

# CONCRETE STRUCTURES

## SUMMARY

**Foreword** Any error or contribution should be reported in the form of an issue, or a pull request for those who can use git and L<sup>A</sup>T<sub>E</sub>X, to  
[https://github.com/mbataillou/Summaries/tree/master/Caminos/Estructuras\\_Hormigon](https://github.com/mbataillou/Summaries/tree/master/Caminos/Estructuras_Hormigon)  
You can notice that there is always place for improvement and your help is therefore welcome.

Version 0.2

January 15, 2018

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# Contents

<b>1 General</b>	<b>3</b>
<b>2 Inertia</b>	<b>3</b>
2.1 Before cracking . . . . .	3
2.1.1 Aproximation . . . . .	3
2.2 Cracked . . . . .	3
2.2.1 Approximation . . . . .	3
<b>3 Material deformation</b>	<b>3</b>
<b>4 Durability requirements</b>	<b>4</b>
<b>5 Load combinations</b>	<b>6</b>
5.1 Ultimate limit state . . . . .	6
5.2 Service state . . . . .	6
<b>6 Longitudinal reinforcement</b>	<b>6</b>
6.1 Aproximations . . . . .	6
6.1.1 Design . . . . .	6
6.2 Spacing . . . . .	6
6.3 Optimisation . . . . .	7
<b>7 Crack width control</b>	<b>8</b>
7.1 Reinforced . . . . .	8
7.2 Prestressed . . . . .	9
<b>8 Shear reinforcement</b>	<b>9</b>
8.1 Optimisation . . . . .	10
<b>9 Columns</b>	<b>10</b>
9.1 Second order effects . . . . .	10
9.2 Longitudinal reinforcement . . . . .	11
9.3 Transversal reinforcement . . . . .	11
<b>10 Detailing and layout</b>	<b>11</b>
<b>Bibliografia</b>	<b>12</b>

# 1 General

$$KPa = \frac{KN}{m^2} \Leftrightarrow MPa = \frac{N}{mm^2}$$

$$I_c = I_{rectangle} = \frac{bd^3}{12}$$

$$I_{circle} = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}$$

$I_{G+x} = I_G + Ax^2$  Steiner ( $x$  perpendicular to the ref axis)

where

$$\frac{x_{cracked}}{d} = \alpha \rho \left( 1 + \frac{\rho^t}{\rho} \right) \left[ -1 \sqrt{1 + \frac{2 \left( 1 + \frac{d^t \rho^t}{d \rho} \right)}{\alpha \rho \left( 1 + \frac{\rho^t}{\rho} \right)}} \right]$$

$$I_c = I_{rectangle/bottom} = \frac{bd^3}{3}$$

## 2.2.1 Approximation

$$I \approx \frac{bd^3}{3} + \alpha [A_s(y_s - x)^2 + A_s^t(y_s^t - x)^2] \quad (3)$$

where

$$\frac{x_{cracked}}{d} \approx \frac{0.18 + 1.8\alpha\rho}{1 + \frac{d^t \rho^t}{d \rho}}$$

# 3 Material deformation

## 1. Elastic deformation

$$\varepsilon_i(t_1) = \frac{\sigma(t_1)}{E_c(t_1)} \quad (4)$$

where

$\sigma(t_1)$ : is the tension at  $t_1$

$E_c(t_1)$ : is the elastic modulus at  $t_1$

## 2. Creep

$$\varepsilon_{cc}(t_2, t_1) = \varphi(t_2, t_1) \frac{\sigma(t_1)}{E_c(28)} \quad (5)$$

where

$\varphi(t_2, t_1)$ : is the creep coefficient from  $t_1$  to  $t_2$  see material PP

$\varphi(\infty, t_1)$ : see Figures 1,2

## 3. Shrinkage

# 2.2 Cracked

$$I = \sum_{i=1}^n \alpha_i [I_i + A_i(y_i - x)^2] =$$

$$I_c + A_c(y_c - x)^2 + \alpha [I_s + I_s^t + A_s(y_s - x)^2 + A_s^t(y_s^t - x)^2] \quad (2)$$

$$\varepsilon_{cs} = \varepsilon_{ca} + \varepsilon_{cd} \quad (6)$$

where

$$\varepsilon_{ca} = \begin{cases} \varepsilon_{ca}(t) & \text{is the autogenous shrinkage} \\ \beta_{as}(t) = 1 - e^{-0.2t^{0.5}} \\ \varepsilon_{ca}(\infty) = 2.5(f_{ck} - 10) \cdot 10^{-6} \end{cases}$$

$$\varepsilon_{cd} = \begin{cases} \varepsilon_{cd} = \beta_{ds}(t, t_s) k_h \varepsilon_{cd,0} \\ \beta_{ds}(t, t_s) = \frac{t-t_s}{t-t_s+0.04\sqrt{h_0^3}} \end{cases} \quad \text{is the drying shrinkage}$$

$h_0$  is the shape coefficient

$k_h$  see Table 1

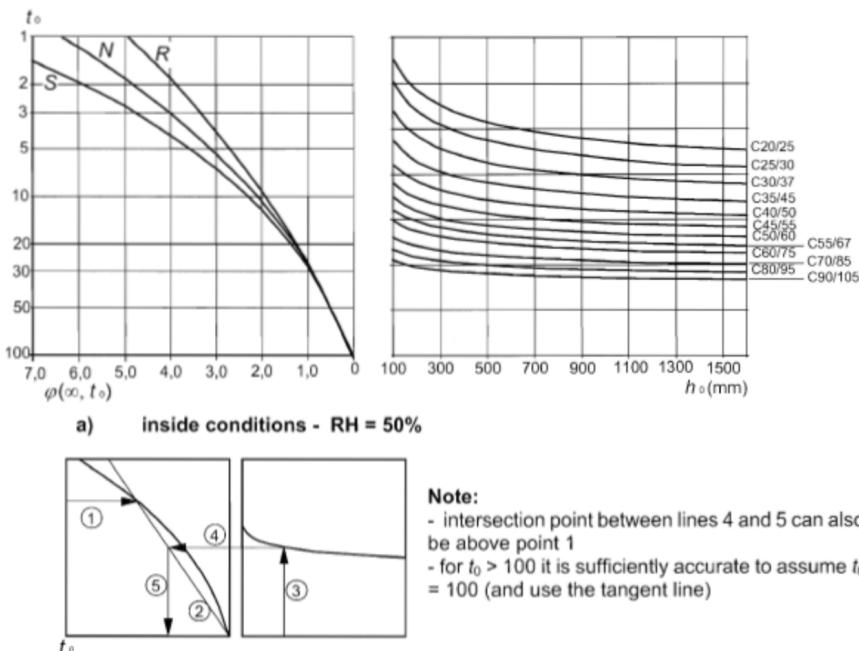


Figure 1: Model for infinite creep coefficient RH = 50

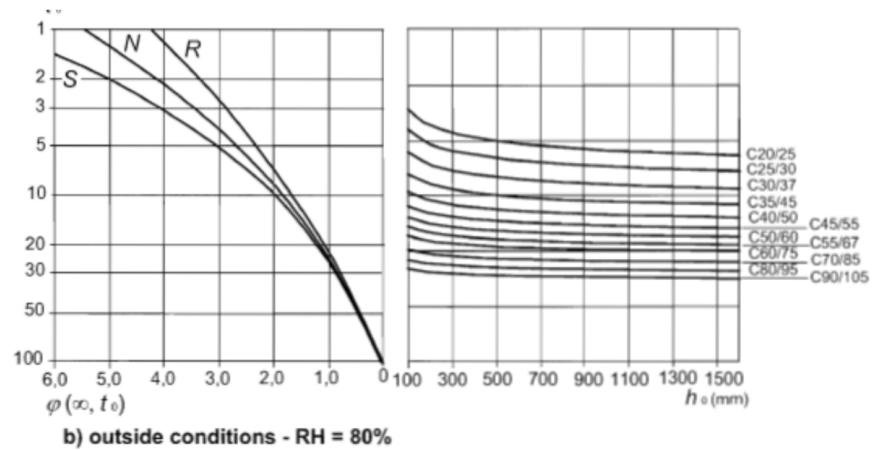


Figure 2: Model for infinite creep coefficient RH = 80

$h_0$	$k_h$
100	1
200	0.85
300	0.75
$\leq 500$	0.7

Table 1:  $k_h$  depending on  $h_0$

## 4 Durability requirements

1. Find exposure class in Table 2
2. Define concrete class with Table 3
3. Compute structural class with Table 4, taking into account that  $S_{min} = S_1$  and  $S_{ref,T=50y} = S_4$
4. Find minimal cover in Table 5
5. Add tolerance

$$C_{dur} = C(Table 5) + \Delta c$$

$$C_{dur} = C(Table 5) + \Delta c \quad (7)$$

where

$$\Delta c \in [0, 10] \rightarrow 5mm$$

Class designation	Description of the environment	Informative examples where exposure classes may occur
<b>1 No risk of corrosion or attack</b>		
X0	For concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack	
	For concrete with reinforcement or embedded metal: very dry	Concrete inside buildings with very low air humidity
<b>2 Corrosion induced by carbonation</b>		
XC1	Dry or permanently wet	Concrete inside buildings with low air humidity Concrete permanently submerged in water
XC2	Wet, rarely dry	Concrete surfaces subject to long-term water contact Many foundations
XC3	Moderate humidity	Concrete inside buildings with moderate or high air humidity External concrete sheltered from rain
XC4	Cyclic wet and dry	Concrete surfaces subject to water contact, not within exposure class XC2
<b>3 Corrosion induced by chlorides</b>		
XD1	Moderate humidity	Concrete surfaces exposed to airborne chlorides
XD2	Wet, rarely dry	Swimming pools Concrete components exposed to industrial waters containing chlorides
XD3	Cyclic wet and dry	Parts of bridges exposed to spray containing chlorides Pavements Car park slabs
<b>4 Corrosion induced by chlorides from sea water</b>		
XS1	Exposed to airborne salt but not in direct contact with sea water	Structures near to or on the coast
XS2	Permanently submerged	Parts of marine structures
XS3	Tidal, splash and spray zones	Parts of marine structures
<b>5. Freeze/Thaw Attack</b>		
XF1	Moderate water saturation, without de-icing agent	Vertical concrete surfaces exposed to rain and freezing
XF2	Moderate water saturation, with de-icing agent	Vertical concrete surfaces of road structures exposed to freezing and airborne de-icing agents
XF3	High water saturation, without de-icing agents	Horizontal concrete surfaces exposed to rain and freezing
XF4	High water saturation with de-icing agents or sea water	Road and bridge decks exposed to de-icing agents Concrete surfaces exposed to direct spray containing de-icing agents and freezing Splash zone of marine structures exposed to freezing
<b>6. Chemical attack</b>		
XA1	Slightly aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water
XA2	Moderately aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water
XA3	Highly aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water

Table 2: Exposure class

	Exposure Classes according to Table 4.1										
Corrosion		Carbonation-induced corrosion				Chloride-induced corrosion		Chloride-induced corrosion from sea-water			
		XC1	XC2	XC3	XC4	XD1	XD2	XD3	XS1	XS2	XS3
Indicative minimum strength class		C20/25	C25/30		C30/37		C30/37		C35/45	C30/37	C35/45
<b>Damage to Concrete</b>											
	No risk	Freeze/Thaw Attack			Chemical Attack						
	X0	XF1	XF2		XF3		XA1	XA2		XA3	
Indicative minimum strength class	C12/15	C30/37	C25/30		C30/37		C30/37		C35/45		

Table 3: Concrete class depending on exposure class

Criterion	Exposure Class according to Table 4.1						
	X0	XC1	XC2 / XC3	XC4	XD1	XD2 / XS1	XD3 / XS2 / XS3
Design Working Life of 100 years	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2
Strength Class <sup>1/2)</sup>	≥ C30/37 reduce class by 1	≥ C30/37 reduce class by 1	≥ C35/45 reduce class by 1	≥ C40/50 reduce class by 1	≥ C40/50 reduce class by 1	≥ C40/50 reduce class by 1	≥ C45/55 reduce class by 1
Member with slab geometry (position of reinforcement not affected by construction process)	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1
Special Quality Control of the concrete production ensured	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1

Table 4: Structural class

Structural Class	Exposure Class according to Table 4.1						
	X0	XC1	XC2 / XC3	XC4	XD1 / XS1	XD2 / XS2	XD3 / XS3
S1	10	10	10	15	20	25	30
S2	10	10	15	20	25	30	35
S3	10	10	20	25	30	35	40
S4	10	15	25	30	35	40	45
S5	15	20	30	35	40	45	50
S6	20	25	35	40	45	50	55

Table 5: Minimal required cover depending on exposure and structural class

## 5 Load combinations

### 5.1 Ultimate limit state

$$\sigma_{ULS} = \gamma_G \sum G_i + \gamma_P P + \gamma_Q Q_1 + \gamma_Q \psi_0 \sum_{i \neq 1} Q_i \quad (8)$$

where

$\gamma_G = 1,35$ : is the security factor for permanent loads

$\gamma_Q = 1,5$ : is the security factor for live loads

$\gamma_P$ : is the security factor for prestressing load

$\psi_0$ : is a factor accounting for the frequency (probability)

$Q_1$ : is the principal live load

$Q_i, i \neq 1$ : is a secondary live load

$G_i$ : is a permanent load

### 5.2 Service state

❖ Characteristic

$$\sigma_{CHA} = \sum G_i + P + Q_1 + \psi_0 \sum_{i \neq 1} Q_i \quad (9)$$

❖ Frequent

$$\sigma_{FRE} = \sum G_i + P + \psi_1 Q_1 + \psi_2 \sum_{i \neq 1} Q_i \quad (11)$$

where

$\psi_1$ : is a factor accounting for the frequency (probability)

$\psi_2$ : is a factor accounting for the frequency (probability)

❖ Quasi-permanent

$$\sigma_{QUA} = \sum G_i + P + \psi_2 \sum_{i \neq 1} Q_i \quad (12)$$

$$(13)$$

## 6 Longitudinal reinforcement

### 6.1 Aproximations

#### 6.1.1 Design

$$d \approx 0,9h$$

1. if  $f_y k = 400$

$$M_{lim} \approx 0,369 f_{cd} b d^2 \quad (14)$$

$$x_{lim} \approx 0,668d \quad (15)$$

$$y_{lim} \approx 0,534d \quad (16)$$

where

$M_{lim}$ : is the max moment resisted without comp reinforcement

$x_{lim}$ : is the  $x$  where steel starts yielding

$y_{lim}$ : is the effective comp zone

2. if  $f_y k = 500$

$$M_{lim} \approx 0,375 f_{cd} b d^2 \quad (17)$$

$$x_{lim} \approx 0,617d \quad (18)$$

$$y_{lim} \approx 0,5d \quad (19)$$

### 6.2 Spacing

1. Define  $\phi_t$ , see Table 6

$$(10) \quad 2. \text{ Compute } n = \frac{A_s}{\frac{\phi_t^2}{4}}$$

3. Compute

$$(20) \quad s = \frac{b - 2c - 2\phi_t - n\phi_t}{n - 1}$$

where

$s > \max(30mm(1,25 \cdot \phi_{arid}); \phi_{long})$   $b$ : is the width

$c$ : is the minimal cover

$\phi_t$ : is the transversal reinforcement diameter

Diameter [mm]	Area [mm <sup>2</sup> ]
12	113
(14)	154
16	200
20	314
25	490

Table 6: Common diameters for longitudinal reinforcement

### 6.3 Optimisation

1. Choose reduction factor  $n$
2. Find  $x$  where  $M_{d,x} = nM_d$
3. if  $\sigma_s \neq f_{yd} = 435MPa$  Compute

$$l_{b,rqd} = l_{table} \frac{\sigma_s}{f_{yd}} \quad (21)$$

where

$l_{table}$ : see Table 7

4. else  $l_{b,rqd} = l_{table}$
5. Apply correction for layout
  - (a) Anchoring

$$l_0 = l_{b,rqd} \prod_{i=1}^5 \alpha_i \approx k l_{b,rqd} \quad (22)$$

where

$\alpha_i$ : see Figures 34

$$k = \begin{cases} 1 & \text{if straight} \\ 0,7 & \text{if hook} \end{cases}$$

$$l_0 = \max(l_0, l_{b,min}) \quad (23)$$

where

$l_{b,min}$ : see Table 8

- (b) Connecting

$$l_0 = l_{b,rqd} \prod_{i=1, i \neq 4}^6 \alpha_i \quad (24)$$

where

$\alpha_i, i \neq 3$ : see Figure 3

$$\alpha_3 : \sum A_{s,min} = A_s \frac{\sigma_{sd}}{f_{yd}}, \quad A_s \text{ the area of one lapped bar}$$

See more details in PP

$$l_0 = \max(l_0, l_{min}) \quad (25)$$

where

$$l_{0,min} = \max(0, 3\alpha_6 l_{b,rqd}; 15\phi; 200mm)$$

6. Cut the bars at  $x = x + d + l_{b,rqd}$

$\emptyset$	C25/30	C30/37	C40/50	C50/60	C60/75
6	242	217	174	150	140
8	322	290	232	200	187
10	403	362	290	250	234
12	483	435	348	300	281
16	644	580	464	400	374
20	805	725	580	500	468
25	1006	906	725	625	584
32	1288	1159	928	800	748
40	1610	1449	1159	1000	935

Table 7:  $l_{b,rqd}$  for  $\sigma_s = f_{yd} = 435MPa$

$\emptyset$	Minimum length in tension					Minimum length in compression				
	C25/30	C30/37	C40/50	C50/60	C60/75	C25/30	C30/37	C40/50	C50/60	C60/75
6	100	100	100	100	100	145	130	104	100	100
8	100	100	100	100	100	193	174	139	120	112
10	121	109	100	100	100	242	217	174	150	140
12	145	130	120	120	120	290	261	209	180	168
16	193	174	160	160	160	386	348	278	240	224
20	242	217	200	200	200	483	435	348	300	281
25	302	272	250	250	250	604	543	435	375	351
32	386	348	320	320	320	773	696	557	480	449
40	483	435	400	400	400	966	870	696	600	561

Table 8:  $l_{min}$  depending on  $\phi$  and  $f_{ck}$

Influencing factor	Type of anchorage	Reinforcement bar	
		In tension	In compression
Shape of bars	Straight	$\alpha_1 = 1,0$	$\alpha_1 = 1,0$
	Other than straight (see Figure 8.1 (b), (c) and (d))	$\alpha_1 = 0,7$ if $c_d > 3\phi$ otherwise $\alpha_1 = 1,0$ (see Figure 8.3 for values of $c_d$ )	$\alpha_1 = 1,0$
Concrete cover	Straight	$\alpha_2 = 1 - 0,15(c_d - \phi)/\phi$ $\geq 0,7$ $\leq 1,0$	$\alpha_2 = 1,0$
	Other than straight (see Figure 8.1 (b), (c) and (d))	$\alpha_2 = 1 - 0,15(c_d - 3\phi)/\phi$ $\geq 0,7$ $\leq 1,0$ (see Figure 8.3 for values of $c_d$ )	$\alpha_2 = 1,0$
Confinement by transverse reinforcement not welded to main reinforcement	All types	$\alpha_3 = 1 - K\lambda$ $\geq 0,7$ $\leq 1,0$	$\alpha_3 = 1,0$
Confinement by welded transverse reinforcement*	All types, position and size as specified in Figure 8.1 (e)	$\alpha_4 = 0,7$	$\alpha_4 = 0,7$
Confinement by transverse pressure	All types	$\alpha_5 = 1 - 0,04p$ $\geq 0,7$ $\leq 1,0$	-
where:			
$\lambda = (\Sigma A_{st} - \Sigma A_{st,min})/A_s$			
$\Sigma A_{st}$ cross-sectional area of the transverse reinforcement along the design anchorage length $l_{bd}$			
$\Sigma A_{st,min}$ cross-sectional area of the minimum transverse reinforcement = 0,25 $A_s$ for beams and 0 for slabs			
$A_s$ area of a single anchored bar with maximum bar diameter			
$K$ values shown in Figure 8.4			
$p$ transverse pressure [MPa] at ultimate limit state along $l_{bd}$			
* See also 8.6: For direct supports $l_{bd}$ may be taken less than $l_{b,min}$ provided that there is at least one transverse wire welded within the support. This should be at least 15 mm from the face of the support.			
Percentage of lapped bars relative to the total cross-section area	< 25%	33%	50% > 50%
$\alpha_6$	1	1,15	1,4
Note: Intermediate values may be determined by interpolation.			

Figure 3:  $\alpha_i$  depending on the layout

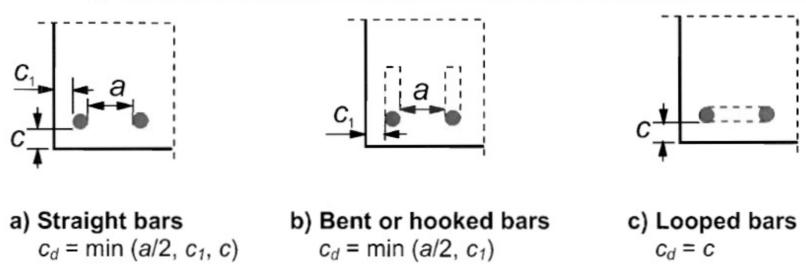


Figure 4:  $c_d$  depending on the layout

## 7 Crack width control

### 7.1 Reinforced

1. Compute  $M_{quasi}$
2. Compute  $x_{cracked}$  see 2.2
3. Compute

$$\sigma_s = \frac{M_{quasi}}{(d - \frac{x_{cracked}}{3}) A_s} \quad (26)$$

where

$$d = h - c - \phi_t - k\phi_l$$

$k$  : is a parameter depending on number of layers

4. Check allowable crack width in Table 9
5. Compute  $\sigma_{crack} = \max(\sigma(Table 10), \sigma(Table 11))$
6. If  $\sigma_{crack} > \sigma_s$

$$A_{s,crack} = \frac{\sigma_{crack}}{\sigma_s} A_s \quad (27)$$

where

$A_{s,crack}$  : is the area needed to fulfill crack width

Exposure Class	Reinforced members and prestressed members with unbonded tendons	Prestressed members with bonded tendons
	Quasi-permanent load combination	Frequent load combination
X0, XC1	0,4 <sup>1</sup>	0,2
XC2, XC3, XC4		0,2 <sup>2</sup>
XD <sub>2</sub> XD1, XD2, XD3, XS1, XS2, XS3 <sub>(ACI)</sub>	0,3	Decompression

**Note 1:** For X0, XC1 exposure classes, crack width has no influence on durability and <sub>(ACI)</sub> this limit is set to give generally acceptable appearance. In the absence <sub>(ACI)</sub> of appearance conditions this limit may be relaxed.

**Note 2:** For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.

Table 9: Maximum allowable crack width depending on technology and environmental conditions

$\omega = 0,2 \text{ [mm]}$	$\omega = 0,3 \text{ [mm]}$	$\omega = 0,4 \text{ [mm]}$	Máxima tensión $\sigma \text{ [Mpa]}$
25	32	40	160
16	25	32	200
12	16	20	240
8	12	16	280
6	10	12	320
5	8	10	360
4	6	8	400
-	5	6	450

Table 10: Diámetro (en mm) de barra en función  $\sigma_{max}$  y  $\omega$

$\omega = 0,2 \text{ [mm]}$	$\omega = 0,3 \text{ [mm]}$	$\omega = 0,4 \text{ [mm]}$	Máxima tensión $\sigma \text{ [Mpa]}$
200	300	300	160
150	250	300	200
100	200	250	240
30	150	200	280
-	100	150	320
-	50	100	360

Table 11: Spacing (en mm) en función  $\sigma_{max}$  y  $\omega$

## 7.2 Prestressed

We need to find  $(P, e)$  that fullfills:

$$\begin{cases} \sigma_{comp,const} = \frac{P}{A} - \frac{Pev_t}{I} + \frac{M_g v_t}{I} \leq \sigma_{max,comp} = 0,6 f_{ck} \\ \sigma_{trac,const} = -\frac{P}{A} + \frac{Pev_t}{I} + \frac{M_g v}{I} \leq \sigma_{max,trac} = 3 \text{ MPa} \\ \sigma_{comp} = \frac{P}{A} - \frac{Pev_t}{I} + \frac{M_{characteristic} v_t}{I} \leq \sigma_{max,comp} = 0,6 f_{ck} \\ \sigma_{trac} = -\frac{P}{A} + \frac{Pev_t}{I} + \frac{M_{frequent} v}{I} \leq \sigma_{max,trac} = 3 \text{ MPa} \end{cases} \quad (28)$$

where

$v$  : is the distance to the c.g from the bottom

$v_t$  : is the distance to the c.g from the top

Signs we supposed the resulting moments compress the top and prestressed ones the bottom

$M_g$  : is the moment in a construction phase

## 8 Shear reinforcement

Procedure:

1. Compute  $V_d$
2. Compute

$$V_c = \max \begin{cases} V_{R_d,c} = \left( C_{Rd,c} k (100 \rho_l f_{ck})^{1/3} + k_1 \sigma_{cp} \right) b_w d \\ V_{R_d,c,min} = \left( 0,035 \cdot k^{3/2} f_{ck}^{1/2} + k_1 \sigma_{cp} \right) b_w d \end{cases} \quad (29)$$

where

$$C_{Rd,c} = \frac{0,18}{\gamma_c} = 0,12$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2 \text{ in mm}$$

$$\rho_l = \frac{A_{s,long}}{b_w d} \leq 0,02$$

$$k_1 = 0,15$$

$$\sigma_{cp} = \frac{N + P}{b_w d} \leq 0,2 f_{cd}$$

3. If  $V_d > V_c \rightarrow$  reinforcement needed

4. Compute

$$V_{R_d,max} = a_{cw} b_w z v_1 f_{cd} \frac{1}{\cot(\theta) + \tan(\theta)} \quad (30)$$

where

$$\nu_1 = \begin{cases} 0,6 & \text{if } f_{ck} \leq 60 \text{ MPa} \\ 0,9 - \frac{f_{ck}}{200} > 0,5 & \text{if } f_{ck} > 60 \text{ MPa} \end{cases}$$

$$\alpha_{cw} = \begin{cases} 1 & \text{non prestressed structure} \\ 1 + \sigma_{cp}/f_{cd} - \frac{f_{ck}}{200} > 0,5 & \text{if } \sigma_{cd} \leq 0,25 f_{cd} \\ 1,25 & \text{if } 0,25 f_{cd} < \sigma_{cd} \leq 0,5 f_{cd} \\ 2,5(1 - \sigma_{cp}/f_{cd}) & \text{if } 0,5 f_{cd} < \sigma_{cd} \leq f_{cd} \end{cases}$$

$$z \approx 0,9d \approx 0,8h$$

$$\cot(\theta) = 2,5$$

5. If  $V_{R_d,max} < V_d$

(a) Compute

$$\nu = \frac{V_d}{a_{cw} b_w z \nu_1 f_{cd}}$$

(b) If  $\nu > 0,5$  increase  $b_w$  or  $z$ , and/or  $f_{cd}$

(c)

$$\cot(\theta) = \begin{cases} 2,5 & \text{if } \nu \leq 0,34 \\ 5,6 - 9,2\nu & \text{if } \nu > 0,34 \end{cases} \in [1, 2,5] \quad (31)$$

6. Compute

$$\frac{A_{sw}}{s} = \frac{V_d}{f_{ywd} z \cot(\theta)} \quad (32)$$

where

$$f_{ywd} = \min\left(\frac{f_{ywk}}{\gamma_s}, 400\right)$$

7. If  $\frac{A_{sw}}{s} < \frac{A_{sw,min}}{s} = \frac{0,08\sqrt{f_{ck}}}{f_{ywd} b_w}$

$$\frac{A_{sw}}{s} = \frac{A_{sw,min}}{s}$$

8. Define

$$A_{sw} = n \frac{\phi^2}{4} \quad (33)$$

where

$\phi$  : is the diameter of the bars, see Table 12

$n$  : the number of bars along the height of the section

it should be choosed in order to have  $s_{section} < 500 \text{ mm}$

9. Compute the spacing  $s$

Diameter [mm]	Area [mm <sup>2</sup> ]
6	28
8	50
10	78
12	113

Table 12: Common diameters for transversal reinforcement

## 8.1 Optimisation

Process

1. Choose  $x$
2. Compute  $V_{d,x+d}$

$$n = \frac{V_{d,x+d}}{V_{d,max}}$$

3. Compute  $s_{x+d} = ns_{min}$

## 9 Columns

### 9.1 Second order effects

1. Compute

$$\varphi_{ef} = \frac{M_{quasi}}{M_d} \varphi \quad (34)$$

where

$\varphi$  : is the creep coefficient

$M_{quasi}$  : the quasi-permanent combination (realistic load case)

2. Compute

$$E_{cd} I_{cd} = \frac{0,3}{1 + \varphi_{ef}} E_{cd} I_c \quad (35)$$

where

$$E_{cd} = \frac{E_c}{1,2}$$

$$I_c = I_{gross}$$

3. Compute  $L_p$ , see Figure 5

$$4. \text{ Compute } N_B = \left(\frac{\pi}{L_p}\right)^2 E_{cd} I_{cd}$$

5. Compute

$$M_{d,2} = \frac{M_d}{1 - \frac{N_d}{N_B}} \quad (36)$$

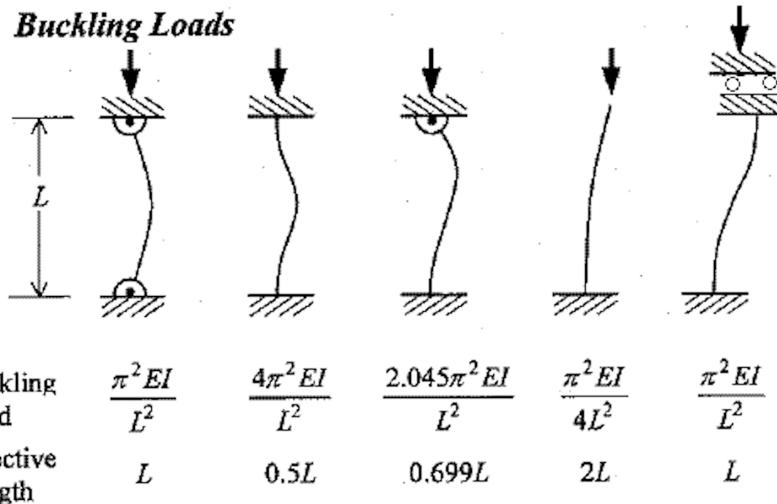


Figure 5: Buckling length and load depending on the support

## 9.2 Longitudinal reinforcement

1. Compute  $\nu = \frac{N_d}{A_c f_{cd}}$
2. Compute  $\mu = \frac{M_{d,2}}{A_c D f_{cd}}$   
where  $D$  : is a characteristic length
3. Find  $w(\nu, \mu) = \frac{A_s f_{yd}}{A_c f_{cd}}$
4. Check minimal quantity

$$w = \max(w; w_{min} = 0,01)$$

## 9.3 Transversal reinforcement

1. Compute  $\tau_c = \frac{V_c}{bd}$
2. If  $\tau_c < 0,5 MPa$  (can be resisted by concrete we should put the minimum)

$$\begin{cases} \phi_t > \phi_{min} = \frac{\phi_{long}}{4} \\ s_t < 15\phi_{long} \end{cases} \quad (37)$$

## 10 Detailing and layout

- Along the section the maximal distance between bar axes should be  $d_{max} = 300mm$

## References

- [1] Jesús Bairan. *Lecture notes in Concrete Structures*. 2017.