Groundwater Hydrology

Summary

Foreword Any error or contribution should be reported in the form of an issue, or a pull request for those who can use `git` and LaTeX, to

https://github.com/mbataillou/Summaries/tree/master/Caminos/Hidrologia

You can notice that there is always place for improvement and your help is therefore welcome.

Version 3.0

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Contents

1 General

Types of soil:

- *Aquifer*: capable to store and transmit water
- *Aquiclude*: not capable to transmit but able to store water
- *Aquitard*: transmit very slowly not store

Aquifer types:

- *Confined*: recharge from other areas (flow)
- *Unconfined*: there is recharge and $\phi \approx 0,3 \in [0,25;0,4]$

Relation river aquifer

- "Efluente": receives water from the aquifer
- "Afluente": gives water to the aquifer
- Disconnected: doesn't interact with the aquifer

2 Darcys law (Momentum continuity equation)

$$\begin{cases} q = -\mathrm{K}\nabla h \\ v = \frac{q}{\phi} \end{cases}$$

where

$$\mathrm{K} = \begin{pmatrix} \mathrm{K}_{xx} & \mathrm{K}_{xy} & \mathrm{K}_{xz} \\ \mathrm{K}_{yx} & \mathrm{K}_{yy} & \mathrm{K}_{xz} \\ \mathrm{K}_{zx} & \mathrm{K}_{xy} & \mathrm{K}_{zz} \end{pmatrix}$$

$\phi :$ *is the porosity*

3 Flow equation

3.1 General expression

$$\mathrm{W} - \nabla \cdot q = \mathrm{S}_s \frac{\partial}{\partial t} h$$

Where $h = \frac{p}{\gamma} + z$ and S_s the specific storage. where

$h = \dfrac{p}{\gamma} + z :$ *is the piezometric height*

$\mathrm{S}_s :$ *is the specific storage*

3.1.1 $\mathbb{R}^2$

Suposing that the transversal (depth) conductivity is constant over layers, we obtain

$$r - \nabla \cdot (\mathrm{T}\nabla h) = \mathrm{S}\frac{\partial}{\partial t} h$$

where

$r = \mathrm{W}b :$ *is the recharge*

$\mathrm{S} = \mathrm{S}_s b :$ *is the storage*

$\mathrm{T} = \mathrm{K}b :$ *is the transmissivity*

3.2 Solutions

3.2.1 $\mathbb{R}$ / no recharge / steady state / homogeneity

$$h(x) = h_0 - \left(\frac{h_\mathrm{L} - h_0}{\mathrm{L}}\right)$$

3.2.2 $\mathbb{R}$ / no recharge / steady-state / heterogeneous

$$\begin{cases} \frac{d}{dx}\left(\mathrm{T}(x)\frac{d}{dx}h\right) = 0 & x \in \Omega \\ h(0) = h_0 & x \in \partial\Omega \\ h(\mathrm{L}) = h_\mathrm{L} \end{cases}$$

$$\Leftrightarrow h(x) = h_0 + \frac{h_\mathrm{L} - h_0}{\int_0^\mathrm{L} \frac{d}{d\mathrm{T}(u)} u} \int_0^x \frac{d}{d\mathrm{T}(u)} u$$

3.2.3 $\mathbb{R}$ / recharge / steady-state / homogeneous

$$\begin{cases} \mathrm{T}\frac{d^2}{dx^2}h + r = 0 & x \in \Omega \\ h(x) = h_0 & x \in \partial\Omega \end{cases}$$

$$\Leftrightarrow h(x) = -\frac{r\mathrm{L}}{2\mathrm{T}}x^2 + \frac{r\mathrm{L}}{2\mathrm{T}}x + \frac{r\mathrm{L}}{2\mathrm{T}}h_0$$

3.2.4 $\mathbb{R}$ / no recharge / transient / homogeneous

$$\begin{cases} \mathrm{T}\frac{d^2}{dx^2}h = \mathrm{S}\frac{d}{dt}h & x \in \Omega \\ h(x, t = 0) = h_0 & t = 0 \end{cases}$$

$\Leftrightarrow$ Numerical Methods

3.2.5 $\mathbb{R}^2$ *(radial symetry)* / no recharge / steady-state / homogeneous

$$\begin{cases} \mathrm{T}_x\frac{d^2}{dx^2}h + \mathrm{T}_y\frac{d^2}{dy^2}h = 0 & x \in \Omega \\ \frac{d}{dr}h|_{r=r_w} = \mathrm{Q} \\ \mathrm{T}_x = \mathrm{T}_y \end{cases}$$

Polar transformation $f : (x, y) \in \mathbb{R}^2 \to (r, \theta) \in \mathbb{R} \times [0, 2\pi]$

$$\begin{cases} \frac{d}{dr}\left(r\frac{d}{dr}h\right) = 0 & x \in \Omega \\ \frac{d}{dr}h|_{r=r_w} = \mathrm{Q} \\ s(r = \mathrm{R}) = 0 & x \in \partial\Omega \end{cases}$$

$$\Leftrightarrow s(r) = \frac{\mathrm{Q}}{2\pi\mathrm{T}}\ln\left(\frac{\mathrm{R}}{r}\right)$$

where

$s(r) = h_0 - h(r, t) :$ *is the drowdown*

$\mathrm{R} :$ *is the influence ratio* $\approx 1000m \in [800m, 1500]$

3.2.6 $\mathbb{R}^2$ *(radial symetry)* / no recharge / transient / homogeneous

$$\begin{cases} \frac{d}{dr}\left(r\frac{d}{dr}h\right) = 0 & x \in \Omega \\ s(r, 0) = 0 & t = 0 \\ \frac{d}{dr}h|_{r=r_w} = \mathrm{Q} \\ \lim_{x\to\infty} s(x, t) = 0 & x \in \partial\Omega \end{cases}$$

$$\Leftrightarrow s(r) = \frac{\mathrm{Q}}{4\pi\mathrm{T}}\int_{\frac{r^2 s}{4\pi\mathrm{T}t}}^{\infty} \frac{e^{-x}}{x}\,dx$$

3.2.7 $\mathbb{R}^2$ / no recharge / steady-state / homogeneous [1]

$$\begin{cases} \mathrm{T}_x\frac{d^2}{dx^2}h + \mathrm{T}_y\frac{d^2}{dy^2}h = 0 & x \in \Omega \\ \mathrm{T}_x = \mathrm{T}_y \\ h(x_i, y_i) = h_i & x \in \partial\Omega \end{cases}$$

$$\Leftrightarrow \frac{d^2}{dx^2}h + \frac{d^2}{dy^2}h = 0$$

4 Groundwater Budget

Water Balance

$$\begin{cases} \Delta\mathrm{S} = \frac{\mathrm{A}\Delta h\phi}{\Delta t} & \textit{water balance} \\ \mathrm{Q}_i = n\mathrm{T}\Delta\mathrm{H}\frac{a_i}{c_i} & \textit{flow through an isoline segment} \end{cases}$$

where

$a_i :$ *length along the segment*

$c_i :$ *distance between isolines*

$n :$ *number of flow tubes*

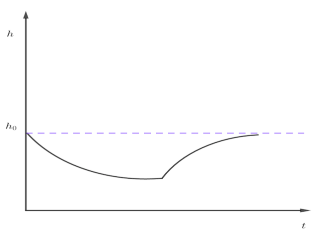
[1] This solution is usually solved with graphic methods

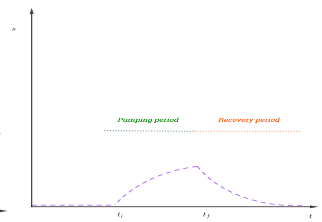
Inputs	Outputs
Boundaries [2]	Boundaries
Recharge [3]	Wells, springs
	Surface water bodies

5 Pumping test

Procedure

- Measure water level at each point
- Turn on the pump
- Stop pumping (Generally after 1 day)
- Continue the measurements for about 4-5 days





And we have the following relations [4]

$$\begin{cases} m = \frac{d}{d\log(t)} s = 0.183\frac{Q}{T} \\ S = \frac{2{,}25 T t_i}{r^2} \end{cases}$$

6 Well hydraulics

6.1 Superposition theory

The contribution of n wells [5] at $t = t_0$

$$s((x_0, y_0), t_0) = \sum_{i=1}^{n} s_i((x_0, y_0), t_0)$$

Steady

$$s(x_0, y_0) = \sum_{i=1}^{n} \frac{Q_i}{2\pi T} \ln\left(\frac{R}{r_i}\right)$$

Transitory

$$s((x_0, y_0), t) = \sum_{i=1}^{n} \frac{Q_i}{4\pi T} \ln\left[\frac{2,25T}{r_i^2 s}(t - t_i)\right]$$

6.2 Boundaries

6.2.1 Impervious boundary

Steady

$$s(x_0, y_0) = \frac{Q}{2\pi T} \ln\left(\frac{R^2}{rr'}\right)$$

Transitory

$$s((x_0, y_0), t) = \frac{Q}{2\pi T} \ln\left[\frac{2,25T}{rr's}(t - t_i)\right]$$

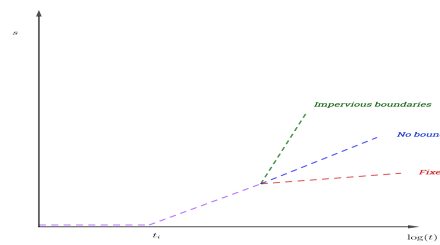
6.2.2 Fixed head boundary

Steady

$$s(x_0, y_0) = \frac{Q}{2\pi T} \ln\left(\frac{r'}{r}\right)$$

Transitory

$$s((x_0, y_0), t) = s(x_0, y_0) = \frac{Q}{2\pi T} \ln\left(\frac{r'}{r}\right)$$



7 Pollution

7.1 Advection

$$\begin{cases} v = \frac{q}{\phi} = -\frac{K}{\phi}\frac{\partial}{\partial x} h \\ F = v\phi C \\ \frac{\partial}{\partial t} C = -v \frac{\partial}{\partial x} C \end{cases}$$

where

v : *is the average velocity in the flow direction*

$$\begin{cases} \frac{\partial}{\partial t} C = D \cdot \Delta C & \textit{difusion + dispersion eq} \\ \frac{\partial}{\partial t} C = D \cdot \Delta C - v \frac{\partial}{\partial x} C & \textit{advection eq} \end{cases}$$

where

D : *is the difusivity vector*

$$\begin{cases} C_{max} = \frac{M}{\phi b 4\pi t \sqrt{D_L D_T}} \\ C = \frac{M}{\phi b 4\pi t \sqrt{D_L D_T}} \exp\left[\frac{-(x - vt)^2}{4 D_L t} - \frac{-y^2}{4 D_T t}\right] \end{cases}$$

where

$$M = \frac{C_0 V_0}{M_{molar}}$$

7.2 Hydrodinamic dispersion

1. Molecular difusion (Ficks laws)
2. Mechanical dispersion

$$\begin{cases} F = -D_d \frac{\partial}{\partial x} C & \textit{Ficks first law} \\ \frac{\partial}{\partial t} C = -D_d \frac{\partial^2}{\partial x^2} C & \textit{Ficks second law} \end{cases}$$

where

F : *is the amount of solute per unit area*

D_d : *is the diffusivity*

C : *is the solute concentratios*

$$\begin{cases} D_L = \alpha_L v_L + D_d \\ D_T = \alpha_T v_T + D_d \end{cases}$$

where

α_L : *is the dispersivity*

L, D : *account for longitudinal and transversal*

<table>
<tr><td>K (cm/s)</td><td>10²</td><td>10¹</td><td>10⁰=1</td><td>10⁻¹</td><td>10⁻²</td><td>10⁻³</td><td>10⁻⁴</td><td>10⁻⁵</td><td>10⁻⁶</td><td>10⁻⁷</td><td>10⁻⁸</td><td>10⁻⁹</td><td>10⁻¹⁰</td></tr>
<tr><td>K (ft/day)</td><td>10⁵</td><td>10,000</td><td>1,000</td><td>100</td><td>10</td><td>1</td><td>0.1</td><td>0.01</td><td>0.001</td><td>0.0001</td><td>10⁻⁵</td><td>10⁻⁶</td><td>10⁻⁷</td></tr>
<tr><td>Relative Permeability</td><td colspan="4">Pervious</td><td colspan="4">Semi-Pervious</td><td colspan="5">Impervious</td></tr>
<tr><td>Aquifer</td><td colspan="5">Good</td><td colspan="4">Poor</td><td colspan="4">None</td></tr>
<tr><td>Unconsolidated Sand & Gravel</td><td colspan="2">Well Sorted Gravel</td><td colspan="3">Well Sorted Sand or Sand & Gravel</td><td colspan="4">Very Fine Sand, Silt, Loess, Loam</td><td colspan="4"></td></tr>
<tr><td>Unconsolidated Clay & Organic</td><td colspan="4"></td><td colspan="2">Peat</td><td colspan="3">Layered Clay</td><td colspan="4">Fat / Unweathered Clay</td></tr>
<tr><td>Consolidated Rocks</td><td colspan="4">Highly Fractured Rocks</td><td colspan="3">Oil Reservoir Rocks</td><td colspan="2">Fresh Sandstone</td><td colspan="2">Fresh Limestone, Dolomite</td><td colspan="2">Fresh Granite</td></tr>
</table>

Conversion:

$$1\,ft = 0,3048m \qquad \frac{1}{86,4}\frac{m^3}{day} = \frac{L}{s}$$

[2] Surface water bodies

[3] Must be data

[4] if $S \approx 0,2 - 0,4 \rightarrow$ free aquifer, if $S \approx 10^{-4} - 10^{-3} \rightarrow$ confined aquifer

[5] Only the wells where $|(x_0, y_0) - (x, y)| < R|$

References

[1] Xavier Sanchez Vila. *Lecture notes in Groundwater Hydrology*. 2017.