# Concrete Structures

## Summary

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# Contents

# 1 General

$$KPa = \frac{KN}{m^2} \Leftrightarrow MPa = \frac{N}{mm^2}$$
$$I_c = I_{rectangle} = \frac{bd^3}{12}$$
$$I_{circle} = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}$$
$$I_{G+x} = I_G + Ax^2 \; Steiner \; (x \; perpendicular \; to \; the \; ref \; axis)$$

# 2 Inertia

## 2.1 Before cracking

$$I = \sum_{i=1}^{n} \alpha_i \left[ I_i + A_i (y_i - \bar{y})^2 \right] = I_c + A_c (y_c - \bar{y})^2 + \alpha \left[ I_s + I_s^t + A_s (y_s - \bar{y})^2 + A_s^t (y_s^t - \bar{y})^2 \right] \quad (1)$$

where

$$\bar{y} = \frac{\sum y_i A_i \alpha_i}{\sum A_i \alpha_i} = \frac{y_c A_c + y_s A_s \alpha + y_s^t A_s^t \alpha}{A_c + A_s \alpha + A_s^t \alpha}$$
$$\alpha_i = \frac{E_i}{E_j} = \alpha = \frac{E_s}{E_c}$$

### 2.1.1 Aproximation

$$I_{gross} = \frac{bd^3}{12} \approx 0.9I$$
$$E_c = 22 \left( \frac{f_{ck} + 8}{10} \right)^{0,3}$$

## 2.2 Cracked

$$I = \sum_{i=1}^{n} \alpha_i [I_i + A_i (y_i - x)^2] = I_c + A_c (y_c - x)^2 + \alpha [I_s + I_s^t + A_s (y_s - x)^2 + A_s^t (y_s^t - x)^2] \quad (2)$$

where

$$\frac{x_{cracked}}{d} = \alpha\rho \left( 1 + \frac{\rho^t}{\rho} \right) \left[ -1 \sqrt{1 + \frac{2\left(1 + \frac{d^t \rho^t}{d\rho}\right)}{\alpha\rho \left(1 + \frac{\rho^t}{\rho}\right)}} \right]$$
$$I_c = I_{rectangle/bottom} = \frac{bd^3}{3}$$

### 2.2.1 Approximation

$$I \approx \frac{bd^3}{3} + \alpha [A_s (y_s - x)^2 + A_s^t (y_s^t - x)^2] \quad (3)$$

where

$$\frac{x_{cracked}}{d} \approx \frac{0.18 + 1.8\alpha\rho}{1 + \frac{d^t \rho^t}{d\rho}}$$

# 3 Material deformation

1. Elastic deformation

$$\varepsilon_i(t_1) = \frac{\sigma(t_1)}{E_c(t_1)} \quad (4)$$

where

$$\sigma(t_1): \; is \; the \; tension \; at \; t_1$$
$$E_c(t_1): \; is \; the \; elastic \; modulus \; at \; t_1$$

2. Creep

$$\varepsilon_{cc}(t_2, t_1) = \varphi(t_2, t_1) \frac{\sigma(t_1)}{E_c(28)} \quad (5)$$

where

$$\varphi(t_2, t_1): \; is \; the \; creep \; coeficient \; from \; t_1 \; to \; t_2 \; see \; material \; \text{PP}$$
$$\varphi(\infty, t_1): \; see \; Figures \; 1,2$$

3. Shrinkage

$$\varepsilon_{cs} = \varepsilon_{ca} + \varepsilon_{cd} \quad (6)$$

where

$$\varepsilon_{ca} = \begin{cases} \varepsilon_{ca} = \beta_{as}(t)\varepsilon_{ca}(\infty) \\ \beta_{as}(t) = 1 - e^{-0.2t^{0.5}} \\ \varepsilon_{ca}(\infty) = 2.5(f_{ck} - 10) \cdot 10^{-6} \end{cases} \quad \textit{is the autogenous shrinkage}$$

$$\varepsilon_{cd} = \begin{cases} \varepsilon_{cd} = \beta_{ds}(t,t_s)k_h\varepsilon_{cd,0} \\ \beta_{ds}(t,t_s) = \frac{t-t_s}{t-t_s+0.04\sqrt{h_0^3}} \end{cases} \quad \textit{is the drying shrinkage}$$

*$h_0$ is the shape coefficient*

*$k_h$ see Table 1*

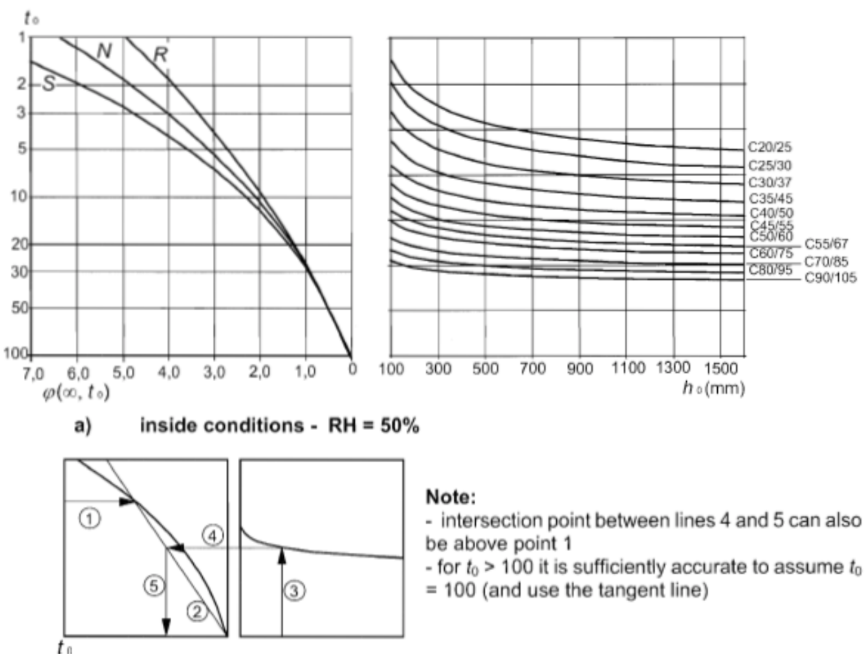



Figure 1: Model for infinite creep coefficient RH =50

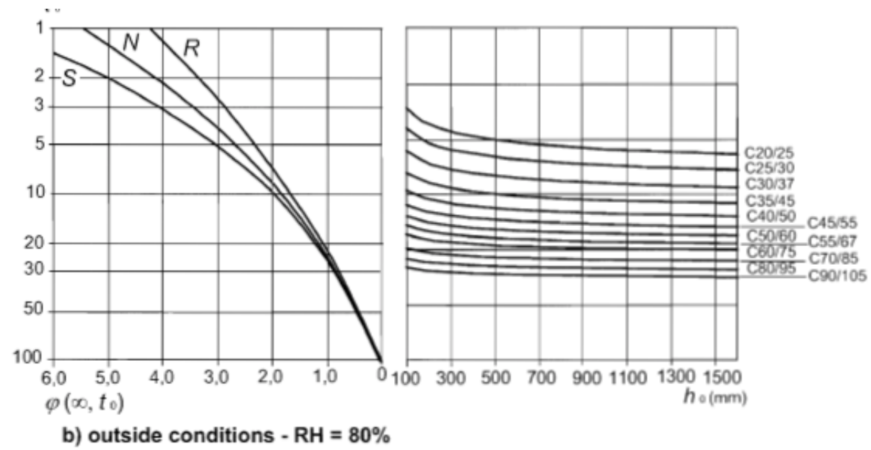



Figure 2: Model for infinite creep coefficient RH =80

| $h_0$ | $k_h$ |
|---|---|
| 100 | 1 |
| 200 | 0.85 |
| 300 | 0.75 |
| $\leqslant 500$ | 0.7 |

Table 1: $k_h$ depending on $h_0$

## 4 Durability requirements

1. Find exposure class in Table 2
2. Define concrete class with Table 3
3. Compute structural class with Table 4, taking into account that $S_{min} = S_1$ and $S_{ref,T=50y} = S_4$
4. Find minimal cover in Table 5
5. Add tolerance

$$C_{dur} = C(Table\ 5) + \Delta c$$

$$C_{dur} = C(Table\ 5) + \Delta c \tag{7}$$

where

$$\Delta c \in [0, 10] \rightarrow 5mm$$

| Class designation | Description of the environment | Informative examples where exposure classes may occur |
|---|---|---|
| **1 No risk of corrosion or attack** | | |
| X0 | For concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack<br>For concrete with reinforcement or embedded metal: very dry | Concrete inside buildings with very low air humidity |
| **2 Corrosion induced by carbonation** | | |
| XC1 | Dry or permanently wet | Concrete inside buildings with low air humidity<br>Concrete permanently submerged in water |
| XC2 | Wet, rarely dry | Concrete surfaces subject to long-term water contact<br>Many foundations |
| XC3 | Moderate humidity | Concrete inside buildings with moderate or high air humidity<br>External concrete sheltered from rain |
| XC4 | Cyclic wet and dry | Concrete surfaces subject to water contact, not within exposure class XC2 |
| **3 Corrosion induced by chlorides** | | |
| XD1 | Moderate humidity | Concrete surfaces exposed to airborne chlorides |
| XD2 | Wet, rarely dry | Swimming pools<br>Concrete components exposed to industrial waters containing chlorides |
| XD3 | Cyclic wet and dry | Parts of bridges exposed to spray containing chlorides<br>Pavements<br>Car park slabs |
| **4 Corrosion induced by chlorides from sea water** | | |
| XS1 | Exposed to airborne salt but not in direct contact with sea water | Structures near to or on the coast |
| XS2 | Permanently submerged | Parts of marine structures |
| XS3 | Tidal, splash and spray zones | Parts of marine structures |
| **5. Freeze/Thaw Attack** | | |
| XF1 | Moderate water saturation, without de-icing agent | Vertical concrete surfaces exposed to rain and freezing |
| XF2 | Moderate water saturation, with de-icing agent | Vertical concrete surfaces of road structures exposed to freezing and airborne de-icing agents |
| XF3 | High water saturation, without de-icing agents | Horizontal concrete surfaces exposed to rain and freezing |
| XF4 | High water saturation with de-icing agents or sea water | Road and bridge decks exposed to de-icing agents<br>Concrete surfaces exposed to direct spray containing de-icing agents and freezing<br>Splash zone of marine structures exposed to freezing |
| **6. Chemical attack** | | |
| XA1 | Slightly aggressive chemical environment according to EN 206-1, Table 2 | Natural soils and ground water |
| XA2 | Moderately aggressive chemical environment according to EN 206-1, Table 2 | Natural soils and ground water |
| XA3 | Highly aggressive chemical environment according to EN 206-1, Table 2 | Natural soils and ground water |

Table 2: Exposure class

| | Exposure Classes according to Table 4.1 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Corrosion** | | | | | | | | | | |
| | Carbonation-induced corrosion | | | | Chloride-induced corrosion | | | Chloride-induced corrosion from sea-water | | |
| | XC1 | XC2 | XC3 | XC4 | XD1 | XD2 | XD3 | XS1 | XS2 | XS3 |
| Indicative minimum strength class | C20/25 | C25/30 | C30/37 | | C30/37 | | C35/45 | C30/37 | C35/45 | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Damage to Concrete** | | | | | | | |
| | No risk | Freeze/Thaw Attack | | | Chemical Attack | | |
| | X0 | XF1 | XF2 | XF3 | XA1 | XA2 | XA3 |
| Indicative minimum strength class | C12/15 | C30/37 | C25/30 | C30/37 | C30/37 | | C35/45 |

Table 3: Concrete class depending on exposure class

| **Structural Class** | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Criterion** | **Exposure Class according to Table 4.1** | | | | | | |
| | X0 | XC1 | XC2 / XC3 | XC4 | XD1 | XD2 / XS1 | XD3 / XS2 / XS3 |
| Design Working Life of 100 years | increase class by 2 | increase class by 2 | increase class by 2 | increase class by 2 | increase class by 2 | increase class by 2 | increase class by 2 |
| Strength Class [1) 2)] | ≥ C30/37 reduce class by 1 | ≥ C30/37 reduce class by 1 | ≥ C35/45 reduce class by 1 | ≥ C40/50 reduce class by 1 | ≥ C40/50 reduce class by 1 | ≥ C40/50 reduce class by 1 | ≥ C45/55 reduce class by 1 |
| Member with slab geometry (position of reinforcement not affected by construction process) | reduce class by 1 | reduce class by 1 | reduce class by 1 | reduce class by 1 | reduce class by 1 | reduce class by 1 | reduce class by 1 |
| Special Quality Control of the concrete production ensured | reduce class by 1 | reduce class by 1 | reduce class by 1 | reduce class by 1 | reduce class by 1 | reduce class by 1 | reduce class by 1 |

Table 4: Structural class

| **Environmental Requirement for $c_{min,dur}$ (mm)** | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Structural Class** | **Exposure Class according to Table 4.1** | | | | | | |
| | X0 | XC1 | XC2 / XC3 | XC4 | XD1 / XS1 | XD2 / XS2 | XD3 / XS3 |
| S1 | 10 | 10 | 10 | 15 | 20 | 25 | 30 |
| S2 | 10 | 10 | 15 | 20 | 25 | 30 | 35 |
| S3 | 10 | 10 | 20 | 25 | 30 | 35 | 40 |
| S4 | 10 | 15 | 25 | 30 | 35 | 40 | 45 |
| S5 | 15 | 20 | 30 | 35 | 40 | 45 | 50 |
| S6 | 20 | 25 | 35 | 40 | 45 | 50 | 55 |

Table 5: Minimal required cover depending on exposure and structural class

# 5 Load combinations

## 5.1 Ultimate limit state

$$\sigma_{ULS} = \gamma_G \sum G_i + \gamma_P P + \gamma_Q Q_1 + \gamma_Q \psi_0 \sum_{i \neq 1} Q_i \tag{8}$$

where

$\gamma_G = 1,35$ : *is the security factor for permanent loads*
$\gamma_Q = 1,5$ : *is the security factor for live loads*
$\gamma_P$ : *is the security factor for prestressing load*
$\psi_0$ : *is a factor accounting for the frequence (probability)*
$Q_1$ : *is the principal live load*
$Q_i\ i \neq 1$ : *is a secondary live load*
$G_i$ : *is a permanent load*

## 5.2 Service state

- Characteristic

$$\sigma_{CHA} = \sum G_i + P + Q_1 + \psi_0 \sum_{i \neq 1} Q_i \tag{9}$$

$$\tag{10}$$

- Frequent

$$\sigma_{FRE} = \sum G_i + P + \psi_1 Q_1 + \psi_2 \sum_{i \neq 1} Q_i \tag{11}$$

where

$\psi_1$ : *is a factor accounting for the frequence (probability)*
$\psi_2$ : *is a factor accounting for the frequence (probability)*

- Quasi-permanent

$$\sigma_{QUA} = \sum G_i + P + \psi_2 \sum Q_i \tag{12}$$

$$\tag{13}$$

# 6 Longitudinal reinforcement

## 6.1 Aproximations

### 6.1.1 Design

$$d \approx 0,9h$$

1. if $fyk = 400$

$$M_{lim} \approx 0,369 f_{cd} b d^2 \tag{14}$$
$$x_{lim} \approx 0,668d \tag{15}$$
$$y_l im \approx 0,534d \tag{16}$$

where

$M_{lim}$ : *is the max moment resisted without comp reinforcement*
$x_{lim}$ : *is the x where steel starts yielding*
$y_{lim}$ : *is the effective comp zone*

2. if $fyk = 500$

$$M_{lim} \approx 0,375 f_{cd} b d^2 \tag{17}$$
$$x_{lim} \approx 0,617d \tag{18}$$
$$y_{lim} \approx 0,5d \tag{19}$$

## 6.2 Spacing

1. Define $\phi_l$, see Table 6
2. Compute $n = \frac{A_s}{\frac{\phi^2}{4}}$
3. Compute

$$s = \frac{b - 2c - 2\phi_t - n\phi_l}{n-1} \tag{20}$$

where

$s > \max(30mm(1,25 \cdot \phi_{arid}); \phi_{long})$ $b$ : *is the width*
$c$ : *is the minimal cover*
$\phi_t$ : *is the transversal reinforcement diameter*

| Diameter $[mm]$ | Area $[mm^2]$ |
|---|---|
| 12 | 113 |
| (14) | 154 |
| 16 | 200 |
| 20 | 314 |
| 25 | 490 |

Table 6: Common diameters for longitudinal reinforcement

## 6.3 Optimisation

1. Choose reduction factor $n$
2. Find $x$ where $M_{d,x} = nM_d$
3. if $\sigma_s \neq f_{yd} = 435MPa$Compute

$$l_{b,rqd} = l_{table}\frac{\sigma_s}{f_{yd}} \tag{21}$$

   where

$$l_{table}: \ \textit{see Table 7}$$

4. else $l_{b,rqd} = l_{table}$
5. Apply correction for layout
   (a) Anchoring

$$l_0 = l_{b,rqd}\Pi_{i=1}^{5}\alpha_i \approx kl_{b,rqd} \tag{22}$$

   where

$$\alpha_i: \ \textit{see Figures 34}$$

$$k = \begin{cases} 1 & \text{if } \textit{straight} \\ 0,7 & \text{if } \textit{hook} \end{cases}$$

$$l_0 = \max(l_0, l_{b,min}) \tag{23}$$

   where

$$l_{b,min}: \ \textit{see Table 8}$$

   (b) Connecting

$$l_0 = l_{b,rqd}\Pi_{i=1,i\neq 4}^{6}\alpha_i \tag{24}$$

   where

$$\alpha_i, i \neq 3: \ \textit{see Figure 3}$$

$$\alpha_3: \sum A_{s,min} = A_s\frac{\sigma_{sd}}{f_{yd}}, \quad A_s \textit{ the area of one lapped bar}$$

   *See more details in PP*

$$l_0 = \max(l_0, l_{min}) \tag{25}$$

   where

$$l_{0,min} = \max(0,3\alpha_6 l_{b,rqd}; 15\phi;\ 200mm)$$

6. Cut the bars at $x = x + d + l_{b,rqd}$

| ∅ | C25/30 | C30/37 | C40/50 | C50/60 | C60/75 |
|---|---|---|---|---|---|
| 6 | 242 | 217 | 174 | 150 | 140 |
| 8 | 322 | 290 | 232 | 200 | 187 |
| 10 | 403 | 362 | 290 | 250 | 234 |
| 12 | 483 | 435 | 348 | 300 | 281 |
| 16 | 644 | 580 | 464 | 400 | 374 |
| 20 | 805 | 725 | 580 | 500 | 468 |
| 25 | 1006 | 906 | 725 | 625 | 584 |
| 32 | 1288 | 1159 | 928 | 800 | 748 |
| 40 | 1610 | 1449 | 1159 | 1000 | 935 |

Table 7: $l_{b,rqd}$ for $\sigma_s = f_{yd} = 435MPa$

Minimum length in tension

| ∅ | C25/30 | C30/37 | C40/50 | C50/60 | C60/75 |
|---|---|---|---|---|---|
| 6 | 100 | 100 | 100 | 100 | 100 |
| 8 | 100 | 100 | 100 | 100 | 100 |
| 10 | 121 | 109 | 100 | 100 | 100 |
| 12 | 145 | 130 | 120 | 120 | 120 |
| 16 | 193 | 174 | 160 | 160 | 160 |
| 20 | 242 | 217 | 200 | 200 | 200 |
| 25 | 302 | 272 | 250 | 250 | 250 |
| 32 | 386 | 348 | 320 | 320 | 320 |
| 40 | 483 | 435 | 400 | 400 | 400 |

Minimum length in compression

| ∅ | C25/30 | C30/37 | C40/50 | C50/60 | C60/75 |
|---|---|---|---|---|---|
| 6 | 145 | 130 | 104 | 100 | 100 |
| 8 | 193 | 174 | 139 | 120 | 112 |
| 10 | 242 | 217 | 174 | 150 | 140 |
| 12 | 290 | 261 | 209 | 180 | 168 |
| 16 | 386 | 348 | 278 | 240 | 224 |
| 20 | 483 | 435 | 348 | 300 | 281 |
| 25 | 604 | 543 | 435 | 375 | 351 |
| 32 | 773 | 696 | 557 | 480 | 449 |
| 40 | 966 | 870 | 696 | 600 | 561 |

Table 8: $l_{min}$ depending on $\phi$ and $f_{ck}$

| Influencing factor | Type of anchorage | Reinforcement bar | |
|---|---|---|---|
| | | In tension | In compression |
| Shape of bars | Straight | $\alpha_1 = 1{,}0$ | $\alpha_1 = 1{,}0$ |
| | Other than straight (see Figure 8.1 (b), (c) and (d) | $\alpha_1 = 0{,}7$ if $c_d > 3\phi$ otherwise $\alpha_1 = 1{,}0$ (see Figure 8.3 for values of $c_d$) | $\alpha_1 = 1{,}0$ |
| Concrete cover | Straight | $\alpha_2 = 1 - 0{,}15\,(c_d - \phi)/\phi$ $\geq 0{,}7$ $\leq 1{,}0$ | $\alpha_2 = 1{,}0$ |
| | Other than straight (see Figure 8.1 (b), (c) and (d)) | $\alpha_2 = 1 - 0{,}15\,(c_d - 3\phi)/\phi$ $\geq 0{,}7$ $\leq 1{,}0$ (see Figure 8.3 for values of $c_d$) | $\alpha_2 = 1{,}0$ |
| Confinement by transverse reinforcement not welded to main reinforcement | All types | $\alpha_3 = 1 - K\lambda$ $\geq 0{,}7$ $\leq 1{,}0$ | $\alpha_3 = 1{,}0$ |
| Confinement by welded transverse reinforcement* | All types, position and size as specified in Figure 8.1 (e) | $\alpha_4 = 0{,}7$ | $\alpha_4 = 0{,}7$ |
| Confinement by transverse pressure | All types | $\alpha_5 = 1 - 0{,}04p$ $\geq 0{,}7$ $\leq 1{,}0$ | - |

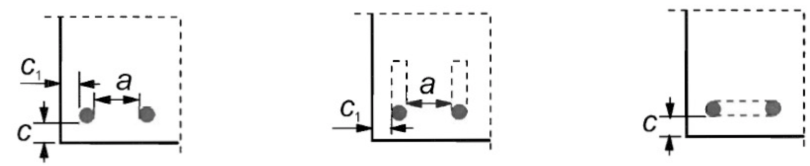
where:
$\lambda$ $= (\Sigma A_{st} - \Sigma A_{st,min})/ A_s$
$\Sigma A_{st}$ cross-sectional area of the transverse reinforcement along the design anchorage length $l_{bd}$
$\Sigma A_{st,min}$ cross-sectional area of the minimum transverse reinforcement $= 0{,}25\,A_s$ for beams and 0 for slabs
$A_s$ area of a single anchored bar with maximum bar diameter
$K$ values shown in Figure 8.4
$p$ transverse pressure [MPa] at ultimate limit state along $l_{bd}$

* See also 8.6: For direct supports $l_{bd}$ may be taken less than $l_{b,min}$ provided that there is at least one transverse wire welded within the support. This should be at least 15 mm from the face of the support.

| Percentage of lapped bars relative to the total cross-section area | < 25% | 33% | 50% | >50% |
|---|---|---|---|---|
| $\alpha_6$ | 1 | 1,15 | 1,4 | 1,5 |

**Note:** Intermediate values may be determined by interpolation.

Figure 3: $\alpha_i$ depending on the layout




**a) Straight bars** $c_d = \min\,(a/2,\ c_1,\ c)$ **b) Bent or hooked bars** $c_d = \min\,(a/2,\ c_1)$ **c) Looped bars** $c_d = c$

Figure 4: $c_d$ depending on the layout

# 7 Crack width control

## 7.1 Reinforced

1. Compute $M_{quasi}$
2. Compute $x_{cracked}$ see 2.2
3. Compute

$$\sigma_s = \frac{M_{quasi}}{\left(d - \frac{x_{cracked}}{3}\right) A_s} \tag{26}$$

where

$$d = h - c - \phi_t - k\phi_l$$
$$k: \ \textit{is a parameter depending on number of layers}$$

4. Check allowable crack width in Table 9
5. Compute $\sigma_{crack} = \max(\sigma(Table\ 10), \sigma(Table\ 11))$
6. If $\sigma_{crack} > \sigma_s$

$$A_{s,crack} = \frac{\sigma_{crack}}{\sigma_s} A_s \tag{27}$$

where

$$A_{s,crack}: \ \textit{is the area needed to fullfill crack width}$$

| Exposure Class | Reinforced members and prestressed members with unbonded tendons | Prestressed members with bonded tendons |
|---|---|---|
| | Quasi-permanent load combination | Frequent load combination |
| X0, XC1 | 0,4[1] | 0,2 |
| XC2, XC3, XC4 | 0,3 | 0,2[2] |
| XD1, XD2, XD3, XS1, XS2, XS3 | | Decompression |

**Note 1:** For X0, XC1 exposure classes, crack width has no influence on durability and this limit is set to give generally acceptable appearance. In the absence of appearance conditions this limit may be relaxed.
**Note 2:** For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.

Table 9: Maximum allowable crack width depending on technology and environmental conditions

| $\omega = 0,2$ [mm] | $\omega = 0,3$ [mm] | $\omega = 0,4$ [mm] | Máxima tensión $\sigma$ [Mpa] |
|---|---|---|---|
| 25 | 32 | 40 | 160 |
| 16 | 25 | 32 | 200 |
| 12 | 16 | 20 | 240 |
| 8 | 12 | 16 | 280 |
| 6 | 10 | 12 | 320 |
| 5 | 8 | 10 | 360 |
| 4 | 6 | 8 | 400 |
| - | 5 | 6 | 450 |

Table 10: Diámetro (en $mm$) de barra en función $\sigma_{max}$ y $\omega$

| $\omega = 0,2$ [mm] | $\omega = 0,3$ [mm] | $\omega = 0,4$ [mm] | Máxima tensión $\sigma$ [Mpa] |
|---|---|---|---|
| 200 | 300 | 300 | 160 |
| 150 | 250 | 300 | 200 |
| 100 | 200 | 250 | 240 |
| 30 | 150 | 200 | 280 |
| - | 100 | 150 | 320 |
| - | 50 | 100 | 360 |

Table 11: Spacing (en $mm$) en función $\sigma_{max}$ y $\omega$

## 7.2 Prestressed

We need to find $(P,e)$ that fullfills:

$$\begin{cases} \sigma_{comp,const} = \frac{P}{A} - \frac{Pev_t}{I} + \frac{M_g v_t}{I} \leqslant \sigma_{max,comp} = 0,6fck \\ \sigma_{trac,const} = -\frac{P}{A} + \frac{Pev_t}{I} + \frac{M_g v}{I} \leqslant \sigma_{max,trac} = 3MPa \\ \sigma_{comp} = \frac{P}{A} - \frac{Pev_t}{I} + \frac{M_{characteristic} v_t}{I} \leqslant \sigma_{max,comp} = 0,6fck \\ \sigma_{trac} = -\frac{P}{A} + \frac{Pev_t}{I} + \frac{M_{frequent} v}{I} \leqslant \sigma_{max,trac} = 3MPa \end{cases} \tag{28}$$

where
$v$ : *is the distance to the c.g from the bottom*
$v_t$ : *is the distance to the c.g from the top*
*Signs we suposed the resulting moments compress the top and prestressed ones the bottom*
$M_g$ : *is the moment in a construction phase*

# 8 Shear reinforcement

Procedure:

1. Compute $V_d$
2. Compute

$$V_c = \max \begin{cases} V_{R_d,c} = \left(C_{Rd,c} k (100 \rho_l f_{ck})^{1/3} + k_1 \sigma_{cp}\right) b_w d \\ V_{R_d,c,min} = \left(0,035 \cdot k^{3/2} f_{ck}^{1/2} + k_1 \sigma_{cp}\right) b_w d \end{cases} \tag{29}$$

where

$$C_{Rd,c} = \frac{0,18}{\gamma_c} = 0,12$$
$$k = 1 + \sqrt{\frac{200}{d}} \leqslant 2 \ in \ mm$$
$$\rho_l = \frac{A_{s,long}}{b_w d} \leqslant 0,02$$
$$k_1 = 0,15$$
$$\sigma_{cp} = \frac{N+P}{b_w d} \leqslant 0,2 f_{cd}$$

3. If $V_d > V_c \rightarrow$ reinforcement needed
4. Compute

$$V_{R_d,max} = a_{cw} b_w z v_1 f_{cd} \frac{1}{\cot(\theta) + \tan(\theta)} \tag{30}$$

where

$$\nu_1 = \begin{cases} 0,6 & \text{if } f_{ck} \leqslant 60MPa \\ 0,9 - \frac{f_{ck}}{200} > 0,5 & \text{if } f_{ck} > 60MPa \end{cases}$$

$$\alpha_{cw} = \begin{cases} 1 & \textit{non prestressed structure} \\ 1 + \sigma_{cp}/f_{cd} - \frac{f_{ck}}{200} > 0,5 & \text{if } \sigma_{cd} \leqslant 0,25 f_{cd} \\ 1,25 & \text{if } 0,25 f_{cd} < \sigma_{cd} \leqslant 0,5 f_{cd} \\ 2,5(1 - \sigma_{cp}/f_{cd}) & \text{if } 0,5 f_{cd} < \sigma_{cd} \leqslant f_{cd} \end{cases}$$

$$z \approx 0,9d \approx 0,8h$$

$$\cot(\theta) = 2,5$$

5. If $V_{R_d,max} < V_d$
   (a) Compute
   $$\nu = \frac{V_d}{a_{cw} b_w z \nu_1 f_{cd}}$$
   (b) If $\nu > 0,5$ increase $b_w$ or $z$, and/or $f_{cd}$
   (c)
   $$\cot(\theta) = \begin{cases} 2,5 & \text{if } \nu \leqslant 0,34 \\ 5,6 - 9,2\nu & \text{if } \nu > 0,34 \end{cases} \in [1, 2.5] \tag{31}$$
6. Compute
   $$\frac{A_{sw}}{s} = \frac{V_d}{f_{ywd} z \cot(\theta)} \tag{32}$$
   where
   $$f_{ywd} = \min(\frac{f_{ywk}}{\gamma_s}, 400)$$
7. If $\frac{A_{sw}}{s} < \frac{A_{sw,min}}{s} = \frac{0,08\sqrt{f_{ck}}}{f_{ywd} b_w}$
   $$\frac{A_{sw}}{s} = \frac{A_{sw,min}}{s}$$
8. Define
   $$A_{sw} = n\frac{\phi^2}{4} \tag{33}$$
   where

   $\phi$ : *is the diameter of the bars, see Table 12*

   $n$ : *the number of bars along the height of the section*

   *it should be choosed in order to have* $s_{section} < 500mm$
9. Compute the spacing $s$

| Diameter $[mm]$ | Area $[mm^2]$ |
|---|---|
| 6 | 28 |
| 8 | 50 |
| 10 | 78 |
| 12 | 113 |

Table 12: Common diameters for transversal reinforcement

## 8.1 Optimisation

Process

1. Choose $x$
2. Compute $V_{d,x+d}$
   $$n = \frac{V_{d,x+d}}{V_{d,max}}$$
3. Compute $s_{x+d} = n s_{min}$

# 9 Columns

## 9.1 Second order effects

1. Compute
   $$\varphi_{ef} = \frac{M_{quasi}}{M_d}\varphi \tag{34}$$
   where

   $\varphi$ : *is the creep coeficient*

   $M_{quasi}$ : *the quasi-permanent combination (realistic load case)*
2. Compute
   $$E_{cd} I_{cd} = \frac{0,3}{1 + \varphi_{ef}} E_{cd} I_c \tag{35}$$
   where
   $$E_{cd} = \frac{E_c}{1,2}$$
   $$I_c = I_{gross}$$
3. Compute $L_p$, see Figure 5
4. Compute $N_B = \left(\frac{\pi}{L_p}\right)^2 E_{cd} I_{cd}$

5. Compute

$$M_{d,2} = \frac{M_d}{1 - \frac{N_d}{N_B}} \tag{36}$$

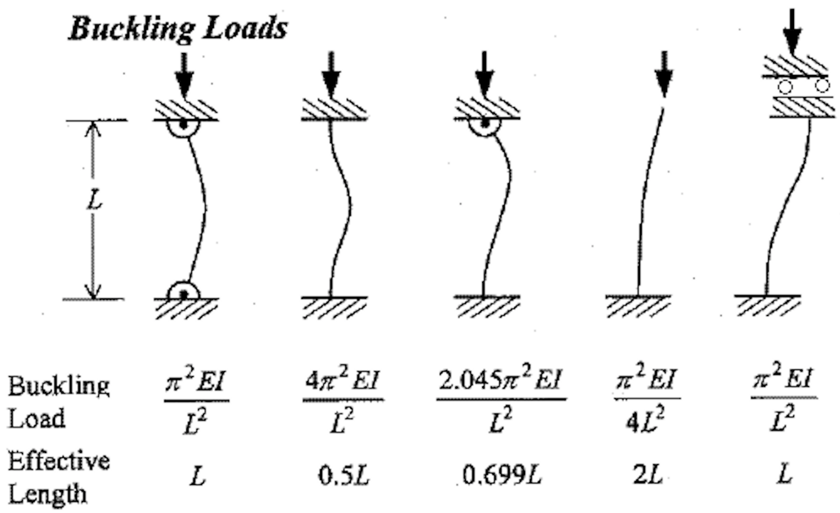



Figure 5: Buckling length and load depending on the support

### 9.2 Longitudinal reinforcement

1. Compute $\nu = \frac{N_d}{A_c f_{cd}}$
2. Compute $\mu = \frac{M_{d,2}}{A_c D f_{cd}}$
   where $D$ : *is a charasteristic length*
3. Find $w(\nu, \mu) = \frac{A_s f_{yd}}{A_c f_{cd}}$
4. Check minimal quantity

$$w = \max(w; w_{min} = 0,01)$$

### 9.3 Transversal reinforcement

1. Compute $\tau_c = \frac{V_c}{bd}$
2. If $\tau_c < 0,5MPa$ (can be resisted by concrete we should put the minimum)

$$\begin{cases} \phi_t > \phi_{min} = \frac{\phi_{long}}{4} \\ s_t < 15\phi_{long} \end{cases} \tag{37}$$

## 10 Detailing and layout

- Along the section the maximal distance between bar axes should be $d_{max} = 300mm$

# References

[1] Jesús Bairan. *Lecture notes in Concrete Structures*. 2017.