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6.042 Problem Set 7

Problem 1 *(Collaborators: None)*

Part 1(a)

Independence means that $P(X|Y) = P(X)$ for all values of X and Y.

If $C = 3$, the following table shows the decimal probabilities for each event: $\begin{bmatrix} H1 & 1.0 \\ H2 & 1.0 \\ H3 & 1.0 \\ M & 1.0 \\ S & 1.0 \end{bmatrix}$

If $C = 1$, the following table shows the decimal probabilities for each event: $\begin{bmatrix} H1 & 0.33 \\ H2 & 0.33 \\ H3 & 0.33 \\ M & 0.0 \\ S & 1.0 \end{bmatrix}$

If $C = 0$, the following table shows the decimal probabilities for each event: $\begin{bmatrix} H1 & 0.0 \\ H2 & 0.0 \\ H3 & 0.0 \\ M & 1.0 \\ S & 0.0 \end{bmatrix}$

For each variable, depending on the value of C, probabilities can change. Therefore, none of the variables are independent of C.

Part 1(b)

M and S will be pairwise independent is the following is True:

$Pr[M \cap S] = Pr[M] * Pr[S]$

$Pr[M \cap S] = 0.125$

$Pr[M] * Pr[S] = .25 * 0.5 = 0.125$

Since the independence formula holds True, M and S are pairwise independent.

Part 1(c)

$Pr[H1 \cap H2 \cap H3] = Pr[H1 \cap H2 \cap S] = Pr[H2 \cap H3 \cap S] = Pr[H1 \cap H3 \cap S] = 1/8$

Therefore, H1, H2, H3, and S are 3-wise independent.

However,

In the case that H1, H2, and H3 all equal 1: Pr[C] = 1

In the case that H1, H2, and H3 all equal 0: Pr[C] = 0

Therefore, H1, H2, H3, and S are NOT mutually independent.

Problem 2 *(Collaborators: Albert Garcia)*

Part 2(a)

$$\text{PDF}_{x+1}(a) -> \begin{Bmatrix} 2 & 1/3 \\ 1 & 2/3 \end{Bmatrix}$$

$$\text{PDF}_{y}(a) -> \begin{Bmatrix} 1 & 1/6 \\ 2 & 1/6 \\ 3 & 1/6 \\ 4 & 1/6 \\ 5 & 1/6 \\ 6 & 1/6 \end{Bmatrix}$$

Because X and Y are independent, we can calculate the probability density function Z by multiplying corresponding matrix elements.

$$\text{PDF}_{Z}(a) -> \begin{bmatrix} 1 & 1/9 \\ 2 & 1/6 \\ 3 & 1/9 \\ 4 & 1/6 \\ 5 & 1/9 \\ 6 & 1/6 \\ 8 & 1/18 \\ 10 & 1/18 \\ 12 & 1/18 \end{bmatrix}$$

Part 2(b)

$$\text{CDF}_{D}(a) -> \begin{bmatrix} 1 & Pr[D <= 1]\ 0.6666 \\ 2 & Pr[D <= 2]\ 0.8888 \\ 3 & Pr[D <= 3]\ 0.9629 \\ 4 & Pr[D <= 4]\ 0.9876 \\ 5 & Pr[D <= 5]\ 0.9958 \\ 6 & Pr[D <= 6]\ 0.9986 \\ 7 & Pr[D <= 7]\ 0.9995 \\ 8 & Pr[D <= 8]\ 0.9998 \\ 9 & Pr[D <= 9]\ 0.9999 \\ 10 & Pr[D <= 10] = 1 \end{bmatrix}$$

For all a's where a is in range of D, $\sum PDF_D(a) = 1$

Therefore, the probability of rolling a 10 is $1 - \sum_1^9 PDF_D(a)$ approximately equals 0.0000508

Problem 3 *(Collaborators: None)*

Part 3(a)

Part 3(b)

Part 3(c)

Problem 4 *(Collaborators: None)*

Part 4(a)

Part 4(b)

Part 4(c)