导入一张测试图像

In[ ]:=
```
img = ExampleData[{"TestImage", "Aerial"}]
```

Out[ ]=

定义变换

In[ ]:=
```
(*光学傅里叶变换 - 中心为低频*)
opticalFourier[data_?MatrixQ]:=
  RotateLeft[Fourier[data],Dimensions[data]/2]
opticalInverseFourier[data_?MatrixQ]:=
  Chop@InverseFourier[RotateRight[data,Dimensions[data]/2]]
```

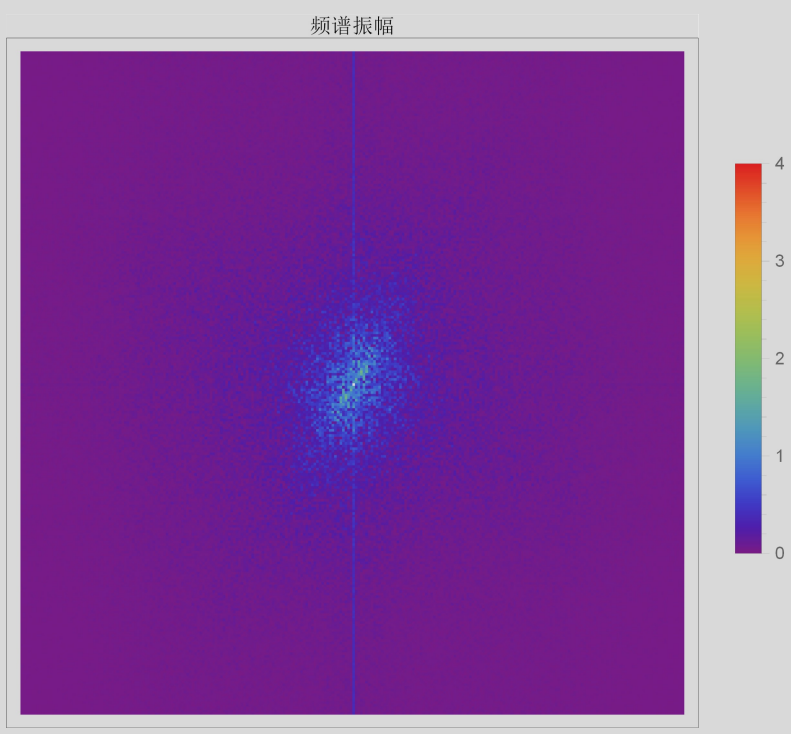
变换是可逆的。如下，变换再进行逆变换后得到的图像与原图像间的差异为0：

In[ ]:=
```
ImageDistance[Image@opticalInverseFourier@opticalFourier@ImageData[img], img]
```

Out[ ]=
```
0.
```

In[ ]:=
```
ArrayPlot[Log[1 + Abs[#]] &@opticalFourier@ImageData[img],
 PlotLegends → Automatic, PlotRange → 4, ColorFunction -> "Rainbow", PlotLabel → "频谱振幅"]
```

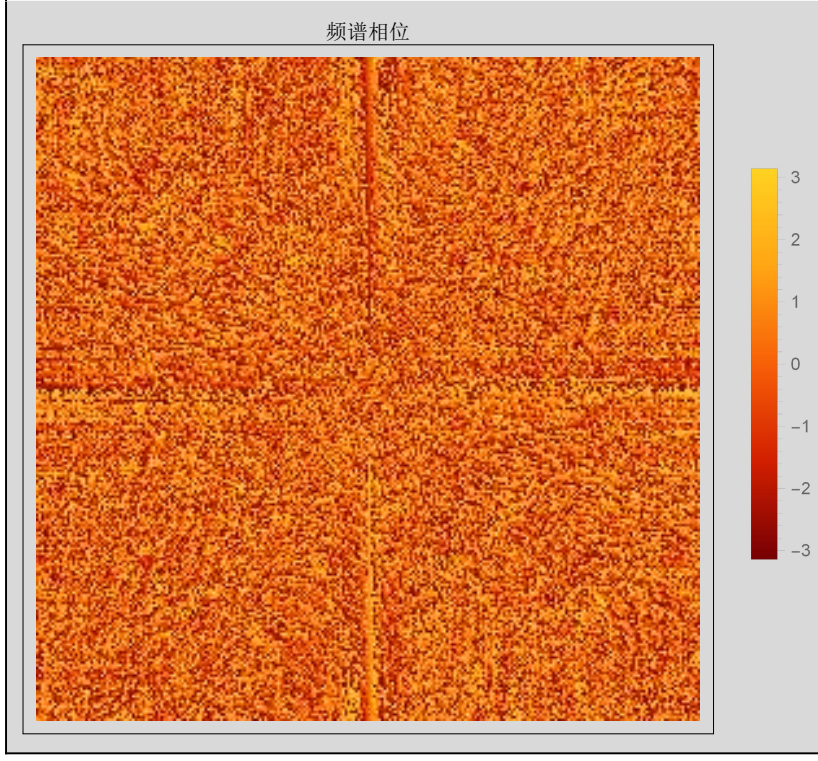
Out[ ]=



傅里叶变换后的振幅如上图所示。从图中可以看到大多数能量都集中在低频部分。水平方向中心有一条竖线，推测应该是由于图片下方边缘出有一条黑色边线造成的。

In[ ]:=
```
ArrayPlot[Arg@opticalFourier@ImageData[img], PlotLegends → Automatic,
 PlotRange → Automatic, ColorFunction → "SolarColors", PlotLabel → "频谱相位"]
```

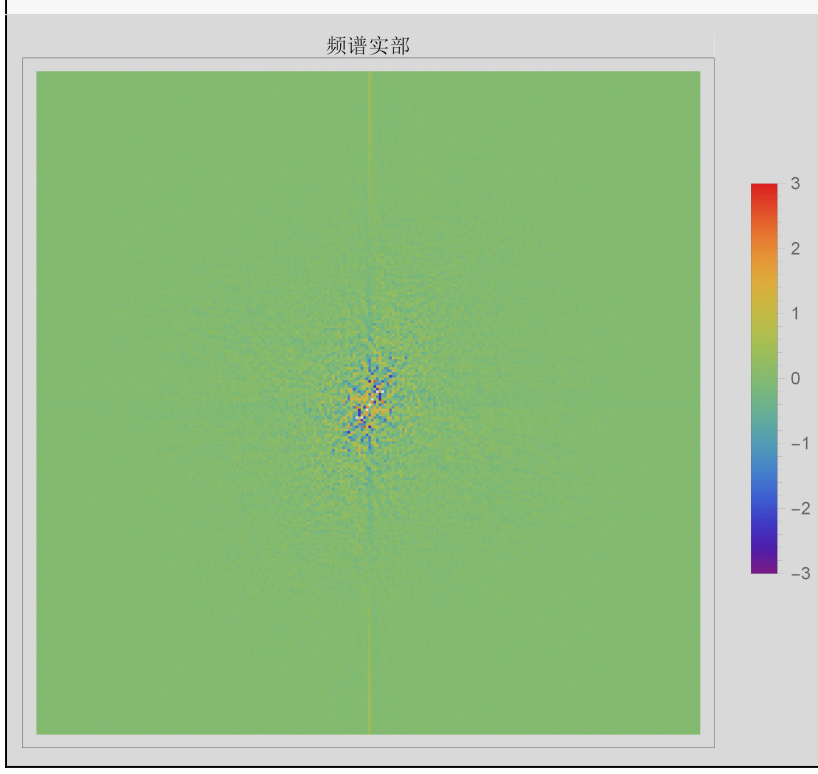
Out[ ]=



相位如上图所示。相位部分看上去没什么规律，但图像的信息主要储藏于相位部分。

In[ ]:=
```
ArrayPlot[Re@opticalFourier@ImageData[img], PlotLegends → Automatic,
 PlotRange → {-3, 3}, ColorFunction -> "Rainbow", PlotLabel → "频谱实部"]
```

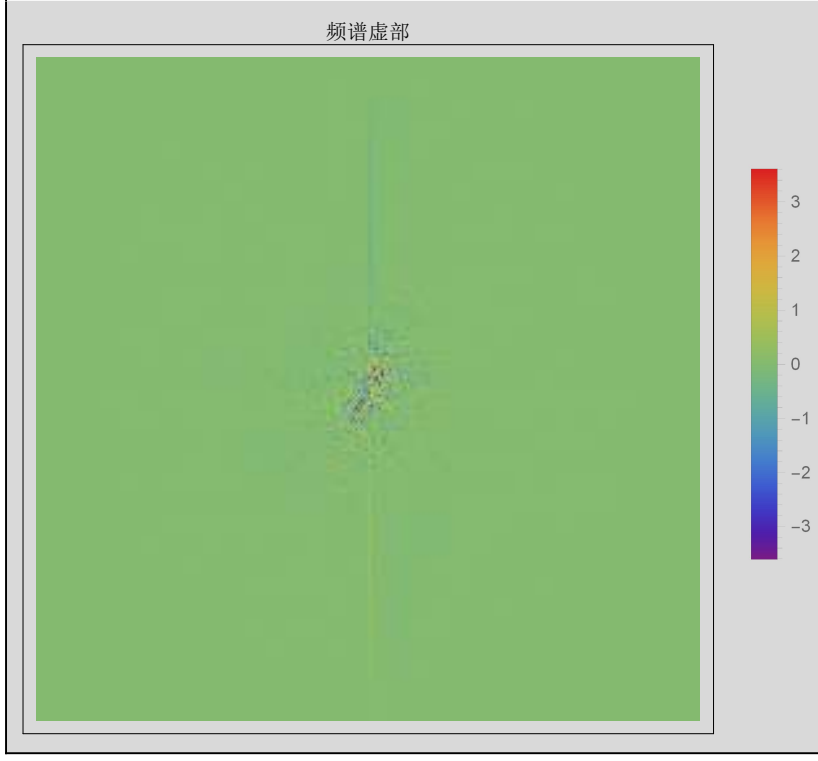
Out[ ]=



实部图像呈中心对称状。

In[ ]:=
```
ArrayPlot[Im@opticalFourier@ImageData[img], PlotLegends → Automatic,
 PlotRange → Automatic, ColorFunction -> "Rainbow", PlotLabel → "频谱虚部"]
```

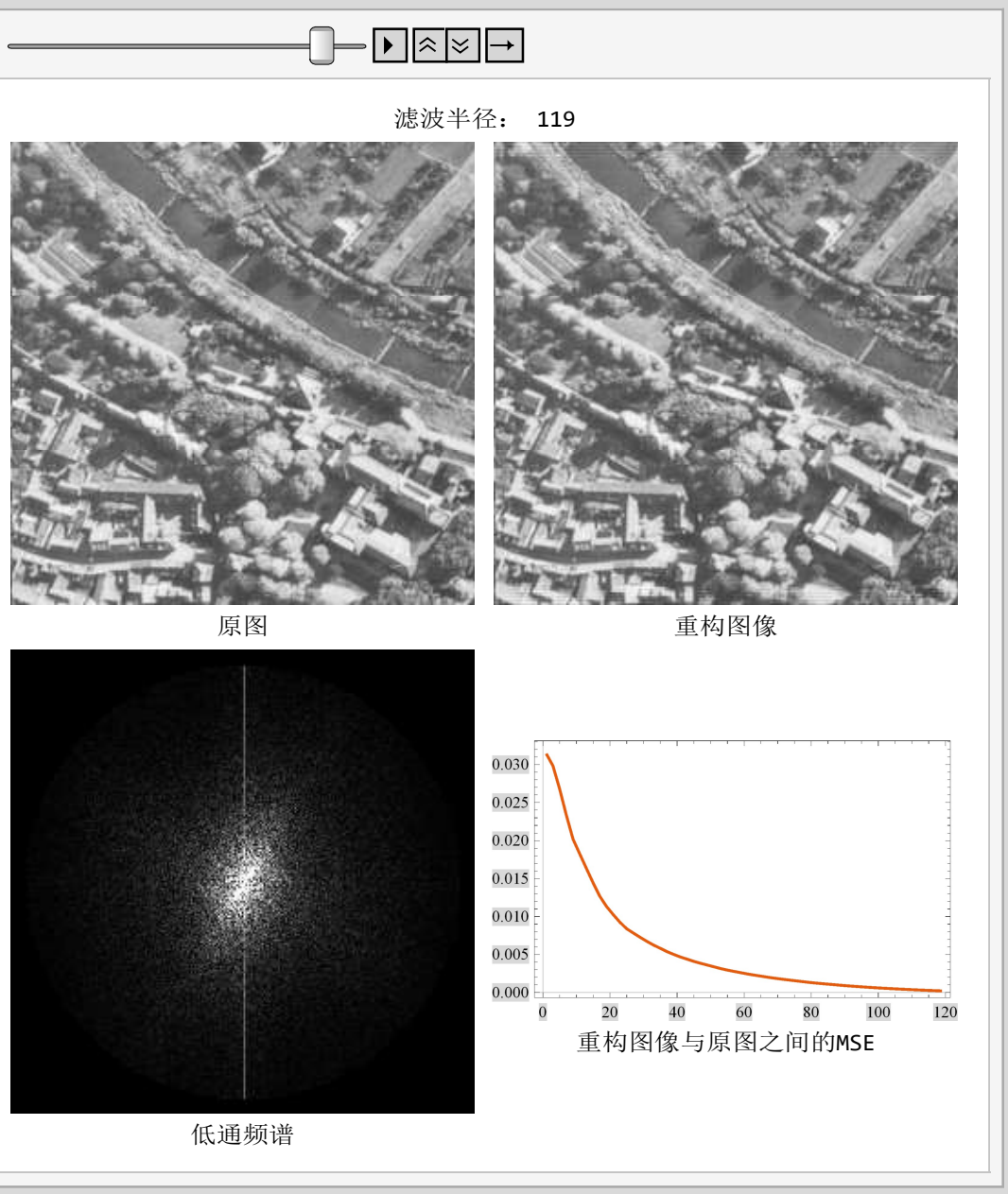
Out[ ]=



从虚部图像大致可以看出虚部是奇对称的。

In[ ]:=
```
lframes = Module[{f = opticalFourier@ImageData[img], rimgds = {}},
   Table[
    With[{ff = f * DiskMatrix[r, Dimensions[f]]},
     rimgds ~AppendTo~
      {r, ImageDistance[img, Image@Abs@opticalInverseFourier[ff], DistanceFunction → "MeanSquaredEuclideanDistance"]};
     Labeled[
      Grid@{{
         Labeled[Image[img, Magnification → 1], "原图"],
         Labeled[Image[Abs@opticalInverseFourier[ff], Magnification → 1], "重构图像"]},
        {Labeled[Image[Log[1 + Abs[ff]], Magnification → 1], "低通频谱"],
         Labeled[
          ListLinePlot[rimgds, PlotTheme → "Scientific", ImageSize → ImageDimensions[img][[1]]], "重构图像与原图之间的MSE"]
        }},
      "滤波半径： " <> ToString[r], Top
     ]
    ],
    {r, 1, 127, 2}
   ]
  ];
ListAnimate[lframes, 8]
Export["lowpass.gif", %, "ControlAppearance" → None, AnimationRepetitions -> Infinity];
```

Out[ ]=



In[ ]:=
```
hframes = Module[{f = opticalFourier@ImageData[img], rimgds = {}},
   Table[
    With[{ff = f * (1 - DiskMatrix[r, Dimensions[f]])},
     rimgds ~AppendTo~
      {r, ImageDistance[img, Image@Abs@opticalInverseFourier[ff], DistanceFunction → "MeanSquaredEuclideanDistance"]};
     Labeled[
      Grid@{{
         Labeled[Image[img, Magnification → 1], "原图"],
         Labeled[Image[Abs@opticalInverseFourier[ff], Magnification → 1], "重构图像"]},
        {Labeled[Image[Log[1 + Abs[ff]], Magnification → 1], "高通频谱"],
         Labeled[
          ListLinePlot[rimgds, PlotTheme → "Scientific", ImageSize → ImageDimensions[img][[1]]], "重构图像与原图之间的MSE"]
        }},
      "滤波半径： " <> ToString[r], Top
     ]
    ],
    {r, 126, 0, -2}
   ]
  ];
ListAnimate[hframes, 8]
Export["highpass.gif", %, "ControlAppearance" → None, AnimationRepetitions -> Infinity];
```

Out[ ]=