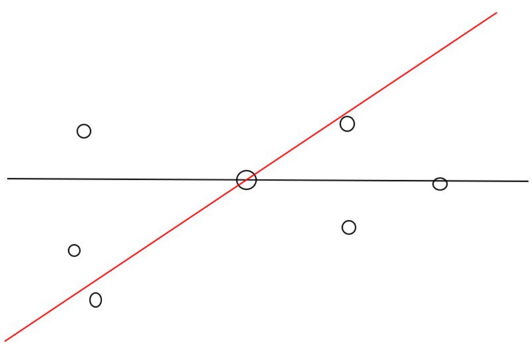
Problem: Given n points and a "magic" point, find the number of triangles that can be formed with three points from the n points given that contain the "magic" point. Let us call the "magic" point p (with coordinates (p_x, p_y))
Assumptions: The magic point has to lie completely inside the triangle. If the magic point is on a line between two points, it is not part of any triangle formed by those two points.

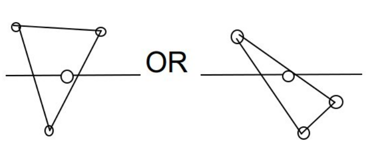
-First off, we can reorient the points so that the "magic" point becomes the origin. This is a simple O(n) operation that only has to be done once, and technically only to make the algorithm cleaner.

-Then, we divide the points into two sets, points with a smaller y than p_y and those with a greater y-coordinate than p_y. Assume that there are no points that have the same y-coordinate as p. However, in the case of points having the exact same y-coordinate as our point p, we can find a line that divides the points into two groups with no points lying on the dividing line. This takes O(n) time as well.

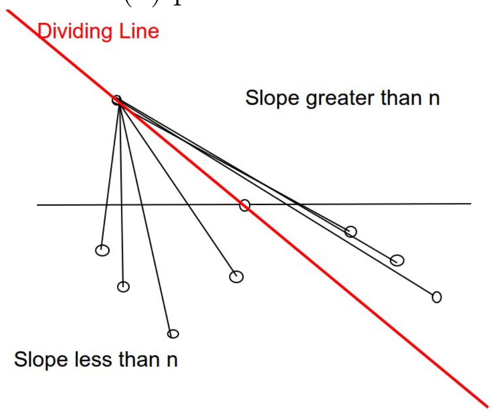


(If we run into that case with the black line, we can rotate the line to get the red line that divides the points into two groups. We know that we can always find such a line since there are a finite number of points and there are an infinite number of possible slopes)

-Now, for making a triangle, we have 3 corner points per triangle. The three corners can either be all three on top of the dividing line, 2 on top and one below, 1 on top, and two below, or three below the dividing line. The two cases with three points either all above or all below the divider can be proven to not contain the magic point p. As for the other two cases of (2 above, 1 below) and (1 above, 2 below), they are symmetrical and can be solved by a very similar second case. Let the points above p_y be the set A and the points below p_y be the set B



Then for each point $a_i \in A$, compute the slope between a_i and $b_j \in B$ as well as the slope from a_i to p. We can figure out which of these points have slope less than, equal to, or greater than $\overline{a_i p}$ in linear time to the number of points in B. Then, discarding all points that have slope equal to $\overline{a_i p}$, the number of triangles that can be formed using 1 point $\in A$ and 2 points $\in B$ using this specific a_i is $\underline{(\#\text{ of points with slope} < \text{slope of } \overline{a_i p}) \cdot (\#\text{ of points with slope} > \text{slope of } \overline{a_i p})}$. The running time for each a_i is O(n), as we can find the slope of each point in O(1) time and there are O(n) points in B.



(n in the picture is the slope of $\overline{a_i p}$)

Now, since there are O(n) points in A, we can do this for every single triplet of points in $\underline{O(n^2)}$ time. The runtime for the problem is O(n) for the reorienting of the points, O(n) for each a_i forming triangles, and with O(n) possible a_i's, this comes out to $O(n^2)$ time.

Sidenote: My original $O(n^2 log(n)^2)$ runtime originated from the fact that I was sorting the elements by slope when it was not necessary to.