Distributed Dual Averaging in Networks

Project Presentation

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The Paper

Agarwal, Alekh, Martin J. Wainwright, and John C. Duchi. "Distributed dual averaging in networks." In Advances in Neural Information Processing Systems, pp. 550-558. 2010.

Outline

Problem Statement

Decentralized optimization

Optimize a global objective formed by a sum of convex functions using local computation and local communication over a network of compute nodes.

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Optimize a global objective formed by a sum of convex functions using local computation and local communication over a network of compute nodes.

Theoretical performance

Provide bounds on algorithm convergence rates as function of network size and topology.

Motivation: Big Data

Classical ML: minimize a loss function over a dataset.
Interesting datasets grow in size faster than innovations in storage capacity of a single computer.

- Google Maps dataset in 2012: 20 PB (20,500 TB)[1]
- Facebook dataset in 2010: 20 PB, in 2011: 30 PB[2]

Non-ML distributed convex minimization: multi-agent coordination, estimation in sensor networks, packet routing...

[1] http://mashable.com/2012/08/22/google-maps-facts/

[2] https://www.facebook.com/notes/paul-yang/moving-an-elephant-large-scale-hadoop-data-migration-at-facebook/10150246275318920

Motivation: Distributed Computation Constraints

In datacenter environments, supercomputers, and ad-hoc distributed networks, available bandwidth of communication is in inverse relationship to distance.

> *Traditionally, inter-cluster connectivity is oversubscribed, with much less bandwidth available between the clusters than within them. This assumes and then dictates that most intra-application communications occur inside the cluster."*[3]

Modelling the network as a graph with nearby connections as edges is convenient.

[3] https://code.facebook.com/posts/360346274145943/introducing-data-center-fabric-the-next-generation-facebook-data-center-network/

Related Work

- N. Tsitsiklis, D. P. Bertsekas, and M. Athans. 1986. Distributed asynchronous deterministic and stochastic gradient optimization algorithms. *Uses shared memory.*
- D. P. Bertsekas, J. N. Tsitsiklis. Parallel and Distributed Computation: Numerical Methods. 1989.
- A. Nedic and A. Ozdaglar. Distributed subgradient methods for multi-agent optimization. 2009.
  Each agent has its own objective function.
- Y. Nesterov. Primal-dual subgradient methods for convex problems. 2009.
  Non-distributed version of the proposed algorithm.

Primal-dual subgradient methods

- Objective function $f(x)$ is non-smooth and not necessarily differentiable.
- Classic: $\min_x\{f(x) : x \in \mathbb{R}^n\} : x_{k+1} = x_k - \alpha_k g_k$
- Convergence and reach: $\alpha_k \to 0, \sum_k \alpha_k = \infty$.
- Lower linear model of the objective function:

$$l_k(x) \stackrel{def}{=} \sum_{i=0}^{k} \alpha_i (f(x_i) + \langle g_i, x - x_i \rangle) / \sum_{i=0}^{k} \alpha_i$$

- New subgradients are added with *decreasing* weights, needed for convergence of primal sequence $\{x_k\}$; but they are more important.

Primal-dual subgradient methods

- Insight: maintain two different sequences of parameters.
- One is responsible for the primal space process and has a vanishing sequence of steps.
- The other is in the dual space of linear function and has a non-decreasing sequence of weights to prioritize newer subgradients.

The Paper's Experimental Results

- The paper's simulations are done using synthetic SVM classification problems with hinge loss and

$$\mathcal{X} = \{x \in \mathbb{R}^d \mid \|x\|_2 \leq 5\}.$$

- Performance is evaluated for different network sizes (n=100, 225, 400, 625, 900) and topology (single cycle, 2-D grid, bounded degree expander).
- "We have observed qualitatively similar behavior for other problem classes".

Our Experiments

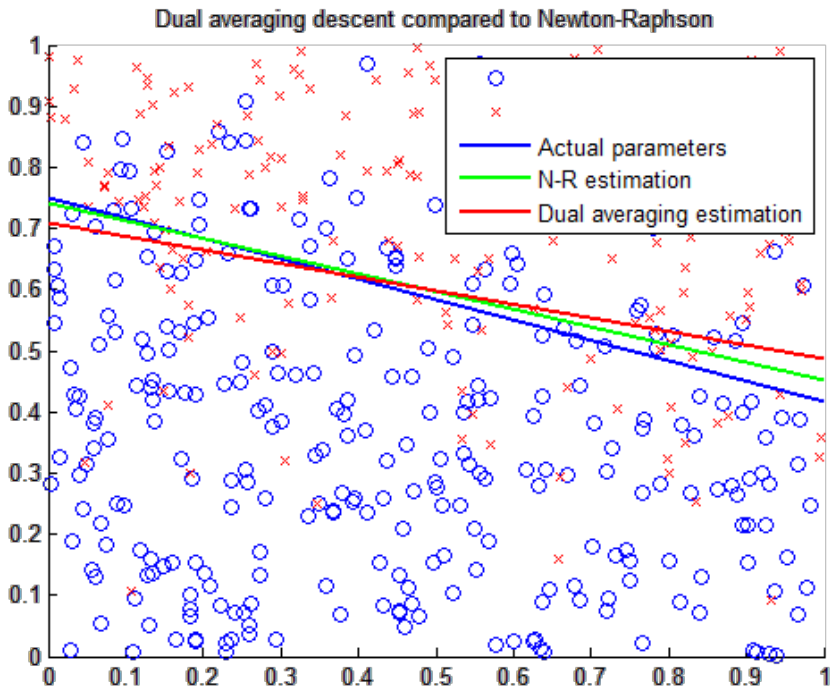
- Choose another class of optimization problems (non-SVM).
- Compare convergence rate of non-distributed primal-dual subgradient method to the proposed distributed method.
- Plot convergence rate vs. network size and parameters for a random geometric graph (mentioned in the paper but not evaluated).[4]
- Try the approach on a real dataset.

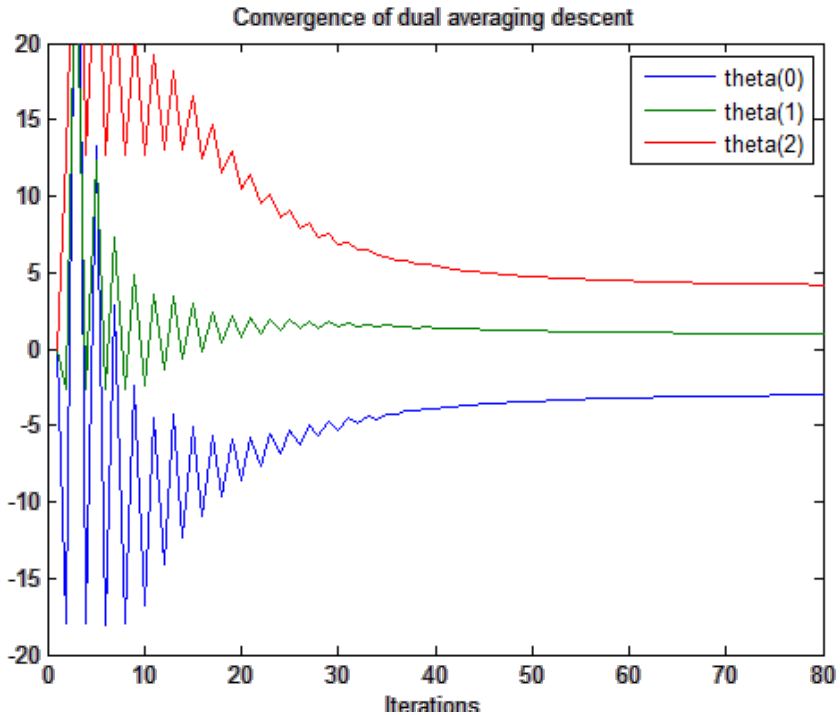
[4] A random geometric graph can model an ad-hoc wireless sensor network with limited communication range.

Experiments

- Implemented Dual Averaging for a 2D logistic regression: simple to understand, fast to simulate, easy to generate unlimited amount of test data.
- $f(\theta) = \min -\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log(1 - h_\theta(x^{(i)})$
- $z(t+1) = z(t) - g(t) = z(t) - [\mathbf{x}(\mathbf{y} - h_\theta(\mathbf{x}))']$
- $\alpha(t) = 1/t, \psi(\theta) = \frac{1}{2}\|\theta\|^2$
- $\theta(t+1) = \Pi^{\psi}(-z(t+1), \alpha(t))$
- $\Pi^{\psi}(z, \alpha) = \text{argmin}_\theta(\langle z, \theta \rangle + \frac{1}{\alpha}\psi(\theta))$
- . . . a bit of algebra . . .
- $\theta(t+1) = -\frac{\alpha}{2} z$

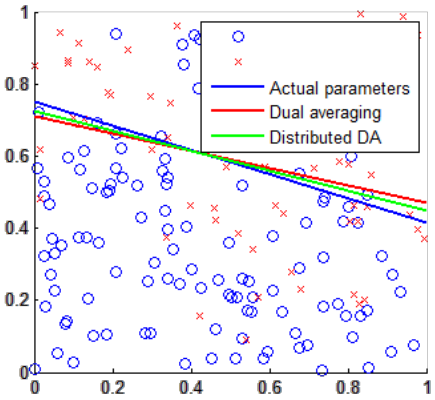
Single-node Dual Subgradient Method

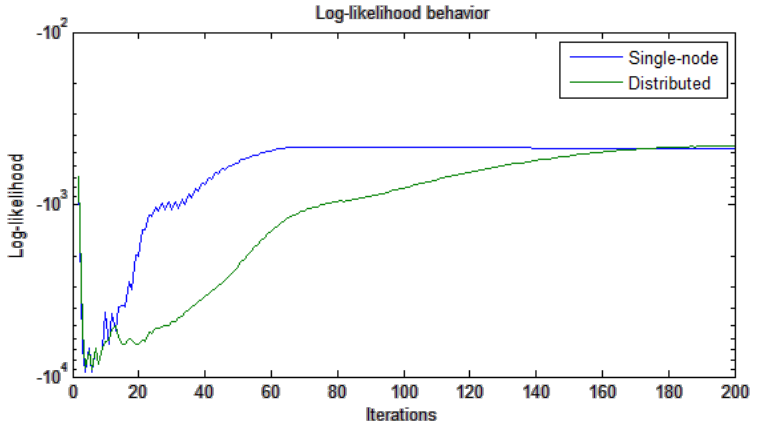


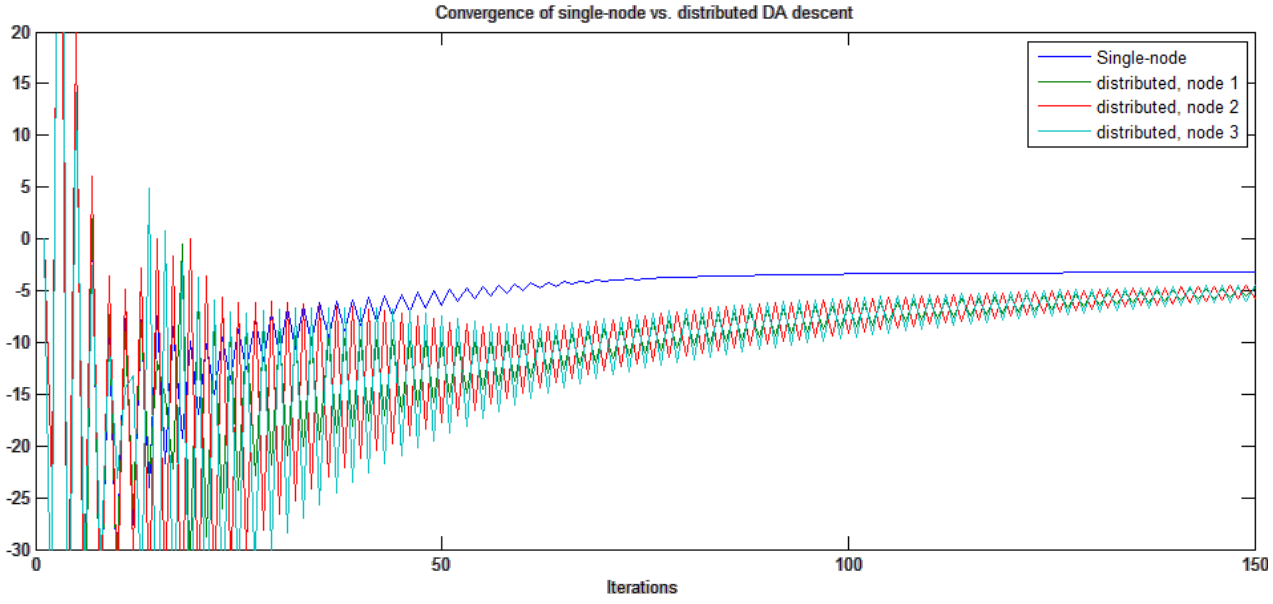


Compared to Newton-Raphson, convergence to same tolerance is much slower (258 iterations vs. 31)

Single-node vs. 9-node Cycle Distributed DA

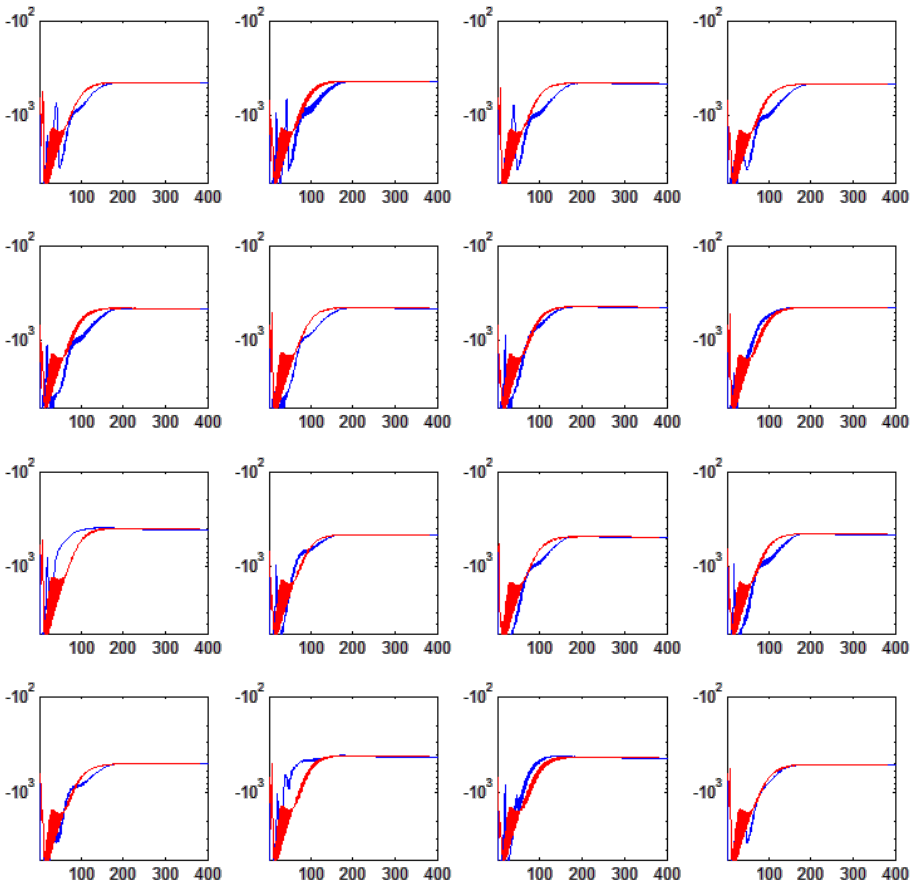


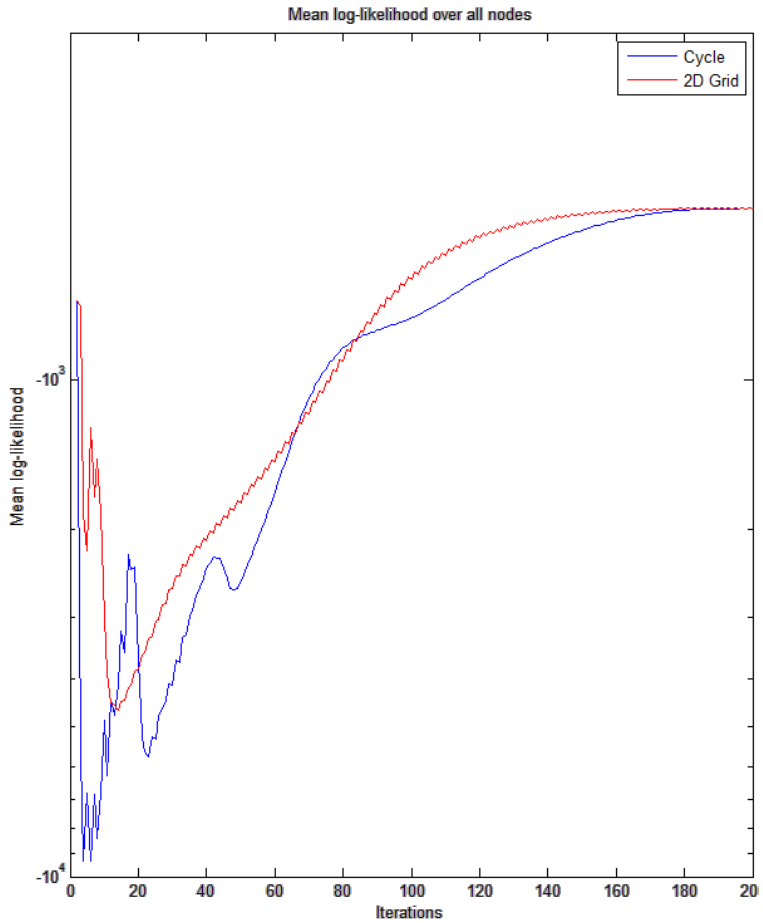




Iterations required: single-node = 261, distributed = 672.

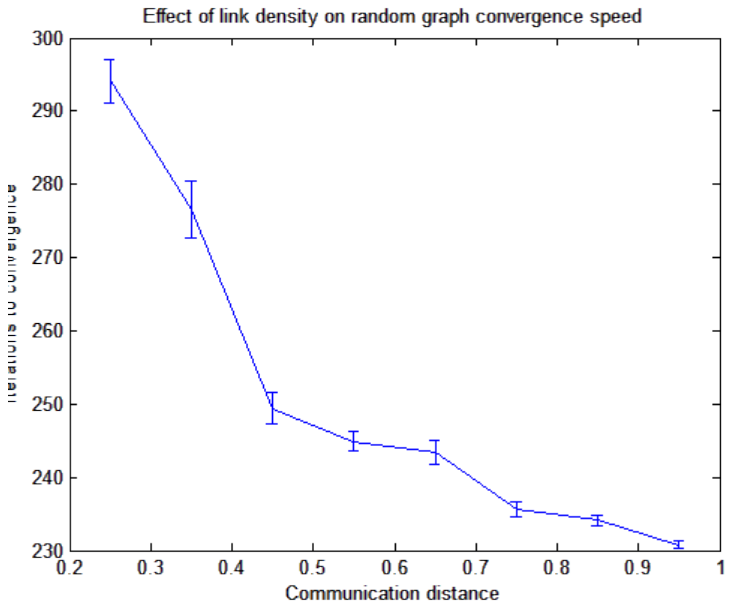
Convergence of a 16-node Cycle vs. 16-node Grid

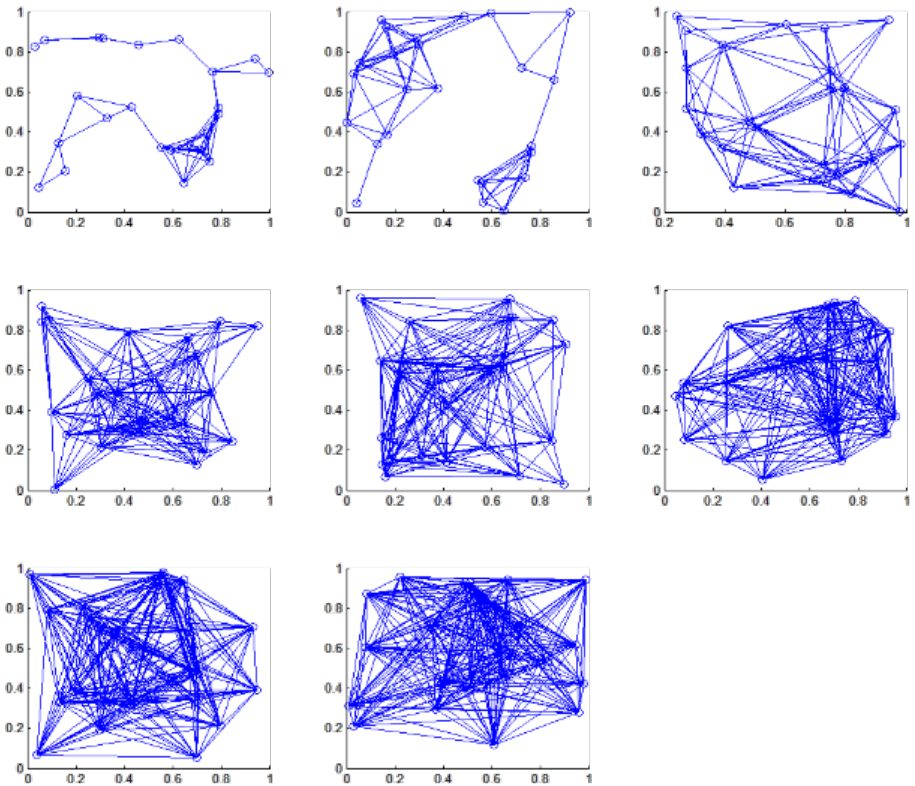




As connectivity improves, convergence speeds up: cycle graph iterations = 744, grid graph iterations = 383.

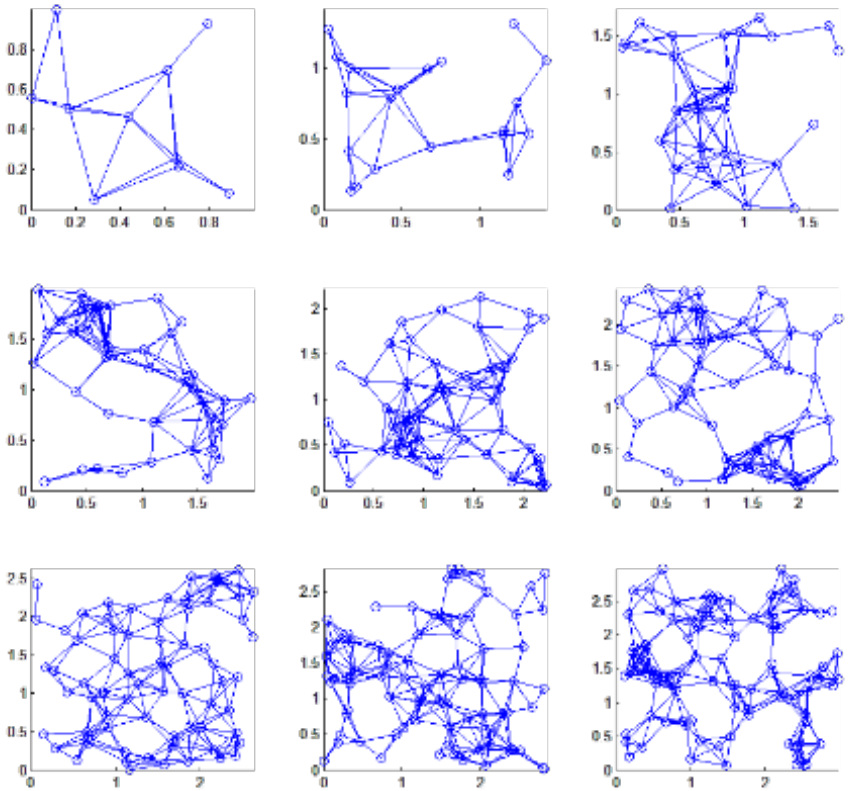
Random Geometric Graph: Density

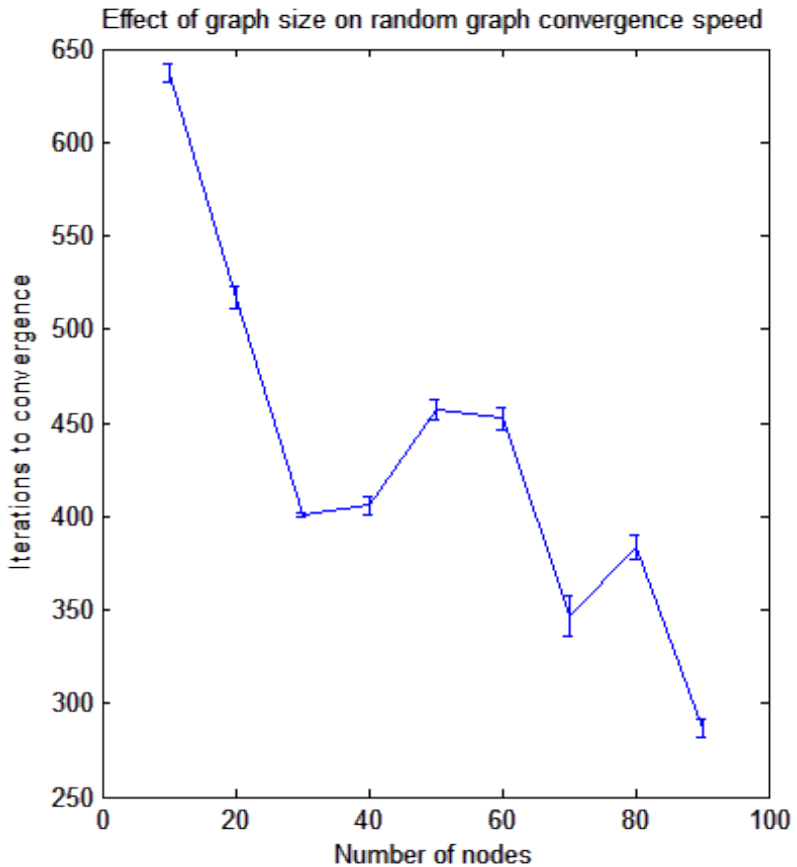




As expected, strong connectivity improves convergence speed.

Random Geometric Graph: Size (10-80 nodes)





The opposite result was expected; it looks like the effects of stronger connectivity outweigh the (quadratic) growth of area covered by the nodes of the graph.

Kernel SVM experiment

- Implement a kernelized SVM dual descent algorithm ("optimal soft margin linear classifier without offset" from HW2) on MNIST 4/9 8x8 dataset.
- Use kernel $@(a, b)(a' * b + 1)$ - fast but shows decent benchmark results with SMO.
- Inhomogenous polynomial kernel works well in a no-constant-term framework.
- The optimization problem is the SVM dual:

$$f(\alpha) = \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y^{(i)} y^{(j)} K(x^{(i)}, x^{(j)}) + \sum_i \alpha_i$$

- Optimal solution for z:

$$z_i = \frac{1 - \frac{1}{2} \sum_{i \neq j} \alpha_j y^{(i)} y^{(j)} K(x^{(i)}, x^{(j)})}{K(x^{(i)}, x^{(i)})}$$

Kernel SVM results

- After 500 iterations with $C = 1000$, $num\_sv = 1107$ and $test\_error = 0.064$.
- SMO result on the same kernel and C (normal / constant term framework): 0.045 with 234 SV.
- Conclusion: likely to be an implementation bug (the projection is now a constrained minimization problem, unlike logistic regression, since $0 \leq \alpha_i \leq \frac{C}{n}$).
- Dual averaging is only efficient when the projection from dual to primal space Π can be efficiently computed.

Conclusions: Tunable Parameters

- The details of the doubly stochastic graph matrix (for example, how much weight to assign to the local node?) influence convergence speed significantly but are not discussed.
- Step choice $\alpha(t)$ and convergence rate: with the suggested formula

$$\alpha(t) = \frac{R\sqrt{1-\sigma_2(P)}}{4L\sqrt{t}}$$

the convergence was extremely slow. Therefore we were unable to test the convergence corollaries for different graph types.

Conclusions: SVM

- Computing the projection is a more complicated task with a constrained problem and is likely an iterative optimization run in its own right.
- The goal of distributed computation in this case is not clear (what is the meaning of averaging of α_i's?)

Questions from the Audience

The Matlab code used to produce the output for this presentation and the presentation itself can be found on GitHub:

`https://github.com/maxvt/science/tree/master/ec719_machine_learning/distributed_dual_averaging`