Ian Miller October 17, 2019

6.042 Problem Set 5

Problem 1 *(Collaborators: None)*

Part 1(a)

Fuller House = (4 cards of one rank) + (2 cards of another rank)

Let P = the chance of getting a Fuller House if you randomly flipped over 6 cards

Let T = the total number of 6-card hands $(52 * 51 * 50 * 49 * 48 * 47/6!)$

The number of possible Fuller House hands is equivalent to:

$P * T$

We already have T so in order to solve the problem, so we only need to calculate P.

$P = (1 * (3/51) * (2/50) * (1/49)) * ((44/48) * (3/47))$

Part 1(b)

I'm going to use the same approach as in part A.

Three Pair = (2 cards of one rank) + (2 cards of another rank) + (2 cards of another rank)

Let T = the total number of 6-card hands $(52 * 51 * 50 * 49 * 48 * 47/6!)$

The number of possible Three Pair hands is equivalent to:

$P * T$

We already have T so in order to solve the problem, so we only need to calculate P.

$P = (1 * (3/51)) * ((48/50) * (3/49)) * ((44/48) * (3/47))$

Part 1(c)

It's a little harder to calculate the probability the same way as in part A and B.

There are 2 variables I'm worried about to find the total number of Imperial Flushes:

1- High card

2- Suit

For each high card in an Imperial Flush, there are 3 others possible because suit can be changed.

Since there are 9 possible high cards (6, 7, 8, 9, 10, J, Q, K, A), there are $4 * 9 = 36$ possible Imperial Flushes.

Part 1(d)

I'm going to use the same approach as in part A and B.

Basically-a-Flush = AT LEAST 5 cards in the hand have the same suit

Let T = the total number of 6-card hands $(52 * 51 * 50 * 49 * 48 * 47/6!)$

The number of possible Basically-a-Flush hands is equivalent to:

$P * T$

We already have T so in order to solve the problem, so we only need to calculate P.

$P = (1 * (12/51) * (11/50) * (10/49)) * (9/48) * 1)$

$P = (12/51) * (11/50) * (10/49)) * (9/48)$

Problem 2 *(Collaborators: None)*

The pigeonholes are the following: for each odd number x in the set $1, 2, 3, ..., 2n$, we can make the set $S_x = x, 2x, 4x, 2^y(x), ...$

This gives us n pigeonholes. The problem dictates that we have to pick n+1 numbers. Any pair of numbers that satisfies the problem constraint (quotient is a power of two) will be in the same set S_x.

Therefore, any pair of numbers will contain one number that divides the other. Each number that we pick is a pigeon. And because of each number will be in the same S_x as another number that divides it, its quotient will be a power of two.

Therefore, we have n+1 pigeons and n pigeonholes.

Problem 3 *(Collaborators: Sophia Chan, Julian Hamelberg)*

Part 3(a)

First, we must construct a bijection to represent donut selection. This bijection will map the set of all possible ways to select n donuts from k possible flavors to the set of distinct n+k-1 bit sequences with n zeroes and k-1 ones.

There are n+k-1 bits regardless of how the zeroes are arranged, assuming there are k-1 ones. That sequence is an element of the set of all distinct n+k-1 bit sequences with n zeroes and k-1 ones.

This is a bijection because every possible arrangement of ones and zeroes is mapped to from one of the elements in the set of all possible ways to select n donuts from k flavors. Therefore, we can say that there are $\binom{n+k-1}{k-1}$

ways to select n donuts from k possible flavors.

Part 3(b)

First, we must construct a bijection to represent donut selection. This bijection will map the set of all possible ways to select 12 donuts from 5 possible flavors to the set of distinct 11 bit sequences with 7 zeroes and 4 ones.

If we are required to select 1 donut in each flavor, that means that 5 of the zeroes are already accounted for. This means that you only have flexibility with those last 7 donuts. The bit length is now 11. In that 11 bit sequence, there will 7 zeroes and 4 ones meaning that there are $\binom{11}{4}$ ways to select 12 donuts from 5 flavors, based on our work from part A.

Part 3(c)

In order for there to be either 0 or an even number of donuts per flavor in our selection, we must pick donuts 2 at a time.

This is equivalent to making the selection size 6 (half of 12). Therefore according to the stars and bars formula in part A, there are $\binom{10}{4}$ ways to make a selection that meet the problem constraints.

Part 3(d)

The number of ways to select 12 donuts from 5 flavors requiring 3 distinct flavors is equivalent to the total number of ways to select 12 donuts from 5 flavors minus the number of ways to select 12 donuts from 5 flavors with at most 2 flavors.

The total number of ways to select 12 donuts from 5 flavors with no constraint is the following:

$$\binom{12+5-1}{5-1}$$

The total number of ways to select 12 donuts from 5 flavors with at most 2 flavors is the following:

$$\binom{12+2-1}{2-1} * \binom{5}{2}$$

Therefore, the total number of ways to select 12 donuts from 5 flavors, if we required at least 3 distinct flavors is

$$\binom{12+5-1}{5-1} - \left(\binom{12+2-1}{2-1} * \binom{5}{2} \right)$$