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# 6.042 Problem Set 7

## Problem 1 *(Collaborators: None)*

### Part 1(a)

Let A represent the size of the set of hands that have at least 2 aces.

Let B represent the size of the set of hands that have at least 1 ace.

$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]}$

The number of hands with exactly 1 ace = C = $4 * (48$ choose $12)$

So, B = (52 choose 13) $-$ (48 choose 13)

So, in A = $B - C$

Therefore, $Pr[A|B] = \frac{(52 choose 13)-(48 choose 13)-(4*(48 choose 12))}{(52 choose 13)-(48 choose 13}$

### Part 1(b)

Let A represent the size of the set of hands that have at least 2 aces (one of which is the ace of spades).

Let B represent the size of the set of hands that have at least the ace of spades.

$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]}$

B = (51 choose 12)

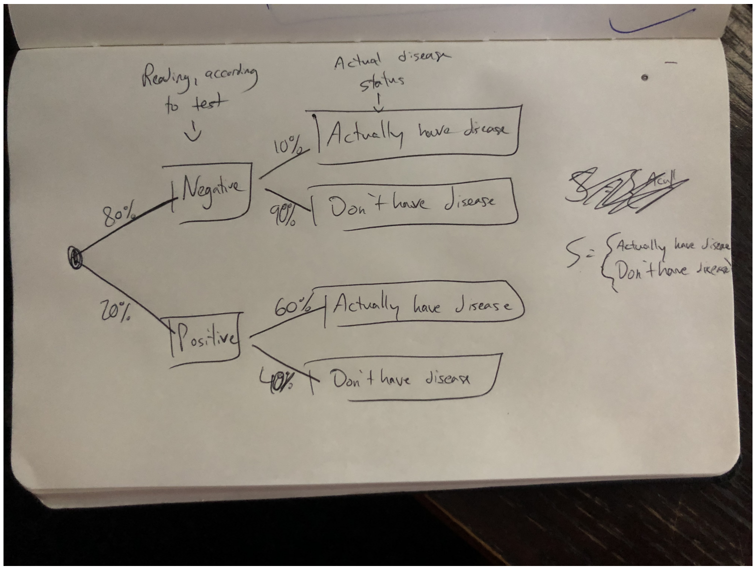
There are (48 choose 12) hands within B that don't have another ace.

So, A = B - (48 choose 12)

Therefore, $Pr[A|B] = \frac{(51 choose 12)-(48 choose 12)}{(51 choose 12)}$

# Problem 2 *(Collaborators: None)*

## Part 2(a)




Pr[Negative Diagnosis but actually have the disease] = 0.08

Pr[Positive Diagnosis and actually have the disease] = 0.12

Pr[Negative Diagnosis and don't have the disease] = 0.72

Pr[Positive Diagnosis but don't have the disease] = 0.08

## Part 2(b)

According to my tree diagram, there are 2 possible situations that a bird has the Aggressive Bird Flu.

1- Goose tested positive and actually has disease.

2- Goose tested negative and actually has disease.

Pr["A goose at MIT has Aggessive Bird Flu"] = Pr[1] + Pr[2]

Pr[1] = 0.2 * 0.6

Pr[2] = 0.8 * 0.1

Pr[1] + Pr[2] = 0.12 + 0.08 = 0.2

## Part 2(c)

Let B represent the following statement: "the bird has Aggressive Bird Flu"

Let A represent the following statement: "the bird will receive a negative diagnosis on Dr. Tim's test"

Pr[A—B] = $\frac{Pr[A \cap B]}{Pr[B]}$

Pr[$A \cap B$] = 1 - 0.12 - 0.72 - 0.08 = 0.08

Pr[B] = 0.2 (as determined in part b)

Therefore, there is a $\frac{0.08}{0.2}$ = 0.4 chance that a bird who has Aggressive Bird Flu will receive a negative diagnosis.

# Problem 3 *(Collaborators: Textbook, page 334)*

## Part 3(a)

### A beats B

Pr[A rolls 4] = 0.67

Pr[B rolls 3] = 1

Pr[A beats B] = 0.67

### B beats C

Pr[B rolls 3] = 1

Pr[C rolls 2] = 2/3

Pr[B beats C] = 0.67

### C beats D

Pr[C rolls 6] = 1/3

Pr[D rolls anything] = 1

Pr[C rolls 2] = 2/3

Pr[D rolls 1] = 1/2

Pr[C beats D] = 1/3 + 1/3 = 0.67

### D beats A

Pr[D rolls 5] = 1/2

Pr[A rolls anything] = 1

Pr[D rolls 1] = 1/2

Pr[A rolls 0] = 1/3

Pr[D beats A] = 1/2 + 1/6 = 0.67

## Part 3(b)

Since you pick first, your opponent will pick only after knowing what die you picked. In part A, we showed that any die can be beat by some other die with a 2/3 probability. So no matter what die you pick, there exists some die that will beat that die 2/3 of the time, according to my answer for part A.

## Part 3(c)

Expected Value for A: $(4 * 1/6) + (4 * 1/6) + (4 * 1/6) + (4 * 1/6) + (0 * 1/6) + (0 * 1/6) = 8/3$

Expected Value for B: $(3 * 1/6) + (3 * 1/6) + (3 * 1/6) + (3 * 1/6) + (3 * 1/6) + (3 * 1/6) = 3$

Expected Value for C: $(6 * 1/6) + (6 * 1/6) + (2 * 1/6) + (2 * 1/6) + (2 * 1/6) + (2 * 1/6) = 10/3$

Expected Value for D: $(5 * 1/6) + (5 * 1/6) + (5 * 1/6) + (1 * 1/6) + (1 * 1/6) + (1 * 1/6) = 3$

According to expectation, the optimal choice is C. This is because C has the highest expected value.

# Problem 4 *(Collaborators: None)*

## Part 4(a)

The following are the probabilities that you'll **win**:

Case 1 $->$ Stay: $2/10 = 0.2$

Case 2 $->$ Switch: $6/10 = 0.6$

## Part 4(b)

The following are the probabilities that you'll **win**:

Case 1 $->$ Stay: $4/8 = 0.5$

Case 2 $->$ Switch: $6/8 = 0.75$

## Part 4(c)

I'll break this problem up into 2 cases:

**Case 1** (6/21 probability) $->$ Initial pair includes a goat

Win prob if stay: $\frac{6}{21} * 1 = \frac{6}{21}$

Win prob if switch: 0

**Case 2** (15/21 probability) $->$ Initial pair does not include a goat

Win prob if stay: 0

Win prob if switch: $\frac{1}{2} * \frac{15}{21} = \frac{15}{42}$

Overall win prob if stay: $\frac{6}{21}$

Overall win prob if switch: $\frac{15}{42}$