Problem Set 7

This problem set is due **at 11:59pm** on **Thursday, October 31**.

Reading Assignment

Chapters 14 and 15

Problem 7-1. Conditional Probability

A 52-card deck is thoroughly shuffled and you are dealt a hand of 13 cards.

(a) If you have at least one ace, what is the probability that you have at least two aces?

(b) If you have the ace of spades, what is the probability that you have at least two aces? Remarkably, the answer is different from part (a).

Problem 7-2. Bayes Rule

A new disease that makes geese more aggressive to humans has been spreading among the geese of Killian Court. In an attempt to stop MIT students from being attacked by geese, Dr. Tim has developed a cheap test that can determine if a goose has contracted Aggressive Bird Flu. However, this test is not perfectly accurate.

80% of MIT's geese test negative according to Dr. Tim's test, but it is known that 10% of geese who test negative actually have Aggressive Bird Flu, and 40% of geese who test positive do not actually have it.

(a) Draw a Tree Diagram for this situation, and use it to specify the corresponding probability space.

(b) What fraction of MIT geese actually have Aggressive Bird Flu? (You may leave your numerical answer unsimplified.)

(c) What is the probability that a goose who has Aggressive Bird Flu will receive a negative diagnosis on Dr. Tim's test? *Hint:* Be sure to interpret this sentence as a conditional probability.

Problem 7-3. Four Weird Dice

Unhappy with the conclusion of the 3 dice game from lecture, you decide to play the same game - but with 4 dice instead! You come across the following four 6-sided dice:

$$A : 4, 4, 4, 4, 0, 0$$
$$B : 3, 3, 3, 3, 3, 3$$
$$C : 6, 6, 2, 2, 2, 2$$
$$D : 5, 5, 5, 1, 1, 1$$

You play the following game against an opponent: you pick any one of the 4 dice, after which your opponent then picks any one of the remaining 3 dice. You each then roll your die, and the die with the higher number showing is declared the winner.

(a) Show that none of these 4 dice is superior to all others. Specifically, show that the probabilities of the following events are all $> 1/2$.

- A beats B
- B beats C
- C beats D
- D beats A

(b) Use the result from part (a) to show that your opponent can always pick a die that makes you lose with probability $\geq 2/3$.

(c) This game is clearly unfair to you, so we now change it slightly to handicap your opponent. You are still allowed to pick the first die, but your opponent picks one of the three remaining dice at random, each with equal probability. As before, the player with the higher roll wins the game.

Show that there is an optimal choice of die for you to maximize your chances of winning this modified game. What is your probability of winning with that die?

Hint: Calculate the probability of winning for each of the 4 choices of the first dice.

Problem 7-4. Monty Hall Variants

Monty Hall has a few cousins with fresh takes on how his game should be played! For each variant below, calculate the probability of winning a car by **staying** with the originally chosen door, and the probability of winning a car by **switching** to one of the remaining unopened doors. What assumptions are you making about how the prizes are placed and/or how the doors are chosen?

(a) Hal Minty's idea is to have 5 doors total, where 1 has a car behind it. After the contestant picks one door, Hal reveals 2 unchosen doors, knowing in advance that both have goats behind them. What is the contestant's chance of winning by **staying**? By **switching**?

(b) Manny Hill's idea is to have 4 doors total, where 2 have cars behind them. Again, a contestant will pick one door, and knowing which doors have the cars behind them, the host will then open an unpicked door with a goat behind it. What is the contestant's chance of winning by **staying**? By **switching**?

(c) Morty Halbertson's idea is to have 7 doors total, where 1 has a car behind it. Now, the contestant chooses **two** doors, and the host reveals one unchosen door that has a goat behind it. The contestant may choose to **stay** with the two original doors, or **switch** to two of the four other unopened doors. The contestant wins if the car lies behind either of their two selected doors. What is the contestant's chance of winning the car by **staying**? By **switching**?