Statistical inference with missing values

Wei Jiang
Ecole Polytechnique

12 Dec. 2019
Spotkania Entuzjastów R @ Wydzial MiNI PW

Missing values

When we attempt to explore data as a source of knowledge, **missing values** lies in the process of obtaining, recording, and preparing the data.

- Unanswered questions in a survey
- loss of data
- machines that fail

"We should be suspicious of any dataset (large or small) which appears perfect." – David J. Hand

Paris Hospitals - TraumaBase dataset

20 000 severely traumatised patients + 250 measurements

	Center	Accident	Age	Sex	Weight	Height	BMI	BP	SBP
1	Beaujon	Fall	54	m	85	NR	NR	180	110
2	Lille	Other	33	m	80	1.8	24.69	130	62
3	Pitie Salpetriere	Gun	26	m	NR	NR	NR	131	62
4	Beaujon	AVP moto	63	m	80	1.8	24.69	145	89
6	Pitie Salpetriere	AVP bicycle	33	m	75	NR	NR	104	86
7	Pitie Salpetriere	AVP pedestrian	30	w	NR	NR	NR	107	66
9	HEGP	White weapon	16	m	98	1.92	26.58	118	54
10	Toulon	White weapon	20	m	NR	NR	NR	124	73
11	Bicetre	Fall	61	m	84	1.7	29.07	144	105

....

	SpO2	Temperature	Lactates	Hb	Glasgow	Transfusion	....
1	97	35.6	NA	12.7	12	yes	
2	100	36.5	4.8	11.1	15	no	
3	100	36	3.9	11.4	3	no	
4	100	36.7	1.66	13	15	yes	
6	100	36	NA	14.4	15	no	
7	100	36.6	NA	14.3	15	yes	
9	100	37.5	13	15.9	15	yes	
10	100	36.9	NA	13.7	15	no	
11	100	36.6	1.2	14.2	14	no	

....

Paris Hospitals - TraumaBase dataset

20 000 severely traumatised patients + 250 measurements

	Center	Accident	Age	Sex	Weight	Height	BMI	BP	SBP
1	Beaujon	Fall	54	m	85	NR	NR	180	110
2	Lille	Other	33	m	80	1.8	24.69	130	62
3	Pitie Salpetriere	Gun	26	m	NR	NR	NR	131	62
4	Beaujon	AVP moto	63	m	80	1.8	24.69	145	89
6	Pitie Salpetriere	AVP bicycle	33	m	75	NR	NR	104	86
7	Pitie Salpetriere	AVP pedestrian	30	w	NR	NR	NR	107	66
9	HEGP	White weapon	16	m	98	1.92	26.58	118	54
10	Toulon	White weapon	20	m	NR	NR	NR	124	73
11	Bicetre	Fall	61	m	84	1.7	29.07	144	105

....

	SpO2	Temperature	Lactates	Hb	Glasgow	Transfusion	....
1	97	35.6	NA	12.7	12	yes	
2	100	36.5	4.8	11.1	15	no	
3	100	36	3.9	11.4	3	no	
4	100	36.7	1.66	13	15	yes	
6	100	36	NA	14.4	15	no	
7	100	36.6	NA	14.3	15	yes	
9	100	37.5	13	15.9	15	yes	
10	100	36.9	NA	13.7	15	no	
11	100	36.6	1.2	14.2	14	no	

....

⇒ Predict the Glasgow score, whether to start a blood transfusion, etc...

⇒ Linear regression /Logistic regression /Random Forests with missing covariates

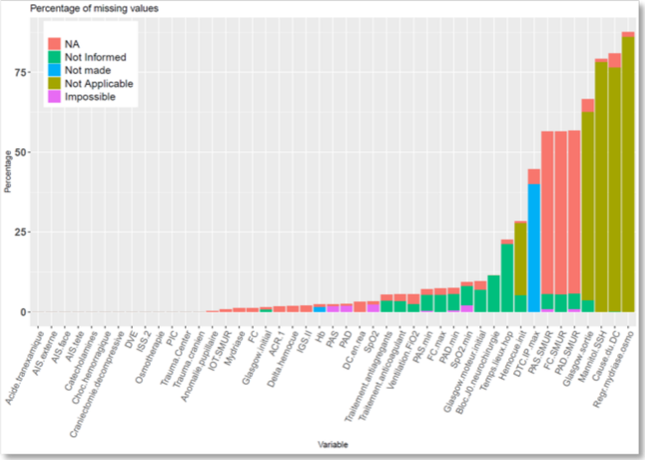
Missing values problematic

List-wise deletion (default `lm` function in R)
$\Rightarrow$ loss of information

Missing values problematic

List-wise deletion (default `lm` function in R)

⇒ loss of information



⇒ less than 10% remained

Single imputation

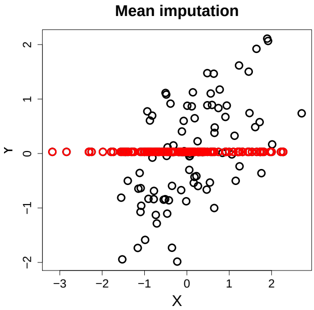
- $(x_i, y_i) \sim \mathcal{N}(\mu, \Sigma)$ *i.i.d.*
- 70% missing entries on y randomly

Date completion by the mean of observed values in y
$\Rightarrow$ Estimate parameters:

Single imputation

- $(x_i, y_i) \sim \mathcal{N}(\mu, \Sigma)$ *i.i.d.*
- 70% missing entries on y randomly

Date completion by the mean of observed values in y
$\Rightarrow$ Estimate parameters:



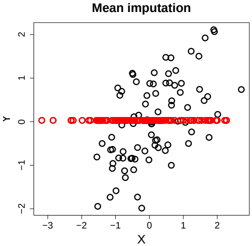
$\mu_y = 0$
$\sigma_y = 1$
$\rho = 0.6$

$\hat{\mu}_y = 0.01$
$\hat{\sigma}_y = 0.5$
$\hat{\rho} = 0.30$

$\Rightarrow$ Biased estimates

Imputation methods

- Mean imputation

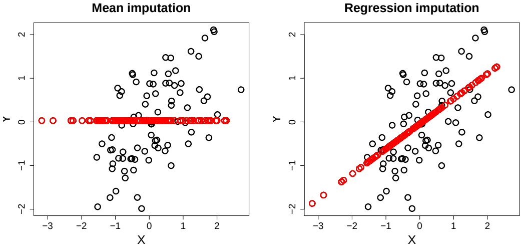


$\mu_y = 0$
$\sigma_y = 1$
$\rho = 0.6$

0.01
0.5
0.30

Imputation methods

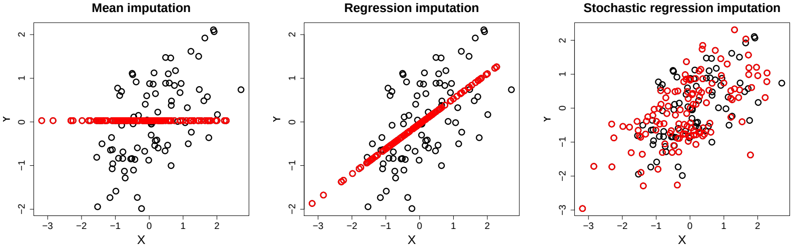
- Mean imputation
- Impute by regression: impute $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$
  $\Rightarrow$ variance underestimated and correlation overestimated.



$\mu_y = 0$	0.01	0.01
$\sigma_y = 1$	0.5	0.72
$\rho = 0.6$	0.30	0.78

Imputation methods

- Mean imputation
- Impute by regression: impute $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$
  $\Rightarrow$ variance underestimated and correlation overestimated.
- Impute by stochastic regression: impute $\hat{y}_i \sim \mathcal{N}\left(x_i\hat{\beta}, \hat{\sigma}^2\right)$
  $\Rightarrow$ preserve distribution

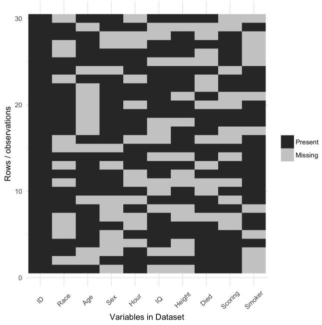


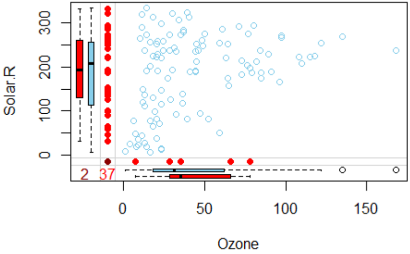
$\mu_y = 0$	0.01	0.01	0.01
$\sigma_y = 1$	0.5	0.72	0.99
$\rho = 0.6$	0.30	0.78	0.59

Missing pattern and mechanism

Dealing with missing values depends on:

- the pattern of missing values
- the mechanism leading to missing values





R packages: VIM, naniar (Matthias Templ, Nick Tierney)
FactoMineR (YouTube):

References on more imputation methods

- based on multivariate Gaussian assumption: $x_i \sim \mathcal{N}(\mu, \Sigma)$
  estimate μ and Σ from an incomplete data with EM
  $\rightarrow$ impute by drawing from $\mathcal{N}\left(\hat{\mu}, \hat{\Sigma}\right)$
  R packages: `Amelia`, `mice`
- k-nearest neighbor
  R packages: `VIM`, `yaImpute`, `impute`
- PCA or MCA
  R package: `missMDA`
- random forest
  R package: `missForest`

$\Rightarrow$ R-miss-tastic (Josse et al.): Methods and references for managing missing data
$\Rightarrow$ Flexible imputation of missing data (Stef van Buuren)

Recommended methods

Modify the estimation process to deal with missing values.
Maximum observed likelihood: EM algorithm to obtain point estimates

Recommended methods

Modify the estimation process to deal with missing values.
Maximum observed likelihood: EM algorithm to obtain point estimates

Eample: Hypothesis $x_i \sim \mathcal{N}(\mu, \Sigma)$, point estimates with EM

```
> library(norm)
> pre <- prelim.norm(as.matrix(don))
> thetahat <- em.norm(pre)
> getparam.norm(pre,thetahat)
```

Recommended methods

Modify the estimation process to deal with missing values.
Maximum observed likelihood: EM algorithm to obtain point estimates

Eample: Hypothesis $x_i \sim \mathcal{N}(\mu, \Sigma)$, point estimates with EM

```
> library(norm)
> pre <- prelim.norm(as.matrix(don))
> thetahat <- em.norm(pre)
> getparam.norm(pre,thetahat)
```

$\Rightarrow$ Natural model selection procedure!
$\Rightarrow$ One specific algorithm for each statistical method.
$\Rightarrow$ Not many implementations even for simple models.
Package `misaem` : logistic regression with missing values (Jiang et al., 2018)

Recommended methods

Modify the estimation process to deal with missing values.
Maximum observed likelihood: EM algorithm to obtain point estimates

Eample: Hypothesis $x_i \sim \mathcal{N}(\mu, \Sigma)$, point estimates with EM

```
> library(norm)
> pre <- prelim.norm(as.matrix(don))
> thetahat <- em.norm(pre)
> getparam.norm(pre,thetahat)
```

$\Rightarrow$ Natural model selection procedure!
$\Rightarrow$ One specific algorithm for each statistical method.
$\Rightarrow$ Not many implementations even for simple models.
Package `misaem` : logistic regression with missing values (Jiang et al., 2018)

Specialized focus on: Model selection with missing covariates,

e.g. Measurements $\xrightarrow{\text{Predict}}$ Platelet

joint work with Malgorzata Bogdan, Julie Josse, Blazej Miasojedow, Veronika Rockova

Model selection in high-dimension

Linear regression model: $$y = X\beta + \varepsilon,$$

- $y = (y_i)$: vector of response of length n
- $X = (X_{ij})$: a standardized design matrix of dimension $n \times p$
- $\beta = (\beta_j)$: regression coefficient of length p
- $\varepsilon \sim \mathcal{N}(0, \sigma^2 I_n)$

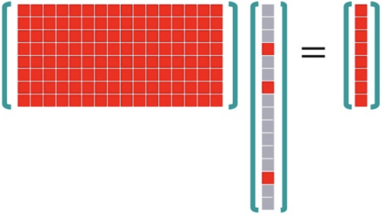
Model selection in high-dimension

Linear regression model: $y = X\beta + \varepsilon,$

- $y = (y_i)$: vector of response of length n
- $X = (X_{ij})$: a standardized design matrix of dimension $n \times p$
- $\beta = (\beta_j)$: regression coefficient of length p
- $\varepsilon \sim \mathcal{N}(0, \sigma^2 I_n)$

Assumptions:

- high-dimension: p large (including $p \geq n$)
- β is **sparse** with $k < n$ nonzero coefficients



l_1 penalization methods

- LASSO (Tibshirani, 1996)

$$\hat{\beta}_{LASSO} = \text{argmin}_{\beta \in \mathbb{R}^p} \frac{1}{2}\|y - X\beta\|^2 + \lambda\|\beta\|_1,$$

detects important variables with high probability but includes many **false positives**.

l_1 penalization methods

- LASSO (Tibshirani, 1996)
$$\hat{\beta}_{LASSO} = \text{argmin}_{\beta \in \mathbb{R}^p} \frac{1}{2}\|y - X\beta\|^2 + \lambda\|\beta\|_1,$$
detects important variables with high probability but includes many **false positives**.

- SLOPE (Bogdan et al., 2015) penalizes larger coefficients more stringently
$$\hat{\beta}_{SLOPE} = \text{argmin}_{\beta \in \mathbb{R}^p} \frac{1}{2}\|y - X\beta\|^2 + \sigma \sum_{j=1}^{p} \lambda_j |\beta|_{(j)},$$
where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p \geq 0$ and $|\beta|_{(1)} \geq |\beta|_{(2)} \geq \cdots \geq |\beta|_{(p)}$.

l_1 penalization methods

- LASSO (Tibshirani, 1996)
$$\hat{\beta}_{LASSO} = \text{argmin}_{\beta \in \mathbb{R}^p} \frac{1}{2}\|y - X\beta\|^2 + \lambda\|\beta\|_1,$$
detects important variables with high probability but includes many **false positives**.

- SLOPE (Bogdan et al., 2015) penalizes larger coefficients more stringently
$$\hat{\beta}_{SLOPE} = \text{argmin}_{\beta \in \mathbb{R}^p} \frac{1}{2}\|y - X\beta\|^2 + \sigma \sum_{j=1}^{p} \lambda_j |\beta|_{(j)},$$
where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p \geq 0$ and $|\beta|_{(1)} \geq |\beta|_{(2)} \geq \cdots \geq |\beta|_{(p)}$.

To control **False Discovery Rate (FDR)** at level q:
$\lambda_{BH}(j) = \phi^{-1}(1 - q_j), \quad q_j = \frac{jq}{2p}, \quad X^T X = I,$ then

$$FDR = \mathbb{E}\left[\frac{\#\text{False rejections}}{\#\text{Rejections}}\right] \leq q$$

Bayesian SLOPE

Problem: λ for SLOPE leading to FDR control are typically large.
SLOPE often returns **an inconsistent estimation.**

$\Rightarrow$ improve?

Bayesian SLOPE

Problem: λ for SLOPE leading to FDR control are typically large.
SLOPE often returns **an inconsistent estimation.**

$\Rightarrow$ improve?

SLOPE estimate = MAP of a Bayesian regression with SLOPE prior.

$$\hat{\beta}_{SLOPE} = \text{argmax}_\beta \, \mathrm{p}(y \mid X, \beta, \sigma^2; \lambda) \propto \mathrm{p}(y \mid X, \beta)\mathrm{p}(\beta \mid \sigma^2; \lambda)$$

where the SLOPE prior:

$$\mathrm{p}(\beta \mid \sigma^2; \lambda) \propto \prod_{j=1}^{p} \exp\left(-\frac{1}{\sigma}\lambda_j |\beta|_{(j)}\right)$$

Adaptive Bayesian SLOPE

We propose an adaptive version of Bayesian SLOPE (ABSLOPE), with the prior for β as

$$\mathrm{p}(\beta \mid \gamma, c, \sigma^2; \lambda) \propto c^{\sum_{j=1}^{p} \mathbb{I}(\gamma_j = 1)} \prod_j \exp\left\{ -w_j |\beta_j| \frac{1}{\sigma} \lambda_{r(W\beta, j)} \right\},$$

Interpretation of the model:

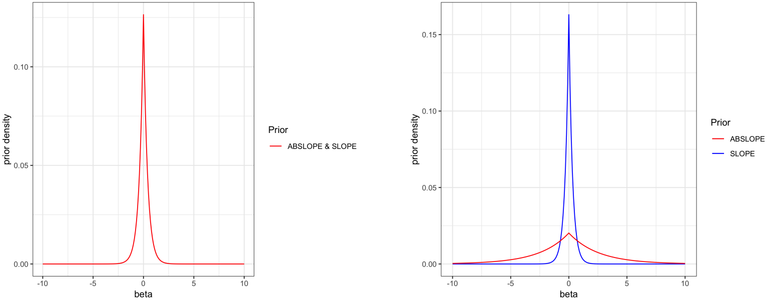
- β_j is large enough $\Rightarrow$ **true signal**; 0 $\Rightarrow$ noise.
- $\gamma_j \in \{0, 1\}$ signal indicator. $\gamma_j | \theta \sim \mathit{Bernoulli}(\theta)$ and θ the **sparsity**.
- $c \in [0, 1]$: the inverse of **average signal magnitude**.
- $W = \mathrm{diag}(w_1, w_2, \cdots, w_p)$ and its diagonal element:

$$w_j = c\gamma_j + (1 - \gamma_j) = \begin{cases} c, & \gamma_j = 1 \\ 1, & \gamma_j = 0 \end{cases}.$$

Adaptive Bayesian SLOPE

Advantage of introducing W:

- when $\gamma_j = 0$, $w_j = 1$, i.e., the null variables are treated with the regular SLOPE penalty
- when $\gamma_j = 1$, $w_j = c < 1$, i.e, **smaller penalty** $\lambda_{r(W\beta,j)}$ for true predictors than the regular SLOPE one



(a) Null β (b) Non-null β

Rysunek: comparison of SLOPE prior and ABSLOPE prior

Model selection with missing values

Decomposition: $X = (X_{\text{obs}}, X_{\text{mis}})$

Pattern: matrix M with $M_{ij} = \begin{cases} 1, & \text{if } X_{ij} \text{ is observed} \\ 0, & \text{otherwise} \end{cases}$

Assumption 1: Missing mechanism - Missing at random (MAR)

$\mathrm{p}(M \mid X_{\text{obs}}, X_{\text{mis}}) = \mathrm{p}(M \mid X_{\text{obs}}) \quad \Rightarrow \quad$ ignorable missing patterns

e.g. People at older age didn't tell his income at larger probability.

Assumption 2: Distribution of covariates

$X_i \sim_{\text{i.i.d.}} \mathcal{N}_p(\mu, \Sigma), \quad i = 1, \cdots, n.$

Model selection with missing values

Decomposition: $X = (X_{\text{obs}}, X_{\text{mis}})$

Pattern: matrix M with $M_{ij} = \begin{cases} 1, & \text{if } X_{ij} \text{ is observed} \\ 0, & \text{otherwise} \end{cases}$

Assumption 1: Missing mechanism - Missing at random (MAR)

$\mathrm{p}(M \mid X_{\text{obs}}, X_{\text{mis}}) = \mathrm{p}(M \mid X_{\text{obs}}) \quad \Rightarrow \quad$ ignorable missing patterns

e.g. People at older age didn't tell his income at larger probability.

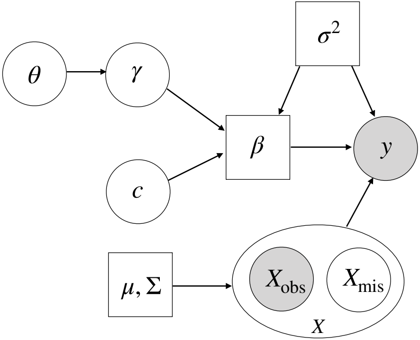
Assumption 2: Distribution of covariates

$X_i \sim_{\text{i.i.d.}} \mathcal{N}_p(\mu, \Sigma), \quad i = 1, \cdots, n.$

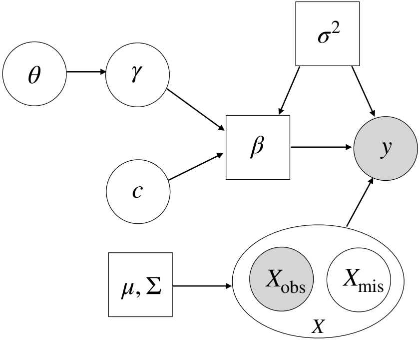
Problem: With NA, only a few methods are available to select a model, and their performances are limited. For example,

- (Claeskens and Consentino, 2008) adapts AIC to missing values $\Rightarrow$ Impossible to deal with high dimensional analysis.
- (Loh and Wainwright, 2012) LASSO with NA
  $\Rightarrow$ Non-convex optimization; requires to know bound of $\|\beta\|_1$
  $\Rightarrow$ difficult in practice

ABSLOPE with missingness: Summary



ABSLOPE with missingness: Summary



$$\begin{aligned}\ell_{\text{comp}} &= \log \mathrm{p}(y, X, \gamma, c;\ \beta, \theta, \sigma^2) + pen(\beta)\\ &= \log\left\{\mathrm{p}(X;\ \mu, \Sigma)\,\mathrm{p}(y \mid X;\ \beta, \sigma^2)\,\mathrm{p}(\gamma;\ \theta)\,\mathrm{p}(c)\right\} + pen(\beta)\end{aligned}$$

Objective: Maximize $\ell_{\text{obs}} = \iiint \ell_{\text{comp}}\, dX_{\text{mis}}\, dc\, d\theta\, d\gamma$.

EM algorithm

- *E step:* evaluate

$$Q^t = \mathbb{E}(\ell_{\mathrm{comp}}) \quad \text{wrt} \quad \mathrm{p}(X_{\mathrm{mis}}, \gamma, c, \theta \mid y, X_{\mathrm{obs}}, \beta^t, \sigma^t, \mu^t, \Sigma^t).$$

- *M step:* update

$$\beta^t, \sigma^t, \mu^t, \Sigma^t = \text{argmax}\, Q^t$$

Problem: The function Q is not tractable. $\Rightarrow$

1. Monte Carlo EM ? (Wei and Tanner 1990)

EM algorithm

- *E step:* evaluate

$$Q^t = \mathbb{E}(\ell_{\text{comp}}) \quad \text{wrt} \quad \mathrm{p}(X_{\text{mis}}, \gamma, c, \theta \mid y, X_{\text{obs}}, \beta^t, \sigma^t, \mu^t, \Sigma^t).$$

- *M step:* update

$$\beta^t, \sigma^t, \mu^t, \Sigma^t = \text{argmax}\, Q^t$$

Problem: The function Q is not tractable. $\Rightarrow$

1. ~~Monte Carlo EM ?~~
   Expensive to generate a large number of samples.
2. Stochastic Approximation EM (book, Lavielle 2014)
   - One sample in each iteration;

Adapted SAEM algorithm

- *E step:*
  $Q^t = \mathbb{E}(\ell_{\mathrm{comp}}) \quad \text{wrt} \quad \mathrm{p}(X_{\mathrm{mis}}, \gamma, c, \theta \mid y, X_{\mathrm{obs}}, \beta^t, \sigma^t, \mu^t, \Sigma^t)$.
  - *Simulation:* draw one sample $(X^t_{\mathrm{mis}}, \gamma^t, c^t, \theta^t)$ from

    $$\mathrm{p}(X_{\mathrm{mis}}, \gamma, c, \theta \mid y, X_{\mathrm{obs}}, \beta^{t-1}, \sigma^{t-1}, \mu^{t-1}, \Sigma^{t-1});$$

    [**Gibbs sampling**]
  - *Stochastic approximation:* update function Q with

    $$Q^t = Q^{t-1} + \eta_t \left(\ell_{\mathrm{comp}} \Big|_{X^t_{\mathrm{mis}}, \gamma^t, c^t, \theta^t} - Q^{t-1} \right).$$

Adapted SAEM algorithm

- *E step:*
  $Q^t = \mathbb{E}(\ell_{\text{comp}}) \quad \text{wrt} \quad \mathrm{p}(X_{\text{mis}}, \gamma, c, \theta \mid y, X_{\text{obs}}, \beta^t, \sigma^t, \mu^t, \Sigma^t)$.
  - *Simulation:* draw one sample $(X^t_{\text{mis}}, \gamma^t, c^t, \theta^t)$ from
    $$\mathrm{p}(X_{\text{mis}}, \gamma, c, \theta \mid y, X_{\text{obs}}, \beta^{t-1}, \sigma^{t-1}, \mu^{t-1}, \Sigma^{t-1});$$
    [**Gibbs sampling**]
  - *Stochastic approximation:* update function Q with
    $$Q^t = Q^{t-1} + \eta_t \left(\ell_{\text{comp}} \Big|_{X^t_{\text{mis}}, \gamma^t, c^t, \theta^t} - Q^{t-1} \right).$$
- *M step:* $\beta^{t+1}, \sigma^{t+1}, \mu^{t+1}, \Sigma^{t+1} = \text{argmax}\, Q^{t+1}$.
  [**Proximal gradient descent, Shrinkage of covariance**]

Details of initialization, generating samples and optimization are in the draft (arXiv:1909.06631)

R package: ABSLOPE

Install package:

```
library(devtools)
install_github("wjiang94/ABSLOPE")
```

R package: ABSLOPE

Install package:

```
library(devtools)
install_github("wjiang94/ABSLOPE")
```

Main algorithm:

```
lambda = create_lambda_bhq(ncol(X),fdr=0.10)
list.res = ABSLOPE(X, y, lambda)
```

R package: ABSLOPE

Install package:

```
library(devtools)
install_github("wjiang94/ABSLOPE")
```

Main algorithm:

```
lambda = create_lambda_bhq(ncol(X),fdr=0.10)
list.res = ABSLOPE(X, y, lambda)
```

A fast and simplified algorithm (Rcpp):

```
list.res.slobe = SLOBE(X, y, lambda)
```

R package: ABSLOPE

Install package:

```
library(devtools)
install_github("wjiang94/ABSLOPE")
```

Main algorithm:

```
lambda = create_lambda_bhq(ncol(X),fdr=0.10)
list.res = ABSLOPE(X, y, lambda)
```

A fast and simplified algorithm (Rcpp):

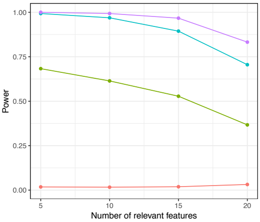
```
list.res.slobe = SLOBE(X, y, lambda)
```

Values:

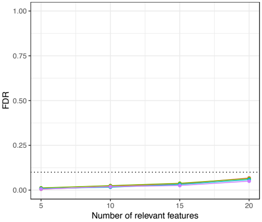
```
list.res$beta
list.res$gamma
```

Simulation study (200 rep. $\Rightarrow$ average)

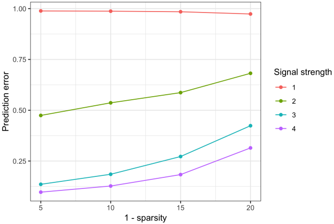
$n = p = 100$, no correlation and 10% missingness



(a) Power



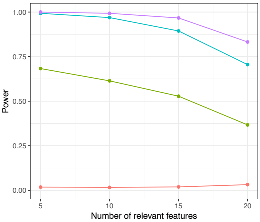
(b) FDR



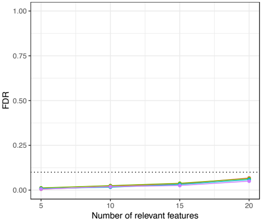
(c) Prediction error

Simulation study (200 rep. $\Rightarrow$ average)

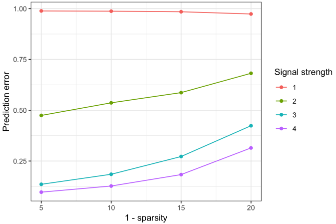
$n = p = 100$, no correlation and 10% missingness



(g) Power

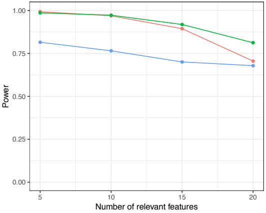


(h) FDR

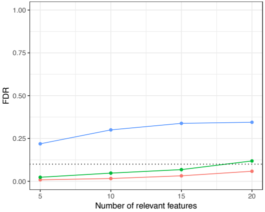


(i) Prediction error

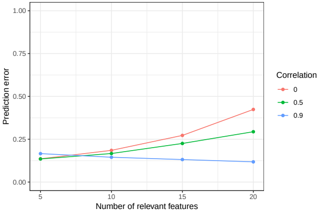
$n = p = 100$, with 10% missingness and strong signal



(j) Power



(k) FDR



(l) Prediction error

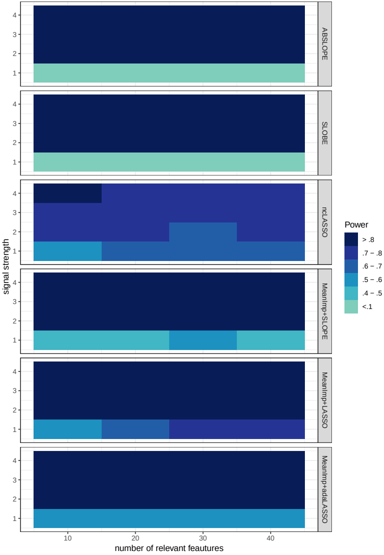
Method comparison

- **ABSLOPE** and **SLOBE**
- **ncLASSO:** non convex LASSO (Loh and Wainwright, 2012)
- **MeanImp + SLOPE:** Mean imputation followed by SLOPE with known σ
- **MeanImp + LASSO:** Mean imputation followed by LASSO, with λ tuned by cross validation
- **MeanImp + adaLASSO:** Mean imputation followed by adaptive LASSO (Zou, 2006)

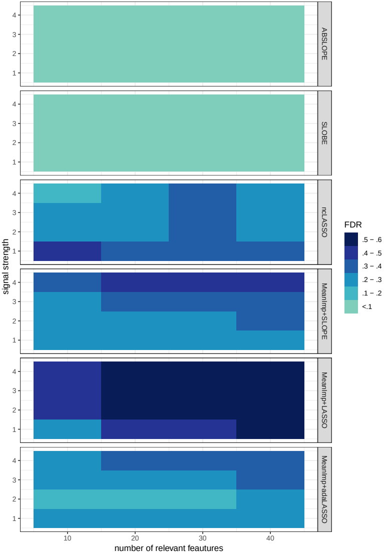
In the SLOPE type methods, $\lambda =$ BH sequence which controls the FDR at level **0.1**

Method comparison (200 rep. ⇒ average)

500×500 dataset, 10% missingness, with correlation



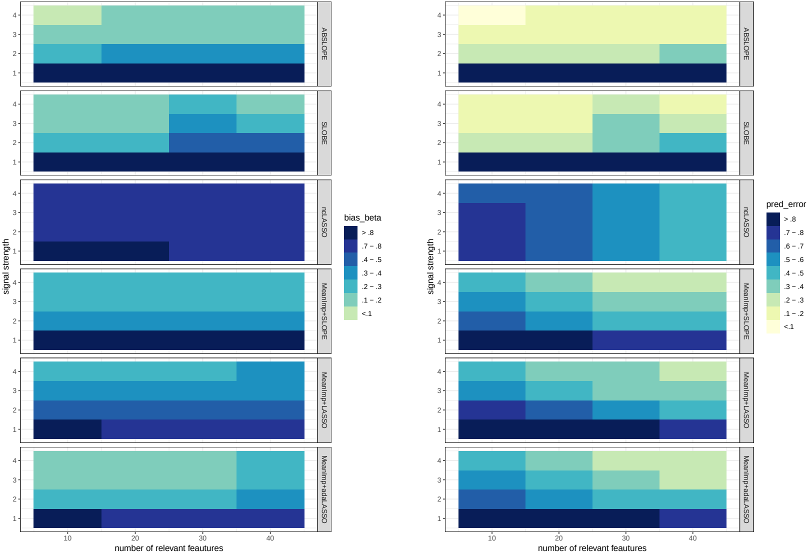
(m) Power



(n) FDR

Method comparison (200 rep. $\Rightarrow$ average)

500×500 dataset, 10% missingness, with correlation



(a) Bias of β (b) Prediction error

Computational cost

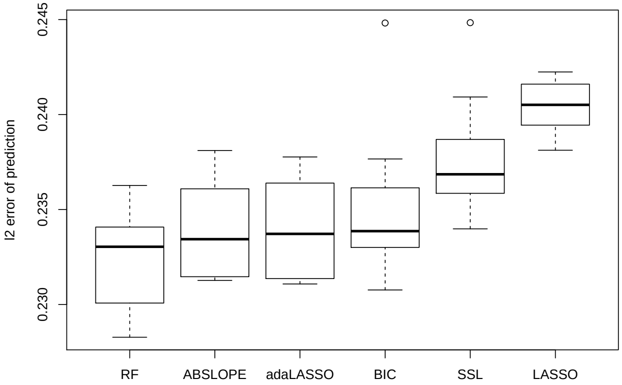
Execution time (seconds) for one simulation	$n = p = 100$			$n = p = 500$		
	min	mean	max	min	mean	max
ABSLOPE	12.83	14.33	20.98	646.53	696.09	975.73
SLOBE	0.31	0.34	0.66	14.23	15.07	29.52
ncLASSO	16.38	20.89	51.35	91.90	100.71	171.00
MeanImp + SLOPE	0.01	0.02	0.09	0.24	0.28	0.53
MeanImp + LASSO	0.10	0.14	0.32	1.75	1.85	3.06

[**Fast implementation:** Parallel computing + Rcpp (C++)]

More on the real data...

TraumaBase: Measurements $\xrightarrow{\text{Predict}}$ Platelet

Cross-validation: random splits to training and test sets $\times$ 10



- Comparable to random forest
- Interpretable model selection and estimation results

Conclusion & Future research

Conclusion:

- ABSLOPE penalizes larger coefficients more stringently to **control FDR**, meanwhile it applies a weighting matrix to **improve the estimation**;
- Modeling in a Bayesian framework gives detailed structure of predictors as **sparsity** and **signal strength**;
- Simulation study shows that ABSLOPE is competitive to other methods in terms of power, FDR and prediction error.

Future research:

- Consider categorical or mixed data
- Deal with other missing mechanisms
- Application on genetic dataset

Thank you!
Dziękuję