E_0: PFA

TEO, PRD 88, 045019 (2013)

$$E \approx E_0 + E_1$$

$$E_0 = -\frac{\hbar c R}{4\pi L^2}\int_0^\infty d\tilde{\xi}\int_{\tilde{\xi}}^\infty dx \sum_{s=1}^\infty \frac{1}{s^2} e^{-2sx} \sum_p r_p^{2s}$$

$$E_1 = -\frac{\hbar c R}{4\pi L^2}\int_0^\infty d\tilde{\xi}\int_{\tilde{\xi}}^\infty dx \sum_{s=1}^\infty \frac{1}{s^2} e^{-2sx} \left[\sum_p r_p^{2s}\left(A + C_p + D_p\right) + XB \right]$$

Def: L: surface-to-surface separation

R: radius

ω_p: plasma frequency

$\tau = \sqrt{1-\left(\frac{\tilde{\xi}}{k}\right)^2}$ $\qquad \xi = \frac{\tilde{\xi} L}{c}$

$\ell = R\tau k = \frac{R}{L}\tau x$

$\varepsilon(\xi) = 1 + \frac{\omega_p^2}{\xi^2}$ plasma model, $\beta = \sqrt{(\varepsilon - 1)\left(\frac{\xi}{c}\right)^2 + k^2}$,

$$r_{TE} = \frac{\beta - k}{\beta + k}$$

$$r_{TM} = \frac{\varepsilon k - \beta}{\varepsilon k + \beta}$$

$$B = \frac{1-\tau^2}{2\ell\tau s}, \quad X = s\left[\underbrace{\frac{2\left(r_{TE} r_{TM} + r_{TM}^2\right)}{r_{TE}^2 - r_{TM}^2}}_{= X_1} r_{TE}^{2s} - \underbrace{\frac{2\left(r_{TE} r_{TM} + r_{TE}^2\right)}{r_{TE}^2 - r_{TM}^2}}_{= X_2} r_{TM}^{2s}\right]$$

$$= s\left(X_1 r_{TE}^{2s} - X_2 r_{TM}^{2s}\right)$$

$$A = \left(\frac{R}{L}\right)^2 \frac{\ell\tau}{3}\left(s^3 + 2s\right) + \frac{1}{3}\frac{L}{R}\left((\tau^2 - 2)s^2 - 3\tau s + 2\tau^2 - 1\right) + \frac{\tau^4 + \tau^2 - 12}{12\ell\tau}s + \frac{(1+\tau)(1-\tau^2)}{2\ell\tau} - \frac{\tau(1-\tau^2)}{3\zeta}$$

$$C_V = -\frac{L}{R}\frac{\tau}{3}\left(s^3 + 2s\right) + \frac{1-\tau^2}{6\ell}s^2 + \frac{\tau}{2\ell}s + \frac{1-4\tau^2}{12\ell}$$

$$C_J = -\frac{L}{R}\frac{\tau}{6}\left(s^3 - s\right) + \frac{1}{12\ell}\left(s^2 - 1\right)$$

$$V = \varepsilon(1-\tau^2)+\tau^2 \qquad VS = \sqrt{V}$$

$$K_1^{TE} = -\frac{2\tau}{VS} \qquad K_1^{TM} = \frac{2\varepsilon\tau(1-\tau^2)}{VS\,(\varepsilon+\tau^2)} \qquad \omega_1^{TE} = 2K_1^{TE} \qquad \omega_1^{TM} = 2K_1^{TM}$$

$$K_2^{TE} = -\frac{\varepsilon(1-\tau^2)}{V\cdot VS} + \frac{2\tau^2}{V}, \qquad K_2^{TM} = \frac{\varepsilon^2(1-\tau^2)^2}{V\cdot VS\,(\varepsilon+\tau^2)} - \frac{\tau^2(\varepsilon^2(1-\tau^2)+\varepsilon+1)}{V(\varepsilon+\tau^2)}$$

$$+ \frac{\tau^2(\varepsilon VS+1)^2}{V\,(VS+\varepsilon)^2}$$

$$C^{TE} = C_V K_1^P + C_J W_1^P = C_V K_1^P + 2C_J K_1^P = (C_V + 2C_J)\,K_1^P$$

$$D_{VV} = \frac{\tau}{12e}(s^3-2s^2+2s-1) \qquad D_{VJ} = \frac{\tau}{12e}(s^3-s)$$

$$D_J = \frac{\tau}{12e}(s^2-1) \qquad D_{JJ} = \frac{\tau}{48e}(s^3-2s^2-s+2) \qquad D_V = \frac{\tau}{6e}(2s^2-3s+1)$$

$$\omega_2^{TE} = \frac{8\tau^2+4\tau^4+4\varepsilon-4\varepsilon\tau^4}{V\cdot VS} + \frac{4(1-\tau^2)^2(\varepsilon+VS)^2}{\tau^2 V\,(VS+1)^2} - \frac{4(1-\tau^2)(\tau^2+\varepsilon)}{\tau^2 V}$$

$$\omega_2^{TM} = -\frac{\varepsilon(1-\tau^2)(8\tau^2+4\tau^4+4\varepsilon-4\varepsilon\tau^4)}{(\varepsilon+\tau^2)\,V\cdot VS} + \frac{4(1-\tau^2)^2\varepsilon^2(1+VS)^2}{\tau^2 V\,(VS+\varepsilon)^2}$$

$$- \frac{4\varepsilon^2(1-\tau^2)^3}{\tau^2(\tau^2+\varepsilon)\,V}$$

$$Y_2^{TE} = -\frac{\tau}{VS} - \frac{8\varepsilon\tau^2-3\varepsilon-5\varepsilon\tau^4+9\tau^2+5\tau^4}{12V}$$

$$Y_2^{TM} = \frac{\varepsilon\tau(1-\tau^2)}{(\varepsilon+\tau^2)\,VS} - \frac{7\varepsilon^2\tau^4-4\varepsilon^2\tau^2-3\varepsilon^2-5\varepsilon\tau^6+13\varepsilon\tau^4-18\varepsilon\tau^2+5\tau^6-3\tau^4}{12(\varepsilon+\tau^2)\,V}$$