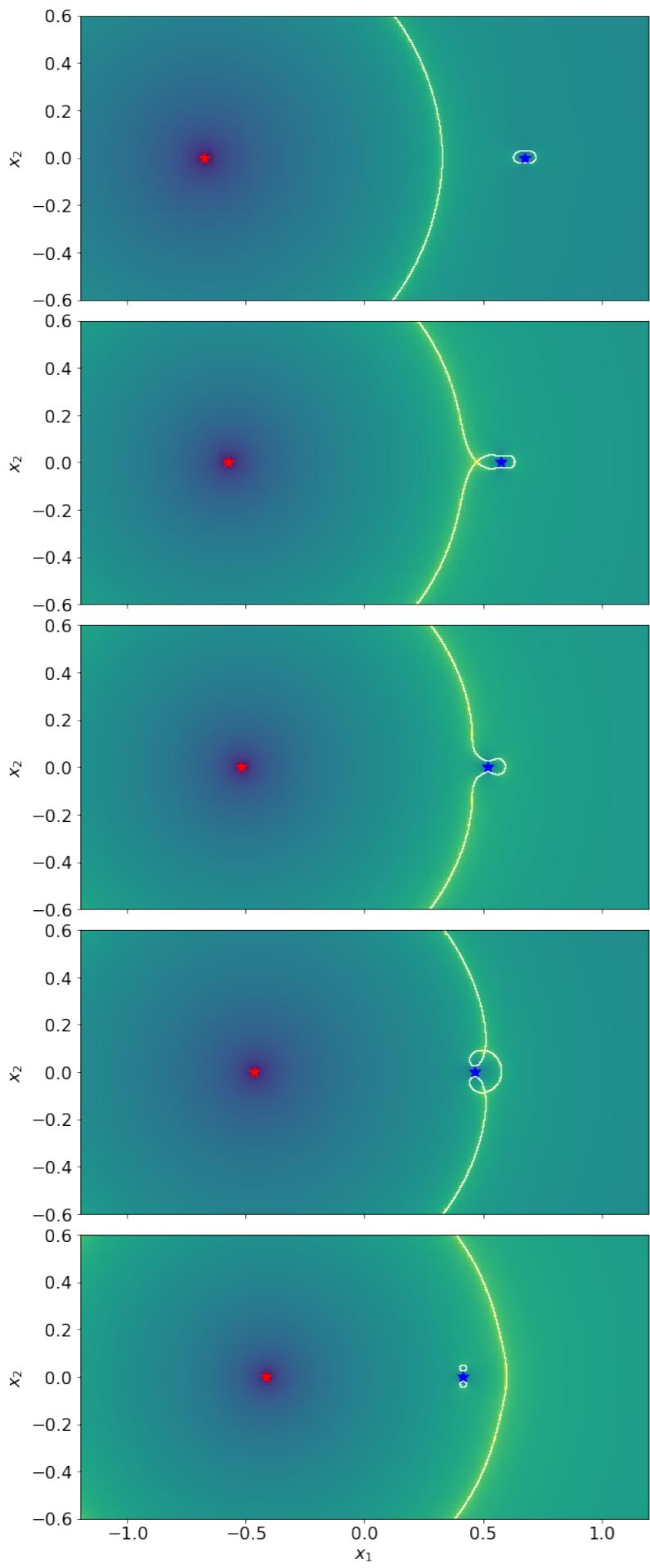
---

*Massimo Meneghetti*  
AA 2018-2019

**a****b**

Caustics

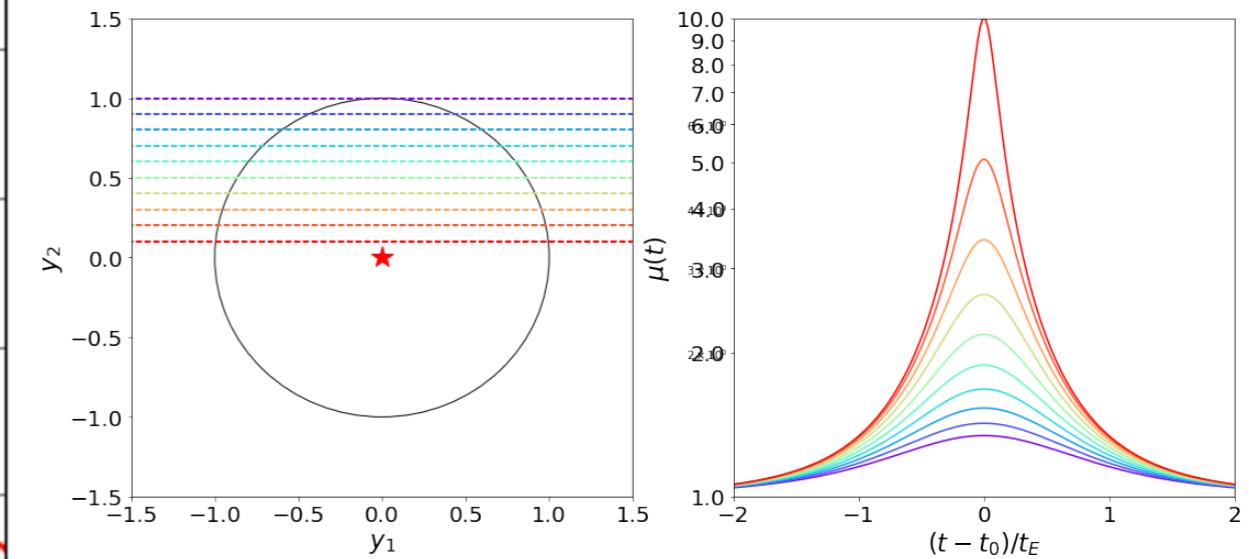
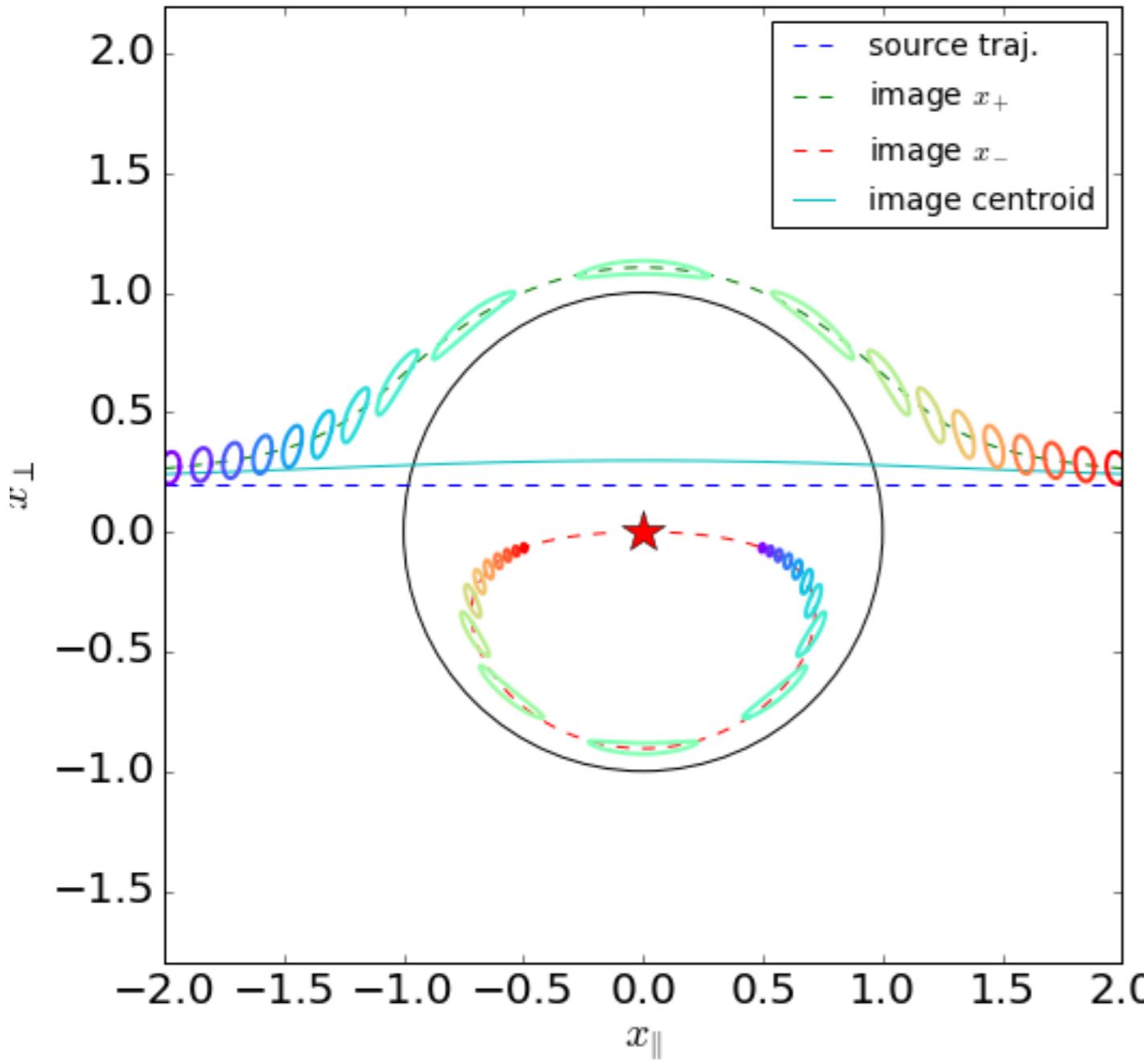
- Intermediate
- Wide
- Close



# ANOTHER WAY TO LOOK AT THE LIGHT CURVES: POINT MASS LENS

---

*Lensing of an extended circular source*



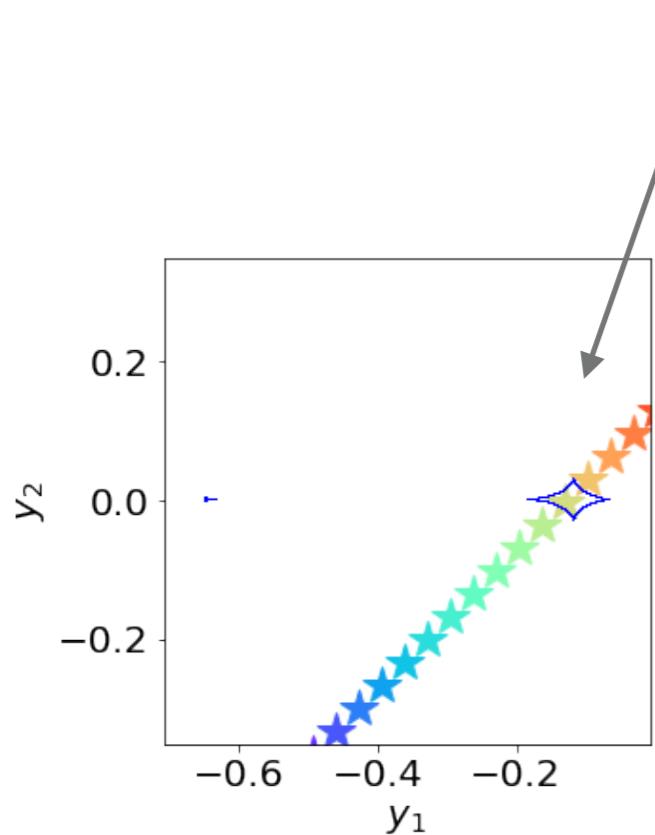
$$x_{\pm} = \frac{1}{2} \left[ y \pm \sqrt{y^2 + 4} \right]$$

*Always two images, largest magnifications near  $t_0$*

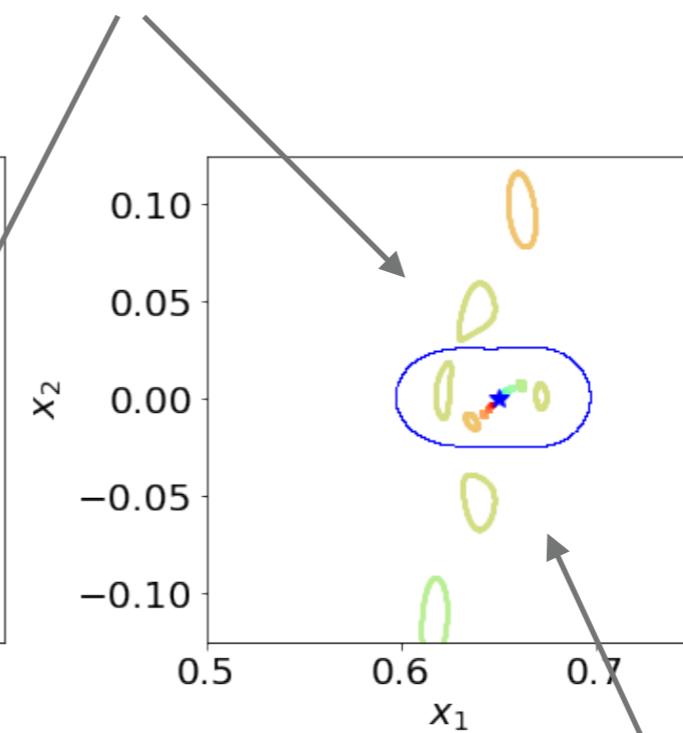
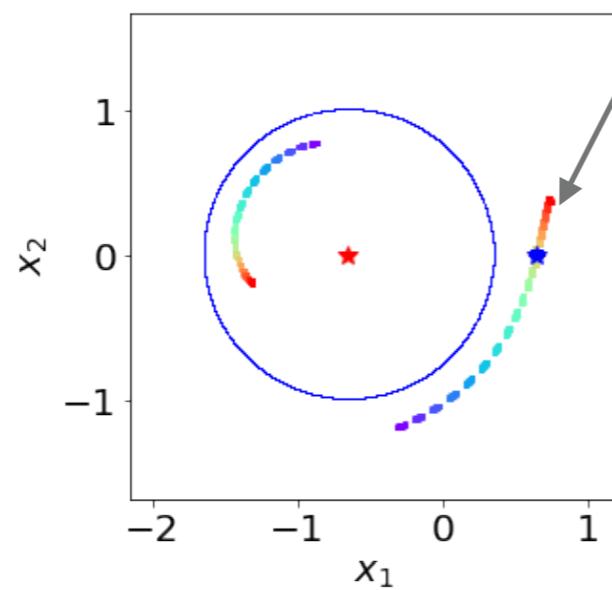
# PLANETARY PERTURBATIONS AS PERTURBATIONS OF SINGLE IMAGES

---

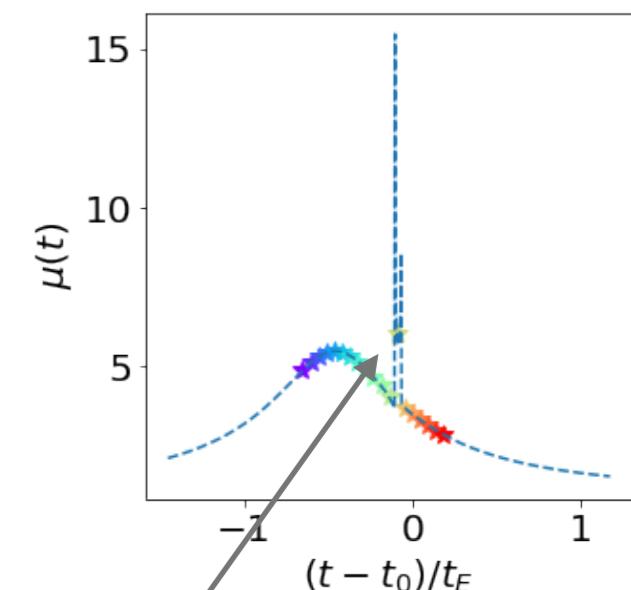
*Planetary caustics*



*Planetary critical lines*



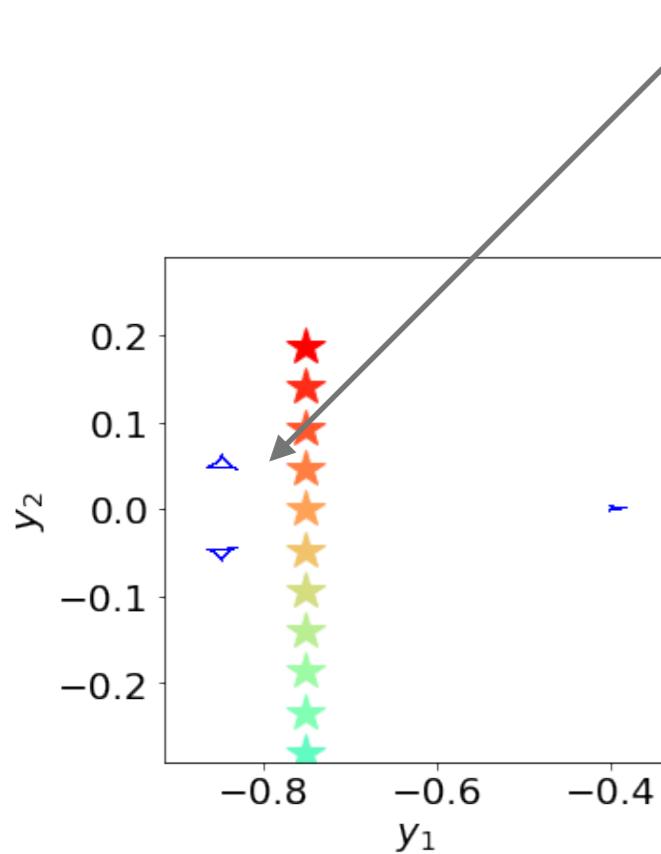
*magnified outer image*



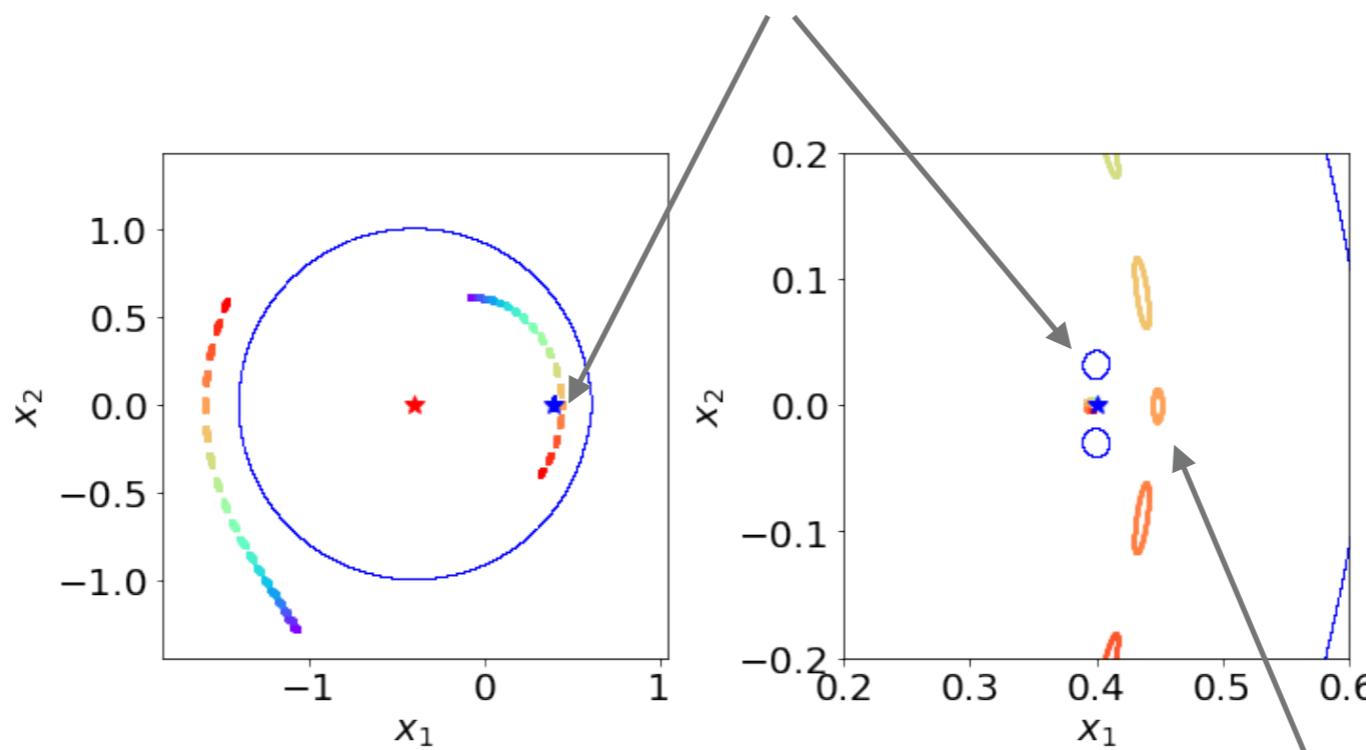
# PLANETARY PERTURBATIONS AS PERTURBATIONS OF SINGLE IMAGES

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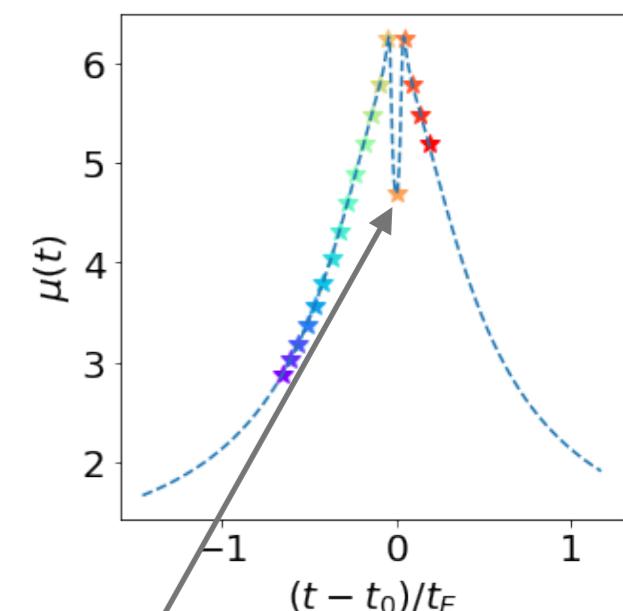
*Planetary caustics*



*Planetary critical lines*



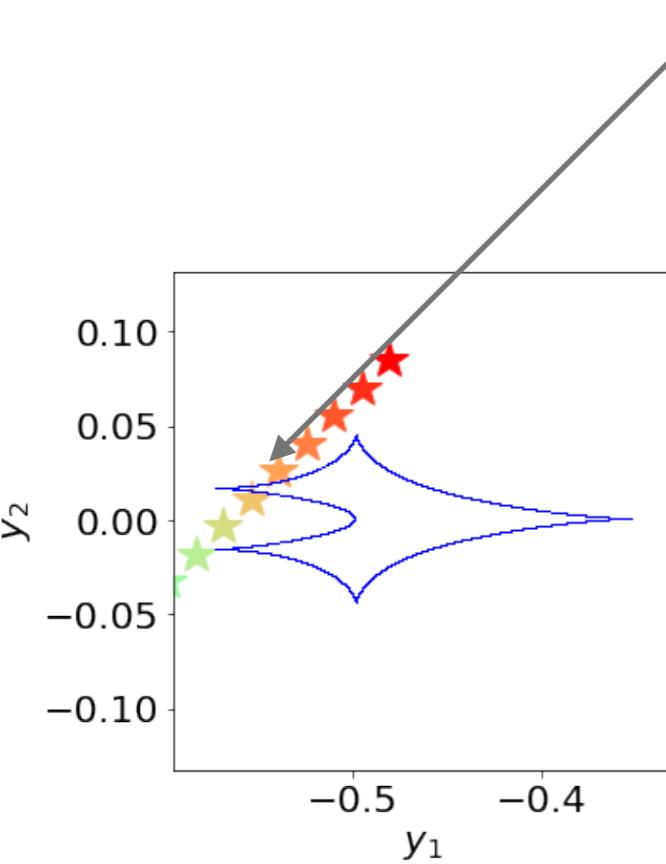
*Demagnified inner image*



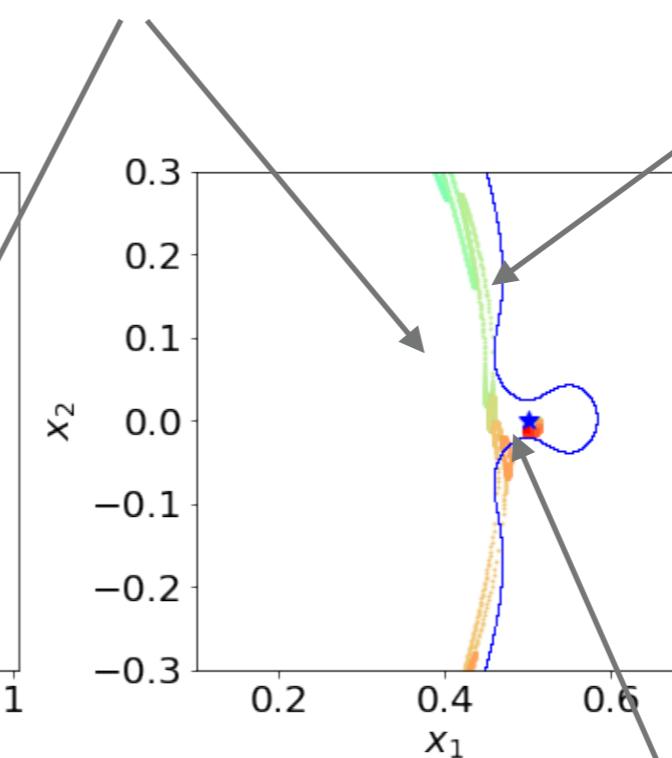
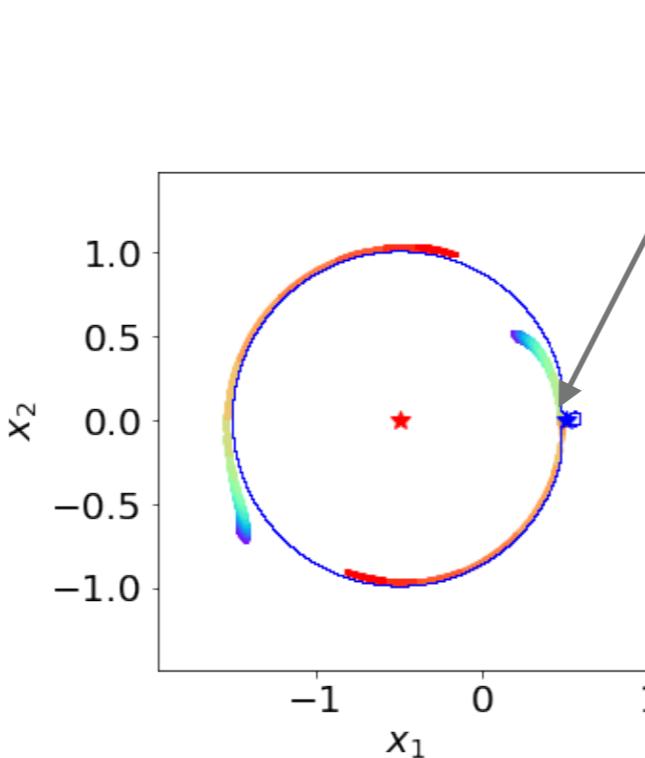
# PLANETARY PERTURBATIONS AS PERTURBATIONS OF SINGLE IMAGES

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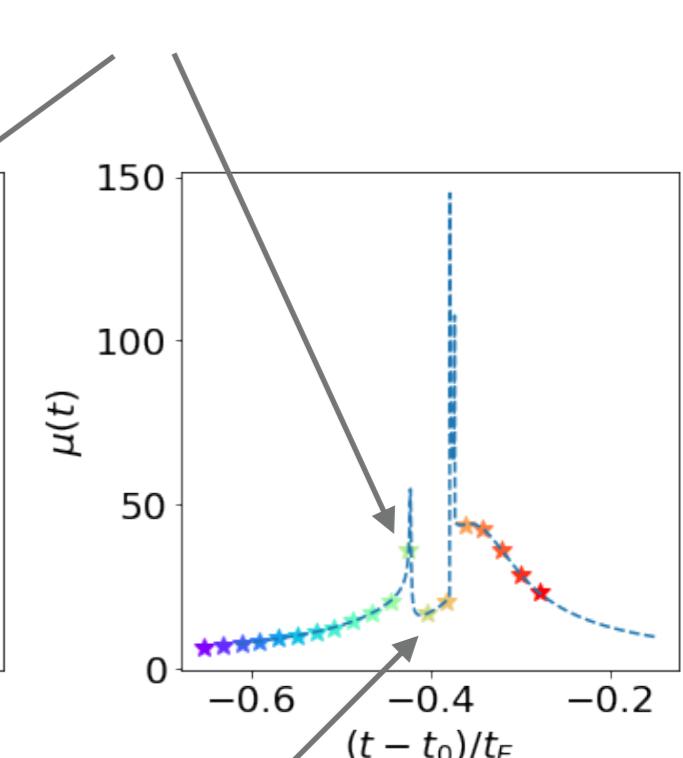
*Planetary caustics*



*Planetary critical lines*



*Magnified inner image*

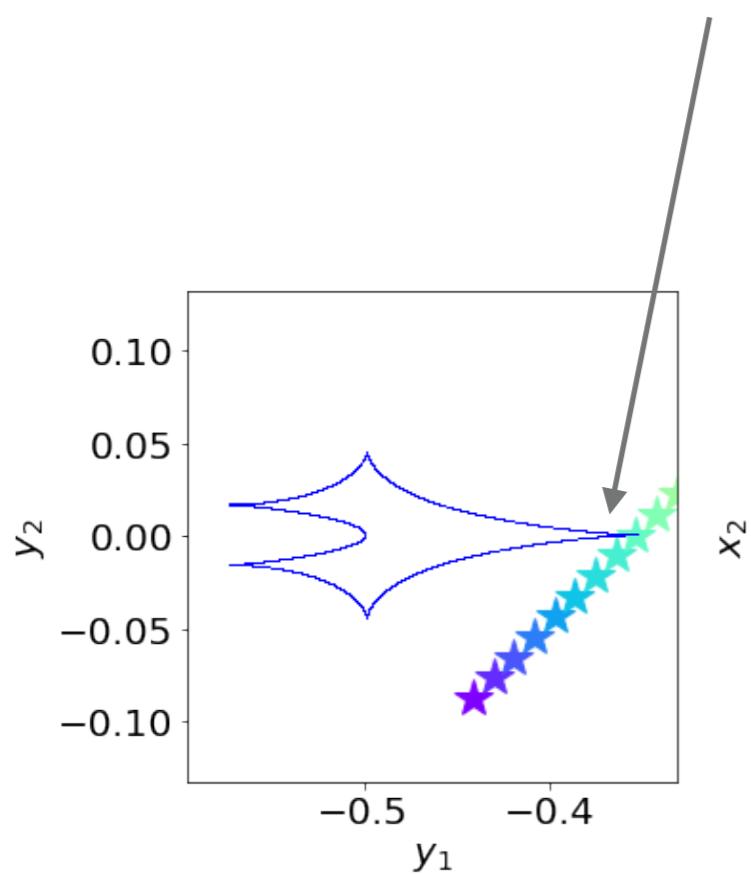


*Demagnified inner image*

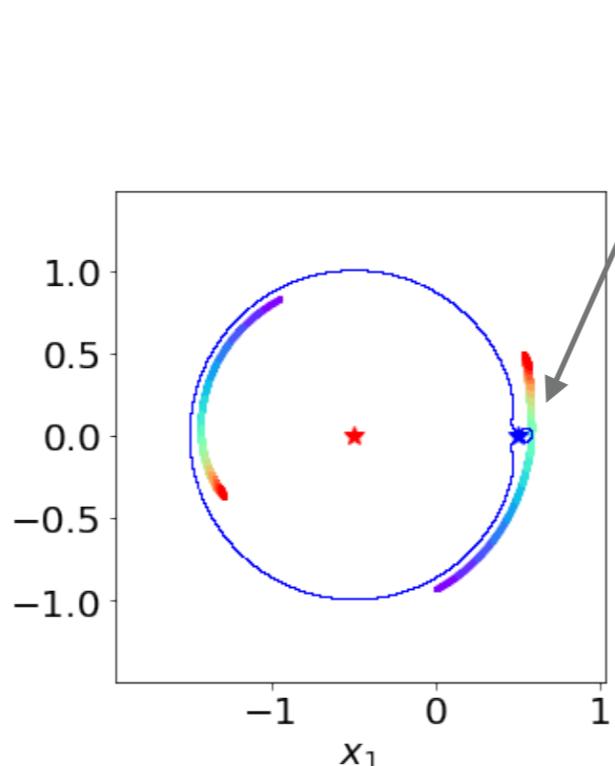
# PLANETARY PERTURBATIONS AS PERTURBATIONS OF SINGLE IMAGES

---

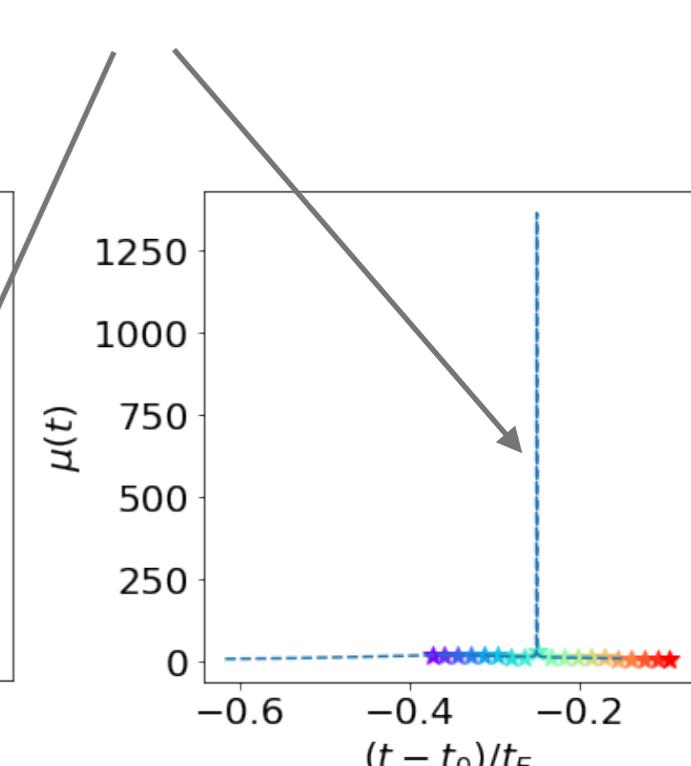
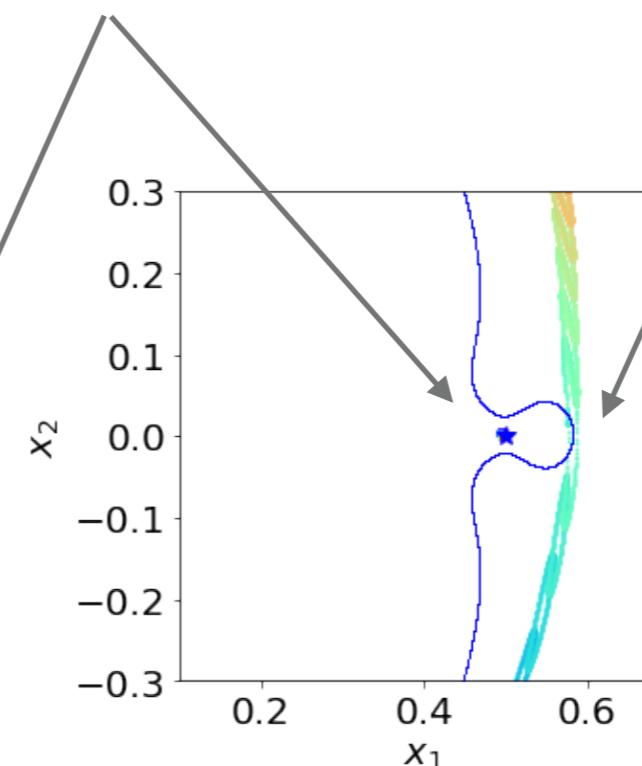
*central caustic*



*Planetary critical lines*



*Magnified outer image*



# INTERPRETING THE LIGHT CURVES

---

*If we notice a planetary caustic perturbation, it means that the planet is located at the position of one of the images:*

$$|x_{\pm}| = d$$

*Consequently the caustic which is being crossed has a position which can be derived from the lens equation (which is satisfied by the images)*

$$x_{\pm} = \frac{1}{2} \left[ y \pm \sqrt{y^2 + 4} \right]$$

# INTERPRETING THE LIGHT CURVES

---

If we notice a planetary caustic perturbation, it means that the planet is located at the position of one of the images:

$$|x_{\pm}| = d$$

$$x_{\pm} = \frac{1}{2} \left[ y \pm \sqrt{y^2 + 4} \right]$$

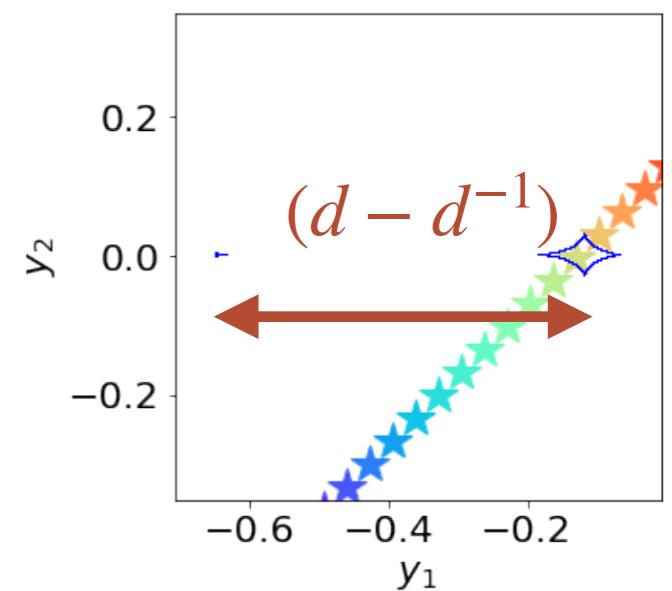
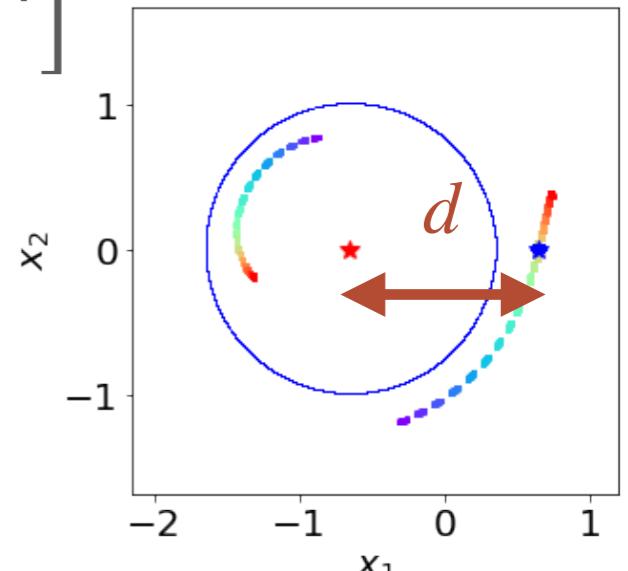
Case 1: perturbation of the outer image ( $x_+ > 0$ ):

$$d = \frac{1}{2}y + \frac{1}{2}\sqrt{y^2 + 4}$$

$$\frac{1}{2}\sqrt{y^2 + 4} = \left( d - \frac{1}{2}y \right)$$

$$y^2 + 4 = 4 \left( d^2 + \frac{y^2}{4} - yd \right) \Rightarrow yd = d^2 - 1$$

$$y = (d - d^{-1})$$



# INTERPRETING THE LIGHT CURVES

---

If we notice a planetary caustic perturbation, it means that the planet is located at the position of one of the images:

$$|x_{\pm}| = d$$

$$x_{\pm} = \frac{1}{2} \left[ y \pm \sqrt{y^2 + 4} \right]$$

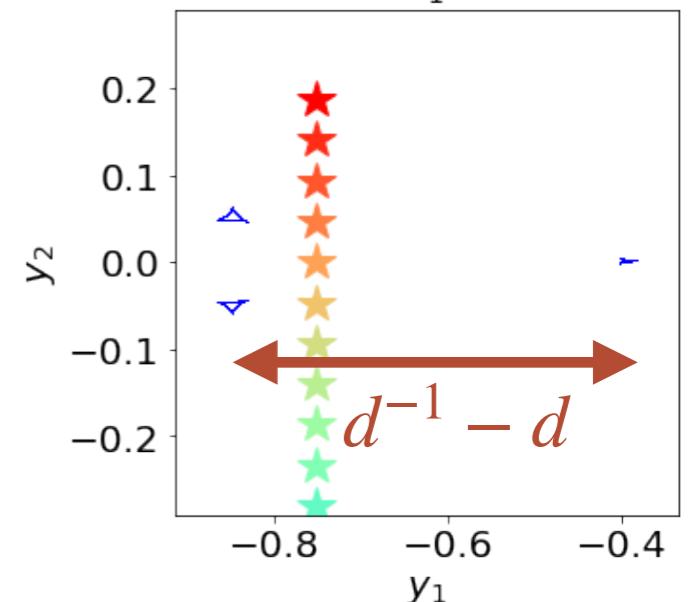
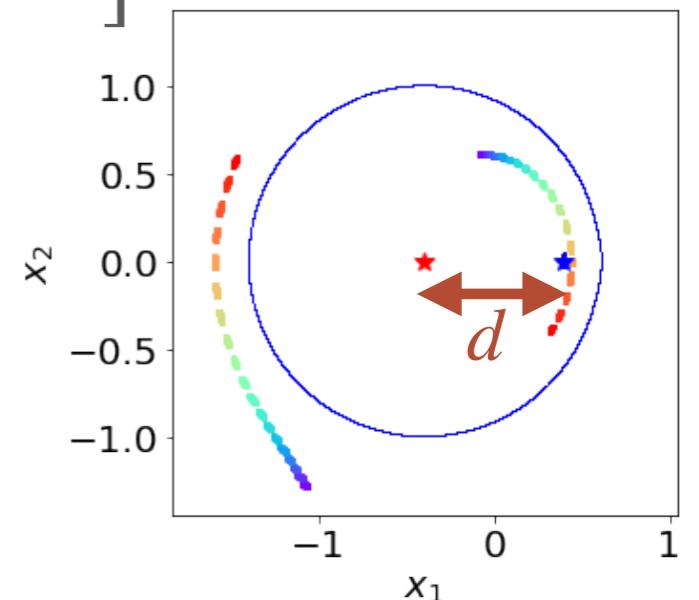
Case 2: perturbation of the inner image ( $x_- < 0$ ):

$$d = - \left( \frac{1}{2}y - \frac{1}{2}\sqrt{y^2 + 4} \right)$$

$$\frac{1}{2}\sqrt{y^2 + 4} = \left( \frac{1}{2}y + d \right)$$

$$y^2 + 4 = 4 \left( \frac{y^2}{4} + yd + d^2 \right) \Rightarrow yd = 1 - d^2$$

$$y = (d^{-1} - d)$$

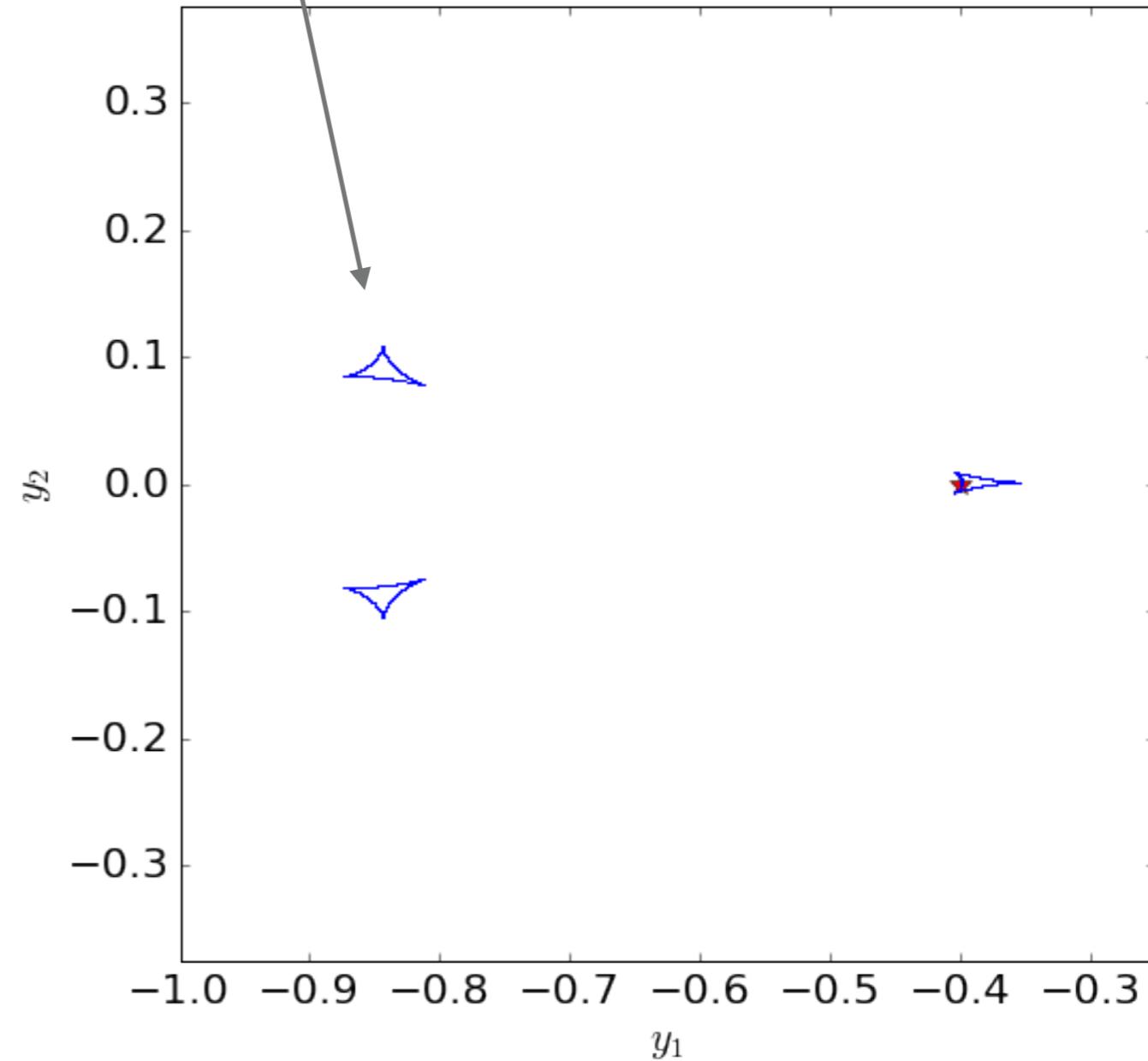
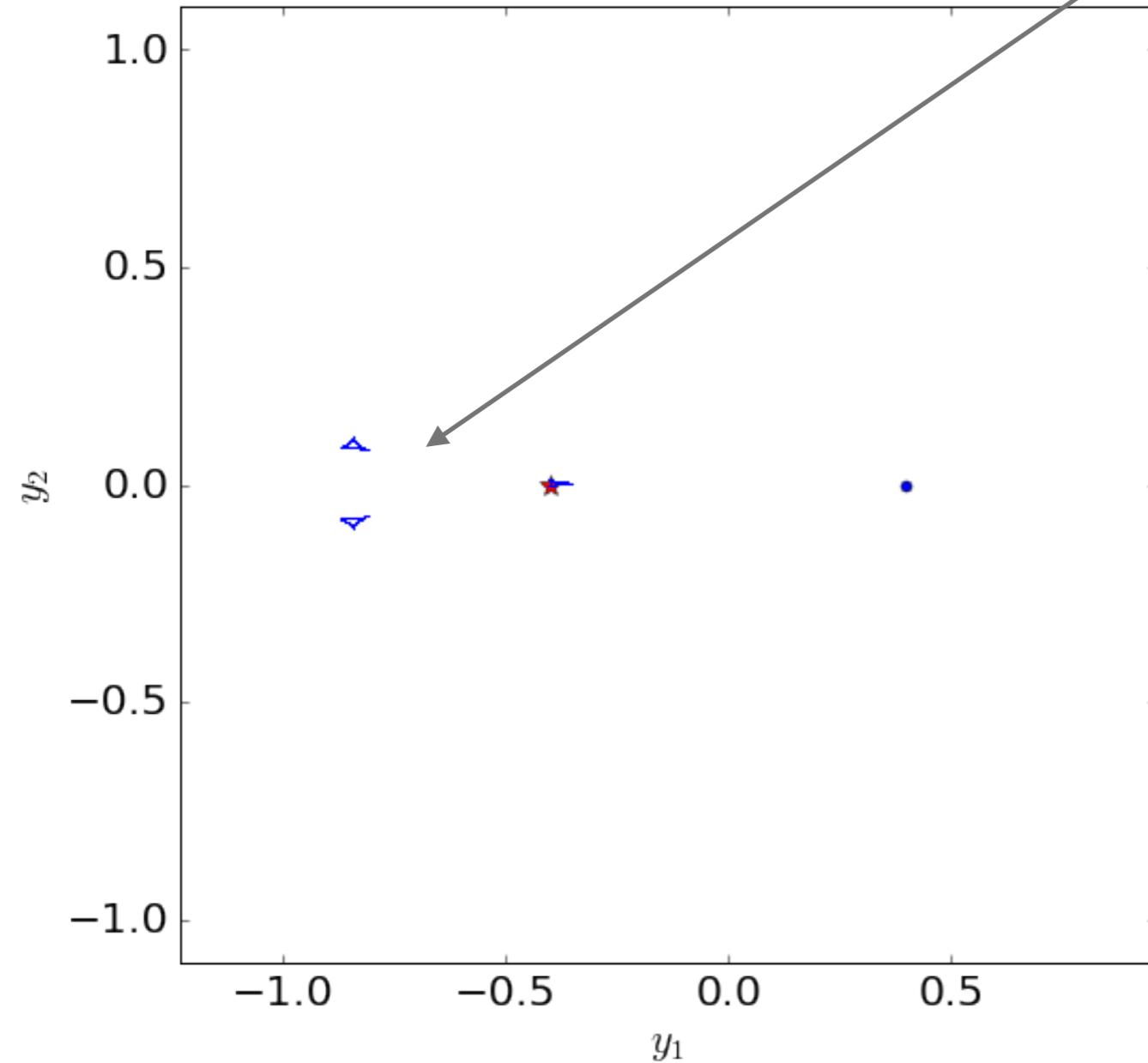


# PLANETARY CAUSTICS IN CLOSE TOPOLOGIES

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*planetary caustics*

*Han 2006*



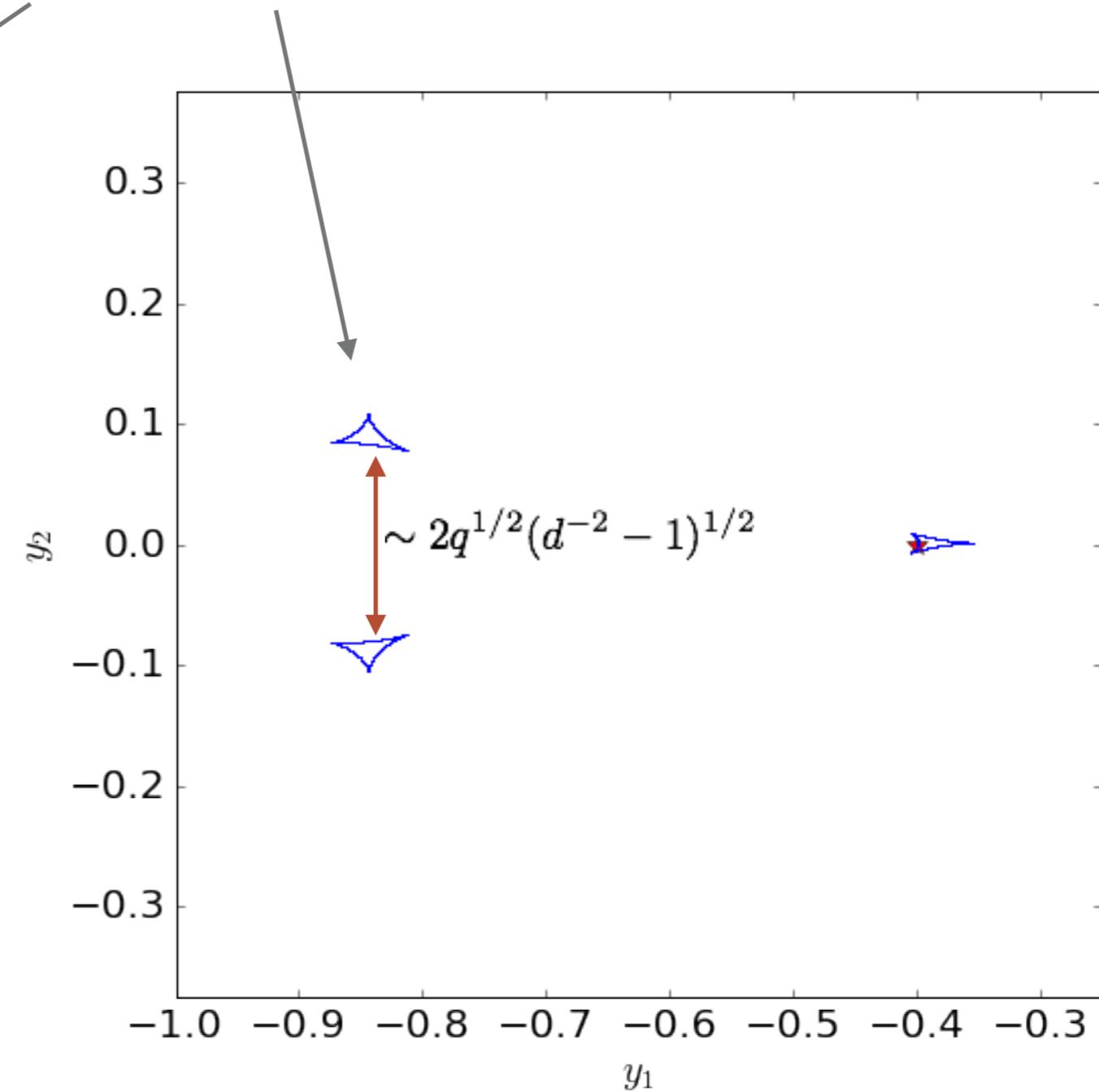
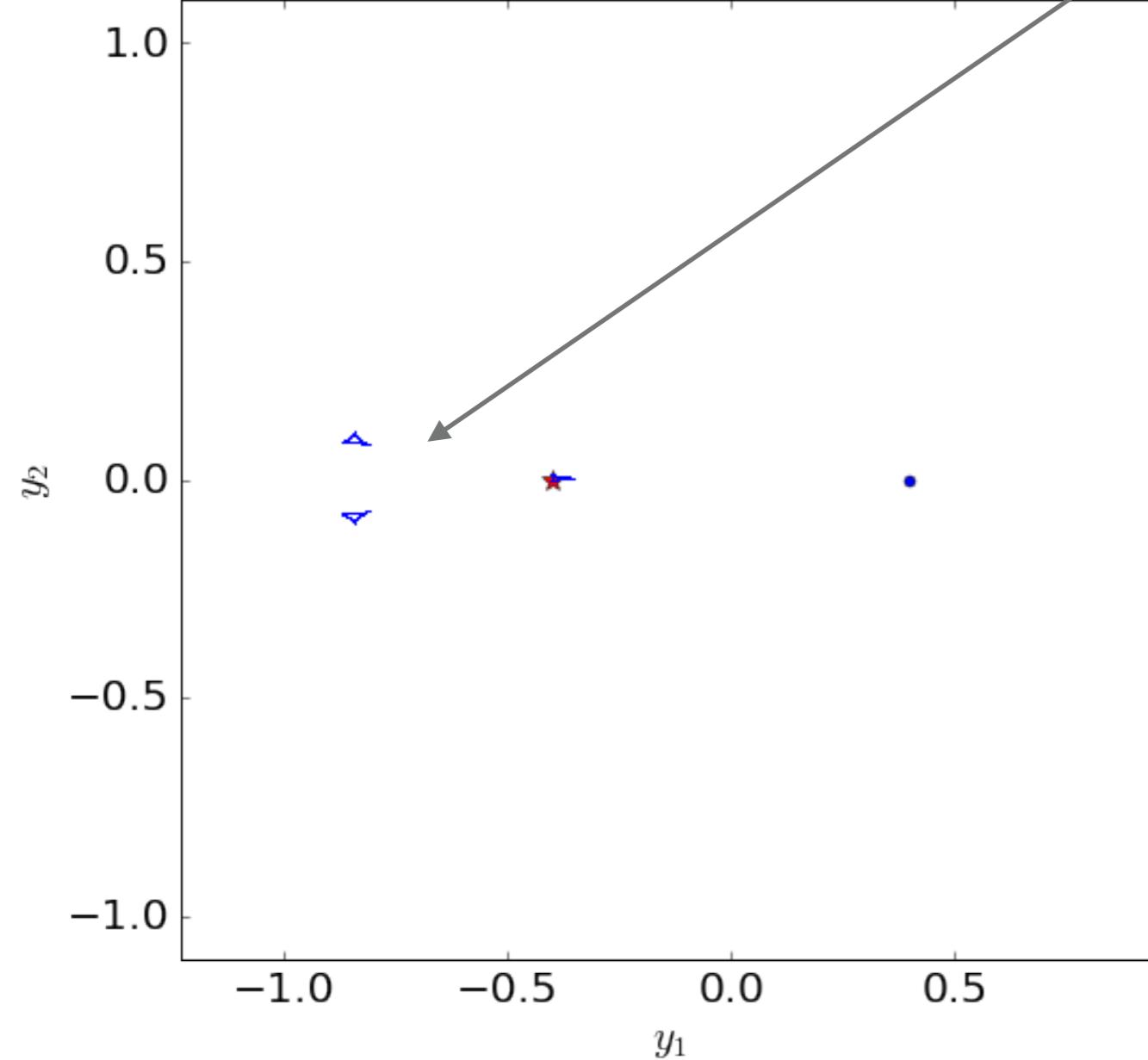
*Recommended reading: Han, C., 2006, ApJ, 638, 1080*

# PLANETARY CAUSTICS IN CLOSE TOPOLOGIES

---

*planetary caustics*

*Han 2006*



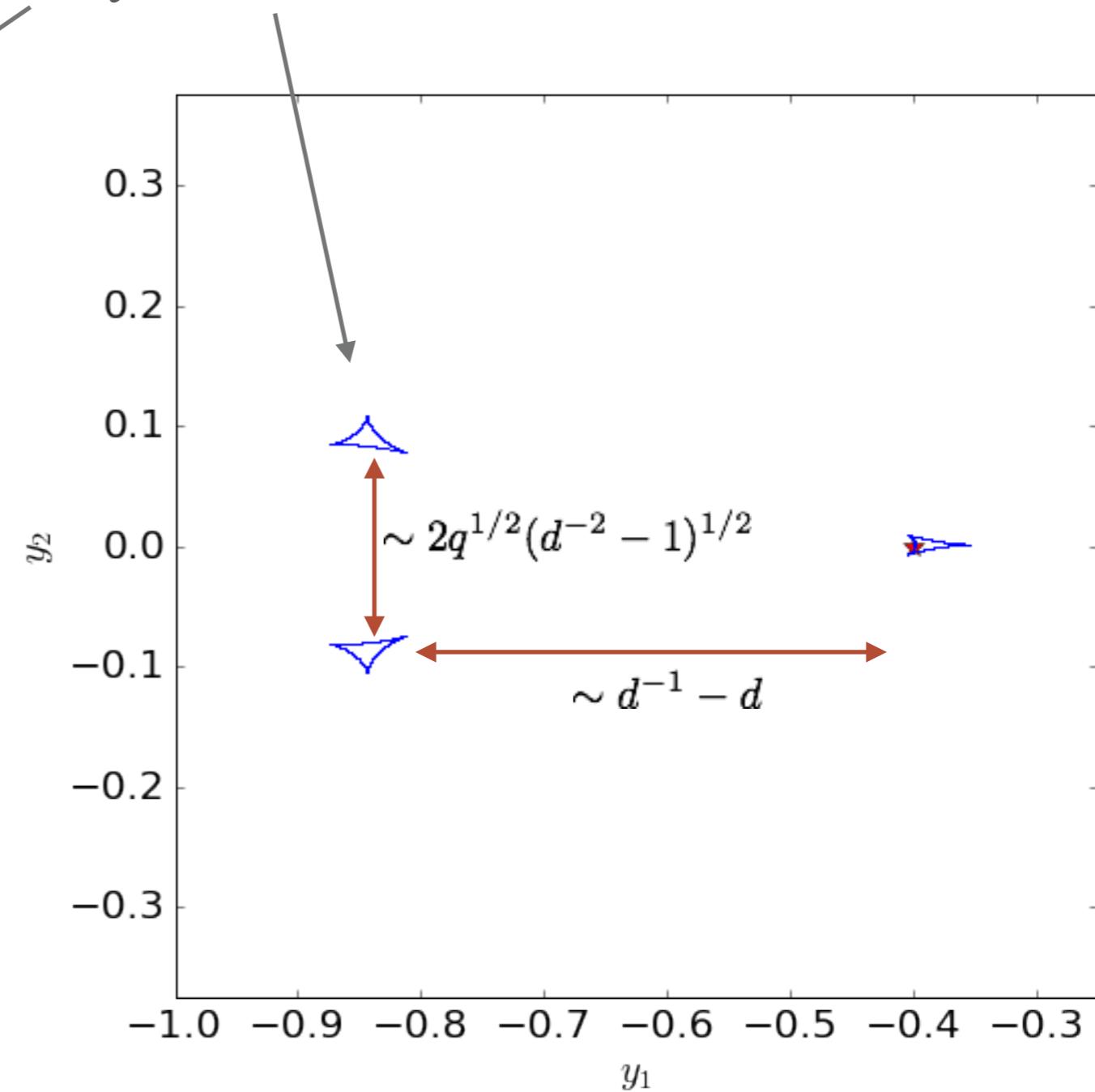
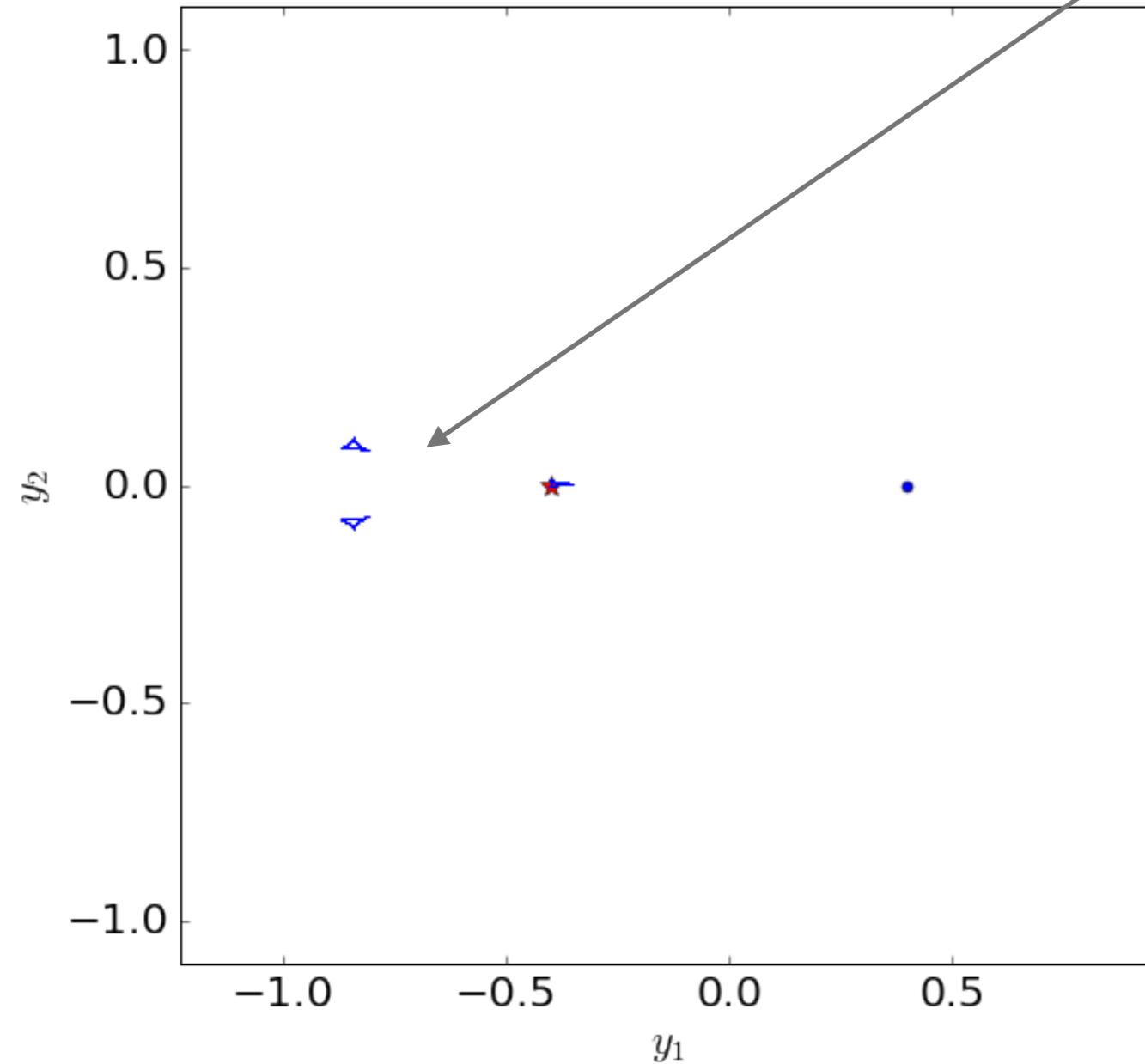
*Recommended reading: Han, C., 2006, ApJ, 638, 1080*

# PLANETARY CAUSTICS IN CLOSE TOPOLOGIES

---

*planetary caustics*

*Han 2006*



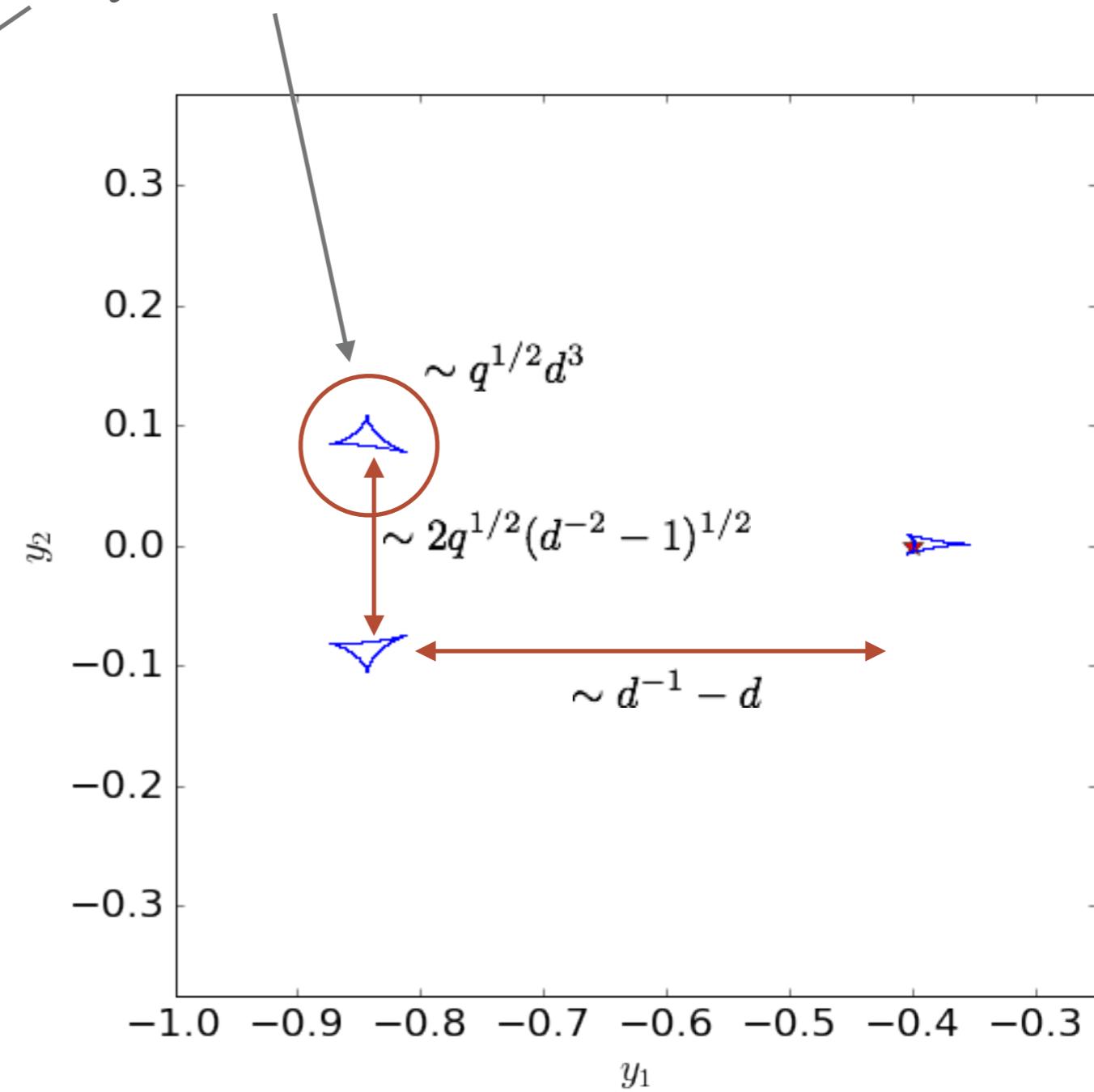
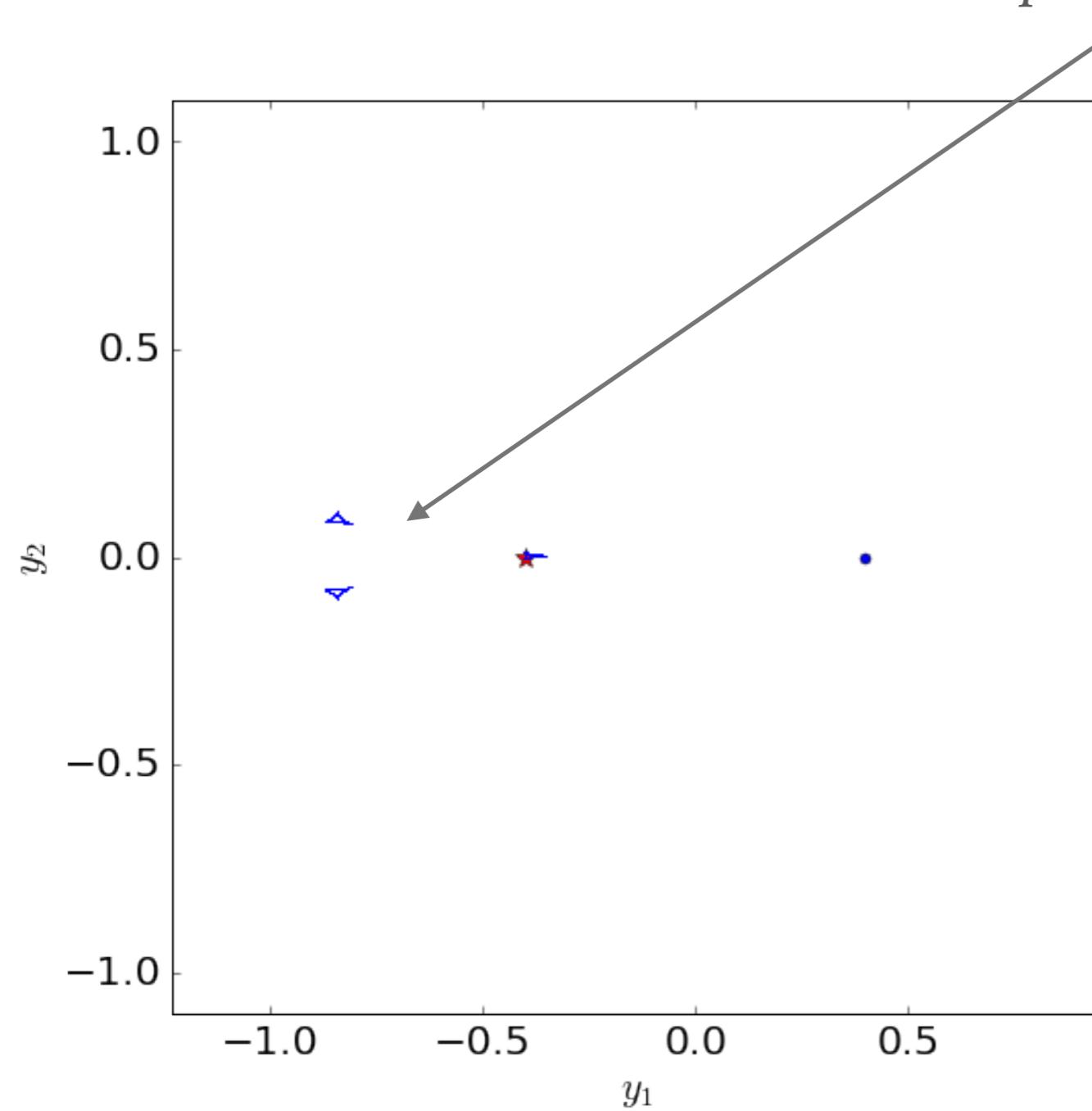
*Recommended reading: Han, C., 2006, ApJ, 638, 1080*

# PLANETARY CAUSTICS IN CLOSE TOPOLOGIES

---

*planetary caustics*

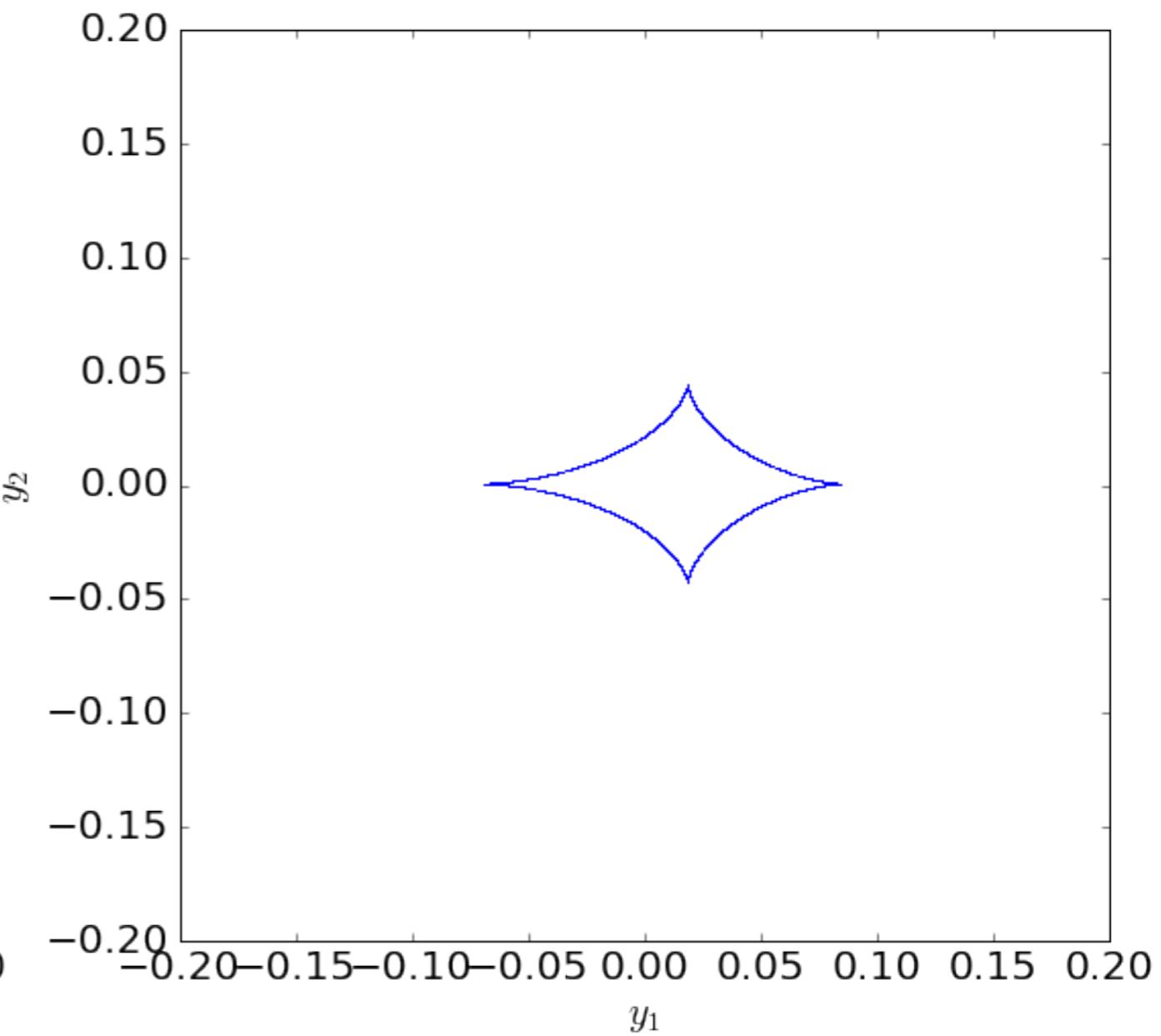
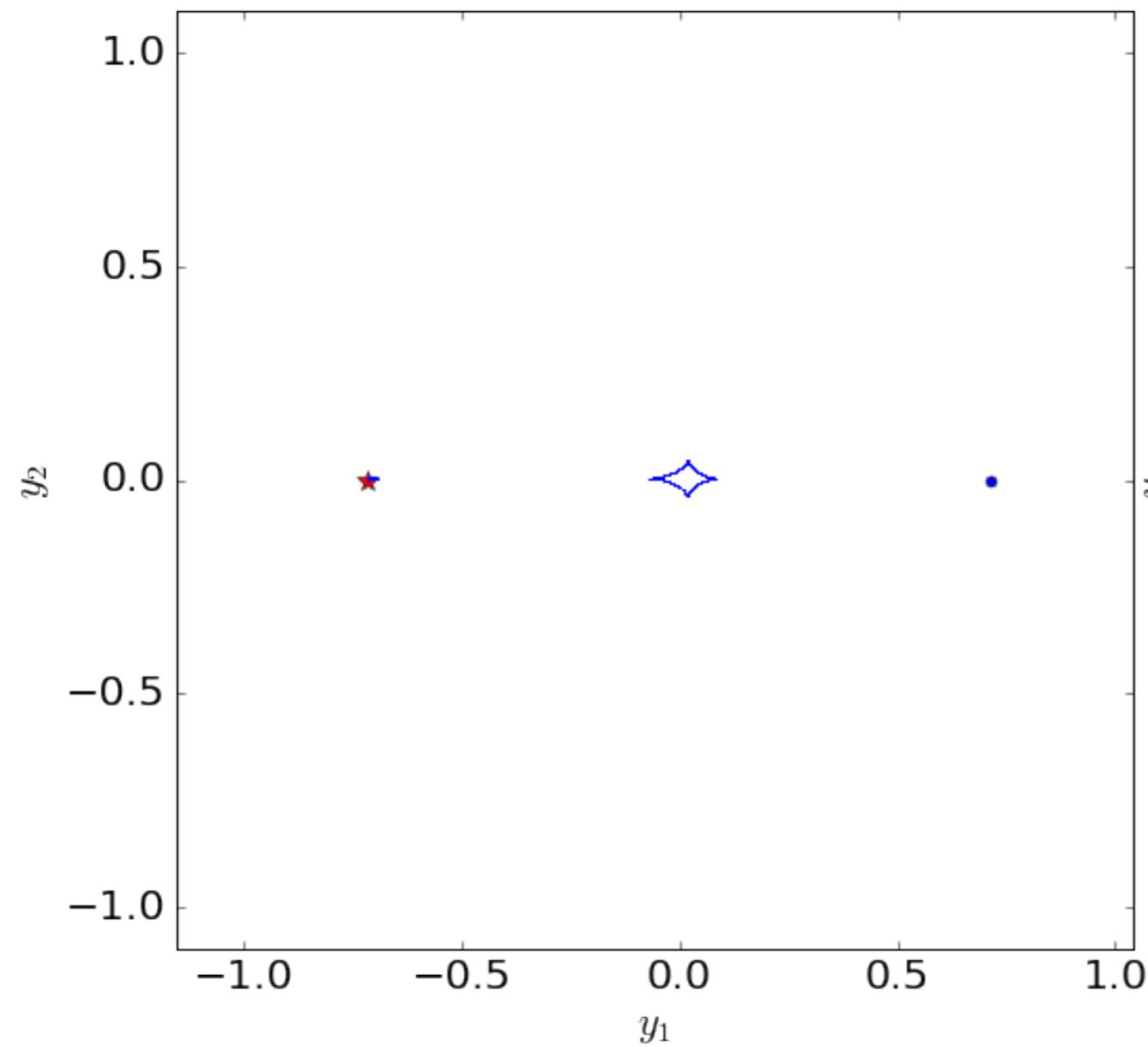
*Han 2006*



*Recommended reading: Han, C., 2006, ApJ, 638, 1080*

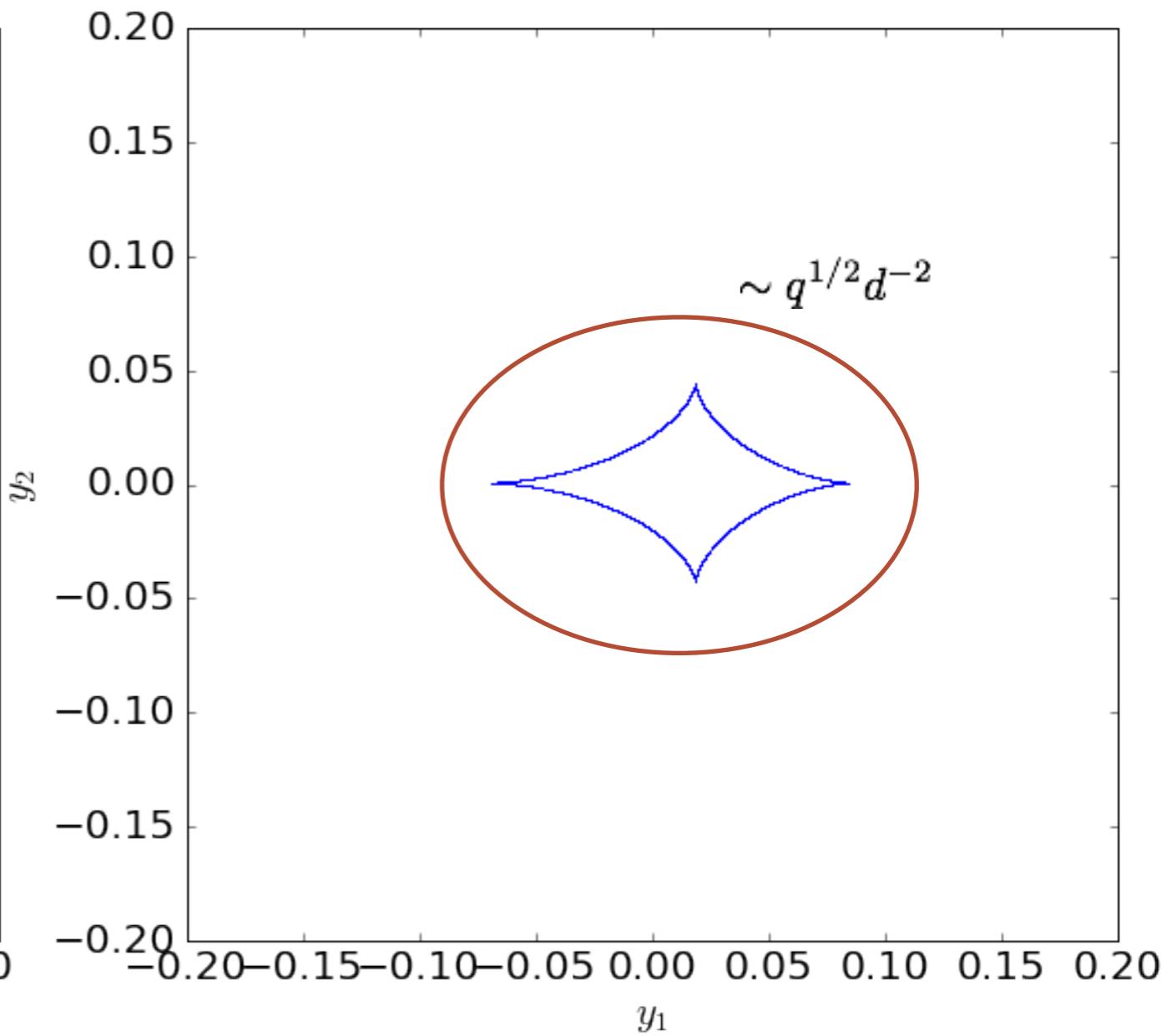
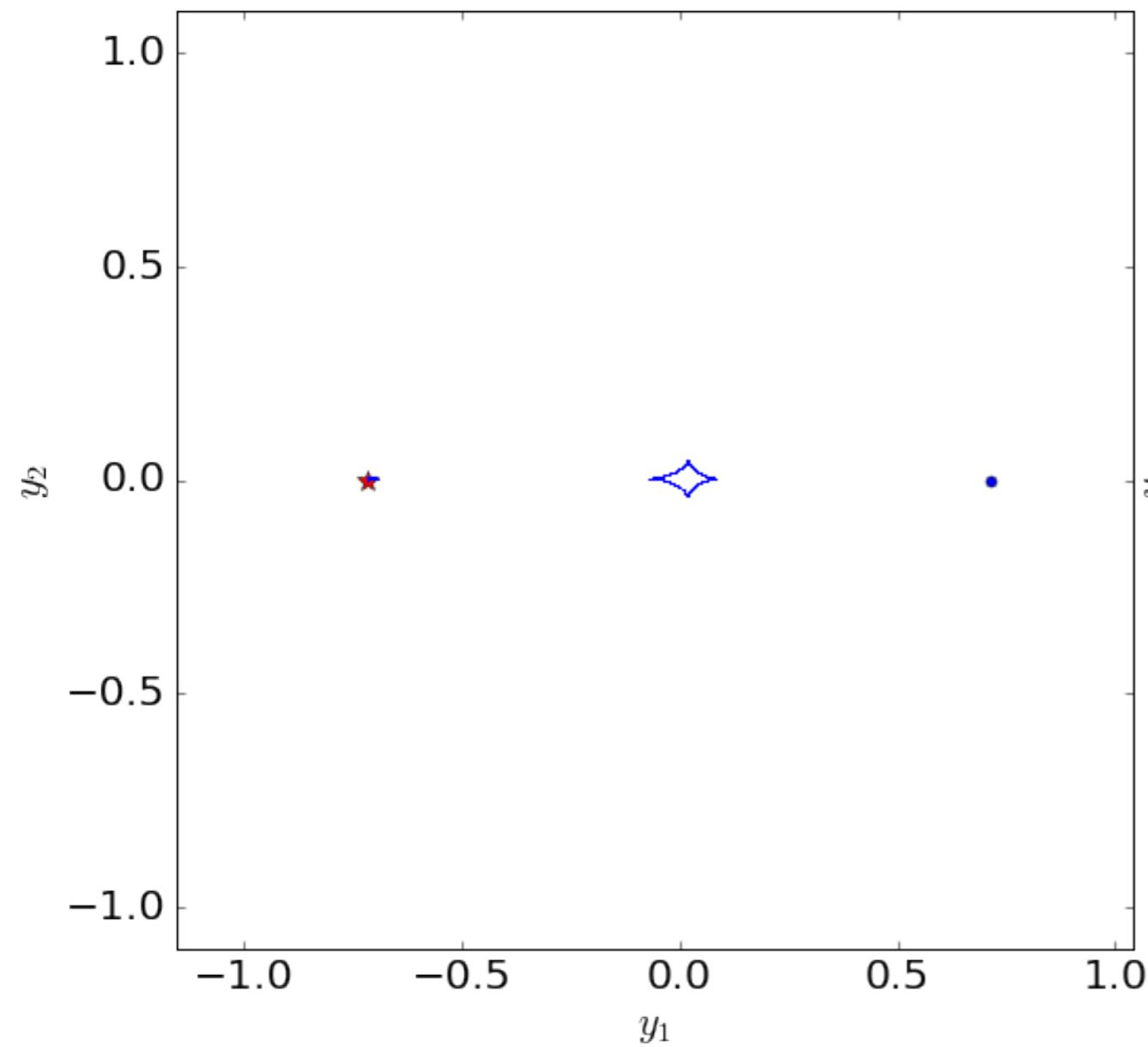
# PLANETARY CAUSTICS IN WIDE TOPOLOGIES

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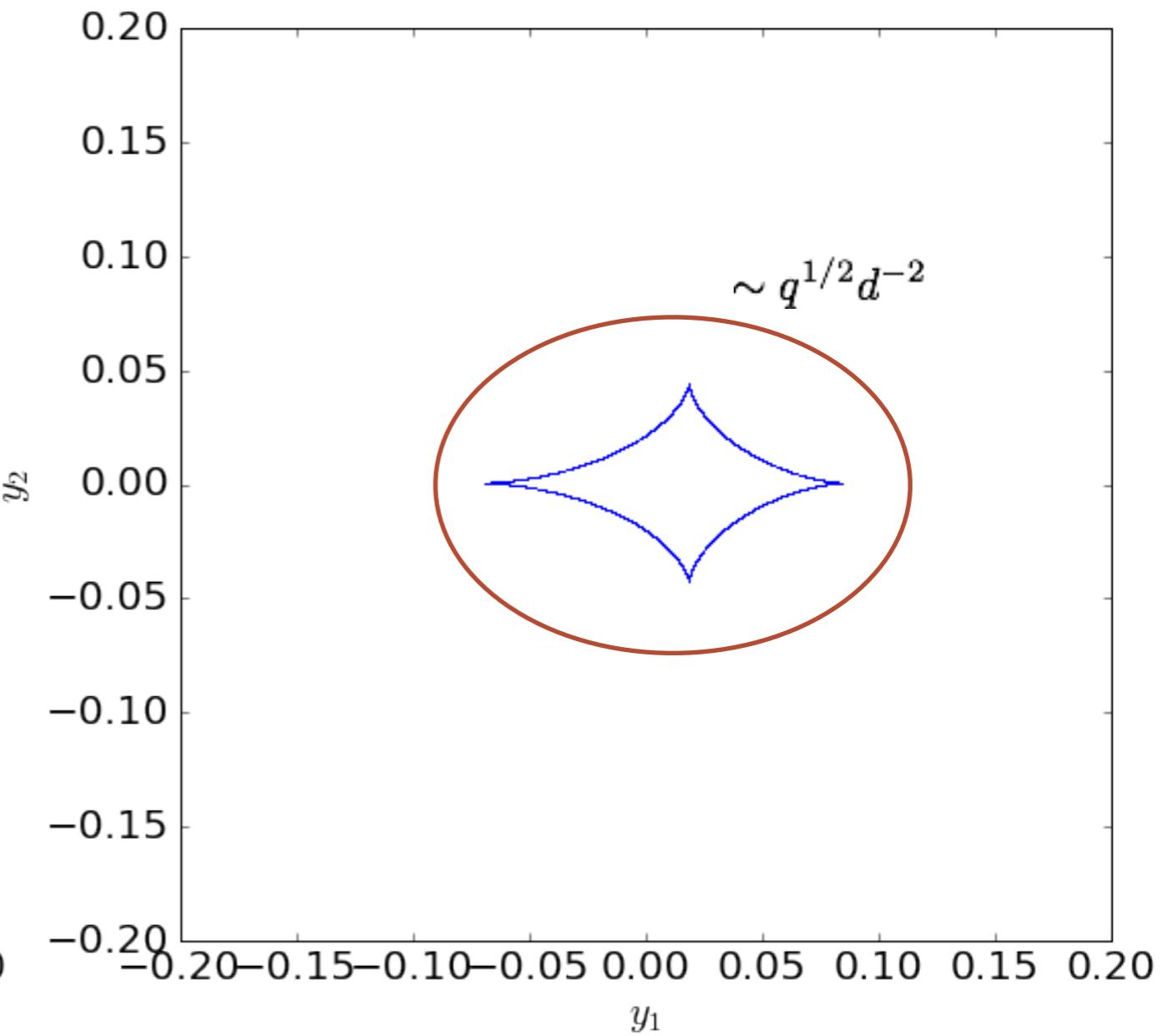
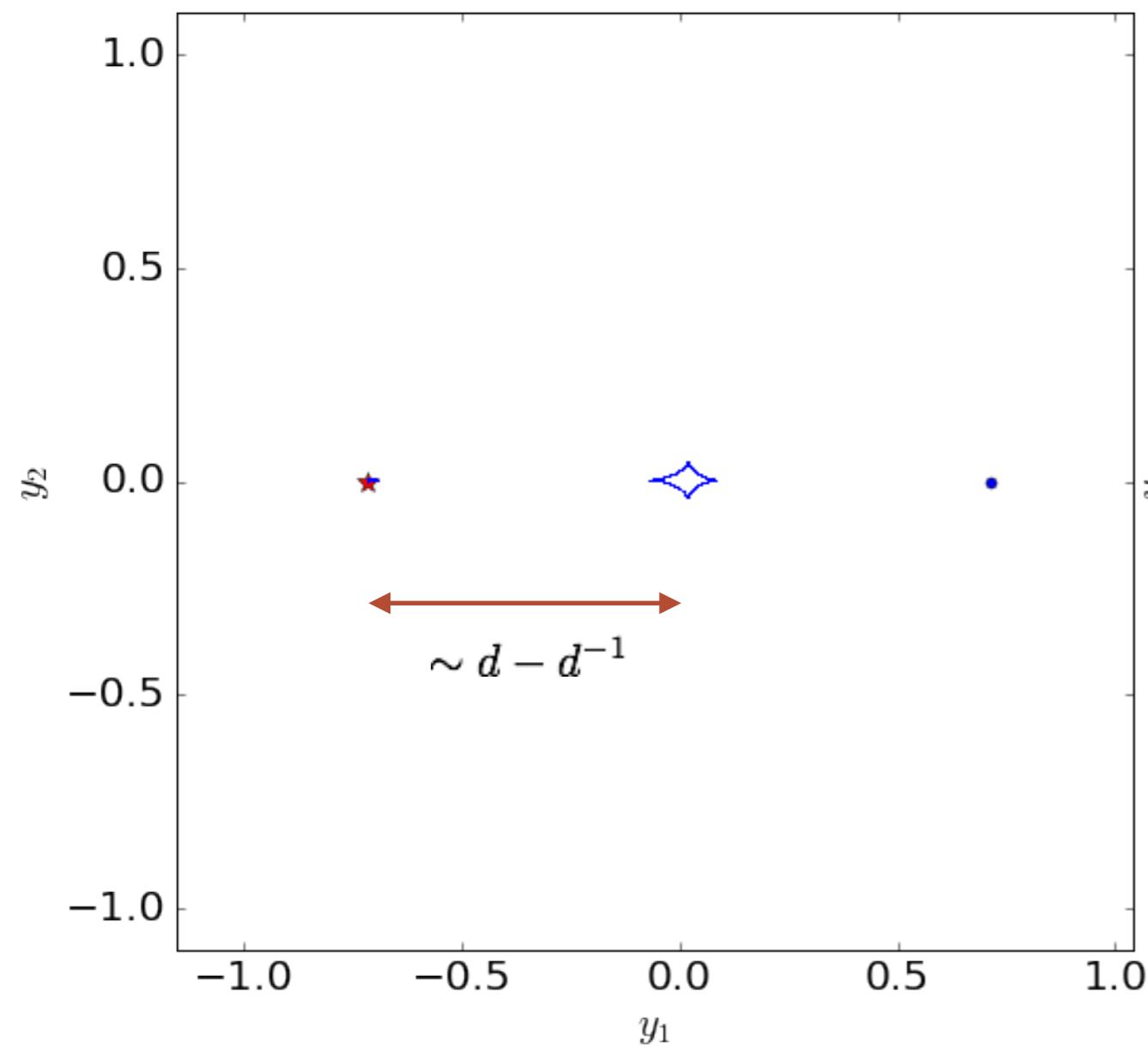
# PLANETARY CAUSTICS IN WIDE TOPOLOGIES

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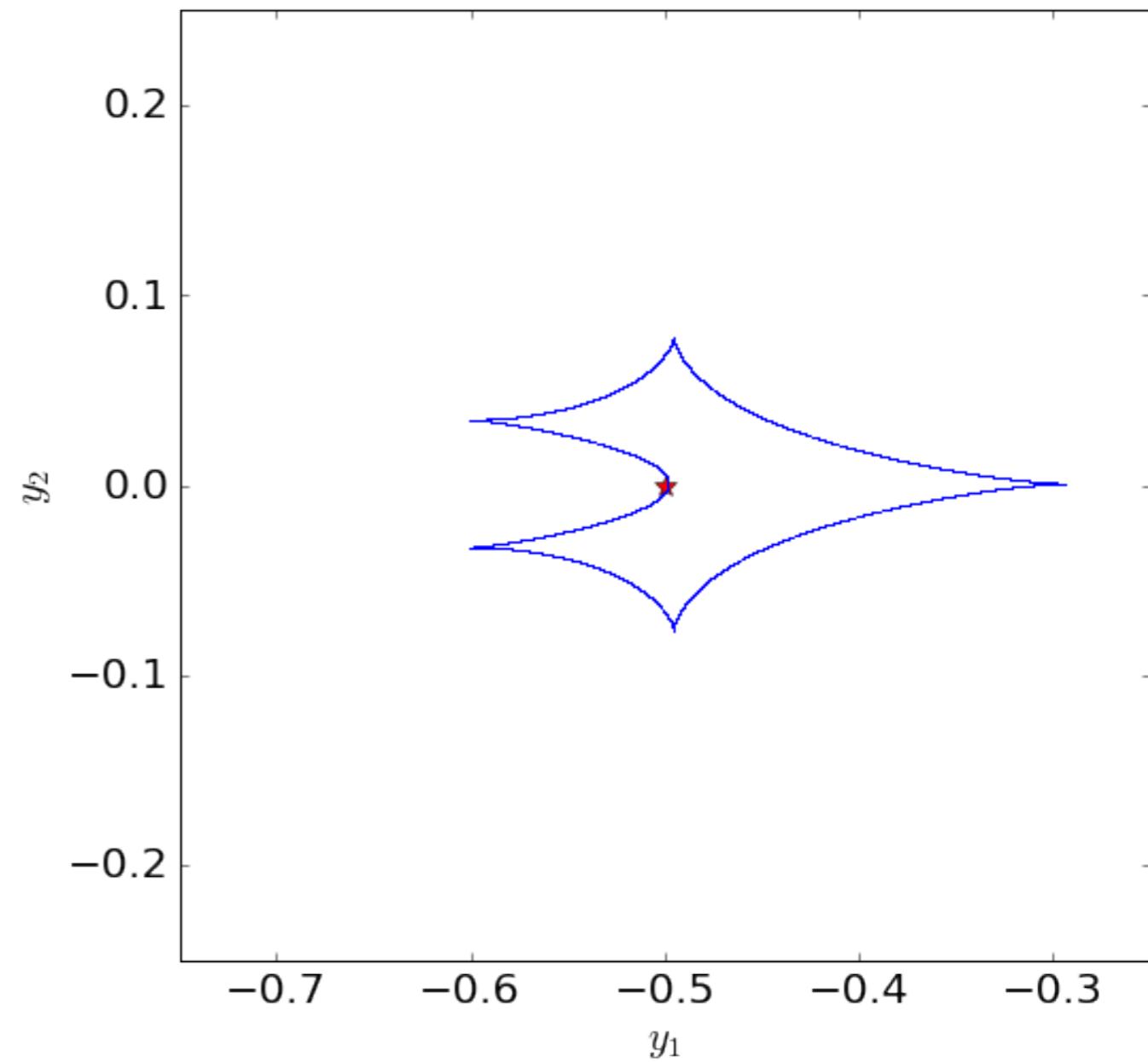
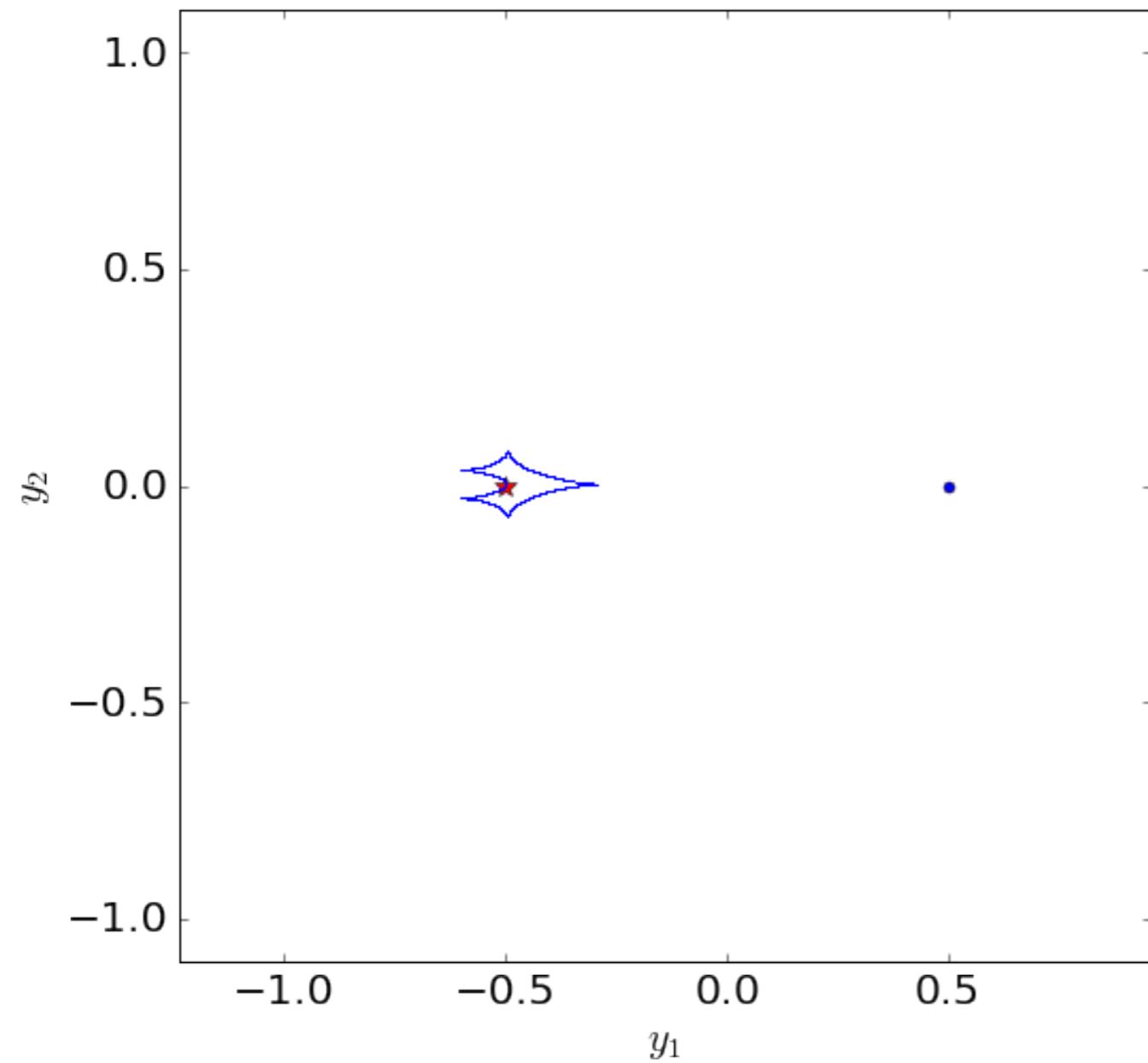
# PLANETARY CAUSTICS IN WIDE TOPOLOGIES

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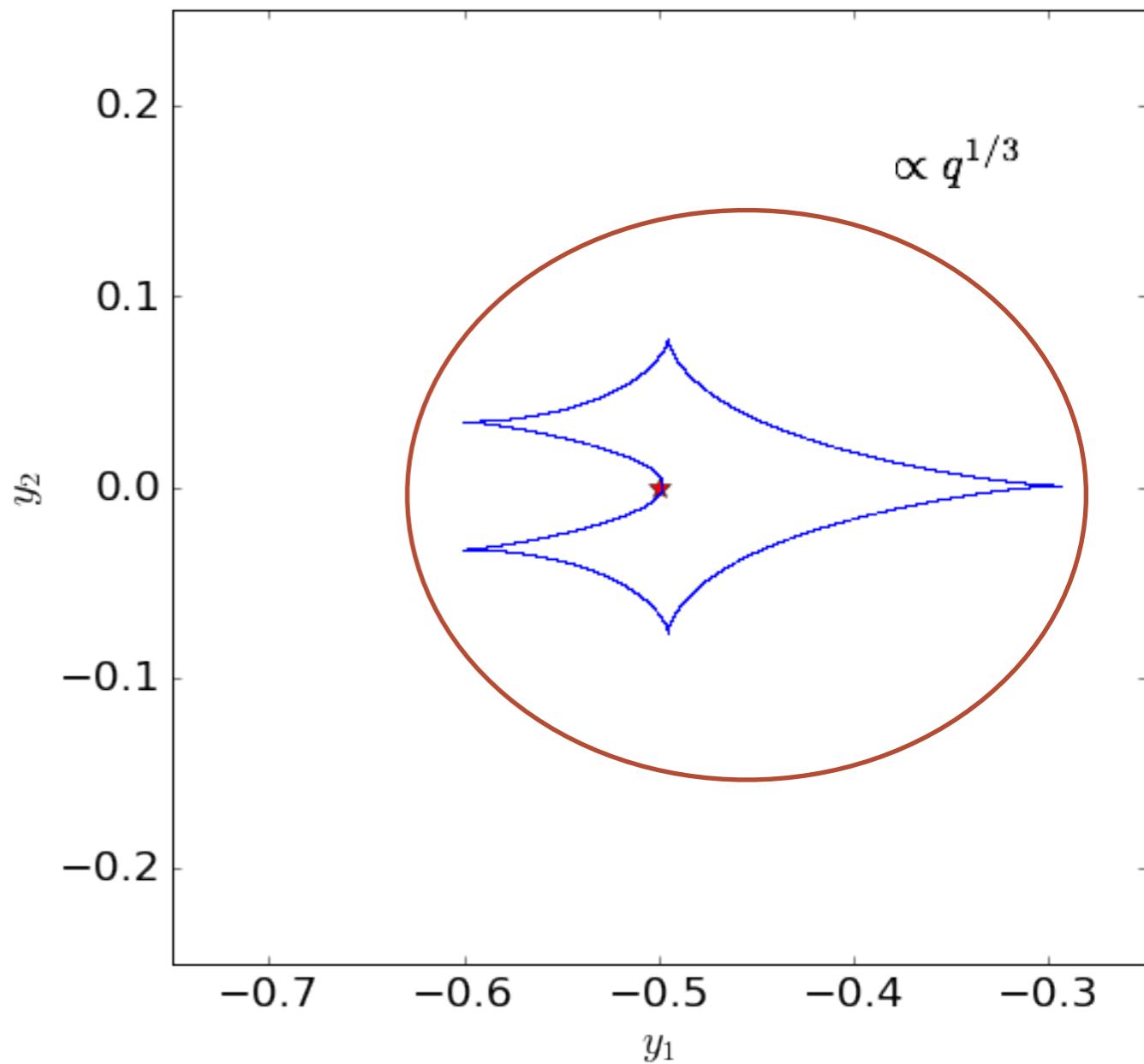
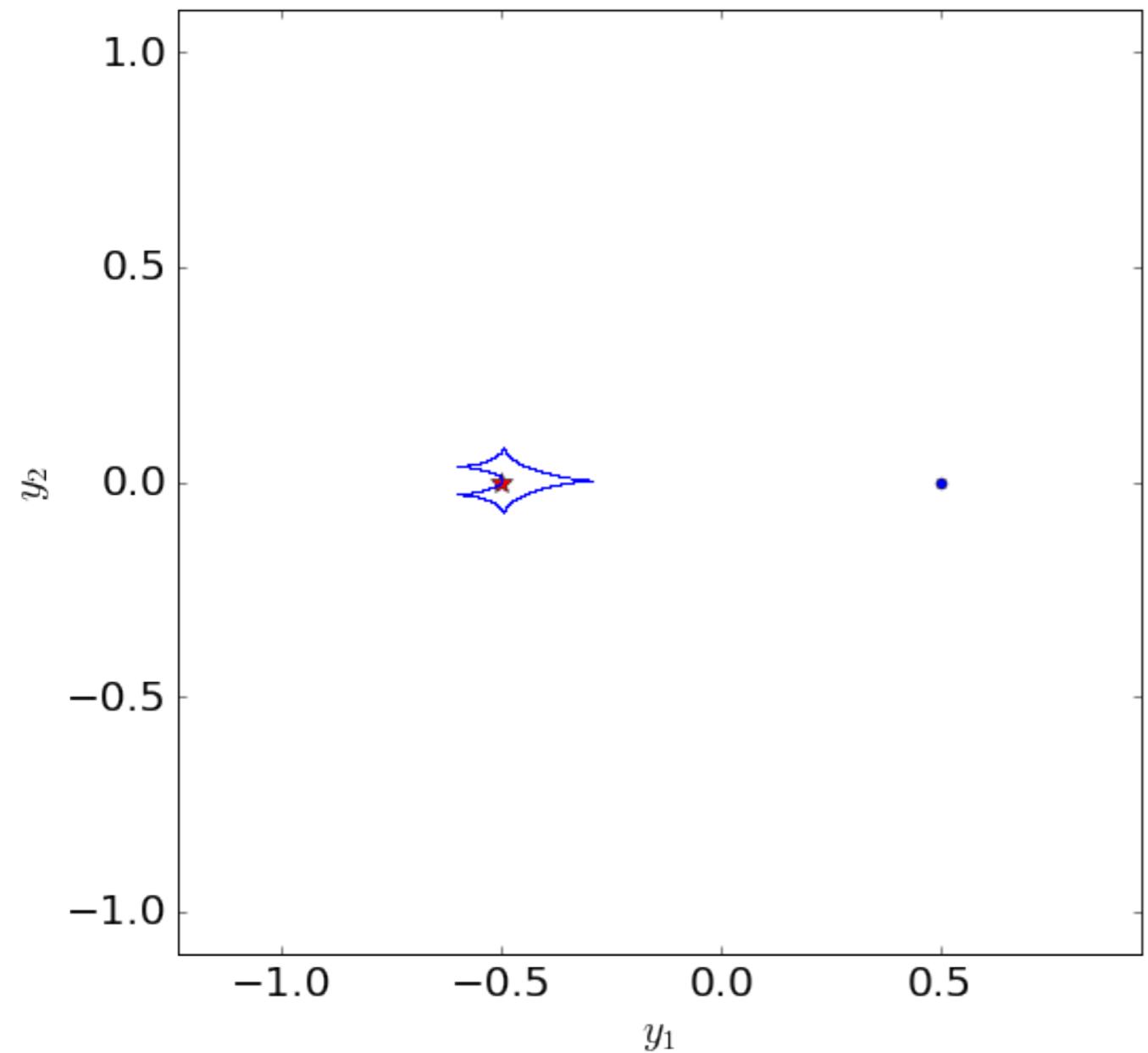
# PLANETARY CAUSTICS IN INTERMEDIATE TOPOLOGIES

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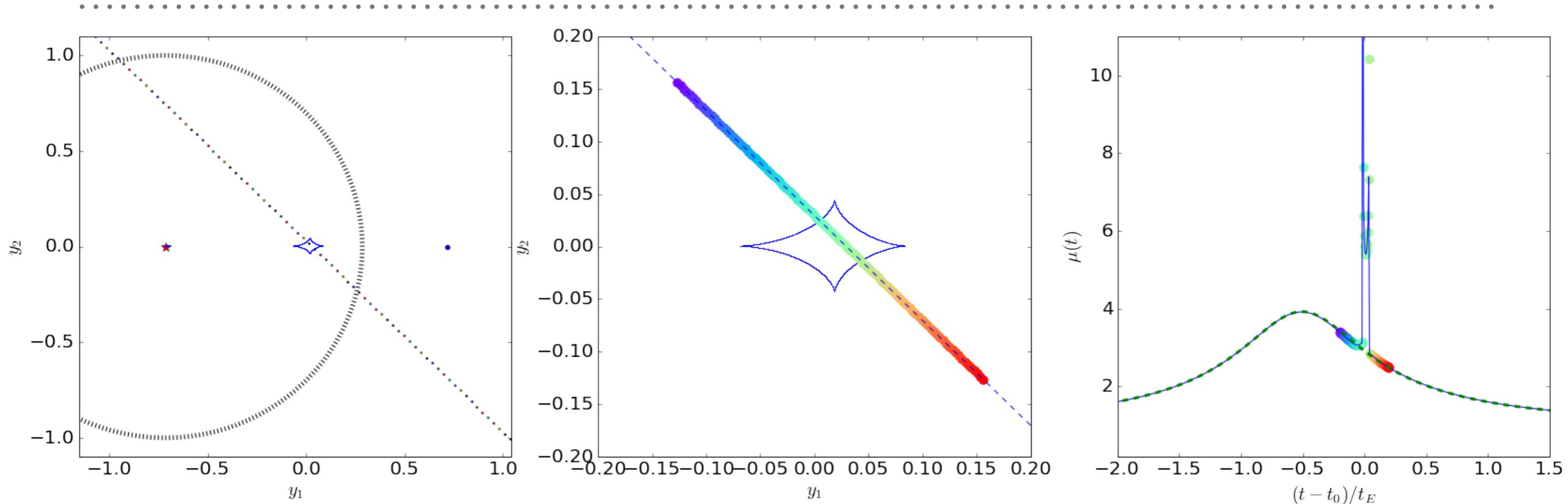


# PLANETARY CAUSTICS IN INTERMEDIATE TOPOLOGIES

---

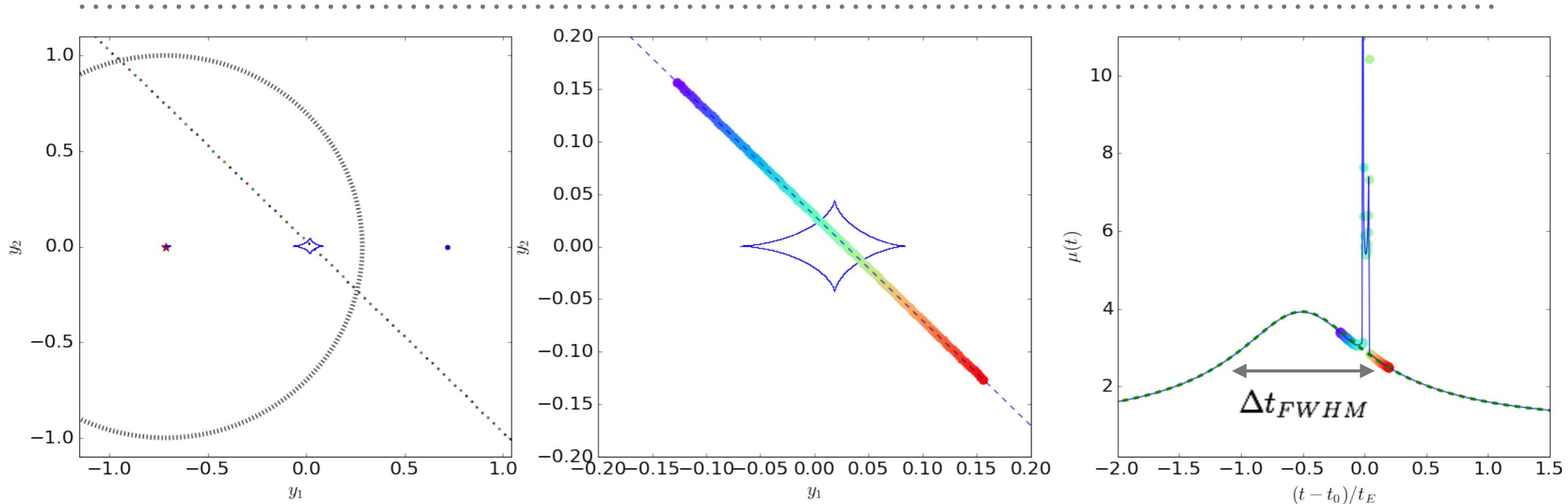


# PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



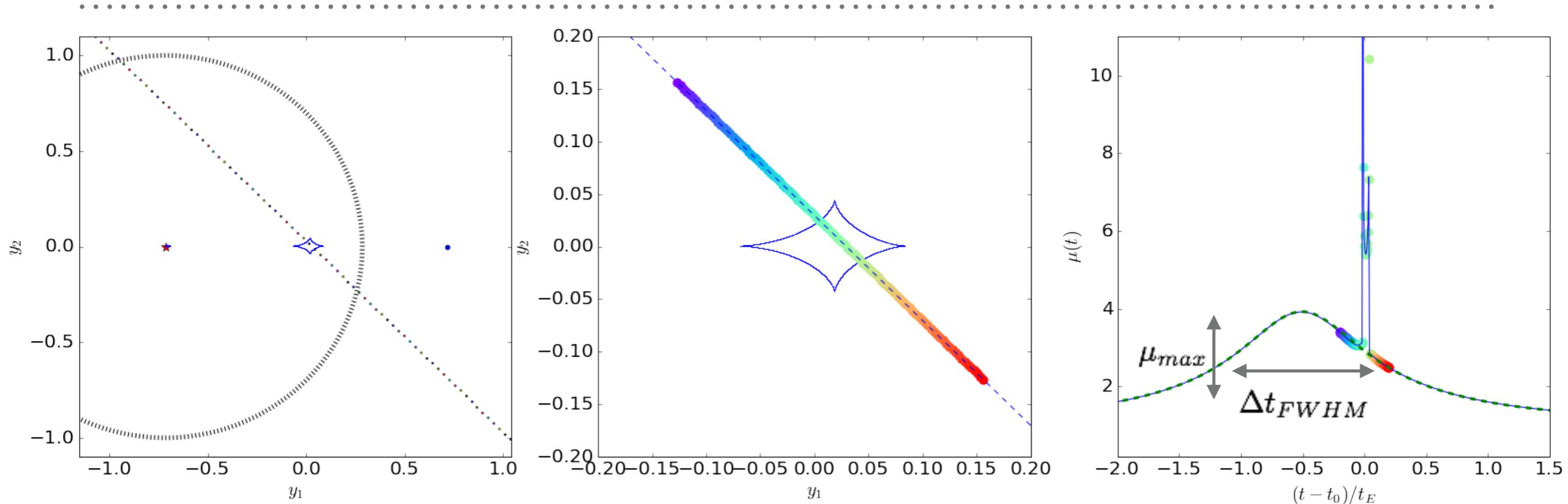
- primary event:
- planetary perturbation:

# PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



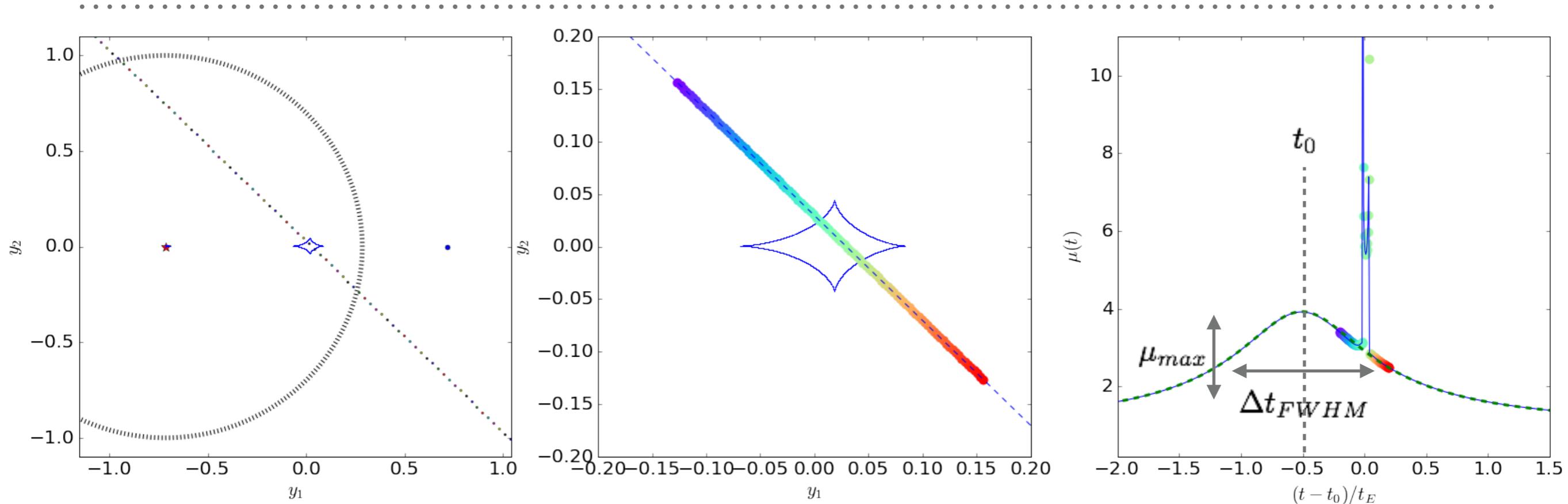
- primary event:  $\Delta t_{FWHM}$
- planetary perturbation:

# PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



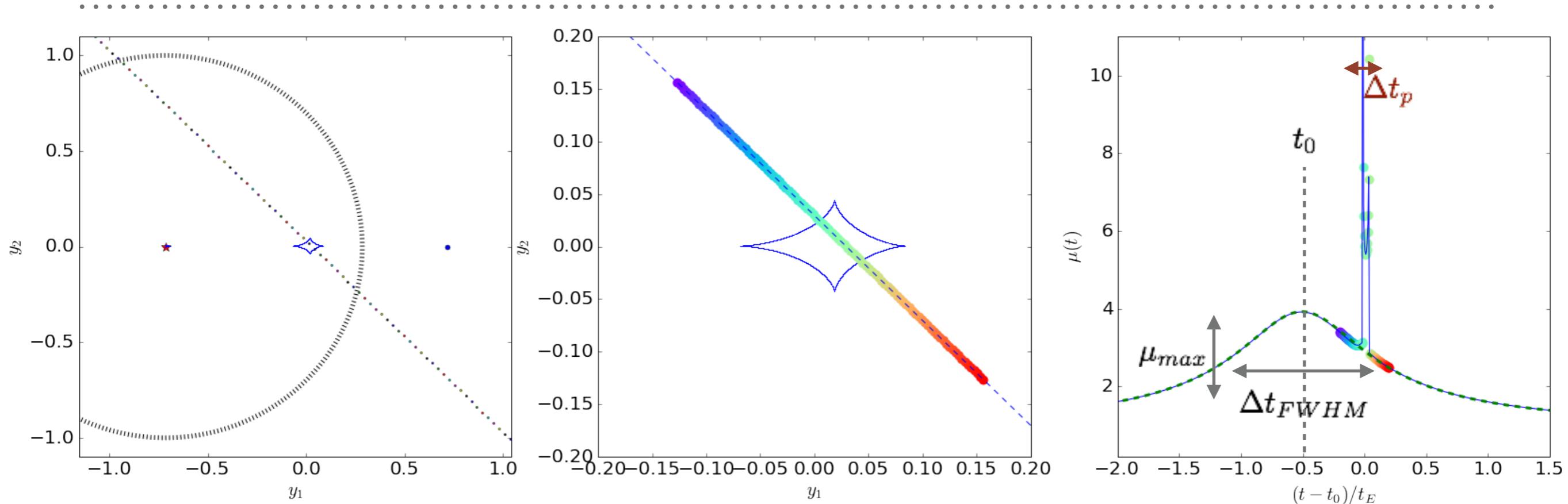
- primary event:  $\Delta t_{FWHM}$      $\mu_{max}$
- planetary perturbation:

# PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



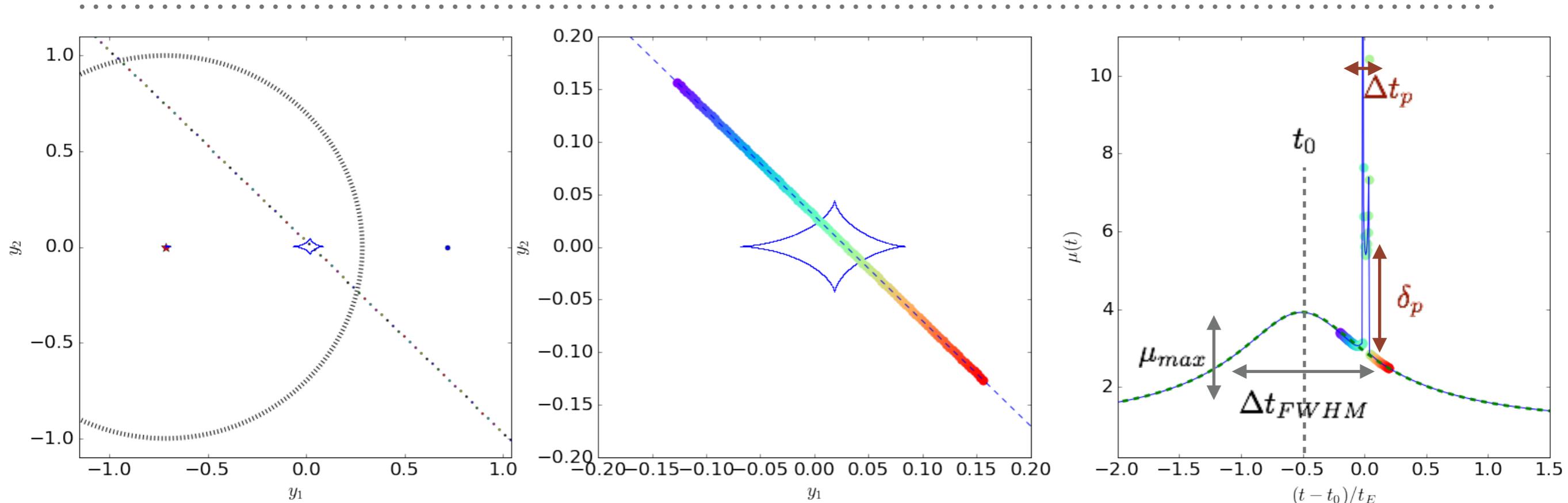
- primary event:  $\Delta t_{FWHM}$     $\mu_{max}$     $t_0$
- planetary perturbation:

# PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



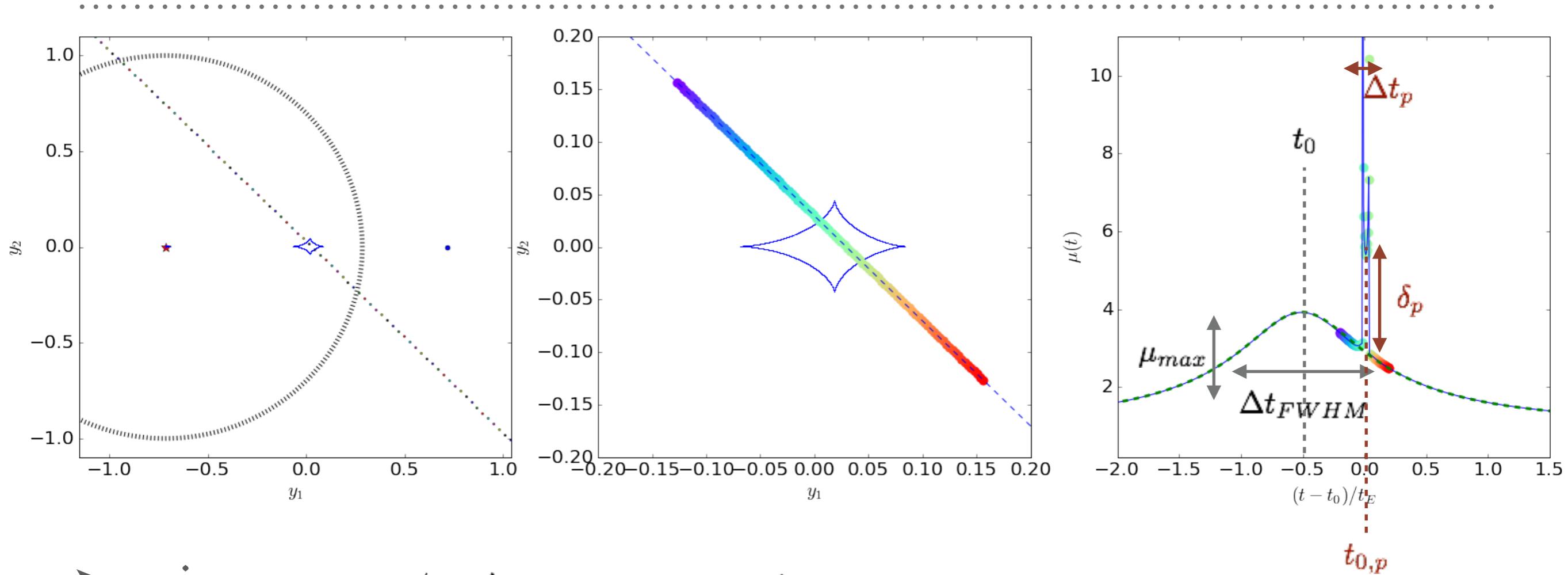
- primary event:  $\Delta t_{FWHM}$     $\mu_{max}$     $t_0$
- planetary perturbation:       $\Delta t_p$

# PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



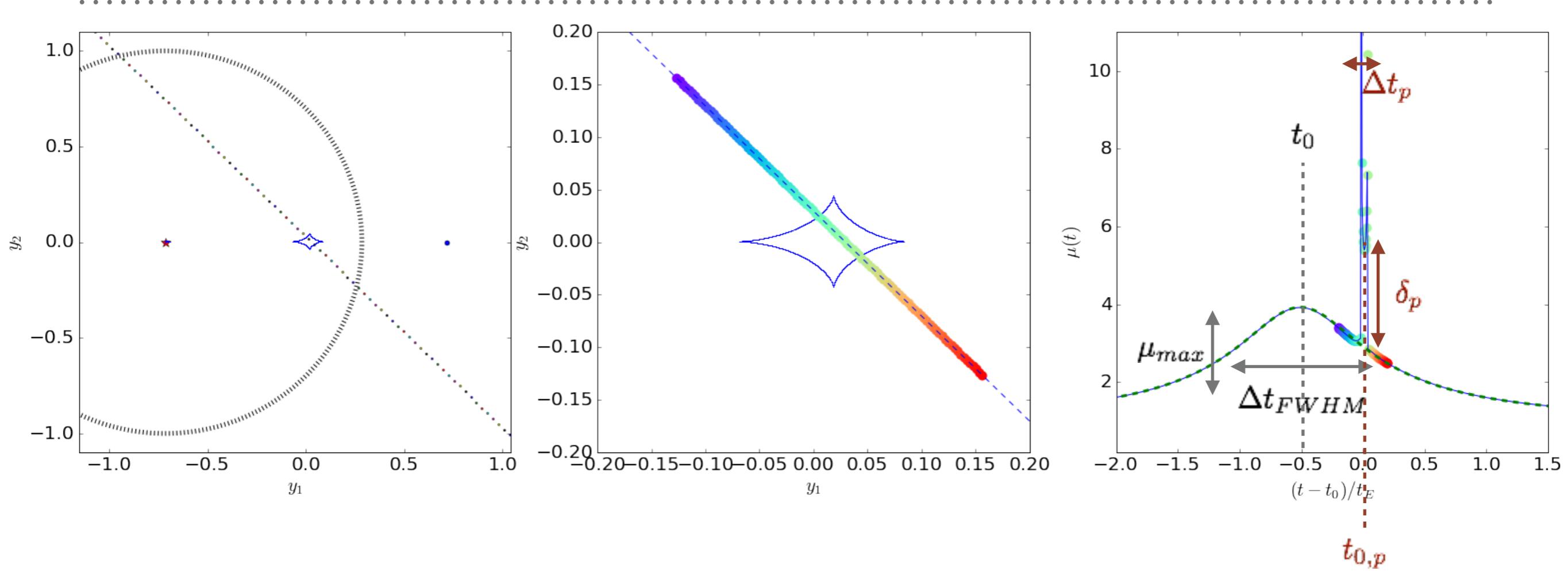
- primary event:  $\Delta t_{FWHM}$     $\mu_{max}$     $t_0$
- planetary perturbation:       $\Delta t_p$     $\delta_p$

# PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



- primary event:  $\Delta t_{FWHM}$     $\mu_{max}$     $t_0$
- planetary perturbation:    $\Delta t_p$     $\delta_p$     $t_{0,p}$

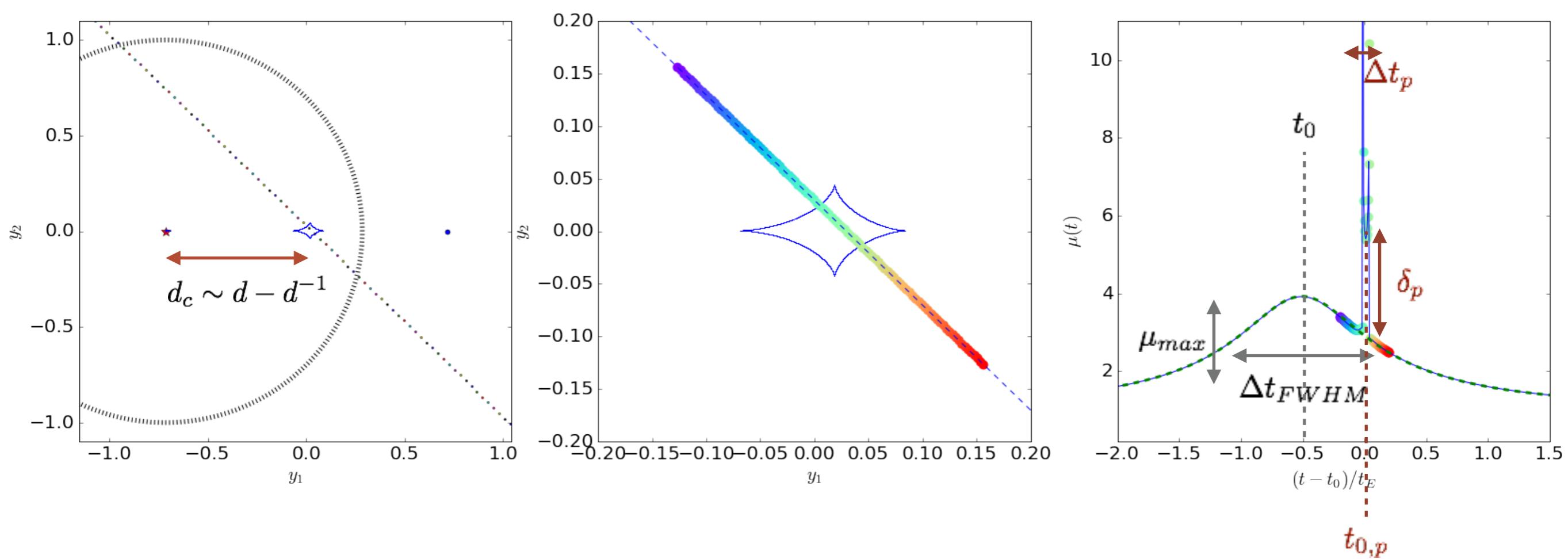
# PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



$$\Delta t_{FWHM}, \mu_{max}, t_0 \Rightarrow \mu(y) = \frac{y^2 + 2}{y\sqrt{y^2 + 4}} \quad y(t) = \sqrt{y_0^2 + \left(\frac{t - t_0}{t_E}\right)^2}$$

$$\Rightarrow \quad y_0 \quad t_E$$

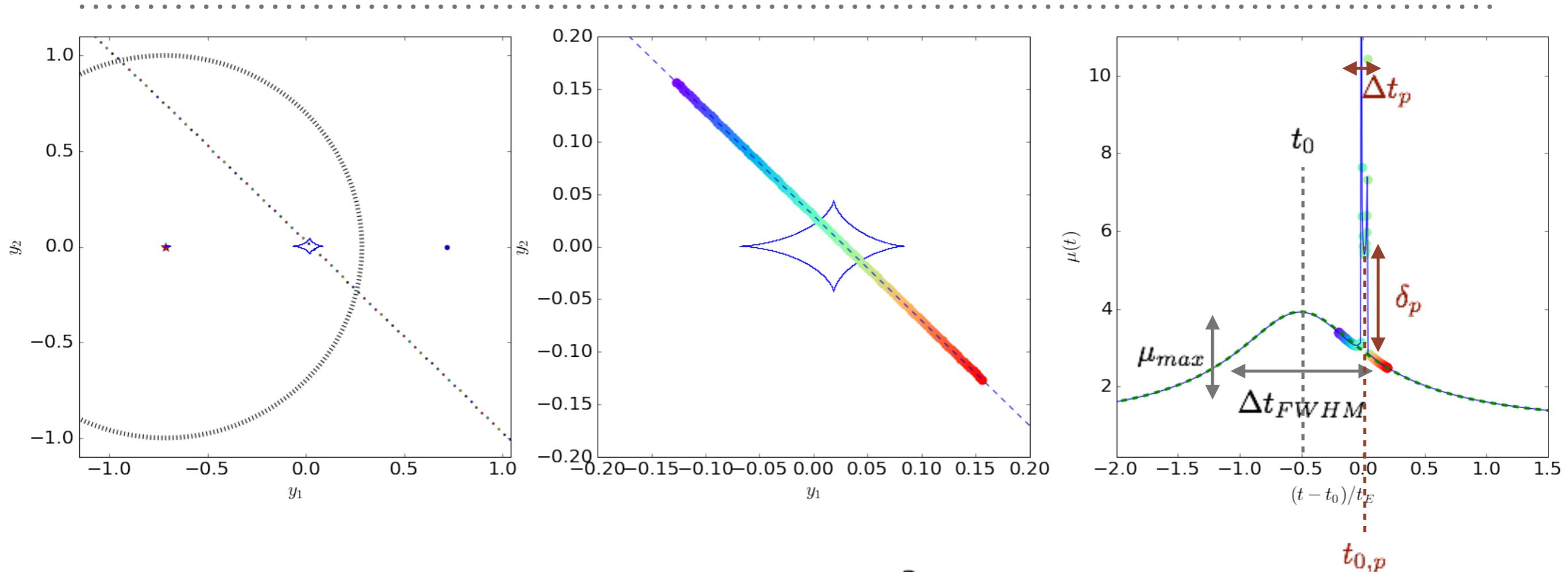
# PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



$$t_{0,p} \Rightarrow y_p = \sqrt{y_0^2 + \left( \frac{t_{0,p} - t_0}{t_E} \right)^2}$$

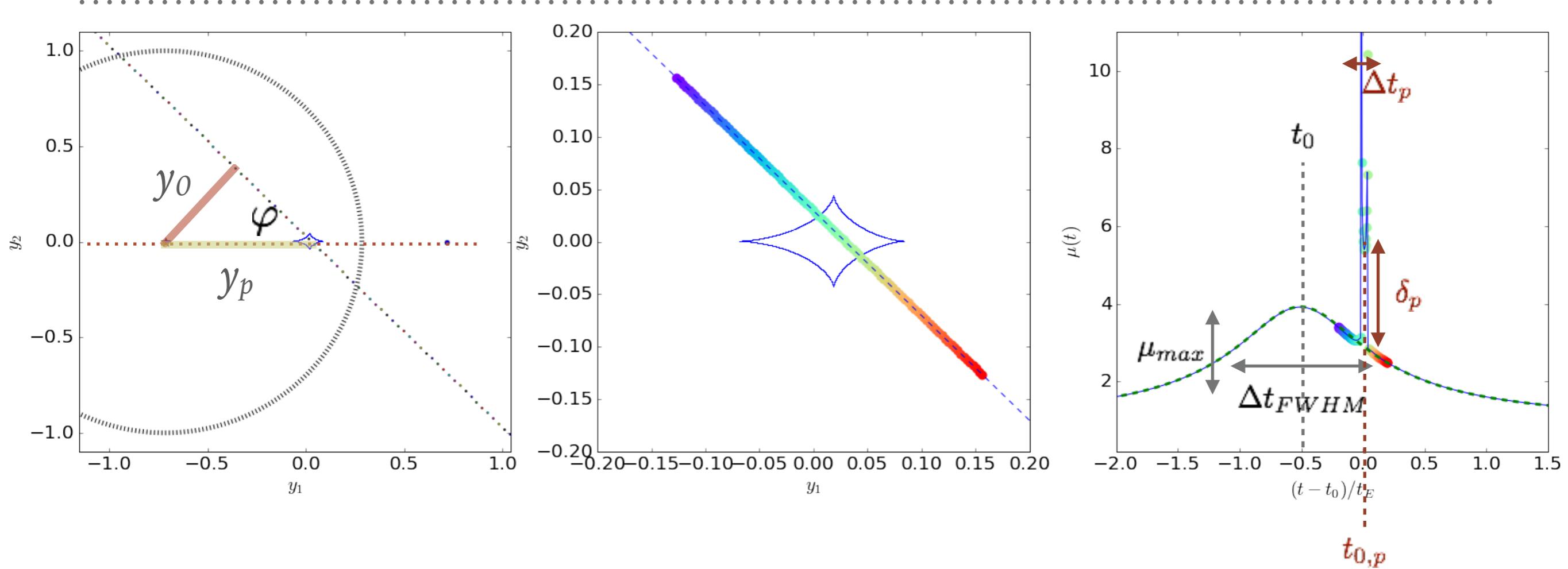
$$\Rightarrow d \sim \frac{1}{2} \left( y_p + \sqrt{y_p^2 + 4} \right)$$

# PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



$$\Delta t_p \sim t_{E,p} \Rightarrow t_E \Rightarrow q = \left( \frac{t_{E,p}}{t_E} \right)^2$$

# PLANET PROPERTIES “READ OFF” OF THE LIGHT CURVES



$$y_0, y_p \Rightarrow \varphi = \sin^{-1} \frac{y_0}{y_p}$$

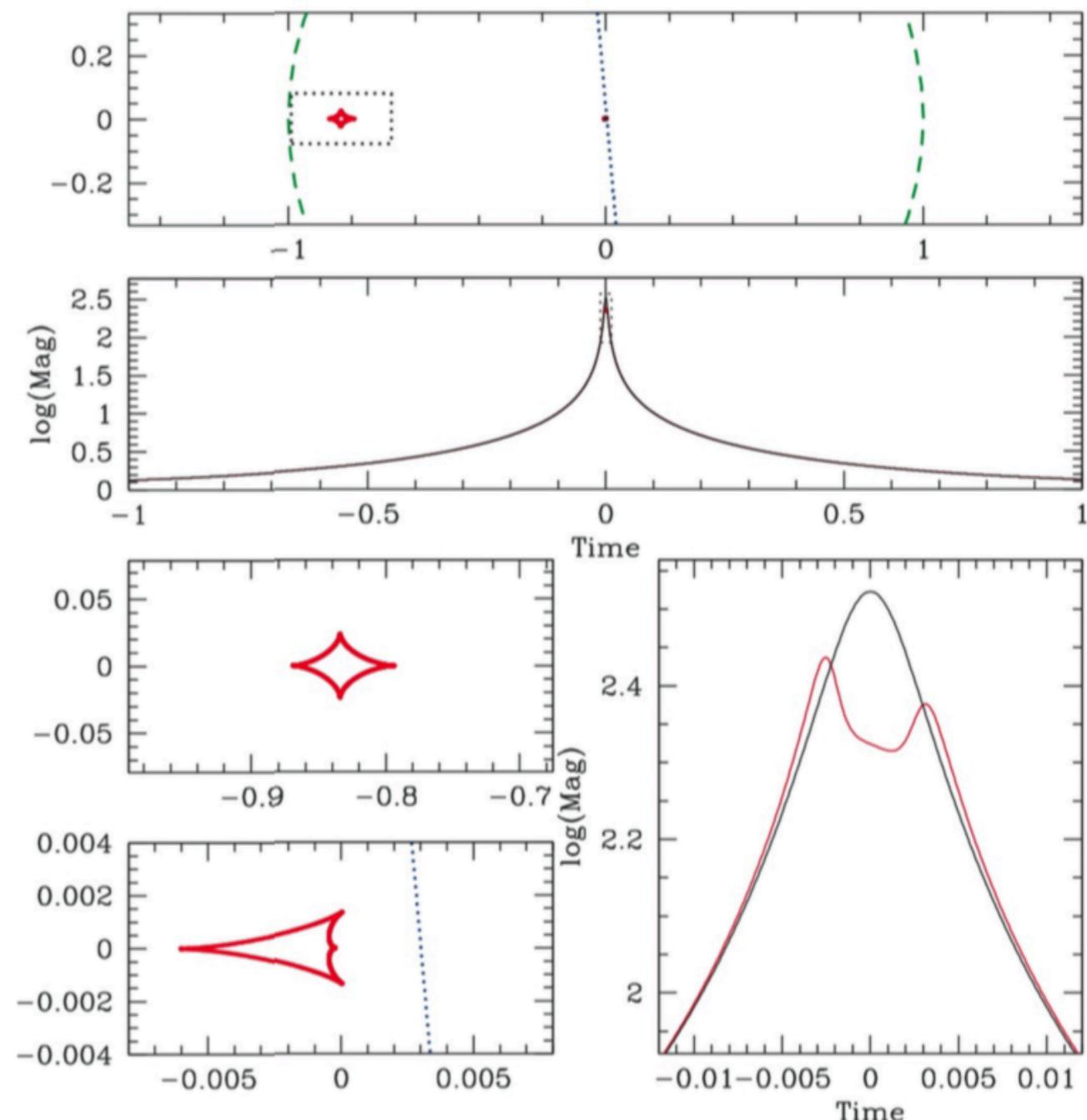
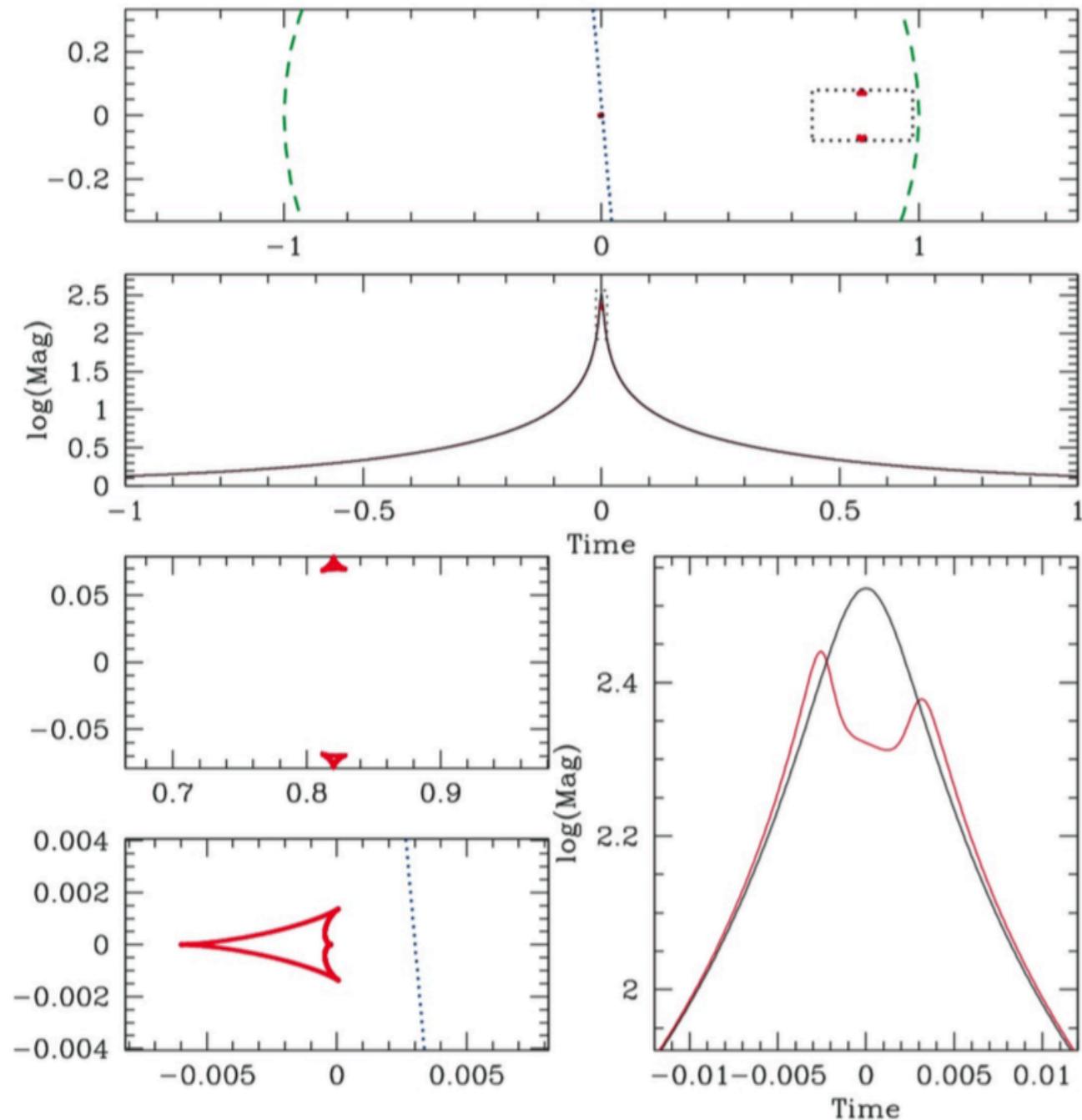
# TO SUMMARIZE

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- different caustic topologies give rise to different kind of perturbations on the light curves
- planets can be detected in only a few qualitatively different ways:
  - at relatively low magnification of the primary, if the source crosses the planetary caustics from close or wide planets
  - near the peak of the light curve, if the source has a small impact parameter, in both cases of wide and close planets
  - at modest to high-magnification, through the perturbations from the resonant caustic.
  - in the case of free-floating planets, as single, short time-scale events.
- Light curves can be used to extract parameters of the primary and of the planet
- As seen for single point masses, degeneracies can be broken (finite source effects, microlensing parallax)
- Some cases are more difficult than others...

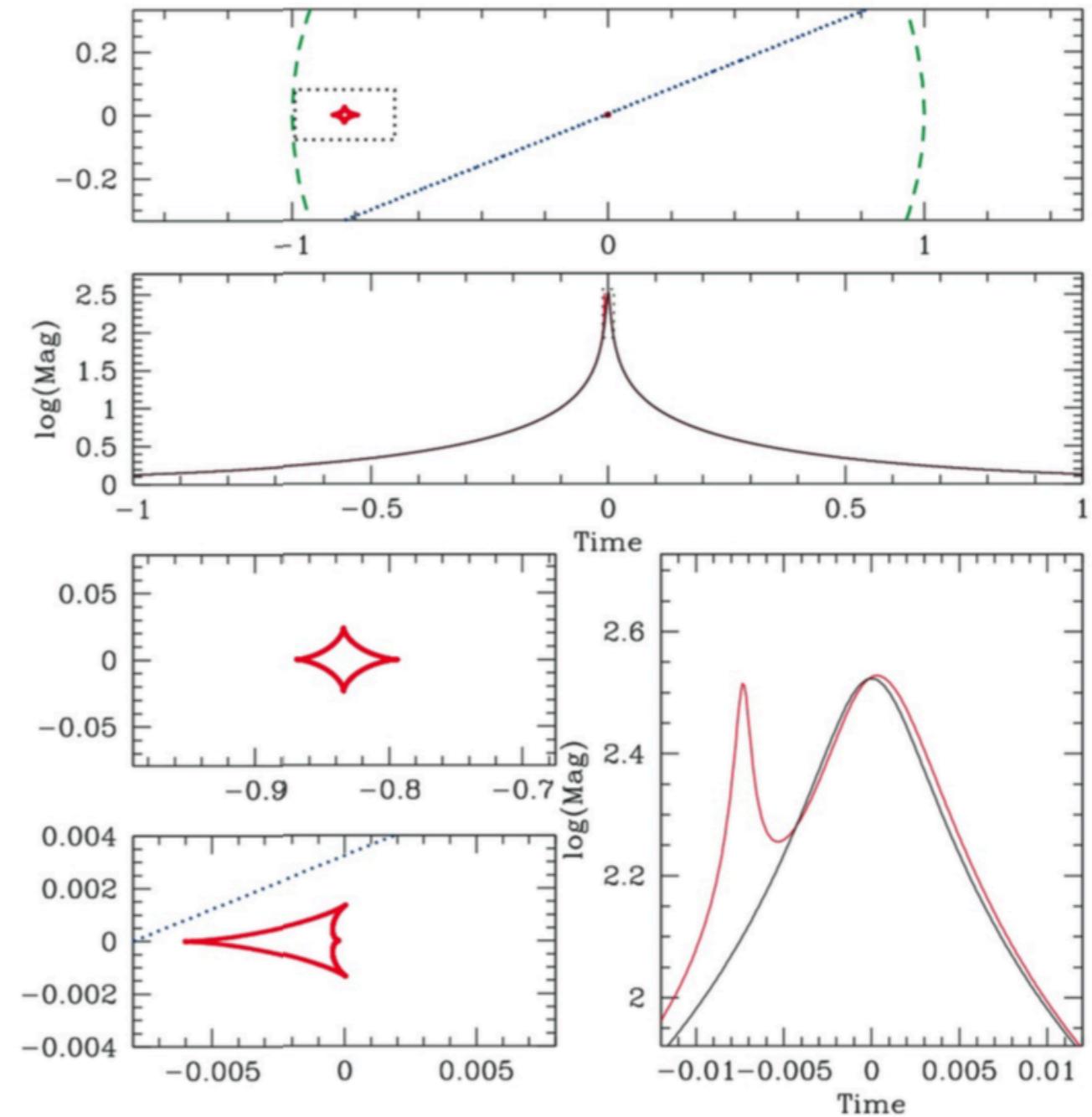
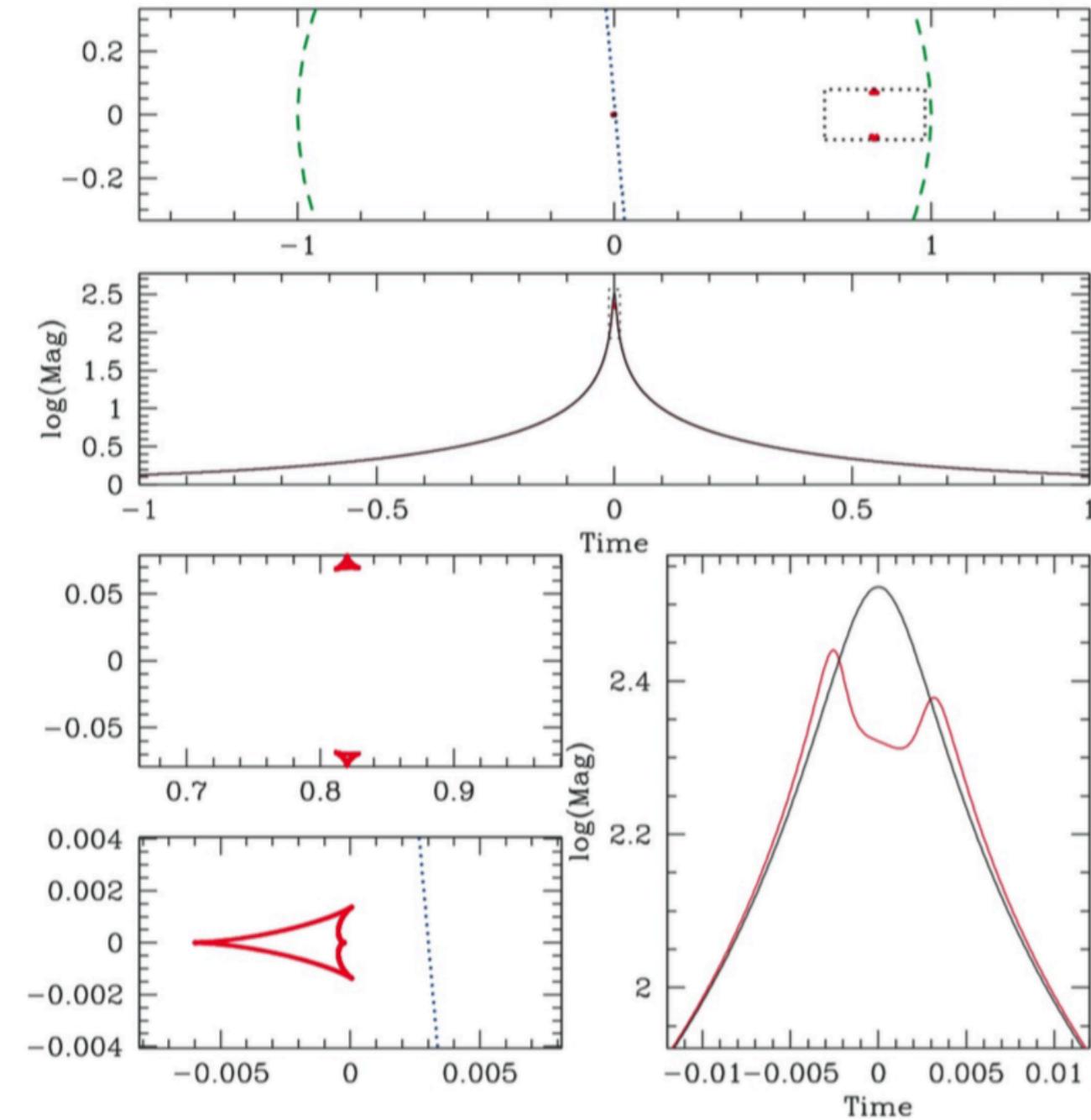
# CLOSE-WIDE DEGENERACY

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# DOUBLE HORN OR BUMP

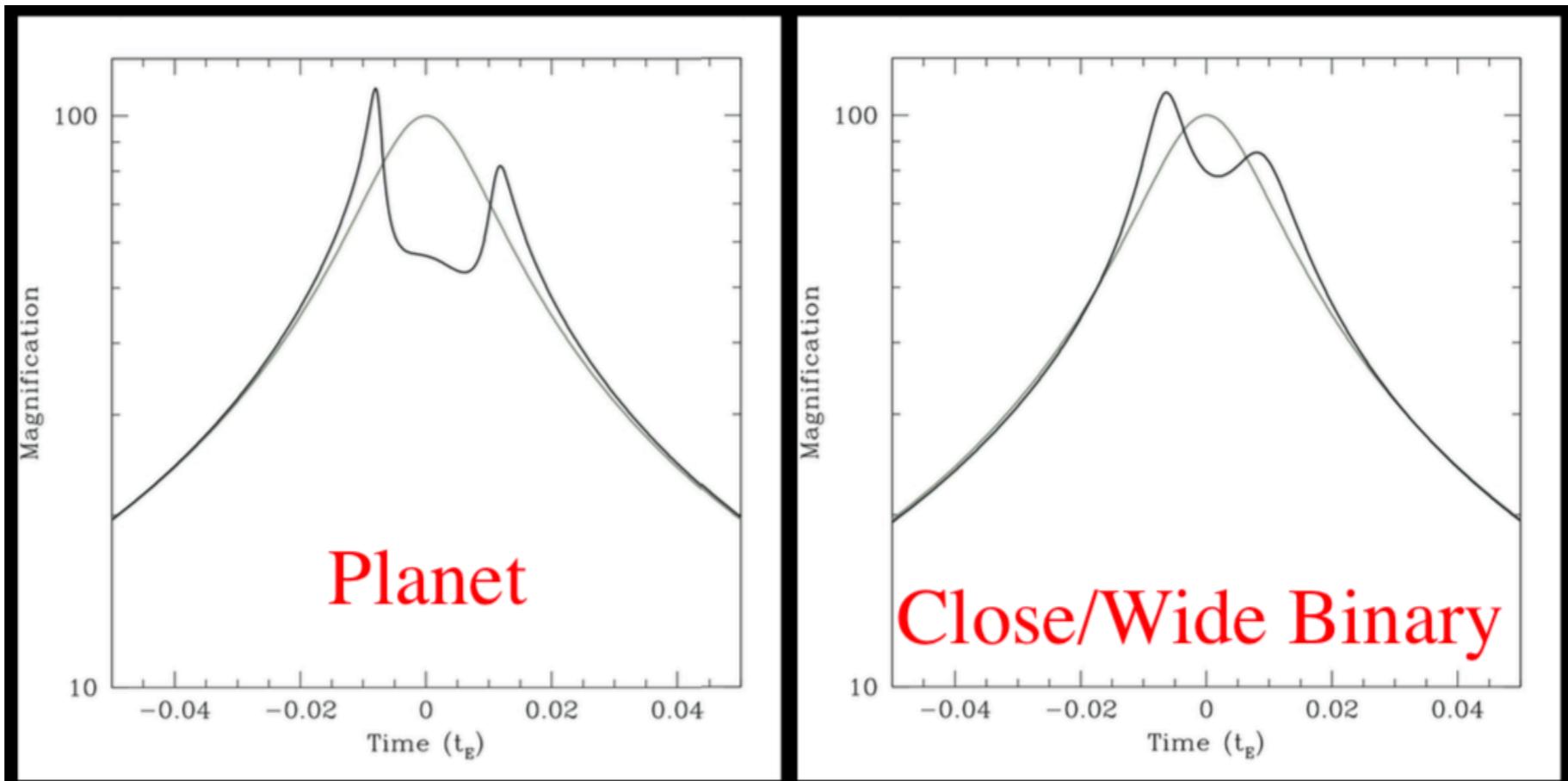
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# UNCERTAINTIES: SOME FEATURES CAN BE EXPLAINED IN DIFFERENT WAYS

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*Short duration deviations at the peak:*



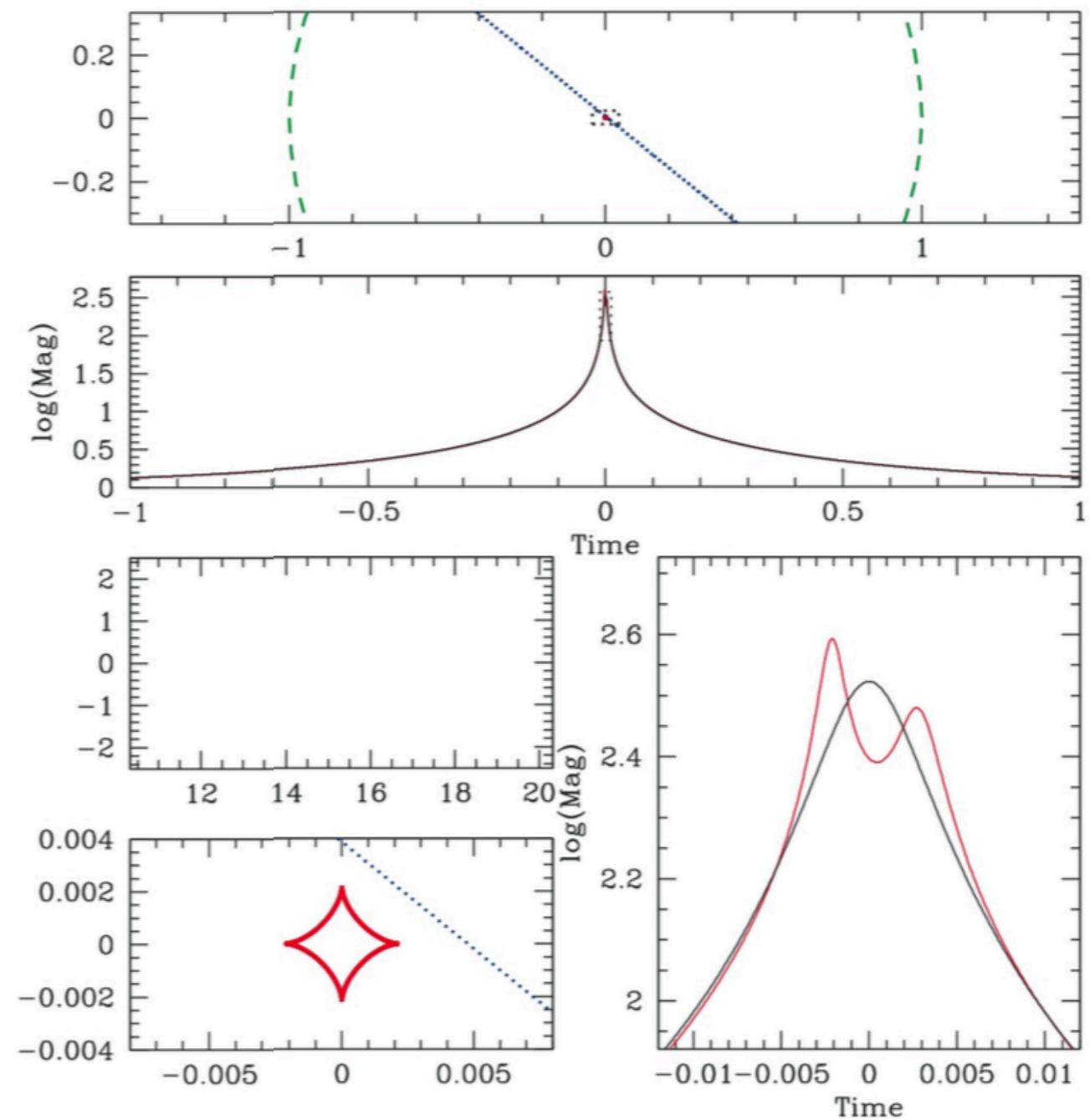
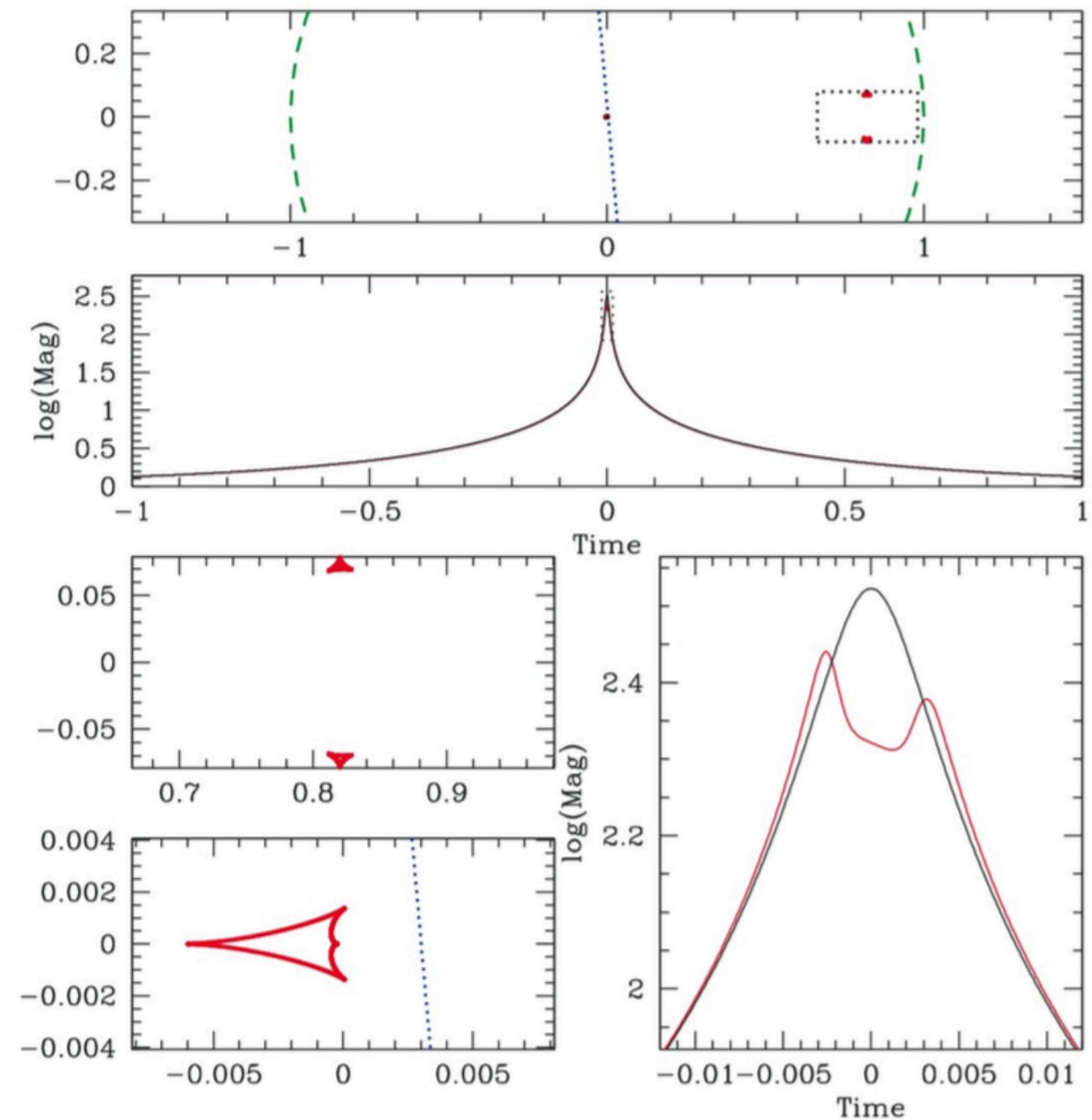
*Can be caused by:*

- Planets
- very close or very wide binary lenses

*Rule: look for the central bump in the middle of the horns*

# CLOSE BINARIES VS PLANETS

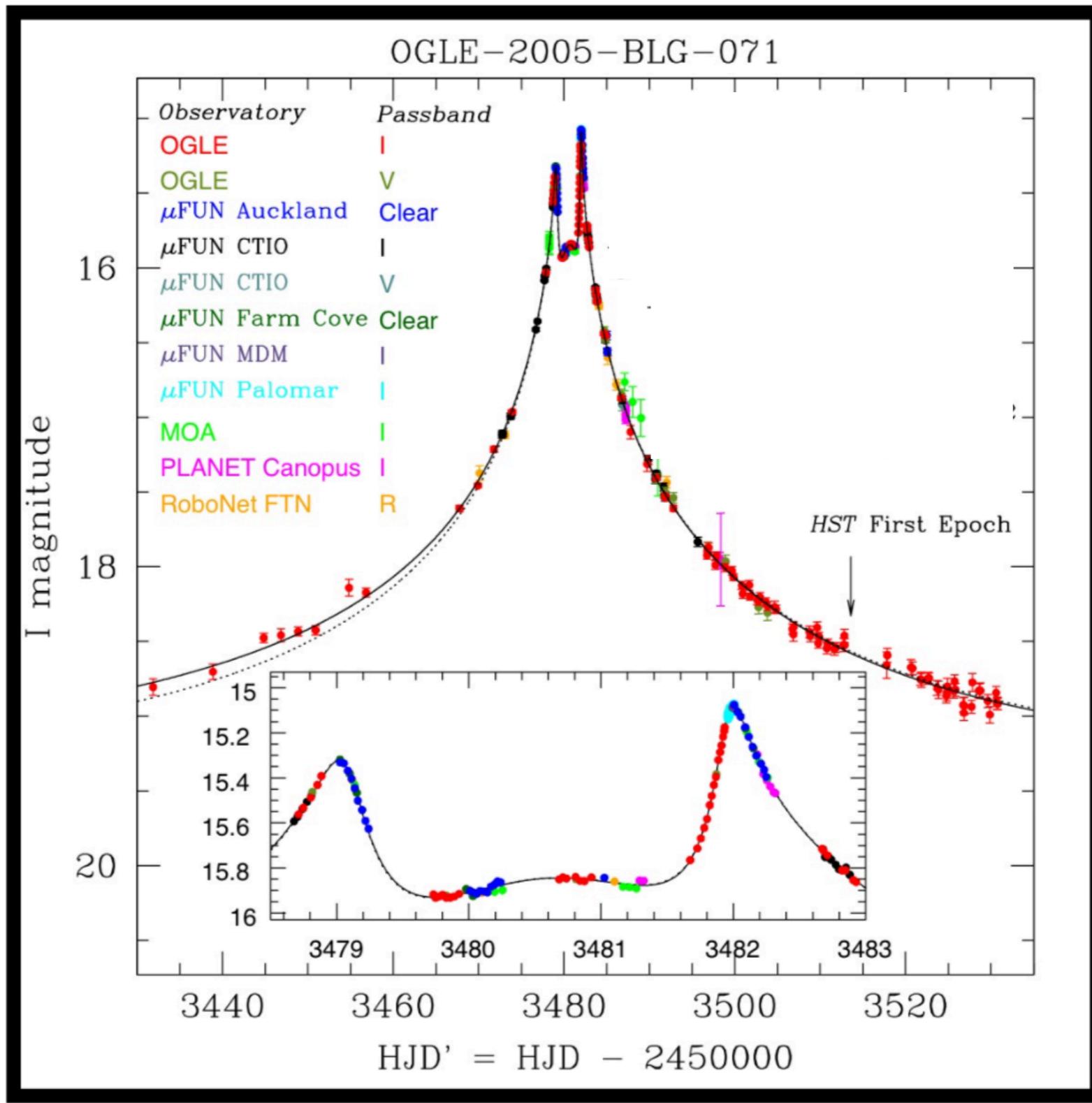
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# QUIZ

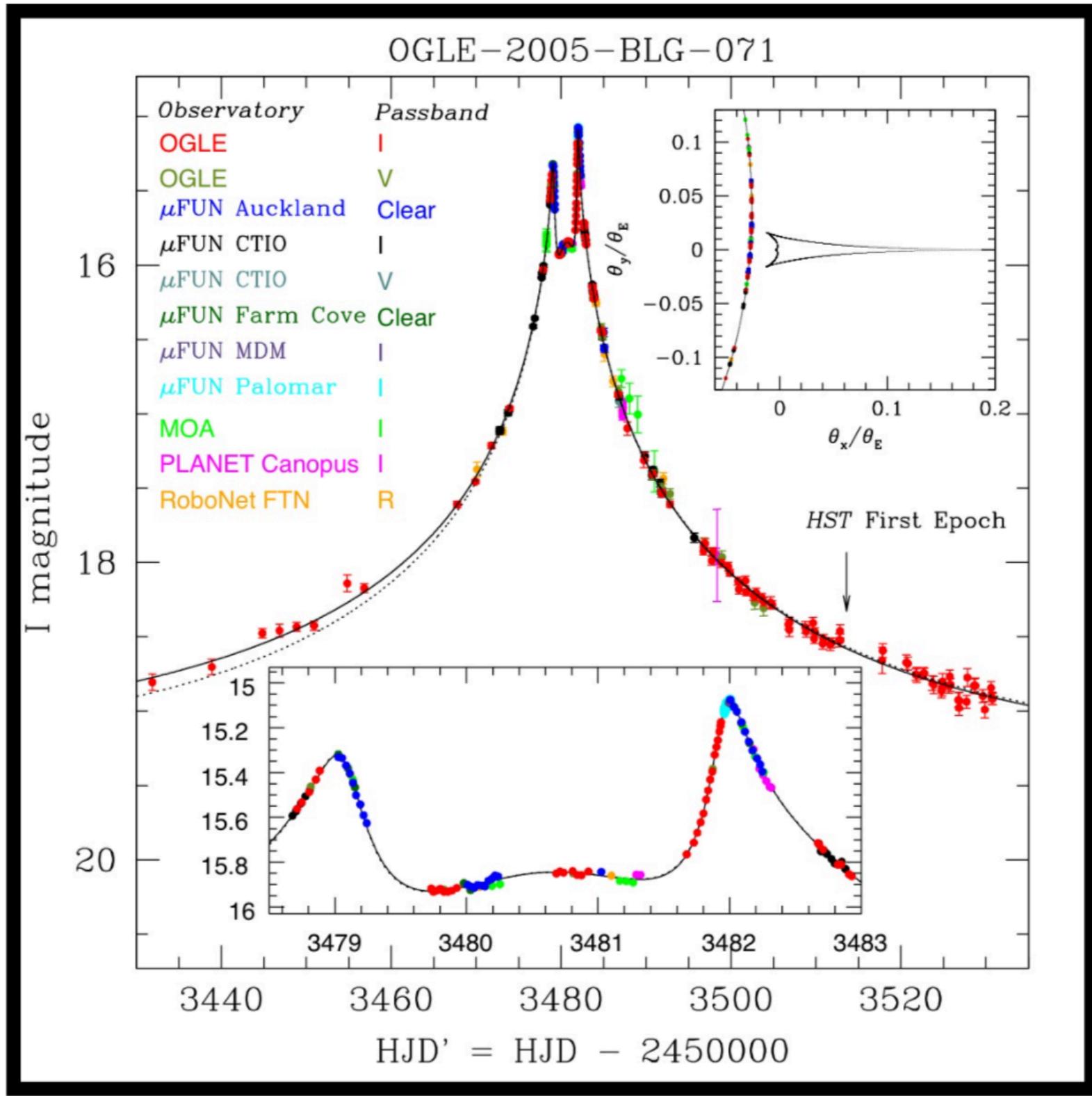
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Udalski et al. 2005, Dong et al. 2009



# QUIZ

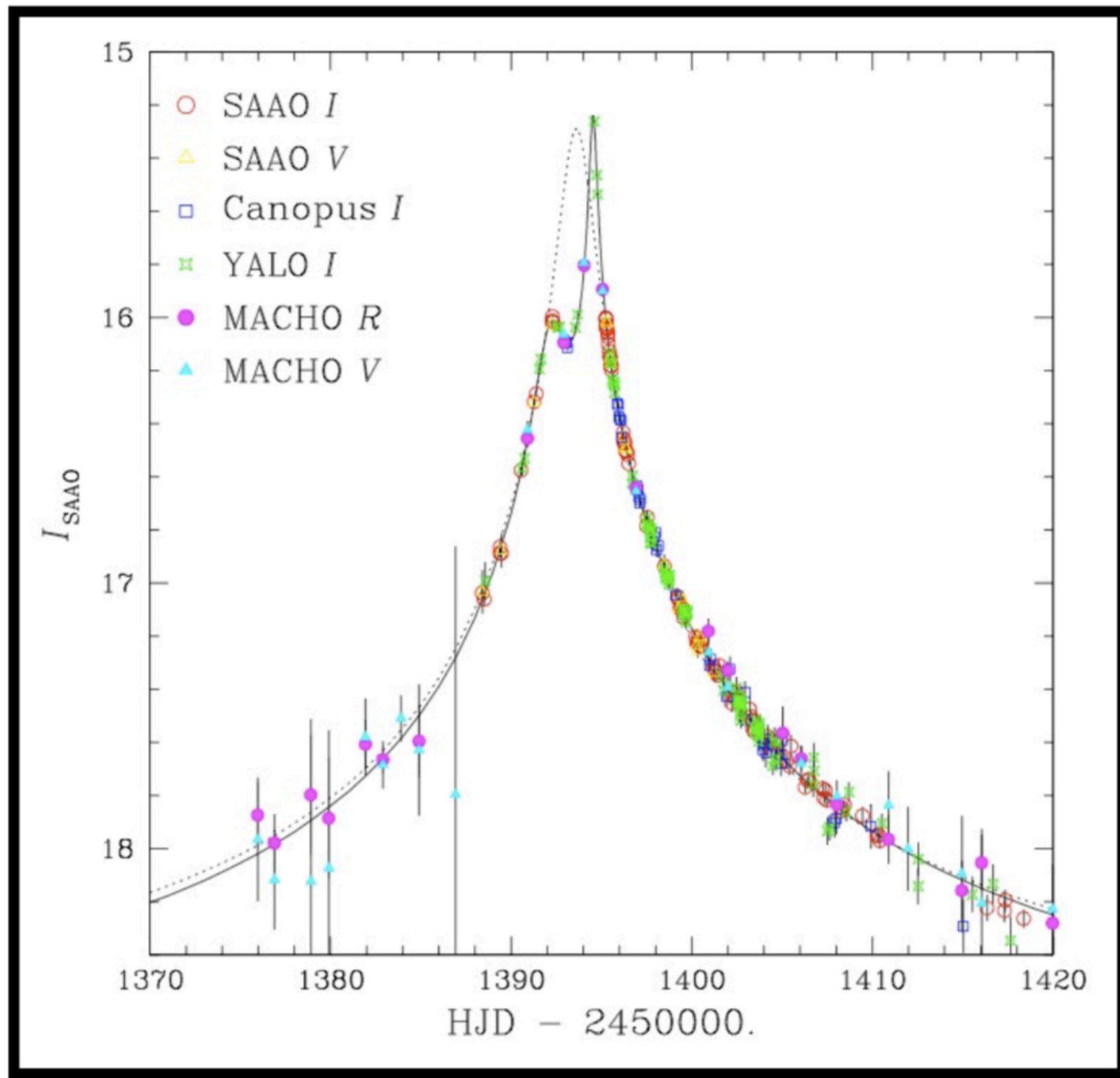
Udalski et al. 2005, Dong et al. 2009



# QUIZ

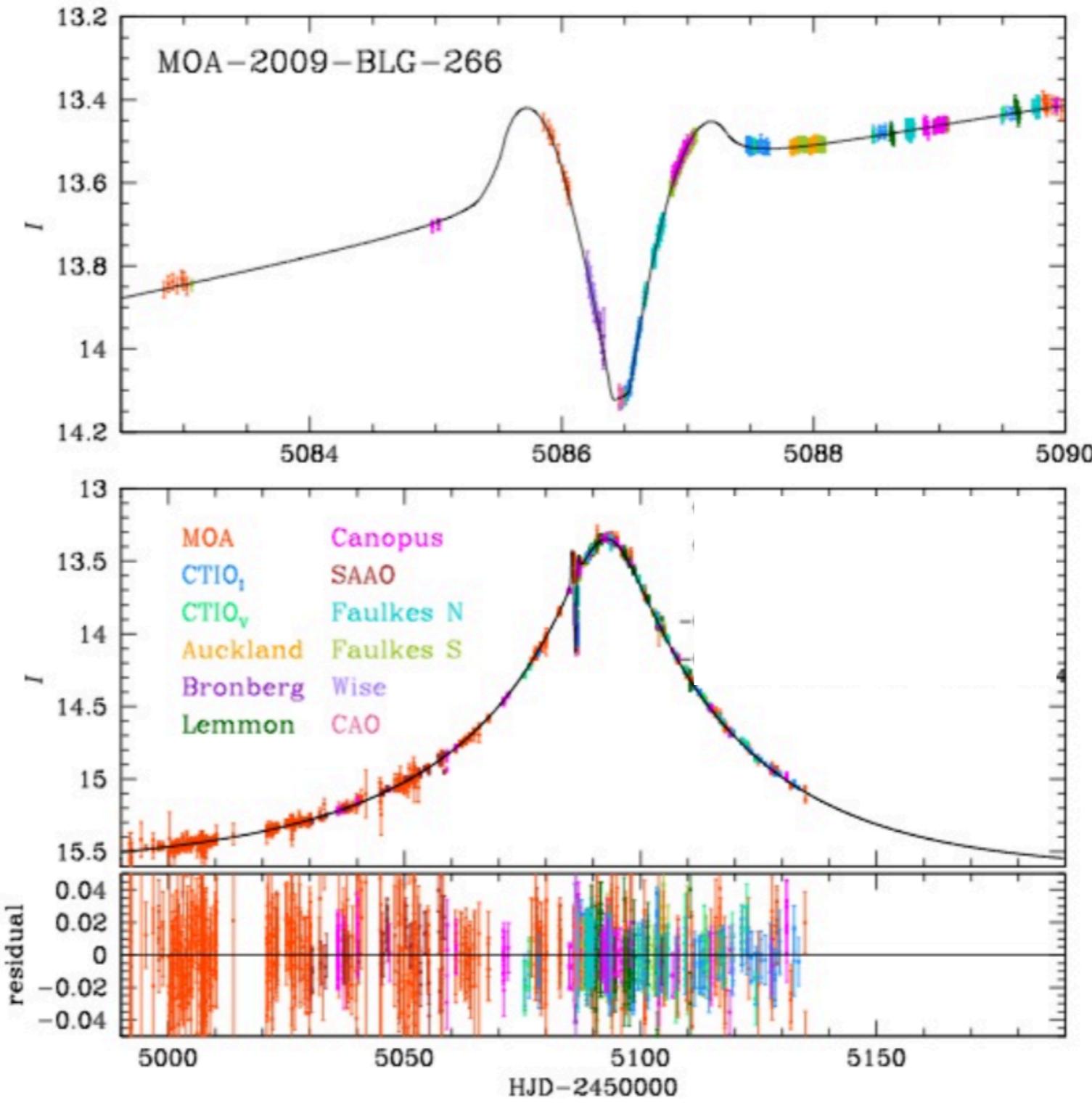
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Albrow et al. 2002



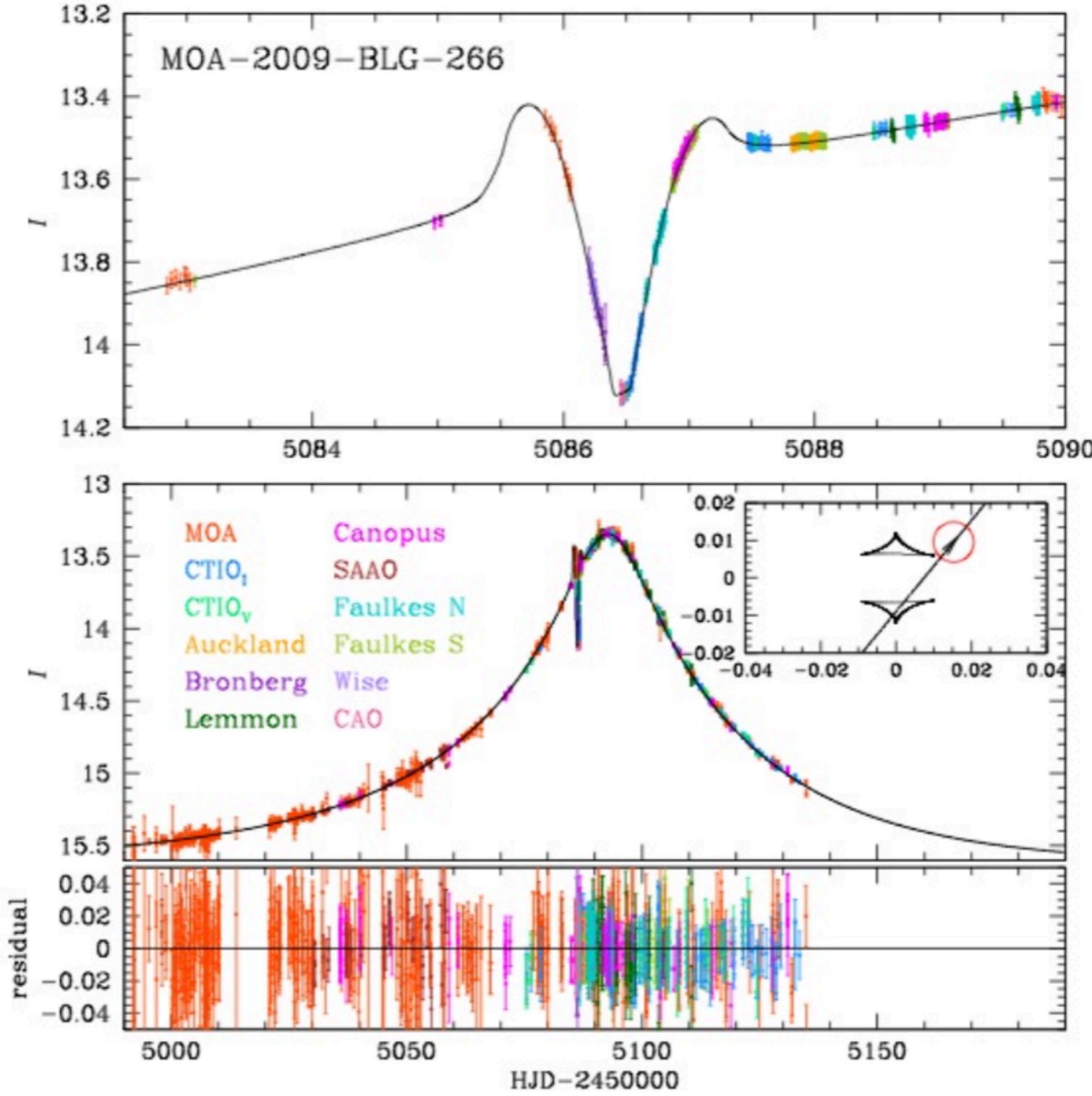
# QUIZ

(Muraki et al. 2011)



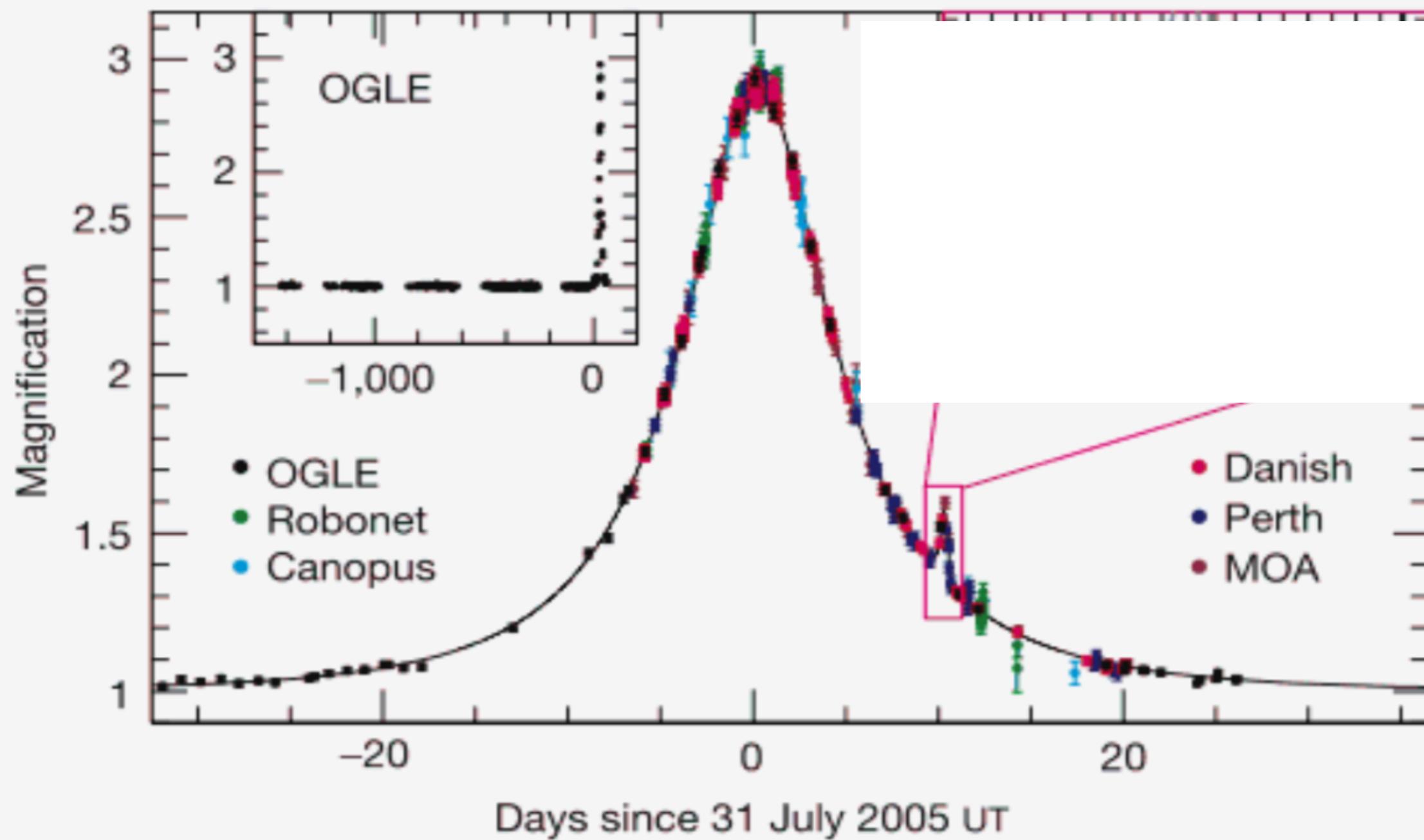
# QUIZ

(Muraki et al. 2011)



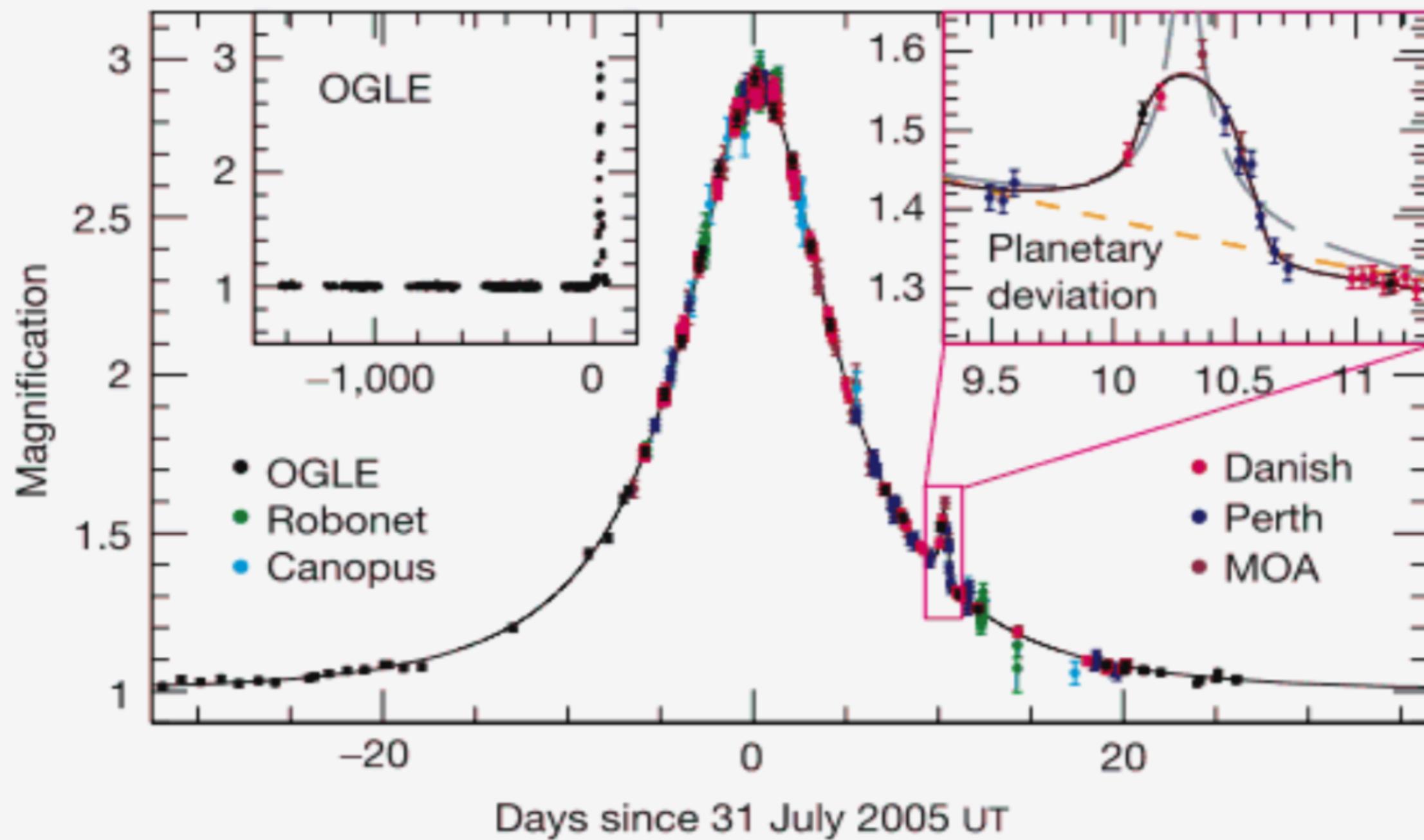
# QUIZ

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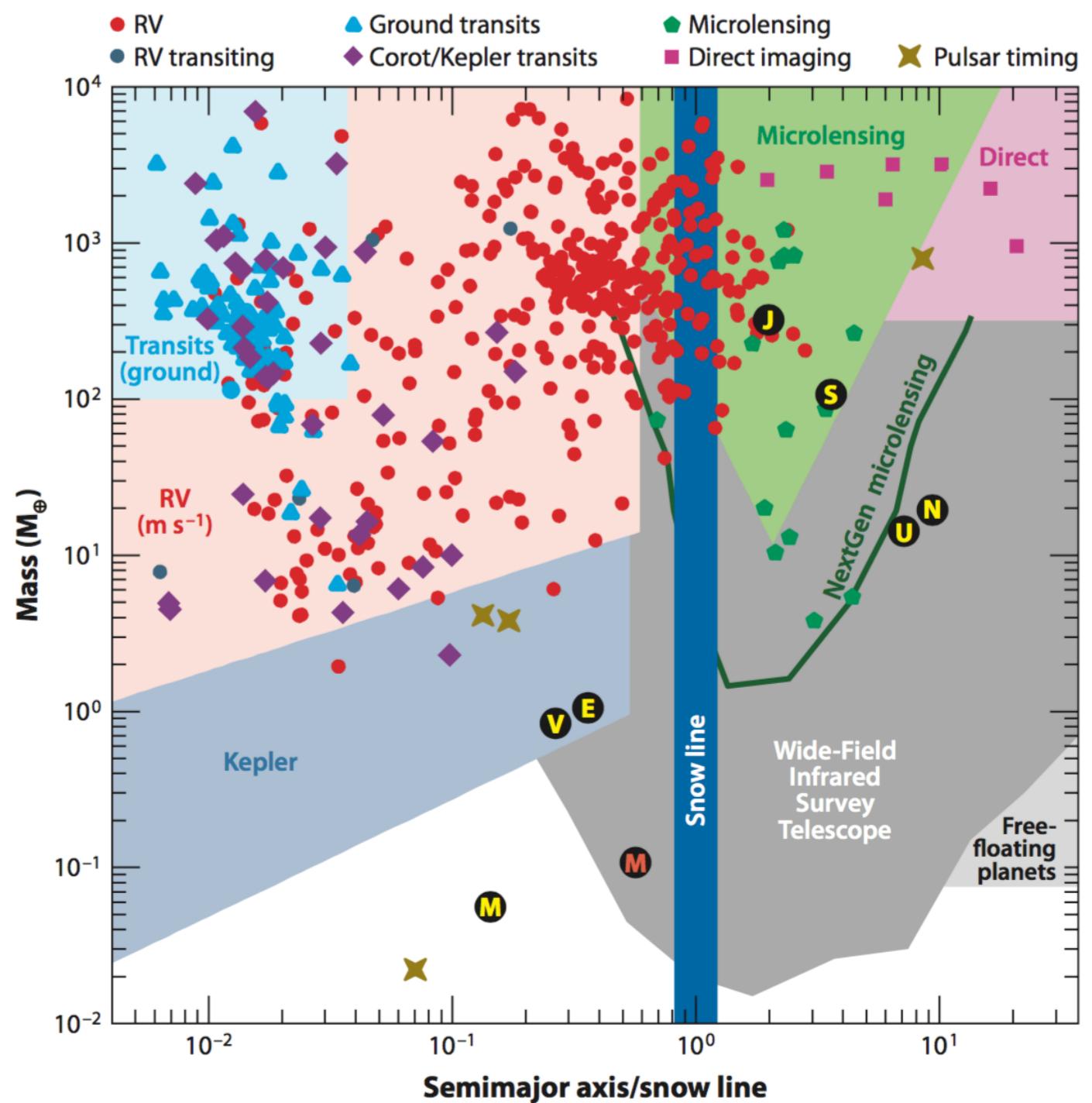
# QUIZ

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# ADVANTAGES OF USING MICROLENSING FOR PLANET SEARCHES

- ~90 planets discovered via microlensing so far
- $d_{\min} = 0.66 \text{ AU}$
- bulk of planets at  $d \sim 3 \text{ AU}$
- wide range of masses
- complementary technique to others that are most sensitive to planets near their host stars (transits, radial velocity)



# ADVANTAGES OF USING MICROLENSING FOR PLANET SEARCHES

- planets are most easily identified when they are at a distance  $\sim ER$
- example: 1 mas at  $\sim 5\text{kpc} = 5\text{AU}$
- peak sensitivity beyond the snow line
- the snow line marks a very important region for planet formation! Giant planets can form only beyond the snow line.

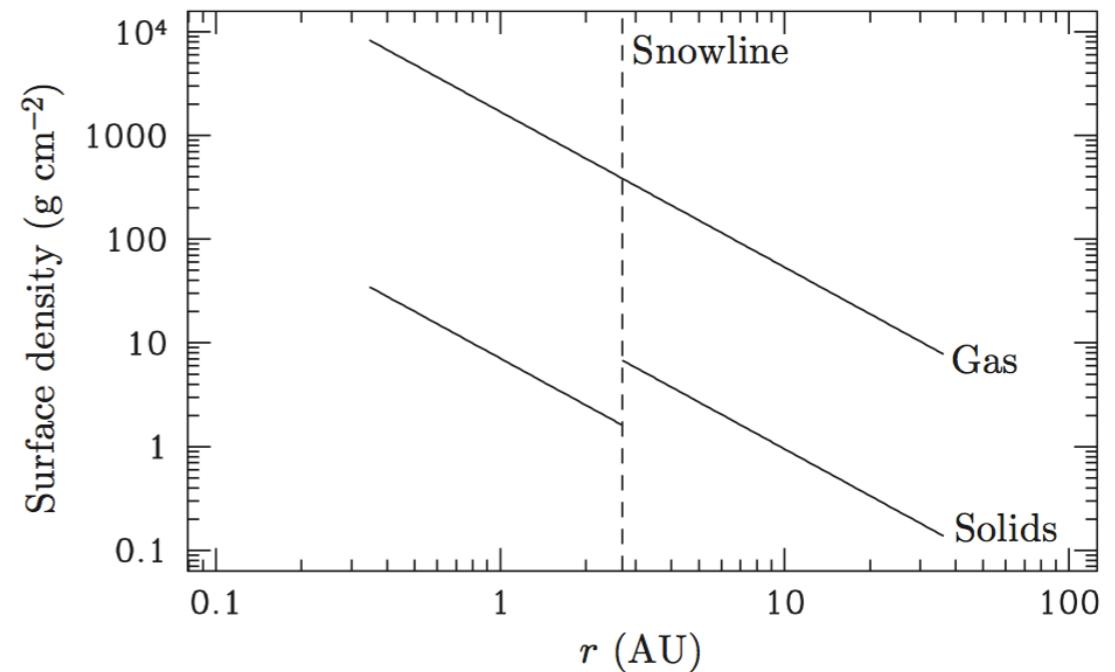
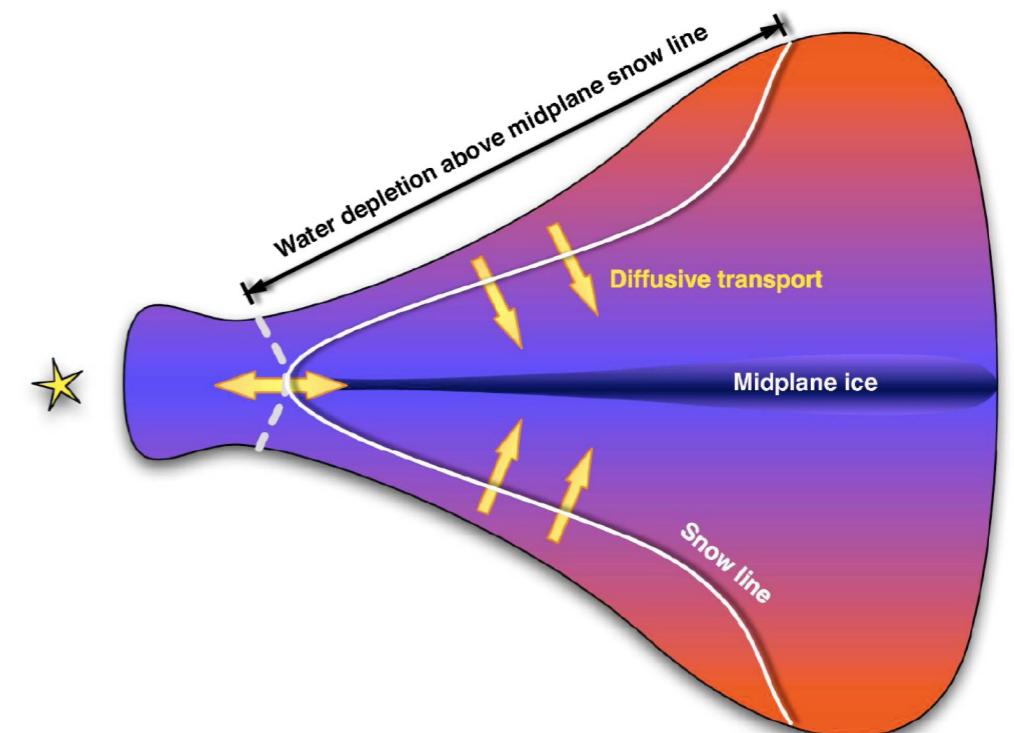


Fig. 1.1. The surface density in gas (upper line) and solids (lower broken line) as a function of radius in Hayashi's minimum mass Solar Nebula. The dashed vertical line denotes the location of the snowline.



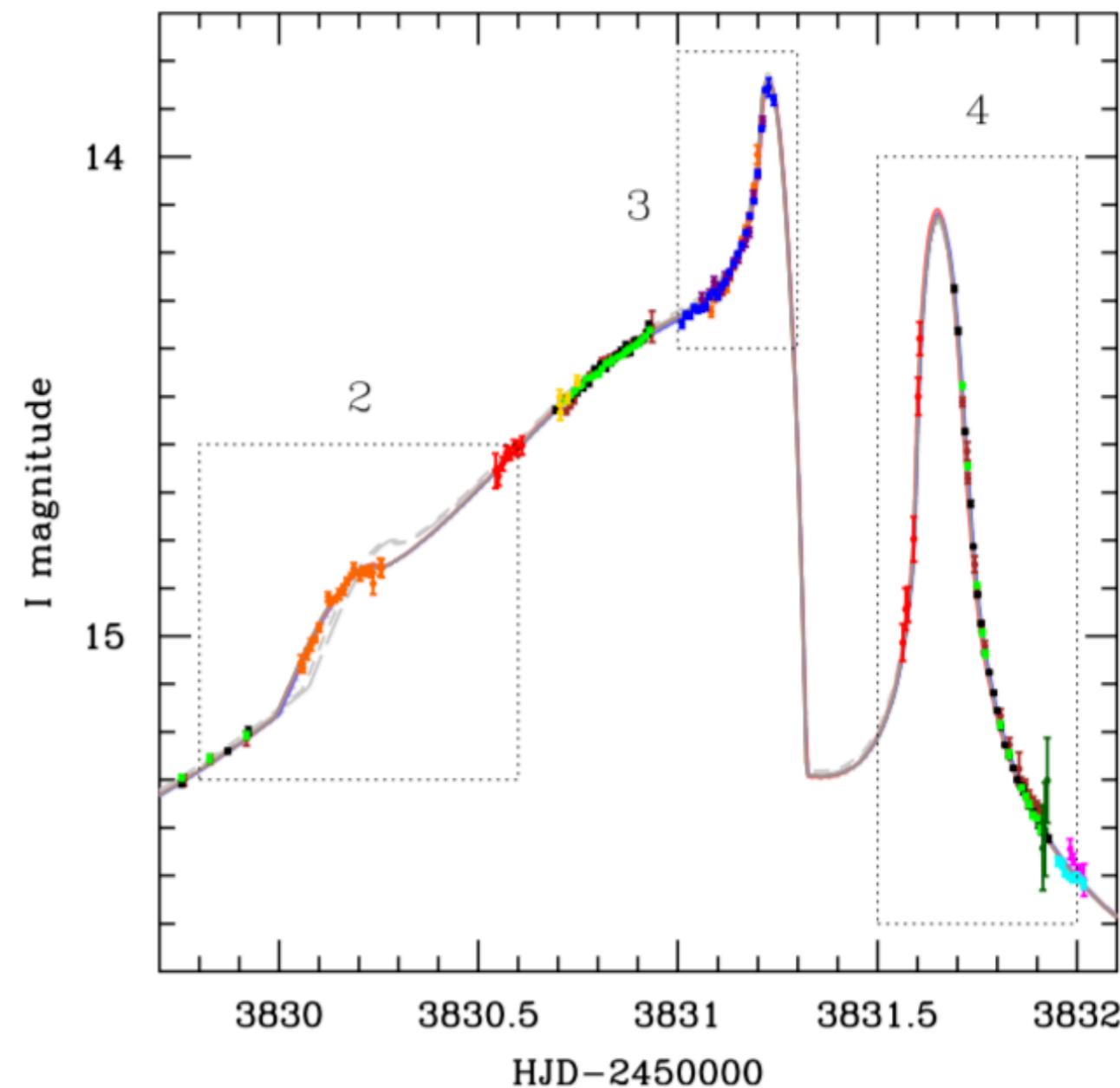
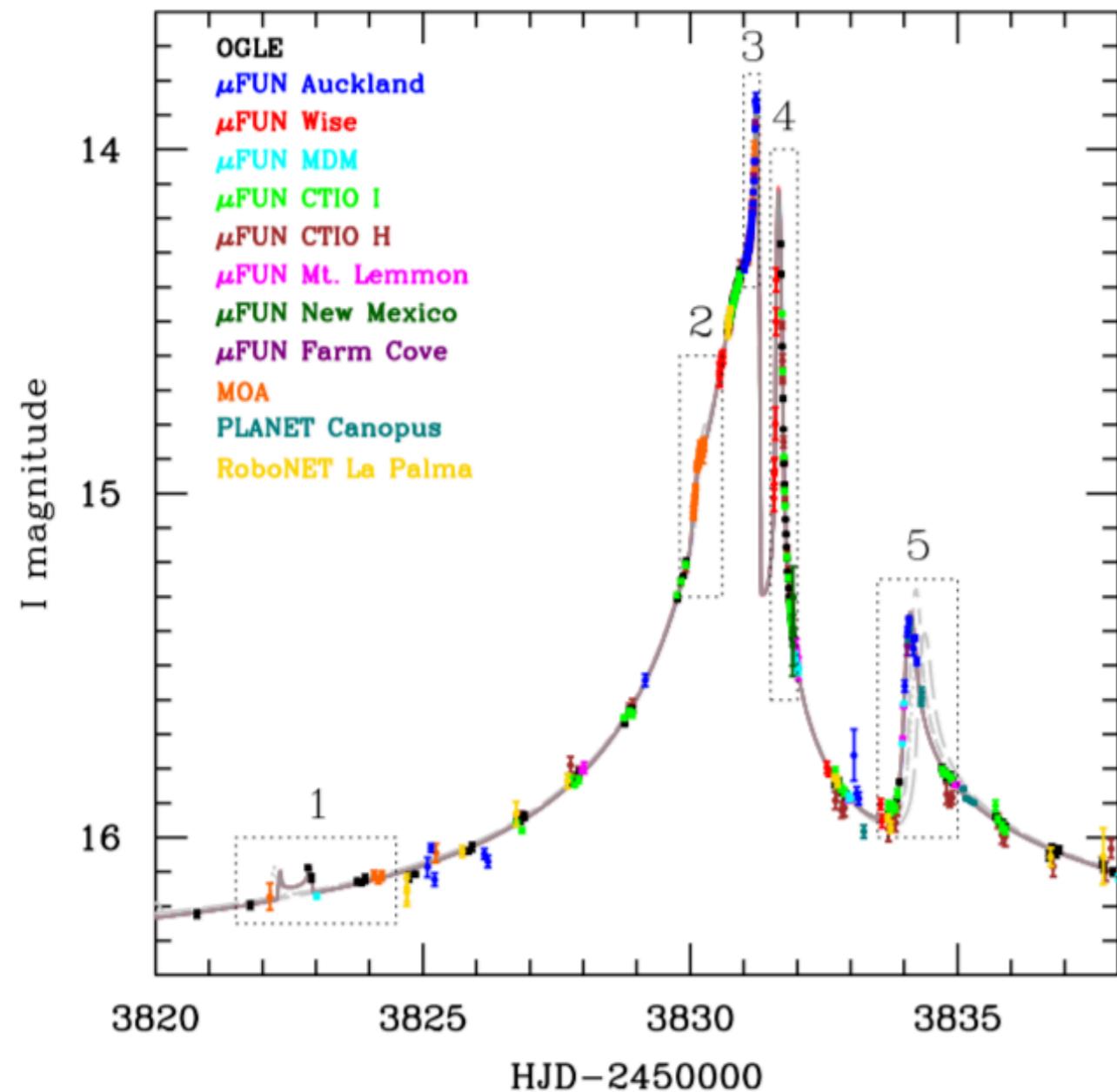
## OTHER ADVANTAGES...

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- sensitivity to low-mass planets
- sensitivity to long period and free-floating planets
- sensitivity to a wide range of host stars over a wide range of galactocentric distances
- sensitivity to multiple planets and orbital parameters...

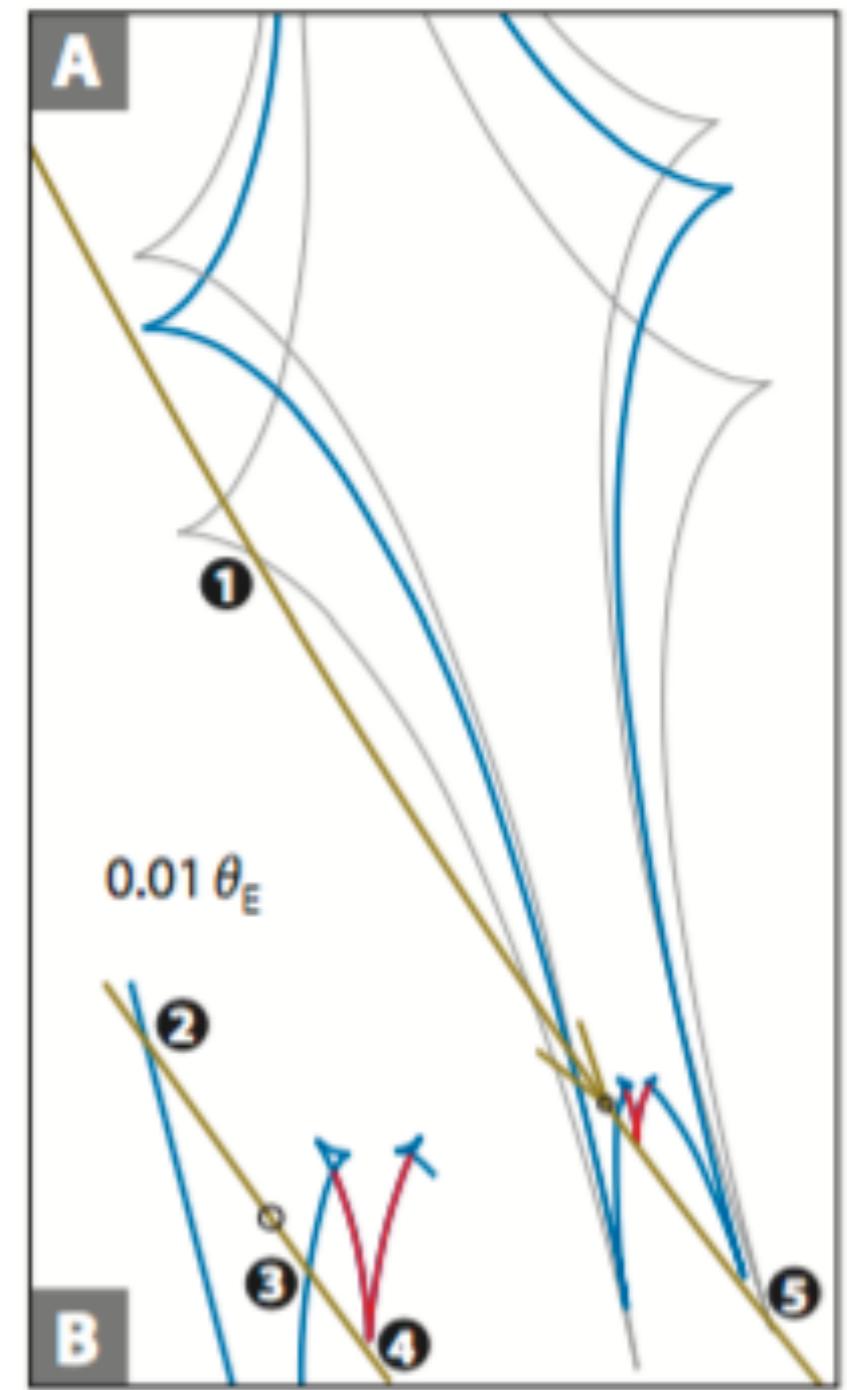
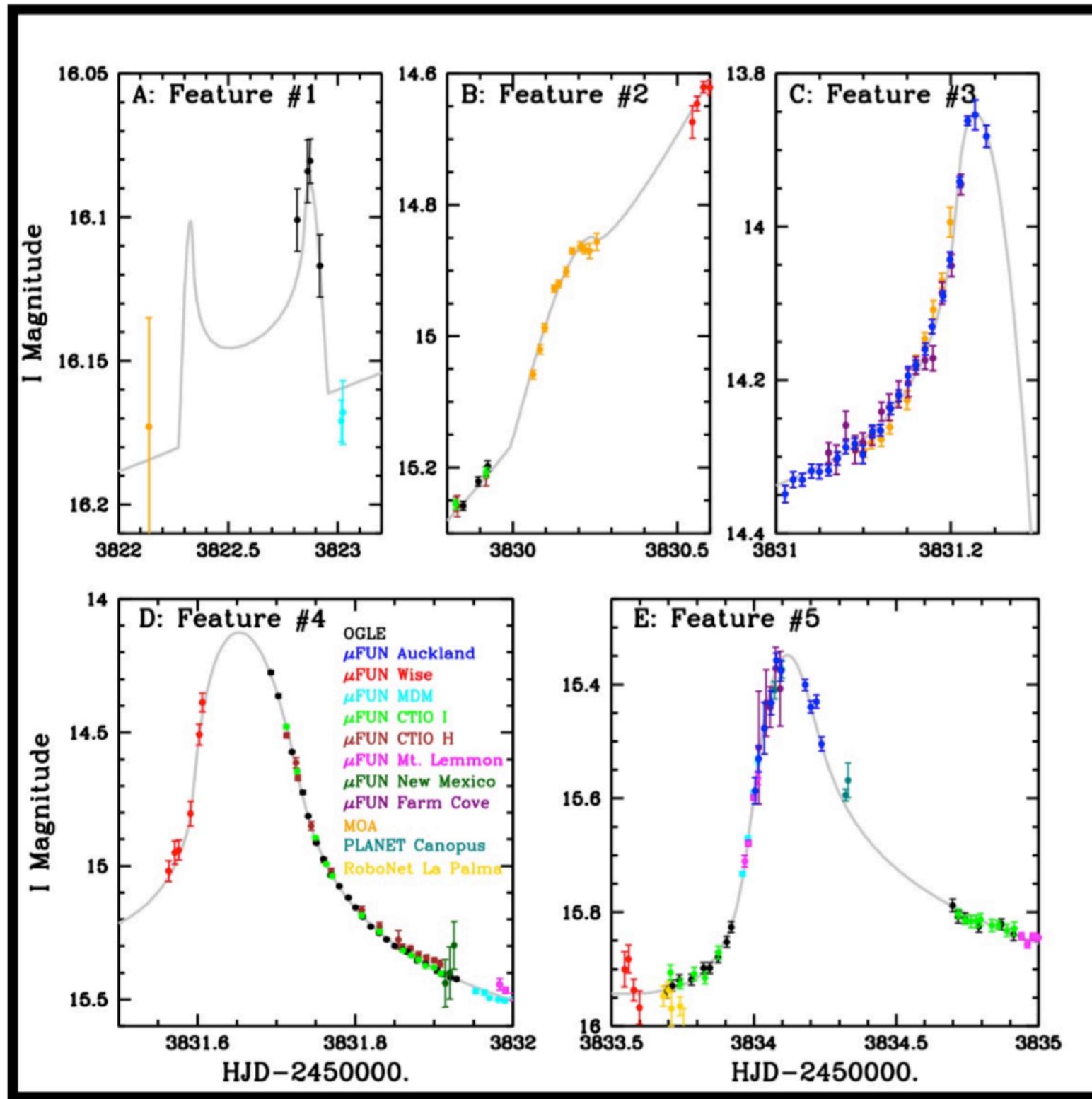
# QUIZ

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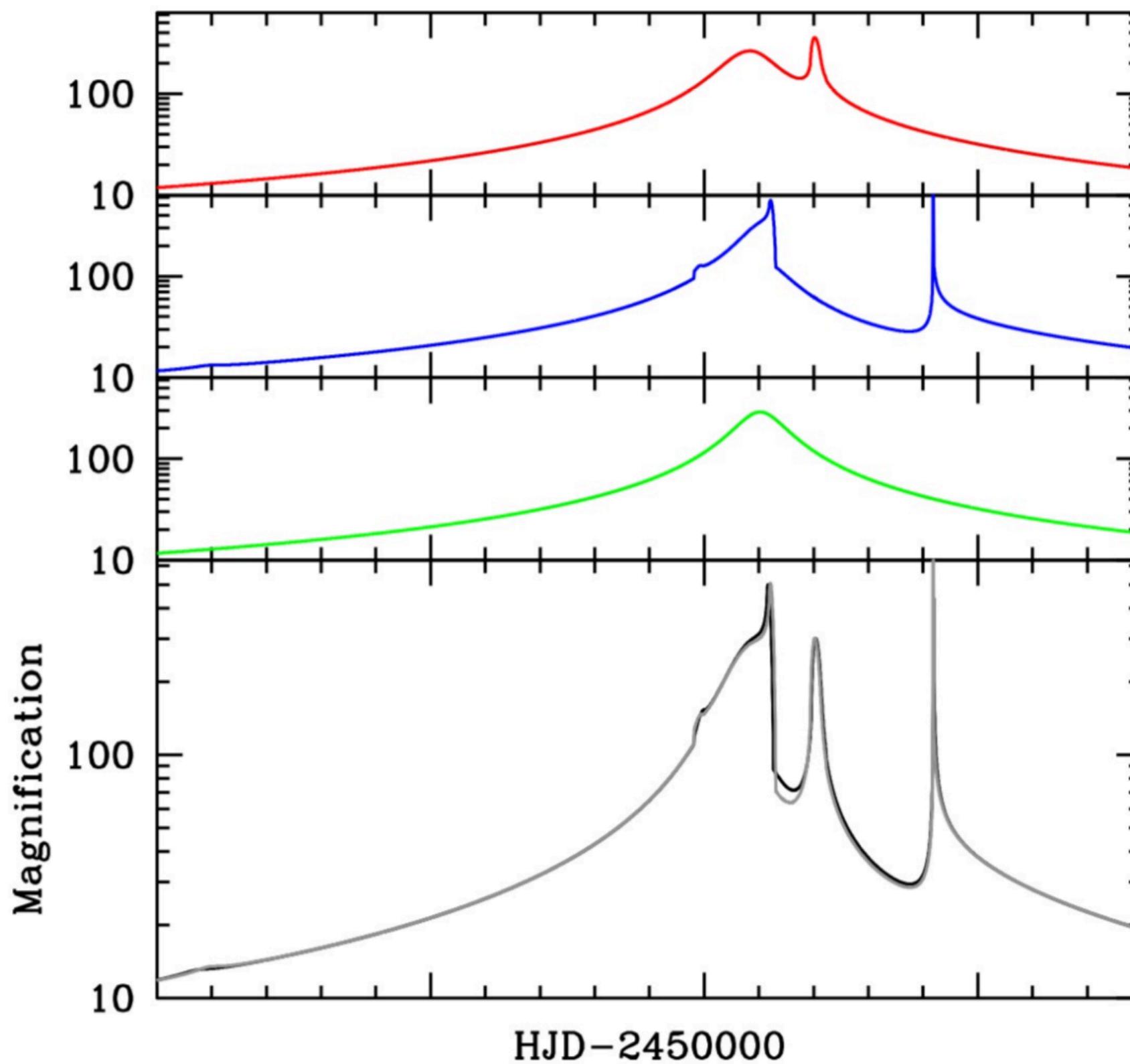
Gaudi *et al.* 2008, Bennett *et al.* 2010 — OGLE-2006-BLG-109

# MULTIPLE PLANETS AND EVOLVING CAUSTIC



# MULTIPLE PLANETS AND EVOLVING CAUSTIC

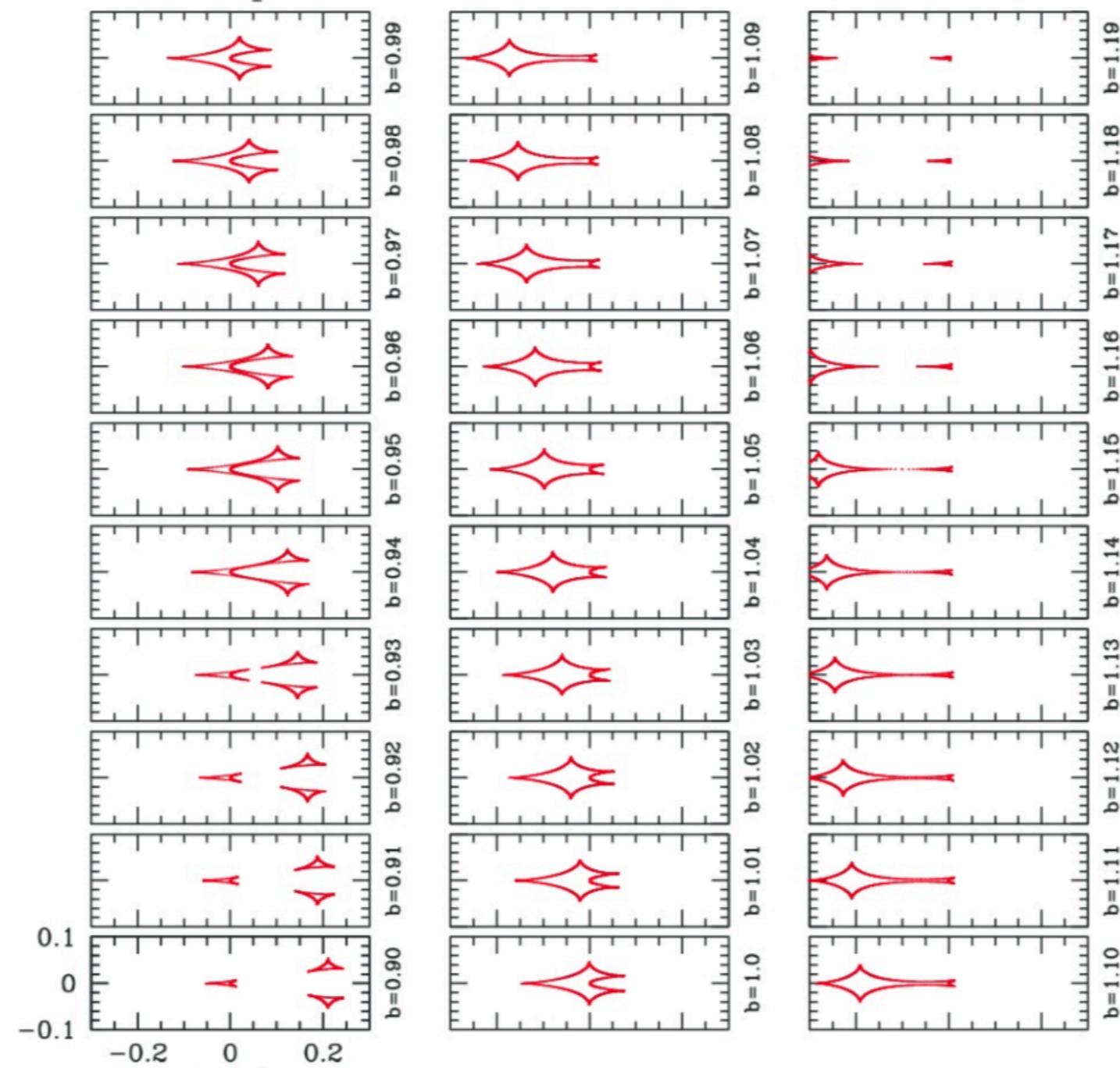
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# SHAPE OF THE RESONANT CAUSTIC VS D: SENSITIVITY TO ORBITAL MOTION

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$$q = 0.001, s = 0.90 - 1.19, \Delta s = 0.29$$



# MULTIPLE PLANETS

*A solar system analog!*

## OGLE-2006-BLG-109

### Model parameters

$m_b/M$	$1.35 \times 10^{-3}$
$m_c/m_b$	0.36
$a_{\perp,b}/a_{\perp,c}$	0.6

### Solar system

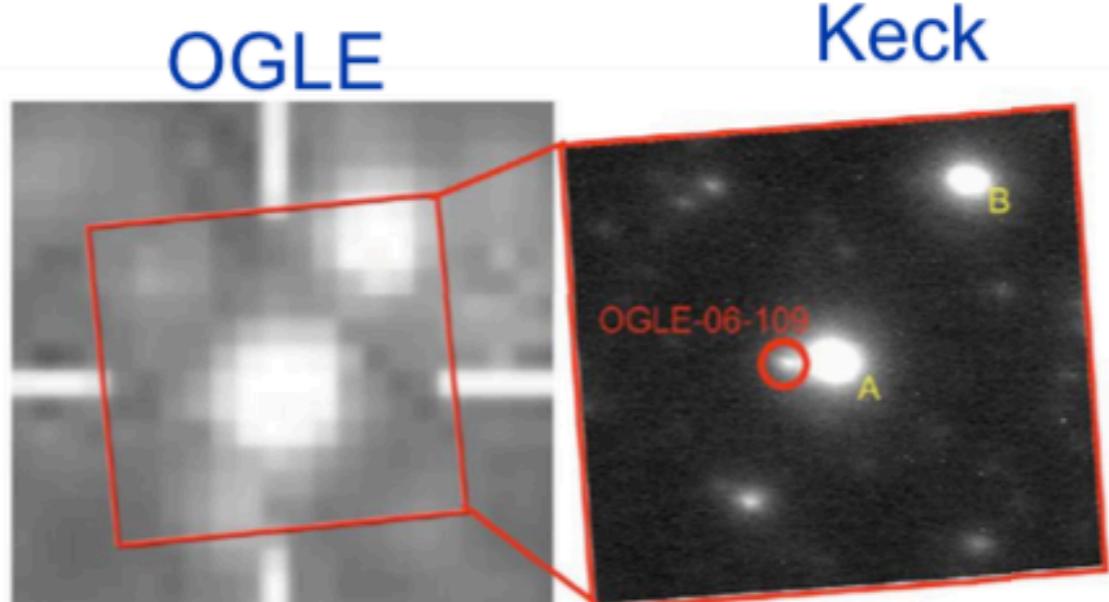
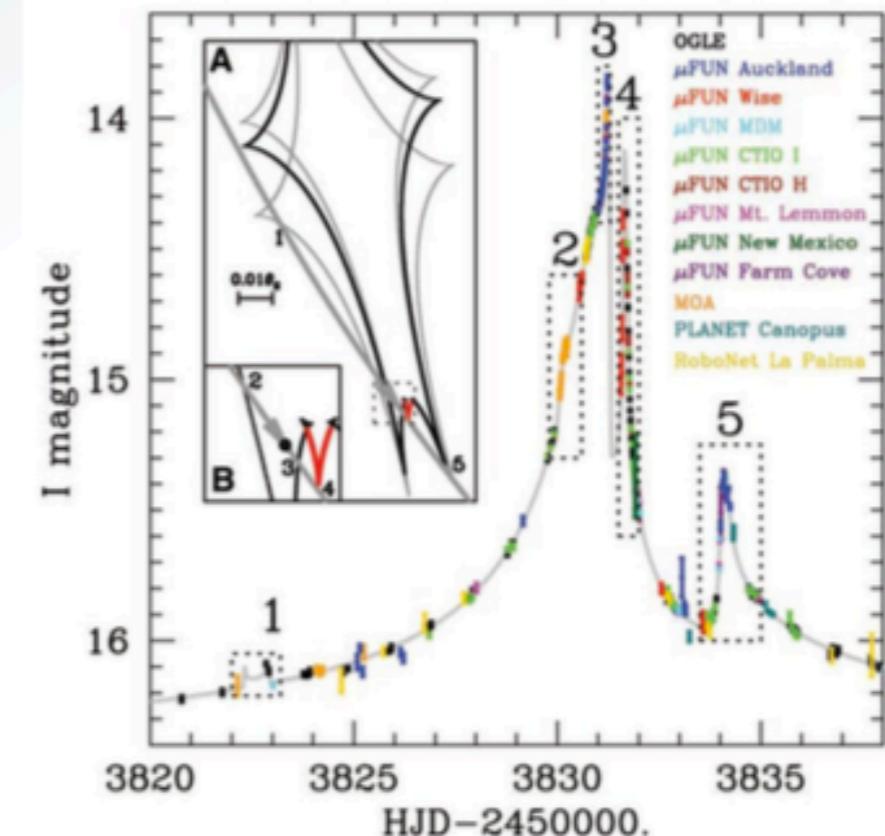
$m_j/M_\odot$	$0.96 \times 10^{-3}$
$m_s/m_j$	0.30
$a_j/a_s$	0.5

### Physical properties

$m_b$	$0.73 \pm 0.06 [M_{Jup}]$
$a_{\perp,b}$	$2.3 \pm 0.5 [\text{AU}]$
$m_c$	$0.75 \pm 0.06 [M_{Sat}]$
$a_{\perp,c}$	$4.5^{+1.1}_{-1.0} [\text{AU}]$
$M_{\text{host}}$	$0.51^{+0.05}_{-0.04} [M_\odot]$

Bennett et al. 2010

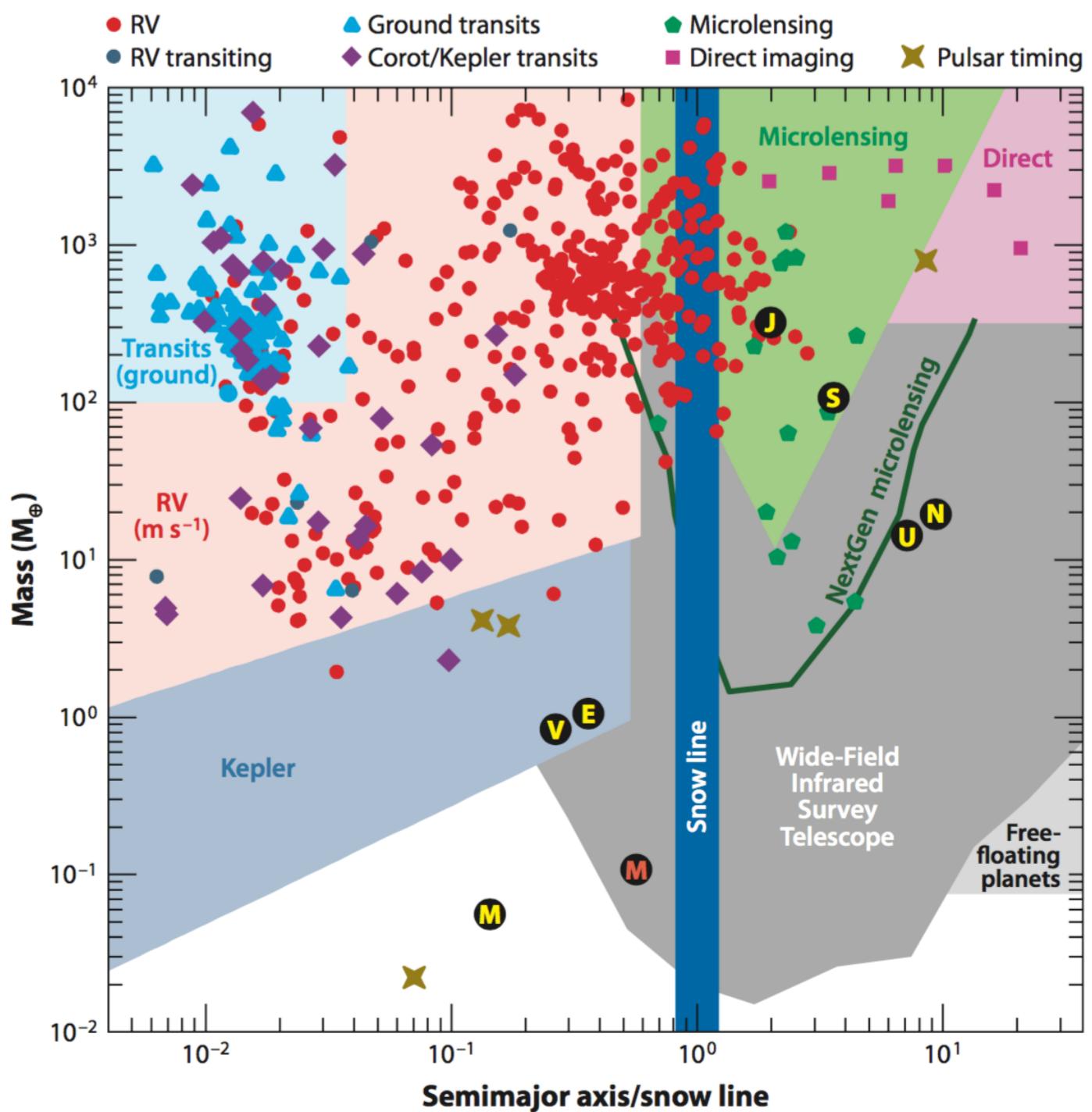
Gaudi et al. 2008



# ...AND DISADVANTAGES

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- small numbers compared to other methods ( $\sim 2000$  exoplanets confirmed to date)
- little sensitivity to the habitable zone
- faint and distant hosts
- limited information about the host and the planet



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